Planetary Sciences Assignment 1
Generating Synthetic RV Curve

Kiran L
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1 Introduction

The present report contains a description of the variation of the radial velocity plot with eccentricity (e) and argument of periastron (ω). For a fixed value of eccentricity, the effect of variation in ω is determined by plotting the radial velocity curves for different values of ω (here, 0^o,30^o,60^o and 90^o). Similarly, the effect of variation in radial velocity curve with eccentricity is observed using different values of eccentricity (here, 0.0, 0.7). The observations made and inferences drawn are described in the below sections. All radial velocity curve plots are plotted as function of phase of orbit, with the phase varying from 0^o to 720^o (which corresponds to two complete cycles). The Figure 3 contains a grid plot of radial velocity curves for different values of e and ω. The value of eccentricity is varied along with row keeping the angle of periastron fixed. Similarly, the value of angle of periastron is varied along with columns with fixed values of eccentricity (along each column). The radial velocity curves are plotted with different colors to improve aesthetics of the plot. All the plots are plotted with following values of orbital parameters:

1. orbital period, P = 5 Earth years
2. length of semi-major axis of star's orbit, $a_* = 0.05$ AU
3. angle of inclination, i = 60^o
4. mass of star, $M_* = 1M_\odot$

All the radial velocity curves are plotted under the assumption that the exo-planet system (so considered) is dynamically stable.

2 Observations

In this section we describe the observations made from the plots and the corresponding inferences drawn with proper reasoning.

1. From Figure 2 we observe that the effect of variation in ω is simply to shift the phase of the radial velocity curves for perfectly circular orbits (e = 0.0).

2. Whereas in case of elliptical orbit ($0 < e < 1$) we observe that the effect of variation in ω is to produce an offset in the mean value of radial velocity over a cycle (a complete orbit) from 0 km/s as well as produce a phase shift by some amount.

3. From Figure 4 we observe that the effect of variation in eccentricity is simply to shift the mean value of radial velocity. It causes an overall increase in the amplitude of radial velocity curves in general, except for the case of perfectly circular orbits.

4. In case of circular orbits, e = 0.0, which implies the term $ecos(\omega)$ in the expression for radial velocity curve (as a function of phase angle) vanishes, produces no effect on the amplitude of the curve.

5. For any two exoplanet system with identical values of orbital period, angle of inclination and length of semi-major axis of the orbit, the system with higher eccentricity has larger amplitude of radial velocity curve.

6. This can also be observed in the set of plots displayed, where, the semi-amplitude of the curve is greater for e = 0.7 ($k_* \approx 0.258$) than for e = 0.0 ($k_* \approx 0.361$) for same values of P, a_* and i.

7. Also, the effect of eccentricity is to vary the amount of offset to the radial velocity curve (since, $v_r \propto ecos(\omega)$). The amount of offset is found to increase monotonically with increase in eccentricity (as shown in the Figure 1, with a_*, P, ω and i being held constant).

What is practically observed is the plot of radial velocity as a function of time, which exhibits high variability in both scale and shape of the plot.

3 Inferences

1. The case of e = 0.0 corresponds to exo-planet system(s) in circular orbit. For this case, the angle of periastron plays no significant role, since, it is not well defined (the separation between the planet and the host star being fixed, there exists no unique periastron). Hence, even though it produces a phase shift in the radial velocity curves, it has no physical significance during observation.

2. The case of e = 0.7 (or in general $0 < e < 1$) corresponds to an elliptical orbit. For this case, unlike in circular orbit, the angle of periastron is well defined and has a physical implication in the observational data. It determines the mean value of radial velocity measured (in analogy to DC part of alternating current in electronics). Thus, it serves as an important

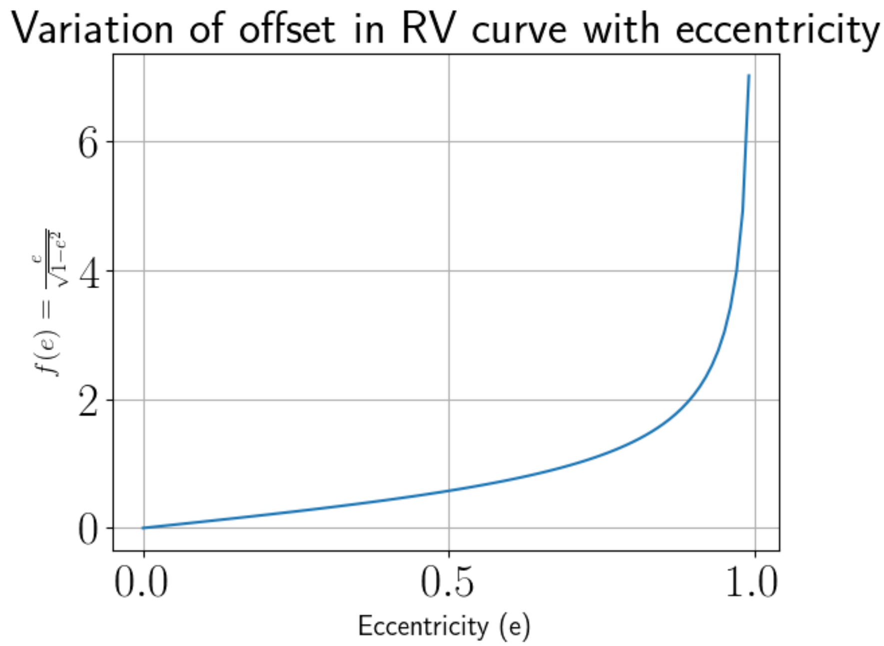

Figure 1: Variation of offset in RV curve with eccentricity

parameter to be fit to the observational data to determine the properties of the exo-planet system.

