

CS221 Spring 2025: Artificial Intelligence: Principles and Techniques

Homework 2: Sentiment Analysis

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By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Advice for this homework:

- Words are simply strings separated by whitespace. Note that words which only differ in capitalization are considered separate (e.g. great and Great are considered different words).
- You might find some useful functions in `util.py`. Have a look around in there before you start coding.

Before you get started, please read the **Assignments** section on the course website thoroughly.

Problem 1: Building intuition

Here are two reviews of *Perfect Blue*, from Rotten Tomatoes:

	Panos Kotzathanasis <i>Asian Movie Pulse</i>	 "Perfect Blue" is an artistic and technical masterpiece; however, what is of utmost importance is the fact that Satoshi Kon never deteriorated from the high standards he set here, in the first project that was entirely his own. Full Review	January 26, 2020
	Derek Smith <i>Cinematic Reflections</i>	 [An] nime thriller [that] often plays as an examination of identity and celebrity, but ultimately gets so lost in its own complex structure that it doesn't end up saying much at all. Full Review Original Score: 2/4	August 19, 2006

Rotten Tomatoes has classified these reviews as “positive” and “negative,” respectively, as

indicated by the intact tomato on the top and the splatter on the bottom. In this assignment, you will create a simple text classification system that can perform this task automatically. We'll warm up with the following set of four mini-reviews, each labeled positive (+1) or negative (−1):

1. (−1) not good
2. (−1) pretty bad
3. (+1) good plot
4. (+1) pretty scenery

Each review x is mapped onto a feature vector $\phi(x)$, which maps each word to the number of occurrences of that word in the review. For example, the second review maps to the (sparse) feature vector $\phi(x) = \{\text{pretty} : 1, \text{bad} : 1\}$. Recall the definition of the hinge loss:

$$\text{Loss}_{\text{hinge}}(x, y, \mathbf{w}) = \max\{0, 1 - \mathbf{w} \cdot \phi(x)y\},$$

where x is the review text, y is the correct label, \mathbf{w} is the weight vector.

- a. [2 points] Suppose we run stochastic gradient descent once for each of the 4 samples in the order given above, updating the weights according to

$$\mathbf{w} \leftarrow \mathbf{w} - \eta \nabla_{\mathbf{w}} \text{Loss}_{\text{hinge}}(x, y, \mathbf{w}).$$

After the updates, what are the weights of the six words (“pretty”, “good”, “bad”, “plot”, “not”, “scenery”) that appear in the above reviews?

- Use $\eta = 0.1$ as the step size.
- Initialize $\mathbf{w} = [0, 0, 0, 0, 0, 0]$.
- The gradient $\nabla_{\mathbf{w}} \text{Loss}_{\text{hinge}}(x, y, \mathbf{w}) = 0$ when margin is exactly 1.

What we expect: A weight vector that contains a numerical value for each of the tokens in the reviews (“pretty”, “good”, “bad”, “plot”, “not”, “scenery”), **in this order**. For example: $[0.1, 0.2, 0.3, 0.4, 0.5, 0.6]$.

Your Solution: Using the code to compute SGD, the new weight vector after the 4th run will be $[0, 0, -0.1, 0.1, -0.1, 0.1]$

- b. [2 points] Given the following dataset of reviews:

1. (−1) bad
2. (+1) good

3. (+1) not bad
4. (-1) not good

Prove that no linear classifier using features of word counts (mapping each word to the number of occurrences of that word in the review) can get zero error on this dataset. Remember that this is a question about classifiers, not optimization algorithms; your proof should be true for any linear classifier of the form $f_{\mathbf{w}}(x) = \text{sign}(\mathbf{w} \cdot \phi(x))$, regardless of how the weights are learned.

Propose a single additional feature for your dataset that we could augment the feature vector with that would fix this problem.

What we expect:

1. A short written proof (~ 3 -5 sentences).
2. A viable feature that would allow a linear classifier to have zero error on the dataset.

