CS010 404 SIGNALS AND COMMUNICATION SYSTEMS

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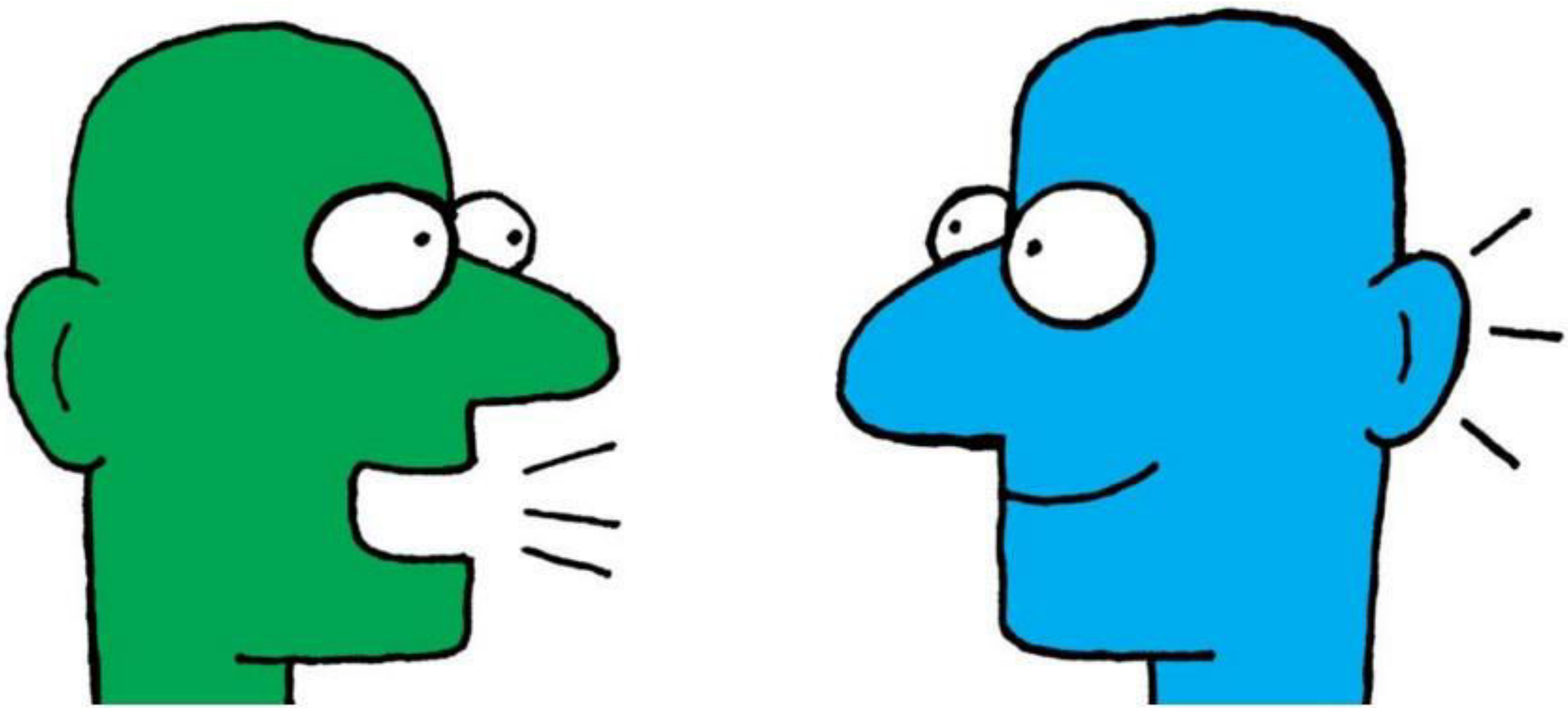
ECE RSET

MODULE I

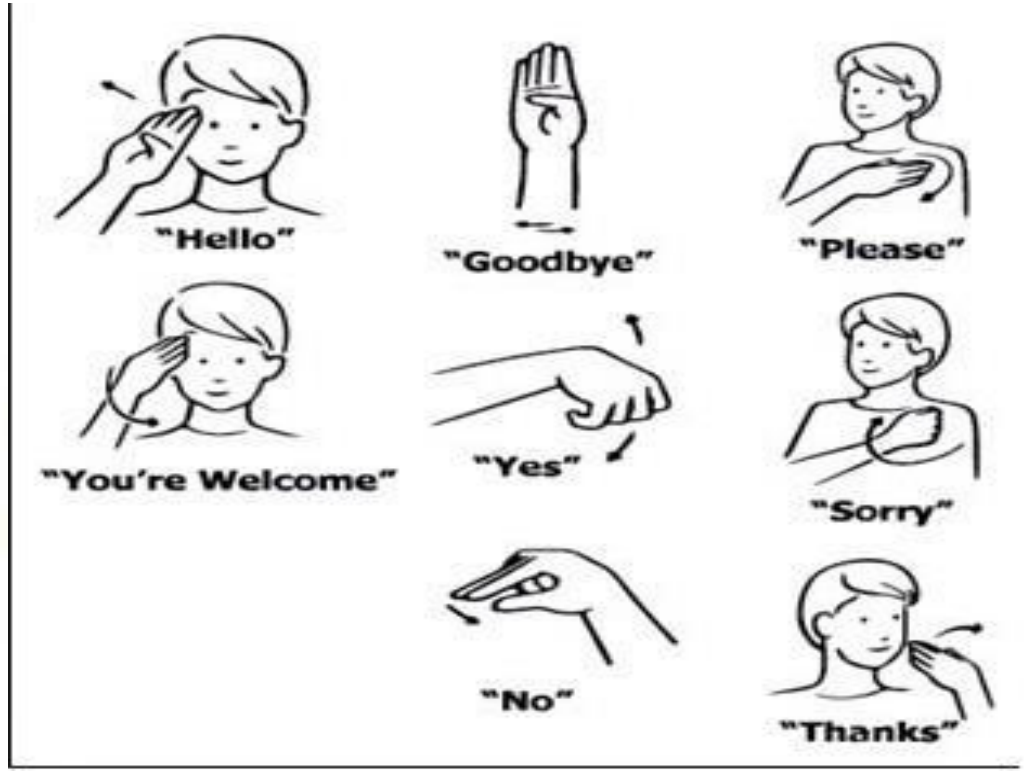
- Introduction to Signals
  - Continuous Time Signals
  - Discrete Time Signals
- Signal Operations
- Properties of Signals
  - Periodicity
  - Symmetry

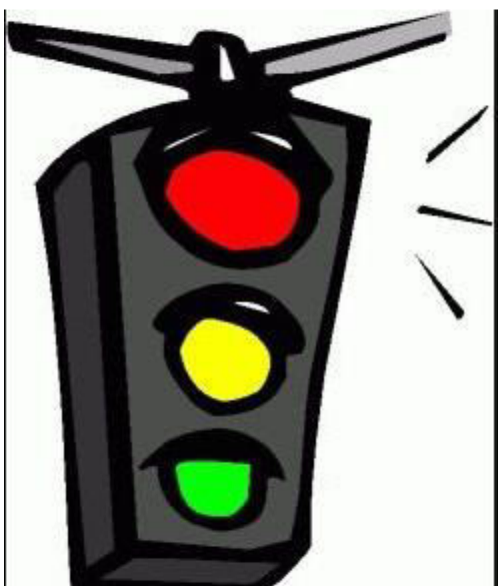
- Frequency Domain Representation of Continuous Time Signals
  - CTFS
  - CTFT
- Concept of Frequency Spectrum
- Sampling- The Sampling Theorem
- Quantisation

Introduction to Signals




Smoke Signals of the Vatican Conclave...
NO POPE
NEW POPE


"Hello"
"Goodbye"
"Please"
"You're Welcome"
"Yes"
"Sorry"
"No"
"Thanks"



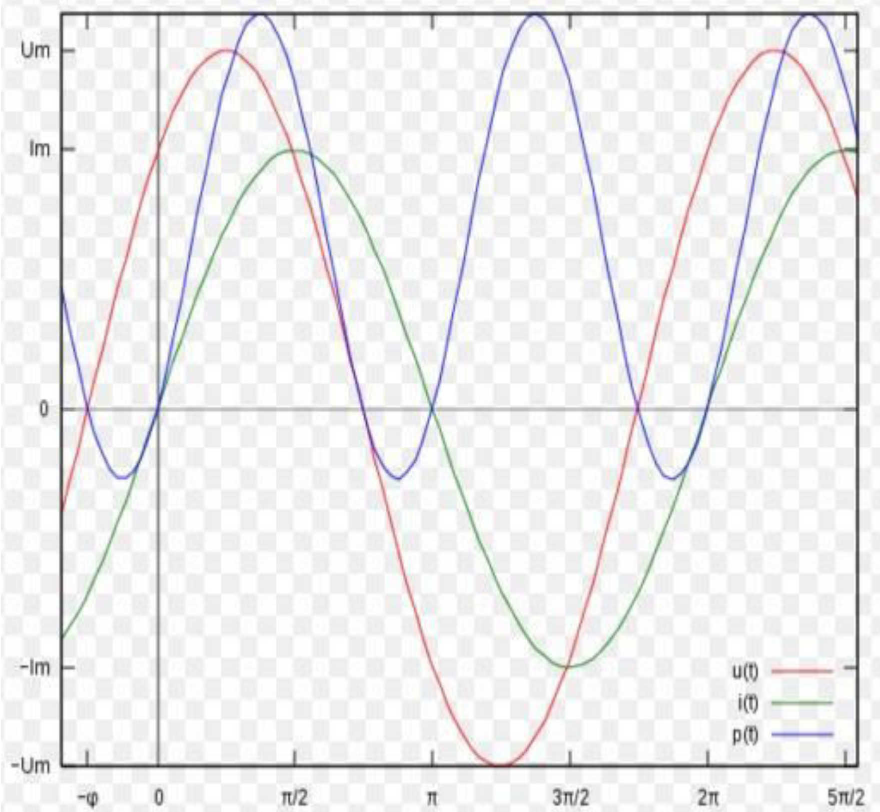
- **Signal** is defined as *physical quantity* that varies with time, space or any other independent variables which conveys some *information*.

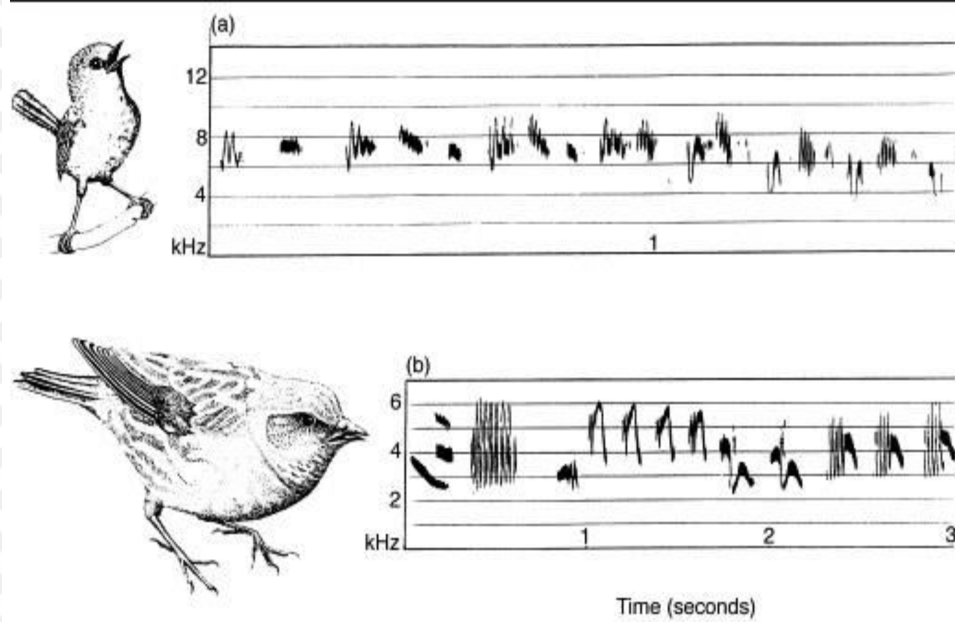
- **Noise** is unwanted message.

