EXPERIMENT NO : CS I /1

TITLE : FAMILIARIZATION WITH MATLAB CONTROL SYSTEM TOOL BOX, MATLAB/SIMULINK TOOL BOX.

OBJECTIVE : To obtain

I. Pole, zero, gain values from a given transfer function
II. Transfer function model from pole, zero, gain values
III. Pole, zero plot of a transfer function

THEORY:

A transfer function is also known as the network function is a mathematical representation, in terms of spatial or temporal frequency, of the relation between the input and output of a (linear time invariant) system. The transfer function is the ratio of the output Laplace Transform to the input Laplace Transform assuming zero initial conditions. Many important characteristics of dynamic or control systems can be determined from the transfer function. The transfer function is commonly used in the analysis of single-input single-output electronic system, for instance. It is mainly used in signal processing, communication theory, and control theory. The term is often used exclusively to refer to linear time-invariant systems (LTI). In its simplest form for continuous time input signal x(t) and output y(t), the transfer function is the linear mapping of the Laplace transform of the input, X(s), to the output Y(s).
Zeros are the value(s) for z where the numerator of the transfer function equals zero. The complex frequencies that make the overall gain of the filter transfer function zero.
Poles are the value(s) for z where the denominator of the transfer function equals zero. The complex frequencies that make the overall gain of the filter transfer function infinite.
The general procedure to find the transfer function of a linear differential equation from input to output is to take the Laplace Transforms of both sides assuming zero conditions, and to solve for the ratio of the output Laplace over the input Laplace.

The transfer function provides a basis for determining important system response characteristics without solving the complete differential equation. As defined, the transfer function is a rational function in the complex variable 's' that is It is often convenient to factor the polynomials in the numerator and the denominator, and to write the transfer function in terms of those factors:

$$G(s) = \frac{N(S)}{D(S)} = K\frac{(s-z_1)(s-z_2)\ldots\ldots\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots\ldots\ldots(s-p_m)}$$

where, the numerator and denominator polynomials, N(s) and D(s).

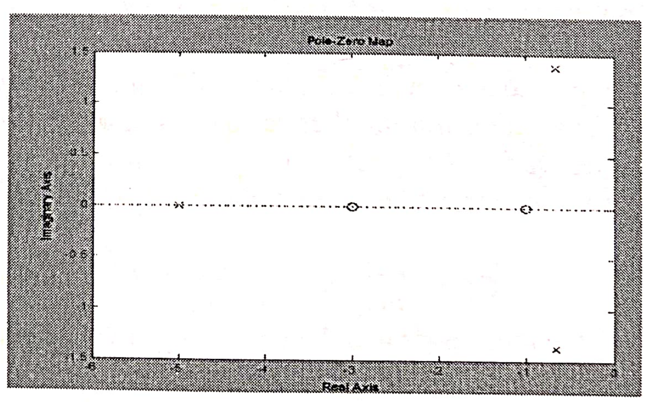
The values of s for which N(S) = 0, are known as zeros of the system. i.e; at s = z_1 , z_2 ,……, z_n.
The values of s for which D(S) = 0, are known as poles of the system. i.e; at s = p_1 , p_2 ,……, p_n.

Example 1: **Obtain pole, zero & gain values of a transfer function** $G(s) = \frac{s^2+4s+3}{(s+5)(s2+4s+7)}$. **Also obtain pole zero plot.**

```
num = [1 4 3]
den= conv([1 5], [3 4 7])
g = tf (num,den)
[z,p,k] = tf2zp(num,den)
pzmap(g)
```

```
Output :

Transfer function:
      s^2 + 4 s + 3
---------------------------

3 s^3 + 19 s^2 + 27 s + 35

z =

    -3
    -1

p =

  -5.0000
  -0.6667 + 1.3744i
  -0.6667 - 1.3744i

k =

   0.3333
```

Pole Zero Plot:



