

UNIT 4/5

COMBINATIONAL LOGIC

Combinational Logic

- **Combinational logic** is a *type of digital logic which is implemented by boolean circuits, where the **output** is a pure function of the **present input only**.*
- No memory to store data.

Combinational circuit:

- Consists of ***n input variables, logic gates, and m output variables.***
- The logic gates accept signals from the inputs and generate signals to the outputs.
- For ***n input variables, there are 2^n possible combinations of binary input values.***
- For ***each possible input combination, there is one and only one possible output combination.***

- A combinational circuit can be described by *m Boolean functions, one for each output variable.*
- Each output function is expressed in terms of the n input variables.
- The n input binary variables come from an external source; the m output variables go to an external destination.



Fig: Block diagram of combinational circuit

Design procedure

- *Starts from the verbal outline of the problem and ends in a logic circuit diagram or a set of Boolean functions from which the logic diagram can be easily obtained.*
- Involves the following steps:
 - **1. Specification**
 - *Write a specification for the circuit if one is not already available*
 - **2. Formulation**
 - *Derive a truth table or initial Boolean equations that define the required relationships between the inputs and outputs, if not in the specification.*
 - *Apply hierarchical design if appropriate*
 - **3. Optimization**
 - *Apply 2-level and multiple-level optimization*
 - *Draw a logic diagram for the resulting circuit using ANDs, ORs, and inverters*
 - **4. Technology mapping**
 - *Map the logic diagram to the implementation technology selected*
 - **5. Verification**
 - *Verify the correctness of the final design manually or using simulation*

- Simply, we can list out the design procedure of combinational circuits as:
 1. *The problem is stated.*
 2. *The number of available input variables and required output variables is determined.*
 3. *The input and output variables are assigned letter symbols.*
 4. *The truth table that defines the required relationships between inputs and outputs is derived.*
 5. *The simplified Boolean function for each output is obtained.*
 6. *The logic diagram is drawn.*

Adder

- Arithmetic operation.
- The most basic arithmetic operation-addition of two binary digits.
- Half-adder/Full adder

1. Half-Adder

- A combinational circuit that performs the addition of two bits.
- Circuit needs **two inputs and two outputs**.
- The input variables designate **the augend (x) and addend (y) bits**; the output variables produce **the sum (S) and carry (C)**.
- Now we formulate a Truth table to exactly identify the function of half-adder.



Fig: Block diagram of Half Adder

| Inputs | | Outputs | |
|--------|---|---------|---|
| x | y | S | C |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

Truth table

- The simplified Boolean functions for the two outputs can be obtained directly from the truth table.
- The simplified sum of products expressions are:

$$S = x'y + xy'$$

$$C = xy$$

| | | | | | |
|----------|----------|----------|----------|----------|----------|
| | | y | | 0 | 1 |
| | | | | | |
| x | 0 | | 0 | | 1 |
| | 1 | | 1 | | 0 |

$$\text{Sum (S)} = xy' + x'y$$

Fig: K-map for SUM

| | | | | | |
|----------|----------|----------|----------|----------|----------|
| | | y | | 0 | 1 |
| | | | | | |
| x | 0 | | 0 | | 0 |
| | 1 | | 0 | | 1 |

$$\text{Carry (C)} = xy$$

Fig: K-map for Carry

Implementation:

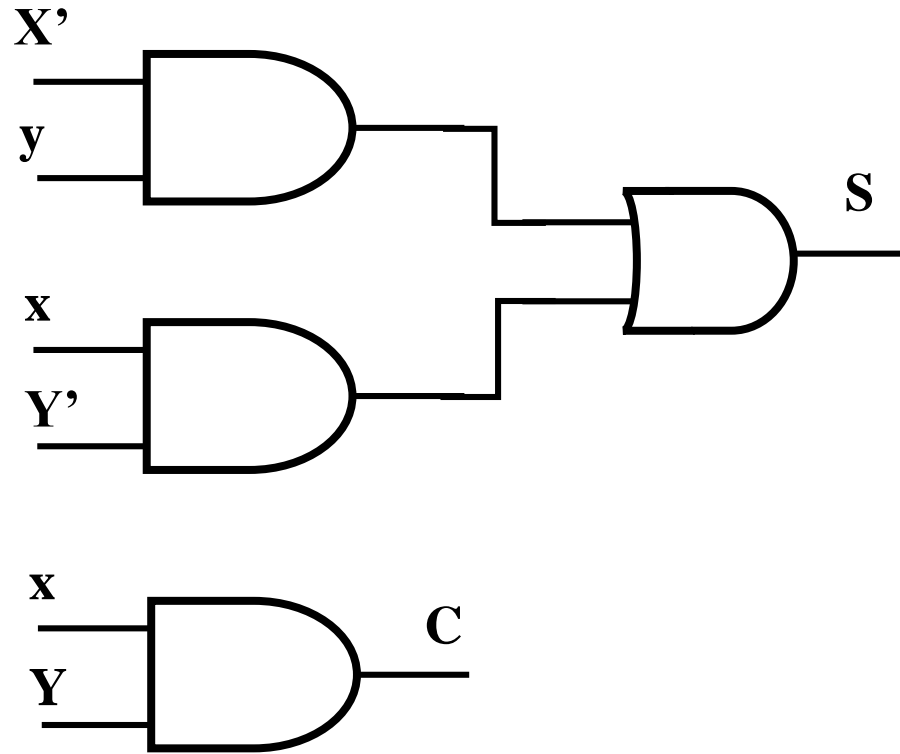


Fig: $S = x'y + xy'$
 $c = xy$

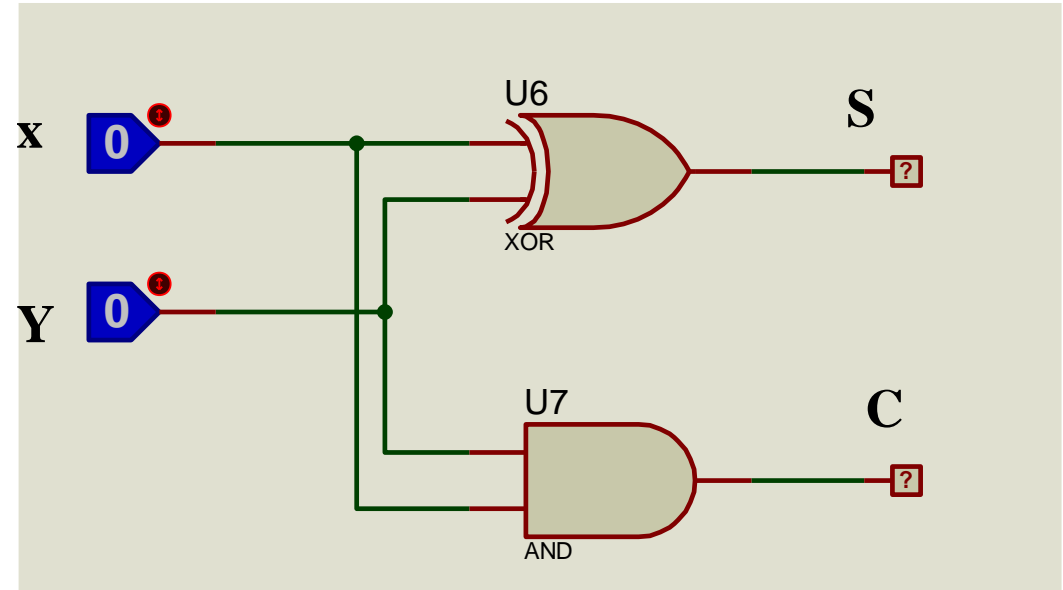


Fig: $S = x \oplus y$
 $c = xy$

2. Full Adder

- Add 3-bits (two inputs and one carry)
- Produces sum and carry



Fig: Block diagram of Full Adder

| Inputs | | | Outputs | |
|--------|---|-----|---------|---|
| x | y | Cin | S | C |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Truth table

Note: Z=Cin

$$\begin{aligned} \text{Sum } (S) &= \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz \\ &= x \oplus y \oplus z \end{aligned}$$

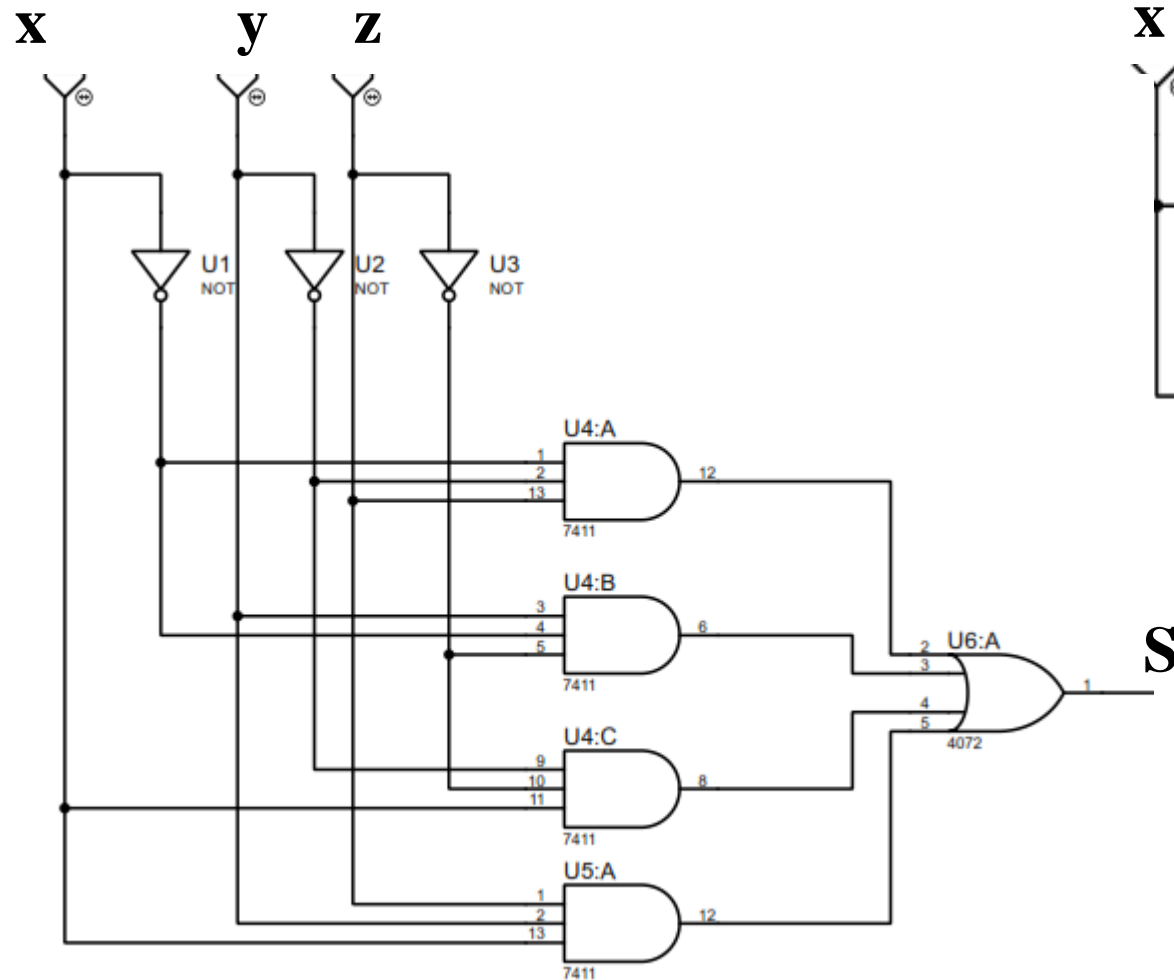
$$\text{Carry } (C) = xy + xz + yz$$

| | | yz | | | |
|---|-------------|------------------|------------|------|------------|
| | | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
| x | \bar{x} 0 | 0 | 1 | 0 | 1 |
| | x 1 | 1 | 0 | 1 | 0 |

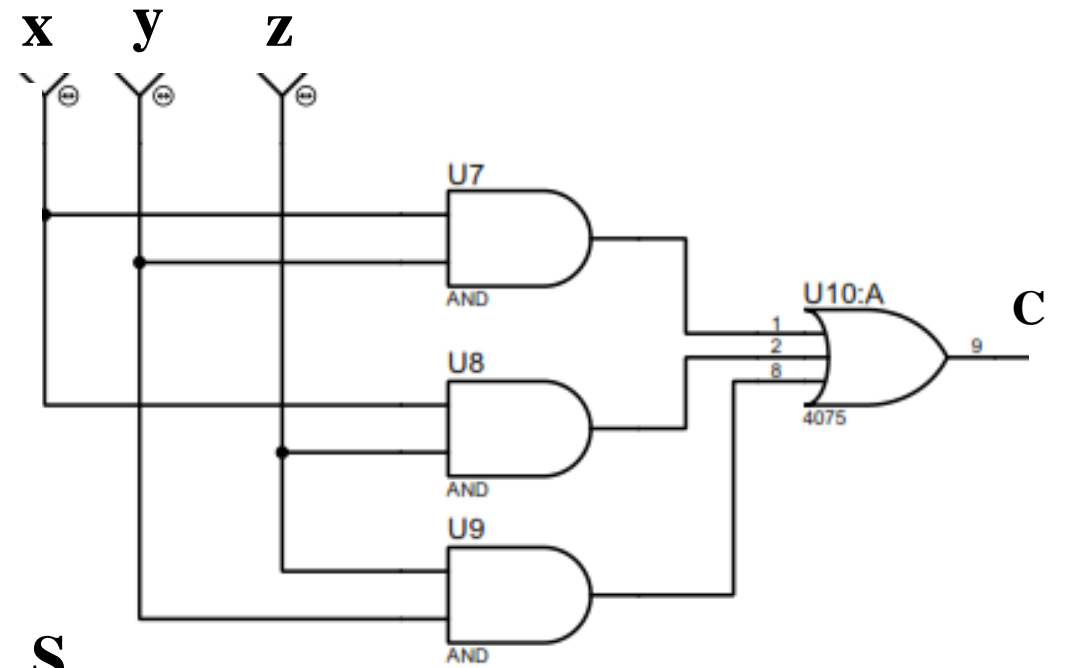
Fig: K-map for SUM

| | | yz | | | |
|---|-------------|------------------|------------|------|------------|
| | | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
| x | \bar{x} 0 | 0 | 0 | 1 | 0 |
| | x 1 | 0 | 1 | 1 | 1 |

Fig: K-map for Carry

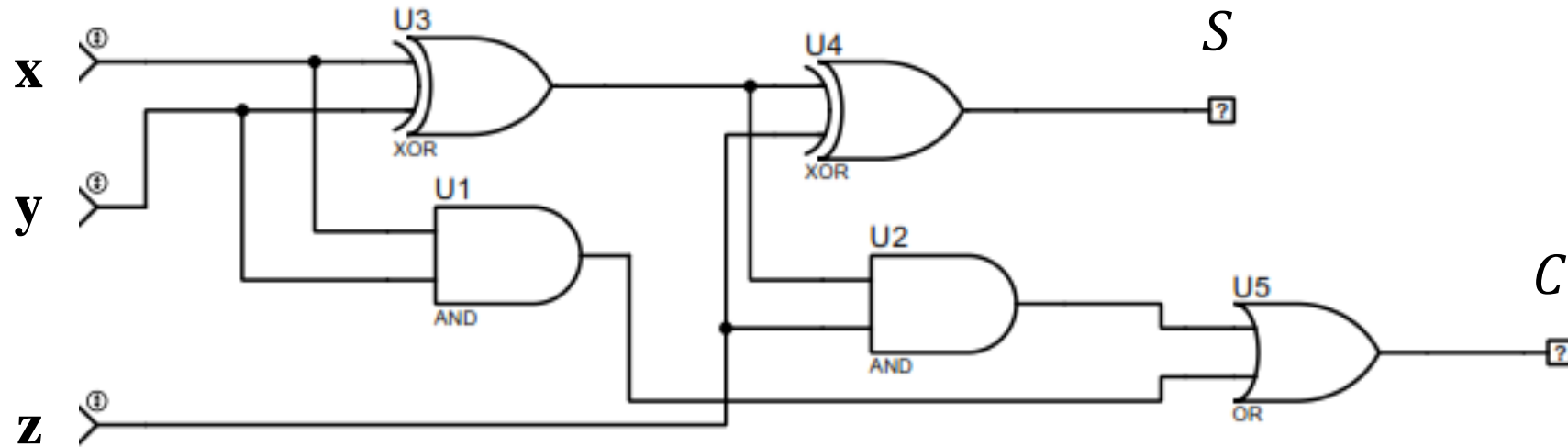


$$\text{Sum (S)} = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$



$$\text{Carry (C)} = xy + xz + yz$$

- *This approach uses two half-adders and a OR gate.*



$$\text{Sum } (S) = x \oplus y \oplus z$$

$$\text{Carry } (C) = xy + (x \oplus y)z$$

Subtractor

- Subtraction of two binary numbers is accomplished by taking the complement of the subtrahend and adding it to the minuend.

$$0-0=0$$

$$0-1=1$$

$$1-0=1$$

$$1-1=0$$

1. Half Subtractor

- Performs subtraction of two numbers of one bit and produces difference.
- Two input as X and Y and two output as difference (D) and borrow (B).