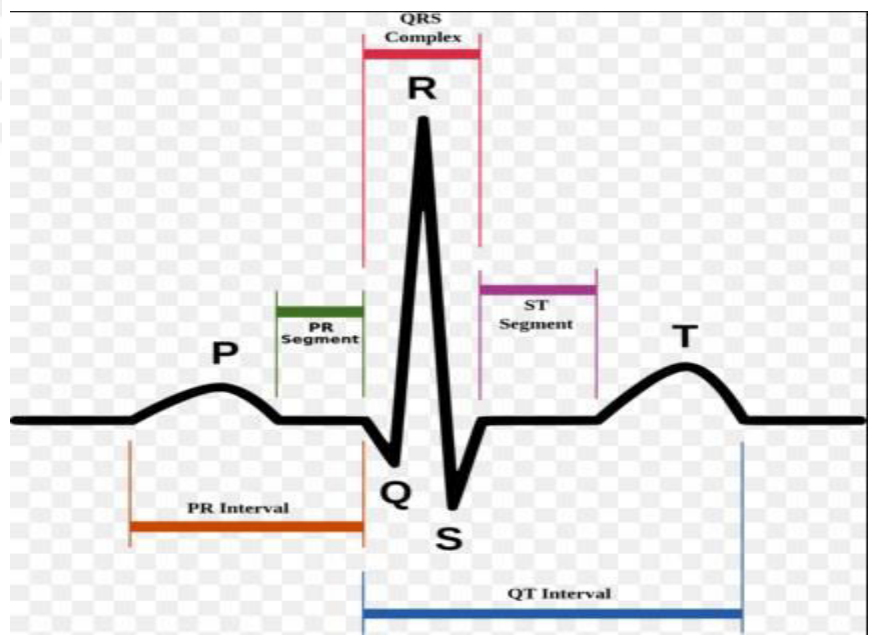
- 1D → 1 independent variable. Eg.?
- 2D → 2independent variable. Eg.?
- MD → Many independent variable

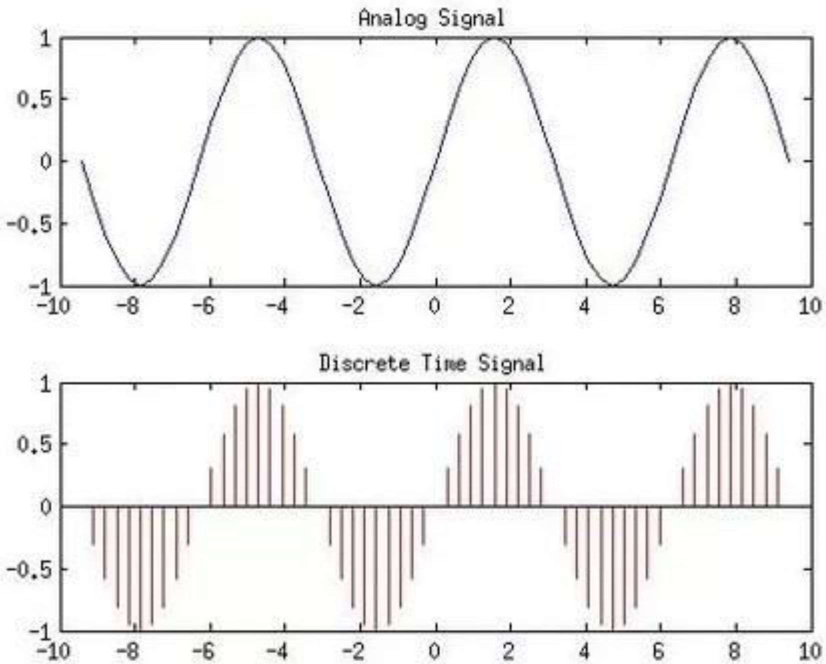
Classification of Signals

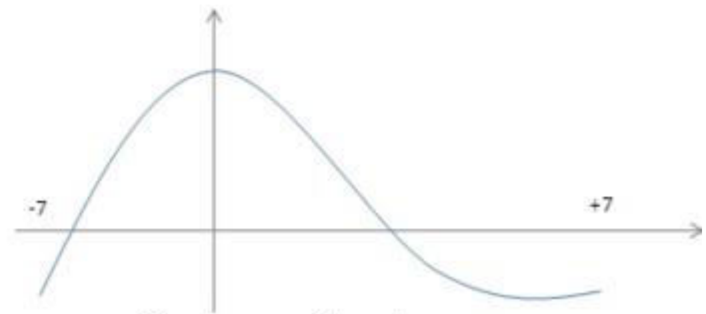
- Continuous Time Signals/ Analog Signals
  - *Signals that are defined for every instant of time.*
- Discrete Time Signals
  - *Signals that are defined for every discrete instant of time*
  - *Continuous in amplitude and discrete in time*
- Digital Signals
  - *Discrete in time and quantized in amplitude*


Um
Im
0
-Im
-Um
−φ
0
π/2
π
3π/2
2π
5π/2
u(t)
i(t)
p(t)


(a)
12
8
4
kHz
1
(b)
6
4
2
kHz
1
2
3
Time (seconds)

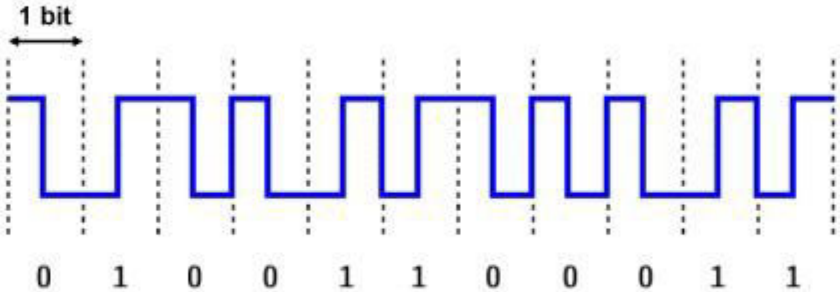

QRS
Complex
R
PR
Segment
ST
Segment
P
T
Q
S
PR Interval
QT Interval

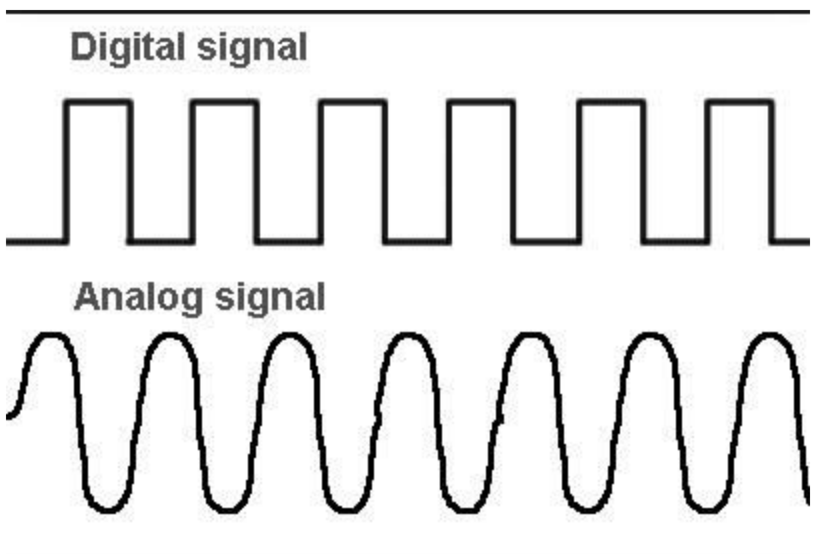




Continuous Signal
(takes values *in the set* [-7,7])

Discrete Signal
(takes values *at the integers* {-7,-6…0…6,7})


1 bit
0 1 0 0 1 1 0 0 0 1 1


Digital signal
Analog signal

Signal Operations

- Operations performed on dependent variables
  - Addition/Subtraction, Multiplication, Integration/Differentiation, Amplitude Scaling.
- Operations performed on independent variables
  - Time Scaling, Time Shifting, Time Reversal.

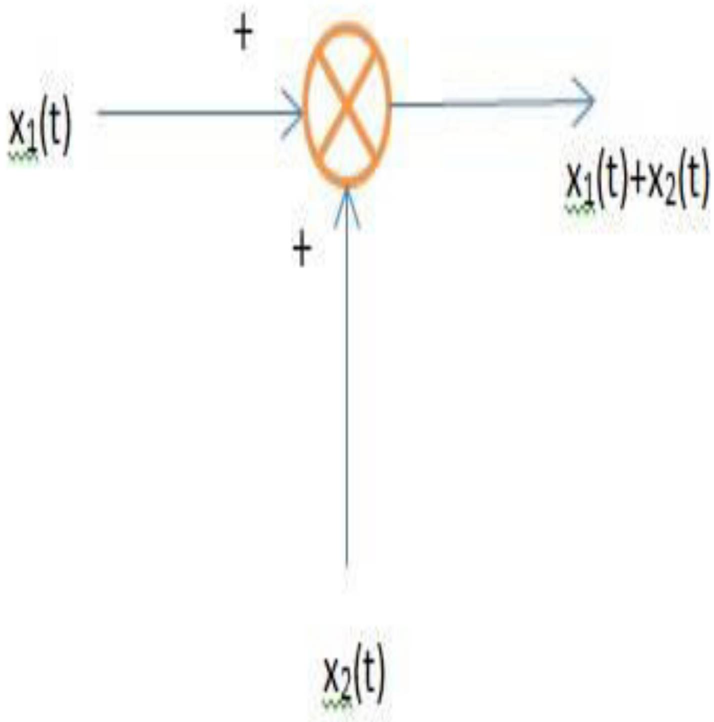
Representation of Signals

- x(t) → Continuous Time Signal

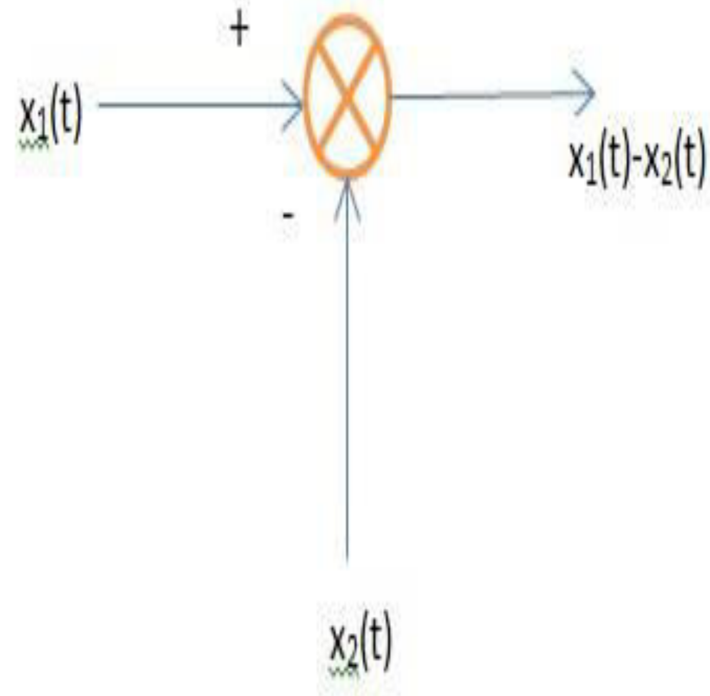
- x(n) → Discrete Time Signal
  - $x(n\tau) = x(t)|_{t\ =\ n\tau}$

Addition / Subtraction

- The sum of two continuous time signal can be obtained by adding their values at every instant.
- Similarly the subtraction is obtained by subtracting the values at every instant.