Example 2: **Write matlab code to obtain transfer function of a system from its pole ,zero, gain values. Assume pole locations are -2, 1, zero at -1 and gain is 7.**

```
Matlab Code :

p= [-2 1]
z= [-1]
```

```
k=7
[num,den]= zp2tf(z',p',k)
g=tf(num,den)
```

Output:

Transfer function:
```
 7 s + 7
-----------
s^2 + s - 2
```

Assignments:

Obtain Pole, zero, gain values of the transfer functions given below. **Also verify your result theoretically**

1. $G(s) = \frac{1}{s^2+s+4}$ 2. $G(s) = \frac{5}{s^2+9}$ 3. $G(s) = \frac{1}{(s^2+3s+5)(s+3)(s+5)}$

Obtain Transfer function of the systems (Theoretically & Practically).

1. Poles = -1+i, -1-i,-4. Zeros = -2,-5, gain = 1
2. Poles = -1+4i, -1-4i,-5. Zeros = -8,-5, gain = .75

DISCUSSION:

1. **What do you mean by poles?**
2. **What is the significance of transfer function?**

EXPERIMENT NO : CS I/2

TILLE : **DETERMINATION OF STEP & IMPULSE RESPONSE FOR A FIRST ORDER UNITY FEEDBACK SYSTEM**

OBJECTIVE : To determine

I. Step response of 1st order system
II. Impulse response of 1st order system

THEORY: A first order system is one in which highest power of s in denominator if transfer function defines order of the system.

For first order system,

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

$$C(s)=\frac{1}{sT+1}R(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Since the laplace transform of the unit step function is 1/s , substituting R(s)=1/s in equation (1)

$$C(s) = \frac{1}{sT+1} * \frac{1}{s}$$

Expanding C(s) into partial fractions gives ,

$$C(s) = \frac{1}{s} - \frac{1}{sT+1}$$

$$C(s) = \frac{1}{s} - \frac{1}{s+(1/T)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Taking the inverse laplace transform of equation (2),we get

$$C(t) = 1- e^{-t/T} \quad \text{for } t \geq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Equation (3) shows that initially (when t=0), the output c(t) is zero and finally ($t \rightarrow \infty$) $e^{-t/T}$ is zero and the output c(t) becomes unity .

At t = T,

$$C(t) = 1 - e^{-1} = 1 - 0.368 = 0.632$$

That's , the output response has reached 63.2 % of it's final value . T is known as the time constant . Thus , the time constant T is defined as the time required for the output response to attain 63.2% of its final value or steady state value .

Equation (3) shows that the response curve is exponential in nature as shown on figure.

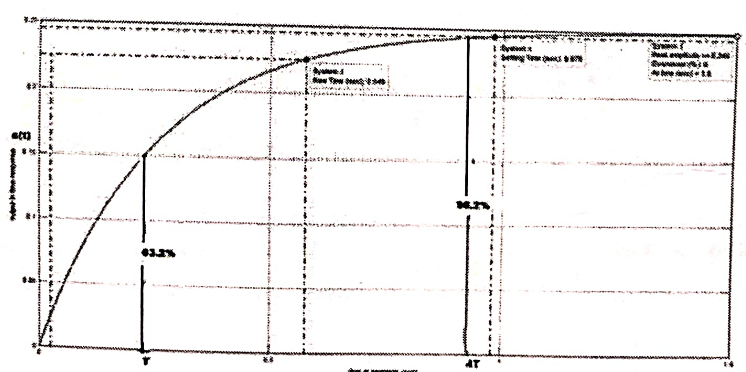


Fig. : Response curve of first order unit step input

Response of the first order system with unit impulse response
For the unit – impulse input
R(s) = 1
C(s) = $\frac{1}{sT+1}$ * R(s) ....(1)
Substituting the value of R(s) = 1 in equation (1),we get
C(s)= = $\frac{1}{sT+1}$ * 1
C(s) = = $\frac{1}{T}$ * = $\frac{1}{s+1/T}$ ....(2)
Taking the inverse laplace transform of the equation of (2) , we get the output response as
C(t) = = $\frac{1}{T}$ e $^{-t/T}$ for t ≥ 0 ....(3)
The output response curve shown in the figure

