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Remark: Since the hypotenuse is the longest side in a right triangle, the value of sin A or cos A is always less than 1 (or, in particular, equal to 1).

Let us consider some examples.

Example 1: Given $\tan A = \frac{4}{3}$, find the other trigonometric ratios of the angle A.

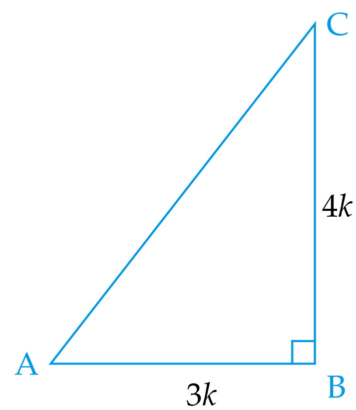

Fig. 8.8

Solution: Let us first draw a right $\triangle ABC$ (see Fig. 8.8).

Now, we know that $\tan A = \frac{BC}{AB} = \frac{4}{3}$.
Therefore, if $BC = 4k$, then $AB = 3k$, where k is a positive number.

Now, by using the Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2 = (3k)^2 + (4k)^2 = 25k^2 \quad \Rightarrow \quad AC = 5k$$

Now, we can write all the trigonometric ratios using their definitions:

$$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5},$$
$$\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

Therefore, $\quad \cot A = \frac{1}{\tan A} = \frac{3}{4}, \quad \csc A = \frac{1}{\sin A} = \frac{5}{4}, \quad \sec A = \frac{1}{\cos A} = \frac{5}{3}$

Example 2: If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.

Solution: Let us consider two right triangles ABC and PQR where $\sin B = \sin Q$ (see Fig. 8.9).

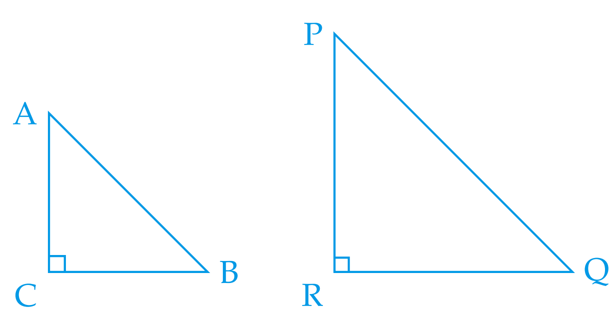

Fig. 8.9

$$\sin B = \frac{AC}{AB}$$

We have

$$\sin Q = \frac{PR}{PQ}$$

Then $$\frac{\mathrm{AC}}{\mathrm{AB}} = \frac{\mathrm{PR}}{\mathrm{PQ}}$$

Therefore, $$\frac{\mathrm{AC}}{\mathrm{PR}} = \frac{\mathrm{AB}}{\mathrm{PQ}} = k, \text{ say} \qquad (1)$$

Now, using Pythagoras theorem,

$$\mathrm{BC} = \sqrt{\mathrm{AB}^2 - \mathrm{AC}^2}$$

and

$$\mathrm{QR} = \sqrt{\mathrm{PQ}^2 - \mathrm{PR}^2}$$

So, $$\frac{\mathrm{BC}}{\mathrm{QR}} = \frac{\sqrt{\mathrm{AB}^2 - \mathrm{AC}^2}}{\sqrt{\mathrm{PQ}^2 - \mathrm{PR}^2}} = \frac{\sqrt{k^2\mathrm{PQ}^2 - k^2\mathrm{PR}^2}}{\sqrt{\mathrm{PQ}^2 - \mathrm{PR}^2}} = \frac{k\sqrt{\mathrm{PQ}^2 - \mathrm{PR}^2}}{\sqrt{\mathrm{PQ}^2 - \mathrm{PR}^2}} = k \qquad (2)$$

From (1) and (2), we have

$$\frac{\mathrm{AC}}{\mathrm{PR}} = \frac{\mathrm{AB}}{\mathrm{PQ}} = \frac{\mathrm{BC}}{\mathrm{QR}}$$

Then, by using Theorem 6.4, $\triangle\mathrm{ACB} \sim \triangle\mathrm{PRQ}$ and therefore, $\angle\mathrm{B} = \angle\mathrm{Q}$.

Example 5: In $\triangle OPQ$, right-angled at P,
$OP = 7\,\text{cm}$ and $OQ - PQ = 1\,\text{cm}$ (see Fig. 8.12).
Determine the values of $\sin Q$ and $\cos Q$.

Solution: In $\triangle OPQ$, we have:

$$OQ^2 = OP^2 + PQ^2$$

i.e., $(1 + PQ)^2 = OP^2 + PQ^2$ (Why?)

i.e., $1 + PQ^2 + 2PQ = OP^2 + PQ^2$

i.e., $1 + 2PQ = 7^2$ (Why?)

i.e., $2PQ = 49 - 1 = 48$

i.e., $PQ = 24\,\text{cm}$ and $OQ = 1 + PQ = 25\,\text{cm}$

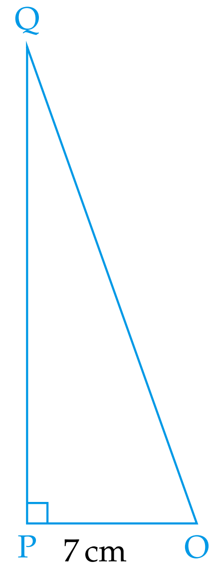

Fig. 8.12

So, $\sin Q = \dfrac{7}{25}$ and $\cos Q = \dfrac{24}{25}$

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