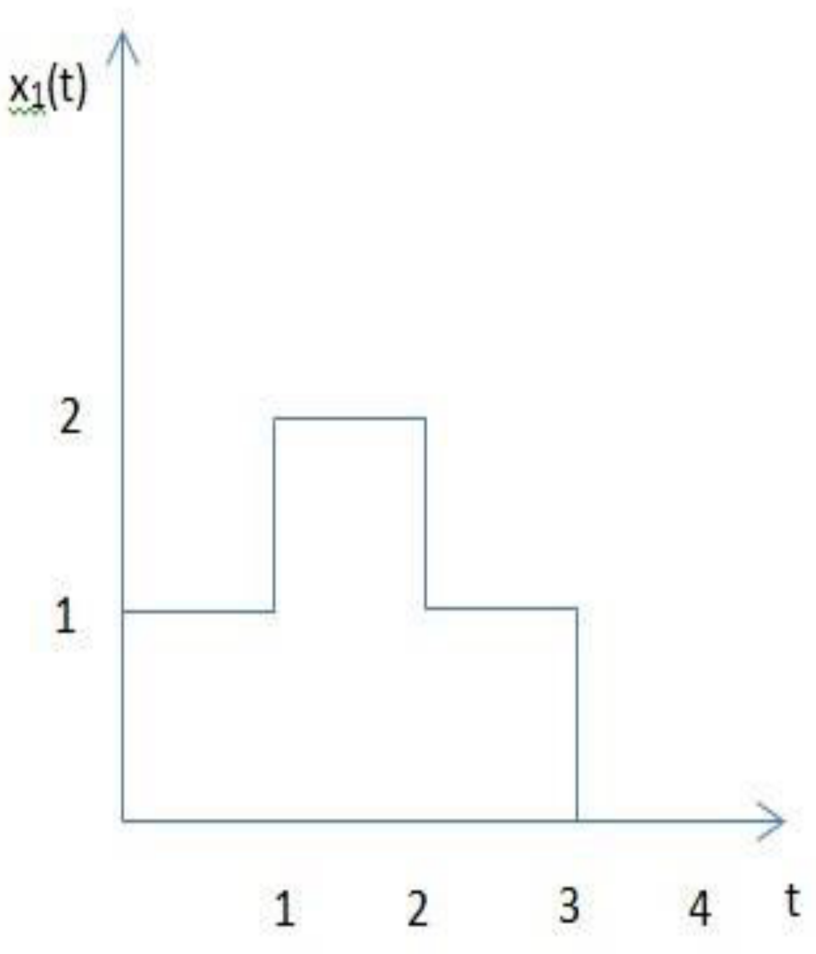


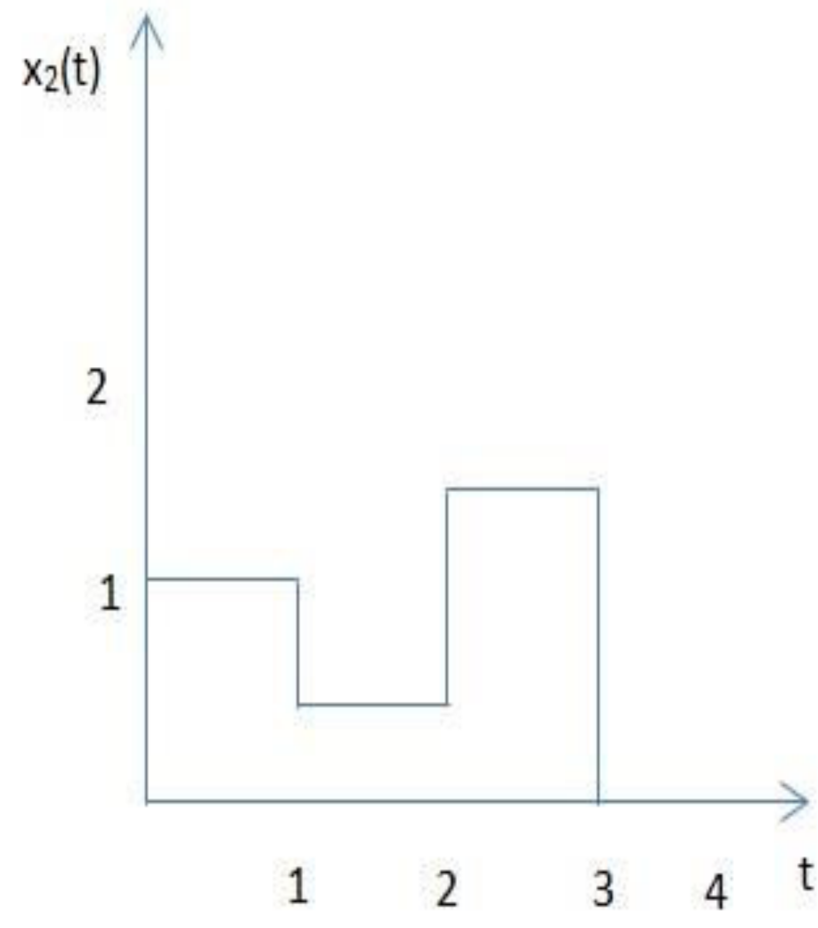
Addition



Subtraction

Perform Addition & Subtraction





Solution

- Addition
  - $0 \leq t \leq 1$ $x_1(t)=1$ $x_2(t)=1$
    - $x_1(t)+ x_2(t)=2$
  - $1 \leq t \leq 2$ $x_1(t)=2$ $x_2(t)=0.5$
    - $x_1(t)+ x_2(t)=2.5$
  - $2 \leq t \leq 3$ $x_1(t)=1$ $x_2(t)=1.5$
    - $x_1(t)+ x_2(t)=2.5$

- Solution for Subtraction???

- Plot the graph!!!!!!!

What about Discrete??

Perform addition and subtraction

$x_1(n)=\{1,3,2,1\}$

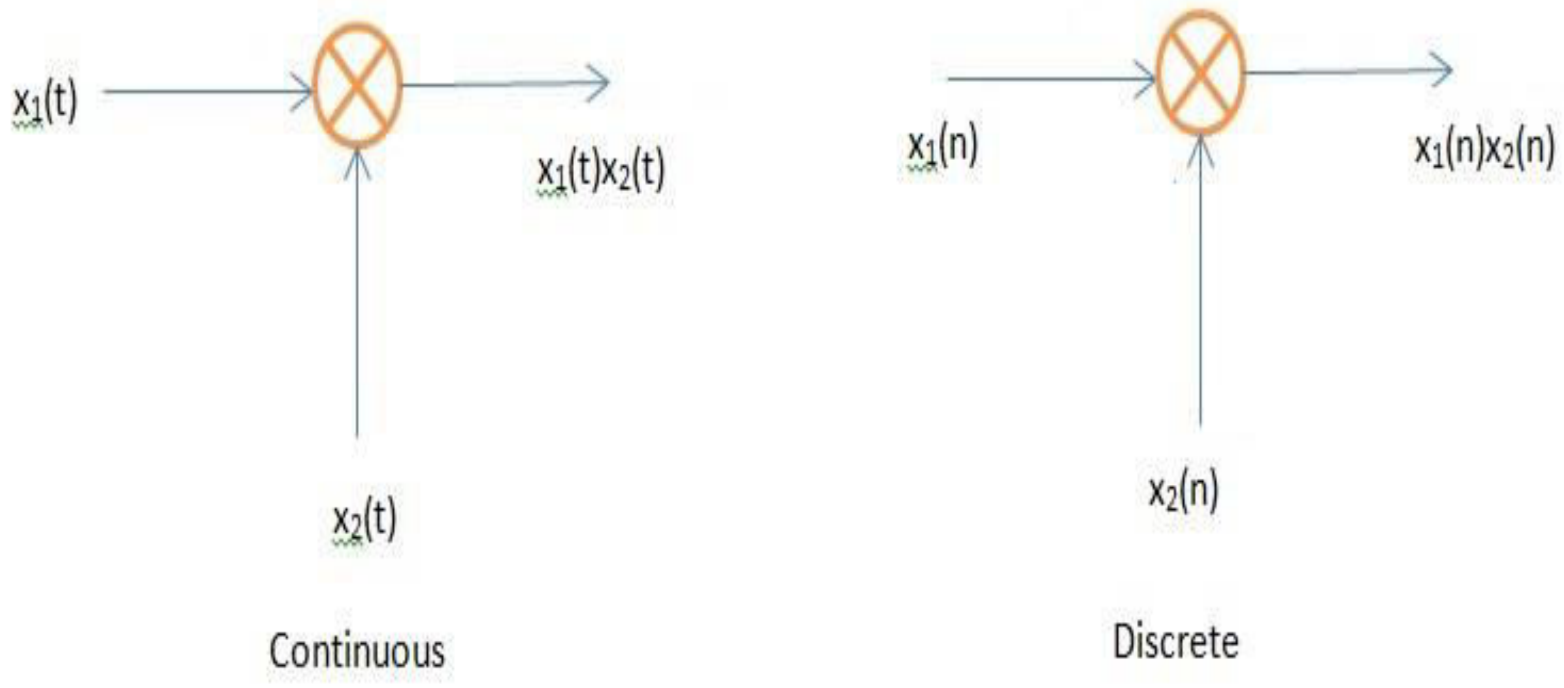
$x_2(n)=\{1,-2,3,2\}$

Addition $x_1(n)+x_2(n)$

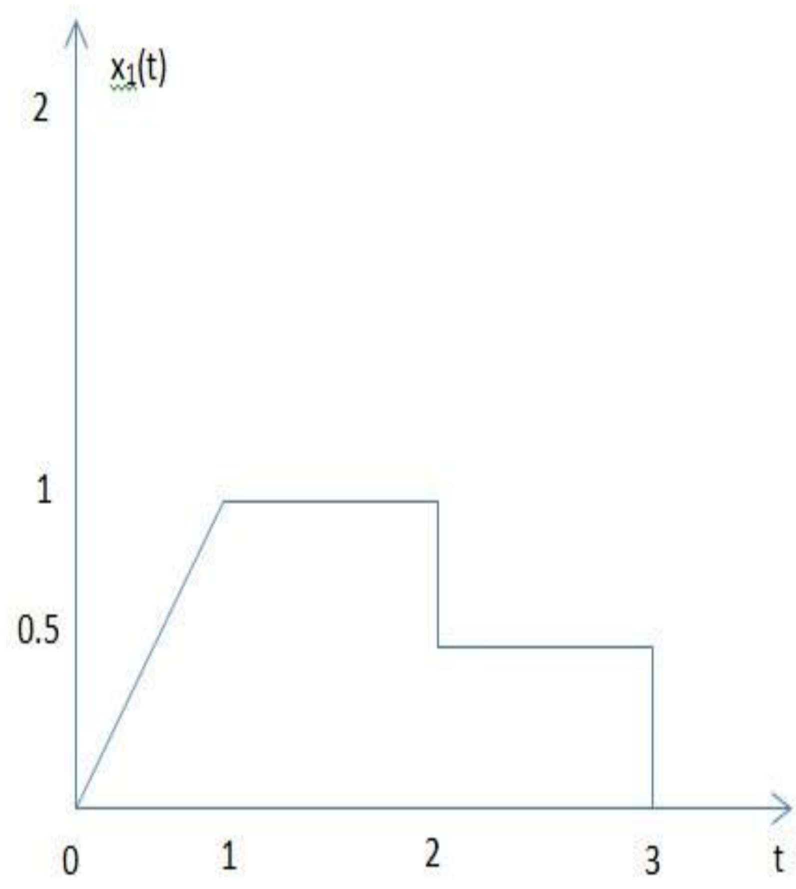
Subtraction $x_1(n)-x_2(n)$

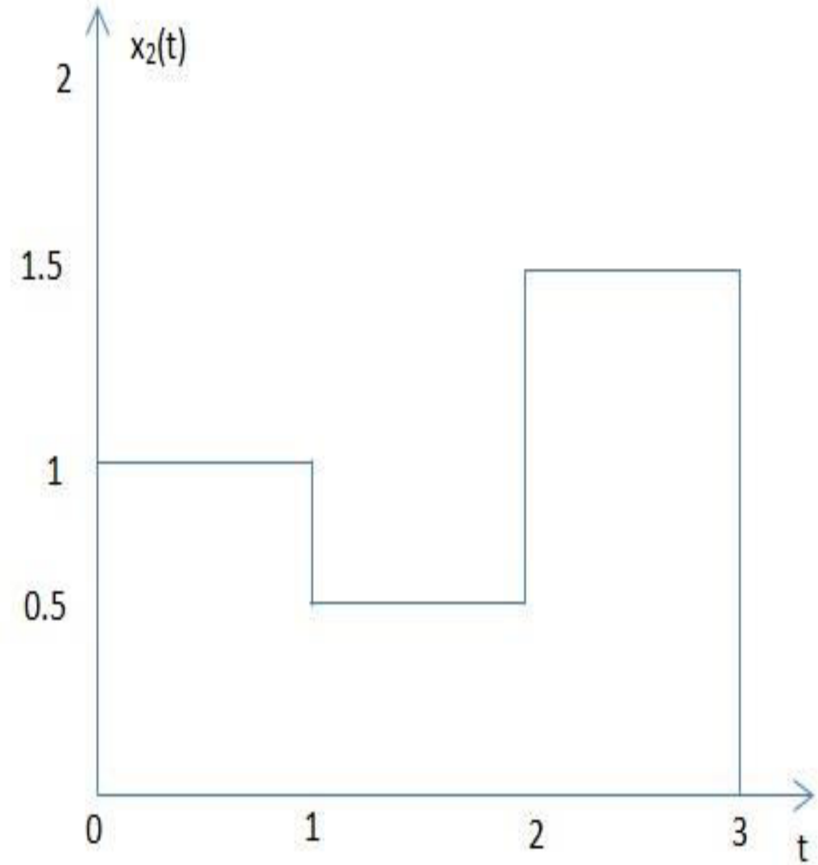
Multiplication

- Multiplication of two signals can be obtained by multiplying values at every instant.