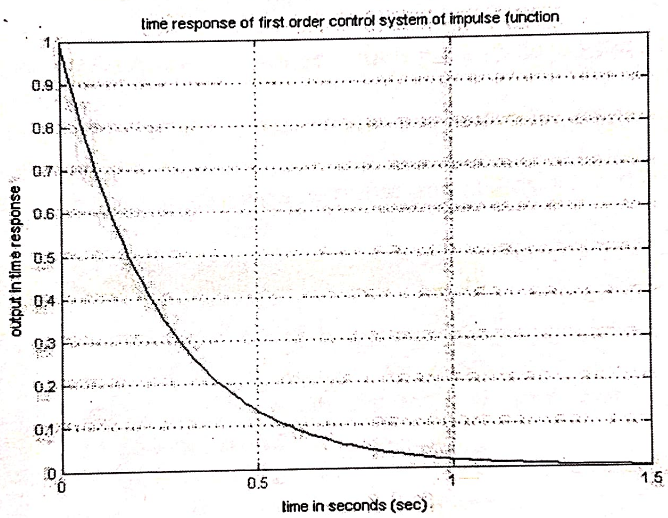


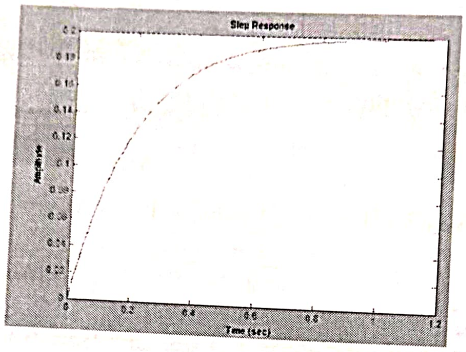
Fig. : unit impulse response of first order system

Example 1: Obtain step response of a unity feedback system having forward path transfer function of
$$G(s) = \frac{1}{s+4}$$

Matlab Code:

```
num = [1];
den = [1 4]
g = tf (num,den)
t = feedback(g,1)
step(t,'r')
```

Output:

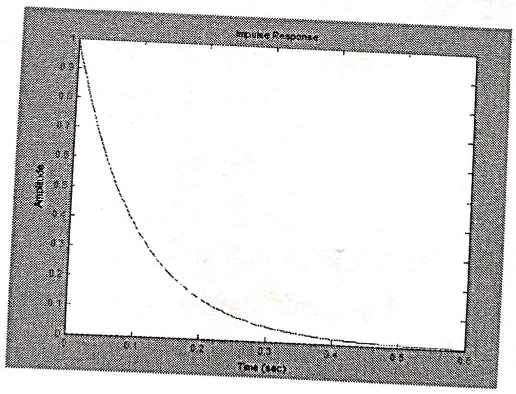


Example 2: Obtain impulse response of a unity feedback system having forward path transfer function of G(s) = $\frac{1}{s+9}$

Matlab Code :

```
num = [1];
den = [1 9]
g = tf (num,den)
t = feedback(g,1)
impulse(t,'r')
```

Output:



Assignments:

Obtain step and impulse response of the following systems with unity feedback connection. Also verify your result theoretically

1. G(s) = $\frac{1}{s+11}$ 2. G(s) = $\frac{1}{s+2}$

DISCUSSION:

1. **Explain why a series RL circuit with high inductance has a slow response?**

EXPERIMENT NO : CS I/3

TILLE : DETERMINATION OF STEP & IMPULSE RESPONSE FOR A SECOND ORDER UNITY FEEDBACK SYSTEM

OBJECTIVE : To determine

I. Step response of 2^{nd} order system
II. Impulse response of 2^{nd} order system

THEORY: The time response has utmost importance for the design and analysis of control systems because these are inherently time domain systems where time is independent variable. During the analysis of response, the variation of output with respect to time can be studied and it is known as time response. To obtain satisfactory performance of the system with respect to time must be within the specified limits. From time response analysis and corresponding results, the stability of system, accuracy of system and complete evaluation can be studied easily. Due to the application of an excitation to a system, the response of the system is known as time response and it is a function of time. The two parts of response of any system:

Transient response; Steady-state response.

