Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

ANS;-

Given Mean=45, Std=8

AS per given condition the work will start after 10min so mean will be 45+10=55

Customer the car will be ready after 1hour(x)=60

Z=(60-55)/8=0.625

From Z tabel

Z value=0.73237

The probability that the service manager cannot meet his commitment=1-0.73237

=0.26763

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.
    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS;- A)

Given m=38,std=6

To find(p>44)

Z=x-mean/std , Z=(44-38)/6=1=84.13%

People above 44 age =100-84.13=15.87%=63 out of 400

Z score for 38=(38-38)/6=0=50%

Hence people between 38&44 age =84.13-50=34.13%=137 out of 400

Hence it is false.

ANS;-B)

Z=(30-38)/6=-1.33=9.15%=36 out of 400

Hence it is true.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS;Hence there is no difference between 2x1 and x1+x2 as both of them have same distribution.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

ANS;-

Z value at 99%=

Stats.norm.ppf(0.995)=2.5758

1st value will be 2.5758*20+100=151.51

2nd value will be(-2.5758)*20+100=48.484

So D is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5th percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:-

Total profit=profit1+profit2

Mean=profit1+profit2=5+7=12
Std=squt(9+7)
=sqrt(25)
=5

Mean in rs=12*45=540
Std in rs=5*45=225

A) Range for 95%:-
Stats.norm.interval(0.95,540,225)
Range is rs(99.008,980.991) in millions

B)The 5th percentile:-

From z score we need to find the value of 0.5000-0.050=0.4500

We are getting the value of -1.645

The 5th percentile of profit =mean+(-1.645)*std

= 540-(1.645*225)

=540-370.125

=169.87=170 in million

C)
Probability of 1st division making loss=stats.norm.cdf(0,5,3)
=0.0479
Probability of 2nd division making loss=stats.norm.cdf(0,7,4)
=0.04005
We can see that 1st division can make more loss compared to 1st division.