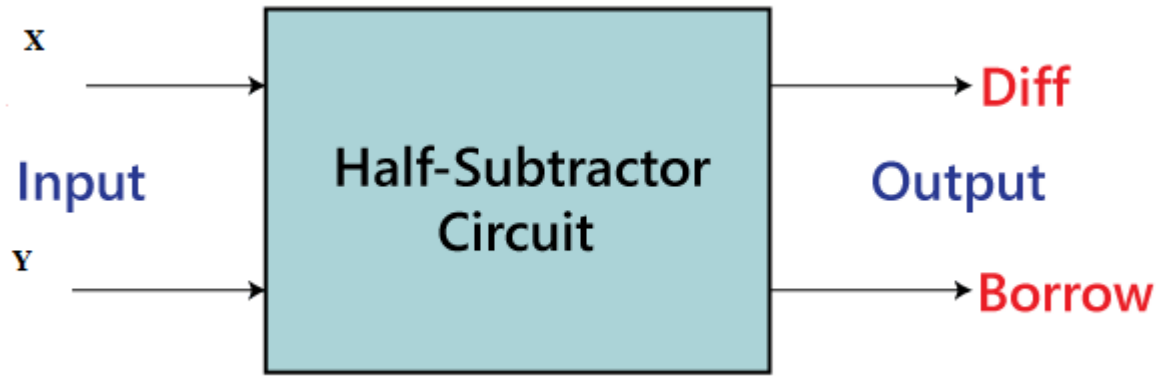


Fig: Block diagram of Half Subtractor

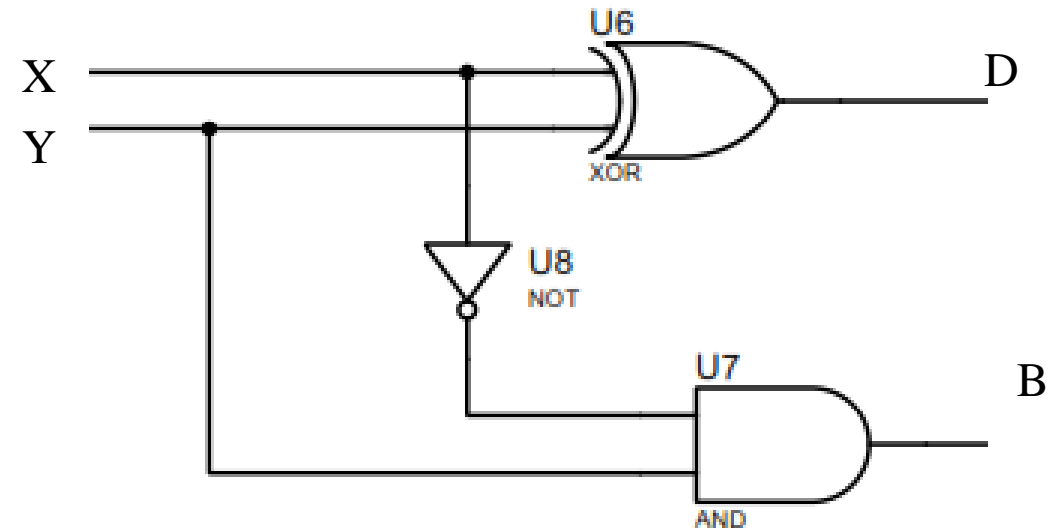
| Inputs | | Outputs | |
|--------|---|---------|---|
| X | Y | B | D |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |

Truth table for (X-Y)

- The Boolean function of the above truth table is:

$$\text{Difference (D)} = \bar{X}Y + X\bar{Y} = X \oplus Y$$

$$\text{Borrow (B)} = \bar{X}Y$$



2. Full Subtractor

- Subtraction of three single bit numbers.

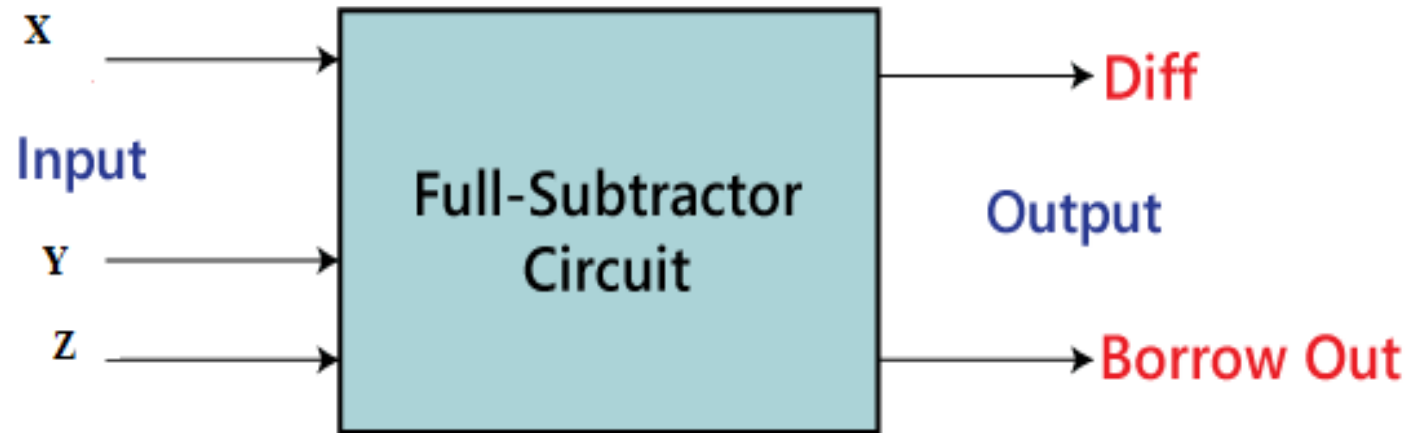


Fig: Block diagram of Full Subtractor

| Inputs | | | Outputs | |
|--------|---|---|---------|---|
| X | Y | Z | B | D |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

Truth table

- The Boolean function from the truth table is:

$$\begin{aligned} \text{Difference (D)} &= \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ \\ &= X \oplus Y \oplus Z \end{aligned}$$

$$\begin{aligned} \text{Borrow (B)} &= \bar{X}Y + \bar{X}Z + YZ \\ &= \bar{X}Y + (\bar{X} \oplus Y).Z \end{aligned}$$

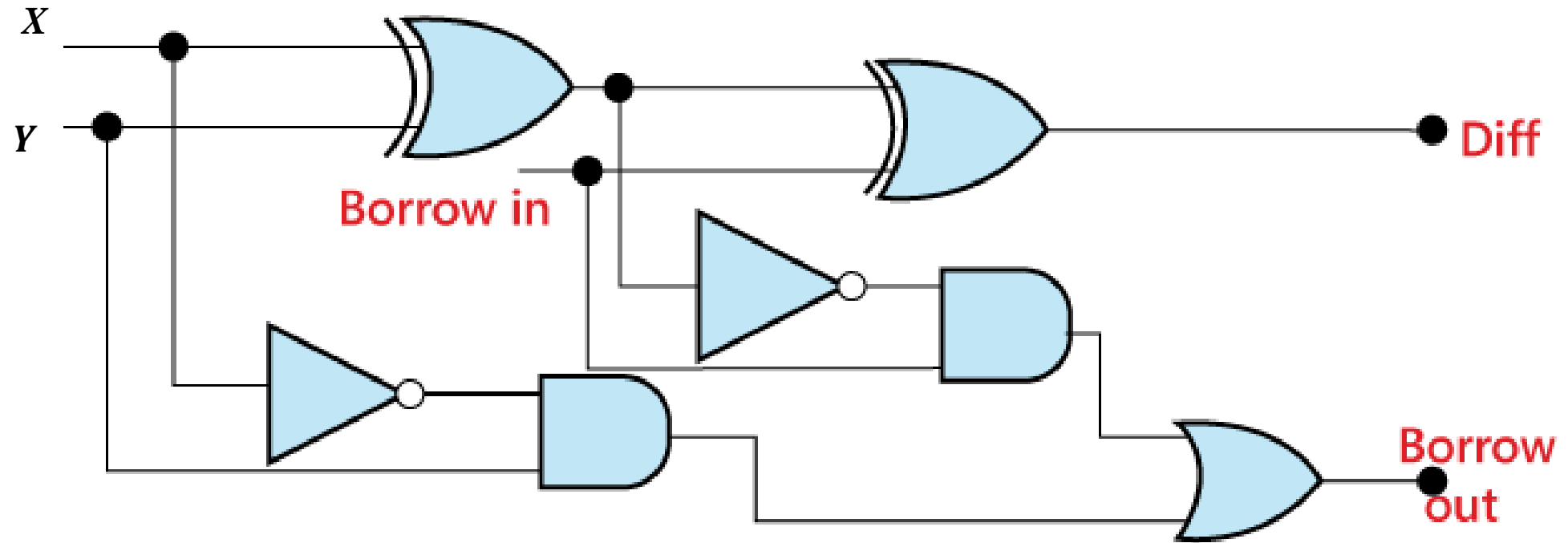
| | | | | | |
|----------|---|-----------|----|----|----|
| | | <i>yz</i> | | | |
| | | 00 | 01 | 11 | 10 |
| <i>x</i> | 0 | 0 | 1 | 0 | 1 |
| | 1 | 1 | 0 | 1 | 0 |

Fig: K-map for Difference

| | | | | | |
|----------|---|-----------|----|----|----|
| | | <i>yz</i> | | | |
| | | 00 | 01 | 11 | 10 |
| <i>x</i> | 0 | 0 | 1 | 1 | 1 |
| | 1 | 0 | 0 | 1 | 0 |

Fig: K-map for Borrow

- This approach uses two half-subtractors and a OR gate.*



Z=Borrow in

Binary Adder

- A logical circuit which is used to perform the addition operation of two binary number of any length.
- Formed with the help of the full-adder circuit.
- The full-adders are cascaded in series, and the output carry of the first adder will be treated as the input carry of the next full-adder.
- **N-bit parallel adder:**
 - To add two n-bit binary numbers rather than only single-bit binary numbers.
 - Need to use n-bit parallel adder.
 - In order to get n-bit parallel adder, we cascade the n number of full adders.

4-bit Binary Adder

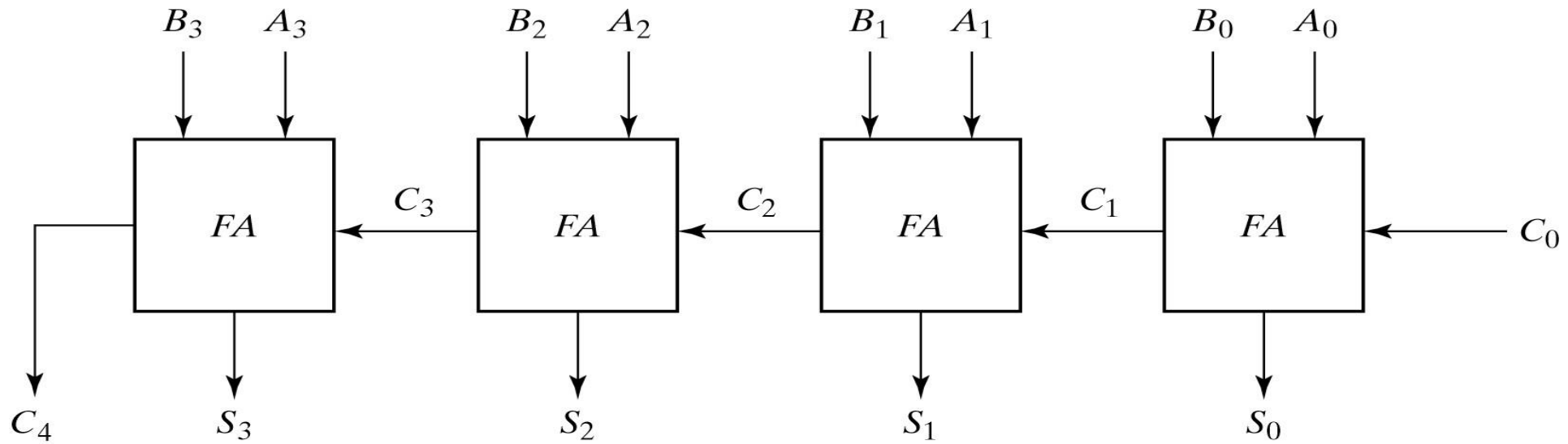
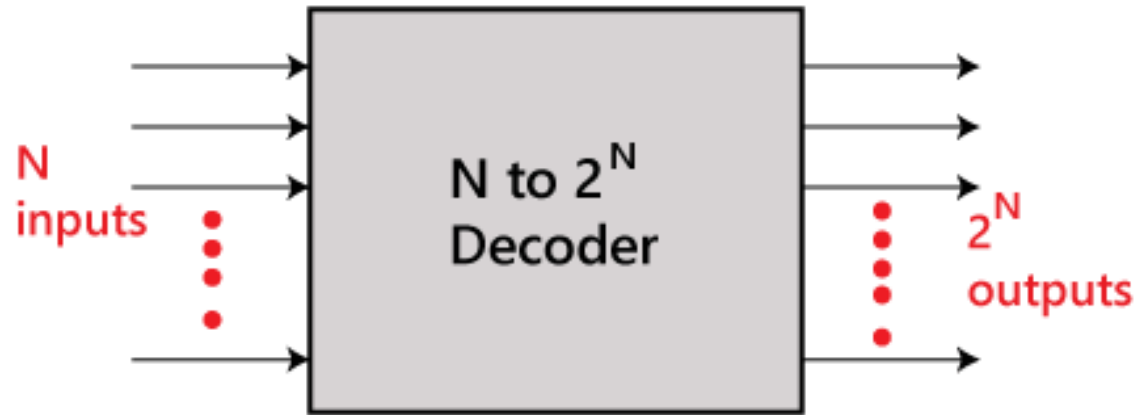


Fig. 4-9 4-Bit Adder

- 'A' and 'B' – augend and addend.
- **C0, C1, C2, and C3** - carry inputs which are connected together as a chain using Full Adder.
- **C4** \rightarrow **C_{out}**
- **S0, S1, S2, and S3** - sum outputs that produce the sum of augend and addend bits.

