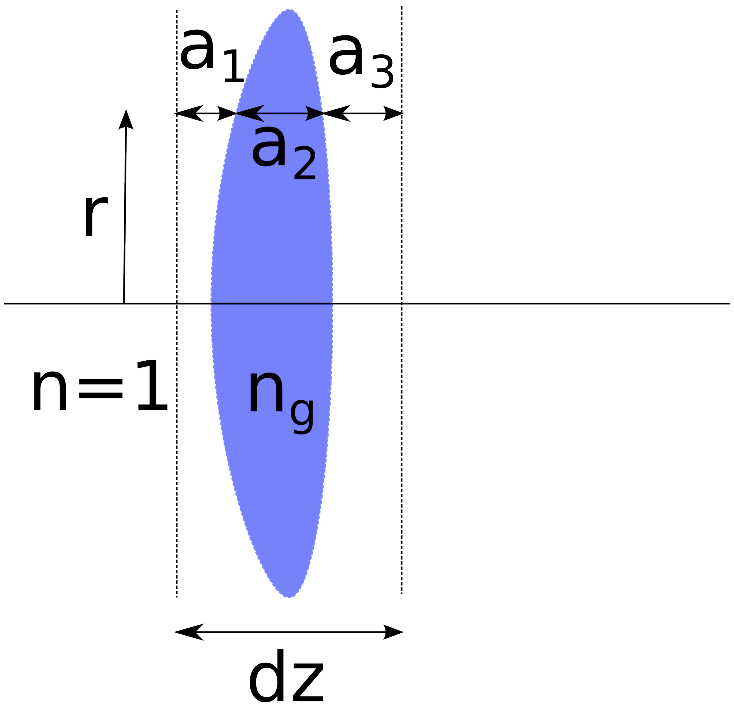
Simple models of "optical elements"

Spectral BPM is mainly appropriate for a homogeneous medium, and thus nearly free-space beam propagation. However, it is often used in problems where the beam goes through various elements, for example phase masks, or it reflects off mirrors comprising an optical resonator cavity. Here we want to summarize the way such beam modifications are usually implemented.

The "elements" to be considered:

- Thin lens
- Curved mirror (as a model, it is equivalent to the above)
- Axicon
- Phase masks (for example to create vortex beams)
- Fresnel lens

The common assumption that serves to include these into BPM is that while the beam "traverses" through the lens, mirror, etc., it does not experience much of diffraction. That of course means that the element must be thin.



Assuming dz is small in comparison with the diffraction lengths of all transverse features in the beam, we neglect diffraction completely as it passes through. Then one evaluates the propagation phase change as a function of the radius r:

$$\Delta\phi(r) = k_0\left[(a_1 + a_3) + a_2(r)n_g\right] = k_0\left[dz - a_2(r) + a_2(r)n_g\right] = k_0 dz + k_0 a_2(r)(n_g - 1)$$

We omit the overall carrier phase change $k_0 dz$, and the effect of this element on the beam profile is just a point-by-point multiplication by a phase mask

$$\text{beam} \rightarrow \text{beam} \times \exp[ik_0 a_2(r)(n_g - 1)]$$

The above can specialized for an axicon and a lens. In the former case

$$a_2(r) \approx a - br$$

and for a lens

$$a_2(r) \approx a - br^2 \ .$$

Again, the constant terms can be neglected, and we are left with linear and quadratic variation of the added phase w.r.t. radius.

Note: It is only in the continuous wave approximation that an effect by a mirror or a lens can be modeled by a phase-shift that depends on the location in the beam. When one works in the temporal domain, simulating optical pulses, this approach must be avoided when the pulse duration is short. In such a case, the phase-shift should be replaced by the *temporal shift* that reflects how much is the pulse delayed by the optical element at different locations in the beam. Of course, even then one assumes that diffraction can be neglected during the transition through the element...

Lens action is usually parametrized in terms of a phase front radius R and the expression obtained is the same as in a Gaussian beam, namely

$$\text{beam} \rightarrow \text{beam} \times \exp[-i\frac{k_0 r^2}{2R}] \; .$$

The same phase-mask can also model the effect of a spherical mirror. The only difference here is that it is implicitly assumed that the beam propagation distance is measured along its propagation direction.

