

Data Structures & Algorithms

PAVAN

Course Outcomes :-

- CO1:- Explain sorting and searching Techniques (K₂)
- CO2:- Demonstrate single linked link, Double linked list (K₃)
- CO3:- Interpret the basic operations on stack's and queues (K₃)
- CO4:- Demonstrate Binary trees and Binary search Trees (K₃)
- CO5:- Compare Binary trees and self balanced trees with appropriate examples (K₄)
- CO6:- Illustrate various graph algorithms (K₃)

Algorithm: It is defined as a step by step procedure to perform a task.

We can write algorithm in any one of the following three ways

- ① Sequence.
- ② Selection / Decision
- ③ Iteration / Repetition

→ Write an algorithm to swap 2 numbers using a third variable.

Step-1:- Start

Step-2:- Declare a,b,t values

Step-3:- Input a,b.

Step-4:- Perform a₂t

$$a = b$$

$$b = t$$

Step-5:- print a,b

Step-6:- Stop.

→ Selection:-

Here a condition is checked if it is true then

some statements are executed otherwise some other statements are executed.

→ Write an algorithm to find largest of 2 numbers

Step-1:- Start

Step-2:- Input a,b.

Step-3:- If ($a > b$)

 print "a is big".

else

 print "b is big".

Step-4:- Stop.

→ Write an algorithm to accept year of birth and check whether the persons birth year is leap year or not.

Step-1:- Start

Step-2:- Declare year

Step-3:- Input y

Step-4:- if ($y \cdot 4 == 0$) & ($y \cdot 100 != 0$) || ($y \cdot 4 == 0$)

 print "Given year is leap year".

 else

 print "Not leap year".

Step-5:- Stop.

Iteration / Repetition :-

If the algorithm want to access more no. of values of a given structure (data structure) then we have to iterate the elements in the structure.

→ write an algorithm to print values from 1 to n

Step-1:- Start

Step-2:- Declare i

Step-3:- Read i=1

Step-4:- Repeat the following steps Until i

Step-5:- print i

Step-6:- calculate i++

Step-7:- Stop.

→ write an algorithm to find sum of digits of n natural numbers.

Step-1:- Start

Step-2:- Declare i, n, sum

Step-3:- Read i, n, sum=0, i=1

Step-4:- Repeat the following steps

 until i <= n

step-5:- sum = sum +
step-6:- calculate i++
step-7:- print sum
step-8:- stop.

Data Structure:- It is defined as the organizing data in a specific manner.

Eg:- Array is an example of data structure
Structures and unions also examples.

Searching:-

Searching can be performed on 2 ways on 1D arrays

(i) linear- If the key element is searched in the given list of array elements then it is called linear search (or) sequential search.

Algorithm:-

Step-1:- Start

Step-2:- Declare a, i, n, Key, found

Step-3:- Read a, i=0, n, key, found=0

Step-4:- Repeat step 5 until i < n

Step-5:- if a[i] matches with key

 print "element found"

 else

 print "element not found".

Step-6:- Stop.

Program:

```
#include <stdio.h>
main()
{
    int a[10], a[i], a[10], i, n, key, found=0;
    printf("enter 'n' size");
    scanf("%d", &n);
    printf("enter array elements");
    for(i=0; i<n; i++)
    {
        . . .
    }
}
```

```

scanf("%d", &a[i]);
}
printf("enter Key value");
scanf("%d", &key);
for(i=0; i<n; i++)
{
    if(a[i] == key)
    {
        printf("Element found");
        found = 1;
        break;
    }
}
if(found == 0)
    printf("element not found");
}

```

Binary searching:-

This searching methodology divides the array in 2 types by calculating the mid value. It performs searching only on sorting array.

```

→ #include<stdio.h>
main()
{
    int i, n, first = 0, last = n - 1, key, a[10], mid;
    printf("enter n value");
    scanf("%d", &n);
    for(i=0; i<n; i++)
    {
        printf("enter elements in a[%d]", i);
        scanf("%d", &a[i]);
    }
    printf("enter key value");
}

```

```

scanf("%d", &key);
while(first <= last)
{
    mid = (first + last) / 2;
    if(a[mid] < key)
        first = mid + 1;
    else if(a[mid] == key)
    {
        printf("element found at %d", mid);
        break;
    }
    if(first > last)
        else
            last = mid - 1;
}
if(first > last)
    printf("element not found");
getch();

```

Data Structure:-

1 Data structure is a group of elements that are put together under one name and which defines a particular way of storing and organizing data in a computer so that it can be used efficiently.

Eg:- Arrays, linked list, stacks, Trees, Queues etc.

Applications of data structures

- ① Compiler Design
- ② Numerical Analysis
- ③ Artificial Intelligence
- ④ O.S
- ⑤ DBMS (Data Base management System)
- ⑥ Graphics

Operations on data structures

- (i) Traversing: It means to access each data item once so that it can be processed.
- (ii) Searching: It is used to find the location of one or more data items that satisfied the given constraint.
- (iii) Insertion: It is used to add new data items in given list of data items.
- (iv) deletion: It is used to remove a particular data item from given collection of data item.
- (v) Sorting: Data items may be arranged in some order like ascending order or descending order.
- (vi) Merging: List of two sort data items can be combined to form a single list of sorted data items.

→ Approaches of designing an algorithm. A complex algorithm is divided into smaller units called modules. This process is called modularization.

The two strategies that are used to write algorithms are

(i) top-down.

(ii) Bottom-up.

Analyzing an algorithm:

We can analyse the efficiency of algorithm using factors

(i) time complexity

(ii) space complexity.

Time Complexity:

It is the running time of the program as function of input size.

Space Complexity:

It is the amount of computer memory that

of input size.

The space needed by a program depends on the following 2 parts (i) fixed part (it includes the space for storing Instructions, constants, variables and structured variables).

(ii) variable part:- It is the space needed for recursion stack and for structured variables that are allocated dynamically.

Worst case time complexity

This denotes the behaviour of an algorithm with respect to the worst possible of the input instance.

It is an upper bound on the running time for any input.

Average case time Complexity:-

It is an estimate of the running time for any average input.

Best case time Complexity:-

It is used to analyze an algorithm under optimal conditions. It is a lower bound on the running time for any input.

Calculating time and space complexity.

The time and space complexity can be expressed using a function $f(n)$ where n is the input size.

If a function is linear without any loop (or) recursion the efficiency of that algorithm (or) running time of algorithm can be given as no. of instructions it contains.

There are different types of loops. If the algorithm contains loops then the efficiency of the algorithm may vary depending upon no. of loops and running time of each loop.

for($i=0$; $i < 10$; $i++$)

In the case of linear loops the no. of iterations executed by the loop is called as loop factor. The $f(n) = \text{loop factor}$.

In the above example $f(n) = 10$

So in general terms the function for linear loops $f(n) = n$.

Eg:- for($i=1$; $i \leq 100$; $i+=2$)

Here the no. of iterations is half the no. of loop.
∴ the efficiency is $f(n) = n/2$

2) Logarithmic Loop:-

The loop controlling variable is either multiplied or divided for each iteration of the loop.

for($i=1$; $i \leq 1000$; $i*=2$)

for($i=1000$; $i/2=1$; $i/2=2$)

Both the loops are iterated 10 times it is expressed in logarithm notation as $\log_{\frac{1}{2}} 1000$, where n is 1000 and the loop controlling variables value for each iteration. Therefore in general terms the function for logarithmic loops is $f(n) = \log n$

3) Nested loops:-

We can find the complexity of nested loops by multiplying iterations in outer loop with no. of iterations in inner loops.

There are 3 types of Nested loops

(i) linear logarithmic loop.

(ii) Eg:- for($i=0$; $i < 10$; $i++$) for($j=0$; $j \leq 10$; $j+=2$)

→ The no. of iterations of outer loop is n . And the no. of iterations of inner loop is $\log n$. In general term

In this loop, the no. of iterations of outer loop are equal to no. of iterations of inner loop. The efficiency here is $f(n) = n \cdot n = n^2$.

Eg:- `for(i=0; i<10; i++)`

`for(j=0; j<10; j++)`

(iii) Dependent Quadratic loop:-

In this loop, the no. of iterations in the inner loop are dependent on outer loop.

Eg:- `for(i=0; i<10; i++)`

`for(j=0; j<=i; j++)` } Floyd's Triangle

We can calculate the average no. of iterations of inner loop by adding all the iterations of inner loop by no. of iterations of inner loop.

$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$
$j=0$	$j=0, 1$	$j=0, 1, 2$	$j=0, 1, 2, 3$						
	1	2	3	4	5	6	7	8	9

$$\rightarrow \frac{1+2+3+4+5+6+7+8+9+10}{10} = 5.5 = \frac{11}{2} = \frac{10+1}{2} = \frac{n+1}{2}$$

$$\therefore f(n) = n \cdot \frac{n+1}{2}$$

Asymptotic Notations :-

There are 3 types of Asymptotic Notations. These are used to determine the behaviour of an algorithm.

Big O Notation:-

This Notation determines the worst case complexity of an algorithm.

Let us assume $f(n)$ and $g(n)$ are two functions and ' c' ' is a constant then the big O Notation is defined as $f(n) = O(g(n))$ if and only if

2) Big Omega Notation-

This notation determines the best case complexity of an algorithm.

Let us assume $f(n)$ and $g(n)$ are two functions and c is a constant, then Big Ω Notation is defined as

$$f(n) = \Omega(g(n))$$

$$f(n) \geq c \cdot g(n) \text{ for } c > 0, n \geq n_0$$

3) Big Theta Notation-

This notation determines the average case complexity of an algorithm. Let us assume $f(n)$ and $g(n)$ as f and g and c_1 and c_2 are two constants then Θ is defined as $f(n) = \Theta(g(n))$ if and only if $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$.

$$\text{i.e. } f(n) \geq c_1 \cdot g(n) \rightarrow \underline{2}$$

$$f(n) \leq c_2 \cdot g(n) \rightarrow \underline{0}$$

Hence Θ lies between $\underline{2}$ and $\underline{0}$.

Sorting Techniques-

Bubble sorting

In Bubble sorting, consecutive adjacent pair of elements in the array are compared with each other. If the element at lower index is greater than the element at higher index then the two elements are interchanged. This process will continue till the list is sorted.

