Unit - 1

Basic Statistics

Central tendency or Averages :-

A single expression representing the hole group is selected which may convey a fairly adopted idea about the hole group. This single expression in statistics is known as Average.

Averages are generally the central part of the distribution and therefore there also called the measure of the control tendency.

These are 5 steps :-

1. Arithematic Mean (A.M)
2. Median
3. Mode
4. Geomatric Mean (G.M)
5. Harmonic Mean (H.M)

1. Arithematic Mean :- (A.M)

Case 1 :-

We know that in an ungrouped data we are given individual items and also w.k.T the avg of 'n' numbers is obtained by finding the sum and then dividing it by n.

let $x_1, x_2 \ldots x_n$ are n observations then their

$\therefore$ A.M (or) $\bar{x}$ = $\boxed{\dfrac{x_1 + x_2 + \cdots x_n}{n}}$

Example :-

find the A.M of the marks obtained by 10 students in Mathematics examination the marks obtained by 25, 30, 21, 55, 47, 10, 15, 17, 45, 35

$$A.M = \frac{25+30+21+55+47+10+15+17+45+35}{10}$$

$$A.M = 30$$

Case 2 :-

If n observations in the ungrouped data consists of n distinct values. $x_1, x_2, x_3 \cdots x_n$ accuring with frequency $f_1, f_2 \cdots f_n$ respectively.

The A.M of variable x is $\dfrac{f_1x_1 + f_2x_2 + \cdots f_nx_n}{f_1+f_2+\cdots f_n}$

(or)

$$\boxed{\frac{\Sigma f_ix_i}{\Sigma f_i}}$$

Problem :-

① find the A.M of the following frequency table

Marks (x_i)	52	58	60	65	68	70	75
No. of Students (f_i)	7	5	4	6	3	3	2

Marks (x_i)	No. of Students (f_i)	f_ix_i
52	7	364
58	5	290
60	4	240
65	6	390
68	3	204
70	3	210
75	2	150
	30	1848

$\therefore$ A.M (or) $\bar{x} = \dfrac{\Sigma f_ix_i}{\Sigma f_i} \Rightarrow \dfrac{1848}{30} \Rightarrow 61.6.$

(2) The following tables given the no.of childrens of 150 families in a villages find the avg. no.of children per family.

No.of children	0	1	2	3	4	5
No.of families	10	21	55	42	15	7

x_i	f_i	$f_i x_i$
0	10	0
1	21	21
2	55	110
3	42	126
4	15	60
5	7	35
	150	352

$\therefore$ A.M (or) $\bar{x} = \frac{352}{150} = 2.346$

(3) find the Mean of the following marks obtained by a class students of a class.

Marks	15	20	25	30	35	40
Students	9	7	12	14	15	6

x_i	f_i	$f_i x_i$
15	9	135
20	7	140
25	12	300
30	14	420
35	15	525
40	6	240
	63	1760

$\bar{x} = \frac{1760}{63}$

$= 27.936$

Case 3 :-

In this case the grouped data is presented in the form of a frequency distribution with class intervels.

NOW. A.M is (or) $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

Where x_i is Mid values of the highest class intervel.

Problems :-

(1) The data of no. of patients attending a hospital in a month given below then find the avg no. of patients attending the hospital per a day.

No. of patients	0-10	10-20	20-30	30-40	40-50	50-60
No. of days	2	6	9	7	4	2

No. of patients	Mid value (x_i)	No. of days (f_i)	$f_i x_i$
0-10	5	2	10
10-20	15	6	90
20-30	25	9	225
30-40	35	7	245
40-50	45	4	180
50-60	55	2	110
		30	860

$\therefore \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{860}{30} = 28.66$

② find the Mean of the following data.

class interval	0-8	8-16	16-24	24-32	32-40
frequency	6	7	10	8	9

class interval	Mid value (x_i)	frequecy (f_i)	f_ix_i
0 - 8	4	6	24
8 - 16	12	7	84
16 - 24	20	10	200
24 - 32	28	8	224
32 - 40	36	9	324
		40	856

$$\therefore \bar{x} = \frac{856}{40} = 21.4$$

③ find the Mean of the following data.

C.I	25-35	35-45	45-55	55-65	65-75
frequency	6	10	8	12	4

C.I	Mid value (x_i)	frequecy (f_i)	f_ix_i
25-35	$\cancel{6}$ 30	6	180
35-45	$\cancel{10}$ 40	10	400
45-55	$\cancel{8}$ 50	8	400
55-65	$\cancel{12}$ 60	12	720
65-75	$\cancel{4}$ 70	4	280
		40	1980

$$\bar{x} = \frac{1980}{40} = 49.5$$

Merits of Mean :-

* It can be easily calculated
* Mean based on all the observations
* It is easy to understand
* It is the best measure to compare two or more Series.

De Merits of Mean :-

* It can not be calculated if all the values are not known.
* It can not be determined for the qualitative data such as love, honesty, Angry etc..

Uses of Mean :-

* It is extensivly used in practical statistics
* A common man uses need for calculating average marks obtained by a students.
* A Business Man uses to find out the operation cost profit per unit of capital, output for man and per Machine average monthly income etc...

Scanned by CamScanner

2. Median :-

Case 1 :-

Median is defined as the Middle most or the central value of the observations, then the observations are arranged in ascending or descending order.

When a Series consists of a even no.of terms than median is the Mean of the two central values

Example :-

(1) The no.of terms runs scored by 11 players of a cricket team are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27

Now We given can arrange the given sequences are ascending order.

6^{th} position.

0, 5, 11, 19, 21, 27, 30, 36, 42, 5, 52.

∴ Given no. of runs scored = 11

Where n is 11

Hence the 'n' is odd number we can use the

formula : $M = \frac{n+1}{2}$

Median $= \frac{11+1}{2} = 12/2 = 6$

∴ 6^{th} position is Median

(2) find the Median of the following data.

6, 10, 3, 4, 9, 11, 28, 18

first We can arrange ascending Order

3, 4, 6, 9, 10, 11, 18, 28

∴ Given no.of position = 8

Where n is 8

Hence the 'n' is even we can use formula $M = N/2$

$$M = \frac{8}{2} = 4.$$

4th and 5th positions are average

$$\frac{9+10}{2} = \frac{19}{2}$$

$$= 9.5$$

③ The following table represents the marks obtained by a batch of 12 Students in a certain class test in statistics and physics.

S.NO	1	2	3	4	5	6	7	8	9	10	11	12
Stat ~~& phy~~	53	54	32	30	60	46	28	25	48	72	33	65
phy	55	41	48	49	27	25	23	20	28	60	43	67

In this case in which subject is the level of achievement is higher.

Let us arrange the marks in two subjects in accending order.

S.NO	1	2	3	4	5	6	7	8	9	10	11	12
Stat	25	28	30	32	33	46	48	53	54	60	65	72
phy	20	23	25	27	28	41	43	48	49	55	60	67

Median marks in statistics $= \frac{46+48}{2} = 47$

Median marks in physics $= \frac{41+43}{2} = 42$

Here the median marks in stat are greater than the Median marks of the physics.

∴ the level of acheivement of the Student higher in statistics.

Scanned by CamScanner

Case 2 :-

* Arrange the values of the variable in ascending or dissending order.

* point in cummulative frequency.

* calculate $\frac{N}{2}$ where $N = \Sigma fi$

* finding cummulative frequency jest $> \frac{N}{2}$ then corresponding variable is it's Median

Problem :-

① Calculate Median from the following table

x :	5	7	9	12	14	17	19	21
f :	6	5	3	6	5	3	2	4

first we have to arrange ascending order.

xi	fi	C.F
5	6	6
7	5	6+5 = 11
9	3	14
12	6	20
14	5	25
17	3	28
19	2	30
21	4	34
	$\Sigma fi = 34$	

Median

$N = \frac{\Sigma fi}{2}$

$= \frac{34}{2} = 17$

Here C.F > N/2

Here 20 is the c.F jest greaterthan N/2.

∴ The corresponding variable of C.F is M required Median.

② Calculate Median from the following data.

Marks (x) :	20	9	25	50	40	80
No. of Students (f) :	6	4	16	7	8	2

first we have to arrange ascending order.

x_i	f_i	C.F
9	6	6
20	4	10
25	16	26
40	7	33
50	8	41
8	2	43
	Σf_i 43	

Median.

$N = \frac{\Sigma f_i}{2}$

$= \frac{43}{2} = \frac{21.5}{2} = 21.5$

Here C.F > N/2

∴ Here 26 is the C.F just greater than N/2

∴ The corresponding variable of C.F is required Median.

Case 3 :-

In this case grouped data given in the table form of a frequency table with class interval — then Median is given by $l + \left(\frac{\frac{N}{2} - CF}{f}\right) \times c$

Where small 'l' is lower limit of class interval in which the Median lies.

$N \longrightarrow \Sigma f_i$ (Sum of the frequency)

$f \longrightarrow$ frequency of the C.I in which the median lies

Scanned by CamScanner

C.F of the class interval preceeding the Median class and c is the width of the class Interval.

problem :-

① The following table gives the weekly expendature of 100 familyes than find the median

Weekly expendature :-	0 - 10	10-20	20-30	30-40	40-50
No.of families :-	14	23	27	21	15

x_i	f_i	C.F
0-10	14	14
10-20	23	37 → CF
20-30	27 → F	64
30-40	21	85
40-50	15	100
	$\Sigma f_i = 100$	

l=20

$$\frac{N}{2} = \frac{\Sigma f_i}{2}$$

$$= \frac{100}{2} = 50$$

Given $N = 100 \Rightarrow N/2 = 50$

$$\text{Medium} = l + \left(\frac{\frac{N}{2} - CF}{f}\right) \times c$$

$$= 20 + \left(\frac{50 - 37}{27}\right) \times 10$$

$$= \cancel{23.22} \quad 24.81$$

② The scores on a reading comprehensive test 1000 students are given below then find its Median

scores :-	0-5	5-10	10-15	15-20	20-25	25-30	30-35
frequency :-	6	12	50	120	225	250	185

35-40	40-45	45-50
110	32	10

Scanned by CamScanner

x_i	f_i	CF
0-5	6	6
5-10	12	18
10-15	50	68
15-20	120	188
20-25	225	413 - CF
25-30	250 - F	663
30-35	185	848
35-40	110	958
40-45	32	990
45-50	10	1000
	$\Sigma f_i = 1000$	

l

$\frac{N}{2} = \frac{\Sigma f_i}{2}$

$= \frac{1000}{2} = 500$

Given $N = 1000$

$N/2 = 500$

Median $= l + \left(\frac{\frac{N}{2} - CF}{f} \right) \times c$

$= 25 + \left(\frac{500 - 413}{250} \right) \times 5$

$= 25 + 1.74$

$= 26.74$

(3) The following table gives the marks obtained by the 50 students in statistics exam. Then find its Median.

Marks :-	10-14	15-19	20-24	25-29	30-34	34-39	40-44
No. of students :-	4	6	10	5	7	3	9

45-49

6

xi	xi	fi	CF
10 - 14	9.5 - 14.5	4	4
15 - 19	14.5 - 19.5	6	10
20 - 24	19.5 - 24.5	10	20
25 - 29	24.5 - 29.5	5	25
30 - 34	29.5 - 34.5	7	32
34 - 39	34.5 - 39.5	3	35
40 - 44	39.5 - 44.5	9	44
45 - 49	44.5 - 49.5	6	50
		Σfi 50	

$$N/2 = 50/2 = 25$$

$$\text{Median} = l + \left(\frac{N/2 - C.F}{f} \right) \times c$$

$$= 29.5 + \left(\frac{25 - 25}{7} \right) \times 4$$

$$= 29.5 + 0$$

$$= 29.5.$$

Merits of Median :-

* It can be easily under
* It is not effected by extrem values
* It can be located graphycally
* It can be easily located even if the class intervels are unequal.

De Merits of Median :-

It can not represents when irregular distribution of series.

It can be estimated in case of a series containing even no. of items.

It doesn't take values of the all items in this series.

It is a positional avg and is based on the Middle item

Uses :-

* It is useful in those causes when numerical meserments are not possible.
* It is also useful from the those causes where marthematically calculations can not be order to obtain the main.

3. Mode :-

Case 1 :-

Mode is the value of a series at which occures most frequently.

ex 1 :-

for example the series is 6, 5, 3, 4, 3, 7, 8, 5, 9, 5, 4 then find it's mode.

Sol :- Here 5 occures most frequent

ex 2 :-

In this series 2, 2, 2, 5, 5, 5, 6, 6, 7, 8, find the mode.

Sol :- 2 and 5 are modes.

Case 2 :- This is also mode on grouped data.

In a frequency distribution mode is the variate which has the max frequency.

ex :- find the mode of the following distribution.

x_i :	4	5	6	7	8	9	10
f_i :	10	15	20	35	16	3	1

find the mode of given data.

from the given table the size 7 has the max frequency $\therefore$ Mode = 7.

Case 3 :-

for grouped data Mode can be calculated from the following formula $l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2} \right) \times c$

Where 'c' is the width of the class interval

'l' is the lower limit Model class

'f_m' is frequency of Model class

'f_1' is frequency of the class preceeding the Model class.

'f_2' is frequency of class succeeding the Model class.

Example :- 1

find the Mode of the following data.