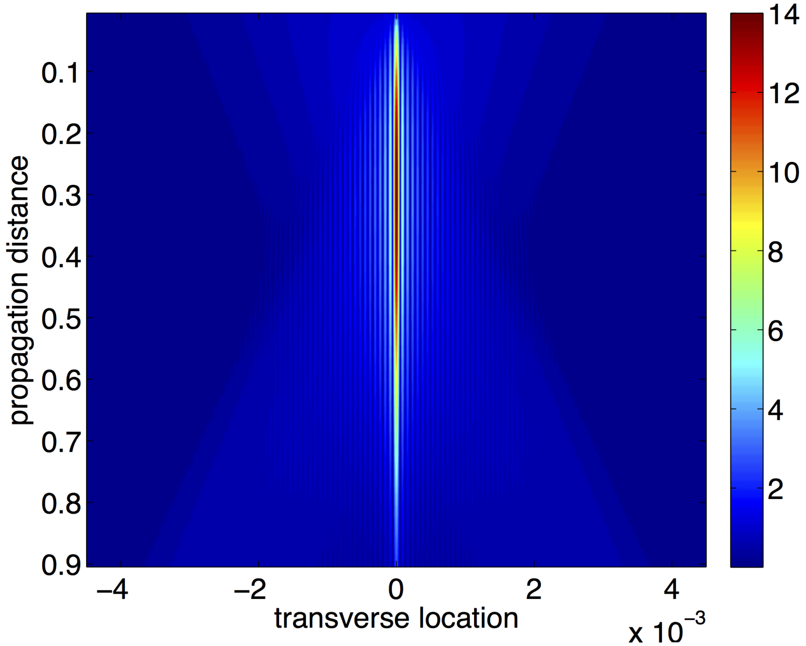
An **axicon** is a "lens" with conical shape, and as a rule it has its cone angle close to 180 degrees. For simulations, it is often convenient to parametrize the corresponding phase mask in terms of the resulting wave-front inclination angle (which is small, normally a fraction of a degree). With α standing for this small angle, the mask can be written as

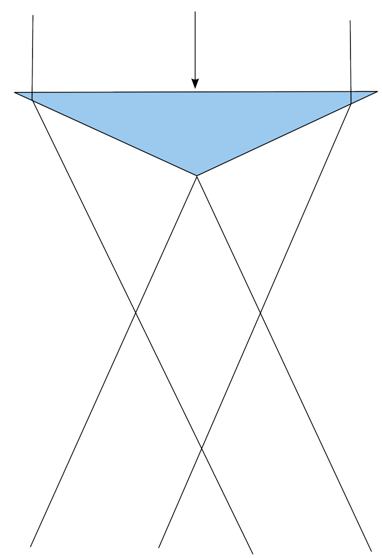
$$\exp[-ik_0\alpha r] \ .$$

Naturally, α can be related to the specific way axicon is characterized by its angle (or half-angle), refractive index and index of refraction of the medium in which it is immersed.

Bessel beam formation with axicon

This exercise package contains several files that are used to model beam propagation. The first example we look at is simulation of experimentally realizable Bessel beam approximation. Instructor's solution is in *pBeamBessel.m*