Decoder

- Combinational circuit that changes the ***N -bit*** binary input information into ***2^N output lines***.



- At a time, only one input line is activated for simplicity.
- The produced 2^N-bit output code is equivalent to the input binary information.
- Process is called ***Decoding***.
- Opposite of ***Decoding*** is ***Encoding***.

1. 2-to-4 line decoder:

- Two inputs- A_0 and A_1
- Four outputs- Y_0, Y_1, Y_2 , and Y_3
- Single enable line- E

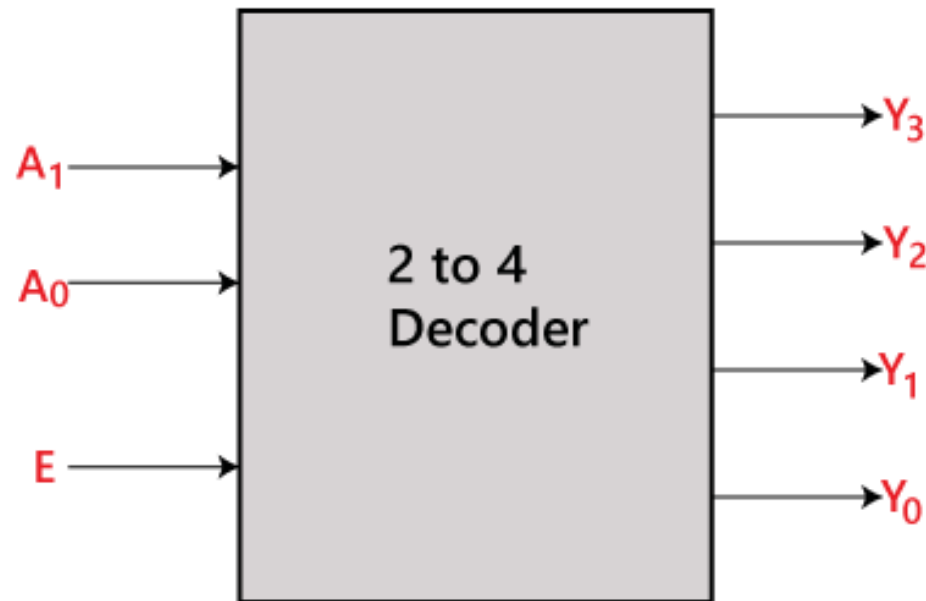


Fig: Block diagram of 2 to 4 Decoder

| Enable | INPUTS | | OUTPUTS | | | |
|--------|--------|-------|---------|-------|-------|-------|
| E | A_1 | A_0 | Y_3 | Y_2 | Y_1 | Y_0 |
| 0 | X | X | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |

Table: Truth Table for 2 to 4 Decoder

The logical expression of the term Y0, Y0, Y2, and Y3 is as follows:

$$Y_3 = E \cdot A_1 \cdot A_0$$

$$Y_2 = E \cdot A_1 \cdot A_0'$$

$$Y_1 = E \cdot A_1' \cdot A_0$$

$$Y_0 = E \cdot A_1' \cdot A_0'$$

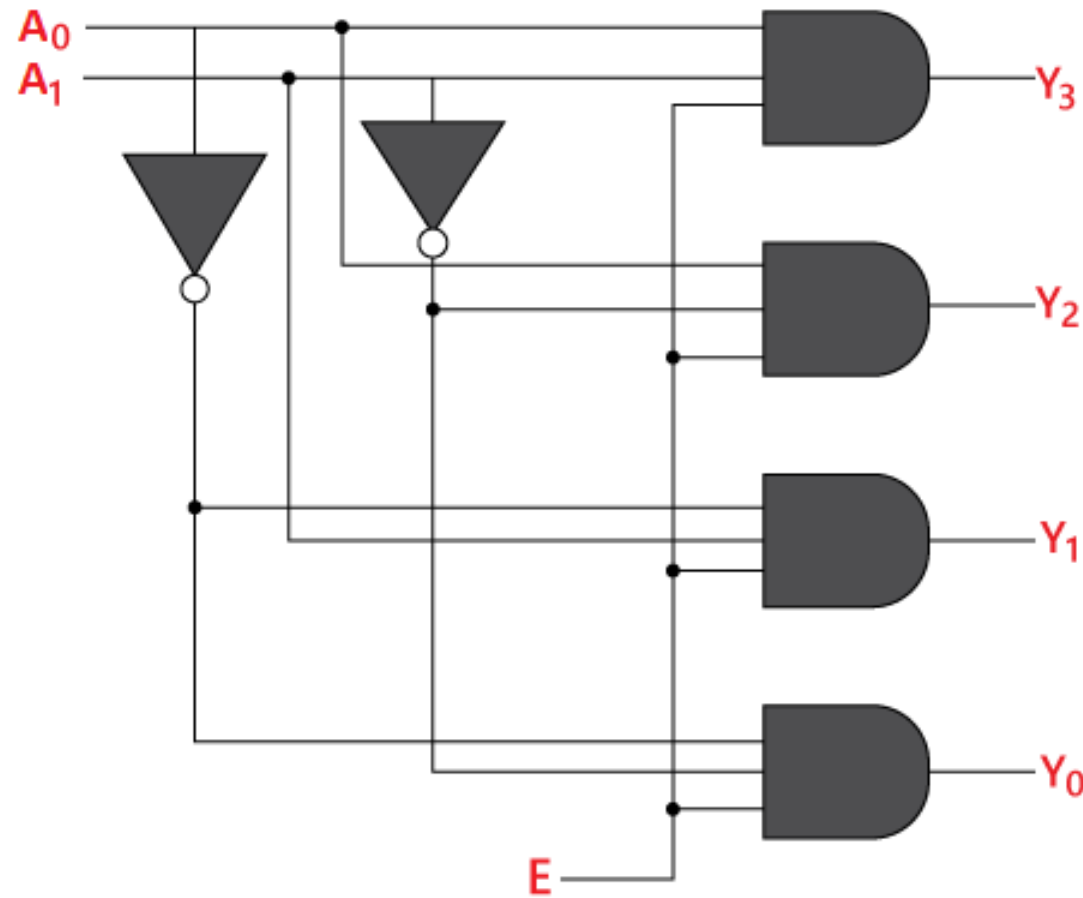


Fig: Logical circuit diagram of 2 to 4 decoder

2. 3-to-8 line Decoder

- Three inputs- A_0 , A_1 and A_2
- Eight outputs- $Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7$
- Single enable line- E

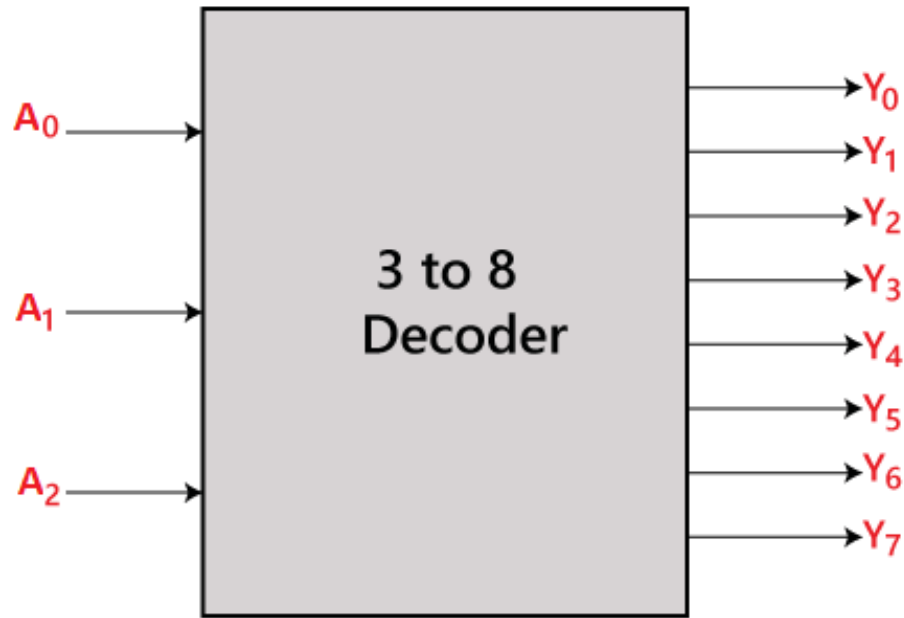


Fig: Block diagram of 3 to 8 Decoder

| Enable | INPUTS | | | Outputs | | | | | | | |
|--------|--------|-------|-------|---------|-------|-------|-------|-------|-------|-------|-------|
| E | A_2 | A_1 | A_0 | Y_7 | Y_6 | Y_5 | Y_4 | Y_3 | Y_2 | Y_1 | Y_0 |
| 0 | x | x | x | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table: Truth Table for 3 to 8 Decoder

The logical expression of the term Y_0 , Y_1 , Y_2 , Y_3 , Y_4 , Y_5 , Y_6 , and Y_7 is as follows:

$$Y_0 = A_0' \cdot A_1' \cdot A_2'$$

$$Y_1 = A_0 \cdot A_1' \cdot A_2'$$

$$Y_2 = A_0' \cdot A_1 \cdot A_2'$$

$$Y_3 = A_0 \cdot A_1 \cdot A_2'$$

$$Y_4 = A_0' \cdot A_1' \cdot A_2$$

$$Y_5 = A_0 \cdot A_1' \cdot A_2$$

$$Y_6 = A_0' \cdot A_1 \cdot A_2$$

$$Y_7 = A_0 \cdot A_1 \cdot A_2$$

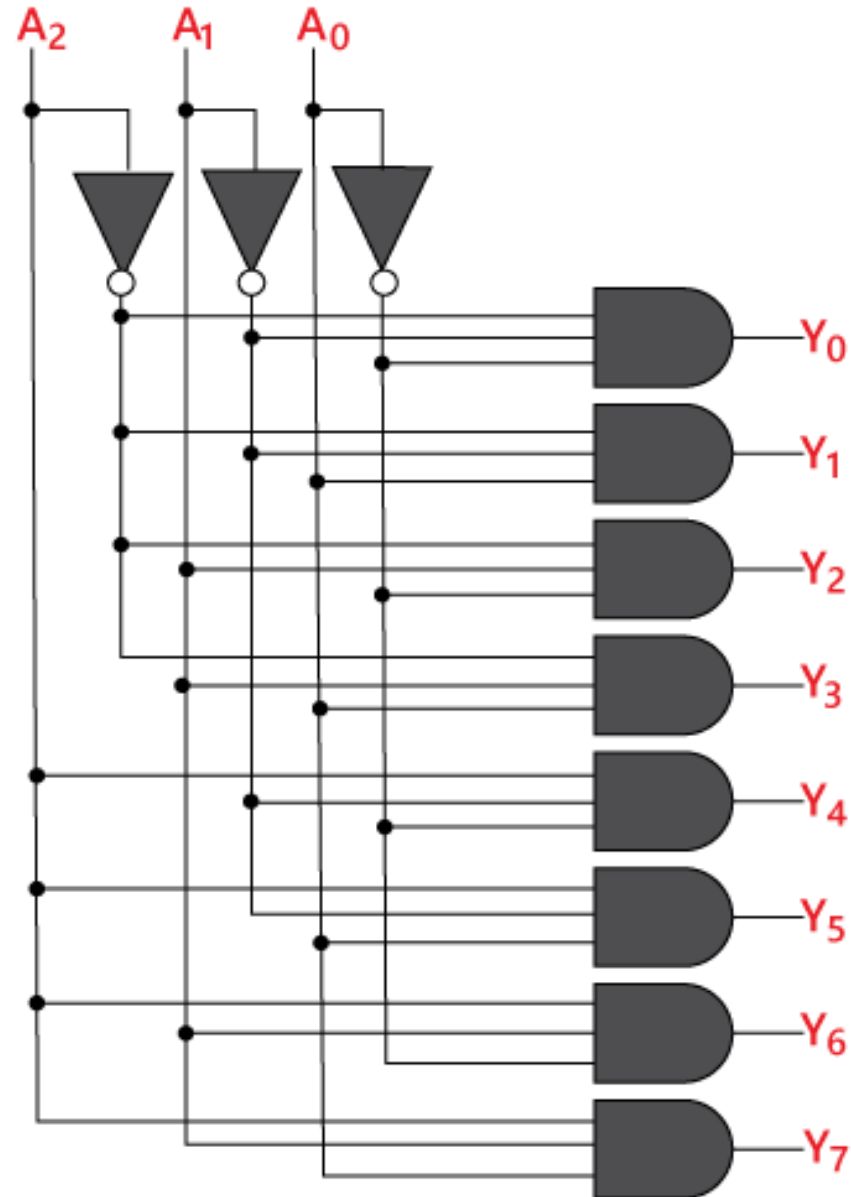


Fig: Logical circuit diagram of 3 to 8 decoder

3. 4 to 16 line Decoder

- Four inputs- A_0, A_1, A_2 and A_3
- Sixteen outputs- $Y_0, Y_1, Y_2, \dots, Y_{15}$
- Can be constructed using either 2 to 4 decoder or 3 to 8 decoder.
- The required number of lower-order decoders is given by,

$$\text{Required number of lower order decoders} = m_2 / m_1$$

For, $m_1 = 8, m_2 = 16$

Required number of 3 to 8 decoders = $16/8=2$

- **Operation:**

- a. When $A_3=0$; the lower decoder will be activated (enabled) and upper decoder will be deactivated. At that time the lower decoder will generate the Minterms from 0000 to 0111 i.e. m_0 to m_7 .
- b. When $A_3=1$; the lower decoder will be deactivated and upper decoder will be activated. At that time the upper decoder will generate the Minterms from 1000 to 1111 i.e. m_8 to m_{15} .

