CS010 404

Signals and Communication Systems

Dept. of Computer Science and Engineering

Course Objectives

1. Module 1:

   To introduce the fundamentals of Analog and Digital Signals, their properties and introduce the relevant transforms used in Communication

   Note:

   More advanced course is waiting for you.

   CS010 504 – Digital Signal Processing

2. Module 2-5:

   To Familiarize the core ideas of Communication Engineering which in turn adds to the study of Computer communication.

Module 1 - Syllabus

- Introduction to Signals:-
- Continuous Time Signals
- Discrete Time Signals
- Signal Operations
- Properties of Signals(Periodicity and Symmetry)
- Frequency Domain Representation of Continuous Time Signals
- Continuous Time Fourier Series(CTFS)
  - Definition- properties Examples,
- Continuous Time Fourier Transform(CTFT)
  - Definition- Properties Examples
- Concept of Frequency Spectrum,
- Sampling- The Sampling Theorem(proof not required)
- Quantization

What are signals

Signal:

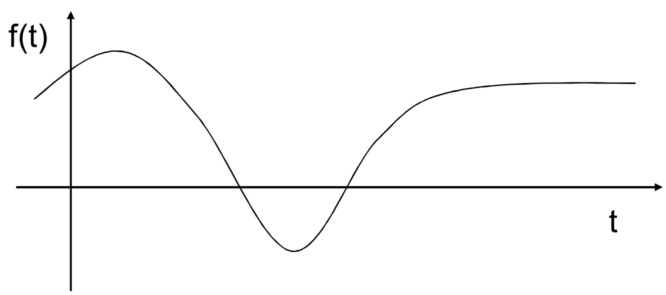
- Function or sequence that represents information.
- A signal is a pattern of variation of some form
- Signals are variables that carry information

Examples of signal include:

Electrical signals - Voltages and currents in a circuit
Acoustic signals - Acoustic pressure (sound) over time
Mechanical signals - Speed of rotation of a wheel
Video signals - Intensity level of a pixel (camera, video) over time

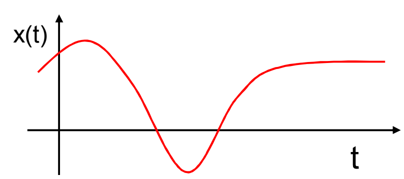
How is a Signal Represented?

- **Mathematically, signals are represented as a function of one or more independent variables.**
- For instance a black & white video signal intensity is dependent on *x*, *y* coordinates and time *t*. $f(x,y,t)$
- On this course, we shall be exclusively concerned with signals that are a function of a single variable: time

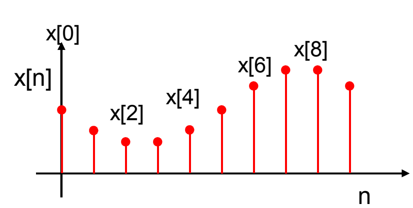


Continuous & Discrete-Time Signals

- **Continuous-Time Signals**
  - Most signals in the real world are continuous in time.
  - Eg voltage, velocity,
  - Denote by $x(t)$ – Symbol **t** for independent variable. Use parenthesis (.)
  - The time interval may be bounded (finite) or infinite



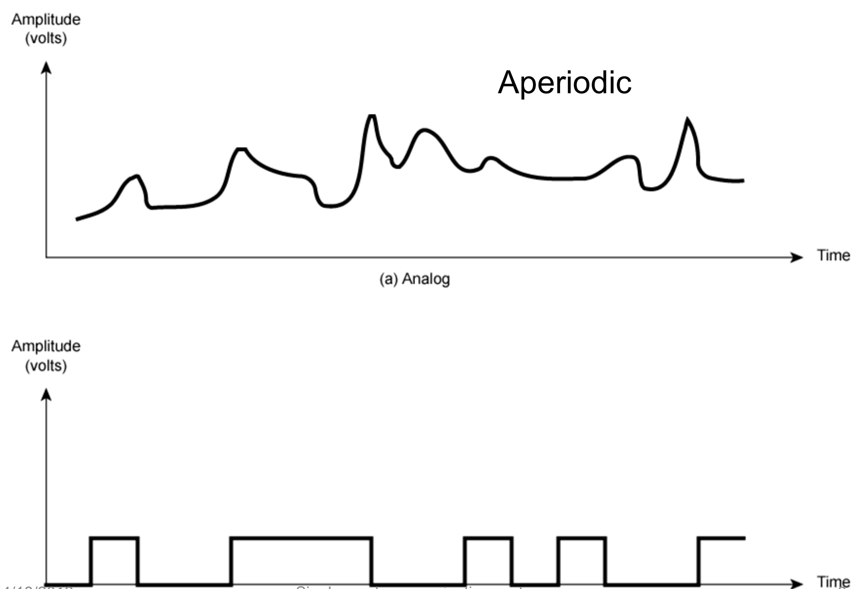
- **Discrete-Time Signals**
  - Some real world and many digital signals are discrete time, as they are sampled
  - E.g. pixels, daily stock price (anything that a digital computer processes)
  - Denote by $x[n]$, where n is independent variable - *n* is an integer value that varies discretely. Use brackets [.]
  - n is bounded



Types of Signal

- **analog signal (Continuous Time Signals)**
  - signal intensity varies smoothly with no breaks
- **digital signal**
  - signal intensity maintains a constant level and then abruptly changes to another level
- **aperiodic signal**
  - pattern not repeated over time
- **periodic signal**
  - signal pattern repeats over time