Transient response: The part of the time response which goes to zero after large interval of time is known as transient response.

Steady state response: The part of response that means even after the transients have died out is said to be steady state response.

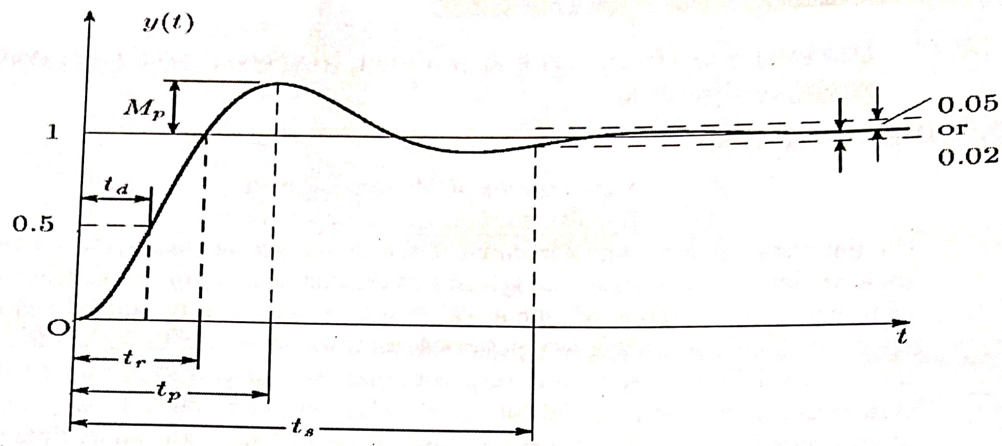
The total response of a system is sum of transient response and steady state response: C(t)=Ctr(t)+Css(t)

Time Response Specification Parameters: The transfer function of a 2-nd order system is generally represented by the following transfer function:

$$\frac{Y(S}{R(S)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

The dynamic behavior of the second-order system can then be described in terms of two parameters: the damping ratio and the natural frequency.

If the dumping ratio is between 0 and 1, the system poles are complex conjugates and lie in the left-half *s* plane. The system is then called **underdamped**, and the transient response is **oscillatory**. If the damping ratio is equal to 1 the system is called **critically damped**, and when the damping ratio is larger than 1 we have **overdamped** system. The transient response of critically damped and overdamped systems do not oscillate. If the damping ratio is 0, the transient response does not die out.



Delay time (td)

The delay time is the time required for the response to reach half the final value the very first time.

Rise time (tr)

The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped second-order systems, the 0% to 100% rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.

Peak time (tp)

The peak time is the time required for the response to reach the first peak of the overshoot.

Maximum (percent) overshoot (Mp)

The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by

$$\frac{y(c_p) - c(\infty)}{\infty} \times 100\%$$

Settling time (ts)

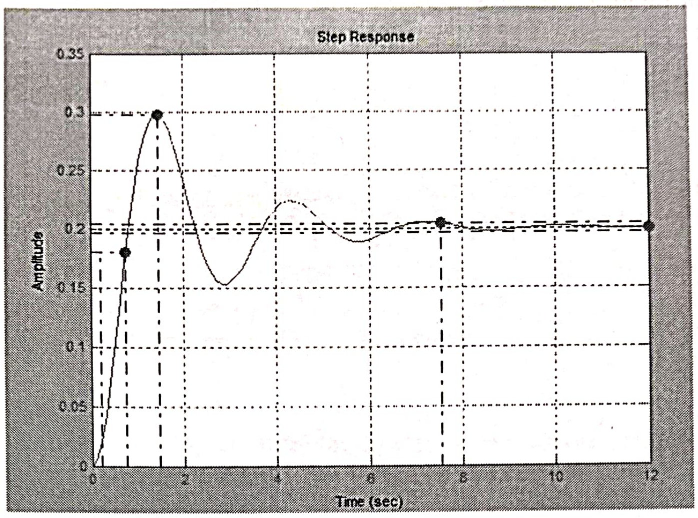
The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%). The settling time is related to the largest time constant of the control system.