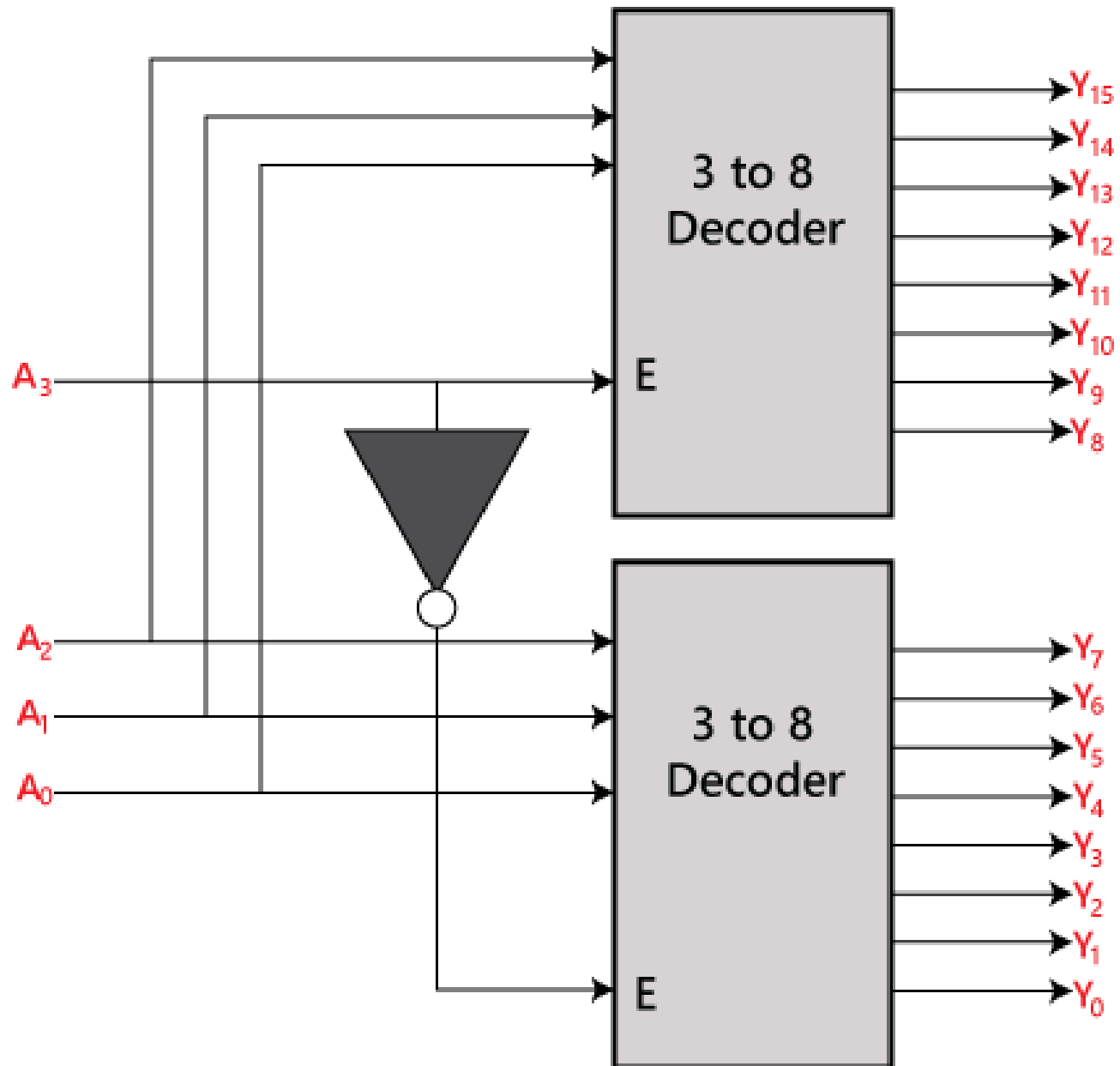
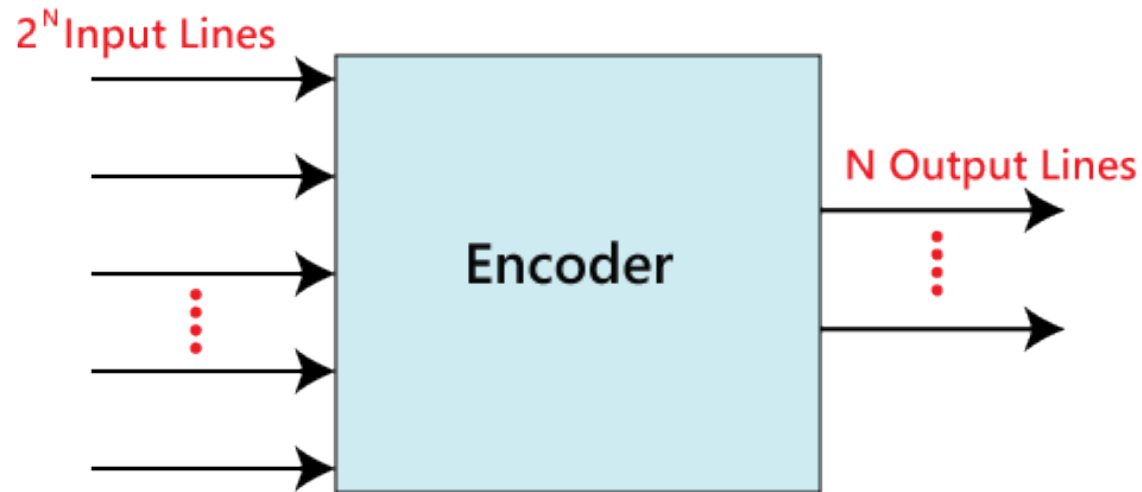


Fig: Logical circuit diagram of 4 to 16 decoder using 3 to 8 decoder

[illegible]

Encoder

- Reverse operation of decoder.
- Has 2^n *input* lines and n *output* lines.
- The output is correspondence to the input line.
- Encodes the information from 2^n input into an n -bit code.



1. 4 to 2 line Encoder:

- Four inputs, *i.e.*, Y_0 , Y_1 , Y_2 , and Y_3 , and two outputs, *i.e.*, A_0 and A_1 .
- One input-line is set to true at a time to get the respective binary code in the output side.

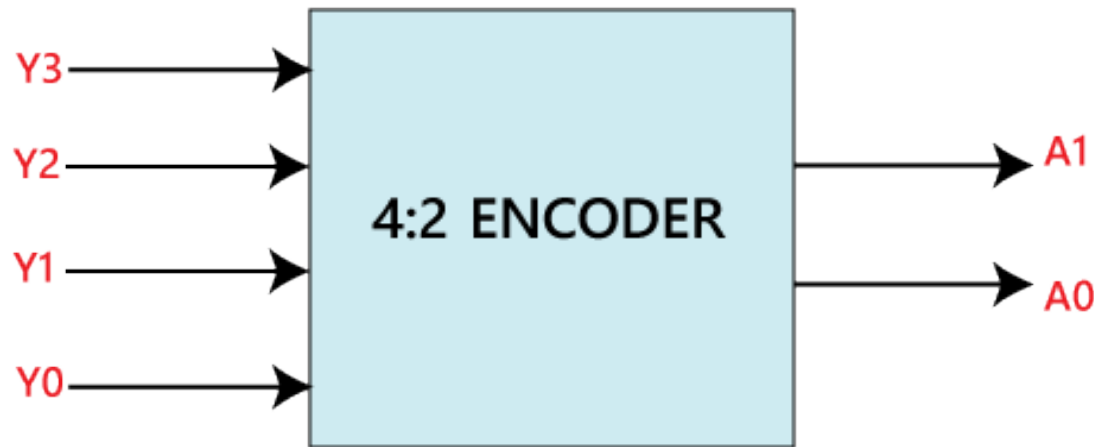


Fig: Block diagram of 4 to 2 Encoder

| Inputs | | | | Outputs | |
|--------|----|----|----|---------|----|
| Y3 | Y2 | Y1 | Y0 | A1 | A0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |

Table: Truth Table foe 4 to 2 Encoder

The logical expression of the term A0 and A1 is as follows:

$$A_1 = Y_3 + Y_2$$

$$A_0 = Y_3 + Y_1$$

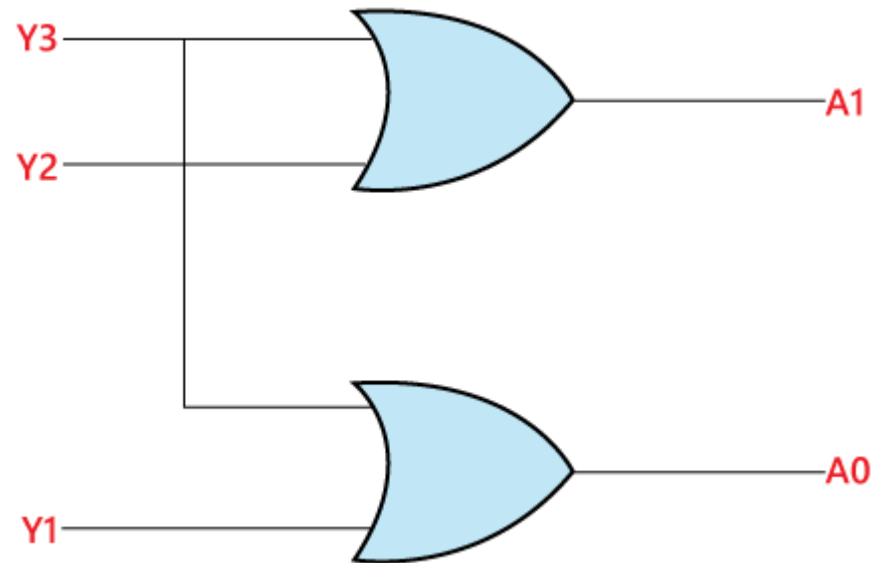


Fig: Logical circuit diagram of 4 to 2 Encoder

Limitation: only one input can be active at a time.

Solution: use parity encoder.

2. Priority Encoder:

- A priority encoder is such that if two or more inputs are active at same time, the input having highest priority will take precedence.

4 to 2 line Priority Encoder:

- The x's designates don't care condition.
- The Y_3 has high and Y_0 has low priority.
- The logical expression of the term A_0 and A_1 can be found using **K-map** as:

$$A_1 = Y_3 + Y_2$$

$$A_0 = Y_3 + Y_2' \cdot Y_1$$

| Inputs | | | | Outputs | |
|--------|----|----|----|---------|----|
| Y3 | Y2 | Y1 | Y0 | A1 | A0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | x | 0 | 1 |
| 0 | 1 | x | x | 1 | 0 |
| 1 | x | x | x | 1 | 1 |

Table: Truth Table for 4 to 2 Priority Encoder

| | | | | | |
|-----------|--|-----------|----|----|----|
| | | $Y_1 Y_0$ | | | |
| $Y_3 Y_2$ | | 00 | 01 | 11 | 10 |
| 00 | | X | 0 | 1 | 1 |
| 01 | | 0 | 0 | 0 | 0 |
| 11 | | x | x | x | x |
| 10 | | 1 | 1 | 1 | 1 |

$$A0 = Y3 + Y2'.Y1$$

| | | | | | |
|-----------|--|-----------|----|----|----|
| | | $Y_1 Y_0$ | | | |
| $Y_3 Y_2$ | | 00 | 01 | 11 | 10 |
| 00 | | X | 0 | 0 | 0 |
| 01 | | 1 | 1 | 1 | 1 |
| 11 | | 1 | 1 | 1 | 1 |
| 10 | | 1 | 1 | 1 | 1 |

$$A1 = Y2 + Y3$$

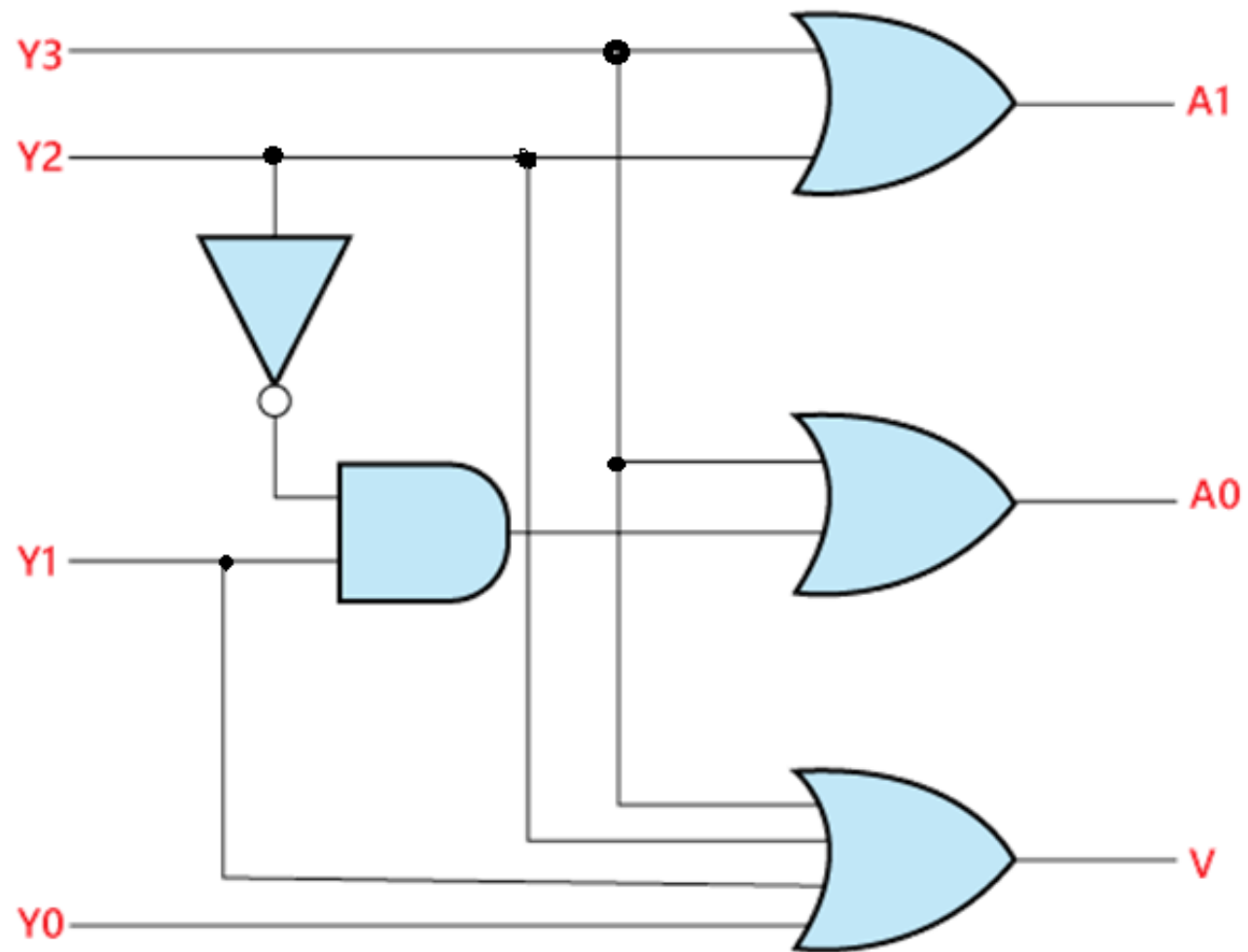


Fig: Logical circuit diagram of 4 to 2 Priority Encoder

3. 8 to 3 line Encoder:

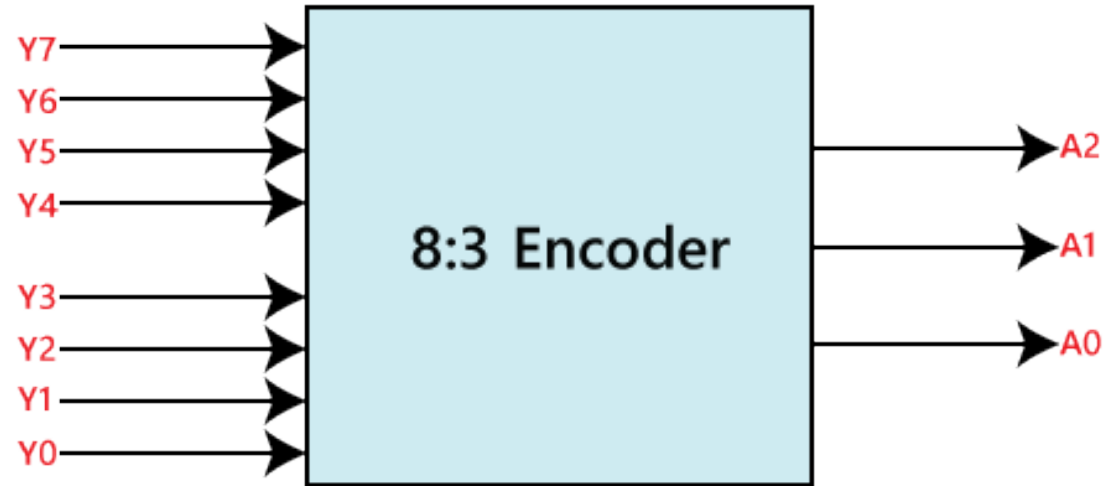


Fig: Block diagram of 8 to 3 Encoder

The logical expression of the term A0, A1, and A2 are as follows:

$$A_2 = Y_4 + Y_5 + Y_6 + Y_7$$

$$A_1 = Y_2 + Y_3 + Y_6 + Y_7$$

$$A_0 = Y_7 + Y_5 + Y_3 + Y_1$$

| INPUTS | | | | | | | | OUTPUTS | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Y ₇ | Y ₆ | Y ₅ | Y ₄ | Y ₃ | Y ₂ | Y ₁ | Y ₀ | A ₂ | A ₁ | A ₀ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

Table: Truth Table for 8 to 3 Encoder

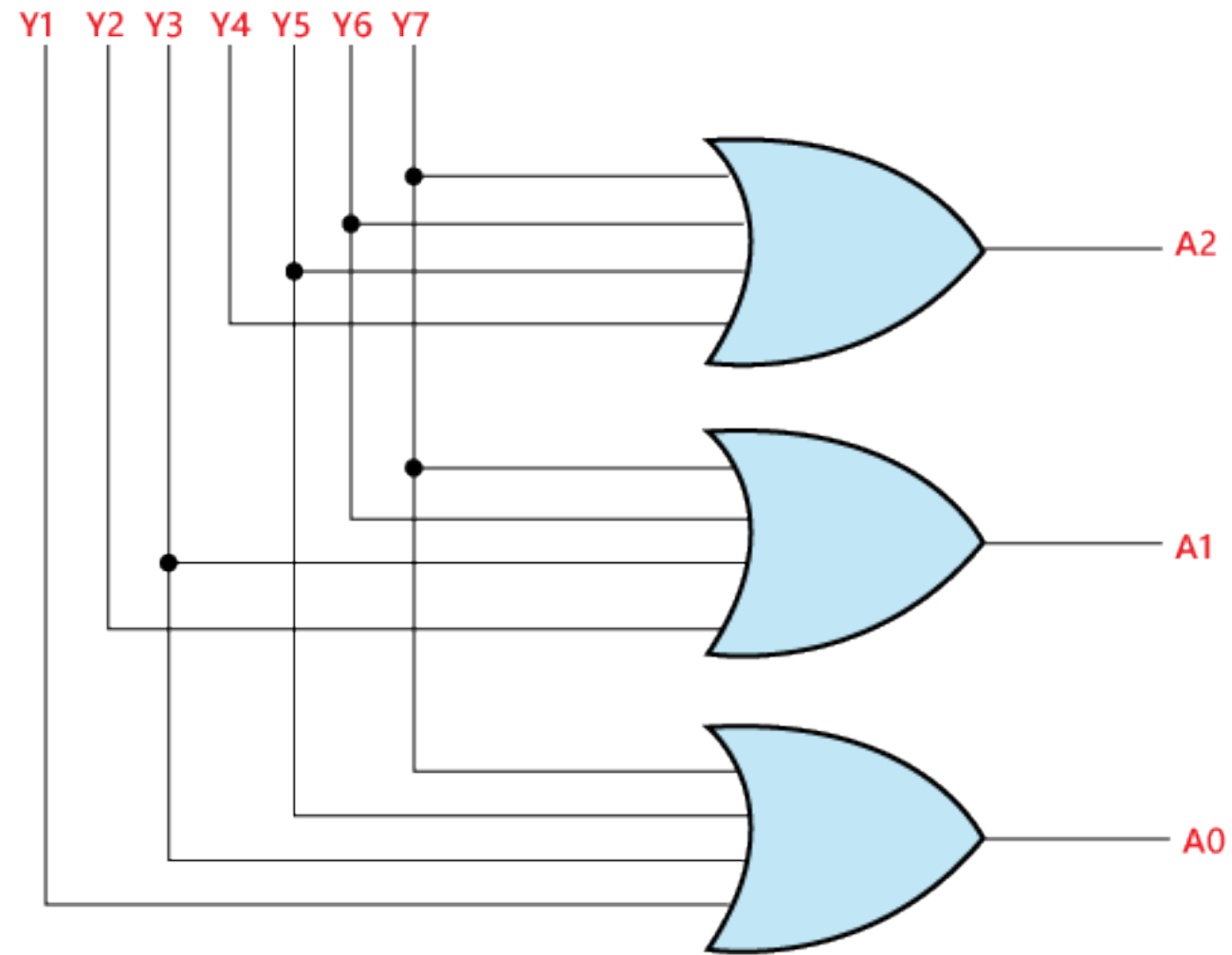


Fig: Logical circuit diagram of 8 to 3 Encoder

4. Decimal to BCD Encoder

- Also known as **10 to 4 line encoder**.
- Total of ten inputs, i.e., Y_0 , Y_1 , Y_2 , Y_3 , Y_4 , Y_5 , Y_6 , Y_7 , Y_8 , and Y_9 and four outputs, i.e., A_0 , A_1 , A_2 , and A_3 .
- In 10-input lines, one input-line is set to true at a time to get the respective **BCD code** in the output side.

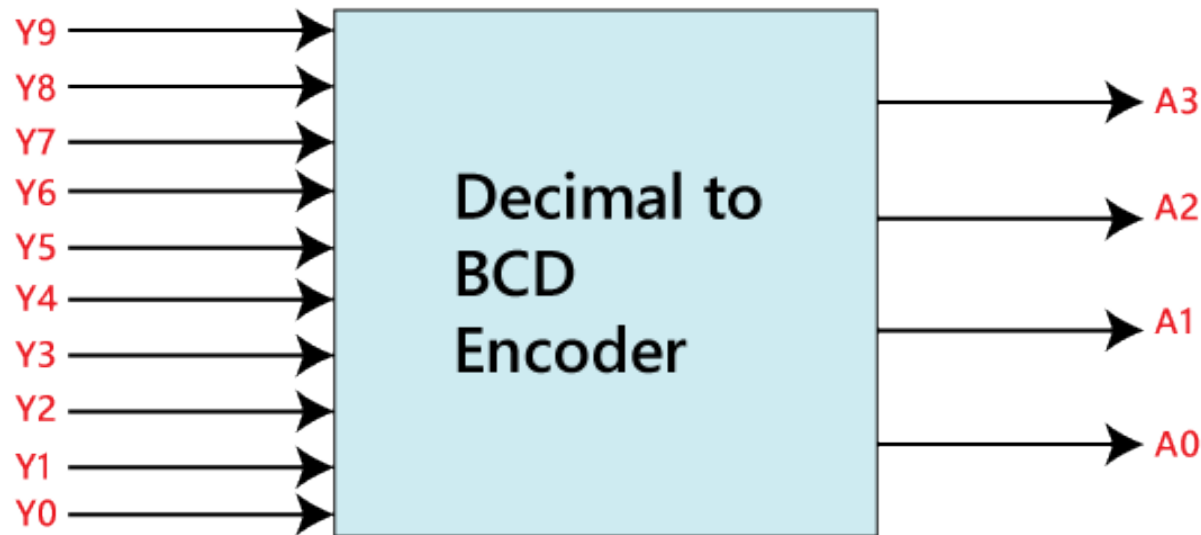


Fig: Block diagram of Decimal to BCD Encoder

The logical expression of the term A_0 , A_1 , A_2 , and A_3 is as follows:

$$A_3 = Y_9 + Y_8$$

$$A_2 = Y_7 + Y_6 + Y_5 + Y_4$$

$$A_1 = Y_7 + Y_6 + Y_3 + Y_2$$

$$A_0 = Y_9 + Y_7 + Y_5 + Y_3 + Y_1$$

| INPUTS | | | | | | | | | | OUTPUTS | | | |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|-------|-------|-------|
| Y_9 | Y_8 | Y_7 | Y_6 | Y_5 | Y_4 | Y_3 | Y_2 | Y_1 | Y_0 | A_3 | A_2 | A_1 | A_0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |

Table: Truth Table for Decimal to BCD Encoder

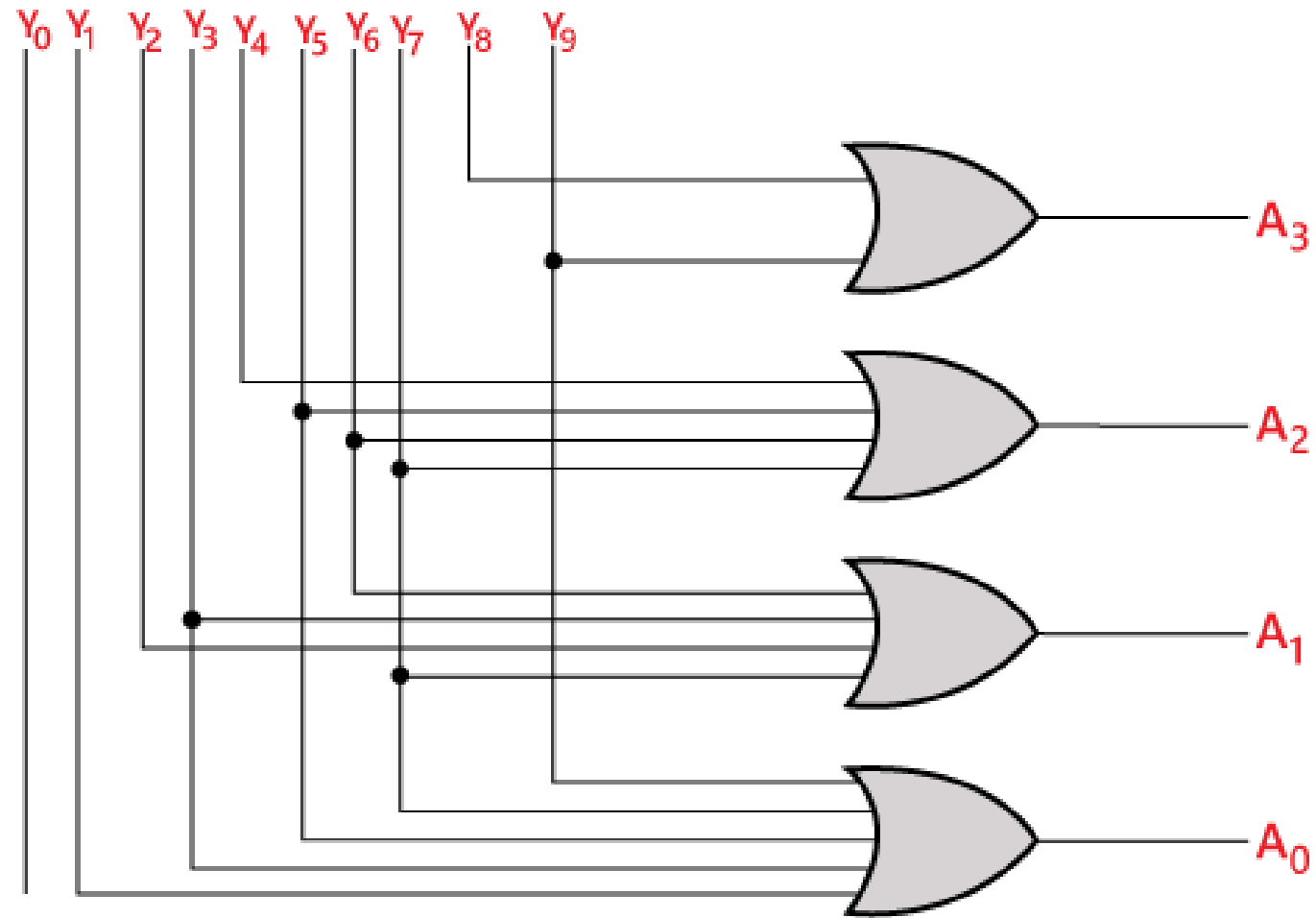


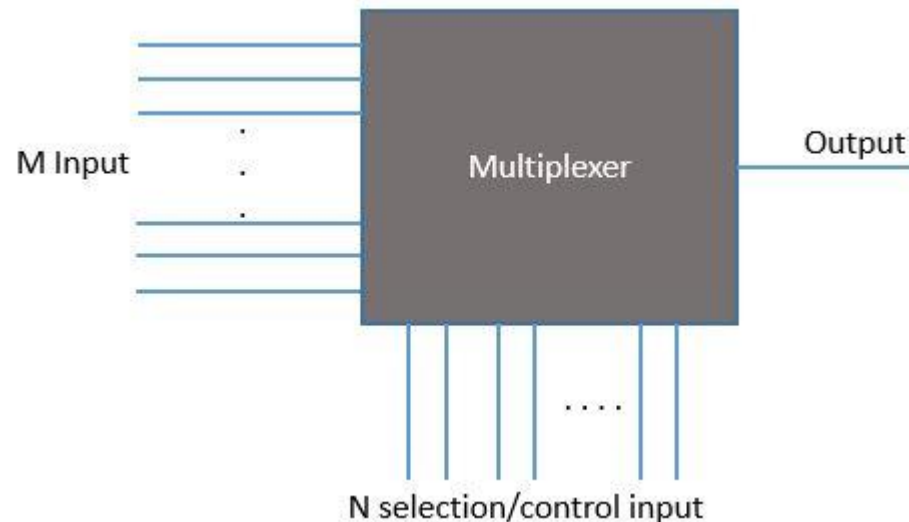
Fig: Logical circuit diagram of Decimal to BCD Encoder

Multiplexers and De-multiplexers

- A combinational circuit that has 2^n *input lines* and a *single output line*.
- Simply, multi-input and single-output combinational circuit.
- On the basis of the values of the **selection lines**, one of these **data inputs will be connected to the output**.
- n *selection lines* and 2^n *input lines*.
- A multiplexer is also treated as **mux**.
- ***De-multiplexer*** is a circuit that performs function exactly *reverse of multiplexer*.
- Generally, both used together, because *communication systems are bi-directional*.