Analog and Digital Signals

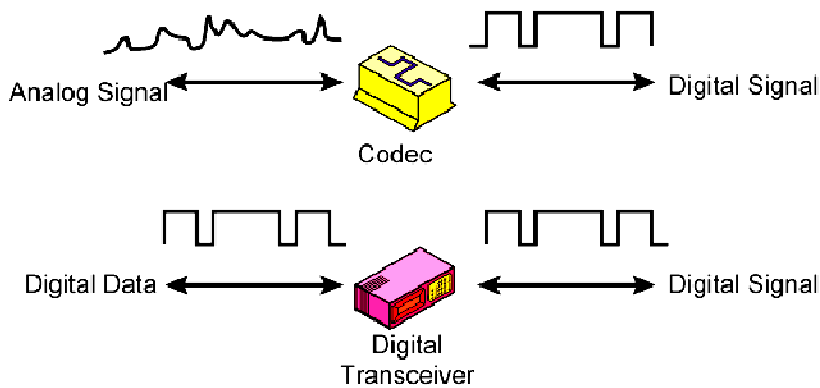


(a) Analog

(b) Digital

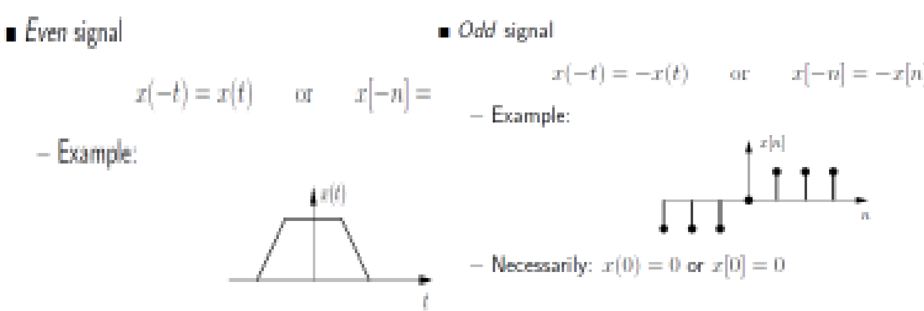
Signal Conversions

Digital Signals: Represent data with sequence of voltage pulses



Even and Odd Signals (Symmetry)

A signal is even, if $x(-t) = x(t)$
(i.e. it can be reflected in the axis at zero). Eg: cos(t)
A signal is odd if $x(-t) = -x(t)$.
(i.e. it can be inverted & reflected in the axis at zero).Eg: sin(t)



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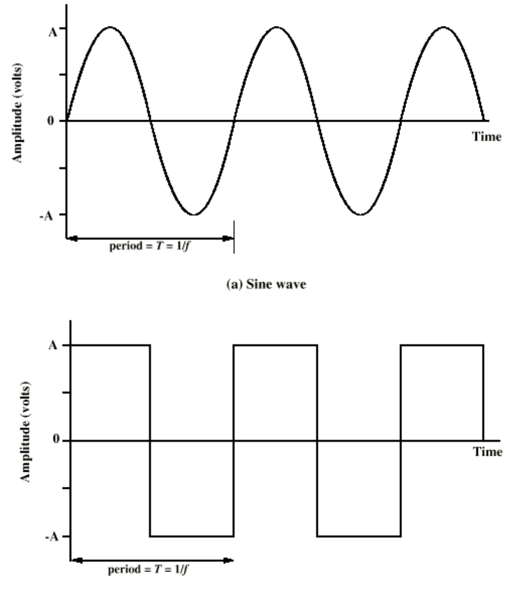
Periodic signals:

- A signal is periodic if it repeats itself after a fixed period T, ie. $x(t) = x(t+T)$ for all t.
- We only need define the signal over one period and we know everything about it
- Eg: A sin(t) signal is periodic



(a) Sine wave

(b) Square wave

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Sine Wave – Periodic Continuous Signal

- **Peak amplitude (A)**
  - maximum strength of signal
  - typically measured in volts
- **Frequency (f)**
  - rate at which the signal repeats
  - Hertz (Hz) or cycles per second
  - period (T) is the amount of time for one repetition
  - T = 1/f
- **Phase (ϕ)**
  - relative position in time within a single period of signal
  - (Demo of Phase impact)

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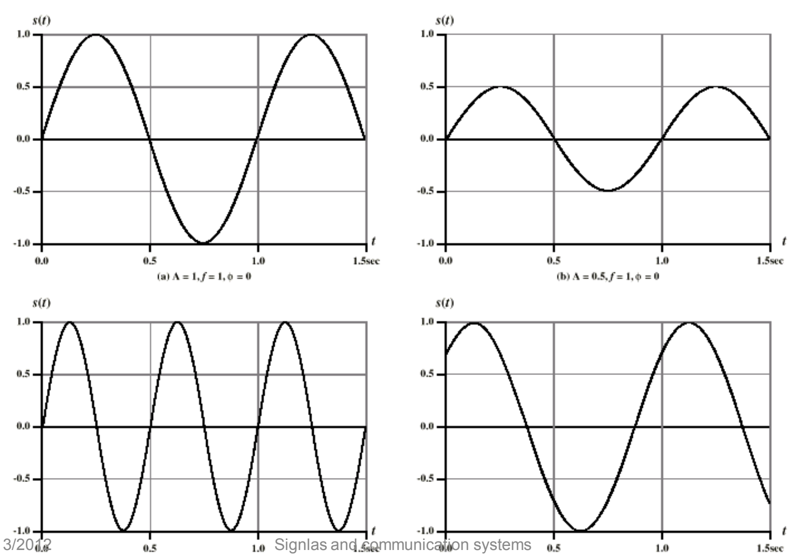
Wavelength (λ)

- is distance occupied by one cycle between two points of corresponding phase in two consecutive cycles
- assuming signal velocity *v* have $\lambda = vT$
- or equivalently $\lambda f = v$
- especially when *v=c*
  - c = $3*10^8$ ms^{-1} (speed of light in free space)