Perform Multiplication





Solution

- Multiplication
  - $0 \leq t \leq 1$ $x_1(t)=t$ $x_2(t)=1$
    - $x_1(t) * x_2(t) = t$
  - $1 \leq t \leq 2$ $x_1(t)=1$ $x_2(t)=0.5$
    - $x_1(t) * x_2(t)=0.5$
  - $2 \leq t \leq 3$ $x_1(t)=0.5$ $x_2(t)=1.5$
    - $x_1(t) * x_2(t)=0.75$
  - Plot the Graph!!!...

What about Discrete??

Perform Multiplication

$x_1(n)=\{1,2,-2,3\}$

$x_2(n)=\{1,0.5,0.5,3\}$

Multiplication $y(n)= x_1(n)*x_2(n)$

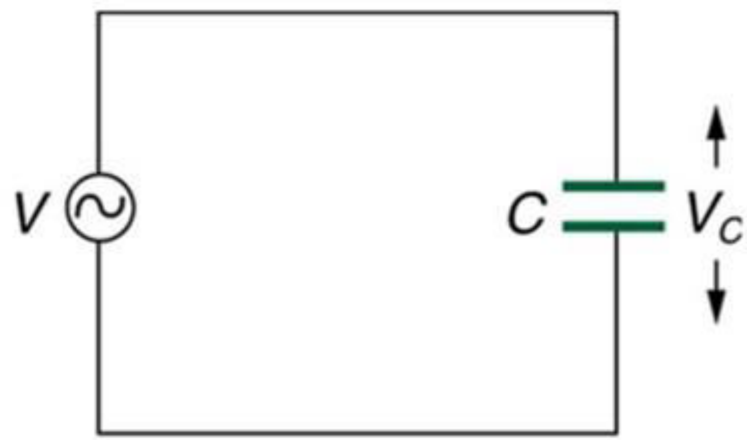
Plot the Graph!!!...

Differentiation

- x(t) → Continuous Time Signal
- $y(t) = \frac{d}{dt}x(t)$
- Eg. Inductor
- $v(t) = L\frac{d}{dt}i(t)$

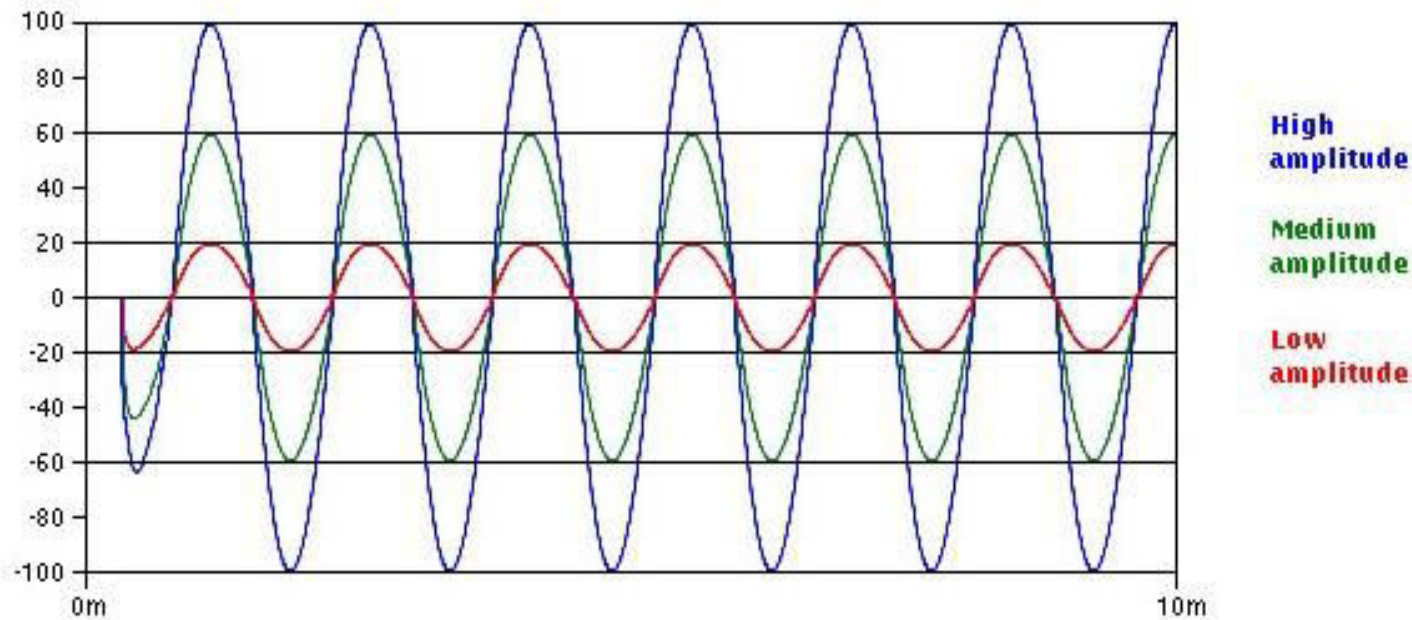
Integration

- x(t) → Continuous Time Signal
- $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$
- Eg. Capacitor
- $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$



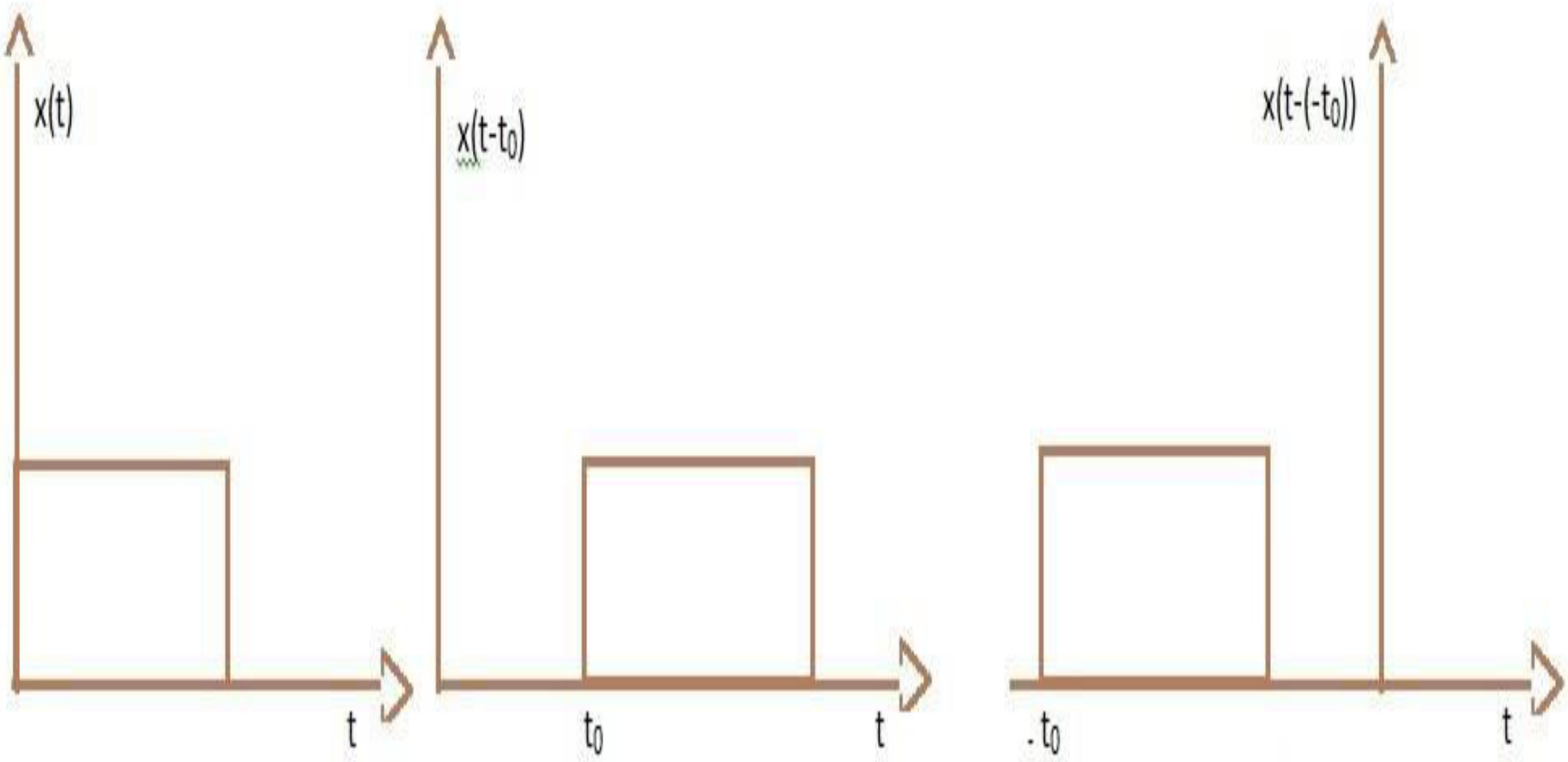
Amplitude Scaling

- x(t) → Continuous Time Signal
- $y(t) = cx(t)$ $\qquad\qquad$ $y(n) = cx(n)$

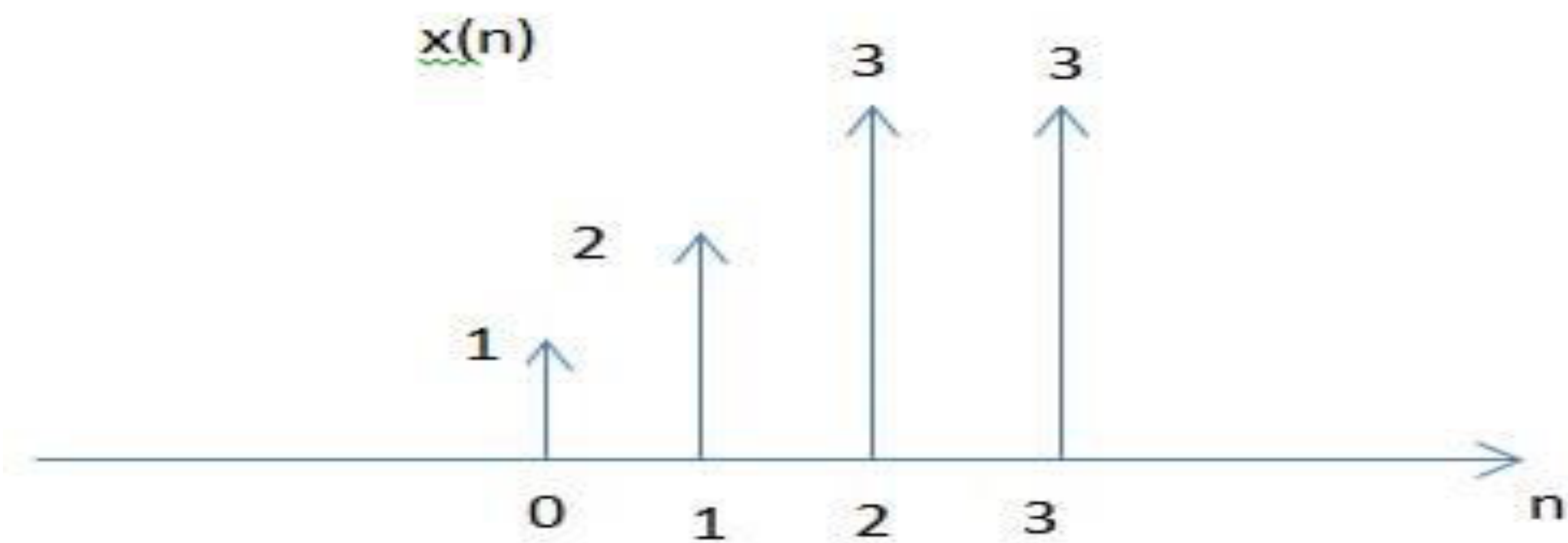


Time Shifting

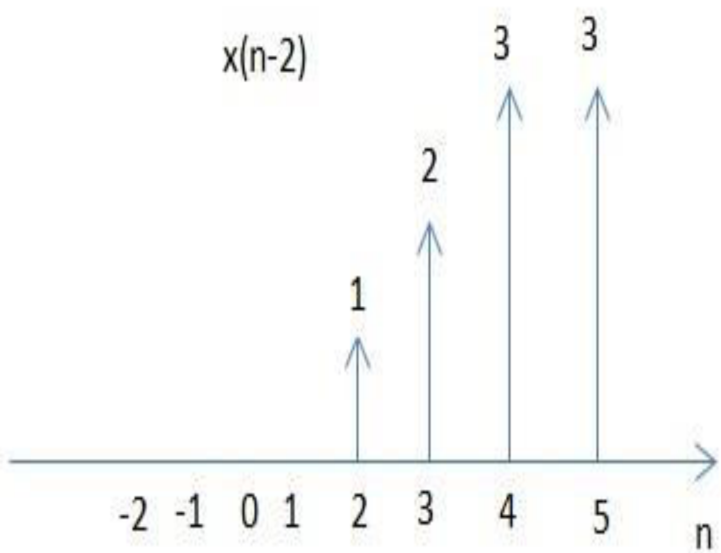
- It may delay or advance the time
- $y(t) = x(t - t_0)$ $\quad$ $y(n) = \mathrm{x}(n - m)$
- If t_0 is positive shifting is towards right which causes delay
- If t_0 is negative shifting is towards left which causes advance