Marks :	1-5	6-10	11-15	16-20	21-25
No.of Students :	7	10	16	32	24

Marks(x_i)	No.of Students (f_i)
0.5-5.5	7
5.5-10.5	10
10.5-15.5	16 f_1
15.5 - 20.5 l	32 f_m
20.5-25.5	24 f_2

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2} \right) \times c$$

$$= 15.5 + \left(\frac{32 - 16}{2(32) - 16 - 24} \right) \times 5$$

$$= 15.5 + \left(\frac{16}{24} \right) \times 5$$

$$= 15.5 + 3.33$$

$$= 18.83$$

Example 2 :-

find the Mode of the following data distribution

production per - day :	21-22	23-24	25-26	27-28	29-30
No. of days :	7	13	22	10	8

Production(xi)	No.of days(fi)
20.5 - 22.5	7
22.5 - 24.5	13 f_1
24.5 - 26.5	22 - f_m
26.5 - 28.5	10 f_2
28.5 - 30.5	8

l

$$\text{Mode} = 24.5 + \left(\frac{22-13}{2(22)-13-10}\right)\times 2$$

$$= 24.5 + \left(\frac{9}{21}\right)\times 2$$

$$= 25.35$$

Merits of Mode :-

* It is easily understood.
* It can be located by the inspection.
* It is not effected by the extreme values.
* It is capable of being determine graphycally.

De Merits of Mode :-

* Sum series have two or more modes.
* Mode for the series with unequal class intervels cannot be calculated.
* The combined mode cannot be calculated for the modes of two series.

Uses :-

* It is extensivly used by business man and commercial managements.
* It is used for the study of most problem fashion.

Standard deviation :- (S.D)

It is the +ve square root of average of square deviations taken from A.M it is denoted with (σ) sigma.

Note :- Standard deviation is also known as root mean - square deviation.

Case 1 :-

Suppose the data is ungrouped without frequencyes let $x_1, x_2 - - - x_n$ are n given observations the nex S.D is given by the formula :

$$S.D = \sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} \quad (\text{or}) \quad \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$\bar{x}$ is mean of given observation

n is no. of observations.

Example :-

find S.D of 16, 13, 17, 22

$$\text{Mean } \bar{x} = \frac{16+13+17+22}{4} = 17$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
16	16-17 = -1	1
13	-4	16
17	0	0
22	5	25
		$\Sigma(x_i-\bar{x})^2 = 42$

$$\therefore S.D = \sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{42}{4}}$$

$$= 3.0403$$

(or)

x	x^2
16	256
13	169
17	289
22	484
$\Sigma x = 68$	$\Sigma x^2 = 1198$

$$\sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\sqrt{\frac{1198}{4} - \left(\frac{68}{4}\right)^2}$$

$$\sqrt{299.5 - 289}$$

$$= 3.2403$$

$\frac{4624}{16}$

1156

Case 2 :-

Suppose the ungrouped data occuring with frequency then the S.D is calculated by the formula.

$$S.D = \sigma = \sqrt{\frac{\Sigma f(x_i - \bar{x})^2}{\Sigma f}} \quad (or) \quad \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

Example :-

find S.D of the following Table.

Size of item x	10	11	12	13	14	15	16
frequency f	2	7	11	15	10	4	1

x_i	f_i	$f_i x_i$
10	2	20
11	7	77
12	11	132
13	15	195
14	10	140
15	4	60
16	1	16
	$\Sigma f_i = 50$	$\Sigma f_i x_i = 640$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{640}{50} = 12.8$$

$$\boxed{\bar{x} = 13}$$

x_i	f_i	$x - \bar{x}$ (10-13)	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
10	2	-3	9	18
11	7	-2	4	28
12	11	-1	1	11
13	15	0	0	0
14	10	1	1	10
15	4	2	4	16
16	1	3	9	9
				$\Sigma f(x-\bar{x})^2 = 92$

$$S.D = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{92}{50}} = 1.356$$

Case 3 :-

The S.D for the grouped data is given by the formula

$$S.D = \sigma = c\sqrt{\frac{\Sigma(fd^2)}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \quad (\text{or}) \quad \sqrt{\frac{\Sigma fm^2}{\Sigma f} - \left(\frac{\Sigma fm}{\Sigma f}\right)^2}$$

m is mid value.

When $d = \frac{x-A}{c}$

Where x_i's are Mid values of given intervals.

'c' is width of the interval

'A' is Assumed Mean i.e generally the mid value of x_i is taken.

Example :-

find S.D of following distribution.

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No.of Students.	5	12	15	20	10	4	2

x	f	Mid value (x_i)	$d = \frac{x-A}{c}$	fd	$f.d^2$	
10-20	5	15	-3	-15	~~225~~	45
20-30	12	25	-2	-24	~~576~~	48
30-40	15	35	-1	-15	~~225~~	15
40-50	20	$=A$ $\boxed{45}$	0	0	~~0~~	0
50-60	10	55	1	10	~~100~~	10
60-70	4	65	2	8	~~64~~	16
70-80	2	75	3	6	~~36~~	18
	68			$\Sigma fd = -30$	~~1226~~	152.

14.28

23.104

4672

$$S.D = \sigma = c\sqrt{\frac{\Sigma(fd^2)}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \Rightarrow 10\sqrt{\frac{152}{68} - \left(\frac{-30}{68}\right)^2}$$

$$= 10\sqrt{2.235 - 4.496} \Rightarrow 10\sqrt{2.761}$$

$$= 14.285$$

Merits

It is based on all observations.

* It is rigidly define.
* It has a greater Mathematical significance and it's capable of further mathematics.
* It is extremly used to

Demerits :

* It is defecult to compute like other mesures of dispertion.
* It is not simple to understand.
* It computes much time.

Uses :-

It is widely used in biological sciences.
* It is used to fitting a normal curve of a frequency distribution.

Varience :-

i.e varience $= (\sigma)^2$

$\Rightarrow \sigma = \sqrt{\text{varience}}$

Coefficient of variation :-

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

Relative Standard devition :-

It is absolute value of cofficient of deviation.

Example :-

1. Find the standard deviation and varience of the following data.

C.I	: 0-10	10-20	20-30	30-40	40-50
frequency	: 7	15	32	16	9

x	f	Mid value (m)	fm	m^2	fm^2
0-10	7	5	35	25	175
10-20	16	15	240	225	~~3375~~ 3600
20-30	32	25	800	625	20000
30-40	16	35	560	1225	19600
40-50	9	45	405	2025	18225
	$\Sigma f: 80$		$\Sigma fm = 2040$		$\Sigma fm^2 = 61,600$

$$\sigma = \sqrt{\frac{\Sigma fm^2}{\Sigma f} - \left(\frac{\Sigma fm}{\Sigma f}\right)^2} \Rightarrow \sqrt{\frac{61,600}{80} - \left(\frac{2040}{80}\right)^2}$$

$$\sqrt{770 - 650.25} = \sqrt{119.75} = 10.943$$

$\sigma = 10.943$

Varience $= (\sigma)^2 \Rightarrow \sigma = \sqrt{\text{varience}}$

$= 119.749$

(2) Calculate the varience and coefficient of the

Height	: 95-105	105-115	115-125	125-135	135-145
No. of child	19	23	36	70	52

x	f	m	fm	m^2	fm^2
95-105	19	100	1900	10000 ~~10000~~	1900
105-115	23	110	2530	12100 ~~6400900~~	278300
115-125	36	120	4320	14400 ~~19662400~~	518400
125-135	70	130	9100	16900	1183000 ~~0337000~~
135-145	52	140	7280	19600	1019200
	200		25130		3188900

$$\sigma = \sqrt{\frac{3188900}{200} - \left(\frac{25130}{200}\right)^2}$$

$$\sigma = 12.51$$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow \frac{25130}{200} = 125.65$$

$$\text{Varience} = (\sigma)^2 = 156.5001$$

$$CV = \frac{\sigma}{x} \times 100 \Rightarrow \frac{12.51}{125.65} \times 100$$

$$= 9.956$$

(3) find S.D of varience & C.V

Age :	10-19	20-29	30-39	40-49	50-59
cases :	1	0	1	10	17

x	f	m	fm	m^2	fm^2
9.5-19.5	1	14.5	14.5	210.25	210.25
19.5-29.5	0	24.5	0	600.25	0
29.5-39.5	1	34.5	34.5	1190.25	1190.25
39.5-49.5	10	44.5	445	1980.25	19802.5
49.5-59.5	17	55.5	943.5	3080.25	52364.25

Relation between

Mode = 3 × Median − 2 × Mean

Example

The value of Mean & Mode are 60, 66 respectively then find it's median.

Mode = 3 × Median − 2 × Mean

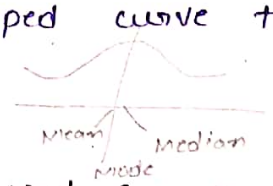
60 = 3 × Median − 2 (66)

Median = ~~Mode~~ $\frac{60 + 2(66)}{3}$ ⇒

Symmetrical Distribution :-

A distribution is said to be symmetrical if its Mean, Mode, Median are identical.

A Symmetrical distribution is plotted on a graph a Perfectly bell shaped curve then it is called a Normal. smiley curve.



Find the Mean, Mode & Median for the following distribution.

x :	1	2	3	4	5	6	7
y :	3	5	6	11	6	5	3

x	f	fx	cf
1	3	3	3
2	5	10	8
3	6	18	14
4 (Mode)	11	44	25
5	6	30	31
6	5	30	36
7	3	21	39
	39	156	

Mean = $\frac{\Sigma f_i x_i}{\Sigma f}$

Mean = $\frac{156}{39}$

Median = $\frac{\Sigma f_i}{2}$

= $\frac{39}{2}$ = 19.5

Median = 4

Here 11 is the higher frequency i.e 4 → Mode

∴ Mean, Mode & Median relation is Same.

Skewness :

The words skewness gives the lack of symmetrical (or) Absence of symmetric. It helps us to study the shape of the distribution.

Note :-

the distribution is said to be skew.

1. Mean, Median & Mode are fall at different points
mean ≠ median ≠ mode

2. the g

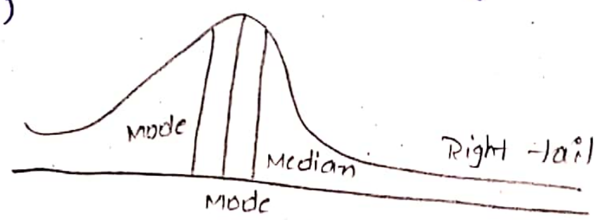
3. The curve drawn with the help of the symmetrical but streched more than the other side.

∴ The skewness is either +ve or -ve.

4. The frequencies are others either side of the mode is unequal.

Positively skewed distribution :-

More items are occured to the right side highest ordinate (mode)

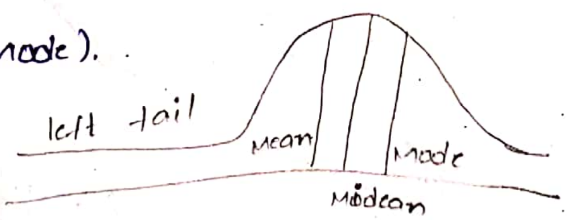


i.e Mode < Median < Mean

→ the frequency curve has a step raise with and a snow fall with a long tail with right side.

Negitevely skewed distribution :-

More items are occured to the left of the highest ordinate (mode).



i.e Mean < Median < Mode.

→ the frequency curve has a slow raise suddenly / severly fall with left tail.

Test for Skewness :-

1. Mean, Median, & Mode are not Coinside are ~~test for the~~ but satisfy the following relations

2. The frequincies are other either side of Mode Unequal.

3. The numerical Measure has been developed to evoluate the Skewness of distribution is called "Karl pearson" distribution.

Case 1 :-

Karl pearson's distribution is

$$B_1 = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$$

B_1 value lies btw 1 & -1 then it is -vely Skewness distribution.

Case 2 :-

Suppose the mode is not well diffined i.e More than 2 Modes occured then the karl pearson of skeness is given by the formula.

$$B_1 = \frac{3(\text{Mean} - \text{Median})}{S.D}$$

This B_1 value lies btw is -ve, then the distrn of Karl pearson of Skeness is called -vely Skeness distrn, otherwise it is calley +vely.

If f_1 is 0, then the distrn is called Symmetrical distribution

Find the Cofficient of Skeness from the following data 25, 15, 23, 40, 27, 25, 23, 25, 20.

Given data is ungrouped here 25 repeats 3 times.