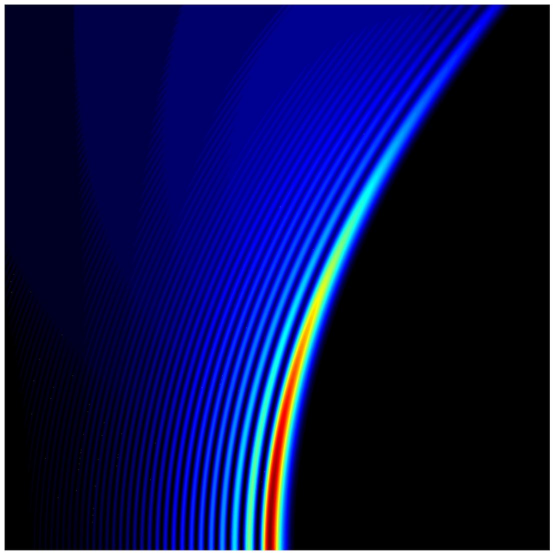


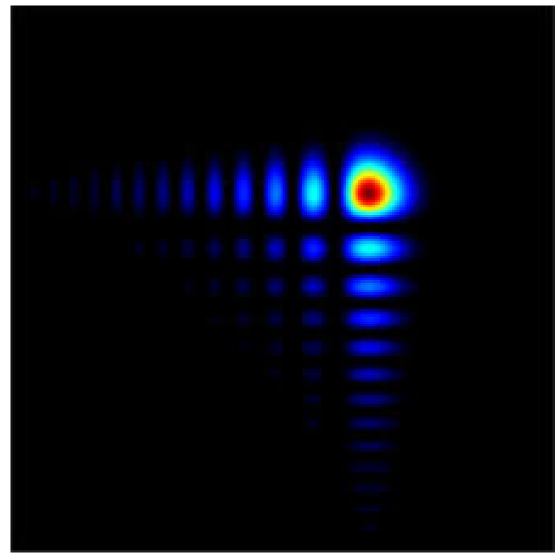


Intensity distribution in an experimentally realizable Bessel beam (left). The Bessel signature arises in the region made up of the intersecting beams created by the axicon (right).

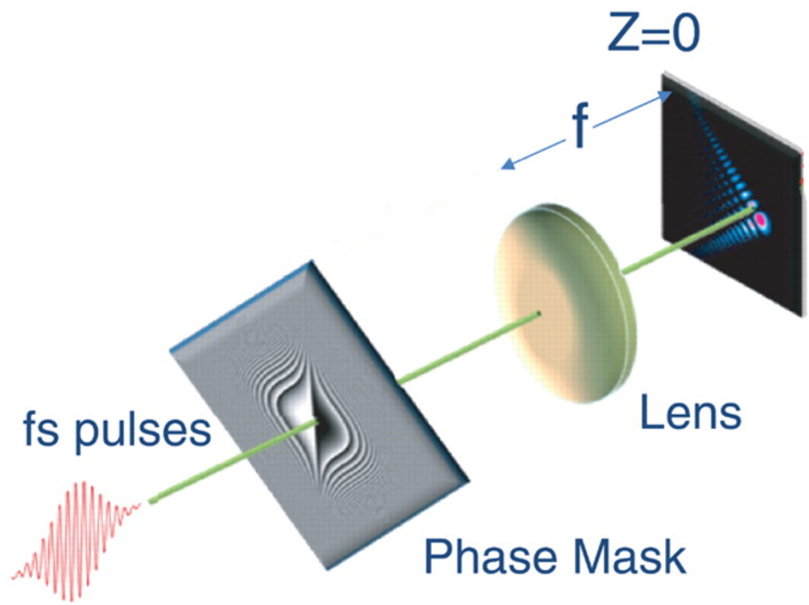
Airy beam formation

This is an example of a beam simulation that is relatively demanding, computationally. Airy function Ai is another special function that can be realized through paraxial beam propagation.