Eg:-

5	4	3	2	1
a[0]	a[1]	2	3	4

$$a[0] > a[1]$$

5 > 4 then swap

4	5	3	2	1
0	1	2	3	4

$$a[1] > a[2]$$

5 > 3

4	3	5	2	1
0	1	2	3	4

$$a[2] > a[3]$$

5 > 2

4	3	2	5	1
0	1	2	3	4

$$a[3] > a[4]$$

5 > 1

4	3	2	1	5
0	1	2	3	4

cycle-1 $a[0] > a[1]$
 $4 > 3$

3	4	2	1	5
---	---	---	---	---

$a[1] > a[2]$
 $4 > 2$

3	2	4	1	5
---	---	---	---	---

$a[2] > a[3]$

3	2	1	4	5
---	---	---	---	---

$a[3] > a[4]$
 $4 > 5$ (wrong)

Program:-

```
#include<stdio.h>
void main()
{
    int a[20], i, j, t, n;
    printf("enter n value");
    scanf("%d", &n);
    printf("enter array elements");
    for(i=0; i<n; i++)
        scanf("%d", &a[i]);
}
```

```
for(i=0; i<n; i++)
{
    for(j=0; j<n-i-1; j++)
        if(a[j] > a[j+1])
            {
```

```
        t = a[j];
        a[j] = a[j+1];
        a[j+1] = t;
    }
```

3
3

```
printf("elements after sorting");
for(i=0; i<n; i++)
    printf("%d", a[i]);
```

cycle-3 $\{3 | 2 | 4 | 5\}$

$a[0] > a[1]$
 $3 > 2$

2	3	1	4	5
---	---	---	---	---

$a[1] > a[2]$
 $3 > 1$

2	1	3	4	5
---	---	---	---	---

$a[2] > a[3]$
 $3 > 4$ (W)

$a[3] > a[4]$
 $4 > 5$ (W)

Cycle-4:-

2	1	3	4	5
---	---	---	---	---

$a[0] > a[1]$
 $2 > 1$

1	2	3	4	5
---	---	---	---	---

$a[1] > a[2]$ (L)

$a[2] > a[3]$ (L)

$a[3] > a[4]$ (L)

$a[4] > a[5]$ (L)

complexity is n^2 .

In insertion sort technic the second element is compared to first element then interchange elements then the third element will be compared to 2nd element then the fourth element will be compared to 3rd element and so on.

In insertion sort technique, 2nd element compare with 1st element, if 2nd element $<$ 1st element then it is swapped, then 3rd element compared with 2nd element, then 4th element compared with 3rd element and placed the element in 1st place. This process is repeated till end of array.

Ex:-	$\begin{array}{ c c c c c c } \hline & 1 & 2 & 3 & 4 & 5 \\ \hline 7 & 3 & 5 & 4 & 2 & 6 \\ \hline \end{array}$	$\begin{array}{ c c c c c } \hline 3 & 5 & 7 & 4 & 2 & 6 \\ \hline t = a[1] = 3 \\ t < a[0] \\ 3 < 7 \\ \hline \end{array}$	$\begin{array}{ c c c c c } \hline 3 & 4 & 5 & 7 & 2 & 6 \\ \hline t = a[2] = 4 \\ t < a[1] \\ 4 < 7 \\ \hline \end{array}$	$\begin{array}{ c c c c c c } \hline 3 & 5 & 4 & 7 & 2 & 6 \\ \hline t < a[1] \\ 4 < 5 \\ \hline \end{array}$	$\begin{array}{ c c c c c } \hline 3 & 4 & 5 & 7 & 2 \\ \hline t < a[0] \\ t < 3 \\ 4 < 3(N) \\ \hline \end{array}$	$\begin{array}{ c c c c c } \hline 3 & 2 & 4 & 5 & 6 \\ \hline t < a[0] \\ t < 3 \\ \hline \end{array}$	$\begin{array}{ c c c c c } \hline 2 & 3 & 4 & 5 \\ \hline \end{array}$
	$\begin{array}{ c c c c c } \hline 2 & 3 & 4 & 5 & 7 & 6 \\ \hline t = a[5] = 6 \\ t < a[4] \\ 6 < 7 \\ \hline \end{array}$	$\begin{array}{ c c c c c } \hline 2 & 3 & 4 & 5 & 6 & 7 \\ \hline t < a[3] (F) \\ t < a[2] (F) \\ t < a[1] (F) \\ t < a[0] (F) \\ \hline \end{array}$					

Program: Selection Sort

for ($i=1$; $i < n$; $i++$)

{
 $t = a[i]$;

$j = i - 1$;

 while ($j \geq 0$ & $a[j] > t$)

{

$a[j+1] = a[j]$;

$j = j - 1$;

}

$a[j+1] = t$;

}

$a[0]$	1	2	3	4	5
7	3	5	4	2	6

$t = a[1]$, $j = 0$.

$a[j] > t$

$a[0] > a[1]$

$7 > 3$

$a[1] = a[0]$;

3	7	5	4	2	6
---	---	---	---	---	---

$t = a[2]$, $j = 1$;

while ($2 \geq 0$ & $a[1] > a[2]$)

$7 > 5$

$a[2] = a[1]$

$j = 1 - 1 = 0$

$a[2] = 7$

$a[j+1] \neq t$

$t = a[0+1]$

$t = a[1] = 5$

3	5	7	4	2	6
---	---	---	---	---	---

$t = a[3]$, $j = 2$

$a[2] > a[3]$

$7 > 4$

$a[3] = a[2]$, $j = 1$

$7 = 4$

$a[2] \neq t$

$t = 4$

0

1

2

3

4

5

6

7

3	5	7	4	2	6
---	---	---	---	---	---

$t = a[4]$, $j = 3$

$a[3] > a[4]$

$a[4] \geq a[3]$, $j = 2$

$2 = 7$
 $a[4] = 7$, $a[3] = t$
 $t = 7$

3	5	4	2	7	6
---	---	---	---	---	---

$t = a[5]$, $j = 4$

$a[4] > a[5]$, $j = 3$

$7 > 6$
 $a[5] = a[4]$, $a[4] = t$
 $a[4] = 6$
 $a[5] = 7$

3	5	4	2	6	7
---	---	---	---	---	---

$a[6] \neq t$, $j = 1$

$a[1] > a[2]$

$5 > 4$

$a[2] = a[1]$ | $j = 0$
 $a[1] = 5$ | $a[1] = 4$

$a[2] = 5$ | $a[1] = 4$

3	4	5	2	6	7
---	---	---	---	---	---

$a[3] \neq t$, $j = 2$

$a[2] > a[3]$

$5 > 2$
 $a[3] = a[2]$, $a[2] = t$
 $a[3] = 5$

3	4	2	5	6	7
---	---	---	---	---	---

$a[4] > a[3]$

$4 > 2$, $a[3] = t$

$a[3] = 4$

3	2	4	5	6	7
---	---	---	---	---	---

$t = a[0], j=0$

$a[0] > a[1]$

$3 > 2$
 $a[i] = a[0], j = i - 1$
 $= 0 - 1 = -1$

$a[i] \geq 3$

$a[j+1] = t \Rightarrow t = a[0]$
 $\Rightarrow a[0] \geq 2$

2	3	4	5	6	7
---	---	---	---	---	---

Selection sorting:-

It is the process of finding minimum element every time and interchange it with index element. For every iteration the smaller comes to the beginning of the array.

Code:-

```

for(i=0; i<n-1; i++)
{
    small=i;
    for(j=i+1; j<n; j++)
    {
        if(a[j]<a[small])
            small=j;
    }
    t=a[i];
    a[i]=a[small];
    a[small]=t;
}

```

Eg:-

9	6	7	2	1	3
---	---	---	---	---	---

$i=0, 0 < 1; 6-1=5$

\downarrow

$; \quad small=0;$
 $j=1, 1 < 6$

$\therefore a[1] < a[0]$

$6 > 9$

$small=1$

$j=2$

$a[j] < a[small]$

$a[2] < a[1]$

$7 < 6 (W)$

$j=3, 3 < 6$

$a[i] < a[small]$

$a[3] < a[1]$

$2 < 6$

$small=3$

$j=4, 4 < 6$
 $a[j] < a[small]$
 $a[4] < a[3]$.
 $i < j$
 $small = 4$

0	1	2	3	4	5
1	6	7	2	9	3

$j=5, 5 < 6$
 $a[j] < a[small]$
 $a[5] < a[4]$
 $3 < 1 (W)$

$i=1, 1 < 5$,
 \downarrow
 $small = 1$
 $j=2, 2 < 6$
 $a[j] < a[small]$
 $a[2] < a[1]$
 $7 < 6 (W)$

0	1	2	3	4	5
9	2	7	6	9	3

$j=3, 3 < 6$
 $a[3] < a[small]$
 $a[3] < a[1]$
 $2 < 6$
 $small = 3$.

$j=4, 4 < 6$
 $a[4] < a[small]$
 $a[5] < a[3]$
 $9 < 2 (W)$
 $3 < 2 (W)$

$i=2, 2 < 5$,
 \downarrow
 $small = 2$.
 $j=3, 3 < 6$
 $a[j] < a[small]$
 $a[3] < a[2]$
 $6 < 7$
 $small = 3$.

$j=4, 4 < 6$
 $a[j] < a[small]$
 $a[4] < a[3]$
 $9 < 6 (W)$
 $3 < 6 (T)$
 $small = 5$.

1	2	3	6	9	7
---	---	---	---	---	---

$i=3$
 $j=3+1$
 $small = 3$
 $a[4] < a[3]$
 $6 < 3 (W)$

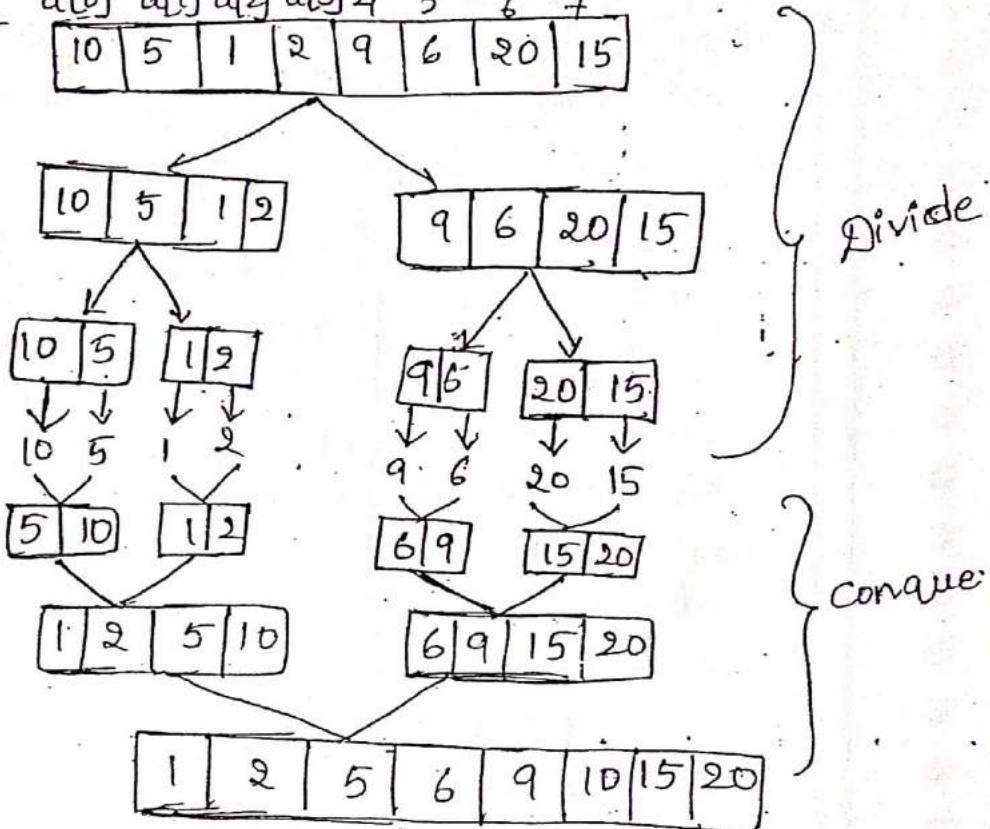
$i < n-1$
 $i < 5 \quad 5 < 5$

$i=4$
 $j=5$
 $small = 4$
 $a[j] < a[small]$
 $a[5] < a[4]$
 $i=5 \quad 5 < 5 (F) \quad 4 < 9 (T)$
 $small = 5$.

