Tutorial 3

CS 337 Artificial Intelligence & Machine Learning, Autumn 2019

Week 3, August, 2019

Problem 1. Given a set of data points $\{x_n\}$, we can define the convex hull to be the set of all points x given by

$$x = \sum_n \alpha_n x_n$$

where $\alpha_n \geq 0$ and $\sum_n \alpha_n = 1$. Consider a second set of points $\{y_m\}$ together with their corresponding convex hull. By definition, the two sets of points will be linearly separable if there exists a vector $\hat{w}$ and a scalar w_0 such that $\hat{w} \cdot x_n + w_0 > 0$ for all x_n and $\hat{w} \cdot x_n + w_0 < 0$ for all y_m. Show that if their convex hulls intersect, the two sets of points cannot be linearly separable, and conversely that if they are linearly separable, their convex hulls do not intersect. [From PRML 2006, Chapter 4.]

Problem 2. Let X have a uniform distribution over integers in an interval $[0, \theta)$:

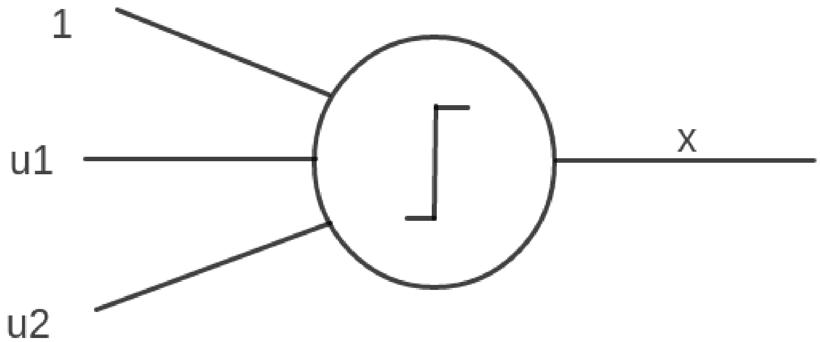
$$p(X = x; \theta) = \begin{cases} \frac{1}{\theta} & \text{if } \ 0 \leq x < \theta \\ 0 & \text{otherwise} \end{cases}$$

Suppose n samples $x_1, \ldots, x_n$ are drawn i.i.d based on $p(x; \theta)$. What is the MLE estimate of θ?

Problem 3. Computing power of perceptrons. Perceptrons can only separate Linearly separable data as discussed in class. Given n variables we can have 2^{2^n} boolean functions, but not all of these can be represented by a perceptron. For example when n=2 the XOR and XNOR cannot be represented by a perceptron. Given n boolean variables how many of 2^{2^n} boolean functions can be represented by a perceptron?

Problem 4. Consider a perceptron for which $u \in R^2$ and

$$f(a) = \begin{cases} 1 & a > 0 \\ 0 & a = 0 \\ -1 & a < 0 \end{cases}$$



Let the desired output be 1 when elements of class A = {(1,2),(2,4),(3,3),(4,4)} is applied as input and let it be -1 for the class B = {(0,0),(2,3),(3,0),(4,2)}. Let the initial connection weights $w_0(0) = +1, w_1(0) = -2, w_2(0) = +1$ and learning rate be h = 0.5.

This perceptron is to be trained by perceptron convergence procedure, for which the weight update formula is $w(t+1) = w(t) + \eta(y^k - x^k(t))u^k$

1. (a) Mark the elements belonging to class A with x and those belonging to class B with o on input space.
   (b) Draw the line represented by the perceptron considering the initial connection weights w(0).
   (c) Find out the regions for which the perceptron output is +1 and 1
   (d) Which elements of A and B are correctly classified, which elements are misclassified and which are unclassified?
2. If u=(4,4) is applied at input, what will be w(1) ?
3. Repeat a) considering w(1).
4. If u=(4.2) is then applied at input, what will be w(2)?

5. Repeat 1) considering w(2).

6. Do you expect the perceptron convergence procedure to terminate? Why?

Problem 5. In the class, we discussed the probabilistic binary (class) logistic regression classifier. How will you extend logistic regression probabilistic model to multiple (say K) classes? Are their different ways of extending? What is the intuition behind each? Discuss and contrast advantages/disadvantages in each.