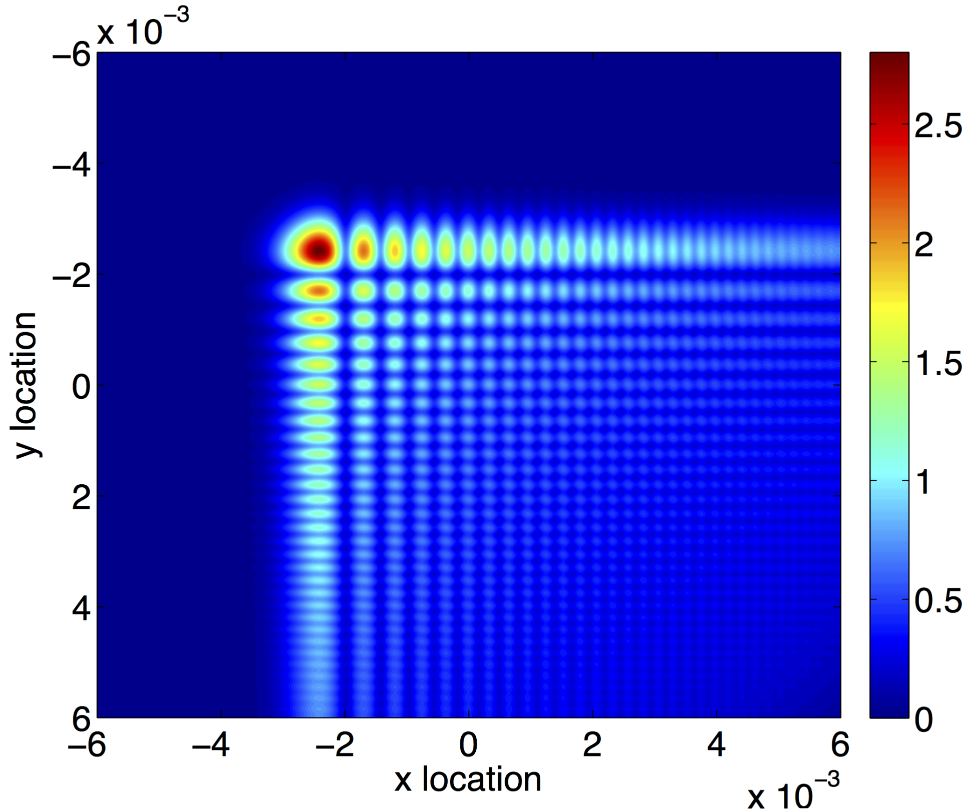


This can be done by taking advantage of the fact that a lens produces a Fourier transform of the incident field in ins focal plane. Airy function is a Fourier transform of a cubic phase profile, so we use this to imprint a phase on a beam. Then the beam passes through the lens, and around its focal plane the Fourier transform property of the lens creates the field profile with the Airy function.



To obtain a two-dimensional beam, the phase mask is created as a direct product of two identical, one-dimensional phase profiles. The resulting beam is then a direct product of two one-dimensional Airy-beam profiles.

Numerical simulation



What makes this and in general all special beams difficult to simulate is that they require a large domain to accommodate the beam. At the same time the domain must have a fine spatial resolution in order to support plane-wave components of the beam which propagate at steep angles...