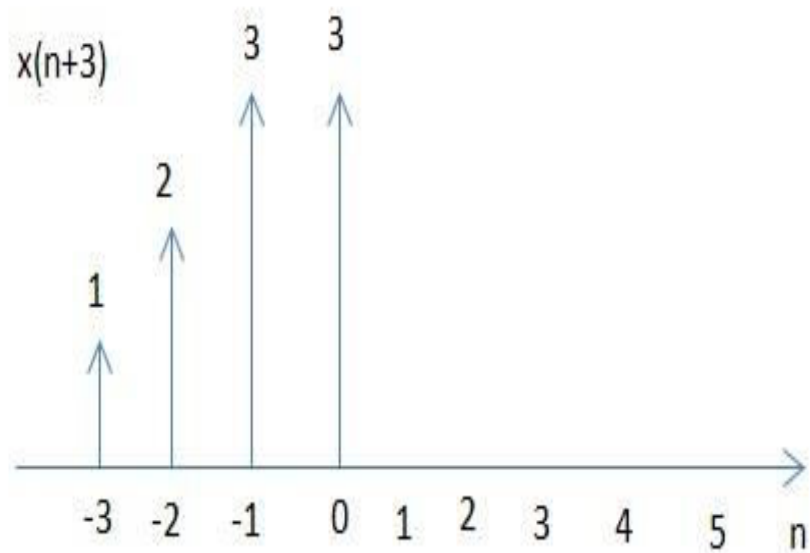

x(t)
t
x(t-t0)
t0
t
x(t-(-t0))
. t0
t

Consider Signal x(n) shown in figure and obtain x(n-2) and x(n+3)



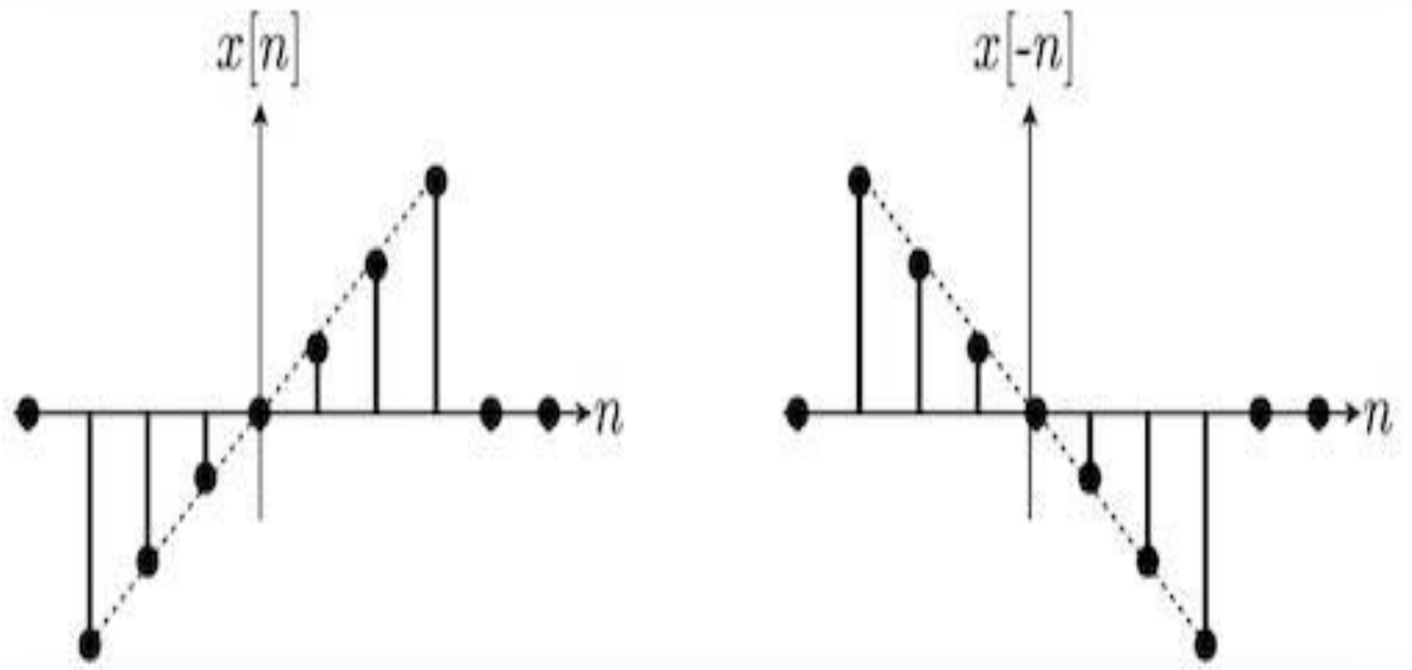
Solution

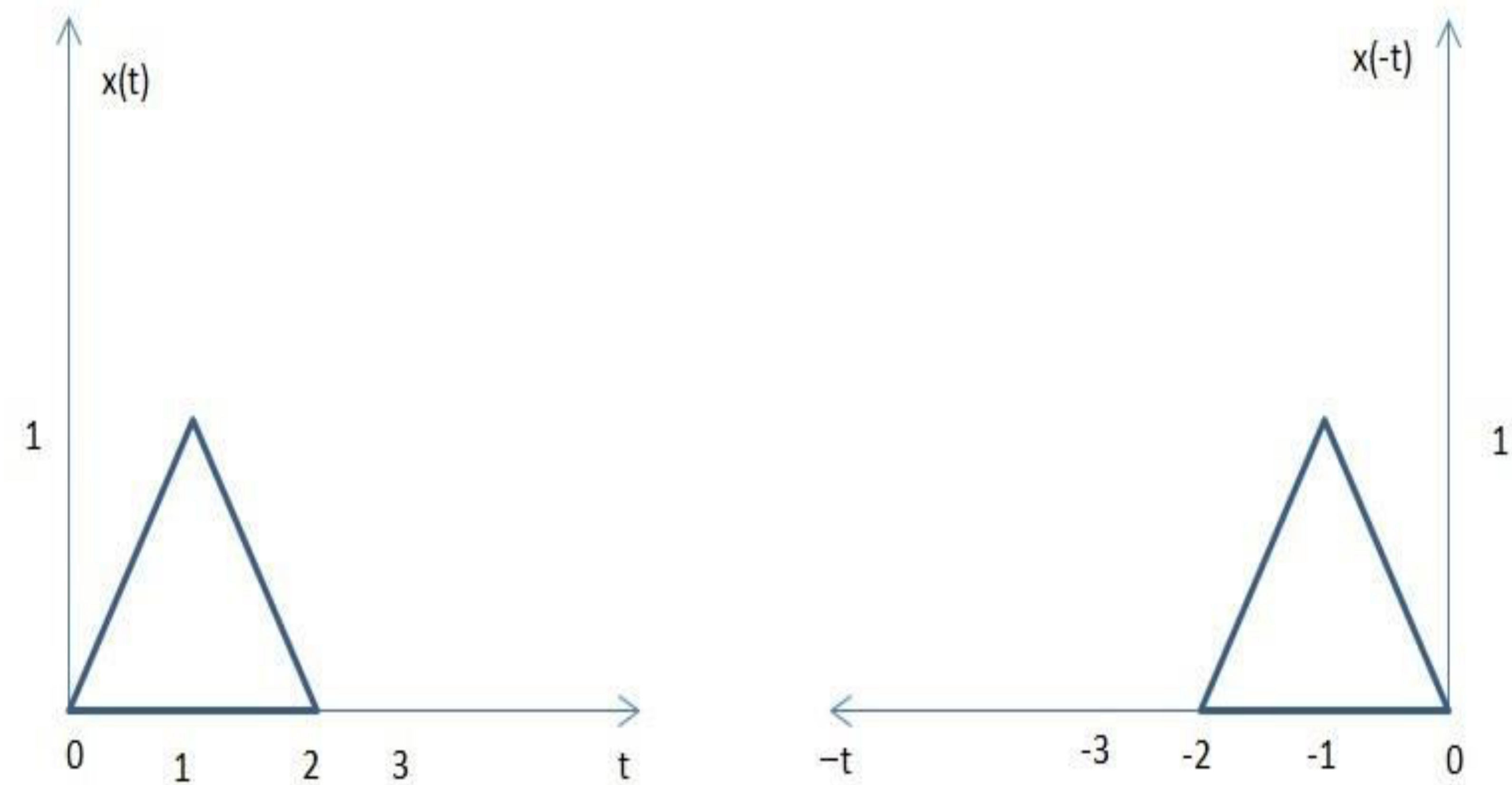




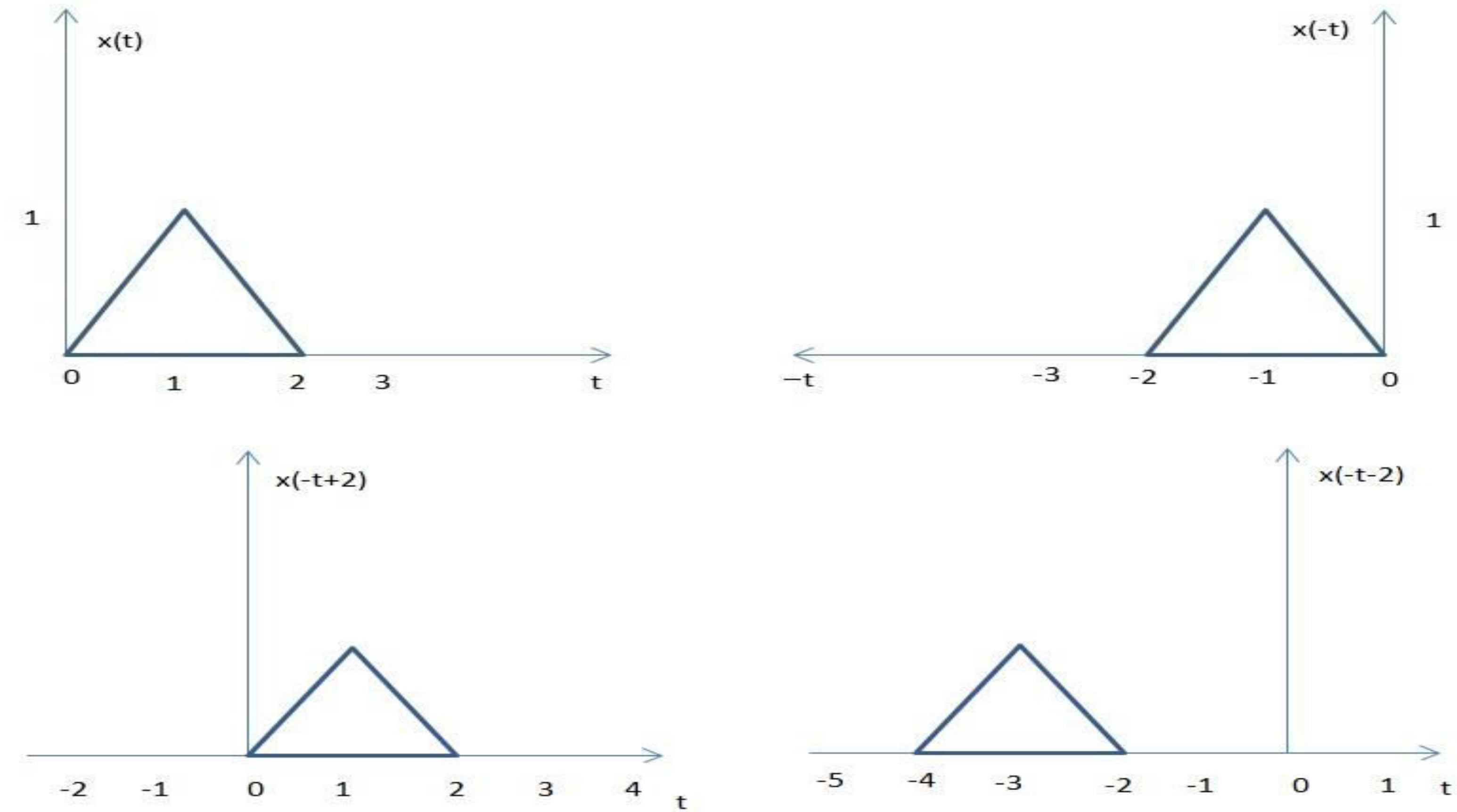
Time Reversal

- Obtained by folding the signal about ZERO.
- $y(t) = x(-t) \qquad y(n) = x(-n)$




x(t)
1
0
1
2
3
t
x(-t)
1
−t
-3
-2
-1
0

Obtain the signal x(-t+2) & x(-t-2)

