Question 3

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1 Question 3)a

Given $I(x)$ (non-discrete) is a real-valued intensity function with a continuous domain within values [0,1]. Image histogram is $h(I)$ with mass 1.

And $h(I)$ is split into two histograms $h_1(I)$ over the domain [0,a] and $h_2(I)$ over the domain (a,1].

$T_1(0) = 0$ and $T_1(a) = a$ for $x \in [0, a]$, similarly $T_2(a) = a$ and $T_2(1) = 1$ for $x \in (a, 1]$ due to histogram equalization in the respective domains.

On performing Histogram equalization on $h_1(I)$ by taking the transformation to preserve histogram mass we get, $h_1(I_1)dI_1 = h_1^{T_1}(I_1)dI_1$, and $h_1^{T_1}(I_1) = h_1^{T1}(constant)$ as the image is equalized.

Integrating both sides for $x \in [0, a]$ we get,

$\int_0^a h_1(I)dI = \int_0^a h_1^{T1}(I)dI$

$\int_0^a h_1(I)dI = h_1^{T1} \int_0^a dI = h_1^{T1}a$

$\alpha = h_1^{T1}a$

And similarly for $x \in (a, 1]$ we get, $h_2(I_2)dI_2 = h_2^{T_2}(I_2)dI_2$, and $h_2^{T_2}(I_2) = h_2^{T2}(constant)$.

By integrating both sides,

$\int_a^1 h_2(I)dI = \int_a^1 h_2^{T2}(I)dI$

$\int_a^1 h_2(I)dI = h_2^{T2} \int_a^1 dI = h_2^{T2}(1 - a)$

$(1 - \alpha) = h_2^{T2}(1 - a)$

Taking average of I_2 we get,

$\Rightarrow < I_2 >= \int_0^a h_1^{T1}IdI + \int_a^1 h_2^{T2}IdI = h_1^{T1} \int_0^a IdI + h_2^{T2} \int_a^1 IdI$

$\Rightarrow < I_2 >= \frac{h_1^{T1}a^2 + h_2^{T2}(1-a^2)}{2}$

Putting the values of h_1^{T1} and h_2^{T2} we get,

$< I_2 >= \frac{a\alpha}{2} + \frac{(1+a)(1-\alpha)}{2} = \frac{1+a-\alpha}{2}$

2 Question 3)b

Given that a = *Median Intensity* of the Original Histogram, and also that the Mean Intensity of the Original image is a, i.e $\int_0^a h(I)IdI = \int_a^1 h(I)IdI$ and $< I >= \int_0^1 h(I)IdI = a$.

Hence we get,

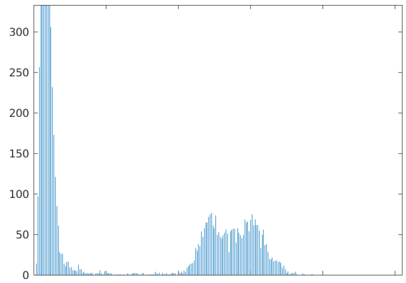
$\alpha = \frac{1}{2}$

Putting this value of α in $< I_2 >$ equation obtained in the above part we get,

$< I_2 >= \frac{0.5+a}{2}$

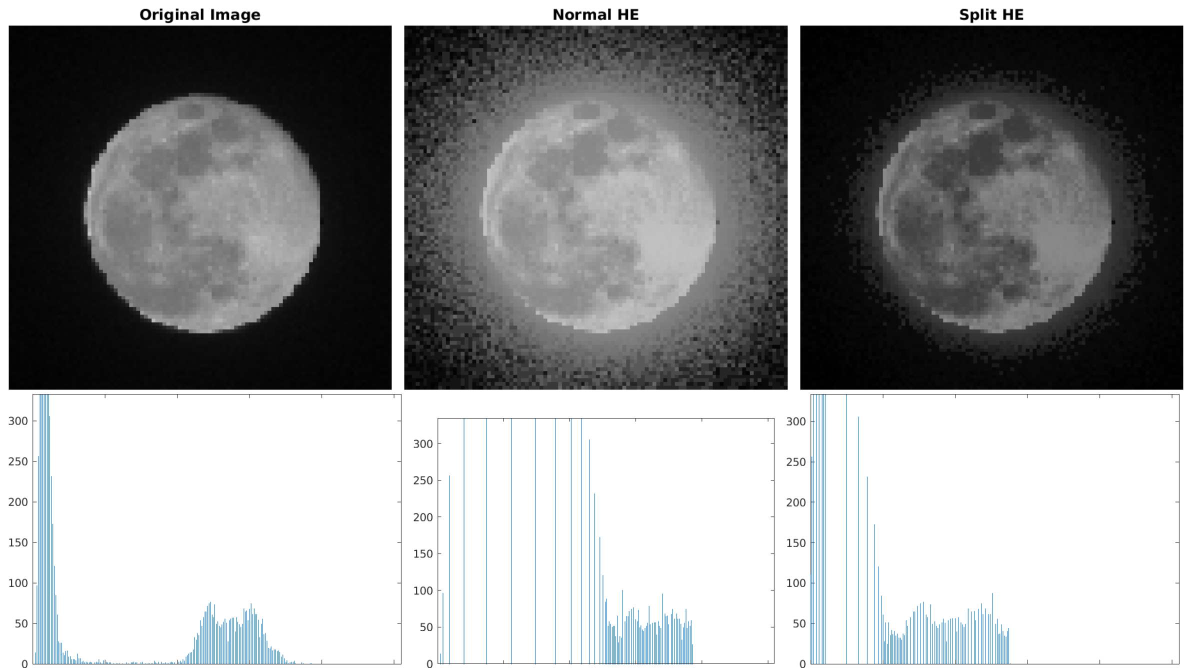
3 Question 3)c

Images whose Histogram are multimodal (i.e. more than one peak) can be better equalized by Split Histogram Equalization with a = *median intensity*, rather than normal Histogram Equalization. Below an example of an image having 3 data peaks in its distribution



In this image if we do normal HE rather than split HE then the dark region(night sky) which is necessary in this photo for contrast with the moon, will also gets brighter. So, split HE preserves the contrast and thus the details in the image.

4 Question 3)d



In this specific example, Split Histogram Equalization would be more effective if the *median intensity* is equal to the value of the intensity near the boundary of the moon.