1	2	3	6	7	9
---	---	---	---	---	---

Merge sort:-
 It is a sorting algorithm that follows
 conquer rule the given array is divided into
 arrays. The two sub arrays are further divided
 into 4 sub arrays. So this process continues until
 individual elements after division the individual
 are sorted into several sub arrays all sub arrays
 are combined and merged to form a single sorted

Eg:- $a[0] \ a[1] \ a[2] \ a[3] \ 4 \ 5 \ 6 \ 7$



Program:-

```
#include <stdio.h>
void mergesort(int[], int, int)
void merge(int[], int, int, int)

main()
{
    int n, i, a[20];
    mergesort(a, 0, n-1);
    printf("enter n value");
    scanf("%d", &n);
    printf("enter array elements");
    for(i=0; i<n; i++)
        . . .
    printf("elements");
}
```

Void merge sort (int a[20], int beg, int end)

```
{  
    int mid;  
    if (beg < end)  
    {  
        mid = (beg + end) / 2;  
        mergesort (a, 0, mid);  
        merge sort (a, mid+1, end);  
        merge (a, beg, mid, end);  
    }  
}
```

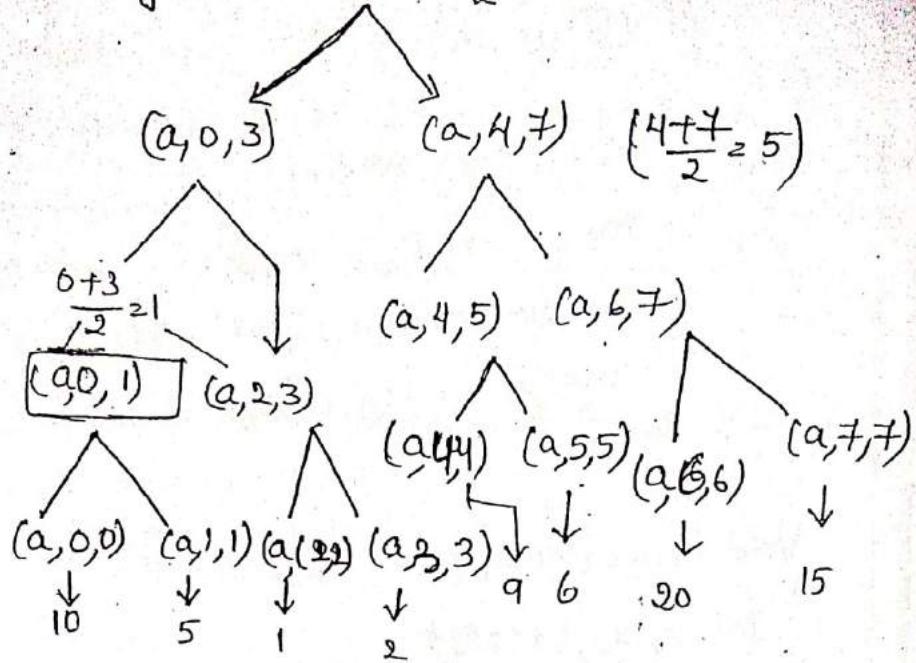
Complexity = $n \log n$

Void merge (int a[20], int beg, int mid, int end)

```
{  
    int l1, l2, i, t[20], k;  
    l1 = beg, l2 = mid + 1, i = beg;  
    while (l1 <= mid && l2 <= end)  
    {  
        if (a[l1] <= a[l2])  
            t[i] = a[l1++];  
        else  
            t[i] = a[l2++];  
        i++;  
    }  
    while (l1 <= mid)  
    {  
        t[i++] = a[l1++];  
    }  
    while (l2 <= end)  
    {  
        t[i++] = a[l2++];  
    }  
    for (k = beg; k <= end; k++)  
        a[k] = t[k];  
}
```

10	5	1	2	9	6	20	15
0	1	2	3	4	5	6	7

merge sort($a, 0, 7$) $\Rightarrow \frac{0+7}{2} = 3$.



Quick Sort :-

This algorithm follows divide and conquer tec which is here called as partitioning. It partition array into smaller arrays based on pivot element procedure:-

- 1) Select an element called pivot in the array el it can be last element (or) first element.
- 2) Rearrange the elements in such a way that elements before pivot are lesser than pivot & elements after pivot are greater than pivot. Such partitioning the pivot is placed in its final position this is called partition operation.
- 3) Recursively sort the two sub arrays the condition for checking pivot is if current element is less than pivot then swap index element and current element and increment the index position by 1.
- 4) The current element is always incremented.

Irrespective of condition current element less than pivot is true or false.

Here current < pivot element are same so we have to check for another condition if index = pivot then ...

if current < pivot
 { swap index = current
 index++
 }
 current++

--

elements before the 'p' are less than pivot & after 'p' are greater than pivot so divide the array into two sub arrays. the elements before 'p' as one array and another elements after 'p' as one array.

--

if c & i are same there is no need to swap.

--

--

so there is no need to swap.

--

a result

16	9	11	8
i=0		P	

If C, i & P are applied on a same variable
pivot value can be neglected.

C	2	1
i=0		P

$$2 < 1 (W)$$

C	2	1
i=0		P

$$\text{if } i > p \\ 2 > 1$$

1	2
---	---

C	16	9	11	8
i=0			P	

$$(16 < 8) (W)$$

C	16	9	11	8
i=0			P	

$$9 < 8 (W)$$

C	16	9	11	8
i=0			P	

$$11 < 8 (W)$$

C	16	9	11	8
i=0			P	

$$i > p$$

$$16 > 8$$

8	9	11	16
---	---	----	----

Algorithm :-

Step-1: start

Step-2 :- Input $\leftarrow n, a[20]$

Step-3 :- Initialize low=0, high=n-1.

Step-4 :- If ($low < high$)

{ }

call partition ($a, low, high$)

call quick sort ()

Alg

1	2	4	7	8
---	---	---	---	---

If pivot interchanging
is not considered.

Eg:-

20	17	4	9	3	11
----	----	---	---	---	----

call quicksort(a, p+1, high)

3

Algorithm for partition:

- 1) Initialize $i=0$, ~~p=p+1~~, $j=0$, $p=a[\text{high}]$
- 2) Repeat step-3 until $j \leq \text{high}$.
- 3) If ($a[j] < a[p]$) or if ($a[j] > p$)
 - (i) then swap $a[i]$ and $a[j]$
 - (ii) Increment i by 1.
- 4) Swap $a[i]$ & pivot.
- 5) Return ~~p+1~~. Pivot.
- 6) Stop.

20	17	4	9	3	11	5	10
----	----	---	---	---	----	---	----

$i=0$

if current < pivot
 $20 < 10$ (T)
current++

\uparrow
P

20	17	4	9	3	11	5	10
----	----	---	---	---	----	---	----

$i=0$

C
if $c < p$
 $17 < 10$ (W)

\uparrow
P

20	17	4	9	3	11	5	10
----	----	---	---	---	----	---	----

$i=0$

C
if $c < p$

\uparrow
P

swap i & c $4 < 10$ (T), $i++$, $c++$

4	17	20	9	3	11	5	10
---	----	----	---	---	----	---	----

$i=1$

C
if ($9 < 10$) (T)

\uparrow
P

4	9	20	17	3	11	5	10
---	---	----	----	---	----	---	----

$i=2$

C
if $c < p \Rightarrow 3 < 10$ (T)

\uparrow
P

4	9	3	17	20	11	5	10
---	---	---	----	----	----	---	----

$i=3$

C
 $11 < 10$ (W)

\uparrow
P

4	9	3	17	20	11	5	10
---	---	---	----	----	----	---	----

$i=3$

C
 $5 < 10$ (T), $i=4$

\uparrow
P

if index > pivot
 $20 > 10$ (T)

4	9	3	5	10	11	17	27
---	---	---	---	----	----	----	----

P

Array-11

4	9	3	5
---	---	---	---

P

$i=0$

if $c < p \Rightarrow 4 < 5$ (T)

5	9	3	4
---	---	---	---

P

here pivot
changed
its pos
so it is placed
in its place.