Example 1: Obtain step response of a unity feedback system having forward path transfer function of $G(s) = \frac{1}{s^2+s+4}$

Matlab Code:

```
num = [1];
den = [1 1 4]
g = tf (num,den)
t = feedback(g,1)
step(t,'r')
```

Output:

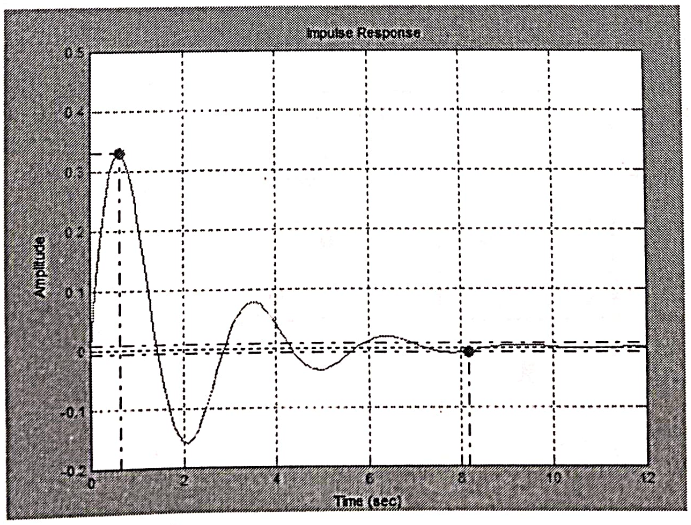


Example 2: Obtain impulse response of a unity feedback system having forward path transfer function of $G(s) = \frac{1}{s^2+s+4}$

Matlab Code :

```
num = [1];
den = [1 9]
g = tf (num,den)
t = feedback(g,1)
impulse(t,'r')
```

Output:



Assignments:

Obtain step and impulse response of the following systems with unity feedback connection. Also verify your result theoretically

1. $G(s) = \frac{19}{(s+4)(s+8)}$ 2. $G(s) = \frac{1}{s^2+3s+4}$

DISCUSSION:

1. What do you mean by rise time? Derive its expression for a unity feedback 2nd order control system.
2. Why is less overshoot desired for practical systems?

EXPERIMENT NO : CS I/4

TILLE : DETERMINATION OF STEP & IMPULSE RESPONSE FOR A TYPE '0', TYPE '1', TYPE '2' SYSTEMS

OBJECTIVE : To determine

I. Step response of Type '0' system
II. Impulse response of Type '0' system
III. Step response of Type '1' system
IV. Impulse response of Type '1' system
V. Step response of Type '2' system
VI. Impulse response of Type '2' system

THEORY:

A real time system can be expressed by its transfer function. Based on presence of poles at origin of s plane , transfer functions can be classified as Type '0', Type '1', Type '2' ….. Systems. Let open loop transfer function a system is expressed as:

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)......}{s^N(1+sT_1)(1+sT_2)......} \quad \text{..............................} \text{ (i)}$$

From equation (i) it is clear that 'N' determines the number of poles at origin.

For a Type '0' system, N = 0
For a Type '1' system, N = 1
For a Type '2' system, N = 2
.
.
.
.
For a Type 'N' system, N = N