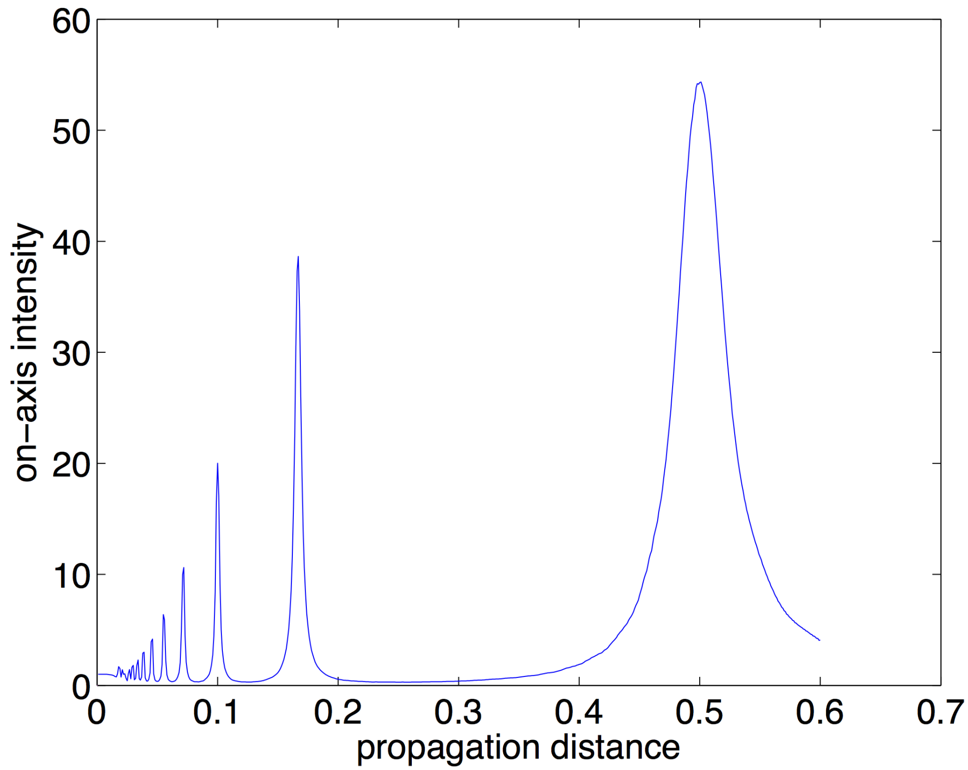
Self-healing beams

A) Airy and Bessel beams belong to the family of beams that have a self-healing property. When a lobe in the beam is destroyed, for example by a droplet, or a small opaque screen, this lobe spontaneously reconstructs after a short propagation distance. This is because the features if this kind of waveforms are continuously formed by interference of waves that propagate at angle w.r.t. each other. An opaque obstacle has a kind of "geometric shadow" behind which its effect diminishes.

Verify through a numerical experiment that this indeed occurs, both in Bessel and Airy beams.

Fresnel lens

Set up a simulation for a Gaussian beam that passes through a Fresnel lens. Design the lens such that its "focus" will be at 0.5 m from the screen. Obtain the on-axis intensity as a function of the propagation distance. Your simulation should produce data similar to this illustration if run with parameters close to those found in instructor's *pBeamFresnel.m*



A) Decide whether the simulation is sound and explain the observed profile of the intensity between the screen and the focus of the Fresnel lens.

B) In this code, we employ a guard in the spectral space. This is often utilized to control waves with high wavenumbers, such as those arising at the sharp edges of the screen used for the simulated Fresnel lens. Experiment with the settings and observe their effects.

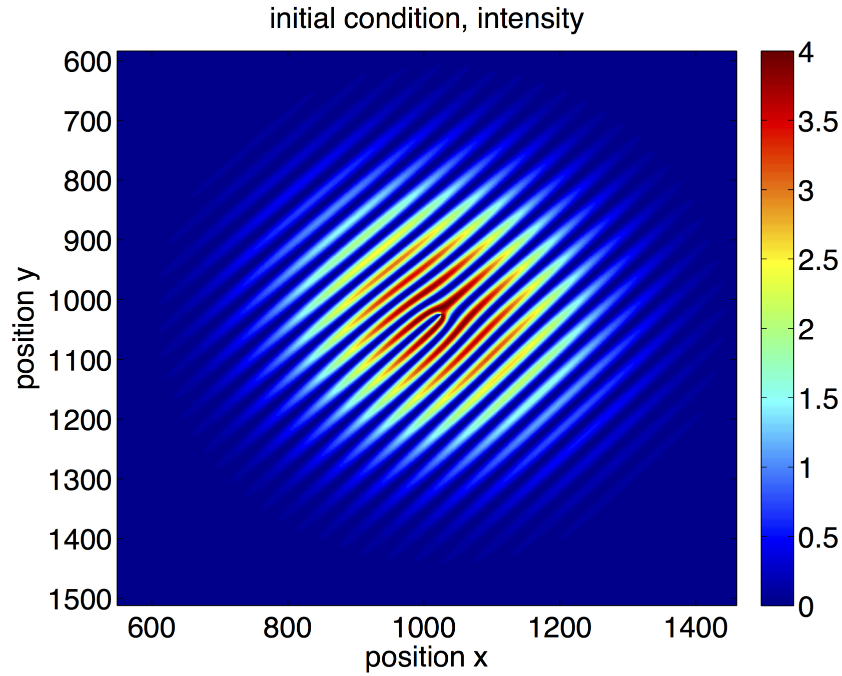
Generation of vortex beams

Simulation of vortex beam generation is relatively straightforward, especially for the first-order beams.

Here we set up a simulation in which the beams are obtained in the far-field behind an intensity mask. The mask is obtained as if from interference from three beams: The central is an ordinary Gaussian, and there are to opposite vortex beams incident at an angle.