9	3	4
---	---	---

P

9	3	4
---	---	---

P

9	3	4
---	---	---

P

3	9	4
---	---	---

P

3	4	9
---	---	---

P

3	9
---	---

P

3	9
---	---

P

3	9
---	---

P

3	9
---	---

P

3	9
---	---

P

3	9
---	---

P

3	9
---	---

P

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P

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P

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P

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P

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P

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---	---

P

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P

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P

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P

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P

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P

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P

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---	---

P

3	9
---	---

P

3	9
---	---

P

3	9
---	---

P

3	9
---	---

P

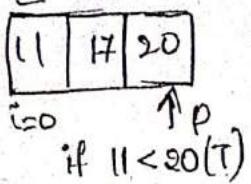
3	9
---	---

P

3	9
---	---

P

3	9
---	---



20	11	17
----	----	----

Here pivot changed its position

11	17
$c_{20}, j=0$	$\uparrow p$

program:

```
#include<stdio.h>
Void main()
{
    int a[10], i, n
    printf("enter n value");
    scanf("%d", &n);
    printf("enter array elements");
    scanf("%d", &a[i]);
    quicksort(a, 0, n-1);
    printf("elements after sorting");
    for(i=0; i<n; i++)
}
```

```
Void quicksort(int[], int, int)
int partition(int[], int, int)
```

```
Void partition sort(a, low, high)
{
    int i = low, j = high, low, p = a[high];
    for(i=0; i<n; i++)
    {
        for(j=0; j<=high; j++)
        {
            if (a[j] < p)
            {
                a[j] = t;
                a[i] = a[j];
                else t = a[i];
            }
        }
    }
}
```

```
printf("%d", a[i]);
getch();
```

}

17
3 4 5

```
Void quick sort (int a[20], int low, int high)
```

```
{ if (low < high)
```

```
{ int pi
```

```
call partition (a, low, high);
```

```
call quick sort (a, low, pi-1);
```

```
call quick sort (a, pi+1, high);
```

```
}
```

```
}
```

```
int partition (int a[20], int low, int high)
```

```
{
```

```
int pivot = a[high];
```

```
int i = low, c, t;
```

```
for (c = low, c <= high-1, c++)
```

```
{
```

```
if (a[c] <= pivot)
```

```
{
```

```
t = a[i];
```

```
a[i] = a[c];
```

```
a[c] = t;
```

```
i++;
```

```
}
```

```
}
```

```
t = a[i];
```

```
a[i] = a[high];
```

```
a[high] = t;
```

```
return i;
```

```
}
```

Radix sort:-

It is a sorting that can be used to sort numbers by its base. To sort the array numbers it needs 10 temporary locations called buckets. Because the base numbers for decimals is 10. If we want to sort English words we need 26 buckets because alphabets are 26. This algorithm is also called as radix sort.

Procedure:-

- 1) Identify the biggest number in the array and no. of digits.
- 2) place 0's before every number in the array like largest number in the array.
- 3) If no. of digits are 3 then their will be 3 pt sorting. If no. of digits are n then their will be n pt of sorting.

Phase-1:- Take buckets from 0 to 9 and search the units place for the numbers according to store the numbers in a temporary array, starting bucket to 9th bucket.

Phase-2:- Search for 10th place and place accordingly numbers in the buckets. Now take the results and perform phase-3.

Phase-3:- Search for 100's place and place the numbers according to buckets. If no. of digits are 3 in number then the array will be sorted completely in this phase. Otherwise places will be increase one place and so on.

Eg:- 10 15 1 60 5 100 25 50

first select largest element = 100

No. of digits in the largest element = 3.

010 015 001 060 005 100 025 050

0	1	2	3	4	5	6	7	8
010	001							
060								
100								
050								

010 060 100 050 001 015 005 025

Phase-2:- (10th place)

0	1	2	3	4	5	6	7	8
001	010	025						
005	015							

100 001 005 010 015 025 050 060

Phase-3:- (100th place)

0	1	2	3	4	5	6	7
001	100						
005							
010							
015							
025							
050							
060							

001 005 010 015 025 050 060 100

→ Remove zero's from the above array

1	5	10	15	25	50	60	100
---	---	----	----	----	----	----	-----

Implement Radix sort in the given list of numbers.

170, 45, 75, 90, 802, 24, 2, 66.

i) first select the largest element in the given array.

largest element = 802.

2) Next count no. of digits in the largest element. And we have to place 0's before every number in the array like the largest number in the array.

170 045 075 090 802 024 002 066

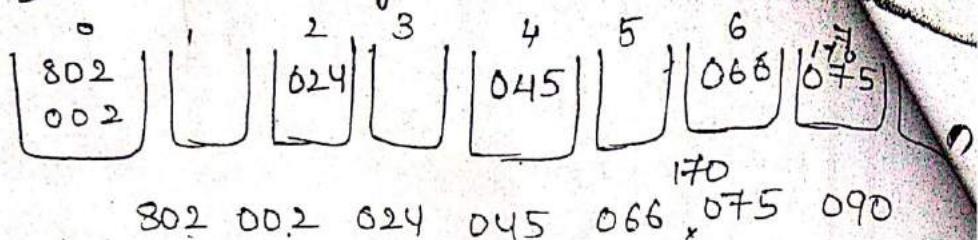
3) After placing 0's in the array phases will start. The no. of phases in the array depends on the no. of digits of the largest element.

Phase-1:- search for units place.

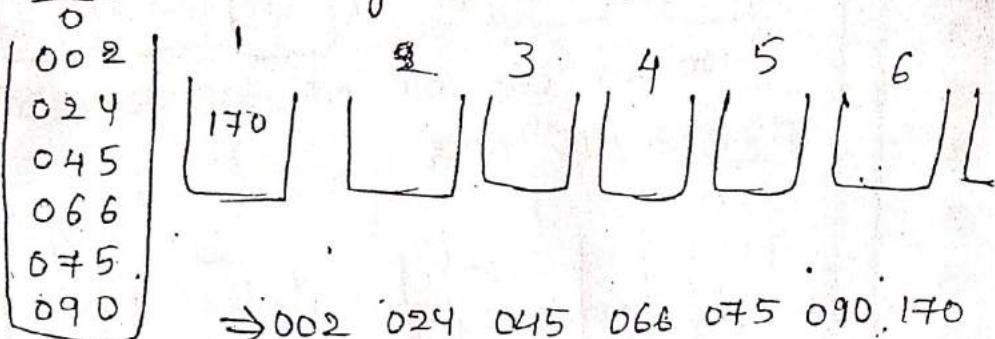
0	1	2	3	4	5	6
170	45	75	90	802	24	002
090				002		

→ 170 002 802 002 024 045 090

Phase-2:- searching for 10's place.



Phase-3:- Searching for 100's place.



Remove zero's added in the above array

2	24	45	66	75	90	170	802
---	----	----	----	----	----	-----	-----

Algorithm:-

Step-1:- Start

Step-2:- ~~Read~~ Input n no. of elements in an

Step-3:- Accept array elements

Step-4:- find the largest element in the arra

Step-4:- find total no. of digits in the largest

Step-5:- pat each element with no. of zero's
that

Step-6:- Initialize bucket j=0 and repeat the next
until j<n.

Step-7:- Compare ith position of each element
array and place it in corresponding buck

7.1 sort the number by Units value

7.2 sort the number by 10's value.

7.3 sort the number by 100's value.

Step-8:- Read the elements of bucket from 0th

Step-9:- Display the sorted array by removing zero before every element.

Hashing

Hashing is an important data structure technique that is used to uniquely identify a specific object from a group of ~~simil~~ similar objects. This hashing technique arranges data in Key Value ^{pairs} based where keys are unique elements. These keys are used as integers in an array and values are used as elements in the array. This hashing is performed in a data structure called "Hash Table" which stores data in an associative manner (key value pairs). In hashing large keys are converted to small keys by using Hash functions. Hash function is defined as $H(x) = [x] \cdot \text{Array size}$

Eg:- Let us assume an array list containing 10 elements as list of 10 11 12 13 14 15 16 17 18 19 20 we can generate Keys by using function as

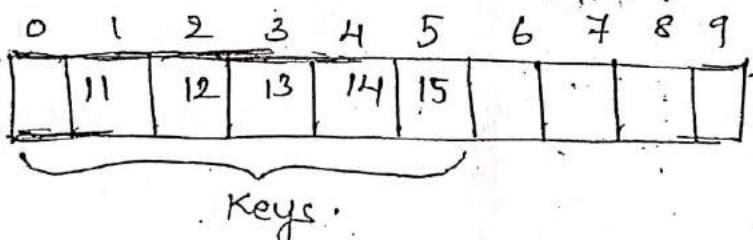
$$H(11) = 11 \% 10 \Rightarrow 1$$

$$H(12) = 12 \% 10 \Rightarrow 2$$

$$H(13) = 13 \% 10 \Rightarrow 3$$

$$H(14) = 14 \% 10 \Rightarrow 4$$

$$H(15) = 15 \% 10 \Rightarrow 5$$



Collision in hashing:-

When two (or) more distinct key elements are mapping to same hash value of index then it is called as collision.

To prevent collisions we are having some collision resolution techniques. Among them (i) linear probing.

Linear probing:-

In this methodology it moves to an next location

free slot.
The function for calculating index

$$\text{probing } H(x,i) = (H(x)+i) \bmod n.$$

where $H(x)$ is the key element.

- n is the size of the array

Develop a C program to create a hash table
perform operations insert, search using linear
technique

```
#define n
```

```
int a[n];
```

```
void insert()
```

```
{
```

```
    int Key, hash, i, index, n;
```

```
    printf("Enter the Key");
```

```
    scanf(".1.d", &Key);
```

```
    hash = Key % n;
```

```
    for (i=0; i<n; i++)
```

```
    {
```

```
        index = (hash + i) % n;
```

```
        if (a[index] == 0)
```

```
        {
```

```
            a[index] = Key;
```

```
            break;
```

```
}
```

```
if (i == n)
```

```
    printf("Element is not inserted");
```

```
}
```

```
void search()
```

```
{
```

```
    int Key, hash, i, index, n;
```

```
    printf("Enter the Key");
```

```
for(i=0; i<n; i++)
{
    index = (hash+i) % n;
    if (a[index] == key)
        printf("element found");
        break;
}
if (i==n)
    printf("element not found");
```

```
3
void disp()
```

```
{  
    int i;  
    for(i=0; i<n; i++)  
        printf("%d", a[i]);  
}
```

```
void main()
```

```
{ int option;  
    while(1)
```

```
{  
    printf("Press 1. Insert\n 2. Display\n 3. search  
          4. Exit")
```

```
    scanf("%d", &option);
```

```
    switch(option)
```

```
{  
    case 1: insert();  
        break;
```

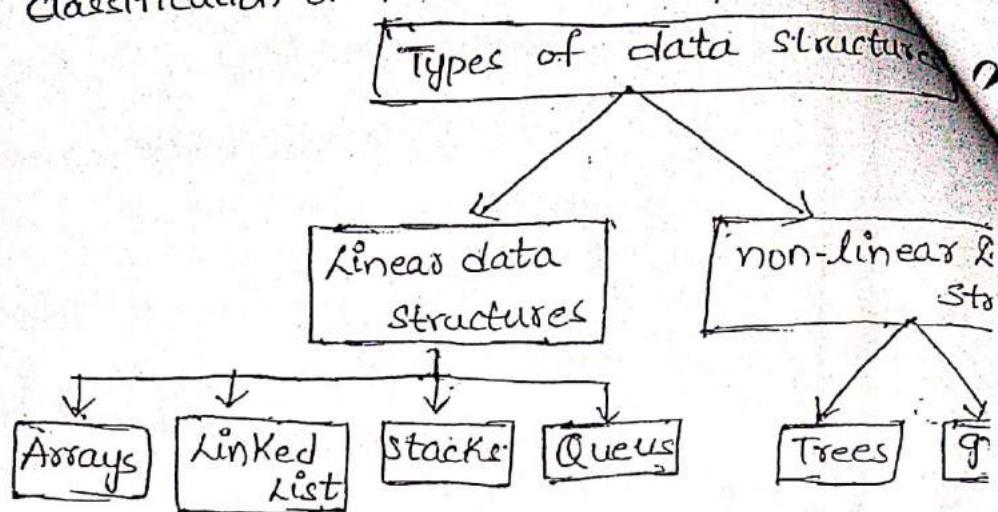
```
    case 2: disp();  
        break;
```

```
    case 3: search();  
        break;
```

```
    case 4: exit(0);  
}
```

2.