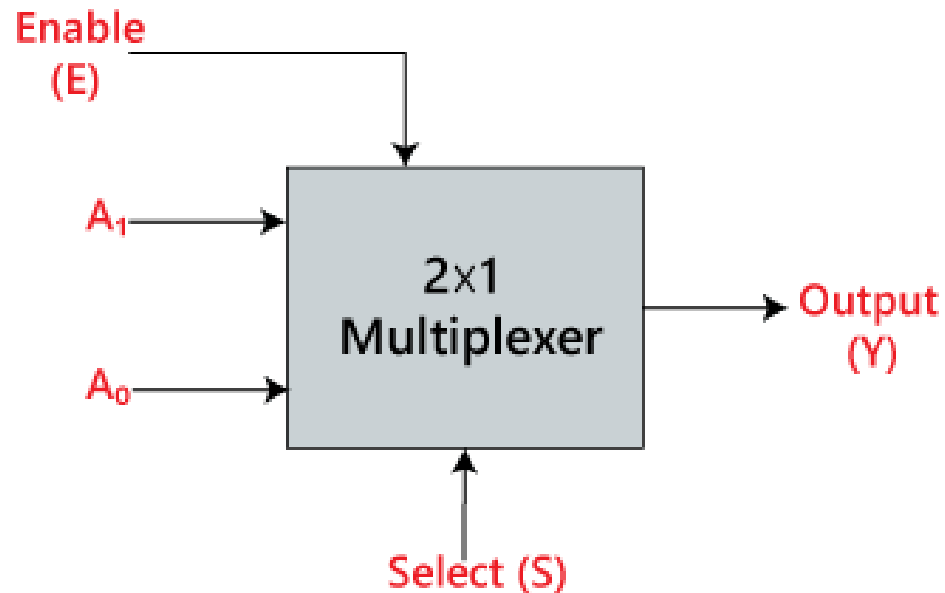
1. Multiplexer/Data selector

- Combinational circuit that select binary information from one of many line and direct it to a single output line.
- Example: *multi-position switch*
- In Digital application it is called *digital multiplexer*.
- Number of selection lines (S)= $\log_2 M$, where $M = \text{no of inputs}$.



2×1 Multiplexer:

- only two inputs, i.e., A_0 and A_1 , 1 selection line, i.e., S_0 and single outputs, i.e., Y .
- On the basis of selection S_0 the input are selected one at a time.



| Inputs | | Selector | Output |
|--------|-------|----------|--------|
| A_1 | A_0 | S_0 | Y |
| - | A_0 | 0 | A_0 |
| A_1 | - | 1 | A_1 |

Table: Truth Table

Fig: Block diagram of 2 x 1 MUX

The logical expression of the term Y is as follows:

$$Y = S_0' \cdot A_0 + S_0 \cdot A_1$$

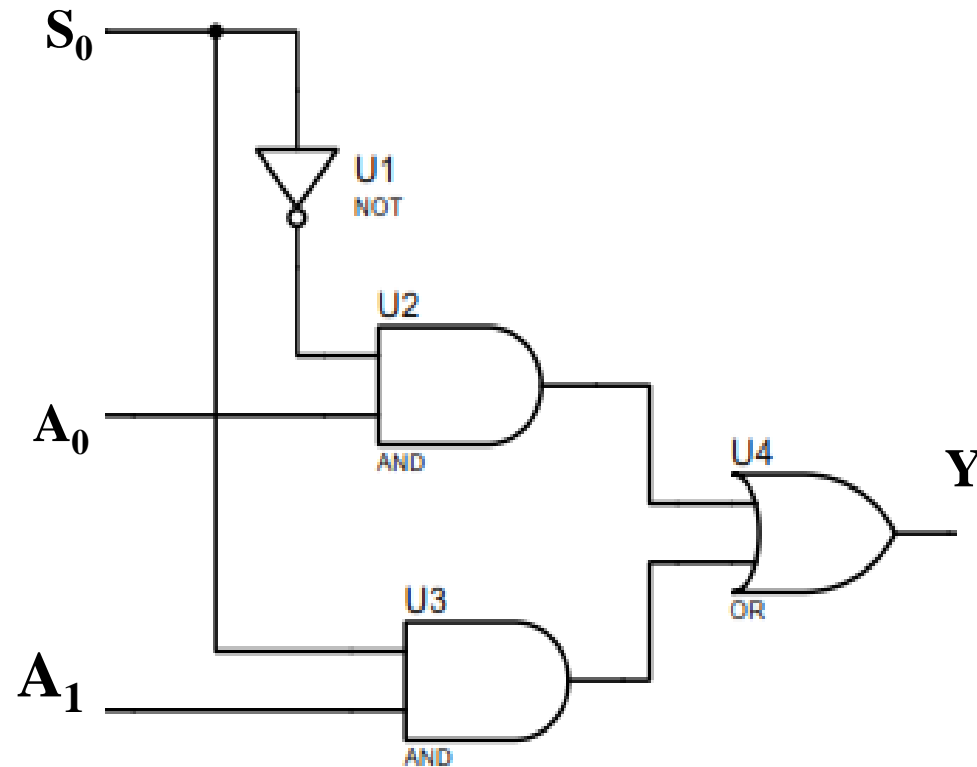
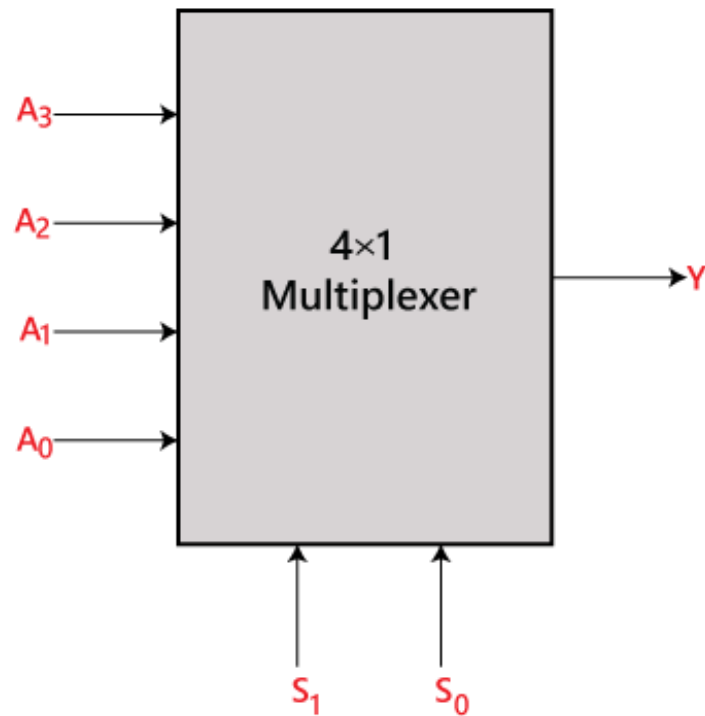


Fig: Logical circuit diagram of 2x1 MUX

4×1 Multiplexer:

- Four inputs, i.E., A_0 , A_1 , A_2 , and A_3 .
- Selection lines, i.E., S_0 and S_1 .
- Single output, i.E., Y .



| Inputs | | | | Selector | | Output |
|--------|-------|-------|-------|----------|-------|--------|
| A_3 | A_2 | A_1 | A_0 | S_1 | S_0 | Y |
| - | - | - | A_0 | 0 | 0 | A_0 |
| - | - | A_1 | - | 0 | 1 | A_1 |
| - | A_2 | - | - | 1 | 0 | A_2 |
| A_3 | - | - | - | 1 | 1 | A_3 |

Table: Truth Table

Fig: Block diagram of 4 x 1 MUX

The logical expression of the term Y is as follows:

$$Y = S_1' S_0' A_0 + S_1' S_0 A_1 + S_1 S_0' A_2 + S_1 S_0 A_3$$

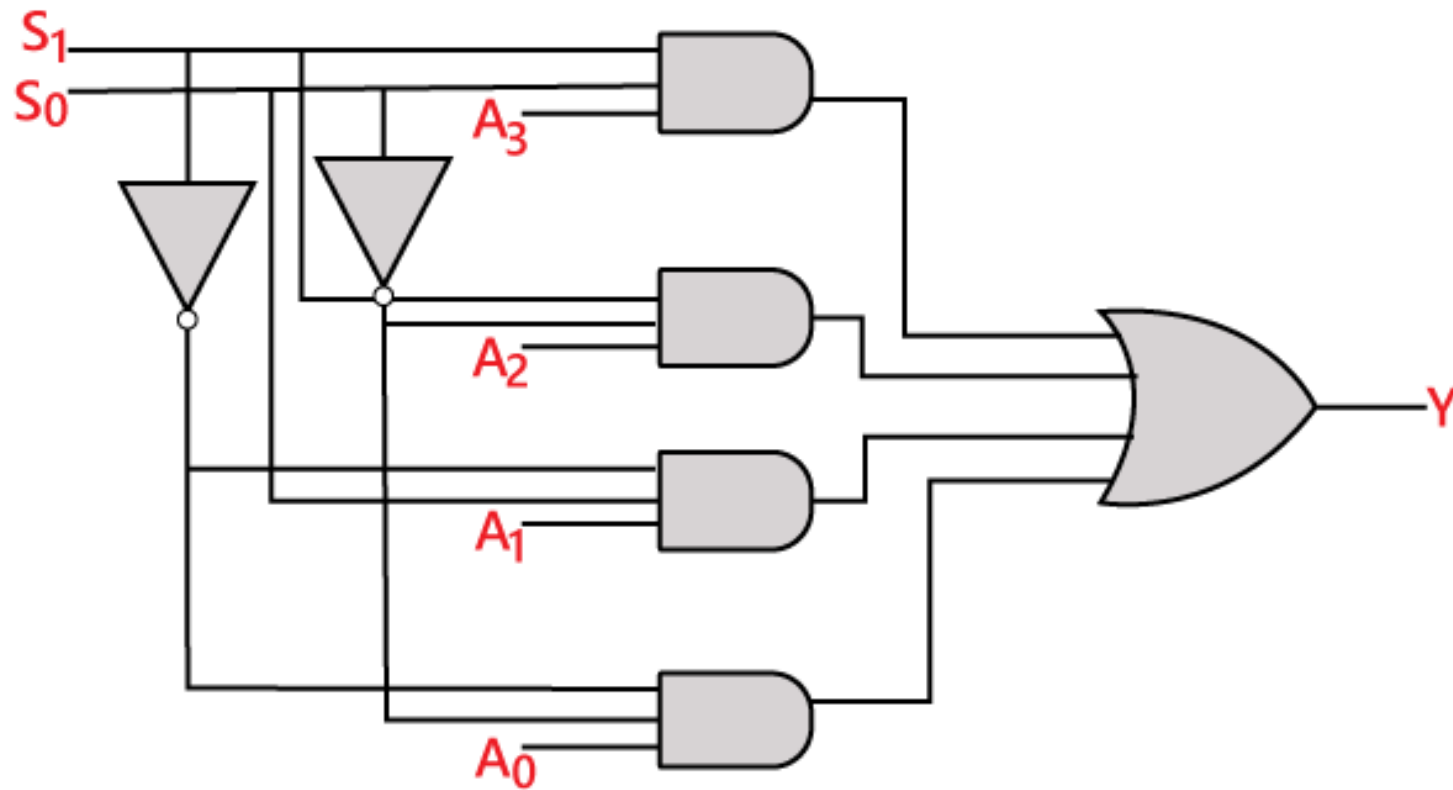


Fig: Logical circuit diagram of 4x1 MUX

8 to 1 Multiplexer

- Eight inputs, i.e., $A_0, A_1, A_2, A_3, A_4, A_5, A_6$ and A_7
- 3 selection lines, i.e., S_0, S_1 and S_2
- Single output, i.e., Y .

The logical expression of the term Y is as follows:

$$Y = S_0' \cdot S_1' \cdot S_2' \cdot A_0 + S_0 \cdot S_1' \cdot S_2' \cdot A_1 + S_0' \cdot S_1 \cdot S_2' \cdot A_2 + S_0 \cdot S_1 \cdot S_2' \cdot A_3 + S_0' \cdot S_1' \cdot S_2 \cdot A_4 + S_0 \cdot S_1' \cdot S_2 \cdot A_5 + S_0' \cdot S_1 \cdot S_2 \cdot A_6 + S_0 \cdot S_1 \cdot S_2 \cdot A_7$$

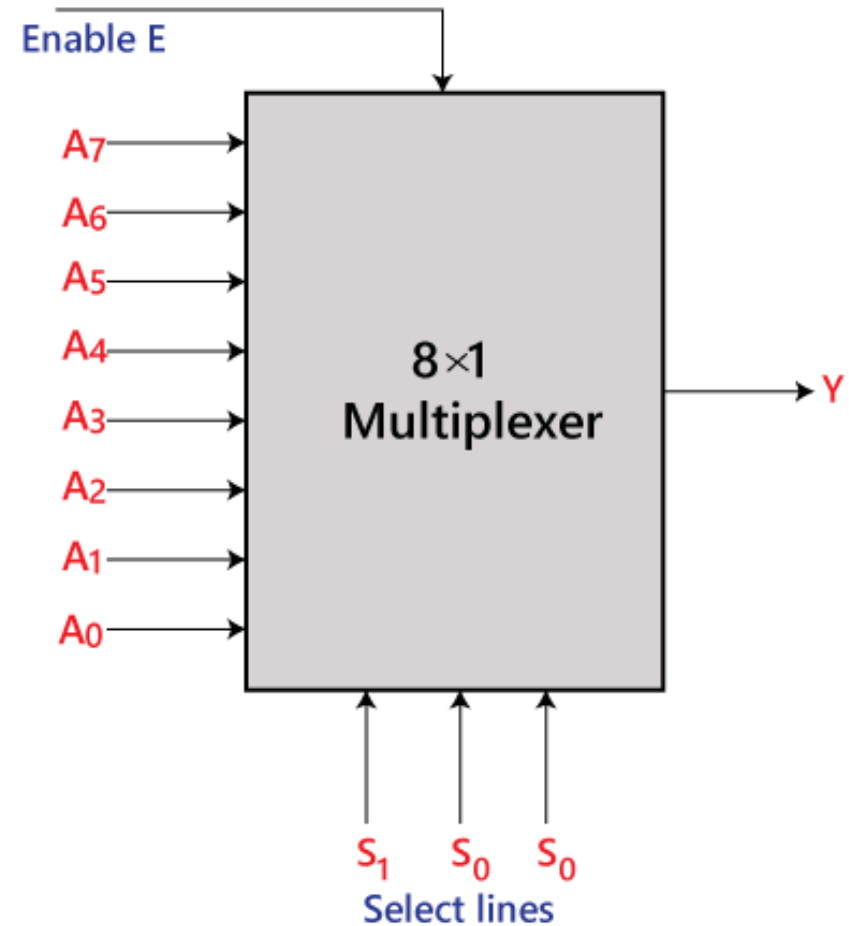


Fig: Block diagram of 8 x 1 MUX

| Inputs | | | | | | | | Selector | | | Output |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| A ₇ | A ₆ | A ₅ | A ₄ | A ₃ | A ₂ | A ₁ | A ₀ | S ₂ | S ₁ | S ₀ | Y |
| - | - | - | - | - | - | - | A ₀ | 0 | 0 | 0 | A ₀ |
| - | - | - | - | - | - | A ₁ | - | 0 | 0 | 1 | A ₁ |
| - | - | - | - | - | A ₂ | - | - | 0 | 1 | 0 | A ₂ |
| - | - | - | - | A ₃ | - | - | - | 0 | 1 | 1 | A ₃ |
| - | - | - | A ₄ | - | - | - | - | 1 | 0 | 0 | A ₄ |
| - | - | A ₅ | - | - | - | - | - | 1 | 0 | 1 | A ₅ |
| - | A ₆ | - | - | - | - | - | - | 1 | 1 | 0 | A ₆ |
| A ₇ | - | - | - | - | - | - | - | 1 | 1 | 1 | A ₇ |

Table: Truth Table For 8x1 MUX

8 ×1 multiplexer using 4×1 multiplexer

- Can be implemented using lower order Multiplexer.
- To implement the 8×1 multiplexer, we need two 4×1 multiplexers.
- The 4×1 multiplexer has 2 selection lines, 4 inputs, and 1 output.
- For getting 8 data inputs, we need two 4×1 multiplexers.
- The two 4×1 multiplexer produces one/one output. So, in order to get the final output, we need a OR gate/2×1 multiplexer.