The steady state error can be found out by the following equation

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

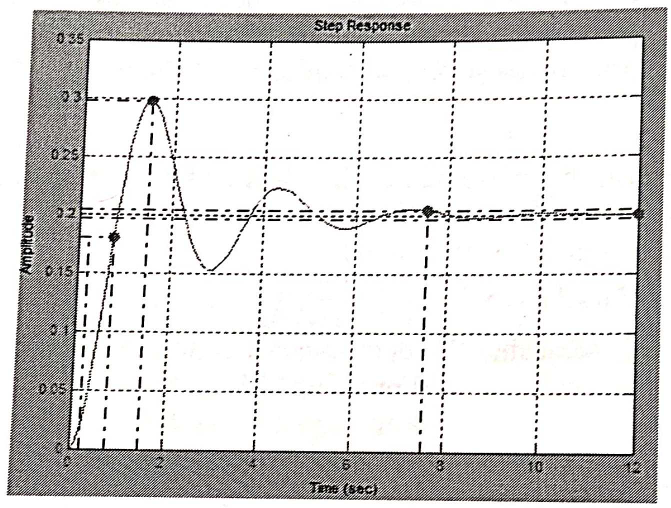
Type	Step Input		Ramp input		Parabolic input	
	K_P	Ess	K_V	Ess	K_a	Ess
0	K	$\frac{1}{1+K}$	0	∞	0	∞
1	∞	0	K	$\frac{1}{K}$	0	∞
2	∞	0	∞	0	K	$\frac{1}{K}$

Example 1: Obtain step response of a Type '0' system **having forward path transfer function of** G(s) = $\frac{1}{s^2+s+4}$

Matlab Code :

```
num = [1];
den = [1 1 4]
g = tf (num,den)
t = feedback(g,1)
step(t,'r')
```

Output:

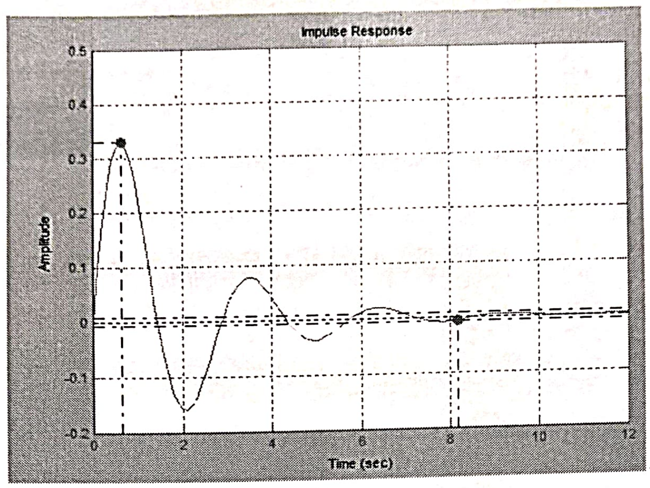


Example 2: Obtain impulse response of a Type '0' system **having forward path transfer function of** G(s) = $\frac{1}{s^2+s+4}$

Matlab Code :

```
num = [1];
den = [1 1 4]
g = tf (num,den)
t = feedback(g,1)
impulse(t,'r')
```

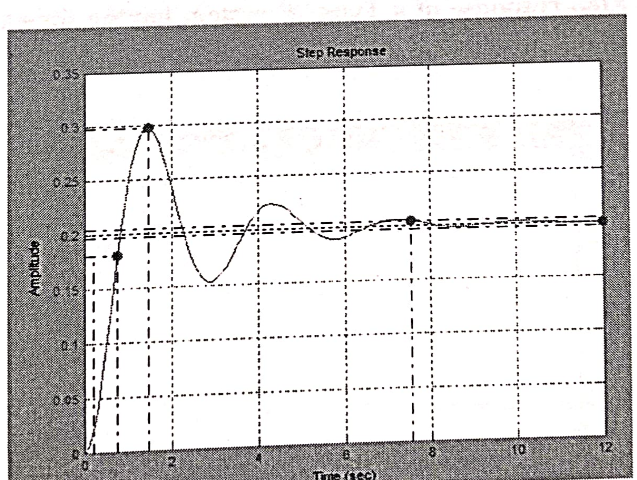
Output:



Example 3: Obtain step response of a Type '1' system **having forward path transfer function of** G(s) = $\frac{1}{s(s^2+s+4)}$

Matlab Code :

```
num = [1];
den = [0 1 1 4]
g = tf (num,den)
t = feedback(g,1)
step(t,'r')

Output:
```