Classification of Data structures.

linear data

- 1) In linear data structures elements are accessed in sequential order.
- 2) These are simple to implement.
- 3) These are operated at single level.
- 4) These data structures memory utilization is ineffective.

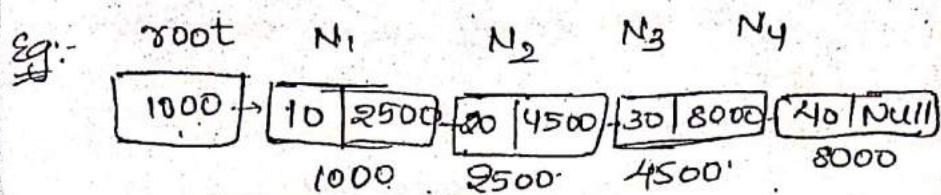
Non linear data

- 1) Elements are stored through sorted condition there will be gap between each and every element.
- 2) These are complex to implement.
- 3) These are operated at multi level.

Linked List :- A linked list is a data structure containing group of elements called nodes which are connected with each other. Each node contains

- (i) 1st part contains data (values)
- (ii) 2nd part contains link which holds the address of next node

The link part of node last node is null.



there are different types of linked list.

(i) single linked list

(ii) Double Linked List

(iii) circular linked list

(iv) single linked list

It contains only one link in each node.

(ii) Double linked list

It contains two links previous link pointing to previous node and next link pointing to next node.

(iii) circular linked list:

the node part
It contains same as Double linked list but is operated in circular manner.

Struct node

```
{  
    int data;  
    struct node * link;  
}
```

Link is a pointer that points to address of another node so the link pointer is of type struct. node.

Operation of single linked list

Insertion: we can insert an element in the beginning of the linked list (or) we can insert an element at the end of the list (or) we can also insert at a specified place in linked list

Deleting:

we can delete an element at the beginning of the list (or) ending of the list (or) we can also delete a specified place.

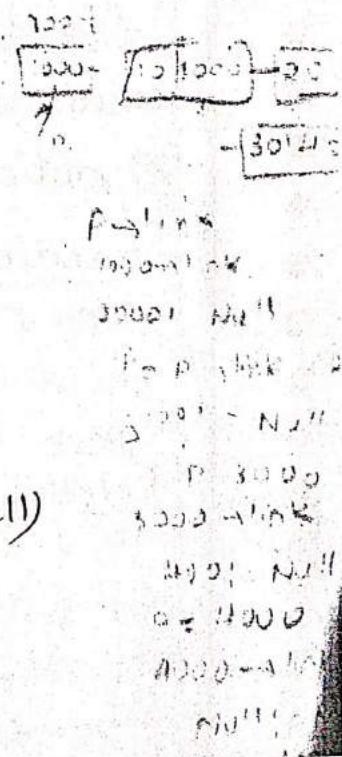
Reversing:

Count:- we can count no. of elements (or) no.

list.

Search:- we can also search for an element in list.
write a C program to perform operations on linked list.

```
struct node
{
    int data;
    struct node * link;
}
struct node * root=NULL;
void main() insert()
{
    struct node * temp;
    temp = (struct node *) malloc (sizeof (str
        printf ("enter data"),
        scanf ("%d", &temp->data),
        temp->link = NULL,
        if (root=NULL)
        {
            root = temp;
        }
        else
        {
            struct node * p,
            p = root;
            while (p->link != NULL)
            {
                p = p->link;
            }
            p->link = temp;
        }
    }
}
```



2

```

int count=0;
Struct node *p;
P=root;
while (p!=NULL !=NULL)
{
    count++;
    P=P->link;
}
return count;
}

void insert after()
{
    struct node *temp,*P; //length: 0 to n
    int loc,length,i=1,node;
    printf("enter location");
    scanf("%d", &location);
    len = counting();
    if (len<loc)
    {
        printf("invalid loc");
    }
    else
    {
        P=root;
        while (i<loc)
        {
            P=P->link;
            i++;
        }
        temp=(struct node *) malloc (sizeof(insert node));
        printf("enter data");
        scanf("%d", &temp->data);
        temp->link = P->link;
        P->link = temp;
    }
}

```

insert after:

i=1
 if ($i < 4$)
 $1000 \rightarrow 101000 \rightarrow 203000 \rightarrow 304000 \rightarrow 405000$
 if ($i < 4$)
 $1000 \rightarrow 101000 \rightarrow 203000 \rightarrow 304000 \rightarrow 50$
 if ($i < 4$)
 $1000 \rightarrow 101000 \rightarrow 203000 \rightarrow 50 \rightarrow 405000$
 if ($i < 4$)
 $1000 \rightarrow 101000 \rightarrow 50 \rightarrow 203000 \rightarrow 405000$
 if ($i < 4$)
 $1000 \rightarrow 50 \rightarrow 101000 \rightarrow 203000 \rightarrow 405000$
 if ($i < 4$)
 $50 \rightarrow 1000 \rightarrow 101000 \rightarrow 203000 \rightarrow 405000$

Void display()

{

struct node * p;

p = root;

while (p != NULL)

{ printf("%d", p->data);

p = p->link

}

}

Deletion of a node:-

point the node as temp.

Remove the left connection of the node by filling the previous nodes link part with current nodes link part. Remove the right connection by placing null in link part of the desired deleted.

Then the node is ready to delete we can get the memory of the desired node by using function free.

Syntax :- free(ptr);

Void delete()

{

struct node * p;

int loc, len = 1;

printf("enter location");

scanf("%d", &loc);

len = counting();

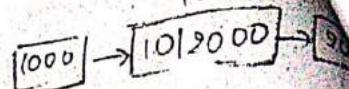
if (len < loc)

printf("invalid location");

else if (loc == 1)

{

p = root;



P: root = 1000

1000 ! = Nu "()"

p->data = 1000

1000 → 10

P = 1000 → link

P: 2000 ! = Nu

2000 → link ⇒ 200

P = 2000 → link

P = 3000 = 300

3000 → data =

P ≠ 3000 → link

P = 4000

4000 → data =

P ≠ 4000 → link

P = 5000

5000 → data =

P ≠ 5000 → link

P = 6000

6000 → data =

P ≠ 6000 → link

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20000 → data =

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P ≠ 21000 → link

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P ≠ 70000 → link

P = 71000

71000 → data =

P ≠ 71000 → link

P = 72000

72000 → data =

P ≠ 72000 → link

P = 73000

73000 → data =

P ≠ 73000 → link

P = 74000

74000 → data =

P ≠ 74000 → link

P = 75000

75000 → data =

P ≠ 75000 → link

P = 76000

76000 → data =

P ≠ 76000 → link

P = 77000

77000 → data =

P ≠ 77000 → link

P = 78000

78000 → data =

P ≠ 78000 → link

P = 79000

</div

```
free(p)
```

```
}
```

```
else
```

```
{
```

```
    struct node *p,*q;
```

```
    p=root;
```

```
    while(i<loc-1).
```

```
{
```

```
    p=p->link;
```

```
    i++;
```

```
y
```

```
    q=p->link
```

```
    p->link=q->link;
```

```
    q->link=NULL;
```

```
    free(q);
```

```
}
```

```
}
```

```
void main()
```

```
{
```

```
    int option,x;
```

```
    while(1)
```

```
{ printf("1.insert\n2.counting\n3.Insert after\n4.display\n5.delete\n6.exit\n");
```

```
    scanf("%d",&option);
```

```
    switch(option)
```

```
        case 1: insert();
```

```
        break;
```

```
        case 2: counting();
```

```
        printf("No of nodes are %d",x);
```

```
        break;
```

```
        case 3: insertafter();
```

```
        break;
```

```
        case 4: display();
```

```
        break;
```

```
        case 5: delete();
```

```
        break;
```

```
        case 6: exit(0);
```

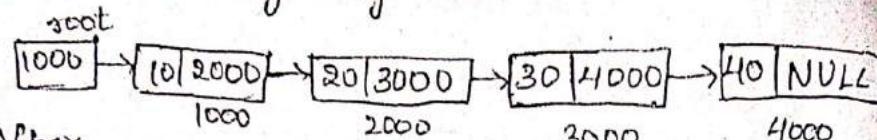
```
z
```

Reversing a single linked list

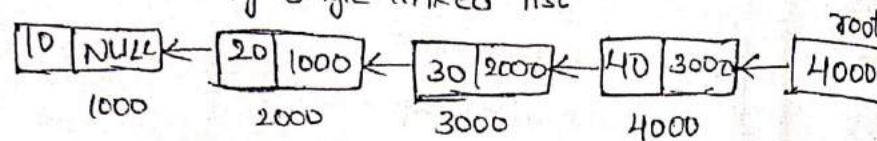
In reversed linked list the root will be last node of the list.

list and every node points to the previous node. The previous node holds the address of the previous node part. The first node contains null in the link part.

Before reversing single linked list



After reversing single linked list



Algorithm:-

Step-1:- Start

Step-2:- Declare 3 pointers *current, *next, *prev

Step-3:- Initialize current=root, prev=NULL.

Step-4:- Repeat step 5,6,7,8 until current=NULL

Step-5:- Set Next=Current → link

Step-6:- Set Current → link = prev

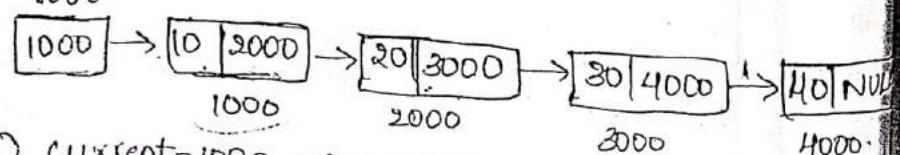
Step-7:- prev=Current

Step-8:- current=next

Step-9:- set prev=root

Step-10:- stop.

root

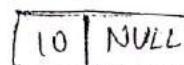


① current=1000, prev=NULL

current !=NULL

1000 !=NULL (T)

Next = current → link



Next = 2000.

2000 → link = prev.