Your Solution:

1. zero error with linear predictor

Lets say wts be w_b , w_g , and w_n for 'bad', 'good' and 'not' respectively. By summing the weights for each word we have.

- (a) when $x = \text{'bad'}$, y is -1. $w_b \leq 1$
- (b) when $x = \text{'good'}$, y is 1, $w_g \geq 1$
- (c) when $x = \text{'not bad'}$, y is 1, $w_n + w_b \geq 1$
- (d) when $x = \text{'not good'}$, y is -1, $w_n + w_g \geq 1$

From above, while (c) can be satisfied for $w_n \geq 1$ because $w_b < 1$. But if w_n is positive, (d) wont be satisfied since left-hand sum of wts can not be less than 1 when $w_n \geq 1$.

2. We can change predictor to be non-linear by defining feature function which will not count word independently. We could either add a feature 'concat.words' or hash of input sentence $\phi(x) = [x, \text{hash}([x])]$.

Problem 2: Predicting Movie Ratings

Suppose that we are now interested in predicting a numeric rating for movie reviews. We will use a non-linear predictor that takes a movie review x and returns $\sigma(\mathbf{w} \cdot \phi(x))$, where $\sigma(z) = (1 + e^{-z})^{-1}$ is the logistic function that squashes a real number to the range $(0, 1)$. For this problem, assume that the movie rating y is a real-valued variable in the range $[0, 1]$.

NOTE: Do not use math software such as Wolfram Alpha to solve this problem.

- a. [2 points] Suppose that we wish to use **squared loss**. Write out the expression for $\text{Loss}(x, y, \mathbf{w})$ for a single datapoint (x, y) .

What we expect: A mathematical expression for the loss. Feel free to use σ in the expression.

Your Solution: Non linear predictor:

$$f_w(x) = \sigma(\mathbf{w} \cdot \phi(x))$$

squared loss is difference between the predicted value and real value as below

$$\text{Loss}(x, y, \mathbf{w}) = (\sigma(\mathbf{w} \cdot \phi(x)) - y)^2$$

- b. [3 points] Given $\text{Loss}(x, y, \mathbf{w})$ from the previous part, compute the gradient of the loss with respect to \mathbf{w} , $\nabla_{\mathbf{w}} \text{Loss}(x, y, \mathbf{w})$. Write the answer in terms of the predicted value $p = \sigma(\mathbf{w} \cdot \phi(x))$.

What we expect: A mathematical expression for the gradient of the loss.

Your Solution: Using $\rho = \sigma(\mathbf{w} \cdot \phi(x))$

$$\nabla_{\mathbf{w}} \text{Loss}(x, y, \mathbf{w}) = \partial \frac{(p-y)^2}{dw}$$

using chain rule for $df/dp \cdot dp/dz \cdot dz/dw$.

$$df/d\rho = 2(\rho - y), \quad d\rho/dz = \sigma(z)(1 - \sigma(z)) \text{ and } dz/dw = \phi(x)$$

$$\begin{aligned} \nabla_{\mathbf{w}} \text{Loss}(x, y, \mathbf{w}) &= 2(\rho - y)(\sigma(\mathbf{w} \cdot \phi(x))(1 - \sigma(\mathbf{w} \cdot \phi(x)))\phi(x) \\ &= 2(\rho - y)(\rho)(1 - \rho)\phi(x) \end{aligned}$$

- c. [3 points] Suppose there is one datapoint (x, y) with some arbitrary $\phi(x)$ and $y = 1$. Specify conditions for \mathbf{w} to make the magnitude of the gradient of the loss with respect to \mathbf{w} arbitrarily small (i.e. minimize the magnitude of the gradient). Can the magnitude of the gradient with respect to \mathbf{w} ever be exactly zero? You are allowed to make the magnitude of \mathbf{w} arbitrarily large but not infinity.

What we expect:

1. 1-2 sentences describing the conditions for \mathbf{w} to minimize the magnitude of the gradient
2. 1-2 sentences explaining whether the gradient can be exactly zero.