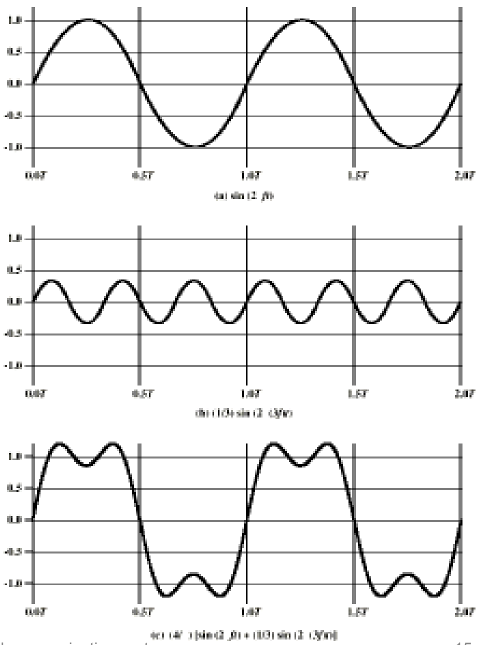
  (We are dealing with Electromagnetic Signals)

Varying Sine Waves
s(t) = A sin(2πft +Φ)

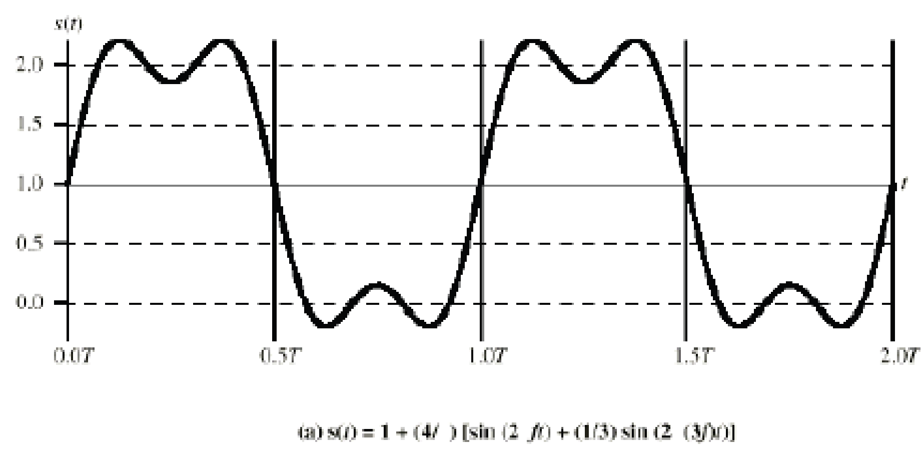


Addition of Frequency Components (T=1/f)

c is sum of *f* & *3f*



Signal with DC component



(a) s(t) = 1 + (4/) [sin (2 ft) + (1/3) sin (2 (3f)t)]

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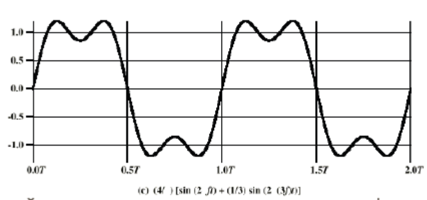
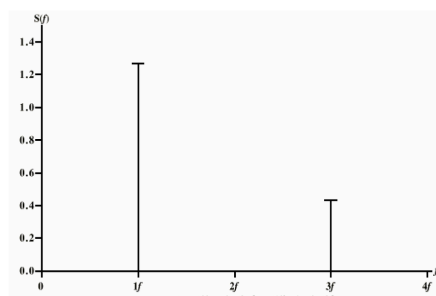
Frequency Domain Concepts

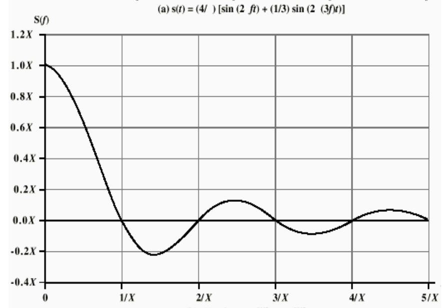
- signals are made up of many components
- components are sine waves of different frequencies, amplitudes and phases
- Fourier analysis can show the components of the signal.
- The sinusoidal components can be plotted with frequency as X-axis and amplitude as Y-axis.
- The above plot is known as frequency domain representation or spectral representation of the signal.

Frequency Domain Representations





frequency domain function ->



frequency domain function

of single square pulse

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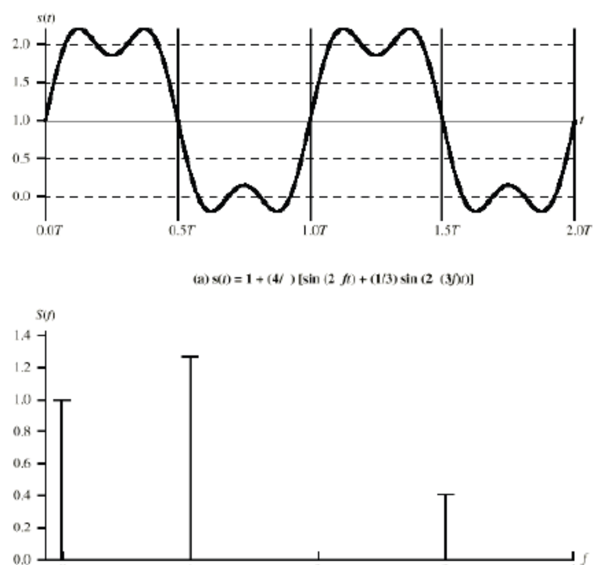
Digital Signals have only 2 levels – What about spectrum ?