$\therefore$ Mode is 25.

$$\text{Mean} = \frac{25+27+15+23+40+25+23+25+20}{9}$$

$$\text{Mean} = \frac{223}{9} = 24.77$$

Standard daviation is $\sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

x	x^2
25	625
15	225
23	529
40	1600
27	729
25	625
23	529
25	625
20	400
223	5887

$$\sqrt{\frac{5887}{9} - \left(\frac{223}{9}\right)}$$

$$\sqrt{654.11 - 613.9}$$

$$\sqrt{40.211} \Rightarrow 6.34$$

$$\beta_1 = \frac{\text{Mean} - \text{Mode}}{\text{S.D}}$$

$$= \frac{24.77 - 25}{6.34}$$

$$= -0.036$$

$\therefore \beta_1$ is -vely skewness distribution.

find the skewness of the following table ungrouped data.

x :	3	4	5	6	7	8	9	10
f :	7	10	14	35	102	136	43	8

x	f	x^2	fx	fx^2
3	7	9	21	63
4	10	16	40	160
5	14	25	70	350
6	35	36	210	1260
7	102	49	714	4998
8	136	64	1088	8704
9	43	81	387	3483
10	8	100	80	800

$$\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

$\Sigma fx = 2610$

$\Sigma fx^2 = 19818$

$\Sigma f = 355.$

$$= \sqrt{\frac{19818}{355} - \left(\frac{2610}{355}\right)^2}$$

$$= \sqrt{55.82 - 54.05}$$

$$= \sqrt{1.7753}$$

$$= 1.3324.$$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f}$$

$$= \frac{2610}{355}$$

$$= 7.352$$

$$B_1 = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$B_1 = \frac{7.352 - 8}{1.33}$$

$$B_1 = -0.4573.$$

Find the Skewness of the following data

x:	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
f:	8	16	30	4	62	32	15	6

x	f	m	fm	m^2	fm^2
10-15	8	12.5	100	156.25	1250
15-20	16	17.5	280	306.25	4900
20-25	30	22.5	675	506.25	15187.5
25-30	4	27.5	110	756.25	3025
30-35	62	32.5	2015	1056.25	65487.5
35-40	32	37.5	1200	1406.25	45000
40-45	15	42.5	637.5	1806.25	27093.75
45-50	6	47.5	285	2256.25	13537.5
	173		$\Sigma fm = 5302.5$		175481

$$\text{S.D} = \sqrt{\frac{\Sigma fm^2}{\Sigma f} - \left(\frac{\Sigma fm}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{5302.5}{173} - \left(\frac{175481.25}{173}\right)^2} \Rightarrow \sqrt{\frac{175,481.25}{173} - \left(\frac{5302.5}{173}\right)^2}$$

$$= \cancel{-10} = 8.6546$$

$$\beta_1 = \frac{\text{Mean} - \text{mode}}{S.D}$$

$$= \frac{30.6502 - 32.5}{8.6546}$$

$$= -0.2137$$

Kurtosis :-

The Kurtosis are flateness (or)- peaked ness ~~are~~ of a frequency curve is discribed by a measure is called Kurtosis,

* A curve which is peaked is called Leptocurtic curve.
* A curve which is flate is called Platycurtic curve.
* A curve which is normal bell shaped is called Mesocurtic curve.

The numerical ~~curtices~~ measurements of ~~by the given~~ to measure Kurtosis formula

$$\beta_2 = \frac{\mu_4}{s^4} \quad \text{where } \mu_4 = \frac{1}{N}\Sigma f(x-\bar{x})^4$$

$$\text{and } s^4 = \left[\frac{1}{N}\Sigma f(x-\bar{x})^2\right]^2$$

$\bar{x}$ = Mean

N = Σf

Note :-

1. If $\beta_2 = 3$ than the curve is called normal curve or Mesocurtic curve.
2. If $\beta_2 > 3$ than the curve is called leptocurtic curve,
3. If $\beta_2 < 3$ " " " " " Platy curtic curve

Example :

find the Kurtosis of the following table :

x :	0-10	10-20	20-30	30-40
f :	1	3	4	2

x	f	fx	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^4$	$f(x-\bar{x})^2$	$f(x-\bar{x})^4$
5	1	5	-17	289	83521	289	83521
15	3	45	-7	49	2401 $\cancel{147649}$ $\cancel{343}$	147	7203
25	4	100	3	9	$\cancel{27}$ 81	36	324
35	2	70	13	169	$\cancel{2197}$ 28561	338	57122
$N = \Sigma f = 10$		220				810	148170

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{220}{10}$$

$$\bar{x} = 22$$

$$M_4 = \frac{1}{N}\Sigma f(x-\bar{x})^4 = \frac{1}{10}(148170) = 14817$$

$$s^4 = \left(\frac{1}{N}\Sigma f(x-\bar{x})^2\right)^2 = \left(\frac{1}{10}(810)\right)^2 = 6561$$

$$\beta_2 = \frac{14817}{6561} = 2.2583 \quad \therefore \beta_2 = 2 \quad \therefore 2 < 3$$

$\therefore$ Curve is platy curve.

(2) find the kurtosis of the following data

x :	1	2	3	4	5
f :	2	3	5	4	1

x	f	fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$	$(x-\bar{x})^4$	$f(x-\bar{x})^4$
1	2	2	2	4	8	16	32
2	3	6	1	1	3	1	3
3	5	15	0	0	0	0	0
4	4	16	-1	1	4	1	4
5	1	5	-2	4	4	16	16
	$\Sigma f = 15$	44					55

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f}$$

$$= \frac{44}{15} = 2.93 \Rightarrow 3$$

$$\mu_4 = \frac{1}{N} \Sigma f (x-\bar{x})^4$$

$$= \frac{1}{15}(55)$$

$$= 3.6666$$

$$s^4 = \left[\frac{1}{N} \Sigma f (x-\bar{x})^2\right]^2$$

$$= \left(\frac{1}{15}(19)\right)^2$$

$$= 1.5376$$

$$\beta_2 = \frac{\mu_4}{s^4} = \frac{3.666}{1.5896}$$

$$= 2.3095 < 3$$

Scanned by CamScanner

Unit - 2

Basic Probabilitys

Basic probability :-

A numarical measure of uncertainity is provided by a very important branch of Marthemetics is called theory of probability.

Random experiment :-

If in each trile of an experiment conducted under identi-cal condⁿ the outcome is not uniqu but may be any one of the possible outcomes than an such an experiment is called Random experiment.

Example :-

Random experements are tossing a coin, throwing dice, selecting a card from pack of cards, selecting a family out of a given group of familys etc...

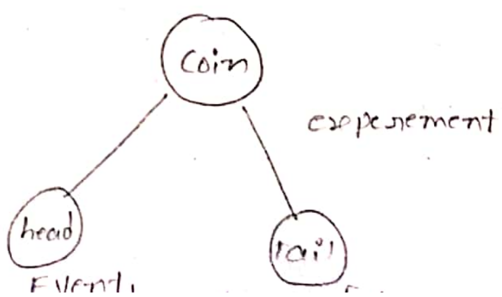
Note :-

→ The result of random experment is called an <u>outcome.</u>

→ Any particular performance of a random experement is called a <u>trile</u> and outcome (or) combination of outcom called <u>events.</u>

ex :-

If a coin is tossed reapeatly than the result is not unique. you may get any of the two facess head or tail then the tossing of the random experement are tail and getting of a head or tail is an event. generaly event is denoted by 'E'.



Note :-

In throwing of a to dial the possible outcomes are 1,2,3...6 in this same experiment the possible event is also be showed on " odd no.of coins, even no.of coins, getting a coin 4 and soon" ...

Composite Events :-

* A single possible exp outcome of the experiment is called simple event otherwise it's called compound (or) composite event.

ex: In a tossing of a single die the event of getting 6 the simple event but the event of getting an even no. is a composite event.

Exhaustive Events :-

The total no.of possible outcomes of a random experiment is called Exhaustive events.

Example:

1. If tossing of a coin there are two exhaustive cases i.e head and tail. for (The possibility of coins ~~bein~~ standing on an edge being ignore).

2. In throwing a die there are 6 exhaustive elementary events i.e 1(or) 2(or) 3(or) 4(or) 5(or) 6.

3. In drawing 3 balls outof 9 balls in a box there are $^9c_3$ eh exhaustive

favourible events

The no. of cases fouvourable to an event. in a experiment is the no.of outcomes which enteraimly happening of the event.

example :-

In throwing of two dies the no.of cases fauvourable to getting the same sum 5 are (1,4) (4,1) (3,2) (2,3)

Classical defination for probability :

In a random experiment having 'n' exhaustive, Mutually exclusive and equally likely outcomes, outof which 'm' are fauvourable to the occurrens of an event 'E'. then the

Probability P of an occu... and it is given by $P(E) = \frac{\text{no. of favourable events}}{\text{Total no. of exhaustive events}}$

$$P(E) = \frac{m}{n}$$

Note :-

→ If $\bar{E}$ denotes event of non occurrence of E then the no. of elementary events in $\bar{E}$.

$$P(\bar{E}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E)$$

$$\text{i.e } P(\bar{E}) + P(E) = \Rightarrow 1$$

→ P(E) is possibility of success

→ $P(\bar{E})$ is " " failure.

→ Since $m \geq 0$ & $n > 0$ and $m \leq n$

then $0 \leq \frac{m}{n} \leq 1$

$= 0 \leq P(E) \leq 1$

→ If $P(E) = 1$ then E is certain event.

→ And $P(E) = 0$ then E is impossible or uncertain events

→ [P(Sample space)] $P(S) = 1$ & $P(\phi) = 0$

Sample space :-

* The set of all possible simple events in a trail is called a Sample Space. Each element of a sample space is called a Sample point. Any subset of sample space is an event.

* Sample space is denoted by 'S'.

* The no. of sample spaces points in the sample space is denoted with $n(S)$.

Example :-

1. Random experiment, tossing a coin

Sample space $S = \{H \quad T\}$
$\quad\quad\quad\quad\quad\quad\quad\quad\downarrow \quad \downarrow$
$\quad\quad\quad\quad\quad\quad\quad\quad E_1 \quad E_2$

Also $n(S) = 2$

ex :-

(2) Two coins are tossed

Sample space $S = \{HT, HT, TT, TH\}$

$n(S) = 4 = 2^2$

(3) Three coins are tossed

Sample space $S = \{(HHH), (H,H,T), (H,T,T), (T,H,H), (HTH), (T,HT), (TTT), (TTH)\}$

~~(HTTT)~~

$n(S) = 2^3 = 8$

(4) Throwing a die

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

(5) Throwing two dies

Sample Space $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1) \ldots (6,6)\}$

$n(S) = 6^2 = 36$

Random Variable :-

A Random Variable is ~~to~~ assign any function that assigns to numarical value that each possible out-come

→ A Random variable is a real valued function defined on sample space S of a random expeiement. Such that for each point of the sample space. Generally random variable are denoted by 'X'

→ The Random Variable is also called "Stochastic Variable" or "chance Variable".

Example :-

(1) Tossing a coin twice

Sample space $S = \{\underset{s_1}{HH}, \underset{s_2}{HT}, \underset{s_3}{TH}, \underset{s_4}{TT}\}$

Define random variable $X : S \to R$ by $x(s)$ = no. of heads

$x(s_1) = 2$

$x(s_2) = 1$

$x(s_3) = 1$

$x(s_4) = 0$

i.e $X(S) = \{ x(s) \mid s \in S \}$

$= \{0, 1, 2\}$ is called range of X

(2) Throwing of two dies at a time

$S = [(1,1), (1,2) \ldots (6,6)]$ define ~~x from~~

$X : S \to R$ by $x(s)$ = sum of no's appearing on the faces of dies.

i.e, $x(s) = \{ (i,j) \mid = i+j \,/\, (i,j) \in S \}$

→ Types of Random Variables :- TWO TYPES

→ This 1. Descreat Random variable and 2. Continues Random variable.

1. Descreat Random Variable.

A Random Variable X Which can take only a finate no. of descreat values in an intervel of domain is called a descreat Random Variable.

Example :- The no. of printing mistake in each page of book, the no. of telephone calls recieved a telephone manager.

(1) Tossing 3 coins ~~at~~ a time ~~x(s) = no. of heads~~.

$x(s)$ = no. of heads

2. Continues Random Variable :-

A random varible is said to be continues if we take all possible values in a given intervel is called a Continues Random Variable.

Examples :- Height, weight, age, temperature etc. are examples of continues R

Probability function of a discreat random Variable :

Let x be the discreat random variable the real valued function 'p' is such 'P' capitial is $P(X)$ is such that $P(X) = p(x)$

Then $p(x)$ is called probability function or probability density function of a distance random variable x.

Propeorties :-

1. $p(x) > 0 \ \forall x$
2. $\Sigma p(x) = 1$ (equals to one) i.e the sum of all probability is or x is 1.

Discreart probability Distribution.

i) Suppose x be the discreat random variable with +ve outcomes $(x_1, x_2 -- x_n)$ then the probability of each possible outcome x_i is $P_i = p(x$ and satisfies 1. $p(x_i) > 0 \ \forall x_i$ 2. $\Sigma p(x_i) = 1$. then $p(x)$ is called probability mass function of the random varible 'x'. The probability of random variable 'x' is given by the following table.

~~X~~ x :	~~0~~	~~1~~	~~2~~
$P(X=x)$:	~~1/4~~		

$X = x_i$:	x_1	x_2	---	x_n
$P(X=x_i)$:	p_1	p_2	---	p_n

Ex :- Random experement of tossing two coins.

Sample space $S = \{HH, HT, TH, TT\}$

2, 1, 1, 0

$X(S)$ = no. of heads.

$X = x$:	0	1	2
$P(X=x)$:	1/4	2/4	1/4

2. Throwing of two dies

Sample space (S) : $\{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$

$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$

$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$

$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$

$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$

$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$X(S)$ = Sum of two no's btw 2 to 12

$X = x_i$	:	x_1	x_2	----	x_n
$P(X=x_i)$	:	p_1	p_2	---	p_n

$X = x_i$	:	2	3	4	5	6	7	8	9	10	11	12
$P(X=x_i)$	:	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

clearly $\Sigma p(x_i) = p_1 + p_2 + p_3 + \cdots + p_n = 1$

Also $P(X \geq 9) = \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36} \Rightarrow \frac{5}{18}$

$P(X \leq 6) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36}$

$P(7 < X < 12) =$

Probability distribution function :

The probability distribution function of random variable x is defined as $F(x) = F_X(x) = P(X \leq x)$ (or) $P\{s / X(s) \leq x\}$, $-\infty < x < \infty$.