HINT: Try to understand intuitively what is going on and what each part of the expression contributes. If you find yourself doing too much algebra, you're probably doing something suboptimal.

MOTIVATION:

1. the reason why we're interested in the magnitude of the gradients is because it governs how far gradient descent will step. For example, if the gradient is close to zero when \mathbf{w} is very far from the optimum, then it could take a long time for gradient descent to reach the optimum (if at all). This is known as the *vanishing gradient problem* when training neural networks.

Your Solution: In sigmoid function, $\frac{1}{1+e^{-z}}$ when z is a very high value the e^{-z} becomes small and the whole expression evaluates close to 1 (similarly it approaches 0 when for high negative values of z). In our example $z = \mathbf{w}\phi(x)$. Thus larger the weights (positive or negative) will give smaller magnitude of the gradient, however, it depends on the sign of $\text{sign}(\mathbf{w}\phi(x))$, the gradient will be either positive or negative depending on the sign of weights and inputs.

2. The sigmoid function flattens very quickly but never approaches 0. The gradient of sigmoid is $\sigma(1 - \sigma)$ and sigma never equals 0.

Problem 3: Sentiment Classification

In this problem, we will build a binary linear classifier that reads movie reviews and guesses whether they are “positive” or “negative.”

NOTE: Do not import any outside libraries (e.g. numpy) for any of the coding parts. Only standard python libraries and/or the libraries imported in the starter code are allowed. In this problem, you must implement the functions without using libraries like Scikit-learn.

HINT: look at the provided ‘util.py’ for some helpful utility functions that you are able to use when implementing your code. You should use the provided utility methods ‘dotProduct’ and ‘increment’ instead of modifying Python dictionaries directly.



- [2 points] Implement the function `extractWordFeatures`, which takes a review (string) as input and returns a feature vector $\phi(x)$, which is represented as a `dict` in Python.
- [4 points] Implement the function `learnPredictor` using stochastic gradient descent and minimize the hinge loss. Print the training error and validation error after each epoch to make sure your code is working. You must get less than 4% error rate on the training set and less than 30% error rate on the validation set to get full credit.
- [2 points] Write the `generateExample` function (nested in the `generateDataset` function) to generate artificial data samples.

Use this to double check that your `learnPredictor` works! You can do this by using `generateDataset()` to generate training and validation examples. You can then pass in these examples as `trainExamples` and `validationExamples` respectively to `learnPredictor` with the identity function `lambda x:x` as a `featureExtractor`.

- [2 points] Some languages are written without spaces between words. So is splitting the words really necessary or can we just naively consider strings of characters that stretch across words? Implement the function `extractCharacterFeatures` (by filling in the `extract` function), which maps each string of n characters to the number of times it occurs, ignoring whitespace (spaces and tabs).
- [3 points] Run your linear predictor with feature extractor `extractCharacterFeatures`. Experiment with different values of n to see which one produces the smallest validation error. You should observe that this error is nearly as small as that produced by word features. Why is this the case?

Construct a review (one sentence max) in which character n -grams probably outperform word features, and briefly explain why this is so.

NOTE: There is a function in `submission.py` that will allow you add a test to `grader.py` to test different values of n . Remember to write your final written solution here.

What we expect:

1. A short paragraph (4-6 sentences). In the paragraph state which value of n produces the smallest validation error, why this is likely the value that produces the smallest error.
2. A one-sentence review and explanation for when character n -grams probably outperform word features.

Your Solution: It is possible that my code may not be right, but in my test runs I saw smaller validation errors for $n=3, 4, 5$. They are just marginally small. If I take this value.

Problem 4: Toxicity Classification and Maximum Group Loss

Recall that models trained (in the standard way) to minimize the average loss can work well on average but poorly on certain groups. One way to mitigate this issue is by minimizing the maximum group loss instead. In this problem, we will compare the average loss and maximum group loss objectives on a toy setting inspired by a problem with real-world toxicity classification models.

