frame

MATRIX Problem

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February 14, 2019

Question

Let k be an integer such that the triangle with vertices $\begin{bmatrix} k \\ -3k \end{bmatrix}$, $\begin{bmatrix} 5 \\ k \end{bmatrix}$, $\begin{bmatrix} -k \\ 2 \end{bmatrix}$, has area 28. Find the orthocentre of this triangle.

Answer

Let the points of the triangle be: A= $\begin{bmatrix} k \\ -3k \end{bmatrix}$ B= $\begin{bmatrix} 5 \\ k \end{bmatrix}$ C= $\begin{bmatrix} -k \\ 2 \end{bmatrix}$

Area of triangle = | **AB x AC** | /2

AB= $\begin{bmatrix} 5-k \\ 4k \end{bmatrix}$, **AC**= $\begin{bmatrix} -2k \\ 2+3k \end{bmatrix}$

Hence | **AB x AC** |= $5k^2 + 13k + 10$

Given, Area of triangle = 28

By solving the equations, we get the following quadratic Equation in 'k':

$5k^2 + 13k - 46 = 0$

Hence k=2 or k=-4.6

K=2

A= $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$, B= $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$, C= $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$

Let P,Q,R be the points of intersection of points with sides BC,CA,AB respectively.

Equation of AP: $\begin{bmatrix} n_{BC}^T \end{bmatrix} \begin{bmatrix} x & -A \end{bmatrix} = 0$

Equation of BQ: $\begin{bmatrix} n_{CA}^T \end{bmatrix} \begin{bmatrix} x & -B \end{bmatrix} = 0$

The orthocentre "H" is obtained from th pt of intersection of AP and BQ lines

The equations of AP and BQ are written as:

$\begin{bmatrix} n_1^T \end{bmatrix}$ x=p1

$\begin{bmatrix} n_2^T \end{bmatrix}$ x=p2

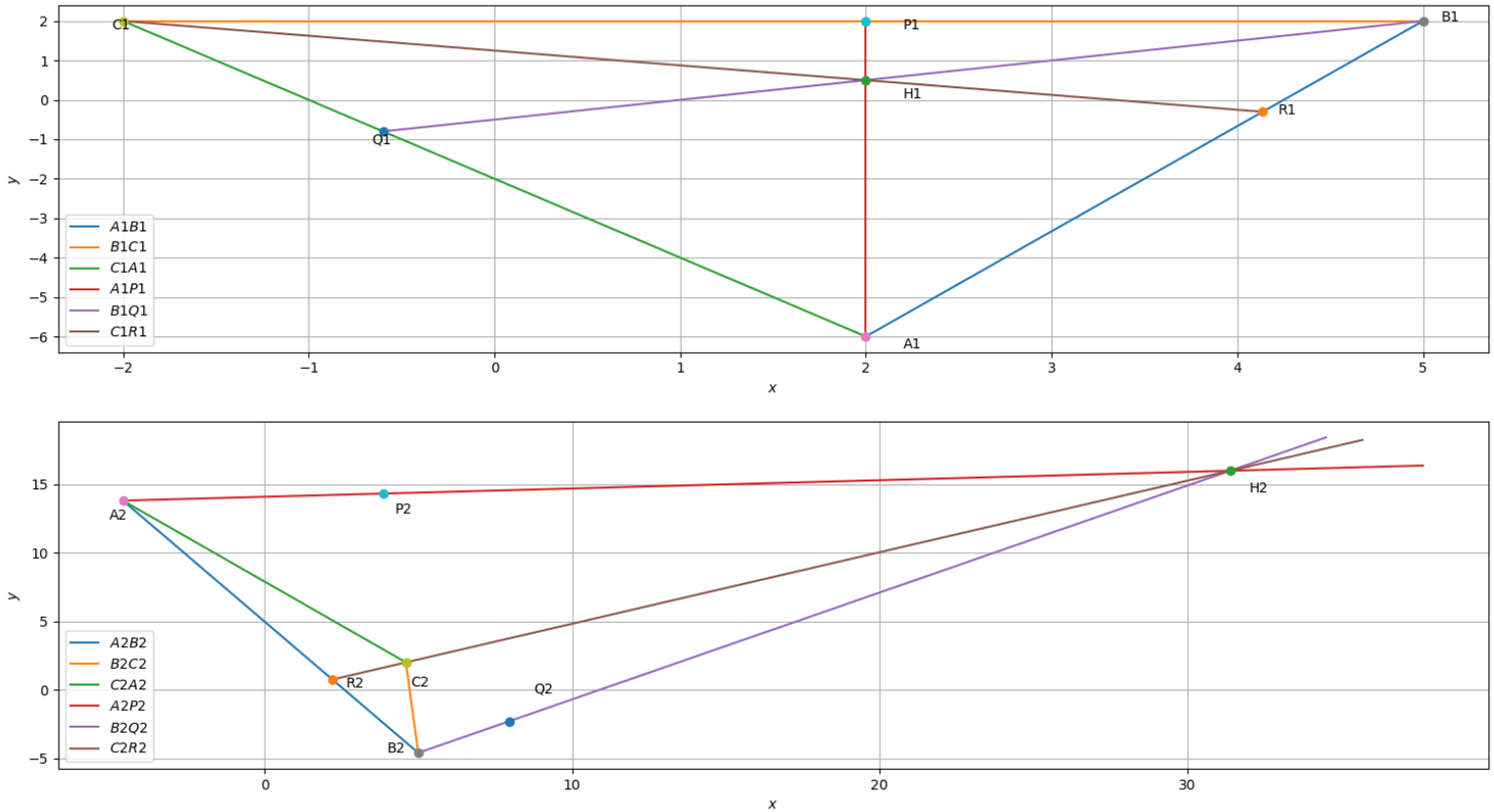
These equations can be written as: $\begin{bmatrix} n_1^T \\ n_2^T \end{bmatrix}$ x=p or $\begin{bmatrix} N^T \end{bmatrix}$ x=p

The point of intersection is : x= $\begin{bmatrix} N^{-T} \end{bmatrix}$ p

Hence Orthocentre: H1= $\begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$

Similarly for k=-4.6 , we get H2= $\begin{bmatrix} 31.39 \\ 15.98 \end{bmatrix}$

Output


C1
P1
B1
H1
R1
Q1
A1
A1B1
B1C1
C1A1
A1P1
B1Q1
C1R1
x
y
A2
P2
H2
R2
C2
Q2
B2
A2B2
B2C2
C2A2
A2P2
B2Q2
C2R2