Properties :-

1. $P(a < X \leq b) = F(b) = F(a)$
2. $P(a \leq X \leq b) = F(b) - F(a) + P(X=a)$
3. $P(a < X < b) = F(b) - F(a) - P(X=b)$
4. $P(a \leq X < b) = F(b) - F(a) - P(X=b) + P(X=a)$

if $p(X=a)$ and $p(X=b)$ then above all properties are equal.

Probabity distribution of Commulative function :-

It is denoted with $F(x)$.

$$F(x) = \sum_{x=0}^{x} p(x) \text{ (or) } \sum_{x=0}^{x} f(x)$$

Note :-

Suppose X takes only a finite no.of values $x_1, x_2 --- x_n$ then distribution function is given by $F(x) = p(x_1)+p(x_2)+ \text{---} p(x_n)$

Expectation :-

Let X be the discreat random variable assumes values $x_1, x_2 ---- x_n$ with probability $p_1, p_2 -- p_n$ then mathematical expectation (or) mean (or) expected value of X is given by

$$E(X) = \sum_{i=0}^{n} x_i p_i \quad \text{(or)} \quad E(X) = \sum_{i=1}^{n} x_i p_i$$

Similarly $E(X^r) = \sum_{i=1}^{n} x_i^r p_i$

Note :-

In general the expected value of any function $e(x)$ of random variable x given by the formula

$$E(x) = E((g(x)) = \sum_{i=1}^{n} p_i g(x_i)$$

properties :-

1. $E(x+k) = E(x)+k$

W.K.T $E(x) = \sum_{i=1}^{n} x_i p_i$

Similarly $E(X+k) = \sum_{i=1}^{n} (x_i+k) p_i = \sum_{i=1}^{n} x_i p_i + k \sum_{i=1}^{n} p_i$

$= E(X) + k(1) \Rightarrow E(X)+k$

2. $E(K) = k$
3. $E(aX+b) = aE(X)+b$
4. $E(X+Y) = E(X)+E(Y)$
5. $E(XY) = E(X).E(Y)$

Problems:-

1) Let X can be denote the no of heads in a single toss of 4 fair coins. Determine (i) $P(X<2)$
(ii) $P(1<X\leq 3)$

Random experiment : Tossing of 4 coins
i.e $n(S) = 2^4 = 16$

Now Sample space (S) :

$$\begin{bmatrix} H & H & H & H & T & H & H & H \\ H & H & H & T & T & H & H & T \\ H & H & T & H & T & H & T & H \\ H & H & T & T & T & H & T & T \\ H & T & H & H & T & T & H & H \\ H & T & H & T & T & T & H & T \\ H & T & T & H & T & T & T & H \\ H & T & T & T & T & T & T & T \end{bmatrix}$$

Let *

X :	0	1	2	3	4
P(X) :	1/16	4/16	6/16	4/16	1/16

Now, (i) $P(X<2) = 1/16 + 4/16 = 5/16$

(ii) $P(1<X\leq 3) = 6/16 + 4/16 = 10/16 \Rightarrow 5/8$

2) Two dies are thrown let X assign to the each point a, b is S, is the max of its no's i.e $X(a,b) = \max(a,b)$. find the probability distribution of X, is a random variable with $X(S) = \{1,2,3,4,5,6\}$

Write the probability distribution for the given case

→ Random experiment : Throwing of two dies, i.e $n(S)=36$

Sample space (S) = { (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
3 (2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
5 (3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
7 (4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
9 (5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
11 (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

X :	1	2	3	4	5	6
$P(X)$:	$1/36$	$3/36$	$5/36$	$7/36$	$9/36$	$11/36$

(3) A random variable x has the following distribution

X :	1	2	3	4	5	6	7	8
$P(X)$:	K	$2K$	$3K$	$4K$	$5K$	$6K$	$7K$	$8K$

find the value of (i) K
(ii) $P(X\leq 2)$
(iii) $P(2\leq X\leq 5)$

(i) W.K.T $\sum_{i=1}^{n} P_i = 1$

$\sum_{i=1}^{8} P = K+2K+3K+4K+5K+6K+7K+8K = 36K$

$36K = 1$

$K = 1/36.$

(ii) $P(X\leq 2) = K+2K = 3K = \frac{3}{36} = 1/12$

$P(2\leq X\leq 5) = 2K+3K+4K+5K = 14K = 14/36 =$

(4) The probability function of random variable x is

X :	0	1	2	3	4	5	6
$P(X)$:	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$

To find (i) K
(ii) $P(X<4)$, $P(X\geq 5)$, $P(3<X\leq 6)$
(iii) What will be the min value of K. So that $P(X\leq 2) > 0.3$?

(i) $\sum_{i=1}^{n} p_i = 1$

$P = K+3K+5K+7K+9K+11K+13K = 49K$

$49K = 1$

$K = 1/49.$

(ii) $P(X<4) = K+3K+5K+7K \Rightarrow 16K \Rightarrow 16/49$

$24/49$

$P(X\geq 5). = 11K+13K = 24K \Rightarrow 24/49$

$P(3 < x \leq 6) = 9K + 11K + 13K = 33K \Rightarrow 33/49$

(iii) $P(x \leq 2) = K + 3K + 5K = 9K$

$P(x \leq 2) > 0.3$

$9K > 0.3$

$9K > 3/10$

$K = 3/90 \Rightarrow K > 1/30$

$\therefore$ The min value of K is $1/30$

* Mean(μ) :

W.K.T $E(x) = \sum_{i=1}^{n} x_i p_i$

Now Mean $(\mu) = \frac{E(x)}{\sum_{i=1}^{n} p_i} = \frac{\sum_{i=1}^{n} p_i x_i}{1}$

$= E(x)$

* Variance $[V(x)]$:-

Let x be the random variable and corresponding probability distribution is

X :	x_1	x_2	------	x_n
$P(x)$:	P_1	P_2	---	P_n

then $V(x) = E[x - E(x)]^2$

$= E[x - \mu]^2$

$= \sum_{i=1}^{n} (x_i - \mu)^2 p_i$

$= \sum_{i=1}^{n} [x_i^2 + \mu^2 - 2x_i \mu] p_i$

$= \sum_{i=1}^{n} x_i^2 \mu_i + \mu^2 \sum_{i=1}^{n} p_i - 2\mu \sum_{i=1}^{n} x_i p_i$

$= E(x^2) + \mu^2(1) - 2\mu E(x)$

$= E(x^2) + \mu^2 - 2\mu\mu$

$= E(x^2) + \mu^2 - 2\mu^2$

$= E(x^2) - \mu^2$ (or) $E(x^2) - (E(x))^2$

Scanned by CamScanner

* Standard deviation :-

$$\sigma = \sqrt{\text{variance}}$$
$$= \sqrt{E(x^2) - \mu^2}$$
$$= \sqrt{\sum_{i=1}^{n} x_i^2 p_i - \mu^2}$$

problem :-

(1) Let x be the min of two numbers let that the appear when a pair of dice is thrown. then determine (i) Discreat probability distribution

(ii) Expectation

(iii) Varience

(iv) S.D

Random experement : Trowing of two dies

Sample sapace $n(S) = 6^2 = 36$

Sample space $(S) = \{$ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) $\}$
11 9 7 5 3 1

(i) Discreat probability distribution

X :	1	2	3	4	5	6
P(x) :	11/36	9/36	7/36	5/36	3/36	1/36

(ii) Expectation :

$$\text{Expectation } E(X) = \sum_{i=1}^{n} x_i p_i$$
$$= \sum_{i=1}^{6} x_i p_i$$
$$= x_1 p_1 + x_2 p_2 + x_3 p_3 + x_4 p_4 + x_5 p_5 + x_6 p_6$$
$$= 1(11/36) + 2(9/36) + 3(7/36) + 4(5/36) + 5(3/36) + 6(1/36)$$
$$= 91/36$$

(iii) Variance $V(x) = E(x^2) - \mu^2$

$$= \sum_{i=1}^{n} p_i x_i^2 - \mu^2$$

$$= \sum_{i=1}^{6} p_i x_i^2 - \left(\frac{91}{36}\right)^2$$

$$(1)^2 \frac{11}{36} + (2)^2 \frac{9}{36} + (3)^2 \frac{7}{36} + (4)^2 \frac{5}{36} + (5^2) \frac{3}{36} + (6)^2 \frac{1}{36} - \left(\frac{91}{36}\right)^2$$

$$= 8.361 - 6.389$$

$$= 1.972.$$

(iv) Standard Deviation $= \sigma = \sqrt{V(x)}$

$$= \sqrt{1.972} \Rightarrow 1.404$$

(2) A random variable x has following probability distribution.

x :	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Determine the value a

(ii) find $p(x<3)$, $p(x \geq 3)$ & $p(0<x<5)$

(iii) find the distribution function $F(x)$

(i) $\sum_{i=1}^{n} p_i = 1$

$$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 81a$$

$$81a = 1$$

$$a = 1/81.$$

(ii) $p(x<3) \Rightarrow a + 3a + 5a \Rightarrow 9a = 9/81 = 1/9$

$p(x \geq 3) \Rightarrow 7a + 9a + 11a + 13a + 15a + 17a \Rightarrow 72/81 = 8/9$

$p(0<x<5) \Rightarrow 3a + 5a + 7a + 9a + \cancel{11a} = 24/81 = 8/27$

(iii) W.K.T $f(x) = \sum_{i=0}^{x} p(x_i)$

x	$f(x)$
0	$a = 1/8$
1	$4a = 4/81$
2	$9a = 9/81$
3	$16a = 16/81$
4	$25a = 25/81$
5	$36a = 36/81$
6	$49a = 49/81$
7	$64a = 64/81$
8	$81a = 81/81 = 1$

Descrete random variable x of the following distribution function $f(x) = 0, x < 1$

$$f(x) = \begin{cases} 0, & x < 1 \\ 1/3, & 1 \le x < 4 \\ 1/2, & 4 \le x < 6 \\ 5/6, & 6 \le x < 10 \\ 1, & x \ge 10 \end{cases}$$

(i) find
$P(2 < x \le 6)$
$P(x = 5)$
$P(x = 4)$
$P(x \le 6)$
$P(x = 6)$

(i) $P(2 < x \le 6) = f(6) - f(2)$
$= P(x \le 6) - P(x \le 2)$
$= \frac{1}{2} - \frac{1}{3}$
$= \frac{3-2}{6} \Rightarrow 1/6$

(ii) $P(x = 6) = P(x \le 5) - P(x < 5)$
$= P(5) - P(x < 5)$
$= 1/2 - 1/2 \Rightarrow 0$

(iii) $P(x = 4) = P(x \le 4) - P(x < 4)$
$f(4) - P(x < 4)$
$= \frac{1}{2} - \frac{1}{3} \Rightarrow 1/6$

(iv) $P(x \le 6) = 5/6$

(v) $P(x = 6) = P(x \le 6) - P(x < 6)$
$= 5/6 - 1/2$
$= 1/3$

A random sample with replacement of size '2' is taken from $S = \{1, 2, 3\}$ let the random variable x denote the sum of the two numbers taken.

(i) Write the probability distribution of x

(ii) Mean

(iii) Variance

Sol Given that

$$\text{Set } S = \{1, 2, 3\}$$

the no. of sample in $S = n(S) = 3^2 = 9$

i.e sample space $(S) = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$

the range of $x = \{1, 2, 3, 4, 5, 6\}$

x :	2	3	4	5	6
$P(x)$:	1/9	2/9	3/9	2/9	1/9

$$E(x) = \sum_{i=2}^{n} x_i p_i$$

$$= \frac{2}{9} + \frac{6}{9} + \frac{12}{9} + \frac{10}{9} + \frac{6}{9}$$

$$= \frac{\cancel{36}\ 4}{\cancel{9}}$$

$$= 4$$

(ii) Variance $= E(x^2) - \mu^2$

$$= \frac{4}{9} + \frac{18}{9} + \frac{48}{9} + \frac{50}{9} + \frac{36}{9} - 16$$

$$= \frac{156}{9} - 16$$

$$= 17.3 - 16 \Rightarrow 1.33$$

(ii) Variance $= E(x^2) - \mu^2$

$$= \frac{4}{9} + \frac{18}{9} + \frac{48}{9} + \frac{50}{9} + \frac{36}{9} - 16$$

$$= \frac{156}{9} - 16$$

$$= 17.3 - 16$$

$$= 1.33$$

find the distribution function which corresponds to the probability distribution defined by $f(x) = \frac{x}{15}$ for $x = 1,2,3,4,5$

$f(1) = \frac{1}{15}$, $f(2) = \frac{2}{15}$, $f(3) = \frac{3}{15}$, $f(4) = \frac{4}{15}$, $f(5) = \frac{5}{15}$

$F(1) = f(1) = 1/15$

$F(2) = f(1) + f(2)$

$= 1/15 + 2/15 = 3/15 \Rightarrow 1/5$

$F(3) = F(2) + f(3)$

$= 3/15 + 3/15 \Rightarrow 6/15 \Rightarrow 2/5$

$F(4) = F(3) + f(4)$

$= 2/5 + 4/15 \Rightarrow 10/15 \Rightarrow 2/3$

$F(5) = F(4) + f(5)$

$= \frac{2}{3} + \frac{5}{15}$

$= \frac{10+5}{15} \Rightarrow 15/15 \Rightarrow 1$

find the Lord of 5 items containing 3 deflecting a sample of 4 items is drawn at random. let the random variable X denote the no. of defective items in the sample then find the probability distribution of X when the sample drawn without replacement.

