**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:- We have a normal distribution with mean = 45 and std = 8.

Let X be the amount of time it takes to complete the repair on a customer’s car

Pr(X>50) = 1- Pr(X<= 50)

Z = (raw score - mean)/ std

Z = (X – 45)/ 8

Pr(x<= 50) = Pr(Z<= (50-45)/8) = Pr(Z<= 0.625) = 0.734

Probability that the service manager will not meet his demand will be,

= 1 – 0.736

= 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center

is normally distributed with mean  = 38 and Standard deviation  =6. For each statement

below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and

44.

B. A training program for employees under the age of 30 at the center would be

expected to attract about 36 employees.

Ans:- we have a normal distribution with mean = 38 and std = 6

a) Probability of employee’s greater than age of 44 = Pr(X&gt;44):-

Pr(X>44) = 1 – Pr(X<=44)

Z = (X – 38) / 6 = (44 – 38) / 6 = 0.841345 = 84.13%

Hence, Pr(X>44) = 1 – 0.841345 = 0.1587 = 15.87%

Z score for 38 = (38 – 38) / 6 = 0 = 50%

Hence, People between age 38 to 44 = 84.13 – 50 = 34.13%

Hence, More employees at the processing center are older than 44 than

between 38 to 44. Is False

b) Z score for 30 = (30 – 38) / 6 = -1.33 = 9.15% = 36 out of 400

Hence, A training program for employees under the age 30 at the center

would be expected to attract about 36 employees – True

3. If X 1 ~ N(μ, σ 2 ) and X 2 ~ N(μ, σ 2 ) are iid normal random variables, then what is the difference

between 2 X 1 and X 1 + X 2 ? Discuss both their distributions and parameters.

Ans:- We know that if X1 ~ N(μ1 , σ1^ 2 ) and X2 ~ N(μ2 , σ2^ 2 ) are two independent

random variables then X + Y ~ N(μ1 + μ2 , σ1^ 2 + σ2^ 2 ) and X - Y ~ N(μ1 - μ2 , σ1^ 2 + σ2^ 2 )

Similarly, if Z = aX + bY , where X and Y are as defined above, i.e. Z is a linear

combination of X and Y , then Z ~ N(aμ1 + bμ2 , a^2σ1^ 2 +b^2 σ2^ 2 )

2X1 ~ N(2 u , 4 σ^ 2 )

X1 + X2 ~ N( μ + μ , σ^ 2 + σ^ 2 ) - N(2 μ, 2 σ^ 2 )

2X1 – (X1 +X2) = N(4 u , 6 σ^ 2 )

4. Let X ~ N(100, 20 2 ). Find two values, a and b, symmetric about the mean, such that the

probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9

B. 80.2, 119.8

C. 22, 78

D. 48.5, 151.5

E. 90.1, 109.9

Ans:- The probability of getting value between a and b should be 0.99.

So, the probability outside the a and b area is 0.01 (i.e. 1-0.99).

The probability towards left from a = -0.05 (i.e. 0.01 / 2)

The probability towards right from b = -0.05 (i.e. 0.01 / 2)

Z = (X - μ) / σ

For probability 0.05 the Z value is -2.57 (from Z table)

Z \* σ + μ = X

Z (-0.05) \* 20 + 100 = -(-2.57) \* 20 + 100 = 151.4

Z (+0.05) \* 20 + 100 = -(-2.57) \* 20 + 100 = 48.6

So, option D is correct

5. Consider a company that has two different divisions. The annual profits from the two

divisions are independent and have distributions Profit 1 ~ N(5, 3 2 ) and Profit 2 ~ N(7, 4 2 )

respectively. Both the profits are in $ Million. Answer the following questions about the

total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability

for the annual profit of the company.

B. Specify the 5 th percentile of profit (in Rupees) for the company

C. Which of the two divisions has a larger probability of making a loss in a given year?

Given that:

$1 = Rs. 45



Thus,

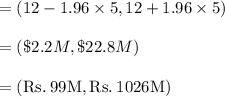
**Company's profit**:



A):

95% of the **probability** **lies**between 1.96 **standard deviations**of the **mean**.

Thus **range**is:



B): **Fifth percentile**is calculated as:



From p **values**of z **score** **table**, we get:



Thus at $3.78M **dollars**, or Rs. 170.1M **amount**, 5th **percentile**of **profit** **lies**.

Or 5th **percentile**of **profit**is Rs. 170.1 Million.

C): Loss is when profit < 0

Thus: p < 0

The first **division** of **company**, thus have **larger probability**of making a loss in a given year.