Toxicity classifiers are designed to assist in moderating online forums by predicting whether an online comment is toxic or not, so that comments predicted to be toxic can be flagged for humans to review [1]. Unfortunately, some models have been shown to misclassify non-toxic comments mentioning demographic identities (e.g., “I am a [demographic identity]”) as toxic [2]. This behavior could arise if we assume that toxic comments in the dataset often mention demographic identities, and as a result, models learn to *spuriously correlate* toxicity with the mention of these identities.

In this problem, we will study a toy setting that illustrates the spurious correlation problem: The input x is a comment (a string) made on an online forum; the label $y \in \{-1, 1\}$ is the toxicity of the comment ($y = 1$ is toxic, $y = -1$ is non-toxic); $d \in \{0, 1\}$ indicates if the text contains a word that refers to a demographic identity; and $t \in \{0, 1\}$ indicates whether the comment includes certain “toxic” words. The comment x is mapped onto the feature vector $\phi(x) = [1, d, t]$ where 1 is the bias term (the bias term is present to prevent the edge case $\mathbf{w} \cdot \phi(x) = 0$ in the questions that follow). To make this concrete, we provide a few simple examples below, where we underline toxic words and words that refer to a demographic identity:

Comment (x)	Toxicity (y)	Presence of demographic mentions (d)	Presence of toxic words (t)
“Stanford <u>sucks</u> !”	1	0	1
“I’m a <u>woman</u> in computer science!”	-1	1	0
“The hummingbird <u>sucks</u> nectar from the flower”	-1	0	1

Suppose we are given the following training data, where we list the number of times each combination (y, d, t) shows up in the training set.

y	d	t	num. examples
-1	0	0	63
-1	0	1	27
-1	1	0	7
-1	1	1	3
1	0	0	3
1	0	1	7
1	1	0	27
1	1	1	63
Total num. examples			200

From the above table, we can see that 70 out of the 100 of toxic comments include toxic words, and 70 out of the 100 non-toxic comments do not. In addition, the toxicity of the comment y is highly correlated with mentions of demographic identities d (again under the assumption that toxic comments target demographic identities) – 90 out of the 100 toxic comments include mentions of demographic identities, and 90 out of the 100 non-toxic comments do not.

We will consider linear classifiers of the form $f_{\mathbf{w}}(x) = \text{sign}(\mathbf{w} \cdot \phi(x))$, where $\phi(x)$ is defined above. Normally, we would train classifiers to minimize either the average loss or the maximum group loss, but for simplicity, we will compare two fixed classifiers (which might not minimize either objective):

- Classifier D: $\mathbf{w} = [-0.1, 1, 0]$
- Classifier T: $\mathbf{w} = [-0.1, 0, 1]$

For our loss function, we will be using the zero-one loss, so that the per-group loss is

$$\text{TrainLoss}_g(\mathbf{w}) = \frac{1}{|\mathcal{D}_{\text{train}}(g)|} \sum_{(x,y) \in \mathcal{D}_{\text{train}}(g)} \mathbf{1}[f_{\mathbf{w}}(x) \neq y].$$

Recall the definition of the maximum group loss:

$$\text{TrainLoss}_{\max}(\mathbf{w}) = \max_g \text{TrainLoss}_g(\mathbf{w}).$$

To capture the spurious correlation problem, let us define groups based on the value of (y, d) . There are thus four groups: $(y = 1, d = 1)$, $(y = 1, d = 0)$, $(y = -1, d = 1)$, and $(y = -1, d = 0)$. For example, the group $(y = -1, d = 1)$ refers to non-toxic comments with demographic mentions.

- a. [2 points] In words, describe the behavior of Classifier D and Classifier T.

What we expect: For each classifier (D and T), an “if-and-only-if” statement describing the output of the classifier in terms of its features when $f_w(x) = 1$.

Your Solution:

- classifier D behavior: The $f_w(x)$ is 1 if and only if $\mathbf{w} \cdot \phi(x) \geq 1$. Output $f_w(x)$ predicts sample x as toxic if and only if it has demographic mention present.
- classifier T behavior: The $f_w(x)$ is 1 if and only if $\mathbf{w} \cdot \phi(x) \geq 1$. Output $f_w(x)$ predicts sample x as toxic if and only if it has presence of toxic words.