Digital signal:

- Maintains a constant level then changes to another constant level
- From computer terminals etc.
- Two DC components (Does it mean that spectrum contains only 2 zero frequency components)
- Transition between the 2 levels introduces high frequency components
- Highest frequency depends on the data rate
- Higher the data rate, higher will be the highest frequency
- So, bandwidth depends on data rate

Spectrum of signal with DC component



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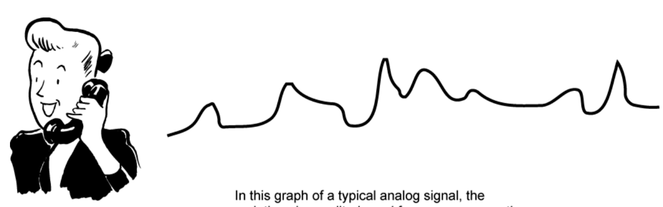
Spectrum & Bandwidth

- ***spectrum***
  - *range of frequencies contained in signal*
- ***absolute bandwidth***
  - *width of spectrum*
- ***effective bandwidth***
  - *often just bandwidth*
  - *narrow band of frequencies containing most energy*
- ***dc component***
  - *component of zero frequency*

Example: - Audio Signals

- frequency range of typical speech is 100Hz-7kHz
- easily converted into electromagnetic signals
- varying volume converted to varying voltage
- can limit frequency range for voice channel to 300-3400Hz



In this graph of a typical analog signal, the variations in amplitude and frequency convey the gradations of loudness and pitch in speech or music. Similar signals are used to transmit television pictures, but at much higher frequencies.

Data Rate and Bandwidth

any transmission system has a limited band of frequencies → this limits the data rate that can be carried on the transmission medium → square waves have infinite components and hence an infinite bandwidth → most energy in first few components → limiting bandwidth creates distortions

There is a direct relationship between data rate and bandwidth.

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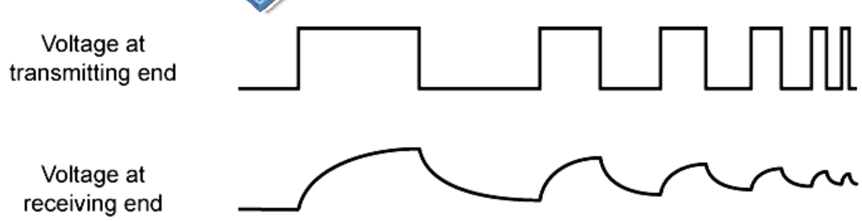
Advantages & Disadvantages of Digital Signals

- cheaper
- less susceptible to noise interference

- suffer more from attenuation
- digital now preferred choice



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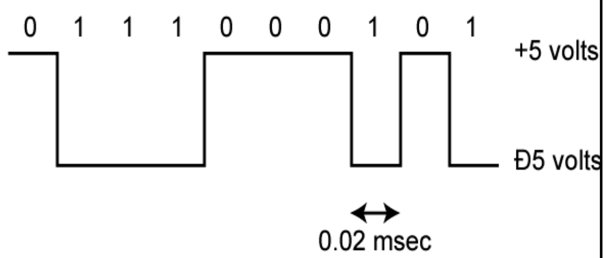
Conversion of PC Input to Digital Signal



User input at a PC is converted into a stream of binary digits (1s and 0s). In this graph of a typical digital signal, binary one is represented by Đ5 volts and binary zero is represented by +5 volts. The signal for each bit has a duration of 0.02 msec, giving a data rate of 50,000 bits per second (50 kbps).

Signal Energy and Power

Signal Energy

– Energy of a possibly complex continuous–time signal $x(t)$ in interval $t_1 \leq t \leq t_2$

$$E(t_1, t_2) = \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

– Energy of a possibly complex discrete–time signal $x[n]$ in interval $n_1 \leq n \leq n_2$

$$E(n_1, n_2) = \sum_{n=n_1}^{n_2} |x[n]|^2$$

– Total energy

$$E_\infty = E(-\infty, \infty) = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E_\infty = E(-\infty, \infty) = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

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Signal Power

– Consider the *time–averaged* signal power

– Average power of $x(t)$ in interval $t_1 \leq t \leq t_2$

$$P(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

– Average power of $x[n]$ in interval $n_1 \leq n \leq n_2$

$$P(n_1, n_2) = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

– Analogously

$$P_\infty = P(-\infty, \infty) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_\infty = P(-\infty, \infty) = \lim_{N \to \infty} \frac{1}{2N + 1} \sum_{n=-N}^{N} |x[n]|^2$$

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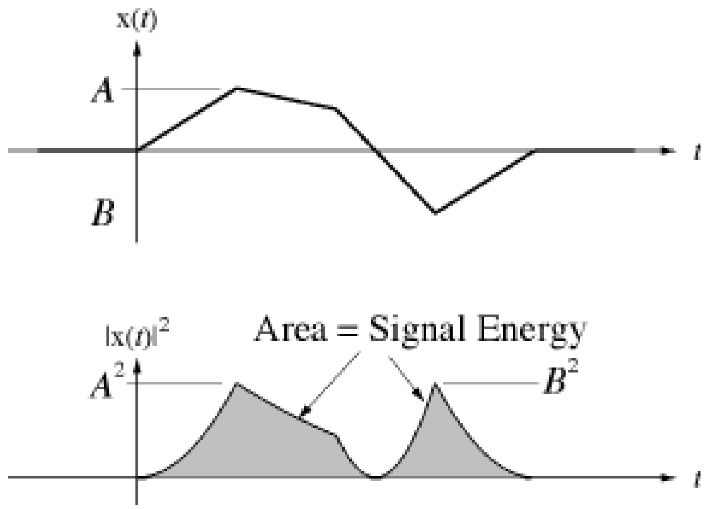
Signal Energy and Power



Signal Energy Example

Example:

Total energy of the discrete–time signal

$$x[n] = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

with $|a| < 1$.