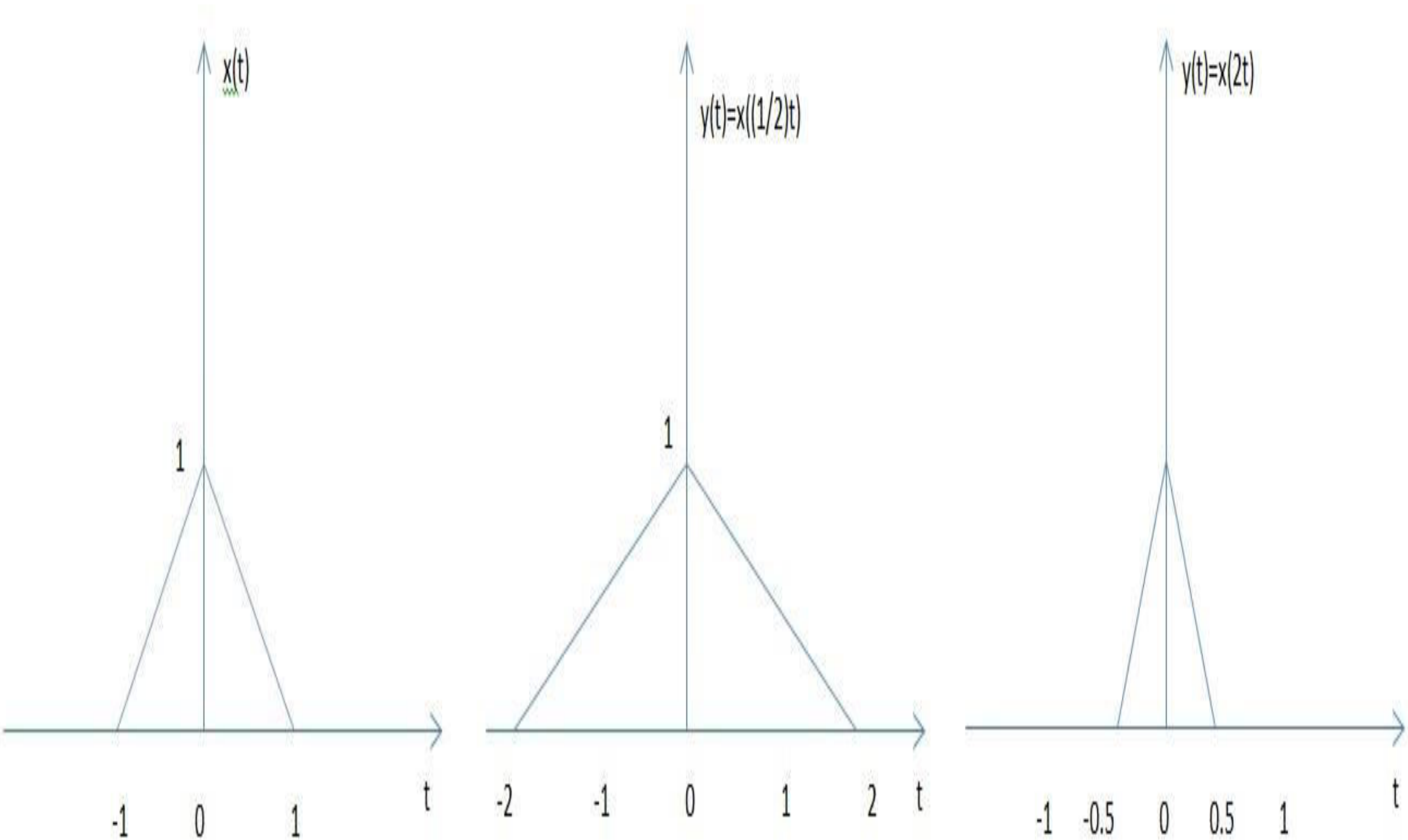


Discrete

- $x(n)=\{2,1.5,1,1.5\}$
- Find $x(-n)$
- Obtain $x(-n+2)$ & $x(-n-2)$
- Plot the graph!!!...

Time Scaling

- Form of Compression / Expansion .
- Accomplished by replacing t by at in the signal x(t).
- $y(t) = x(at)$ $\qquad\qquad$ $y(n) = x(kn)$
- Where *a* and *k* should be *> 0*.


x(t)
1
t
-1
0
1
y(t)=x((1/2)t)
1
t
-2
-1
0
1
2
y(t)=x(2t)
t
-1
-0.5
0
0.5
1

Try for Discrete

$$x(n) = \{1,2,3,\underset{\uparrow}{4},3,2,1\}$$

Find y(n)=x(2n)

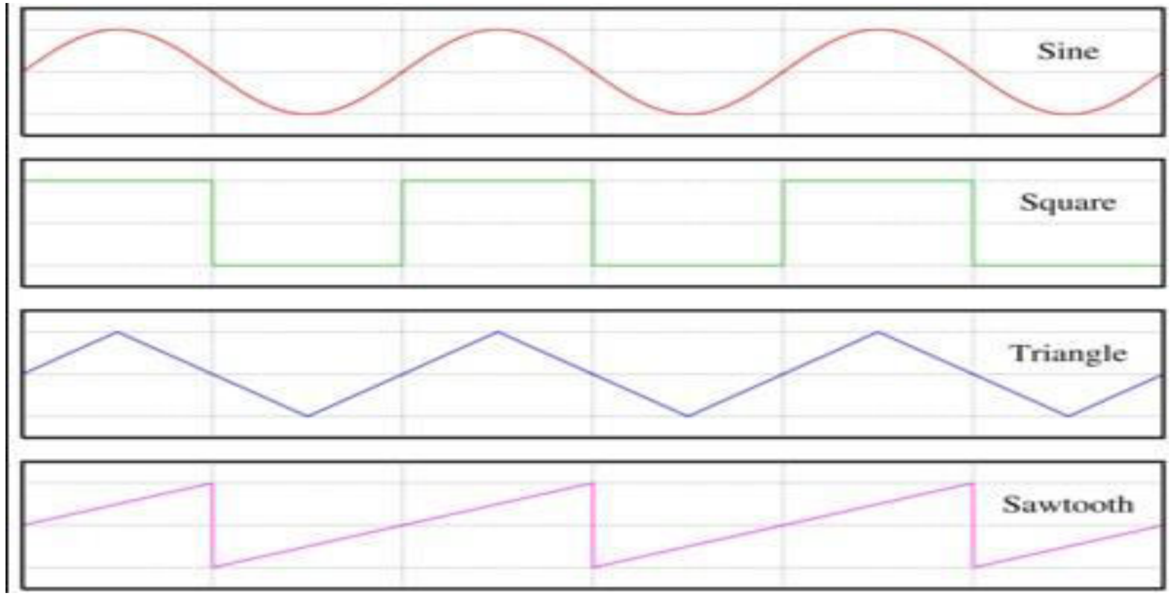
Plot the graph!!!...

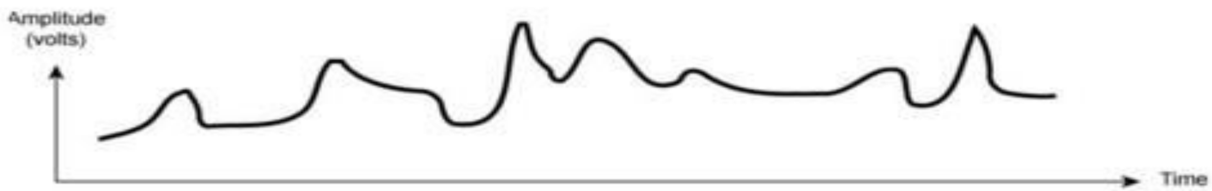
Properties of Signals

- Continuous Time & Discrete Time Signals
- Deterministic & Random Signals
- Periodic & Non periodic/Aperiodic Signals
- Symmetric & Anti symmetric Signals

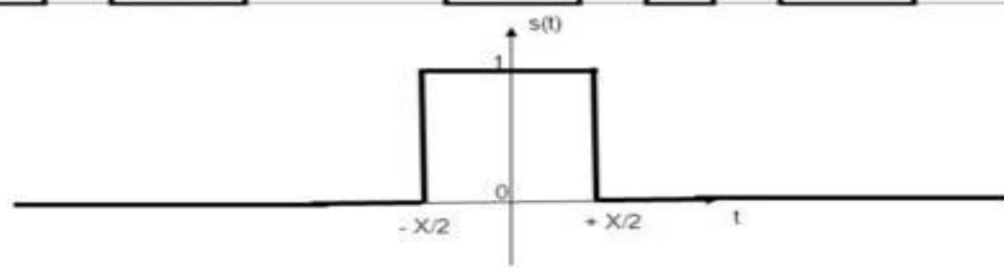
Periodic Signals

- A signal which repeats itself after a specific interval of time.
- $x(t+T) = x(t) \qquad x(n+N) = x(n)$
- T & N are fundamental time period.
- The signal that repeats its pattern over a period.
- They are deterministic→ their value can be determined at any instant.


Sine
Square
Triangle
Sawtooth


Amplitude
(volts)
Time


Amplitude
(volts)
Time


s(t)
1
0
- X/2
+ X/2
t

Consider Sinusoidal Signal

$x(t) = ASin(\omega_0 t + \theta)$ $\rightarrow 1$

A $\rightarrow$ Amplitude

ω_0 $\rightarrow$ Freuency

θ $\rightarrow$ Phase

$x(t) = x(t + T)$ $\rightarrow 2$

Comparing 1 and 2

$$x(t+T) = ASin(\omega_0(t+T)+\theta)$$
$$= ASin(\omega_0 t + \omega_0 T + \theta)$$

- If signal is periodic

$$\omega_0 T = 2\pi$$
$$\therefore T = \frac{2\pi}{\omega_0}$$

For Discrete!!!

$x(n) = ASin(\omega_0 n + \theta)$ →1

A→Amplitude

ω_0→Freuency

θ→Phase

$x(n) = x(n + N)$ →2

Comparing 1 and 2

$$x(n+N) = ASin(\omega_0(n+N)+\theta)$$
$$= ASin(\omega_0 n + \omega_0 N + \theta)$$

- If signal is periodic

$$\omega_0 \mathrm{N} = 2\pi$$
$$\therefore N = \frac{2\pi}{\omega_0}$$

Find the Fundamental Time Period

1. $x(t) = je^{j5t}$
2. $x(t) = 20Cos(10\pi t + \frac{\pi}{6})$

Solution

1. $T = 0.4\pi sec$
2. $T = 0.2sec$

Note!!!

- *Sum of two periodic signals are periodic if their fundamental time period ratio is a rational number.*
- **Check whether the signals are periodic or not??**

1. $x(t) = 2Cos(10t + 1) - Sin(4t - 1)$
2. $x(t) = 3Cos4t + 2Sin\pi t$
3. $x(t) = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}$

Symmetric & Anti Symmetric Signals

- Even and Odd Signals
- A signal is said to be symmetric (**EVEN**) if it satisfies the condition

$$x(-t) = x(t) \qquad \text{for all t}$$

$$x(t) = ACost$$

- A signal is said to be anti symmetric (**ODD**) if it satisfies the condition

$$x(-t) = -x(t) \quad \text{for all t}$$

$$x(t) = ASint$$

Any signal can be expressed as sum of even and odd components

$x(t) = x_e(t) + x_o(t)$ → 1

Replacing t by –t

$$x(-t) = x_e(-t) + x_o(-t)$$

$x(-t) = x_e(t) + [-x_o(t)]$ → 2

Add 1 and 2

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

Subtract 2 from 1

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

For Discrete!!