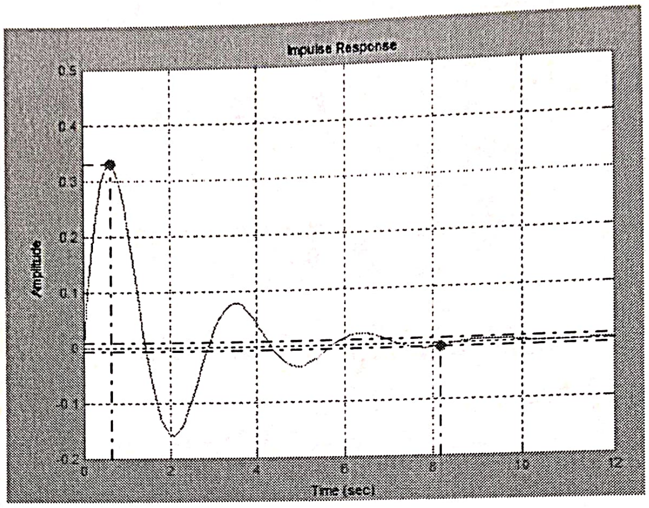


Example 4: Obtain Impulse response of a Type '1' system **having forward path transfer function of** G(s) = $\frac{1}{s(s^2+s+4)}$

Matlab Code :

```
num = [1];
den = [0 1 1 4]
g = tf (num,den)
t = feedback(g,1)
impulse(t,'r')
```

Output:

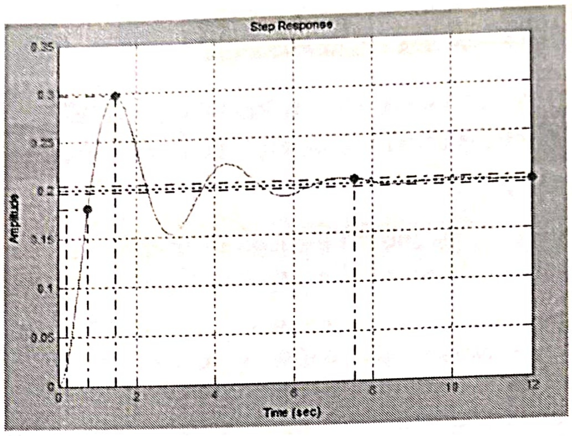


Example 5: Obtain Step response of a Type '2' system **having forward path transfer function of** G(s) = $\frac{1}{s^2(s^2+s+4)}$

Matlab Code :

```
num = [1];
den = [0 0 1 1 4]
g = tf (num,den)
t = feedback(g,1)
step(t,'r')
```

Output:

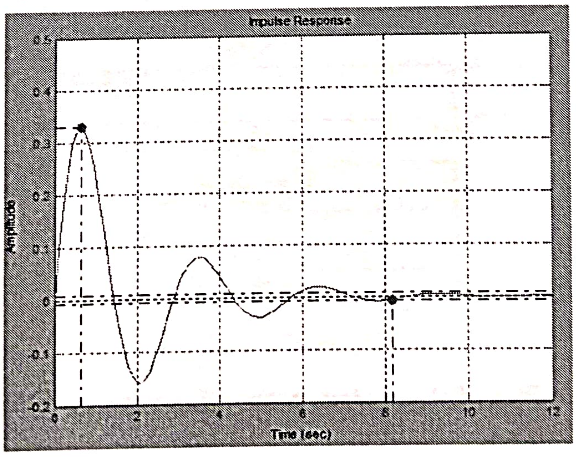


Example 6: Obtain Impulse response of a Type '2' system **having forward path transfer function of** G(s) $= \frac{1}{s^2(s^2+s+4)}$

Matlab Code :

```
num = [1];
den = [0 0 1 1 4]
g = tf (num,den)
t = feedback(g,1)
impulse(t,'r')
```

Output :



DISCUSSION:

1. **What would be steady state error for a type 1 system if unit ramp input is applied?**

EXPERIMENT NO : CS I/5

TILLE : **DETERMINATION OF BODE PLOT USING MATLAB CONTROL SYSTEM TOOLBOX FOR 2ND ORDER SYSTEM & OBTAIN CONTROLLER SPECIFICATION PARAMETERS.**