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} (|a|^2)^n = \frac{1}{1-|a|^2}$$

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Signal Energy and Power

A signal with finite signal energy is called an *energy signal*.

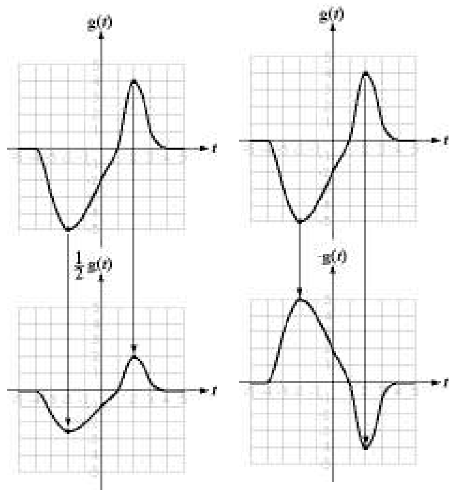
A signal with infinite signal energy and finite average signal power is called a *power signal*.

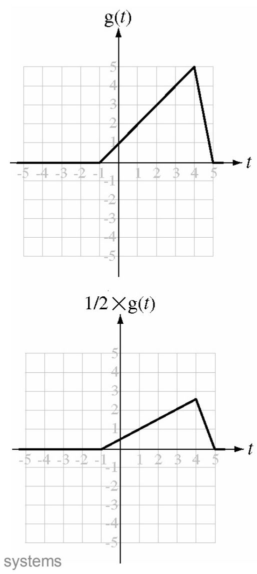
Signal Operations

- Amplitude Scaling
- Time Shift
- Time Scaling
- Time Reversal
- Signal filtering

Amplitude Scaling

$g(t) \to A\,g(t)$



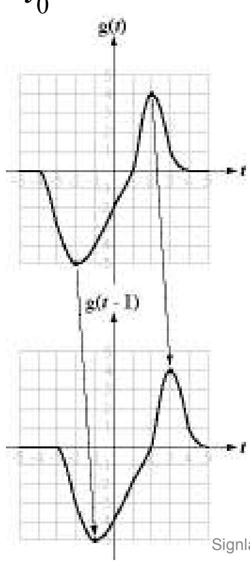


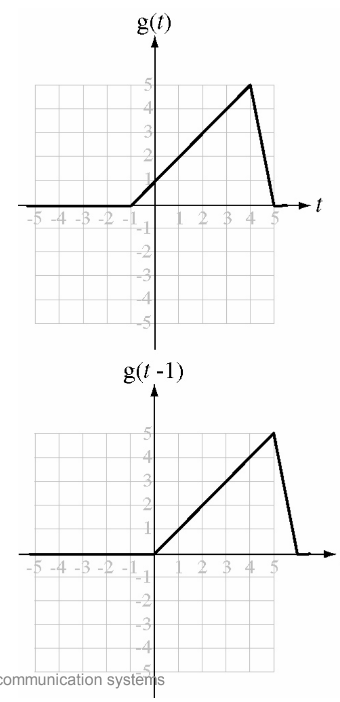
t	$g(t)$
−5	0
−4	0
−3	0
−2	0
−1	0
0	1
1	2
2	3
3	4
4	5
5	0

t	$1/2 \times g(t)$
−5	0
−4	0
−3	0
−2	0
−1	0
0	1/2
1	1
2	3/2
3	2
4	5/2
5	0

Time Shifting

$t \to t - t_0$



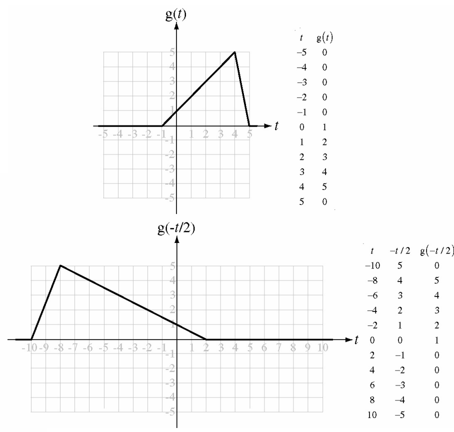


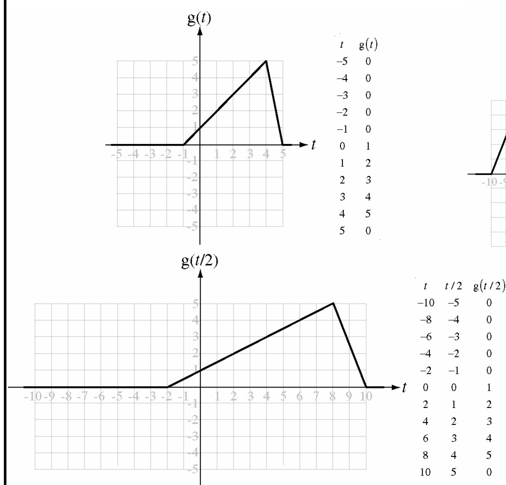
t	$g(t)$
−5	0
−4	0
−3	0
−2	0
−1	0
0	1
1	2
2	3
3	4
4	5
5	0

t	$t-1$	$g(t-1)$
−5	−6	0
−4	−5	0
−3	−4	0
−2	−3	0
−1	−2	0
0	−1	0
1	0	1
2	1	2
3	2	3
4	3	4
5	4	5

Time Scaling

Time scaling, $t \rightarrow \frac{t}{a}$





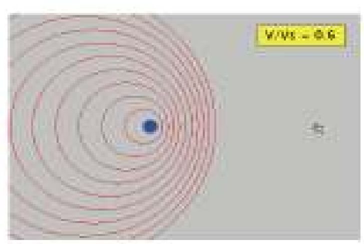
Expansion/contraction of signal along X axis Rotation about Y axis for negative a