- b. [3 points] Compute the following three quantities concerning Classifier D using the dataset above:

1. Classifier D’s average loss
2. Classifier D’s average loss for each group (fill in the table below)
3. Classifier D’s maximum group loss

What we expect: A value for average loss, a complete table (found below) with average loss for each group with the values in the given order, and a value for maximum group loss.

Classifier D	$y = 1$	$y = -1$
$d = 1$	0	1
$d = 0$	1	0

Your Solution:

1. Classifier D’s average loss: $\frac{20}{200}$. By looking at entries with (7,3,3,7) in the table above there are 2 entries with incorrect classification $\{d = 1, y = -1\}, \{d = 0, y = 1\}$ because we are only picking inputs with $d=1$.
2. Classifier D’s maximum group loss
 For each group: $G\{1, 1\}$ has 0 loss, $G\{1, 0\}$ has $\frac{10}{10}$ loss, $G\{-1, 0\}$ has 0 loss, $G\{-1, 1\}$ has $\frac{10}{10}$ loss
 solution: $\max(0, 1, 0, 1) = 1$

- c. [3 points] Now compute the following three quantities concerning Classifier T using the same dataset:

1. Classifier T's average loss
2. Classifier T's average loss for each group (fill in the table below)
3. Classifier T's maximum group loss

Which classifier has lower average loss? Which classifier has lower maximum group loss? **Note the groups are still defined by d , the demographic label.**

What we expect: A value for average loss, a complete table with average loss for each group with the values in the given order, and a value for maximum group loss. Indicate which classifier has lower average loss, then indicate which classifier has lower maximum group loss.

Classifier T	$y = 1$	$y = -1$
$d = 1$	$\frac{27}{100}$	$\frac{3}{10}$
$d = 0$	$\frac{3}{30}$	$\frac{27}{100}$

Your Solution:

- Classifier T's average loss: $\frac{60}{200}$. By looking at entries with (27,3,3,27) in the table above there are 2 entries with incorrect classification $\{t = 1, y = -1\}, \{t = 0, y = 1\}$.
- Classifier T's maximum group loss is the max value from the table above solution: 0.30

Classifier T has higher average loss (0.3) and Classifier D has higher maximum loss of 1.

- d. [4 points] As we saw above, different classifiers lead to different numbers of accurate predictions and different people's comments being wrongly rejected. Accurate classification of a non-toxic comment is good for the commenter, but when no classifier has perfect accuracy, how should the correct classifications be distributed across commenters?

The module on Algorithms and Distribution video, pdf highlights some well-known principles of fairness distribution (note: reviewing the module will help you answer this question well). These ethical frameworks are a great starting point for thinking through how to choose a classifier, but in reality a combination of approaches might be needed to balance potential trade-offs.

$\text{TrainLoss}_{new}(w) = \lambda \cdot \text{TrainLoss}_{avg}(w) + (1 - \lambda) \cdot \text{TrainLoss}_{max}(w)$ where $\lambda \in [0, 1]$ is a hyperparameter.

1. Consider the new loss term above that we want our classifier to minimize. For the case where $\lambda = 1$, describe the optimal classifier according to the ethical frameworks from the module.

What we expect: Describe which ethical framework aligns with the optimal classifier in 1-2 sentences. You may reference classifiers D or T when discussing the optimal classifier, but you must justify your answer using the provided ethical framework to earn full points.

Your Solution: When $\lambda = 1$, only $\text{TrainLoss}_{avg}(w)$ contributes to the training loss. Minimizing the average loss which already lower for D-classifier, will further favor demographic mentions but will perform much worse for toxicity detection in comments. Whereas minimizing T classifier, representation of demographic will be further diminished.

2. Now consider the case where $\lambda = 0$, describe the optimal classifier according to the ethical frameworks from the module.