1000 → link = NULL.

prev = 1000

← ----

(2) current = 2000, prev = NULL
2000! = NULL

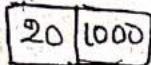
Next = current → link.

Next = 3000.

2000 → link = 1000

prev = 1000

Next = 3000 = current



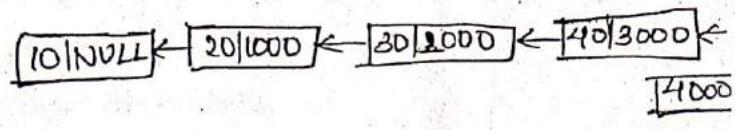
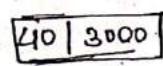
Current = 4000

prev = 3000

Next = 4000 → NULL

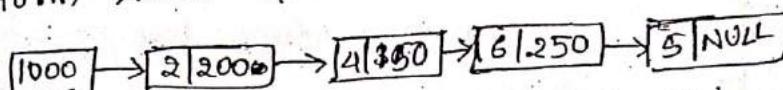
4000 → NULL = 3000

prev = 3000



14000

Perform reverse operation for linked list



current = 100, prev = NULL

current! = NULL

100! = NULL(T)

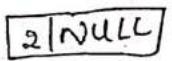
Next = current → link

Next = 200

100 → link = NULL

prev = 100

current = 200



current = 200, prev = NULL

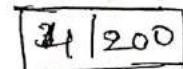
200! = NULL(T)

Next = 300

300 → link = 200

prev = 200

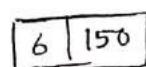
current = 300



current = 150, prev = NULL

150! = NULL

Next = 250



250 → link = 150.

prev = 150

current = 250

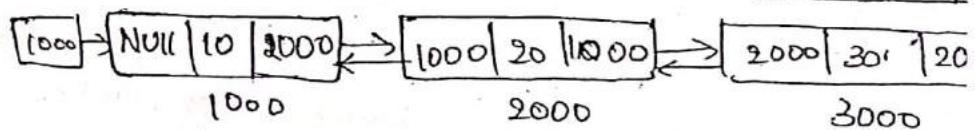
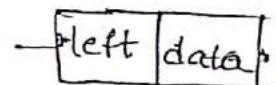
current = 250, prev = NULL

250! = NULL

Next = 9

Double linked list.
It is a two way linked list which points to the previous node as well as a pointer to the next node, in the sequence. In single linked list we can move in forward direction only. Whereas in double linked list we can move in forward and backward direction. Creation of a node in double linked list contains 3 parts - middle part contains data, left pointer to the previous node and right pointer to next node.

Eg:-



Program to perform the operations on Double linked list node

```
{  
    int data;  
    struct node *left;  
    struct node *right;  
}  
  
struct node *root=NULL;  
Void insert()  
{  
    struct node *temp;  
    temp=(struct node*)malloc(sizeof(struct  
    printf("enter data");  
    scanf("%d", &temp->data);  
    temp->right=NULL;  
    if(temp->left=NULL);  
    if(root==NULL)  
    {  
        root=temp;  
    }  
}
```

```
Struct node *p;
```

```
P=root;
```

```
while( P->right != NULL)
```

```
{
```

```
    P = P->right;
```

```
}
```

```
    P->right = temp;
```

```
    temp->left = P;
```

```
}
```

```
}
```

```
int counting()
```

```
{
```

```
    int count=0;
```

```
    struct node *P;
```

```
    P=root
```

```
    while( P->right != NULL)
```

```
{
```

```
    count++;
```

```
    P=P->right;
```

```
}
```

```
return count;
```

```
}
```

```
void display()
```

```
{
```

```
    struct node *P;
```

```
    P=root;
```

```
    while( P!=NULL)
```

```
{
```

```
    printf("%d", P->data);
```

```
    P=P->right;
```

```
}
```

```
}
```

```
void insert after()
```

```
{
```

```
    struct node *P, *temp;
```

```
    int loc, len, i=1, node;
```

```
    printf("enter location");
```

```
len = counting();
if (len < loc)
{
    printf("invalid location");
}
else
{
    p = root;
    while (i < loc)
    {
        p = p->right;
        i++;
    }
    temp = (struct node *) malloc(sizeof(struct node));
    printf("enter data");
    scanf("%d", &temp->data);
    temp->right = p->right;
    p->right->left = temp;
    p->right = temp;
    temp->left = p;
}
```

```
Void delete()
{
    struct node *p,
    int loc, len, i = 1;
    printf("enter loc");
    scanf("%d", &loc);
    len = counting();
    if (len < loc)
        printf("invalid location");
    else if (loc == 1)
    {
        p = root;
        root = p->right;
    }
}
```

$\rightarrow \text{right} = \text{NULL}$,

$p \rightarrow \text{left} = \text{NULL};$

$\text{free}(p);$

}

else

{

struct node *p, *q;

$p = \text{root};$

while ($i < loc - 1$)

{

$p = p \rightarrow \text{right};$

$i++;$

}

$q = p \rightarrow \text{right};$

$p \rightarrow \text{right} = q \rightarrow \text{right};$

$q \rightarrow \text{right} = \text{NULL};$

$\leftarrow [p \rightarrow \text{left} = q \rightarrow \text{left};] \quad \text{q} \rightarrow \text{left}$

$q \rightarrow \text{left} = \text{NULL};$

$\text{free}(q);$

}

}

void main()

{

int x, option;

while (!)

switch (option)

{

case 1: insert();

break;

case 2: d = counting;

printf("No of nodes are %d", x);

break;

case 3: insertafter();

break;

case 6: exit(0)

case 4: display();

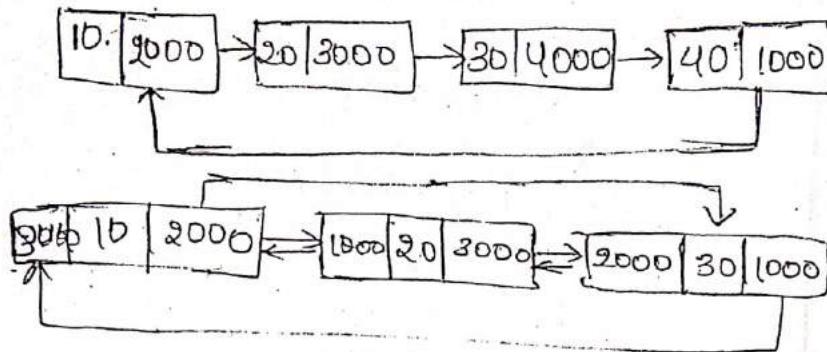
break;

case 5: . . .

}

Circular Linked List:-

It is a linked list where all the nodes point to form a circle. There is no NULL value at the end of the list. A circular linked list can be a singly linked list or double linked list.



Algorithm:-

The operations on circular single linked list :-

- 1) Insertion of a node at beginning
- 2) Insertion of a node at ending.
- 3) Deletion of a node at beginning
- 4) Deletion of a node at ending

Procedure:- Allocate memory for new node

Initialize its data

- 2) Take a pointer variable p which will point to the list.
- 3) move p so that it points to the last node.

Note:- If availability space is NULL then u insert a node in the list.

Step-1:- If avail = NULL (No available memory)

then print error: overflow.

Step-2:- goto step 10.

Allocate memory for temp

Step-3:- Initialize temp → data

Step-4:- set p = root.

Step-6:- $p = p \rightarrow link$.

Step-7:- set $p \rightarrow link = temp$

Step-8:- Set $temp \rightarrow link = root$

Step-9:- set $root = temp$.

Step-10:- Stop.

Algorithm to insert node at end.

Step-1:- Start

Step-2:- If $avail = NULL$ (no available memory space)

then print error: Over flow.

Step-3:- Allocate memory for temp.

Step-4:- Initialize $temp \rightarrow data$.

Step-5:- set $p = root$

Step-6:- Repeat step 7 until $p \rightarrow link \neq root$.

Step-7:- $p = p \rightarrow link$

Step-8:- $p \rightarrow link = temp$.

Step-9:- $temp \rightarrow link = root$

Step-10:- Stop.

Algorithm to delete node at beginning.

Step-1:- Start

Step-2:- If $root == NULL$

print "Underflow"

goto step 10.

Step-3:- step $= root$

Step-4:- Repeat step 5 until $p \rightarrow link \neq root$.

Step-5:- set $p = p \rightarrow link$

Step-6:- Set $p \rightarrow link = root \rightarrow link$

Step-7:- $root \rightarrow link = NULL$.

Step-8:- free($root$)

Step-9:- $root = p \rightarrow link$

Step-10:- stop.

Algorithm to delete node at ending.

Step-1:- start.

Step 2:- if root == NULL
print underflow
goto step 10

Step 3:- Set p = root

Step 4:- Repeat step 5,6 until $p \rightarrow link_1 = \text{root}$

Step 5:- $P = P \rightarrow link_1$

Step 6:- set preptr = p

Step 7:- $\text{preptr} \rightarrow link = \text{root}$

Step 8:- $P \rightarrow link = \text{NULL}$

Step 9:- free(p)

Step 10:- stop.

Applications of linked list

- 1) Implementation of stacks and queues.
- 2) Implementation of graphs. (adjacency representation of graphs is most popular which uses linked list of adjacent vertices)
- 3) Dynamic memory allocation.
- 4) Maintaining directory of names.
- 5) performing arithmetic operations on long integers.
- 6) Manipulation of polynomials by storing coefficients in the node of linked list.
- 7) Representing sparse matrix.

Applications in real world

- 1) Image viewer. (previous & next images are linked to be accessed by using previous & next buttons)
- 2) Music player.
- 3) previous & next pages in web browser.
- 4) circular linked list is used in task management application of operating system.

Consider the Polynomials $x_1 = 7x^4 + 5x^2 + 3x$, $x_2 = 5x^3 + 3x - 8$

$$x_1 = \left\{ \begin{array}{|c|c|c|c|} \hline \text{Coeff} & 7 & 5 & 3 \\ \hline \text{Exp} & 4 & 2 & 1 \\ \hline \end{array} \right\}, x_2 = \left\{ \begin{array}{|c|c|c|c|} \hline \text{Coeff} & 5 & 3 & -8 \\ \hline \text{Exp} & 3 & 1 & 0 \\ \hline \end{array} \right\}$$

Develop a C program to perform addition of 2 polynomials using array.

