CS - 663
H.W - 5

Q.1

Given - $g_1 = f_1 + h_2 * f_2$

and $g_2 = f_2 + h_1 * f_1$; g_1, g_2, h_1, h_2 are known

We have to find 'f_1' and 'f_2'

So, Taking DFT of given equations

$$\mathcal{F}(g_1) = G_1(u,v) = F_1 + H_2 F_2$$

$$\mathcal{F}(g_2) = G_2(u,v) = F_2 + H_1 F_1$$

{ Using convolution theorem $h * f = HF$

where, F_1, F_2, H_1 and H_2 are DFT of f_1, f_2, h_1 and h_2

Now, using above 2 equations

$$G_1 = F_1 + H_2(G_2 - H_1 F_1) = F_1 + H_2 G_2 - H_2 H_1 F_1$$

$$\Rightarrow F_1(1 - H_2 H_1) = G_1 - H_2 G_2$$

$$\Rightarrow \boxed{F_1 = \frac{G_1 - H_2 G_2}{1 - H_2 H_1}}$$

Similarly,

$$\boxed{F_2 = \frac{G_2 - H_1 G_1}{1 - H_1 H_2}}$$

Now, taking IDFT of F_1 and F_2 we get f_1 and f_2

i.e. $\boxed{f_1 = \mathcal{F}^{-1}(F_1)}$ and $\boxed{f_2 = \mathcal{F}^{-1}(F_2)}$

- Now, problem with above formulae is the term in denominator i.e. '$1 - H_1 H_2$' or '$1 - H_2 H_1$' (both are equal)

Let's say (u_0, v_0) are the frequencies for which '$H_1 H_2$' is near to '1' i.e.

$$H_1(u_0, v_0) \cdot H_2(u_0 v_0) \approx 1 \text{ , then}$$

$$F_1(u_0, v_0) \text{ or } F_2(u_0, v_0) \ggg F_1(u, v) \text{ or } F_2(u, v)$$

~~to~~ (u, v) are frequencies other than (u_0, v_0)

Therefore, in images f_1, f_2 the feauters corresponding to (u_0, v_0) frequency will be dominant and other feauters might get lost or weaken.

For e.g. - If (u_0, v_0) are small frequencies, the feature corresponding to higher frequency i.e. 'edges' will get smoothed.