What we expect: Describe which ethical framework aligns with the optimal classifier in 1-2 sentences. You may reference classifiers D or T when discussing the optimal classifier, but you must justify your answer using the provided ethical framework to earn full points.

Your Solution: When $\lambda = 0$, $\text{TrainLoss}_{max}(w)$ with D classifier is 1, which shows very poor performance on toxicity feature. Minimizing it further will help model perform better. Whereas T classifier has lower maximum loss (than d). It seems minimizing this loss further does not necessarily mean that demographic mentions will be better represented. I believe D classifier as described is not suitable for obtaining good results on ethical framework.

3. Suppose λ is still set to 0. We are given another set of training data, where we list the number of times each combination (y, d, t) shows up in the training set.

y	d	t	num. examples
-1	0	0	36
-1	0	1	33
-1	1	0	32
-1	1	1	33
1	0	0	32
1	0	1	33
1	1	0	1
1	1	1	0
Total num. examples			200

Let us again define groups based on the value of (y, d) . Group $(y = 1, d = 1)$ has (sample size = 1) and the other three groups have much larger group sizes (group $(y = 1, d = 0)$ has size 65, group $(y = -1, d = 1)$ has size 65, group $(y = -1, d = 0)$ has size 69). A valid concern might be that particularly small groups should not be weighted the same as much larger groups of individuals. How would you factor in group size in the $\text{TrainLoss}_{\max}(w)$ term to account for this concern?

Note: There are multiple valid answers to this problem.

What we expect: Justify your proposed adjustment to the loss term in 1-2 sentences.

Your Solution: Assuming we will still use only one classifier (either T or P not both together), we may change the classifier T or objective such that $[\beta, d, t]$ has nonzero values for d .

4. What value of λ in $\text{TrainLoss}_{\text{new}}(w)$ (without the adjustment from the previous part) would you deploy in the following scenario? A real online social media platform is using a classifier to flag posts as toxic for review. The platform cares equally about average loss and minimizing the maximum loss per group. Again, make sure that your argument refers to the ethical frameworks mentioned in the module and discusses the trade-offs.

What we expect: There are many ways to answer these questions well; a good answer explains the connection between a classifier, the loss function and the ethical principle clearly and concisely in 3-5 sentences.

Your Solution: One potential adjustment is to take average of 2 models. This involves training models twice (e.g. <https://arxiv.org/html/2309.06717v2>), one biased for demography (D classifier) and another for normal toxicity training (T classifier). Averaging over the losses from D classifiers and T classifiers separately should give us a better outcome. E.g. $\text{TrainLoss}_{\text{new}}(w) = \text{Avg}(\lambda_p \cdot \text{TrainLoss}_{\max}(w, P) + \lambda_t \text{TrainLoss}_{\max}(w, T))$.

Problem 5: K-means Clustering

Suppose we have a feature extractor ϕ that produces 2-dimensional feature vectors, and a toy dataset $\mathcal{D}_{\text{train}} = \{x_1, x_2, x_3, x_4\}$ with

1. $\phi(x_1) = [0, 0]$
 2. $\phi(x_2) = [4, 0]$
 3. $\phi(x_3) = [6, 0]$
 4. $\phi(x_4) = [11, 0]$
- a. [2 points] Run 2-means on this dataset until convergence. Please show your work. What are the final cluster assignments z and cluster centers μ ? Run this algorithm twice with the following initial centers:
- (a) $\mu_1 = \phi(x_1) = [0, 0]$ and $\mu_2 = \phi(x_4) = [11, 0]$
 - (b) $\mu_1 = \phi(x_1) = [0, 0]$ and $\mu_2 = \phi(x_2) = [4, 0]$

What we expect: Show the cluster centers and assignments for each step, and the loss for each final clustering.

Your Solution:

since the points are on same line, we will use x co-ordinates only.