$x(-n) = x(n)$ EVEN

$x(-n) = -x(n)$ ODD

$$x(n) = x_e(n) + x_o(n)$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

Find even and odd component of the following signals

1. $x(t) = Cost + Sint + CostSint$

2. $x(n) = \{-2, 1, \underset{\uparrow}{2}, -1, 3\}$

Frequency Domain & Time Domain Analysis Signals

- Both the domain are used analyze the data in two modes
- Time Domain→ Variation of amplitude of signal with time
- Frequency Domain→ Number of times each event has occurred during the total period of observation

Frequency Domain Analysis of Continuous Time Signals

- Frequency Domain→ Sine Component Analysis of signals

- Study of signals using sinusoidal representation is termed as Fourier Analysis

- Here we represent signals as weighted sum of sinusoids.

$$f(x)$$

There are four distinct Fourier Representation

TIME PROPERTY	PERIODIC	APERIODIC
CONTINUOUS	CTFS FOURIER SERIES	CTFT FOURIER TRANSFORM
DISCRETE	DTFS FOURIER SERIES	DTFT FOURIER TRANSFORM

Continuous Time Fourier Series

- A periodic signal

$$x(t) = x(t+T), \qquad\qquad for\ all\ t$$

$$T = \frac{2\pi}{\omega_0}$$

- Two methods to evaluate Fourier Coefficients
  - Complex Exponential Analysis
  - Trigonometric Analysis

Complex Exponential Analysis

- Exponential Signal ??

$$x(t) = e^{j\omega t}$$

- The importance of complex exponential signal is that complex exponential signal $e^{j\omega t}$, the response of the system is same complex exponential signal with a only change in amplitude

$$x(t) = ae^{j\omega t}$$

- Linear Combination of harmonically related complex exponential signal is of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$\rightarrow 1$

- Multiplying both sides by $e^{-jn\omega_0 t}$

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}$$

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t}$$

$\rightarrow 2$

- Integrating both sides from 0 to T $\quad T = \frac{2\pi}{\omega_0}$

$$\int_{t=0}^{T} x(t)e^{-jn\omega_0 t}\,dt = \int_{t=0}^{T} \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t}\,dt$$

$$\int_{t=0}^{T} x(t)e^{-jn\omega_0 t}\,dt = \sum_{k=-\infty}^{\infty} a_k \int_{t=0}^{T} e^{j(k-n)\omega_0 t}\,dt$$

$\rightarrow 3$

- Case 1 $k = n$

$$\int_{t=0}^{T} x(t)e^{-jk\omega_0 t}\,dt = \sum_{k=-\infty}^{\infty} a_k \int_{t=0}^{T} 1\,dt$$

$$= Ta_k$$

$$\therefore\ a_k = \frac{1}{T}\int_{t=0}^{T} x(t)e^{-jk\omega_0 t}\,dt$$

→4

- Case 2 $k \neq n$

$$\int_{t=0}^{T} x(t)e^{-jn\omega_0 t}dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_{t=0}^{T} e^{-j(n-k)\omega_0 t}\, dt$$

→5

- Integral part in the 5 can be written as

$$\int_{t=0}^{T} e^{-j(n-k)\omega_0 t}\, dt$$

$$= \int_{t=0}^{T} [\cos(n-k)\,\omega_0 t$$

$$- jsin(n-k)\omega_0 t]dt$$

- This is periodic with fundamental time period

$$\frac{T}{|n-k|}$$

$$\int_{t=0}^{T} e^{-j(n-k)\omega_0 t}\, dt = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

Inverse CTFS

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

CTFS

$$a_k = \frac{1}{T} \int_{t=0}^{T} x(t) e^{-jk\omega_0 t}\, dt$$

Solve

- Find the Fourier Coefficients of the given signal

$$x(t) = 1 + sin2\omega_0 t + 2cos2\ \omega_0 t + 3\cos(\omega_0 t + \frac{\pi}{3})$$

Trigonometric Analysis

- A periodic signal can be expressed as trigonometric series

$$x(t) = a_0 + \sum_{k=1}^{\infty}(a_k cosk\omega_0 t + b_k sink\omega_0 t)$$

→1

- a_0 → DC components of the signal
- a_k, b_k → Coefficients of signal, constant

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)dt$$

$$a_k = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\, cosk\omega_0 t\, dt$$

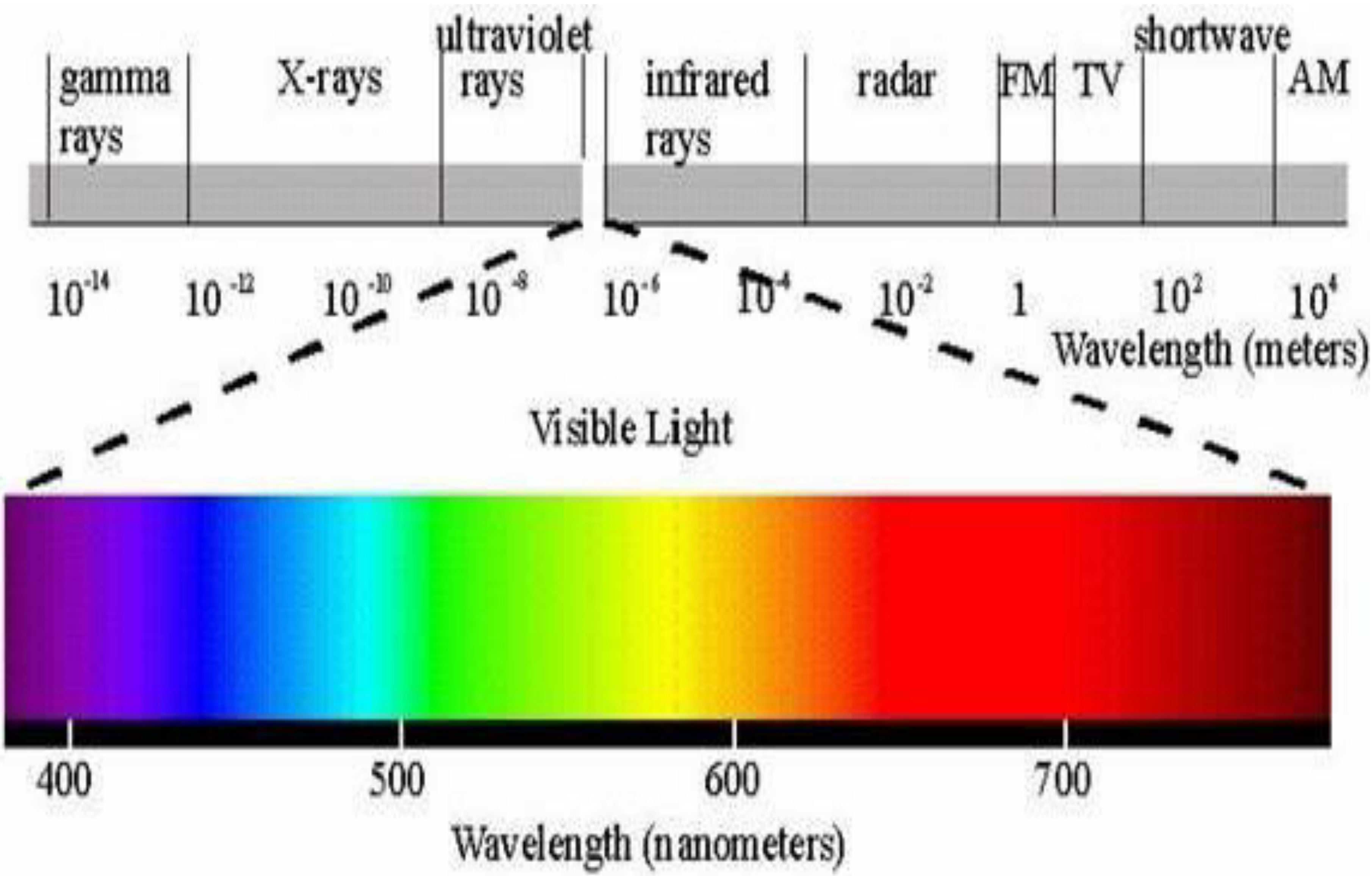
$$b_k = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\, sink\omega_0 t\, dt$$

Continuous Time Fourier Transform

- Applying appropriate conditions in CTFS we get CTFT.
- CTFT is used to represent continuous time aperiodic signals as a superposition of complex sinusoids.
- CTFT represents aperiodic signal having a limit of periodic signal.

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$


gamma rays
X-rays
ultraviolet rays
infrared rays
radar
FM
TV
shortwave
AM
10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 1 10^{2} 10^{4}
Wavelength (meters)
Visible Light
400 500 600 700
Wavelength (nanometers)

Concept of frequency spectrum

- Fourier spectrum of a signal x(t) can be obtained by plotting fourier coefficient versus angular freuency.
  - Amplitude Spectrum
  - Phase Spectrum
- These two combines to form Fourier Frequency Spectrum
- Not conitinuous ∴ also know as Discrete / Line Spectrum

- Now if $X(j\omega)$ is complex value function of ω then it can divided into real and imaginary part.