Given that

Total no. of items = 10

instead of that 3 are bad and 7 are good

the range of $x = \{0, 1, 2, 3\}$

now, $P(X) = 0$

$P(X=0) = \frac{^7C_4}{^{10}C_4} \Rightarrow \dfrac{\frac{7!}{4!3!}}{\frac{10!}{4!7!}} \Rightarrow \frac{7!}{3!} \cdot \frac{7!}{10!}$

$\Rightarrow \frac{7\times6\times5\times4\times3\times2\times1}{3\times2\times1} \times \frac{7\times6\times5\times4\times3\times2\times1}{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}$

$\Rightarrow \frac{21\times4}{9\times8\times2} \Rightarrow \frac{21}{18} \Rightarrow 7/6$

$P(X=1)$:- $\frac{^3C_1 \times {^7C_3}}{^{10}C_4} \Rightarrow \dfrac{\frac{3!}{1!2!} \times \frac{7!}{3!4!}}{10!/4!7!} \Rightarrow \dfrac{\frac{3\times2!}{2!} \times \frac{7\times6\times5\times4!}{3!4!}}{\frac{10\times9\times8\times7!}{4!7!}}$

$\Rightarrow \frac{3\times7\times5}{10\times3} \Rightarrow 105/30$

Hence the probability distribution of x is

x :	0	1	2	3
$P(x)$:	1/6	1/2	3/10	1/30

A sample of 4 items is selected at random from a box containing 12 items. out of which 5 are detected then find the probability distribution of X. when X is the no. of defective items.

Given that

Total no. of items = 12

$$P(X=0) = \frac{^7C_4}{^{12}C_4} \Rightarrow \frac{\frac{7!}{3!\,4!}}{\frac{12!}{8!\,4!}} \Rightarrow \frac{7\times6\times5\times4\times3\times2\times1}{3\times2\times1.\ 4\times3\times2\times1} \cdot \frac{8\times7\times6\times5\times4\times3\times2\times1}{12\times11\times10\times9\times8\times7\times6\times5\times4\times3\ldots}$$

$$\Rightarrow 7/99$$

$$P(X=1) = \frac{^7C_1}{^{12}C_1}$$

Probability of Continues random Variable

When a random variable x takes every value in a interval it gives rise to continuous distribution of x. the distribution define by the varians like

for example : the distribution define by the temperature, hight and weights are continues random variable distributions.

probability density function of continues rand

It is denoted with $F_x(x) = p(X \leq x)$ and defined as

$$F_x(x) = \int_a^b f(x)dx = 1$$

where $(a,b)\int_a^b$ is the range of variable x.

properties :-

1. $f(x) \geq 0 \ \forall x \in R$.
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $p(E) = \int_E f(x)dx$ is well defined for every event E.

Note :-

probability of $p(a < x \leq b) = p(a \leq x \leq b) = p(a \leq x < b) = p(a < x < b) = F(b) - F(a)$

Since the probability of variable the perticular point is always 0. i.e $P(X=a) = p(X=b) = 0$.

Cummulative distribution function :-

$$\boxed{f(x) = p(X \leq x) = \sum_{i=1}^{x} p(x_i) \ (or) \ \sum_{i=1}^{x} f(x_i)}$$

$$F(x) = p(X \leq x) = \int_{-\infty}^{x} f(x)dx$$

properties :-

1. $0 \leq f(x) \leq 1, \ -\infty < x < \infty$
2. $f(-\infty) = 0$
3. $f(\infty) = 1$

Measures of central tendency for continues probability dist^n?

1. Mean (or) expectation ($E(x)$ or μ) :

The $E(x)$ (or) μ is given by $\int_{-\infty}^{\infty} x f(x)dx$

i.e $\boxed{\mu \ (or) \ E(X) = \int_{-\infty}^{\infty} x f(x)dx}$

Suppose x takes the values in $[a,b]$ then

$$\boxed{\mu \text{ (or) } E(x) = \int_a^b x\, f(x)\, dx}$$

The Mean (or) expectation of any function $\phi(x)$ is

$$\mu(\phi(x)) \text{ (or) } E(\phi(x)) = \int_{-\infty}^{\infty} x\, \phi(x)\, dx$$

2. Median :-

The Median is the point which divides the entire distribution into two equal parts in case of continues distribution into two equal parts Median is the point which divides the two equal parts. Median is the point which 'x' is defined from a and b into total divice. thus 'x' is defined from a and b and 'M' is the Median then

$$\int_a^M f(x)\, dx = \int_a^b f(x)\, dx = 1/2$$

Solving for M, then we get Median

3. Mode :-

Mode is the value of 'x' for which $f(x)$ is maximum,

Mode is given $f'(x)=0$ and $f''(x)<0$ for $a<x<b$

4. Variance :-

$$V(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \quad \text{(or)} \quad \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

5. Mean deviation :-

It is given by $\int_{-\infty}^{\infty} (x-\mu)\, f(x)\, dx$

Problems :-

1. If a random variable x has the following probability density function as

$$f(x) = \begin{cases} 2e^{-2x}, & \text{for } x>0 \\ 0, & x \le 0 \end{cases}$$

find the probabilities that it will take the values

1. lessthan 1 and 3
2. greaterthan 0.5

(1) $P(1 \le x \le 3) = \int_1^3 f(x)\,dx$

$$= \int_1^3 2e^{-2x}\,dx \Rightarrow 2\int_1^3 e^{-2x}\,dx \Rightarrow \cancel{2}\left(\frac{e^{-2x}}{-\cancel{2}}\right)_1^3$$

$$\Rightarrow -(e^{-6}-e^{-2}) \Rightarrow e^{-2}-e^{-6}$$

(2) $P(x > 0.5) = \int_{0.5}^{\infty} f(x)\,dx$

$$= \int_{0.5}^{\infty} 2e^{-2x}\,dx \Rightarrow 2\int_{0.5}^{\infty} e^{-2x} \Rightarrow \cancel{2}\left(\frac{e^{-2x}}{-\cancel{2}}\right)_{0.5}^{\infty}$$

$$= -(e^{-2x})_{0.5}^{\infty}$$

(2) If the probabity density of a random variable x ~~having this probability density will take on a value.~~

$$f(x) = \begin{cases} K(1-x^2), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

find the K and probabilities of a random variable x having these probability density will take on a value

(i) btw 0.1 & 0.2

(ii) greater than 0.5

W.K.T $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^{\infty} f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_0^1 K(1-x^2)\,dx + 0 = 1$$

$$\Rightarrow K\int_0^1 (1-x^2)\,dx = 1 \Rightarrow K\left(x-\frac{x^3}{3}\right)_0^1 = 1$$

$$\Rightarrow K\left(1-\frac{1}{3}\right) - 0 = 1$$

$$\Rightarrow K\left(\frac{2}{3}\right) = 1$$

$$\Rightarrow K = 3/2$$

(i) $P(0.1 \le x \le 0.2) = \int_{0.1}^{0.2} f(x)\,dx$

$$= \int_{0.1}^{0.2} K(1-x^2)\,dx \Rightarrow \frac{3}{2}\int_{0.1}^{0.2} (1-x^2)\,dx$$

$$\frac{3}{2}\left(x-\frac{x^3}{3}\right)^{a}$$

(iii) $P(x>0.5) = \int_{0.5}^{\infty} f(x)\,dx$

$$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{1} K(1-x^2)\,dx + 0$$

$$= \frac{3}{2}\left(x-\frac{x^3}{3}\right)_{0.5}^{1}$$

$$= \frac{3}{2}\left[0.5-\frac{\ }{3}\right]-\left[1-\frac{1}{3}\right]$$

$$= \frac{3}{2}\left[\frac{2}{3}-1.5-1.5\right.$$

③ for the continuous probability function $f(x) = Kx^2e^{-x}$ where $x\geq$

then find (i) K

(ii) Mean

(iii) Variance

Given that $f(x) = Kx^2e^{-x}\ \forall\ x\geq 0$

NOW $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx = 1$$

$$= 0 + \int_{0}^{\infty} Kx^2e^{-x}\,dx = 1$$

$$= K\int_{0}^{\infty} x^2e^{-x}\,dx = 1$$

$$= K\left[x^2(-e^{-x}) - (2x)(e^{-x}) + 2(-e^{-x})\right]_{0}^{\infty} = 1$$

$$= K\left[-e^{-x}(x^2+2x+2)\right]_{0}^{\infty} = 1$$

Scanned by CamScanner

$= K[-e^{-\infty}(\infty^2+2\infty+2)-(-e^{-0}(0+0+2))]=1$

$= K[0+2]=1$

$= 2K=1$

$= K=1/2$

(ii) Mean $= \mu$ (or) $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$= \int_{-\infty}^{0} x\, f(x)\,dx + \int_{0}^{\infty} x\, f(x)\,dx$

$\because \int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 - \cdots$

$= 0 + \int_0^{\infty} x\, K x^2 e^{-x}\,dx$

$= K\int_0^{\infty} x^3 e^{-x}\,dx$

$= K\left[x^3(-e^{-x}) - (3x^2)(e^{-x}) + (6x)(-e^{-x}) - 6(e^{-x})\right]_0^{\infty}$

$= \frac{1}{2}\left[-e^{-x}(x^3+3x^2+6x+6)\right]_0^{\infty}$

$= \frac{1}{2}\left[-e^{-\infty}(\infty^3+3\infty^2+6\infty+6)\right] - \left[-e^{-0}(0+0+0+6)\right]$

$= \frac{1}{2}[0+6]$

$= 6/2 \Rightarrow 3$

Mean $(\mu) = 3$

(iii) Variance $V(x) = E(x^2) - \mu^2$

$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$= \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{\infty} x^2 f(x)\,dx - \mu^2$

$= 0 + \int_0^{\infty} x^2 K x^2 e^{-x}\,dx - \mu^2$

$= K\int_0^{\infty} x^4 e^{-x}\,dx - \mu^2$

$= \frac{1}{2}\left[x^4(-e^{-x}) - (4x^3)(e^{-x}) + (12x^2)(-e^{-x}) - 24x(e^{x}) + 24(-e^{x})\right]_0^{\infty}$

$= \frac{1}{2}\left[-e^{-x}(x^4+4x^3+12x^2+24x+24)\right]_0^{\infty} - \mu^2$

$= \frac{1}{2}[0+24] - \mu^2 \Rightarrow (\frac{1}{2}+24) - 9 \Rightarrow 12-9 \Rightarrow 3$

Marks (x_i)	No o…
52	7
58	5
60	4
65	6
68	3
70	3
75	2

$\bar{x} = 52(7) + 58…$

$= \frac{184…}{3…}$

$= 61…$

(4) probability density function of a random variable x is

$$f(x) = \begin{cases} \frac{1}{2}\sin x, & 0 \le x < \pi \\ 0, & \text{otherwise} \end{cases}$$

find the Mean, Mode and median of the distribution and also find the probability btw 0 and $\pi/2$.

$$f(x) = \begin{cases} \frac{1}{2}\sin x, & 0 \le x < \pi \\ 0, & \text{otherwise} \end{cases}$$

1. Mean (μ) $= \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{0} x f(x)dx + \int_{0}^{\pi} x f(x)dx + \int_{\pi}^{\infty} x f(x)dx$$

$$= 0 + \int_{0}^{\pi} x f(x)dx + 0$$

$$= \int_{0}^{\pi} x \frac{1}{2}\sin x\,dx$$

$$= \frac{1}{2}\int_{0}^{\pi} x \sin x\,dx$$

$$= \frac{1}{2}\left[x(-\cos x) - (1)(-\sin x)\right]_0^{\pi}$$

$$= \frac{1}{2}\left[-x\cos x + \sin x\right]_0^{\pi}$$

$$= \frac{1}{2}\left[(-\pi\cos\pi + \sin\pi\right] - \left[0+0\right]$$

$\sin\pi = 0$

$\cos\pi = (-1)^n$

1 in even

-1 in odd

$$= \frac{1}{2}\left[-\pi(-1)\right] = \pi/2$$

Mean (μ) $= \pi/2$

2. Mode :-

Here $f(x) = 1/2 \sin x$

$f'(x) = \frac{1}{2}\cos x$

Now $f'(x) = 0$

$\frac{1}{2}\cos x = 0$

$\cos x = 0$

$\cos x = \cos \pi/2$

i.e $x = \pi/2$

and $f''(x) = -1/2 \sin x$

Now $f''(x)$ $f''(\pi/2) = -1/2 \sin \pi/2$

$= -1/2(1)$

$= -1/2 < 0$

$\therefore f$ has max at $x = \pi/2$.

(iii) Median :-

We know that $\int_a^M f(x)dx = \int_M^b f(x)dx = 1/2$

$= \int_0^M f(x)dx = \int_M^\pi f(x)dx = \frac{1}{2}$

$= \int_0^M \frac{1}{2}\sin x\, dx = 1/2 \Rightarrow \int_0^M \sin x\, dx = 1$

$= [-\cos x]_0^M = 1$

$= -\cos M + \cos 0 = 1$

$= -\cos M + 1 = 1 \Rightarrow -\cos M = 0$

$\Rightarrow \cos M = 0 = \cos \pi/2$

$\Rightarrow M = \pi/2$

$\therefore$ Median $= \pi/2$.