Time Reversal: Replace t with -t

Time Scaling

Example: Doppler effect

Sound heard by firefighters: g(t)
Sound we hear when truck comes: A(t) g(at), A increasing, a>1
Sound we hear when truck goes: B(t) g(bt), B decreasing, b<1



Checkout the website:
http://www.lon-capa.org/~mmp/applist/doppler/d.htm

http://www.lon-capa.org/~mmp/applist/doppler/d.htm

Multiple Transformations

$$g(t) \rightarrow A\,g\left(\frac{t-t_0}{a}\right)$$

A multiple transformation can be done in steps

$$g(t) \xrightarrow[\text{scaling, } A]{\text{amplitude}} A\,g(t) \xrightarrow{t \to \frac{t}{a}} A\,g\left(\frac{t}{a}\right) \xrightarrow{t \to t-t_0} A\,g\left(\frac{t-t_0}{a}\right)$$

The sequence of the steps is significant

$$g(t) \xrightarrow[\text{scaling, } A]{\text{amplitude}} A\,g(t) \xrightarrow{t \to t-t_0} A\,g(t-t_0) \xrightarrow{t \to \frac{t}{a}} A\,g\left(\frac{t}{a}-t_0\right) \neq A\,g\left(\frac{t-t_0}{a}\right)$$

Signal Operations (Cntd) Filtering

Signal filtering

- Filters are used to change the shape of the input signal's spectrum
- The filter let some frequencies through undistorted while other frequencies are blocked
- Filters can be designed to operate on Analog or digital signals
- Frequency selective filters are often classified as:
  1. Low pass filters
  2. High pass filters
  3. Band pass filters

Frequency Response and Filtering

- $y(t)$ and $y[n]$ periodic with the same fundamental period as $x(t)$ and $x[n]$, respectively
  - Fourier series coefficients

$$\boxed{\begin{array}{ll} x(t) \xleftrightarrow{\mathcal{FS}} a_k & x[n] \xleftrightarrow{\mathcal{FS}} a_k \\ y(t) \xleftrightarrow{\mathcal{FS}} a_k H(\mathrm{j}k\omega_0) & y[n] \xleftrightarrow{\mathcal{FS}} a_k H(e^{\mathrm{j}2\pi k/N}) \end{array}}$$

- Process of $\left\{ \begin{array}{ccc} x(t) & \longrightarrow & y(t) \\ a_k & \longrightarrow & a_k H(\mathrm{j}k\omega_0) \end{array} \right\}$ referred to as *filtering*
- Depending on the shape of the frequency response
  - filters change the shape of the *frequency spectrum* (Fourier coefficients) of the input signal
  - and/or pass some frequencies undistorted but suppress undesired frequency components.

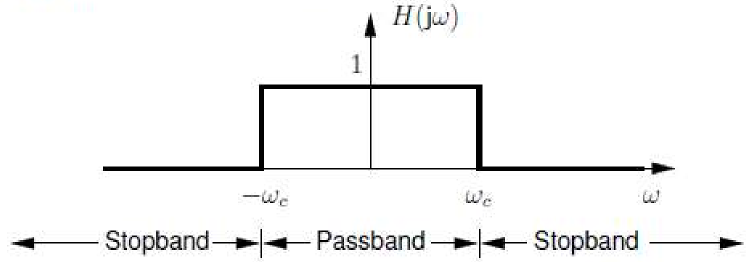
Low pass filter

Ideal Lowpass Filter

– Continuous–time case

$$H(\mathrm{j}\omega) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

with *cut–off frequency* ω_c



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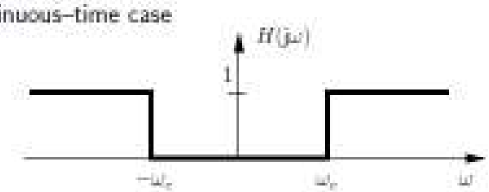
High pass / Band pass filters

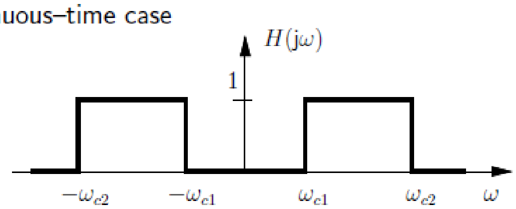
Ideal Highpass Filter

– Continuous–time case



Ideal Bandpass Filter

– Continuous–time case



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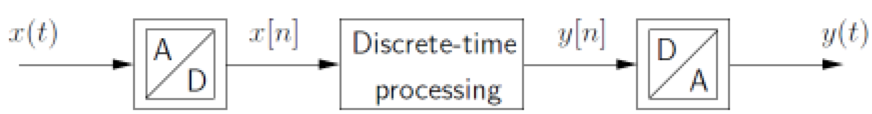
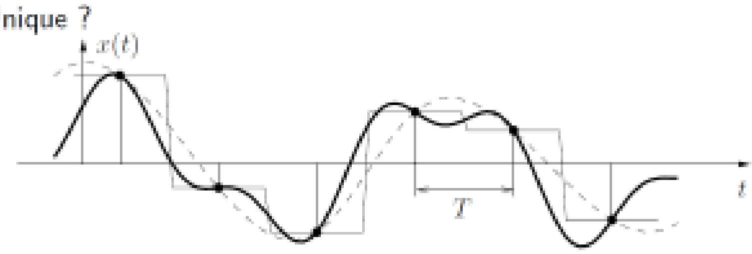
Sampling and Quantization

Sampling

Bridge between continuous–time and discrete–time signals



- Unique ?