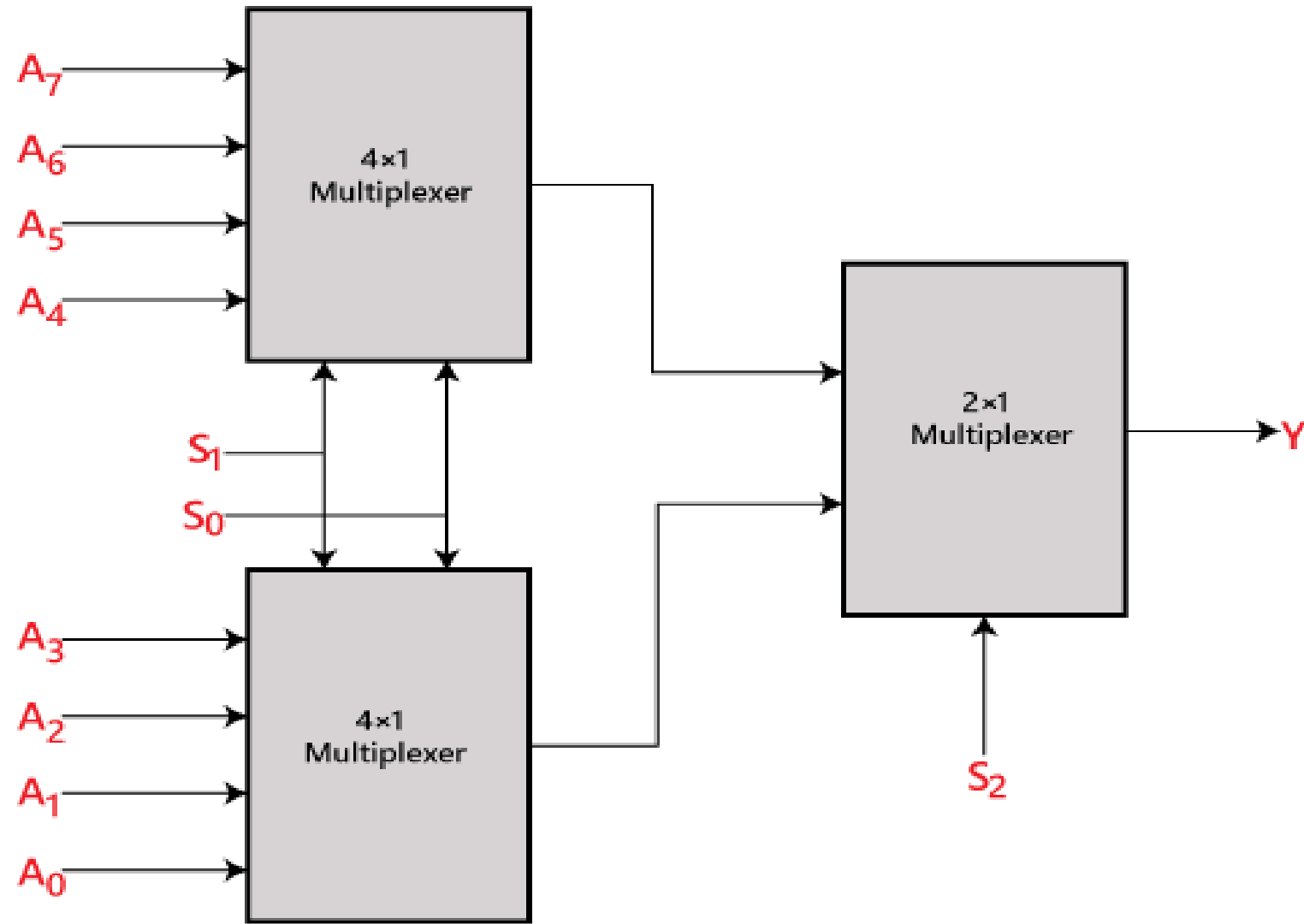


Fig: Block diagram of 8 x 1 MUX using 4x1 MUX

| Inputs | | | | | | | | Selector | | | Output |
|--------|-------|-------|-------|-------|-------|-------|-------|----------|-------|-------|--------|
| A_7 | A_6 | A_5 | A_4 | A_3 | A_2 | A_1 | A_0 | S_2 | S_1 | S_0 | Y |
| - | - | - | - | - | - | - | A_0 | 0 | 0 | 0 | A_0 |
| - | - | - | - | - | - | A_1 | - | 0 | 0 | 1 | A_1 |
| - | - | - | - | - | A_2 | - | - | 0 | 1 | 0 | A_2 |
| - | - | - | - | A_3 | - | - | - | 0 | 1 | 1 | A_3 |
| - | - | - | A_4 | - | - | - | - | 1 | 0 | 0 | A_4 |
| - | - | A_5 | - | - | - | - | - | 1 | 0 | 1 | A_5 |
| - | A_6 | - | - | - | - | - | - | 1 | 1 | 0 | A_6 |
| A_7 | - | - | - | - | - | - | - | 1 | 1 | 1 | A_7 |

Table: Truth Table For 8x1 MUX using 4x1

Case (a): When $S_2=0$ the Lower 4x1 MUX will operate as 2x1 mux selects the lower one and Upper one will be disabled at that time.

Case (b): When $S_2=1$ the Upper 4x1 MUX will operate and Lower one will be disabled.

Applications

- Communication system
- Telephone network
- Computer memory
- Satellite communication
- Television channel selection

Advantages:

- Reduces the no. of wires
- Reduces circuit complexity
- Easy selection of source

De-multiplexers/data distributors

- Single input
- 2^n possible output lines
- Selection is controlled by selection line S .
- Just reverse process of Multiplexer
- *A decoder with enabled input can be function as De-MUX.*

1×2 De-multiplexer

- Only two outputs, i.e., Y_0 and Y_1
- 1 selection lines, i.e., S_0 and
- Single input, i.e., A .

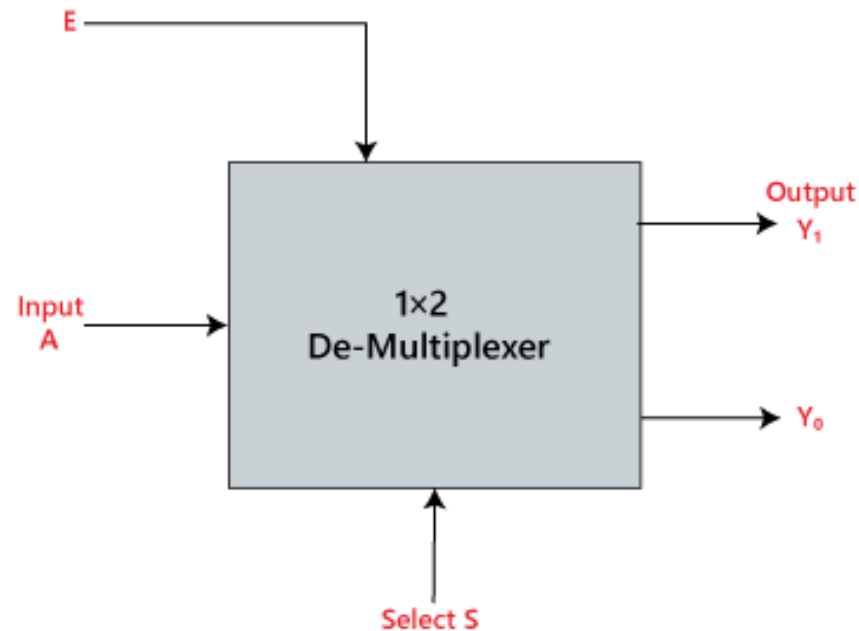


Fig: Block diagram of 1x 2 De-MUX

| INPUTS | | Output | |
|--------|--|--------|-------|
| S_0 | | Y_1 | Y_0 |
| 0 | | 0 | A |
| 1 | | A | 0 |

Table: Truth Table For 1x2 De-MUX

The logical expression of the term Y is as follows:

$$Y_0 = S_0' \cdot A$$

$$Y_1 = S_0 \cdot A$$

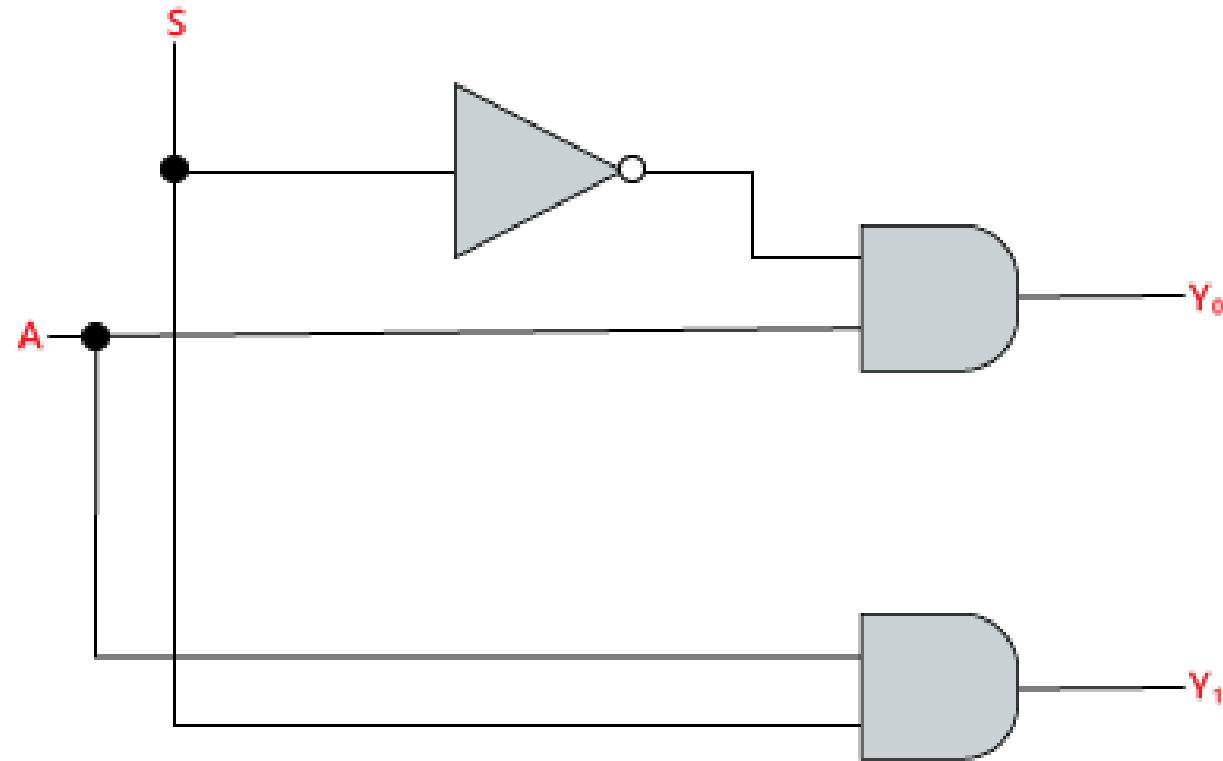
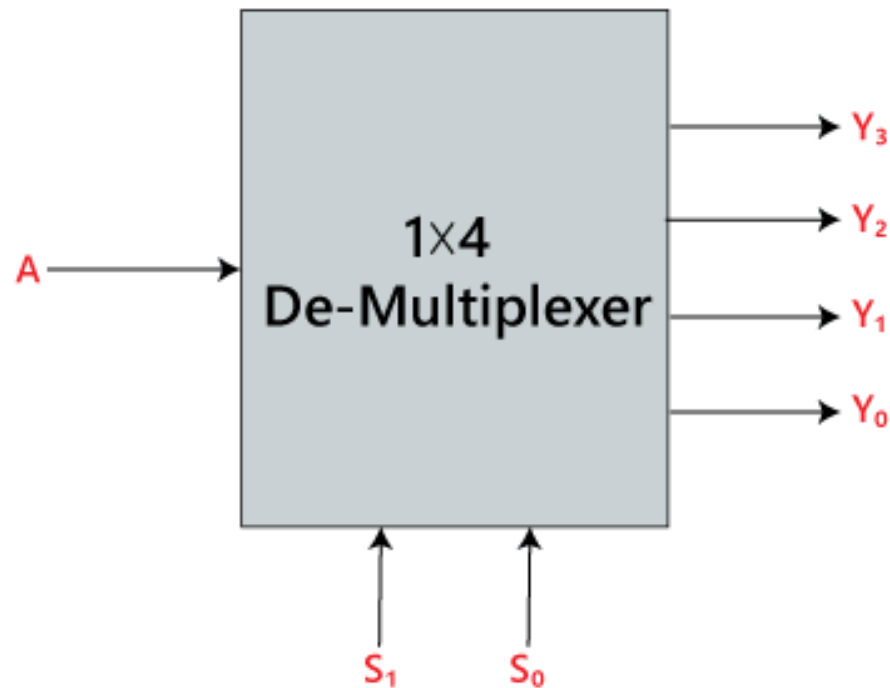


Fig: Circuit diagram of 1x 2 De-MUX

1×4 De-multiplexer:

- Single input, *i.e.*, A
- Four outputs, *i.e.*, Y_0 , Y_1 , Y_2 , and Y_3
- 2 selection lines, *i.e.*, S_0 and S_1



| INPUTS | | Output | | | |
|--------|-------|--------|-------|-------|-------|
| S_1 | S_0 | Y_3 | Y_2 | Y_1 | Y_0 |
| 0 | 0 | 0 | 0 | 0 | A |
| 0 | 1 | 0 | 0 | A | 0 |
| 1 | 0 | 0 | A | 0 | 0 |
| 1 | 1 | A | 0 | 0 | 0 |