$\therefore$ Mean, = Mode = Median = $\pi/2$

(iv) probability of the function :

$= P(0 < x < \pi/2)$

$= \int_0^{\pi/2} f(x)\, dx$

$= \int_0^{\pi/2} \frac{1}{2}\sin x\, dx$

$= \frac{1}{2}(-\cos x)_0^{\pi/2}$

$= \frac{-1}{2}[\cos \pi/2 - \cos 0]$

$= \frac{-1}{2}[0 - 1]$

$= \frac{1}{2}$

If 'x' is a continues random ...

density function $f(x) = \begin{cases} x^2, & a \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$

If $p(a < x < 1) = \frac{19}{81}$ then find "a"

$$P(a < x < 1) = \int_a^1 f(x)\,dx = \int_a^1 x^2\,dx = \frac{19}{81}$$

$$\Rightarrow \left(\frac{x^3}{3}\right)_a^1 = \frac{19}{81}$$

$$\Rightarrow \frac{1}{3} - \frac{a^3}{3} = \frac{19}{81}$$

The density function of a random variable x is

$$f(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

find $E(x)$, $E(x^2)$ & $V(x)$

(i) $E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx = \int_{-\infty}^{0} x\, f(x)\,dx + \int_{0}^{\infty} x\, f(x)\,dx$

$$= 0 + \int_0^\infty x\, e^{-x}\,dx$$

$$= \left[x\left(\frac{e^{-x}}{-1}\right) - (1)\left(e^{-x}\right)\right]_0^\infty$$

$$= \left[-e^{-x}(x+1)\right]_0^\infty = \left(0 + e^{-0}(0+1)\right) \Rightarrow 1$$

(ii) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$= \int_{-\infty}^{0} x^2 f(x)\,dx + \int_0^\infty x^2 f(x)\,dx$$

$$= 0 + \int_0^\infty x^2 f(x)\,dx = \int_0^\infty x^2 e^{-x}\,dx \Rightarrow 2$$

(iii) $V(x) = E(x^2) - [E(x)]^2$

$$= 2 - (1)^2 = 2 - 1 \Rightarrow 1$$

If 'x' is Continues random variable is

$$f(x)\begin{cases} x, & \text{if } 0<x<1 \\ 2-x, & \text{if } 1\le x\le 2 \\ 0, & \text{otherwise}\end{cases}$$

find $E(25x^2+30x-5)$

$= \boxed{25E(x^2)+30E(x)-5}$

$E(ax+b) = a(E(x))+b$

$E(x) = \int_{-\infty}^{\infty} x f(x)dx = \int_{-\infty}^{0} x f(x)dx + \int_{0}^{1} x f(x)dx + \int_{1}^{2} x f(x)dx + \int_{2}^{\infty} x f(x)dx$

$= 0 + \int_0^1 x f(x)dx + \int_1^2 x f(x)dx + 0$

$= \int_0^1 x(x)dx + \int_1^2 x(2-x)dx + 0$

$= \int_0^1 x^2 dx + \int_1^2 2x - x^2 dx$

$= \left(\frac{x^3}{3}\right)_0^1 + \left(\frac{2x^2}{2} - \frac{x^3}{3}\right)_1^2$

$= \left(\frac{1}{3}-0\right) + \left(\frac{8}{2}-\frac{8}{3}\right) - \left(\frac{2}{2}-\frac{1}{3}\right)$

$= \frac{1}{3}+4-\frac{8}{3}-\left(1-\frac{1}{3}\right) \Rightarrow \frac{1}{3}+\left(4-\frac{8}{3}-1+\frac{1}{3}\right)$

$= \frac{1}{3}+\cancel{4}-\frac{\cancel{2}}{3} \Rightarrow \frac{1}{3}+\frac{2}{3} \Rightarrow \frac{3}{3}=1.$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

$= \int_{-\infty}^{0} x^2 f(x)dx + \int_0^1 x^2 f(x)dx + \int_1^2 x^2 f(x)dx + \int_2^{\infty} x^2 f(x)dx$

$= 0 + \int_0^1 x^2 - x + \int_1^2 x^2(2-x) + 0$

$= \left(\frac{x^4}{4}\right)_0^1 + 2\left(\frac{x^3}{3}\right)_1^2 - \left(\frac{x^4}{4}\right)_1^2$

$= \frac{1}{4}+2\left(\frac{8}{3}-\frac{1}{3}\right)-\left(\frac{16}{4}-\frac{1}{4}\right) \Rightarrow \frac{1}{4}+\frac{14}{3}-\frac{15}{4}$

$= 7/6.$

- The probability density function is given by $f(x) = ce^{-|x|}$, $-\infty<x<\infty$. show that $c=1/2$ and find the mean & Variance show that and also find the probability that the variate lies btw 0 & 4

We have $\int_{-\infty}^{\infty} f(x)dx = 1$

$= \int_{-\infty}^{\infty} ce^{-|x|} dx = 1$

Scanned by CamScanner

$$= 2c\int_0^{\infty} e^{-|x|}dx = 1 \quad (\because e^{-|x|} \text{ is elen function})$$

the

$$= 2c\int_0^{\infty} e^{-x}dx = 1 \quad (\because |x| = x \text{ if } 0 < x < \infty)$$

$$= 2c\left(\frac{e^{-x}}{-1}\right)_0^{1} = 1$$

$$= \frac{2c}{-1}\left(e^{-\infty} - e^{-0}\right) = 1$$

$$= -2c(0-1) = 1$$

$$c = 1/2$$

$$\text{Mean}(\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x\, ce^{-|x|}dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} x\, e^{-|x|}dx$$

$$= \frac{1}{2}(0) \quad (\because x \text{ is odd } \& \ e^{-|x|} \text{ is even}$$

$$= 0 \qquad \therefore x \cdot e^{-|x|} \text{ is odd})$$

$$\text{Variance}(x) = E(x^2) - \mu^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)dx - (0)^2$$

$$= \int_{-\infty}^{\infty} x^2\, ce^{-|x|}dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} x^2 e^{-|x|}dx$$

$$= 1/\cancel{2} \times \cancel{2}\int_0^{\infty} x^2 e^{-|x|}dx$$

$$= \int_0^{\infty} x^2 e^{-x}dx = \left[x^2(-e^{-x}) - (2x)(e^{-x}) + 2(-e^{-x})\right]_0^{\infty}$$

$$= \left(-e^{-x}(x^2 + 2x + 2)\right)_0^{\infty}$$

$$= \infty + e^0(0+0+2)$$

$$= 2$$

$$P(0 < x < 4) = \int_0^4 f(x)dx$$

$$= \int_0^4 ce^{-|x|}dx$$

$$= 1/2\int_0^4 e^{-x}dx$$

$= \frac{1}{2}\left(\frac{e^{-x}}{-1}\right)_0^4$

$= \frac{-1}{2}\left(e^{-4} - e^{-0}\right)$

$= \frac{1}{2}\left(1 - e^{-4}\right)$

$= 0.49084$

Unit - 3 Probability Distribution.

* There are 2 types of probability distributions.

1. Descrete theoretical distribution.
   a, Binomial distribution
   b. position distribution
   c, rectangular distribution
   d, Negative binomial distribution
   e, Geometrical distribution
2. Continuous theorical distribution
   a normal distribution
   b Student's distribution
   c Chi - square distribution
   d f - distribution

In this chapter we shall study about 3 distributions binomial, poision and normal distribution.

Descrete uniform distribution:

A random variable x has a discrete uniform distribution if and only if It's probability distribution is given by $P(x) = \frac{1}{K}$, for $x = x_1, x_2 --- x_K$

for example

x :	0	1
$P(x)$:	$1/2$	$1/2$

Bernoullis Distribution:

A random variable x which takes two values 0 & 1 probabilities q & p respectevely i.e $P(X=0) = q$ & $P(X=1) = P$

NOTE :- $\boxed{p+q = 1}$

1. W.K.t $p+q = 1$ i.e $q = 1-p$ is called a Bernoullis discrete random variable and is said to have a Bernoullis distribution.

2. The probabilities function for Bernoullis distribution is given by $p(x) = p^x q^{1-x}$, $x = 0, 1$.

Scanned by CamScanner

Mean : $\mu = \Sigma x_i p_i$

$= 0 \times q + 1 \times p$

$\boxed{\mu = p}$

Varience : $V(x)$: $\sigma^2 = \Sigma x_i^2 p_i - \mu^2$

$= (0)^2 \times q + (1)^2 + p - p^2$

$\sigma^2 = p - p^2$ $\quad [\because q = 1-p]$

$= \cancel{p(p} \; p(1-p)$ $\quad V(x) = pq$

$= pq$ $\quad$ S.D $\sigma = \sqrt{pq}$

$\sigma = \sqrt{pq}$, which is S.D.

Bernoullis Theorem :-

If the probability of occurence of an event (successive) in a single trail is p then the probability that will occur exactly y times out of n independent trails is given by $n_{c_r} p^r q^{n-r}$

Binomial distribution :-

A random variable x has a binomal if it assumes only non -ve values and its probability density function is $p(r) = p(x=r) = \begin{cases} n_{c_r} p^r q^{n-r} & , r = 0,1,2,\ldots n, q = 1-p \\ 0 & , \text{otherwise} \end{cases}$

i.e probability of getting 'r' succesess and n-r failures in trails

Binomial distribution function :-

1. for example no. of post graduates in a group of n men. 2. the no. of deffective goats in a box of n voltes.
3. The no. of heads while tossing n coins at a time.

$F(x) = p(X \leq x) = \sum_{r=0}^{x} n_{c_r} p^r q^{n-r}$

Mean of Binomial Distribution :-

W.K.T μ (or) $E(x) = \Sigma x_i p_i = \sum_{r=0}^{n} r \, p(x=r)$

$= \sum_{r=0}^{n} r \, n_{c_r} p^r q^{n-r}$

$= 0 + nc_1 p^1 q^{n-1} + 2nc_2 p^2 q^{n-2} + \cdots nc_n p^n q^{n-n}$

$= npq^{n-1} + q\frac{n(n-1)}{2!} p^2 q^{n-2} - - - - + np^n$

$= np\left[q^{n-1} + (n-1)q^{n-2}p + \cdots + p^{n-1}\right]$

$= np\left[(n-1)c_0 p^0 q^{n-1} + (n-1)c_1 p^1 q^{n-2} + \cdots + (n-1)c_{n-1} p^{n-1} q^0\right]$

$= np\left[p+q\right]^{n-1}$

$= np(1)^{n-1} = np$

$\mu = np$

Variance of Binomial distribution :-

Variance $V(x)$: $\sigma^2 = E(x^2) - \mu^2$

$= \sum_{r=0}^{n} r^2 P(x=r) - (np)^2$

$= \sum_{r=0}^{n} \left[r(r-1)+r\right] P(x=r) - (np)^2$

$= \sum_{r=0}^{n} r(r-1)P(x=r) + \sum_{r=0}^{n} rP(x=r) - (np)^2$

$= \sum_{r=0}^{n} r(r-1)\, nc_r p^r q^{n-r} + E(x) - (np)^2$

$= 0 + 0 + 2.1\, nc_2 p^2 q^{n-2} + 3.2\, nc_3 p^3 q^{n-3} + \cdots n(n-1)\, nc_n p^n q^{n-n} + np - (np)^2$

$V(x)\,(or)\,\sigma^2 = \cancel{2}\frac{n(n-1)}{\cancel{2}} p^2 q^{n-2} + \cancel{6}\frac{n(n-1)(n-2)}{3\cancel{!}} p^3 q^{n-3} + \cdots n(n-1)p^n + np - (np)^2$

$= n(n-1)p^2\left[q^{n-2} + (n-2)pq^{n-3} + \cdots + p^{n-2}\right] + np - (np)^2$

$= n(n-1)p^2\left[(n-2)c_0 p^0 q^{n-2} + (n-2)c_1 p^1 q^{n-3} + \cdots + (n-2)c_{n-2} p^{n-2} q^0\right] + np - (np)^2$

$= n(n-1)p^2 + np - n^2p^2$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$\sigma^2 = np(1-p) = npq$

Standard deviation :-

$S.D = \sigma = \sqrt{Variance} = \sqrt{npq}$

Recurrence Relation btw Binomial distribution :-

W.K.T $P(X=r) = {}^nC_r\, p^r q^{n-r}$ — ①

Also $P(X=r+1) = {}^nC_{r+1}\, p^{r+1} q^{n-r-1}$ — ②

$\frac{②}{①} \Rightarrow P(r+1) = \frac{(n-r)}{(r+1)}\left(\frac{p}{q}\right) P(x=r)$

$$\frac{{}^nC_{r+1}}{{}^nC_r} \quad \frac{p^{r+1}}{p^r} \quad \frac{q^{n-r-1}}{q^{n-r}}$$

① A fair coin is tossed 6 time find probabity getting 4 heads.