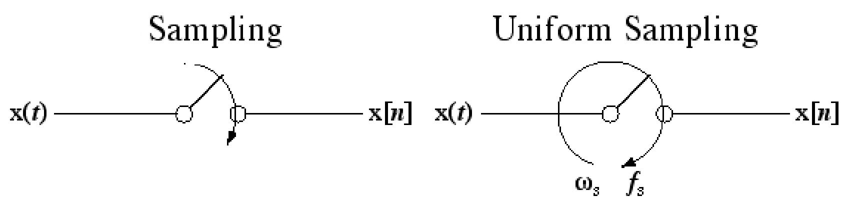


  - Observation: An infinite number of signals can generate a given set of samples.
  - We need additional constraints on continuous-time signal!

Sampling a CT Signal to Create a Discrete-Time (DT) Signal

- *Sampling* is acquiring the values of a CT signal at discrete points in time
- x(*t*) is a CT signal --- x[*n*] is a DT signal

$\mathrm{x}[n] = \mathrm{x}(nT_s)$ where T_s is the time between samples



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Sampling Theorem

The Sampling theorem, also known as Nyquist-Shannon Sampling theorem, states that if a continuous time function f(t) is band-limited with its highest frequency component less than W, then f(t) can be completely recovered from its sampled values, if the sampling frequency is equal to or greater than 2W.

OR

If a signal is sampled at regular intervals at a rate higher than twice the highest signal frequency, the samples contain all the information of the original signal.

— In short: sample with rate more than twice the highest signal frequency
— Voice data limited to below 4000Hz, thus, require 8000 sample per sec
— The samples are analog samples, think of a slice of the signal
— The signal can be reconstructed from the samples using a low pass filter

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Nyquist Frequency

- The frequency 4 kHz, fs/2, is called the Nyquist frequency, after Harry Nyquist, an engineer at Bell Labs who, in the 1920s and 1930s, laid much of the groundwork for digital transmission of information.
- The Nyquist frequency turns out to be a key threshold in the relationship between discrete-time and continuous-time signals, more important even than the sampling frequency.
- Intuitively, this is because if we sample a sinusoid with a frequency twice the Nyquist frequency , then we take at least two samples per cycle of the sinusoid.
- It should be intuitively appealing that taking at least two samples per cycle of a sinusoid has some key significance.
- The two sample minimum allows the samples to capture the oscillatory nature of the sinusoid.
- Fewer than two samples would not do this. However, what happens when fewer than two samples are taken per cycle is not necessarily intuitive. It turns out that the sinusoid masquerades as one of another frequency.

Nyquist sampling theorem - (sampling theorem)

According to the Nyquist sampling theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

OR

If a function $x(t)$ contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced $1/(2B)$ seconds apart.

It can be seen that sampling at the **Nyquist rate** can create a good approximation of the original sine wave.
Oversampling in can also create the same approximation, but it is redundant and unnecessary.
Sampling below the Nyquist rate (**Under sampling**) does not produce a signal that looks like the original sine wave.

Problem 1

A low-pass signal has a bandwidth of 20 kHz. What is the minimum sampling rate for this signal?

4.49

Problem 2

A bandpass signal has a bandwidth of 10 kHz. What is the minimum sampling rate for this signal?

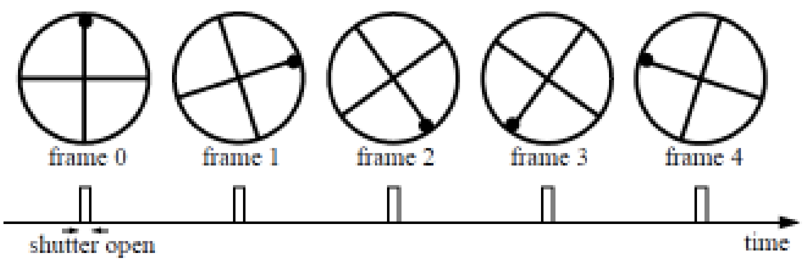
4.50

Wagon Wheel Effect

an example of temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Demo

Aliasing - Definition

- In A/D conversion, Nyquist theorem states that the sampling rate must be at least twice the maximum bandwidth of the analog signal.
- If the sampling rate is insufficient, then higher-frequency components are "undersampled" and appear shifted to lower-frequencies. These frequency-shifted components are called aliases.
- The frequencies that shift are sometimes called "folded" frequencies.

Aliasing

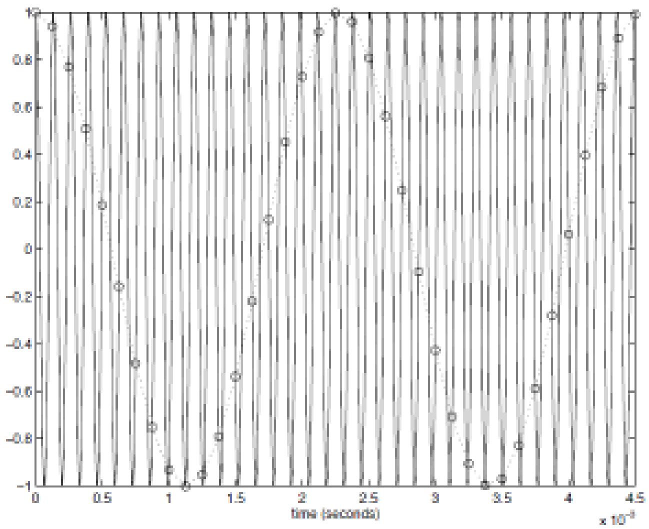


Figure 11.3: A sinusoid at 7.56 kHz and samples taken at 8 kHz.