$$X(j\omega) = X_R(j\omega) + X_I(j\omega)$$

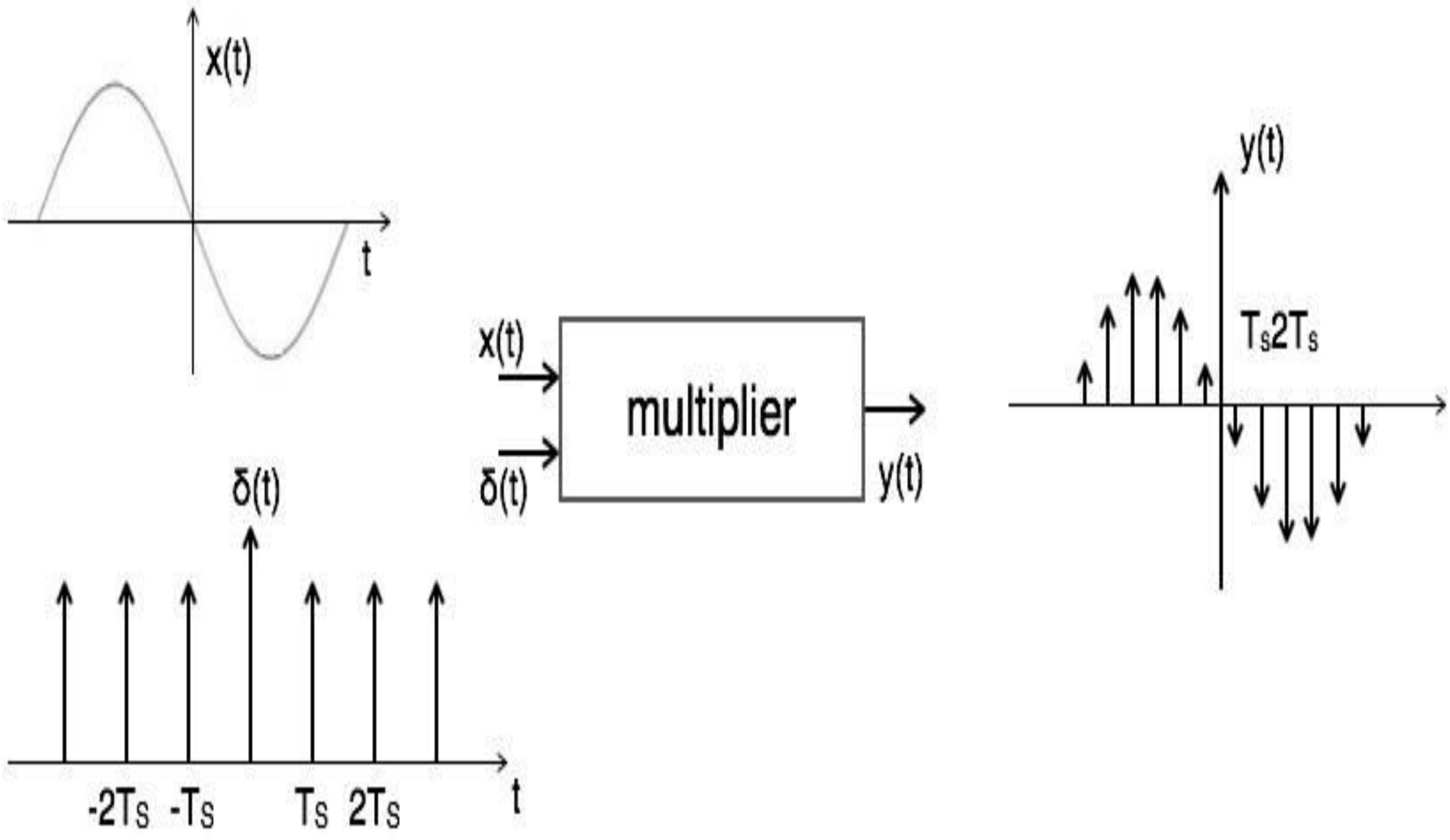
- Magnitude / Amplitude Spectrum

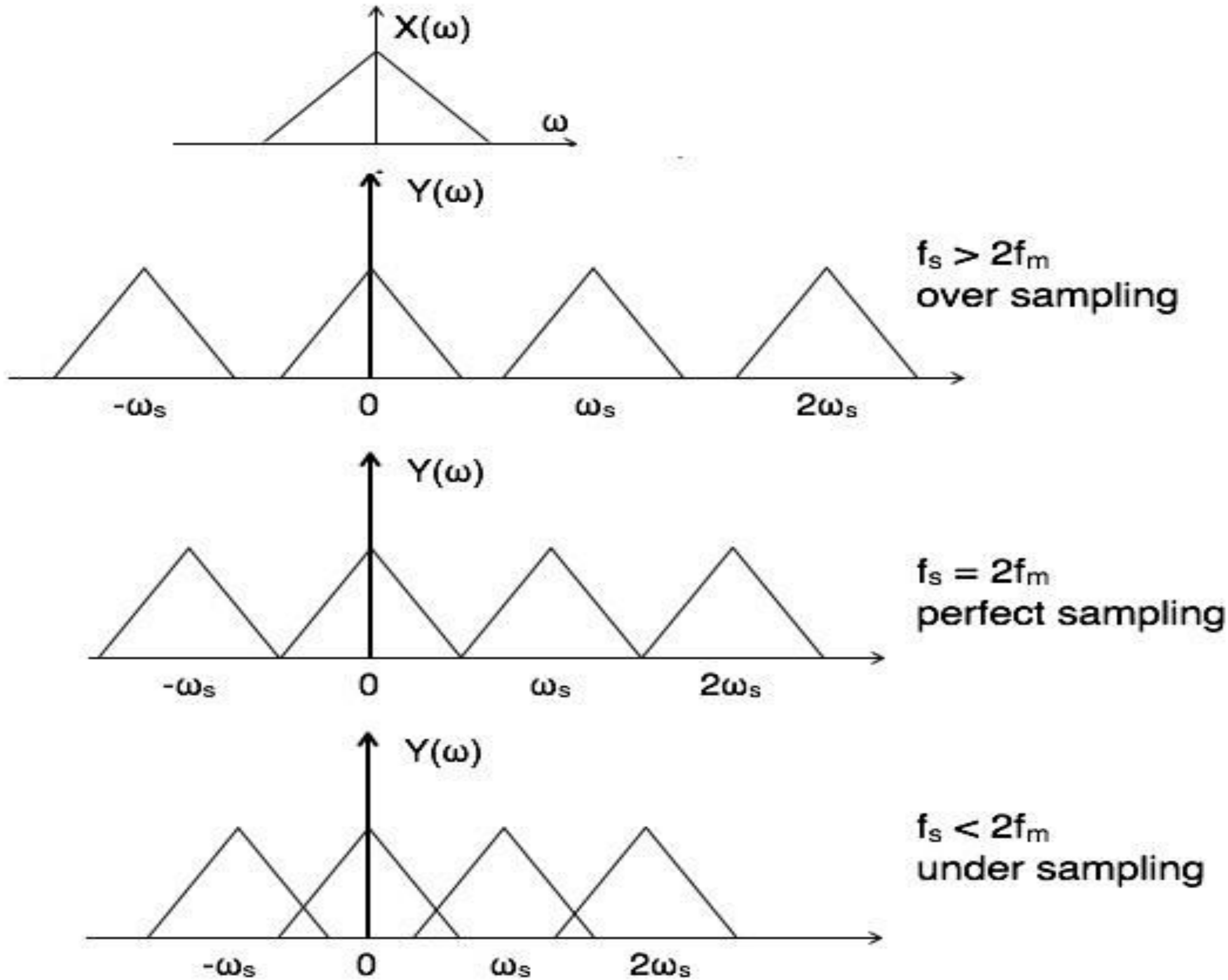
$$|X(j\omega)| = \sqrt{X_R(j\omega)^2 + X_I(j\omega)^2}$$

- Phase Spectrum

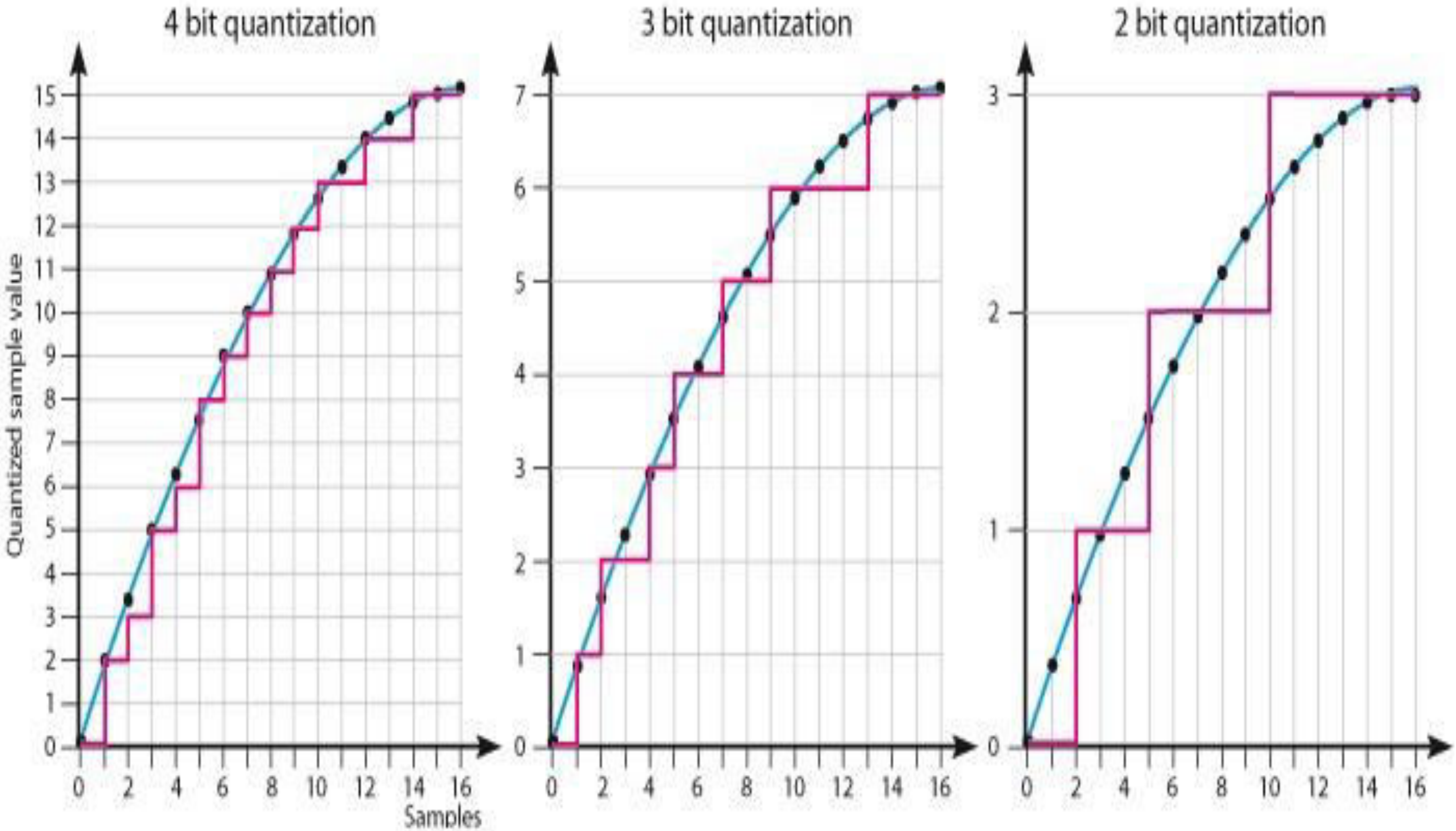
$$\langle X(j\omega) = Tan^{-1}\left[\frac{X_I(j\omega)}{X_R(j\omega)}\right]$$

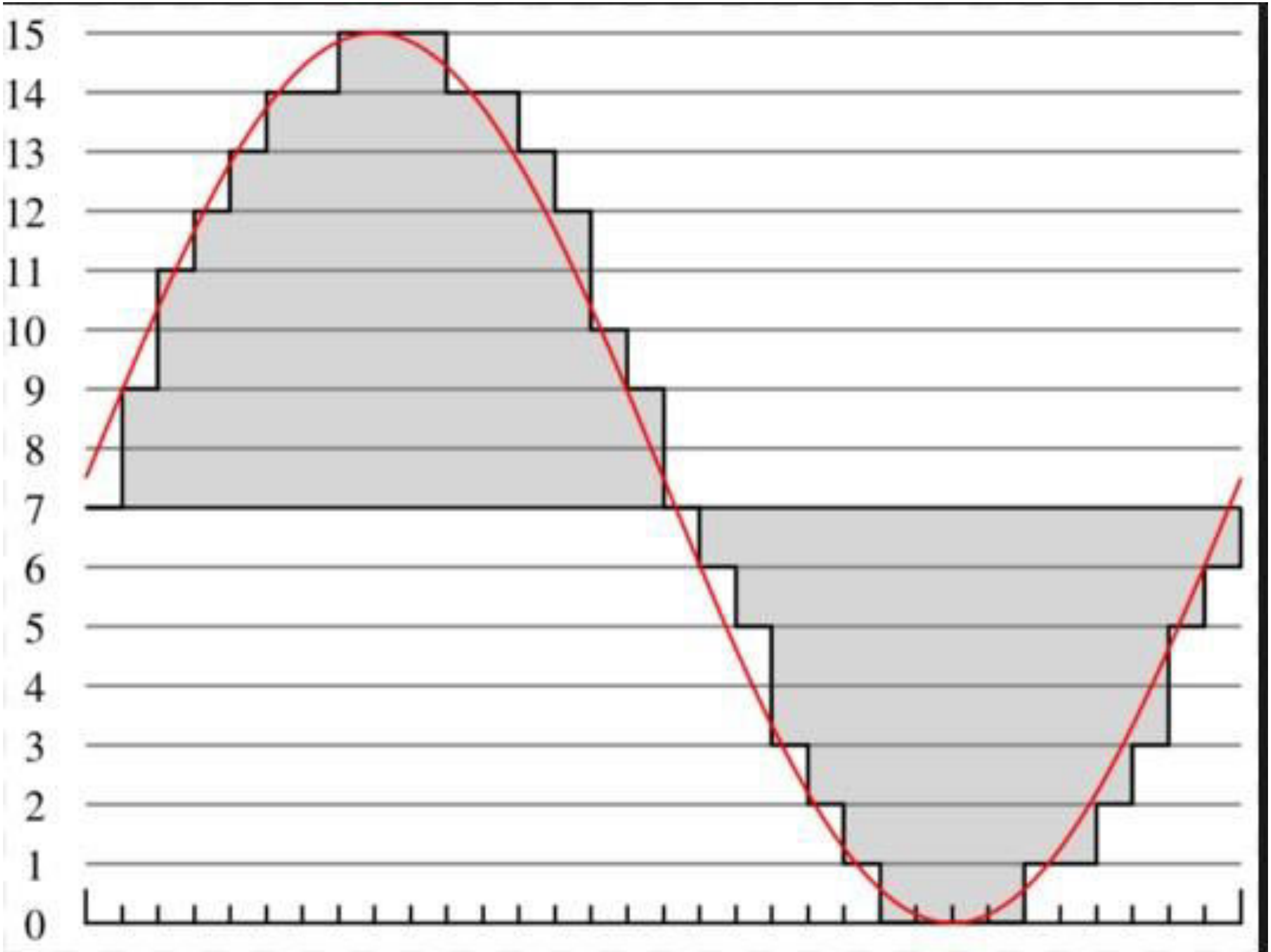
Sampling




X(ω)
ω
Y(ω)
fs > 2fm
over sampling
-ωs
0
ωs
2ωs
Y(ω)
fs = 2fm
perfect sampling
-ωs
0
ωs
2ωs
Y(ω)
fs < 2fm
under sampling
-ωs
0
ωs
2ωs

Quantization




15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0

Two types of quantization: (*a*) midtread and (*b*) midrise.