$n=6$ $\quad$ p = getting head $= 1/2 \Rightarrow q = 1/2$

$r=4$

$$P(x=4) = {}^6C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^{6-4}$$

$$= \frac{6\times5\times4\times3}{4\times3\times2\times1} \times \frac{1}{24} \times \frac{1}{4}$$

$$= \frac{15}{2^6} \Rightarrow 0.234375$$

② 10 coins are thrown in findin simontaneously find the probability of getting atleast 7 heads. (The total no. of)

$n=10, \; r=7$

p = probability of getting heads $= 1/2$ $\qquad = 0.1575$

$q = 1-p = 1-\frac{1}{2} \Rightarrow \frac{1}{2}$

$$P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

$$= {}^{10}C_7\left(\tfrac{1}{2}\right)^7\left(\tfrac{1}{2}\right)^{10-7} + {}^{10}C_8\left(\tfrac{1}{2}\right)^8\left(\tfrac{1}{2}\right)^{10-8} + {}^{10}C_9\left(\tfrac{1}{2}\right)^9\left(\tfrac{1}{2}\right)^1 + {}^{10}C_{10}\left(\tfrac{1}{2}\right)^{10}\left(\tfrac{1}{2}\right)^{10-10}$$

$$= \frac{10!}{7!}\left(\frac{1}{2^7}\right)\left(\frac{1}{2^3}\right) + \frac{10!}{8!}\left(\frac{1}{2^8}\right)\left(\frac{1}{2}\right)^2 + \frac{10!}{9!}\left(\frac{1}{2^9}\right)\left(\frac{1}{2}\right) + 0$$

$$= \frac{10\times9\times8\times7\times6\times5\times4\times3}{8\times7\times6\times5\times4\times3\times2\times1} \quad \frac{10\times9\times8\times7\times6\times5\times4\times3}{8\times7\times6\times5\times4\times3\times2\times1} \quad \frac{10\times9\times8\times}{9}$$

$$= 80\left(\frac{1}{2^7}\right)\left(\frac{1}{2^3}\right) + 45\left(\frac{1}{2^8}\right)\left(\frac{1}{4}\right) + 10\left(\frac{1}{2^9}\right)\left(\frac{1}{2}\right)$$

$80\left(\frac{1}{128}\right)\left(\frac{1}{8}\right) +$ $\qquad \Rightarrow 0.17$

$+$

$0.0781 + 0.04139 + 9.7656 \Rightarrow \left(\frac{1}{2^{10}}\right)$

0.1317

Scanned by CamScanner

find the probability of getting

$n=10$, $r=6$

$$P(x\geq 6) = P(x=6) + 0.17$$

$$= 10c_6\left(\tfrac{1}{2}\right)^6\left(\tfrac{1}{2}\right)^4 + 0.17$$

$$= \frac{10\times 9\times 8\times 7}{4\times 3\times 2} = \frac{1}{2^6}\times\frac{1}{2^4} + 0.17$$

$$= \frac{210}{210} + 0.801$$

$$= 0.376.$$

The 'incidence' of an occupation disease in an industry is such that workers have a 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random of 4 or more will suffer from the disease.

$n=6$ => total number of

p = probability getting without suffering from disease

$= 20\% = \frac{20}{100} = 0.2$

$q = 1-p = 1-0.2 \Rightarrow 0.8$

r = no. of workers.

$$P(x\geq 4) = P(x=4) + P(x=5) + P(x=6)$$

$$= 6c_4(0.2)^4(0.8)^{6-4} + 6c_5(0.2)^5(0.8)^{6-5} + 6c_6(0.2)^6)(0.8)^{6-6}$$

$$= 6c_4(0.2)^4(0.8)^2 + 6c_5(0.2)^5(0.8)^1 + 6c_6(0.2)^6(0.8)^0$$

$$= \frac{6!}{4!2!}(0.2)^4(0.8)^2 + \frac{6!}{5!}(0.2)^5(0.8)$$

$$= 15(0.2)^4(0.8)^2 + 6(0.2)^5(0.8)$$

$$= 0.017.$$

Marks obtained by

8. Two dices are thrown five ~~minutes~~ times, then find the probability getting 7 as sum (i) atleast once
(ii) 2 times
(iii) $p(1<x<5)$

$n=5$
r = sum as 7 $p=\frac{1}{6}$ $q=\frac{5}{6}$

(i) $p(x\geq1) = 1-p(x<1) = 1-p(x=0)$

$= 1-5c_0(\frac{1}{6})^0(\frac{5}{6})^5$

$= 1-[1 \cdot \frac{1}{6} \times (\frac{5}{6})^5]$

$= 1-[\frac{1}{6} \times \frac{5^5}{6^5}]$

$= 0.934$

(ii) $p(x=2) = 5c_2(\frac{1}{6})^2(\frac{5}{6})^3$

$= \frac{5\times \cancel{4}^2}{\cancel{2}} \times \frac{1}{36} \times \frac{5^3}{6^3}$

$= 1.160$

(iii) $p(1<x<5) = p(x=2)+p(x=3)+p(x=4)$

$= 0.160 + 5c_3(\frac{1}{6})^3(\frac{5}{6})^2 + 5c_4(\frac{1}{6})^4(\frac{5}{6})^1$

$= 0.160 + \frac{5\times4\times3}{6^6} \times 25 + \frac{5^2}{6^5}$

$= 0.192 + 3.2\times10^{-3}$

$= 0.195$

The mean and variance of binomial distribution are 4 and 4/3 respectively find $p(x\geq1)$

9/4

Mean $= np = 4$

Variance $= npq = 4/3$

$npq = 4/3 \Rightarrow q\times4 = \frac{4}{3} \Rightarrow q = 1/3$, $p = 4/3$

$n(\frac{2}{3}) = 4$

$n = 6$

$p(x\geq1) = 1-p(x<1) = 1-p(x=0)$

the

$= 1 - {}^6C_0 \left(\frac{4}{3}\right)^1 \left(\frac{1}{3}\right)^6$

$= 1 - \frac{1}{36} = 0.998$

✓In 8 thrown of a die 5 or 6 are considered as success then find mean number of success and its S.D

$n = 8$

$p = \frac{1}{6} + \frac{1}{6} \Rightarrow 1/3$, $q = 2/3$

$S.D = \sqrt{npq}$

$= \sqrt{8 \times \frac{2}{9}} \Rightarrow \sqrt{\frac{16}{9}} = \frac{4}{\sqrt{9}} = \frac{4}{3}$

Mean $= np$

$= 8 \times 1/3$

$= 8/3$

2 dies are thrown 120 times find the avg no. of times in which the num. on the first die exceeds the num on 2nd die.

$n = 120$

$p = \frac{15}{36}$

Mean $= \cancel{120}^{10} \times \frac{\cancel{15}^{5}}{\cancel{36}_{\cancel{12}}}$

$= 50$

**

Binomial frequency distribution:

Let us suppose n trails consitute an experiment then this experiment is repeated 'N' Times. Then the frequency function is Binomial distribution given by the

$f(x) = N\,p(x) = N \sum_{x=0}^{n} {}^nC_x\, p^x q^{n-x}$ (or) $N(p+q)^n$

Mode of Binomial distribution:-

Mode is $y(x)$ value of x at which $p(x)$ of the maximum value. i.e

Mode = $\begin{cases} (n+1)p \text{ and } (n+1)p-1 & \text{if } (n+1)p \text{ is integer} \\ \text{integeral part of } (n+1)p & \text{if } (n+1)p \text{ is not a integer} \end{cases}$

Problem :-

Out of 800 familys with 5 childrens each. How many would you expect to have (i) 3 boys (ii) 5 girls, (iii) either 2 or 3 boys, (iv) atleast one boy, assume equal probability for boys and girls.

Given that $n = 5$, r = no. of boys.

p = probability of getting a boy = $\frac{1}{2}$.

$q = 1 - p \Rightarrow 1 - \frac{1}{2} = \frac{1}{2}$

(i) $P(X=3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$${}^5C_3 \left(\frac{1}{2^3}\right)\left(\frac{1}{2^2}\right)$$

$$\frac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1}\left(\frac{1}{2^3}\right)\left(\frac{1}{2^2}\right)$$

$$10\left(\frac{1}{2^5}\right) \Rightarrow \frac{10}{2^5} \Rightarrow \frac{\cancel{10}^{5}}{\cancel{32}_{16}} \Rightarrow \frac{5}{16},$$

for 800 families, $800 \times \frac{5}{16} = 250$.

(ii) P(getting 5 girls) = P(no boys)

$= P(X=0)$

$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= \frac{1}{32}$

for 800 families $800 \times \frac{1}{32} = 25$

(iii) P(getting either 2 or 3 boys) = $P(2) + P(3)$

$= {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$= \frac{5 \times \cancel{4}^{2}}{\cancel{2} \times 1}\left(\frac{1}{2^2}\right)\left(\frac{1}{2^3}\right) + \frac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1}\left(\frac{1}{2^3}\right)\left(\frac{1}{2^2}\right)$

$= 10\left(\frac{1}{2^2}\right)\left(\frac{1}{2^3}\right) + 10\left(\frac{1}{2^3}\right)\left(\frac{1}{2^2}\right)$

$= 10\left(\frac{1}{2^5}\right) + 10\left(\frac{1}{2^5}\right)$

$= \frac{5}{16} + \frac{5}{16} \Rightarrow \frac{10}{16}$

for 800 familys $800 \times \frac{10}{16} \Rightarrow 500$

(iv) P(getting atleast one boy)

$= p(x \geq 1)$

$= p(x=1) + p(x=2) + p(x=3) + p(x=4) + p(x=5)$

$= 5c_1 (\frac{1}{2}) (\frac{1}{2})^4 + 5c_2 (\frac{1}{2^2}) (\frac{1}{2^3}) + 5c_3 (\frac{1}{2^3}) (\frac{1}{2^2}) + 5c_4 (\frac{1}{2^4})(\ldots$

$+ 5c_5 + (\frac{1}{2^5}) (\frac{1}{2}) \ldots$

$= 5(\frac{1}{32}) + 10(\frac{1}{32}) + 10(\frac{1}{32}) + 5(\frac{1}{32})$

$= \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + 0$

$= \frac{30}{32}$

for 800 families $800 \times \frac{30}{32} = 750$

Poission distribution :-

The poisson Distribution can be derived as a limiting case of a binomial distribution.

* p is probability of occurence the event in very small.
* n is very very large number.
* np is a finite quantity say 'λ' i.e $np = \lambda$. then 'λ' is called parameter of the poission distribution.
* The probability of r successes in a series x is given by. of independent.

$$p(r,\lambda) = P(y=r) = \frac{e^{-\lambda} \lambda^r}{r!}, \quad r = 0,1,2,3 \ldots$$

Definition of poission distribution :-

let 'x' is a discreate random variable x has a poisson distribution if it assumes only non-ve values and it's probability density function is

$$p(r,\lambda) = p(x=r) = \begin{cases} \frac{e^{-\lambda} \lambda^r}{r!}, & r = 0,1,2 \ldots \\ 0, & \text{Otherwise.} \end{cases}$$

Note :- The sum of all probabilities of poisson distribution is 1

ex :-
$$\sum_{r=0}^{\infty} P(x=r) = \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^{r}}{r!}$$
$$= e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^{r}}{r!}$$
$$= e^{-\lambda}\, e^{\lambda}$$
$$= e^{-\lambda+\lambda} \Rightarrow e^{0} = 1$$

for example :-

1. The no. of defective bulbs manufactured by a repeated company.
2. The no. of cars passing a certain point in one minute.
3. The no. of printing mistakes for a page in a large text book.
4. The no. of persons gone blind per year in a large city.

Mean of the poisson's distribution;

$$E(x) \text{ or } \mu = \sum_{r=0}^{\infty} r\, p(x=0)$$
$$= \sum_{r=0}^{\infty} r \frac{e^{-\lambda}\lambda^{r}}{r!}$$
$$= \sum_{r=1}^{\infty} \cancel{r} \frac{e^{-\lambda} e^{r}}{\cancel{r}(r-1)!}$$
$$= e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^{r}}{(r-1)!}$$
$$= e^{-\lambda}\left[\lambda + \frac{\lambda^{2}}{1!} + \frac{\lambda^{3}}{2!} + \cdots\right]$$
$$= e^{-\lambda}\lambda\left[1 + \frac{\lambda}{1!} + \frac{\lambda^{2}}{2!} + \cdots\right]$$
$$= \lambda e^{-\lambda} e^{\lambda}$$
$$= \lambda e^{-\lambda+\lambda}$$
$$= \lambda e^{0}$$
$$= \lambda$$

$$\therefore \mu = \lambda$$

Variance :- $V(x) = E(x^{2}) - \mu^{2}$

Scanned by CamScanner

final the

$$= \sum_{r=0}^{\infty} r^2 (p(x=r)) - \lambda^2$$

$$= \sum_{r=0}^{\infty} r^2 \frac{e^{-\lambda}\lambda^r}{r!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} \cancel{r}^2 \frac{e^{-\lambda}\lambda^r}{\cancel{r}(r-1)!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} \frac{r e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} ((r-1)+1) \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} (r-1) \frac{e^{-\lambda}\lambda^r}{(r-1)!} + \sum_{r=1}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \sum_{r=2}^{\infty} (r-1) \cdot \frac{e^{-\lambda}\lambda^r}{(r-1)(r-2)!} + \sum_{r=1}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \sum_{r=2}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-2)!} + \sum_{r=1}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \lambda^2 e^{-\lambda} \sum_{r=2}^{\infty} \frac{\lambda^{r-2}}{(r-2)!} + e^{-\lambda}\lambda \sum_{r=1}^{\infty} \frac{\lambda^{r-1}}{(r-1)!} - \lambda^2$$

$$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda e^{-\lambda} e^{\lambda} - \lambda^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$V(x) = \lambda$$

Standard deviation :-

$$\sigma = \sqrt{\text{Variance}}$$

$$\sigma = \sqrt{\lambda}$$

Hence in the poissions destribution Mean and Vari are same.