Struct Poly

```
int Coeff, Exp;
```

```
};
```

```
Struct poly P1[10], P2[10], P3[10];
```

```
void readPoly (Struct poly P[], int *);
```

```
int addPoly (Struct Poly R[], Struct poly P[], int t1, int t2);
```

```
void display (Struct poly P[], int t);
```

Void main()

{

```
int t1, t2, t3;
```

```
printf ("Enter no of terms in Polynomial 1");
```

```
scanf ("%d", &t1);
```

```
readPoly (P1, t1);
```

```
printf ("Enter no of terms in polynomial 2");
```

```
scanf ("%d", &t2);
```

```
readPoly (P2, t2);
```

```
printf ("representation of polynomial 1 is ");
```

```
display (P1, t1);
```

```
printf ("representation of polynomial 2 is ");
```

```
display (P2, t2);
```

```
P3 = addPoly (P1, P2, t1, t2);
```

```
printf ("addition of 2 polynomials is ");
```

```
display (P3, t3);
```

```

Void readPoly (Struct Poly P[0], int t)
{
    int i;
    Poly (i = 0; i < t; i++)
}

printf ("In Enter Coefficient %.d", i+1);
scanf ("%d", &P[i].coeff);

printf ("In Enter Exponent %.d", i+1);
scanf ("%d", &P[i].exp);
}

int addPoly (Struct Poly P1[10], Struct Poly P2[10], int t1, int t2)
{
    int i = 0, j = 0, k = 0;
    while (i < t1 && j < t2)
    {
        if (P1[i].exp > P2[j].exp)
        {
            P3[k].coeff = P1[i].coeff;
            P3[k].exp = P1[i].exp;
            i++; k++;
        }
        else if (P1[i].exp < P2[j].exp)
        {
            P3[k].coeff = P2[j].coeff;
            P3[k].exp = P2[j].exp;
            j++; k++;
        }
        else
        {
            P3[k].coeff = P1[i].coeff + P2[j].coeff;
            P3[k].exp = P1[i].exp;
            i++; j++; k++;
        }
    }
}

```

```

while (i < t)
{
    P3[k].coeff = P1[i].coeff;
    P3[k].exp = P1[i].exp;
    i++; k++;
}

while (j < t2)
{
    P3[k].coeff = P2[j].coeff;
    P3[k].exp = P2[j].exp;
    j++; k++;
}

return K;
}

void display(struct poly P[], int t)
{
    int i;
    for (i=0; i < t; i++)
    {
        printf("%d x^%d", P[i].coeff, P[i].exp);
        if (i < t-1)
            printf("+");
    }
}

```

Sparse Matrix: A sparse matrix is a matrix which contains more no of 0's & very few non-zero elements.

Elements

$$\text{Ex: } \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix}$$

When a Sparse matrix is represented in a 2D array we waste a lot of space to represent that matrix because the matrix will contain majority of 0's.

For Example: If a matrix contains 5 rows & 6 columns total 30 elements. Then Compiler allocates $30 \times 4 = 120$ bytes. But the actual no of non zero elements are only 6. That means the size of 24 bytes is enough is that means the size of 24 bytes is enough for matrix & we are wasting 96 bytes also. So Compiler will store all 30 elements. To reduce this drawbacks we can use a Triplet Representation of Sparse matrix.

Triplet Representation: In this we consider only non zero values along with their rows & columns index values. So the resultant matrix contains 3 columns in 0th row (i) Total no of rows (ii) Total no of columns (iii) Total no of non zero elements.

0	1	2	3	4	5
0	0	0	0	9	0
0	8	0	0	0	0
2	4	0	0	2	0
3	0	0	0	0	5
4	0	0	0	0	0

5	6	6
0	4	9
1	1	8
2	0	4
3	3	2
3	5	5
4	2	2

Transpose of Sparse Matrix: The first row of the triplet matrix the row value is changed to column value, column value to row value. From and row check the column value for index 0 & interchanging row to column, column to row value. Non zero value remains. like this repeat the column index checking upto end of matrix.

Triplet

5	6	6
0	4	9
1	1	8
2	0	4
3	3	2
3	5	5
4	2	2

Transpose

6	5	6
0	2	4
1	1	8
2	4	2
3	2	2
4	0	9
5	3	5

Write an algorithm to stored the values in a matrix of order $m \times n$ & then print its triplet representation & also find transpose of matrix.

Step 1: Start

Step 2: Declare m, n, S[0][10], j, i, t, K, Element

Step 3: Read i=0, j=0, m, n, t=0

Step 4: Repeat Step 5 until i < m

Step 5: Repeat Step 6, 7 until j < n

Step 6: Read Element

Step 7: If Element != 0

Step 7.1: Set t + t

Step 7.2: Set S[t][0] = i

Step 7.3: Set S[t][1] = j

Step 7.4: Set S[t][2] = Element

Step 8: S[0][0] = m

Step 9: S[0][1] = n

Step 10: S[0][2] = t

Step 11: Display S[0][10]

Step 12: Stop

* Stack *

A Stack is an important data structure which stores its elements in an ordered manner. It uses a principle called "LIFO" that means the element that was inserted last was the first one to be taken out.

A Stack is used to store function calls.

Ex :- If there are

A Stack is a linear data structure in which elements are added or removed from stack from one end called TOP.

Stack representation :-

A Stack is represented using i) array ii) linked list

(i) Array :- Every Stack has a variable called top which is used to store the address of the top most element of stack. There is another variable called MAX which is used to store the max number of elements the stack can hold. If $\text{TOP} = \text{MAX} - 1$ then we can say that Stack is in overflow condition.

Similarly if $\text{TOP} == -1$ then Stack will be in underflow condition.

Algorithm for push operation in Stack :-

Step 1 : Start

Step 2 : If $TOP == MAX - 1$ then print "overflow" goto step 6

3 : Read val

4 : Set $TOP = TOP + 1$

5 : Set $Stack[TOP] = val$

6 : STOP

Algorithm for pop operation in Stack :-

Step 1 : Start

2 : If $TOP == -1$ then print "underflow" goto step 5

3 : Print $Stack[TOP]$

4 : Set $TOP = TOP - 1$

5 : STOP

Algorithm for peek operation :-

Step 1 : Start

2 : If $TOP == -1$ then print "underflow" go to step 4

3 : Print $Stack[TOP]$

4 : STOP.

```

#define Max 10

int Stack [max], top = -1;
Void Push(); Void Pop(); Void Peek(); Void display();

Void main()
{
    int val, opt;
    while(1)
    {
        printf("1. Enter Element\n2. Push\n3. Peek\n4. display\n5. Exit");
        scanf("%d", &opt);
        switch(opt)
        {
            Case 1: Push();
            break;
            Case 2: Pop();
            break;
            Case 3: Peek();
            break;
            Case 4: display();
            break;
            Case 5: Exit(0);
            default: printf("Overflow");
        }
    }
}

Void Push()
{
    int val;
    if (top == max-1)
        printf("Overflow");
    else
        printf("Enter Element");
        scanf("%d", &val);
        top = top + 1;
}

```

Stack [top] = val

Pointf ("Element is inserted")

{

}

void push()

{ if (top == -1)

Pointf ("Underflow")

else

{

Pointf ("The element deleted is %d", Stack [top]);

top = top - 1;

}

}

void peek()

{ if (top == -1)

Pointf ("Underflow")

else

Pointf ("top most Element is %d", Stack [top])

}

void display()

{

int i,

if (top == -1)

Pointf ("No Element Found")

else

{

for (i = top; i >= 0; i--)

{

Pointf ("%d", Stack [i])

{

}

Stack Representation Using linked list :

Stack is represented as collection of nodes in linked list. Each node contains two consecutive data & link. Data contains elements & link contains address to next node. The elements are added or removed at one end of linked list i.e. at the beginning of link list. The disadvantage of array representation of stack is the array occupies fixed memory so we can't expand the memory during execution.

If the list is empty then "top" contains NULL.

Algorithm for Push operation:

Step 1: Allocate memory for node & name it as top

Step 2: Read temp \rightarrow data

Step 3: If $top = \text{NULL}$ then

3.1: Set $temp \rightarrow \text{link} = \text{NULL}$

3.2: Set $top = temp$

3.3: Else Set $temp \rightarrow \text{link} = top$

3.4: Set $top = temp$

Step 4: STOP

Algorithm for pop operation:

Step 1: If $top = \text{NULL}$ then print "No elements in list"

Step 2:0: Else Set $P = top$

2.1: Set $top = P \rightarrow \text{link}$

2.2: Set $P \rightarrow \text{link} = \text{NULL}$

Step 2: free(p)

Step 3: STOP

Algorithm for Peek operation:-

Step 1: If top == NULL point "No Elements in the list"

Step 2: Else Point top \rightarrow data

Step 3: STOP.

Algorithm for traversing linked list:-

Step 1: If top == NULL point "No Elements in the list"

Step 2: Else

2.1: Set P = top

2.2: While P != NULL

2.3: Set P = P \rightarrow link

2.4: Point P \rightarrow data

Step 3: STOP

Applications of Stacks:-

* Stacks are used for conversion of infix to Post fix expression

* Stack is used to implement Towers of hanoi

Types of Notations:-

(i) Infix : In this notation operator is placed in b/w operands Ex: A + B

(ii) Post fix : It is called as reverse polish notation
The idea of forming post fix notation / prefix notation
is to develop a non-associative Free Expression

In Postfix Notation operators are placed after the operand

$$\text{Ex: } (A+B)*C = (AB+)*C = ABC + C*$$

$$(A-B)*(C-D) = (AB-)* (CD-) = ABC - CD - *$$

(iii) Postfix: It is evaluated from left to right like Infix but difference is operator is placed before operand

$$\text{Ex: } (A+B)*C = (+AB)*C = * + ABC$$

$$(A-B)*(C+D) = (-AB)*(+CD) = * -AB + CD$$

Infix to Postfix: $A - (B/C + (D \% E * F)/G) * H$

The algorithm uses a Stack to temporarily hold operators
§ Postfix Expression holds operators & operators that are popped from Stack. For conversion

Step1: Add "}" at end of expression

$$A - (B/C + (D \% E * F)/G) * H)$$

Input character to Scan Stack Postfix Expression

	(
A	C	A
-	(-	A
((-	A
B	(-C	AB
/	(-CI	AB
C	(-CI	ABC
+	(-C(+	ABC/
((-C+(ABC/
D	(-C+(C	ABC/D
%	(-C+(C%	ABC/D

E	$(-C + (%))$	ABC/DE	$\boxed{*/\cdot + -}$
*	$(-C + (%)*)$	ABC/DE	
F	$(-C + (%*))$	ABC/DEF	
)	$(-C +)$	ABC/DEF-%	
/	$(-C + /)$	ABC/DEF-*%	
G	$(-C + /)$	ABC/DEF-*%G	
)	$(-C$	ABC/DEF-*%G/H	
*	$(-*))$	ABC/DEF-*%G/H	
H	$(-*))$	ABC/DEF-*%G/H+H	
)	Empty	ABC/DEF-*%G/H+H*-	

$$\therefore A - (B/C + (D * E * F) / G) * H = ABC/DEF-*%G/H+H*-$$

Algorithm :

Step 1: Add ")" at the end of infix expression

Step 2: push left bracket "(" on to the Stack

Step 3: Repeat until each character in infix character scanned

3.1: If "(" is encountered push it on to Stack

3.2: If an operand (where a digit or alphabet) is encountered then add into, Post fix Expression

3.3: If an enclosed bracket is encountered

3.3.1: Repeatedly from Stack is add it to Post fix Expression until a ")" is encountered

3.3.2: Discard "(" & ")" & don't add them to Post fix Expression

3.4: If an operator "O" is encountered then repeatedly pop from Stack & add each operator to Post fix Expr

which has highest precedence than '*' :

Step 4: Repeatedly Pop from Stack & add char to Post.

