Calculation of the reflection coefficient for a simple ABC
Idea : Similar to dispersion relation caclulation
1. Create ansatz with a reflected wave
2. Insert into discrete BC update schem
3. Calculate the amplitude of the reflected wave
4. Eliminate angular frequency using dispersion relation for waves

In[3]:=
```
SetDirectory["/Users/kolesik/Ucenie/OPTI-547/pract/EP02-1D-Maxwell"]
```

Out[3]=
```
/Users/kolesik/Ucenie/OPTI-547/pract/EP02-1D-Maxwell
```

In[4]:=
```
eqn1 = -F[0, 1] + F[1, 0] + r ( F[1, 1] - F[0, 0])
```

Out[4]=
$$-F[0,1]+F[1,0]+r\left(-F[0,0]+F[1,1]\right)$$

In[5]:=
```
Ansatz[x_, t_] := 1 Exp[-I k x - I om t] + R Exp[ +I k x - I om t]
```

In[6]:=
```
eqn2 = eqn1 /. F[a_, b_] → Ansatz[a, b]
```

Out[6]=
$$e^{-i\,k}-e^{-i\,om}+e^{i\,k}\,R-e^{-i\,om}\,R+r\left(-1+e^{-i\,k-i\,om}-R+e^{i\,k-i\,om}\,R\right)$$

In[7]:=
```
solR = Simplify[Solve[eqn2 == 0, R][[1]]]
```

Out[7]=
$$\left\{R\to\frac{e^{-i\,k}\left(e^{i\,k}-e^{i\,om}-r+e^{i\,(k+om)}\,r\right)}{-1+e^{i\,(k+om)}+e^{i\,k}\,r-e^{i\,om}\,r}\right\}$$

In[8]:=
```
aux1 = Simplify[ExpToTrig[R /. solR]]
```

Out[8]=
$$\frac{\left(\text{Cos}[k]-i\,\text{Sin}[k]\right)\left(\text{Sin}\left[\frac{k-om}{2}\right]+r\,\text{Sin}\left[\frac{k+om}{2}\right]\right)}{r\,\text{Sin}\left[\frac{k-om}{2}\right]+\text{Sin}\left[\frac{k+om}{2}\right]}$$

In[9]:=
```
aux2 = TrigExpand[aux1];
```

This is where we use the dispersion relation to eliminate omega

In[10]:=
```
aux3 = Simplify[
   (aux2 /. Cos[om/2] → Sqrt[1 - Sin[om/2]^2]) /. Sin[om/2] → dtoverdx Sin[k/2]]
```

Out[10]=
$$\left(\left(2\,\text{dtoverdx}\,(-1+r)\,\text{Cos}\left[\frac{k}{2}\right]+(1+r)\,\sqrt{4-2\,\text{dtoverdx}^2+2\,\text{dtoverdx}^2\,\text{Cos}[k]}\right)\left(\text{Cos}[k]-i\,\text{Sin}[k]\right)\right)\Big/\left(-2\,\text{dtoverdx}\,(-1+r)\,\text{Cos}\left[\frac{k}{2}\right]+(1+r)\,\sqrt{4-2\,\text{dtoverdx}^2+2\,\text{dtoverdx}^2\,\text{Cos}[k]}\right)$$

In[11]:=
```
check0 = Simplify[Simplify[aux3 /. k → 0] /. r → (dtoverdx - 1) / (dtoverdx + 1)]
```

Out[11]=
```
0
```

In[12]:=
```
aux4 = Simplify[(aux3 /. k → 2 Pi / n) /. r → (dtoverdx - 1) / (dtoverdx + 1)]
```

Out[12]=
$$\frac{\left(-2\cos\left[\frac{\pi}{n}\right]+\sqrt{4-2\,\text{dtoverdx}^2+2\,\text{dtoverdx}^2\cos\left[\frac{2\pi}{n}\right]}\right)\left(\cos\left[\frac{2\pi}{n}\right]-i\sin\left[\frac{2\pi}{n}\right]\right)}{2\cos\left[\frac{\pi}{n}\right]+\sqrt{4-2\,\text{dtoverdx}^2+2\,\text{dtoverdx}^2\cos\left[\frac{2\pi}{n}\right]}}$$

Log - scale reflection coefficient :

In[22]:=
```
Plot[Log[10, Abs[aux4 /. dtoverdx → 0.9]], {n, 2, 125}, PlotRange → All]
```

Out[22]=