Aliasing:

If a signal is not band-limited, or if the sampling rate is too low, the spectral components of the signal will overlap each another and this condition is called *aliasing.*
To avoid aliasing, we must increase the sampling rate.
Or
use an Anti-aliasing filter prior to sampling

Demo

Anti-Aliasing filter

- Nyquist frequency is half of the Sampling frequency (Nyquist Frequency is sometimes known as folding frequency)
- If a continuous-time signal contains only frequencies below the Nyquist frequency (fs/2), then it can be perfectly reconstructed from samples taken at sampling frequency fs.
- This suggests that prior to sampling, it is reasonable to filter a signal to remove components with frequencies above fs/2.
- A filter that realizes this is called an anti-aliasing filter.

Demo

Quantization

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\Delta = (\text{max} - \text{min})/L$$

4.55

Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

Demo

4.56

Amplitude Quantization

Amplitude quantization is defined as the process of transforming the sample amplitude x(nTs) of a signal x(t) at time t = nTs into a discrete amplitude X[nTs] taken from a finite set of possible amplitudes.

Quantization process is memory less and instantaneous – Meaning is that the transformation done at time t = nTs is not affected by the earlier and later samples of the signal.

Uniform quantizer – representation levels are uniformly placed. Otherwise non-uniform.

Quantization Error

- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more levels, the smaller Δ which results in smaller errors.
- BUT, the more levels the more bits required to encode the samples -> higher bit rate

Quantization

Quantization - Means Amplitude digitization

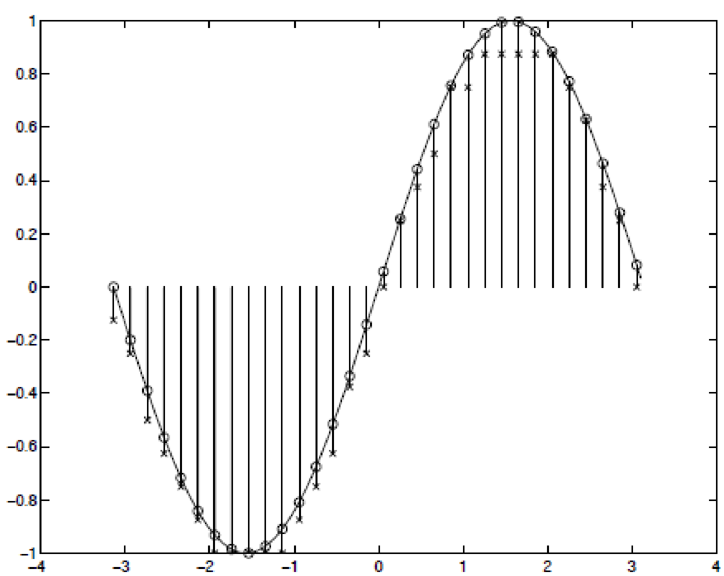
- Converts the analogue sample value to a N bit binary
- Finite number of levels $L = 2^N$
- Analogue sample is approximated to the nearest digital value
- The approximation changes the actual signal level
- Quantization Noise = Analogue value (Virtual) - Digital

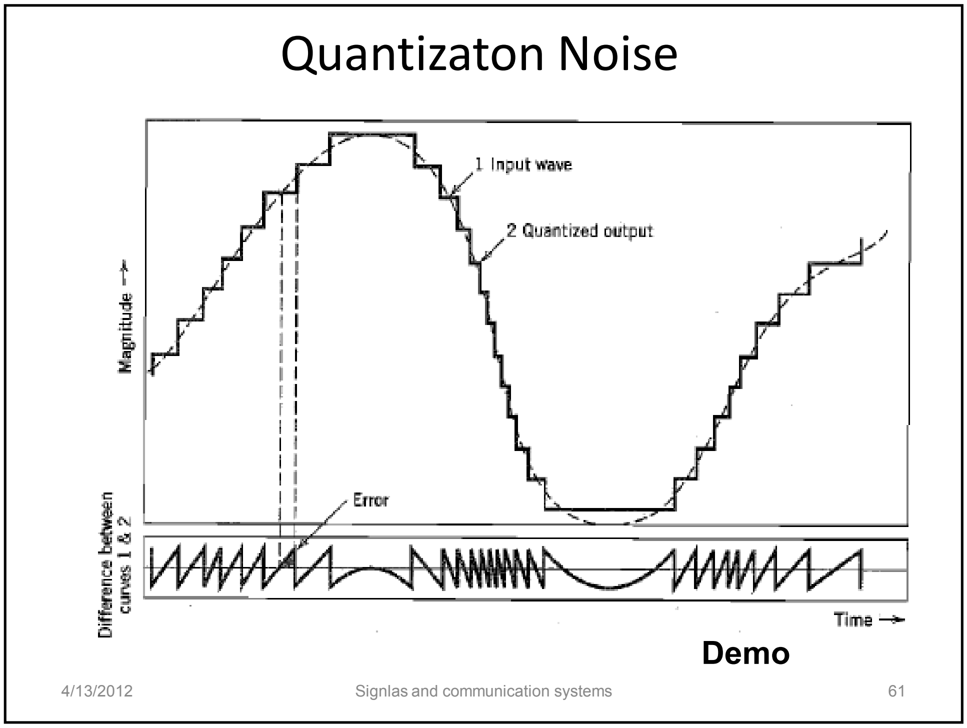
- Signal-to-noise ratio for the quantizing noise can be expressed as

$$\begin{aligned} \text{SNR} &= 20 \log 2^n + 1.76 \text{ dB} \\ &= 6.02n + 1.76 \text{ dB} \end{aligned}$$

- For each additional bit used for quantizing, SNR increases by about 6 dB, or a factor of 4

Quantizaton




Quantizaton Noise
1 Input wave
2 Quantized output
Magnitude →
Error
Difference between curves 1 & 2
Time →
Demo
4/13/2012
Signlas and communication systems
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