Table: Truth Table For 1x4 De-MUX

Fig: Block diagram of 1x 4 De-MUX

The logical expression of the term Y is as follows:

$$Y_0 = S_1' S_0' A$$

$$Y_1 = S_1' S_0 A$$

$$Y_2 = S_1 S_0' A$$

$$Y_3 = S_1 S_0 A$$

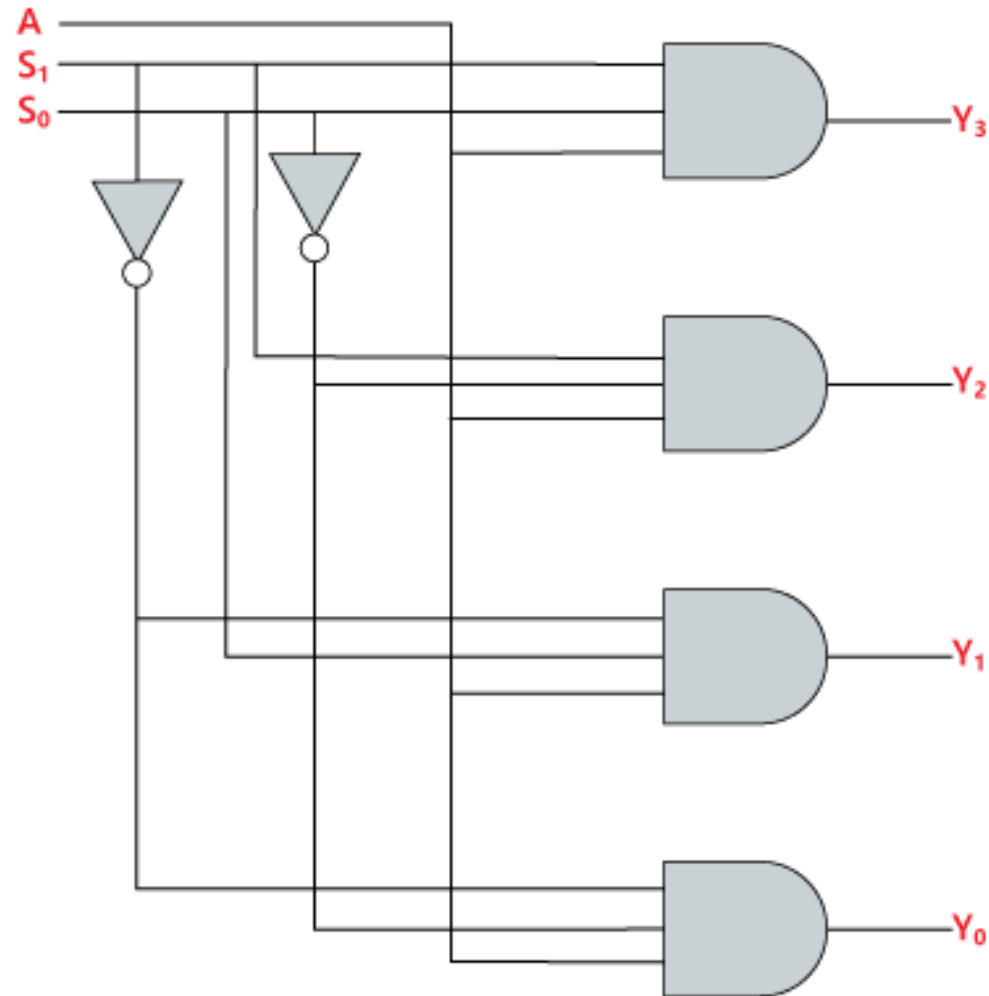


Fig: Circuit diagram of 1x 4 De-MUX

1×8 De-multiplexer

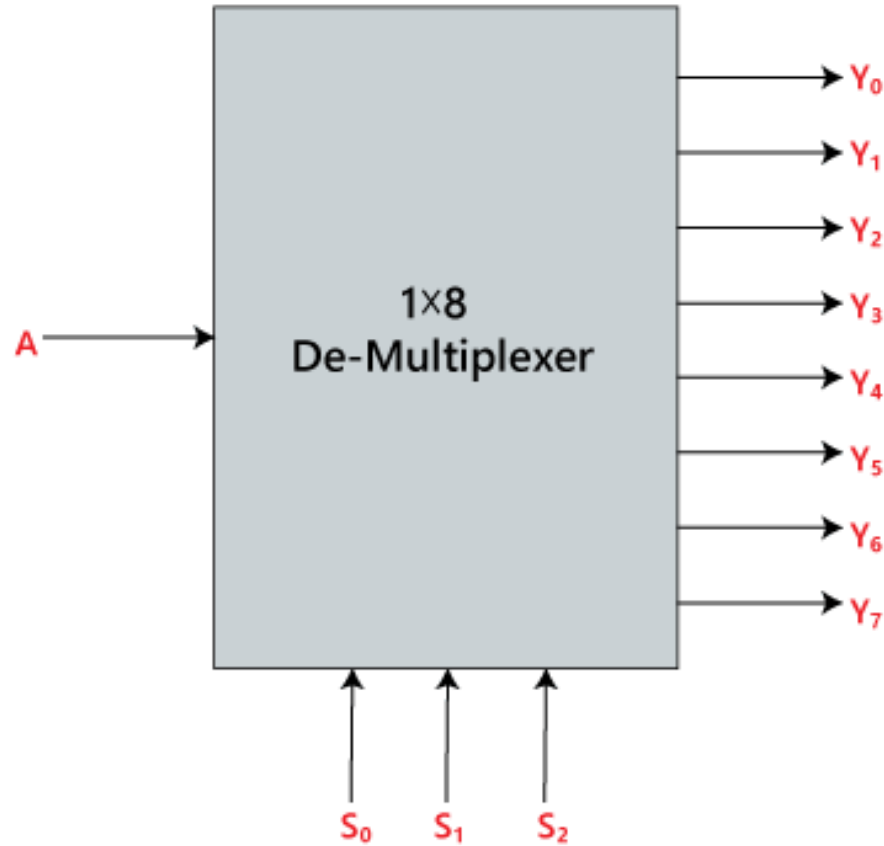


Fig: Block diagram of 1x 8 De-MUX

The logical expression of the term Y is as follows:

$$Y_0 = S_0' \cdot S_1' \cdot S_2' \cdot A$$

$$Y_1 = S_0 \cdot S_1' \cdot S_2' \cdot A$$

$$Y_2 = S_0' \cdot S_1 \cdot S_2' \cdot A$$

$$Y_3 = S_0 \cdot S_1 \cdot S_2' \cdot A$$

$$Y_4 = S_0' \cdot S_1' \cdot S_2 \cdot A$$

$$Y_5 = S_0 \cdot S_1' \cdot S_2 \cdot A$$

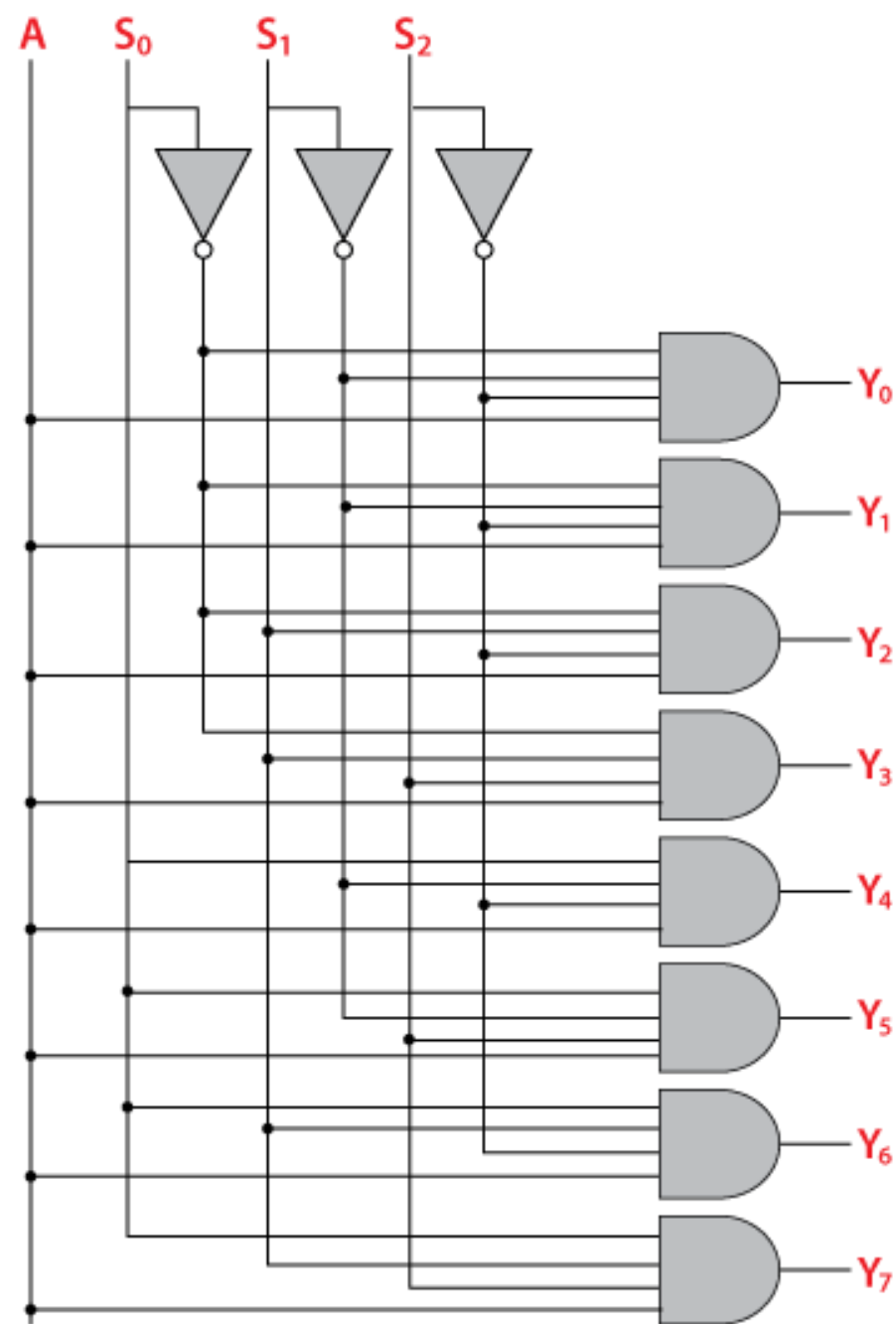
$$Y_6 = S_0' \cdot S_1 \cdot S_2 \cdot A$$

$$Y_7 = S_0 \cdot S_1 \cdot S_2 \cdot A$$

| INPUTS | | | Output | | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| S ₂ | S ₁ | S ₀ | Y ₇ | Y ₆ | Y ₅ | Y ₄ | Y ₃ | Y ₂ | Y ₁ | Y ₀ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | A |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | A | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | A | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | A | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | A | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | A | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | A | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table: Truth Table For 1x8 De-MUX

Fig: Circuit diagram of 1x 8 De-MUX



1×8 De-multiplexer using 1×4 and 1×2 de-multiplexer

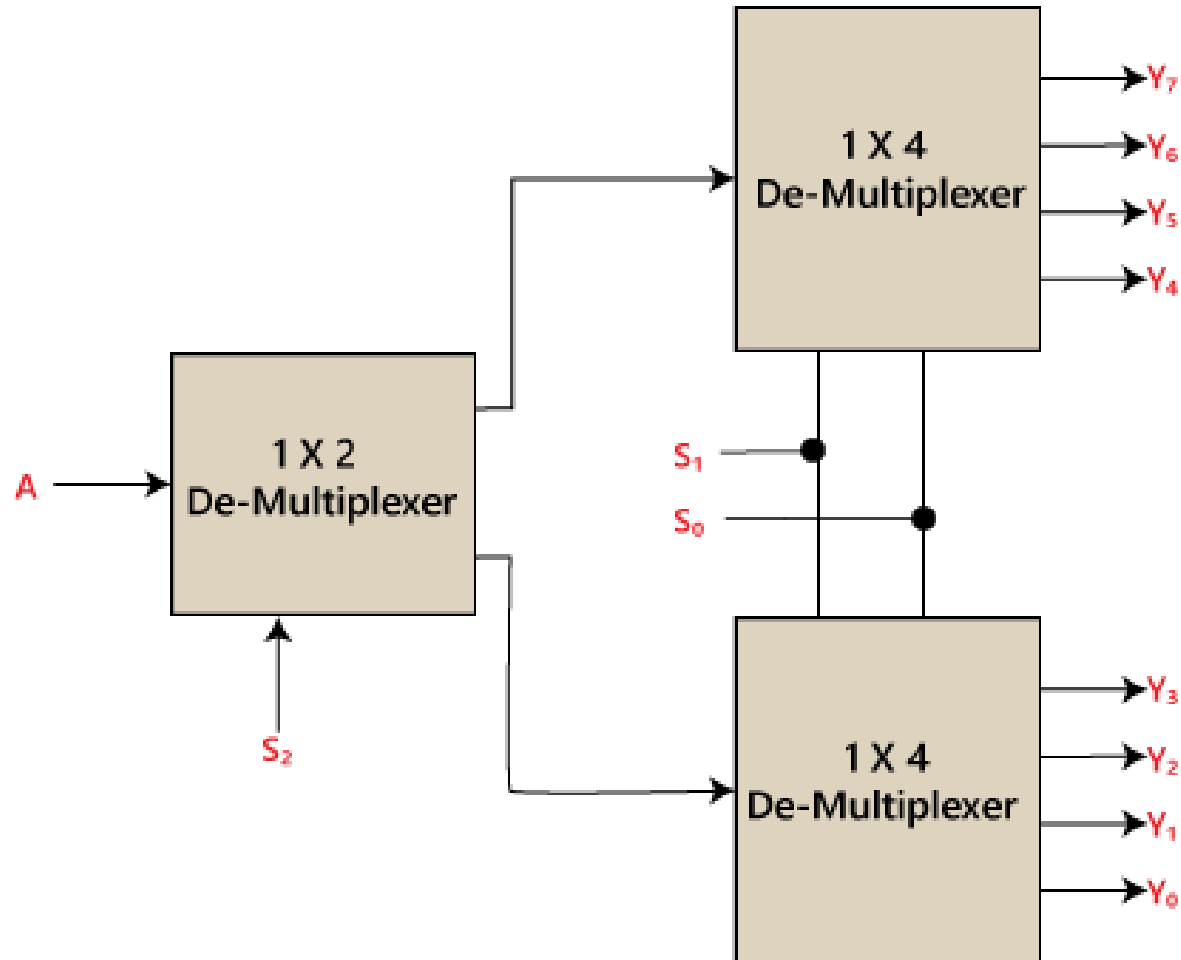


Fig: Block diagram of 1x 8 De-MUX using 1x4 De-MUX