(a) iteration 1: centroids 1 = [0,0], 2 = [11,0]

$$x_1 = \operatorname{argmin}(0 - 0)^2, (0 - 11)^2 = 1$$

$$x_2 = \operatorname{argmin}(4 - 0)^2, (4 - 11)^2 = 1$$

$$x_3 = \operatorname{argmin}(6 - 0)^2, (6 - 11)^2 = 2$$

$$x_4 = \operatorname{argmin}(11 - 0)^2, (11 - 11)^2 = 2$$

(a) iteration 2: cluster 1: {[0,0], [4,0]}, cluster 2: {[6,0], [11,0]} and centroids 1 = [2,0], 2 = [8.5,0].

$$x_1 = \operatorname{argmin}(0 - 2)^2, (0 - 8.5)^2 = 1$$

$$x_2 = \operatorname{argmin}(4 - 2)^2, (4 - 8.5)^2 = 1$$

$$x_3 = \operatorname{argmin}(6 - 2)^2, (6 - 8.5)^2 = 2$$

$$x_4 = \operatorname{argmin}(11 - 2)^2, (11 - 8.5)^2 = 2$$

cluster 1: {[2,0], [4,0]}, cluster 2: {[6,0], [11,0]} and centroids 1 = [2,0], 2 = [8.5,0]. The clusters are converged since there's no change.

case (b) (b) iteration 1: centroids 1 = [0,0], 2 = [4,0]

$$x_1 = \operatorname{argmin}(0 - 0)^2, (0 - 4)^2 = 1$$

$$x_2 = \operatorname{argmin}(4 - 0)^2, (4 - 4)^2 = 2$$

$$x_3 = \operatorname{argmin}(6 - 0)^2, (6 - 4)^2 = 2$$

$$x_4 = \operatorname{argmin}(11 - 0)^2, (11 - 4)^2 = 2$$

(b) iteration 2: cluster 1: {[0,0]}, cluster 2: {[4,0], [6,0], [11,0]} and centroids 1 = [0,0], 2 = [7,0] (21/3).

$$x_1 = \operatorname{argmin}(0 - 0)^2, (0 - 7)^2 = 1$$

$$x_2 = \operatorname{argmin}(4 - 0)^2, (4 - 7)^2 = 2$$

$$x_3 = \operatorname{argmin}(6 - 0)^2, (6 - 7)^2 = 2$$

$$x_4 = \operatorname{argmin}(11 - 0)^2, (11 - 7)^2 = 2$$

cluster 1: {[0,0]}, cluster 2: {[4,0], [6,0], [11,0]} and centroids 1 = [0,0], 2 = [7,0]. The clusters are converged since there's no change.

- b. [5 points] Implement the `kmeans` function. You should initialize your k cluster centers to random elements of `examples`. After a few iterations of k-means, your centers will be very dense vectors. In order for your code to run efficiently and to obtain full credit, you will need to precompute certain dot products for squared distance calculation. As a reference, our code runs in under a second on cardinal, on all test cases. You might find `generateClusteringExamples` in `util.py` useful for testing your code.

NOTE: Do not use libraries such as Scikit-learn.

- c. [2 points] In general, if we scale all dimensions in our initial centroids and data points by some non-zero factor, are we guaranteed to retrieve the same clusters after running k-means (i.e. will the same data points belong to the same cluster before and after scaling)? What if we scale only certain dimensions? If your answer is yes, provide a short explanation; if not, give a counterexample.

What we expect: This response should have two parts. The first should be a yes/no response and explanation or counterexample for the first subquestion (scaling all dimensions). The second should be a yes/no response and explanation or counterexample for the second subquestion (scaling only certain dimensions). Note that you should not only consider the above toy dataset, think generally.

Your Solution:

Submission

Submission is done on Gradescope.

Written: When submitting the written parts, make sure to select **all** the pages that contain part of your answer for that problem, or else you will not get credit. To double check after submission, you can click on each problem link on the right side and it should show the pages that are selected for that problem.

Programming: After you submit, the autograder will take a few minutes to run. Check back after it runs to make sure that your submission succeeded. If your autograder crashes, you will receive a 0 on the programming part of the assignment. Note: the only file to be submitted to Gradescope is `submission.py`.

More details can be found in the Submission section on the course website.