

MODULE 1

SET THEORY

4 Marks Questions

1. Give an example of an uncountable set
2. Define a set? Explain the basic operations and properties of sets
3. Define the following terms with an examples
 - a) Equinumerous sets
 - b) Countable sets
 - c) Uncountable sets
 - d) Countably infinite sets
4. Define
 - a) Injective function
 - b) Surjective function
 - c) Bijective function
5. What are primitive recursive and partial recursive functions
6. Explain the terms
 - a) Bounded Minimalization
 - b) Unbounded Minimalization
7. Define finite and infinite sets
8. Explain the terms
 - a) Computable functions
 - b) Non computable functions
9. Explain chomsky classification of languages
10. Define countable sets. Is the set of all polynomial functions with integer coefficients countable. Explain the reason
11. Show that the union of a countably infinite collection of countably infinite sets is countably infinite
12. Show that the union of finite number of countably infinite sets is countably infinite.
13. Show that the set of all positive rational numbers is a countably infinite set
14. Show that the difference of an uncountable set and countable set is uncountable
15. Is the set $\{0, 2, 4, \dots\}$ is countably infinite. Prove
16. Generate 5 strings generated out of the language:
 - a) $L = \{xx \mid x \text{ element of } \{0, 1\}^*\}$
 - b) $L = \{0^i \mid i \text{ is a prime number}\}$
17. Show that the function $f(x) = x/2$ is partial recursive

12 marks Questions

1. Explain in detail Chomsky hierarchy or Chomsky classification of languages.
2. Explain in detail diagonalization principle. Prove that $2^{\mathbb{N}}$ is not countable.
3. Show that the grammars.

$S \longrightarrow aSb/ab/\epsilon$

$S \longrightarrow aAb/ab$

$S \longrightarrow aAb/\epsilon$ are equivalent. What language do they generate?

4. Explain primitive recursive & partial recursive function with examples.
5. Show that the set of all real numbers in the interval $[0, 1]$ is uncountable.
6. What is primitive recursive fn: & check whether
$$F(x, y) = \begin{cases} 1, & \text{for } x < y \\ 0, & \text{for } x \geq y \end{cases}$$
 is primitive recursive
7. Discuss the concepts of alphabets and languages
8. Show that the set $\mathbb{N} \times \mathbb{N}$ is countably infinite
9. Define primitive recursive function and prove that $f(x, y) = x + y, x, y \in \mathbb{N}$ is primitive recursive

10. Check whether the function given by, $\max(x,y) = \begin{cases} x, & \text{if } x \geq y \\ y, & \text{if } x < y \end{cases}$ is primitive recursive
11. Show that if a function f is defined from 'g' by bounded minimization then f is also primitive recursive
12. Prove that the function $f(x) = x^y$ is primitive recursive
13. Explain strategy set with eg and base functions

MODULE 3 PUSHDOWN AUTOMATA

1. Give the formal definition of PushDown Automata
2. Explain the language acceptability by PDA
3. What is instantaneous description of a PDA
4. Construct a PDA accepting the languages
 - a) $L = \{wcw^R \mid w \text{ in } (0+1)^*\}$ BY EMPTY STACK
 - b) $L = \{ww^R \mid w \text{ in } (0+1)^*\}$ by empty stack
5. Explain the steps in constructing a PDA equivalent to a given CFG
6. What are contextfree grammars
7. Construct a PDA equivalent to the following grammar

$S \rightarrow Aaa$
 $A \rightarrow aA/bS/a$
8. Define the terms a)terminals b)Non-terminals c)Productions
9. How can we reduce a given CFG
10. What are useless symbols
11. How can we eliminate ϵ -productions and unit productions from a given CFG
12. What are the two normal forms of CFG
13. What is Nullable Nonterminal? How can we eliminate ϵ -productions from the grammar ,G? Eliminate ϵ -productions from the given grammar

$S \rightarrow XY$
 $X \rightarrow Zb$
 $Y \rightarrow Bw$
 $Z \rightarrow AB$
 $W \rightarrow Z$
 $A \rightarrow aA/bA/\epsilon$
 $B \rightarrow Ba/Bb/\epsilon$
14. Construct a NPDA that accepts the language generated by $S \rightarrow aSbb/a$
15. Convert the productions into CNF,

$S \rightarrow abAB$
 $A \rightarrow bAB/\epsilon$
 $B \rightarrow BAa/A/\epsilon$
16. Design a PDA which accepts the language, $L = \{w \in \{a,b\}^* \mid w \text{ has equal no. of } a\text{'s and } b\text{'s}\}$
17. Let G be the grammar, $S \rightarrow aB/Ba$
 $A \rightarrow a/aS/bAA$
 $B \rightarrow b/bS/Abb$. For the string 'aaabbabbba' find a) Leftmost derivation b) Rightmost derivation
 c) Parse tree
18. Find a CFG with no useless symbols equivalent to

$S \rightarrow AB/CA$
 $B \rightarrow BC/AB$
 $A \rightarrow a$
 $C \rightarrow aB/b$

- 19.Explain the applications of PDA
- 20.Compare TM with PDA

MODULE 4

TURING MACHINES

- 1.Give the formal definition of Turing machines
- 2.What do you mean by instantaneous description of a TM
- 3.Explain the language acceptability of a TM
- 4.Give short note on universal TM
- 5.What are the different types of TM
- 6.Explain Godelization or Godel numbering
- 7.Explain the terms a)Decidable problems
b)Undecidability
- 8Explain Church thesis
- 9.Explain the halting problem of Turing Machine.Prove that halting problem is undecidable
- 10.Design a TM to accept the language,
 $L=\{0^n1^n / n \geq 1\}$ or
 $L=\{a^n b^n / n \geq 0\}$
- 11.Design a TM to perform proper subtraction
- 12.What are recursively enumerable languages
- 13.Design a TM to accept the language, $L=\{ww^R / w \in \{0+1\}^*\}$
- 14.Design a TM to recognize the following languages, $L=\{0^n1^n 0^n / n \geq 1\}$
- 15.Design a TM to compute the following functions
a) $n!$ b) n^2
- 16.Design a TM that recognizes the set of all strings of 0's and 1's containing atleast one 1.
- 17.Design a TM that replaces every 0 with 1 and every 1 with 0 in the binary string
- 18.Give an example of a problem that cannot be solved by TM
- 19.Write a recursive fn: add(x,y) to add two integers x and y
- 20.Give an example of zero function,succesor function and predecessor fn:
- 21.Design a TM to copy strings of 1's only
- 22.Show that it is undecidable whether a TM halts on all inputs
- 23.Design a TM to add two given integers.

MODULE 5

ALGORITHMIC COMPLEXITY

- 1.What are tractable and intractable problems
- 2.Explain Cook's theorem
- 3.Explain the complexity hierarchy
- 4.Explain the P and NP Classes with example
- 5.Define an NP-Complete problem and give an example
- 6.Explain the polynomial time algorithm,exponential time algorithm and computationally intractable problems
- 7.Explain algorithmic complexity
- 8.Explain the Boolean satisfiability problem
- 9.Prove that 3CNF is NP-Complete