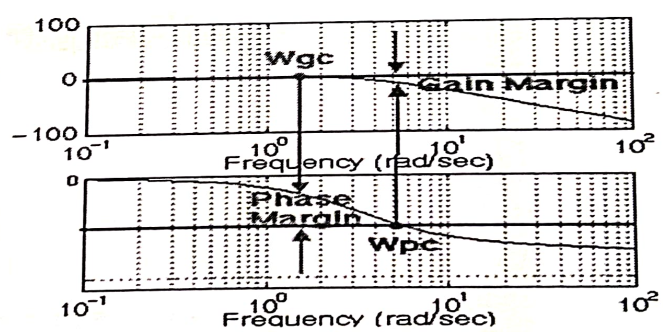
OBJECTIVE : To determine

I. Bode plot of a 2nd order system
II. Frequency domain specification parameters

THEORY: The frequency response method may be less intuitive than other methods you have studied previously. However, it has certain advantages, especially in real-life situations such as modeling transfer functions from physical data. The frequency response of a system can be viewed two different ways: via the Bode plot or via the Nyquist diagram. Both methods display the same information; the difference lies in the way the information is presented. We will explore both methods during this lab exercise. The frequency response is a representation of the system's response to sinusoidal inputs at varying frequencies. The output of a linear system to a sinusoidal input is a sinusoid of the same frequency but with a different magnitude and phase. The frequency response is defined as the magnitude and phase differences between the input and output sinusoids. In this lab, we will see how we can use the open-loop frequency response of a system to predict its behavior in closedloop. To plot the frequency response, we create a vector of frequencies (varying between zero or "DC" and infinity i.e., a higher value) and compute the value of the plant transfer function at those frequencies. If G(s) is the open loop transfer function of a system and ω is the frequency vector, we then plot G(jω) vs. ω . Since G(jω) is a complex number, we can plot both its magnitude and phase (the Bode plot) or its position in the complex plane (the Nyquist plot).

The gain margin is defined as the change in open loop gain required to make the system unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed loop.

The phase margin is defined as the change in open loop phase shift required to make a closed loop system unstable.



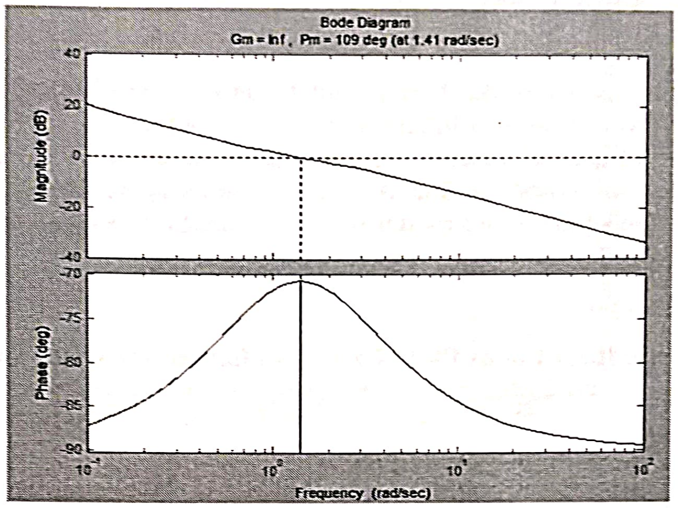
Example 1: Obtain Bode Plot of the system having forward path transfer function of $G(s) = \frac{1+s}{s(1+0.5s)}$

Matlab Code:

```
num = [1 1]
den = conv([1 0],[.5 1])
```

```
g = tf(num,den);
bode(g)
margin(g)
```

Output :



Assignments:

Obtain bode diagram of the following transfer function. Also verify your result theoretically.

$$G(s) = \frac{4(2+s)}{s(5+s)(s+10)}$$

DISCUSSION:

1. What do you mean by GM & PM?
2. How GM & PM affects system?
3. What is gain cross over frequency?