Run until the Stack is empty L.P * / +

Step 5: Stack L.P + -

Expression Evaluation using Stacks :-

Convert the following Expr into Postfix & Evaluate it.

Expression $9 - ((3 * 4) + 8) / 4$

$9 - ((\underline{3 * 4}) + 8) / 4$

$9 - (\frac{3 \underline{4 *} 8}{A} + B) / 4$

$9 - (\frac{3 \underline{4 *} 8}{A} + \frac{B}{B})$

$\frac{9 - 3 \underline{4 *} 8 + 4}{B}$

$9 3 4 * 8 + 4 -$

Input Character to Scan

Stack

9

9

3

9, 3

4

9, 3, 4

*

9, 18 (9, 3 * 4)

8

9, 18, 8

+

9, 20 (9, 18 + 8)

4

9, 20, 4

/

9, 5 (9, 20/4)

-

4 (9-5)

Evaluate the Expression $2 * (4 + 6)$

Input Character/Scan	Stack
2	2
4	2, 4
6	2, 4, 6
+	2, 10
*	20

Algorithm for Evaluation of Postfix Expression:

Step 1: If an operator is encountered push it onto Stack.

Step 2: If an operator "O" is encountered then

2.1: Pop top 2 elements from Stack & name them as A & B. A is top most element, B is element below A.

2.2: Evaluate $B O A$

2.3: Push the result into Stack

Step 3: STOP.

Queue: A Queue is a FIFO data structure.

Containing two ends the left end is called front & right end is called rear. Insertion of element is done from rear & deletion of element done from front.

A Queue can be implemented using

(i) array (ii) linked list.

Queue Representation Using Array

Algorithm for insertion:

Step 1: If Rear = MAX - 1 then

Print "Queue is overflow"

Go to Step 5

Step 2: If Rear == -1 and Front == -1

then Set Front = Rear = 0

Step 3: else

Set * Queue = Rear + 1

Step 3: Read (x) from keyboard or input or file

Step 4: Set Queue [Rear] = x

Step 5: STOP.

Algorithm for deletion:

Step 1: If (Front == -1 || Front > Rear) then
^{elements are deleted}

Print ("Queue is empty");

Go to Step 4.

Step 2: Print Queue [Front]

Step 3: Set Front = Front + 1.

Step 4: STOP

Algorithm for display:

Step 1: If (Front == -1 || Front > Rear) then

Print "Queue is empty"

Go to Step 6

Step 2: Get i = Front

Step 3: Repeat the steps until i < Rear

Step 4: Print Queue [i]

Step 5: Increment i by 1

Step 6: STOP

Queue representation using linked list:

Struct node

{

int data;

Struct node *link;

}

Struct node *front=NULL, *rear=NULL;

Void insert()

{

Struct node *temp;

temp = (Struct node *) malloc (sizeof (Struct node));

Point f ("Enter data");

Scanf ("%d", &temp->data);

temp->link = NULL;

if (rear == NULL)

{ Point f ("Data")

rear = temp; } front = rear = temp;

front = temp;

}

else

{

rear->link = temp;

rear = temp;

}

Void delete()

{

if (front == NULL)

Point f ("List is Empty");

else

{

Point f ("Deleted Element is %d", front->data);

```
Struct Node *P;
```

```
P = Front;
```

```
Front = P->link;
```

```
P->link = NULL;
```

```
Free (P);
```

```
3      void display()
```

```
{ struct node *temp = Front;
```

```
if (Front == NULL)
```

```
    printf ("List is empty");
```

```
else
```

```
    while (temp->link != NULL)
```

```
{
```

```
        printf ("%d", temp->data);
```

```
        temp = temp->link;
```

```
void main()
```

```
{
```

```
int opt;
```

```
while (1)
```

```
{
```

```
    printf ("Enter choice 1. insert 2. delete 3. display");
```

```
scanf ("%d", &opt);
```

```
switch (opt)
```

```
{
```

```
    case 1: insert();
```

```
        break;
```

```
    case 2: delete();
```

```
        break;
```

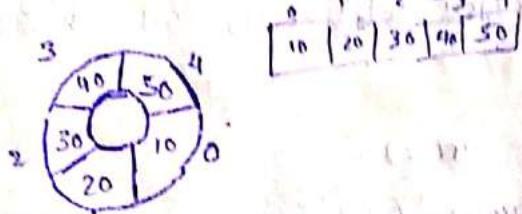
```
    case 3: display();
```

```
        break;
```

```
    case 4: exit (0);
```

Circular Queue :- A Queue is represented in Circular fashion that means we can traverse from last to first & again from last to first.

Circular Queue representation using array will be.



Write a program to implement operations on Circular Queue using array

```
#define size 5
int front = -1, rear = -1, q[size];
void insert()
{
    int val;
    if (size == size + 1 || front == size + 1) // when we crossed array boundary
        cout("Queue is full");
    else if (front == -1 && rear == -1)
        front = rear = 0;
    q[rear] = val;
    else if (rear == size - 1)
        rear = 0;
    q[rear] = val;
}
else
    rear += 1;
q[rear] = val;
```

Void delete()

{

if (Front == -1 && Rear == -1)

printf ("Queue is Empty");

if (Front == Size - 1)

printf ("Element deleted is %d", q[Front]);

Front = 0;

else if (Front == Rear)

printf ("Element deleted is %d", q[Front]);

Rear = Front = -1;

else

{

printf ("Element deleted is %d", q[Front]);

Front += 1;

}

Void display()

{

int i;

if (Front <= Rear)

> if (Front == -1 && Rear == -1)

for (i = Front; i <= Rear; i++) printf ("Queue is empty");

printf ("%d", q[i]);

}

else

{

for (i = 0; i <= Rear; i++)

printf ("%d", q[i]);

for (i = Front; i < Size; i++)

printf ("%d", q[i]);

}

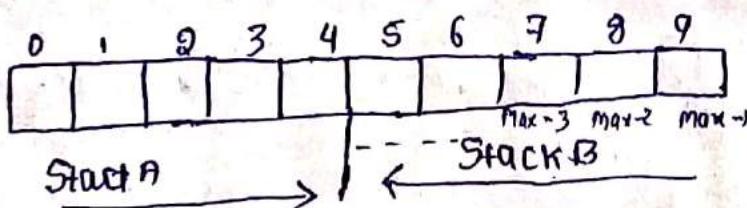
}

(void main)

Multi Stack: It is multiple implementation of stacks in a single array. While implementing a stack using array the size of array must be known in advance. If the stack is allocated less space then frequent overflow condition will be encountered. So in order to increase the efficiency of program & save the memory space then we have a solution called multiple stacks where more than one stack is used in same array.

If 2 stacks are represented as Stack A & Stack B
Stack A will grow from left \rightarrow right & Stack B will increase from right \rightarrow left.

$$\cdot \text{MAX} = 10$$



Algorithm for Push A

Step 1: If $\text{TOPA} = \text{TOPB} = 1$ then

Point "Stack is full"

Go to Step 5

Step 2: Read value

Step 3: Set $\text{TOPA} = \text{TOPA} + 1$

Step 4: $\text{Stack}[\text{TOPA}] = \text{val}$

Step 5: Stop

Algorithm for Push 'B'

~~Step 1:~~ if ($\text{top}_B = \text{top}_B - 1$)

 printf("Stack is full");

 go to step 5;

~~Step 2:~~ Read value;

~~Step 3:~~ set $\text{top}_B = \text{top}_B - 1$

~~Step 4:~~ set $\text{stack}[\text{top}_B] = \text{val}$;

~~Step 5:~~ Stop

Algorithm for Pop 'B'

~~Step 1:~~ if $\text{top}_A == -1$

 printf("Stack is empty")

 go to step 4

~~Step 2:~~ Print $\rightarrow \text{stack}[\text{top}_A]$

~~Step 3:~~ top_A--

~~Step 4:~~ Stop

Algorithm for Pop 'A'

~~Step 1:~~ if $\text{top}_B == \text{MAX} - 1$

 Print \rightarrow "Stack is empty"

 go to step 4

~~Step 2:~~ Print $\rightarrow \text{stack}[\text{top}_B]$

~~Step 3:~~ top_B++

~~Step 4:~~ Stop

Algorithm for display 'A':

Step 1: If ($\text{top A} = -1$)
Point \rightarrow "stack is empty?"
go to step 6

Step 2: ~~Repetition loop~~

Step 2: set $i := \text{TOP A}$

Step 3: Repeat the following steps, until $i >= 0$

Step 4: Point $\rightarrow \text{stack}[i]$

Step 5: Decrement i by 1

Step 6: Stop

Algorithm for display 'B':

Step 1: If ($\text{top B} = \text{MAX}$)
Point \rightarrow "stack is empty?"
go to step 6

Step 2: set $i := \text{TOP B}$

Step 3: Repeat following step until $i < \text{max}$

Step 4: Point $\rightarrow \text{stack}[i]$

Step 5: Increment i by 1

Step 6: Stop

Towers of Hanoi:

It is mathematical problem which contains 3 poles. A is source, C is destination, B is spare pole. Using B we have to shift all rings from A to C. This problem is solved by using recursion. All the recursive calls will be placed inside stack. The condition for problem is larger rings will be at bottom of and smaller rings at top of pole. We can move one ring at a time.

```
Main() {
```

```
    int n;
```

```
    printf("Enter no of rings:");
```

```
    scanf("%d", &n);
```

```
    move(n, 'A', 'C', 'B');
```

```
}
```

```
Void move(int n, char source, char dest, char spare)
```

```
{
```

```
    if (n == 1)
```

```
        printf("Move from %c to %c", source, dest);
```

```
    else
```

```
{
```

```
    move(n - 1, source, spare, dest);
```

```
    move(1, source, dest, spare);
```

```
    move(n - 1, spare, dest, source);
```

```
}
```