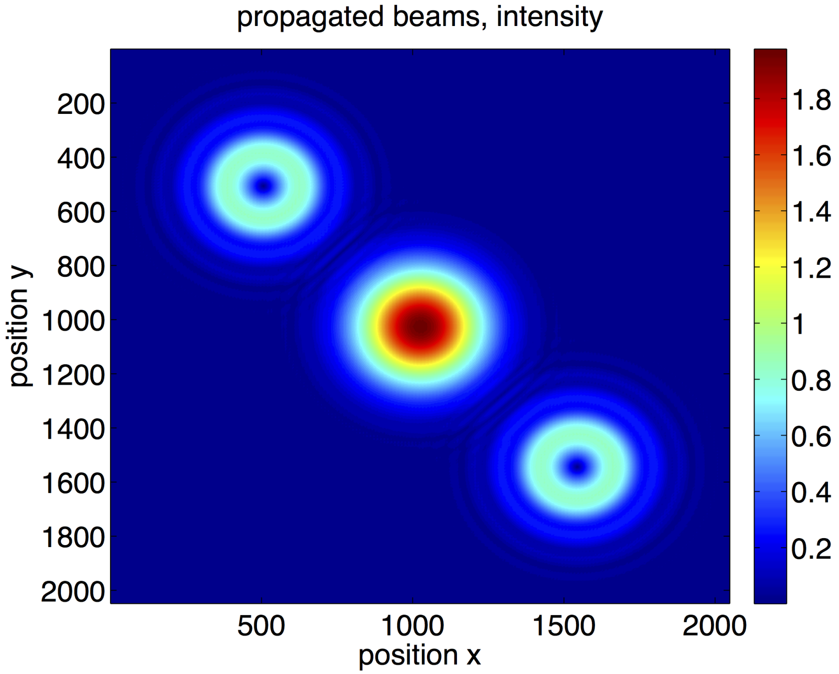
Beam propagation produces a Fourier transform in the far field and as such it reconstructs the three beams that could have created the mask.

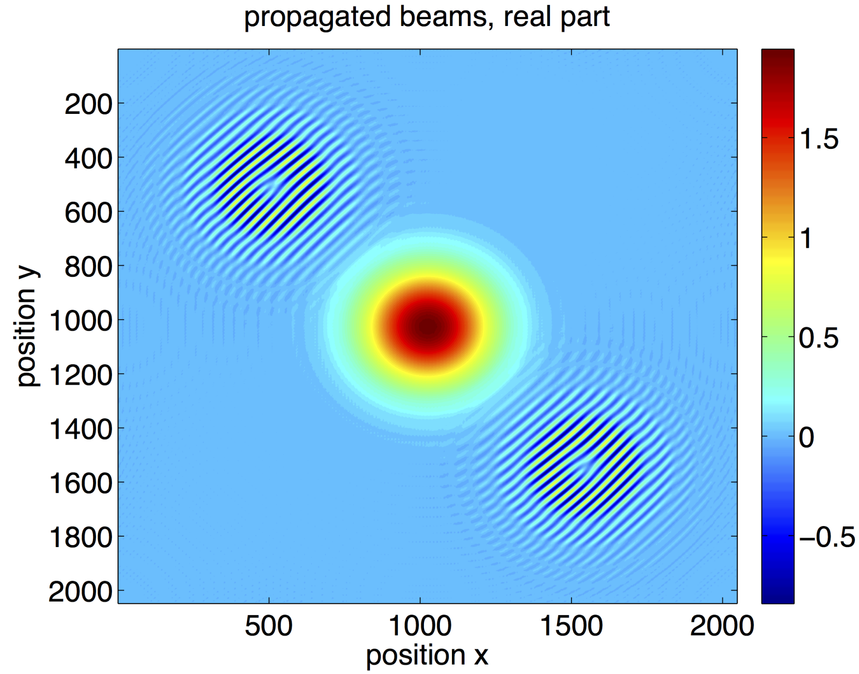
The initial condition



is a Gaussian beam just behind the intensity mask. It has a characteristic "fork" that will generate vortex beams in the far field. The angle of the pattern controls the direction of propagation for the "daughter" beams.

Propagated beams:





A) Experiment with the numerical simulation parameters to get a feel about what is needed for a well-resolved model.

B) Instead of obtaining Fourier transform by propagation into far field, one could use a lens. "Insert" a suitable lens into this simulation in order to create vortex beams in a shorter distance.