Mode of the poission's distribution:

Mode is the value of x for which the proba lity of $p(x)$ is maximum i.e.

$$p(r) \geq p(r+1) \quad \& \quad p(r) \geq p(r-1)$$

$$\frac{e^{-\lambda}\lambda^r}{r!} \geq \frac{e^{-\lambda}\lambda^{r+1}}{(r+1)!} \qquad \Bigg| \qquad \frac{e^{-\lambda}\lambda^r}{r!} \geq \frac{e^{-\lambda}\lambda^{r-1}}{(r-1)!}$$

$$\frac{\lambda^r}{\cancel{r}!} \geq \frac{\cancel{e^{-\lambda}}\lambda^{r+1}}{\cancel{r}(r+1)!} \qquad \Bigg| \qquad \frac{\lambda^r}{r(r-1)!} \geq \frac{\lambda^{r-1}}{r(r-1)!}$$

Scanned by CamScanner

$\lambda^r (r+1)! \geq \lambda^r \cdot \lambda^1$

$r+1 \geq \lambda$

$r \geq \lambda - 1$

$\frac{\lambda^r (r-1)!}{r \, \lambda^r (r-1)!} \geq \lambda^r \lambda^{-1}$

$\frac{1}{r} \geq \frac{1}{\lambda}$

$\lambda \geq r$

Mode lies b/w $(\lambda-1)$ & λ

Note :
1. If λ is an integer than $\lambda-1$ is also an integer. so we have two mode values. and the distribution is called bimodel distribution and the two modes are $\lambda-1$ and λ.

2. if λ is not an integer than the mode of poission distribution is integral part of λ.

Recurance relation of 'poission's distribution:

W.K.T $p(r) = \frac{e^{-\lambda} \lambda^r}{r!}$

also $p(r+1) = \frac{e^{-\lambda} \lambda^{r+1}}{(r+1)!} = \lambda \frac{e^{-\lambda} \lambda^r}{(r+1) r!}$

$= \frac{\lambda}{r+1} \frac{e^{-\lambda} \lambda^r}{r!}$

$\boxed{p(r+1) = \frac{\lambda}{r+1} p(r)} \rightarrow$ (by (1))

$p(r) = \frac{\lambda}{r} p(r-1)$, $r = 1, 2, 3 \ldots$

With this formula we can calculate $p(1)$, $p(2)$ if $p(0)$ is given.

A car hire burn has two cars which it hires out day by day. The number of demands for a car on each day is is disturbed as a persons distribution with mean 1.5.

Calculate the proportion of day (i) On which flore is no demand (ii) on which demand is refused.

Given that

Mean of poission's distribution i.e $\lambda = 1.5$

W.K.T $p(r) = \frac{e^{-\lambda} \lambda^r}{r!}$

(i) $P(X=0) = P(\text{no demand}) = \frac{e^{-1.5}(0.5)^0}{0!}$

$= 0.2231$

∴ no. of days in a year they is not allo demand of car $= 365 \times 0.2231$

$= 81.4315$

$= 81$ days

(ii) $P(\text{demand refused}) = P(X>2)$

$= 1-P(\leq 2)$

$= 1-[P(X=0)+P(X=1)+P(X=2)]$

$= 1-\left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$

$= 0.1911$

The no. of days refused in a year $= 365 \times 0.1911$

$= 69.7709$

$= 70$ days

A hospital switch board received an average emergency calls in 10 minutes interval. What is the probability that (i) they are atmost 2 emergency calls in a 10 minutes interval.
(ii) they are exactly 3 emergencies in a 10 minutes interval.

Given Mean $\lambda = 4$

(i) $P(X \leq 2) = P(\text{atmost 2 emergency calls})$

$= P(X=0) + P(X=1) + P(X=2)$

$= \frac{e^{-4}\lambda^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!}$

$= 0.2381$

(ii) $P(X=3) = \frac{e^{-4}4^3}{3!}$

$= 0.1953$

Scanned by CamScanner

A manufactured of cotton pins knows that 5% of his product is defective. Pins are sold in boxes of 1 he guarantee that not more than 10 pins will be defective. What is the approximate probability that a box will fail to meet a guarantee property.

Given that total pins $n = 100$

the probability of cotton pins to be defective is

5% i.e $p = 5\%$

$= 5/100 \Rightarrow 0.05$

$P(x > 10) = 1 - p(x \leq 10)$

$= 1 - p(x=1) + p(x=2) + p(x=3) + p(x=4) + p(x=5) + p(x=6) + p(x=7)$
$+ p(x=8) + p(x=9) + p(x=10)$

$= 1 - \frac{e^{-5}(5)^0}{0!} + \frac{e^{-5}(5)^2}{2!} + \frac{e^{-5}(5)^3}{3!} + \frac{e^{-5}(5^4)}{4!} + \frac{e^{-5}(5)^5}{5!} + \frac{e^{-5}(5^6)}{6!}$

$+ \frac{e^{-5}(5^7)}{7!} + \frac{e^{-5}(5^8)}{8!} + \frac{e^{-5}(5^9)}{9!} + \frac{e^{-5}(5^{10})}{10!}$

$= 0.0137.$

If a random variable x has a poisson distribution such that $p(1) = p(2)$ then find (i) Mean of the distribution
(ii) $p(4)$
(iii) $p(x \geq 1)$
(iv) $p(1 < x < 4)$

Given

(i) $p(1) = p(2)$

$= \frac{e^{-\lambda}(\lambda)^1}{1!} = \frac{e^{-\lambda}(\lambda)^2}{2!}$

$= 1 = \frac{\lambda}{2}$

$\therefore \lambda = 2$ i.e $mean(\lambda) = 2$

(ii) $P(x=4) = \frac{e^{-2}(2)^4}{4!} \Rightarrow \frac{0.135 \times 16}{24}$

$= 0.0902$

Scanned by CamScanner

$= 0.1353$

(iii) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0) \Rightarrow \frac{e^{-\lambda}(2)^0}{0!} \Rightarrow 1 - 0.1353$

$= 0.8647$

(iv) $P(1 < X < 4) = P(X=2) + P(X=3)$

$= \frac{e^{-2}(2)^2}{2!} + \frac{e^{-2}(2)^3}{3!}$

$= \frac{0.135 \times 4}{2} + \frac{0.135 \times 8}{6}$

$= 0.27 + 0.18$

$= 0.45$

Average no. of accidents on any day on a national highway is 1.8 Determine the probability that the no. of accidents are atleast 1 and atmost 1.

Given $\lambda = 1.8$

(i) $P(\text{atleast}) = P(X \geq 1) \Rightarrow 1 - P(X < 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[\frac{e^{-1.8}(1.8)^0}{0!} + \frac{e^{-1.8}(1.8)^1}{1!}\right]$

$= 1 - \left[\frac{0.1652(1.8)^0}{0!} + \frac{0.1652(1.8)^1}{1!}\right]$

$= \cancel{1-}[0.1652 + 0.4652]$

$= 1 - [0.1652 + 0.2975]$

$= 1 - 0.4627$

$= 0.5373.$

If X is the poisson's variate such that $P(X=0) = P(X=1)$ find $P(X=0)$ and using recursive formula. find the probabilities of $x = 1, 2, 3, 4, 5.$

Given $P(X=0) = P(X=1)$

$\frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-\lambda}(\lambda)^1}{1!} \Rightarrow \boxed{\lambda = 1}$ Mean

(i) $P(x=0) = \dfrac{e^{-\lambda}\lambda^0}{0!} = \dfrac{e^{-1}(1)^0}{0!}$

By recursive relation

$$P(r) = \frac{\lambda}{r} P(r-1)$$

put $r=1$, $P(1) = \frac{\lambda}{1} P(0) \Rightarrow 1 \times 0.3678 \Rightarrow 0.3678$

put $r=2$, $P(2) = \frac{1}{2} P(1) \Rightarrow \frac{1}{2} \frac{e^{-\lambda}(1)^1}{1!} = 0.09196$

put $r=3$ $P(3) = \frac{1}{3} P(2) = 0.02043$

put $r=4$ $P(4) = \frac{1}{4} P(3) = 0.0038$

put $r=5$ $P(5) = \frac{1}{5} P(4) \Rightarrow \frac{1}{5} \frac{e^{-1}(1)^5}{5!} = 0.0006$

Given that $P(x=2) = 9\,P(x=4) + 90\,P(x=6)$ for a poisson variate x. (i) $P(x<2)$

(ii) $P(x>4)$

(iii) $P(x \geq 1)$

(i) $P(x<2) = P(x=0) + P(x=1)$

$P(x=2) = 9\,P(x=4) + 90\,P(x=6)$

$$= \frac{e^{-\lambda}(\lambda)^2}{2!} = 9 \cdot \frac{e^{-\lambda}(\lambda)^4}{4!} + 90 \cdot \frac{e^{-\lambda}(\lambda)^6}{6!}$$

$$= \frac{\lambda}{2} = 9 \cdot \frac{\lambda^2}{24} + \frac{90\lambda^4}{720}$$

$$= \lambda = \frac{9\lambda^2}{12} + \frac{90\lambda^4}{360}$$

$$= 1 = \frac{9\lambda^2}{12} + \frac{90\lambda^4}{360}$$

$$1 = \frac{3\lambda^2 + \lambda}{4}$$

$$4 = 3\lambda^2 + \lambda$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

$$\lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$$

$$\lambda^2(\lambda^2+4) - 1(\lambda^2+4) = 0$$

$$(\lambda^2-1)(\lambda^2+4) = 0$$

$$\lambda^2 = 1$$

$$\lambda = 1$$

(i) $p(x<2) = p(x=0) + p(x=1)$

$$= \frac{e^{-1}(1)^0}{0!} + \frac{e^{-1}(1)^1}{1!} \Rightarrow 0.7357$$

(ii) $p(x>4) = 1 - p(x \leq 4)$

$$= 1 - [p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4)]$$

$$= 1 - \left[\frac{e^{-1}(1)^0}{0!} + \frac{e^{-1}(1)^1}{1!} + \frac{e^{-1}(1)^2}{2!} + \frac{e^{-1}(1)^3}{3!} + \frac{e^{-1}(1)^4}{4!}\right]$$

$$= 0.0037$$

(iii) $p(x \geq 1) = 1 - p(x<1)$

$$= 1 - p(x=0)$$

$$= 1 - \frac{e^{-1}(1)^0}{0!}$$

$$= 0.6321$$

* A random variable x whose mean is μ & variance σ^2 of its PDF (or) probability of distribution is given by

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \; -\infty < x < \infty, -\infty < \mu < \infty, \sigma>$$

→ length and diameter of certain products like pipes, secure &

→ Aggregate marks obtained by the student heights & weig of baby at the time of birth.

Note :-

1. the no. of trails is indefenetly large i.e $n \to \infty$
2. neither p nor q is small
3. the Mean mode median are same i.e given distribution is Symmetrical distribution.

Mean deviation from the Mean of normal distribution

W.K.T Mean deviation $= \int_{-\infty}^{\infty} |x-\mu| f(x) dx$

$$= \int_{-\infty}^{\infty} |x-\mu| \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $\frac{x-\mu}{\sigma} = z \Rightarrow \frac{dx}{\sigma} = dz \Rightarrow dx = \sigma dz$

$$M.D = \int_{-\infty}^{\infty} |dz| \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \sigma\, dz$$

$$= \frac{1}{\sqrt{2}\sqrt{\pi}} \int_{-\infty}^{\infty} |z| \sigma e^{-\frac{z^2}{2}} dz$$

$$= \frac{\sigma}{\sqrt{2}\sqrt{\pi}} \int_{-\infty}^{\infty} |z|\, e^{-z^2/2} dz$$

$$= \frac{\sigma}{\sqrt{2}\sqrt{\pi}} = 2\int_{0}^{\infty} z e^{-z^2/2} dz$$

put $\frac{z^2}{2} = t \Rightarrow \frac{2z\,dz}{2} = dt$

$$M.D = \frac{\sqrt{2}}{\sqrt{\pi}}\sigma \int_0^{\infty} e^{-t} dt$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}}\sigma (-e^{-t})_0^{\infty}$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}}\sigma (-e^{-\infty} + e^{-0})$$

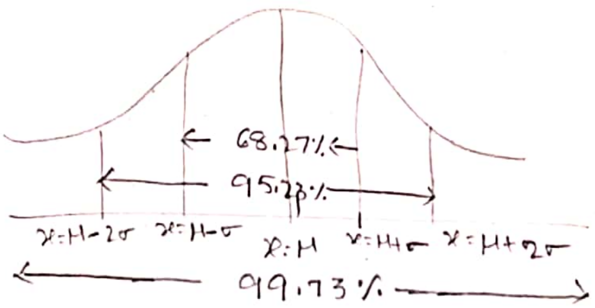
$$= \frac{\sqrt{2}}{\sqrt{\pi}}\sigma = \frac{4}{5}\sigma$$

$$M.D = \frac{4}{5} S.D \Rightarrow \frac{M.D}{S.D} \Rightarrow \frac{4}{5} = 4:5$$

Characteristics of normal distribution :-

* The curve is bell shape and the line symmetrical above the line $x = \mu$ and the tiles in the right and left side of the mean (μ) extends to infinity. The top of the bell shape is directly above the mean (μ).
* Area under the normal curve represents the total population. i.e $\int_{-\infty}^{\infty} f(x)\,dx = 1$
* Mean, Mode, Median of distribution coinside at $x = \mu$. So, this curve is called uni model (has only 1 mode coin)

Area under the normal curve is distributed has following



The

→ The area of normal curve btw $\mu-\sigma$ & $\mu+\sigma$ is 68.27%.

→ The area of the normal curve btw $\mu-2\sigma$ & $\mu+2\sigma$ is 95.93%.

→ The area of the normal curve btw $\mu-3\sigma$ & $\mu+3\sigma$ is 99.73%.

Standard normal distribution

The normal distribution is Mean, $\mu = 0$ and S.D $\sigma = 1$ is known as standard normal distribution.

→ The S.N.D is random variable is denoted by 'Z'

put $Z = \frac{x-\mu}{\sigma}$ in probability distribution function

$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ become $P(Z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(Z)^2}$