4 Results

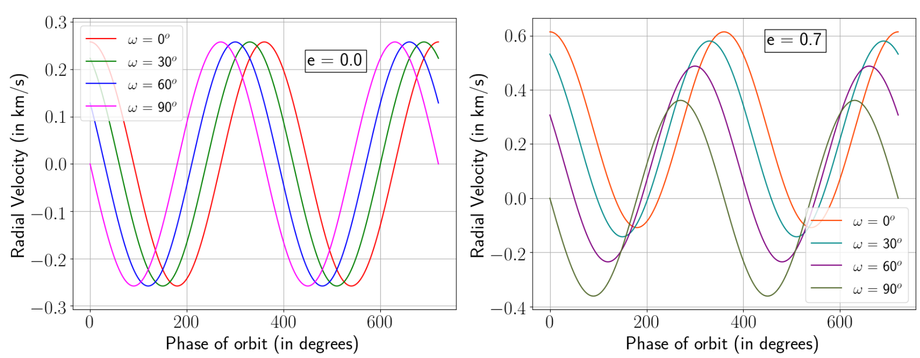

Figure 2: Radial Velocity Curves for Different values of argument of periastron (ω) for given eccentricity (e)

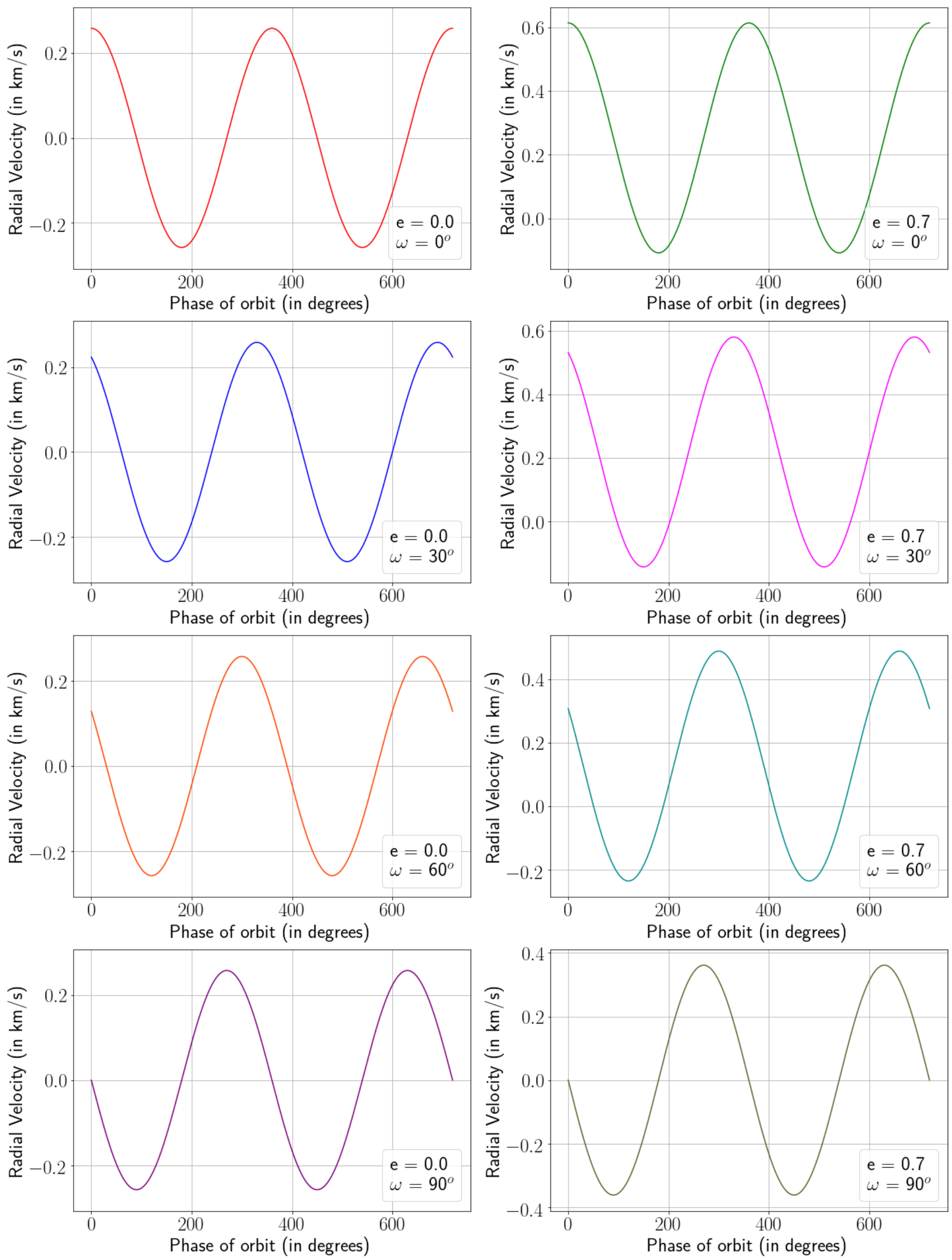

Figure 3: Radial Velocity Curves for Different values of eccentricity (e) and argument of periastron (ω)

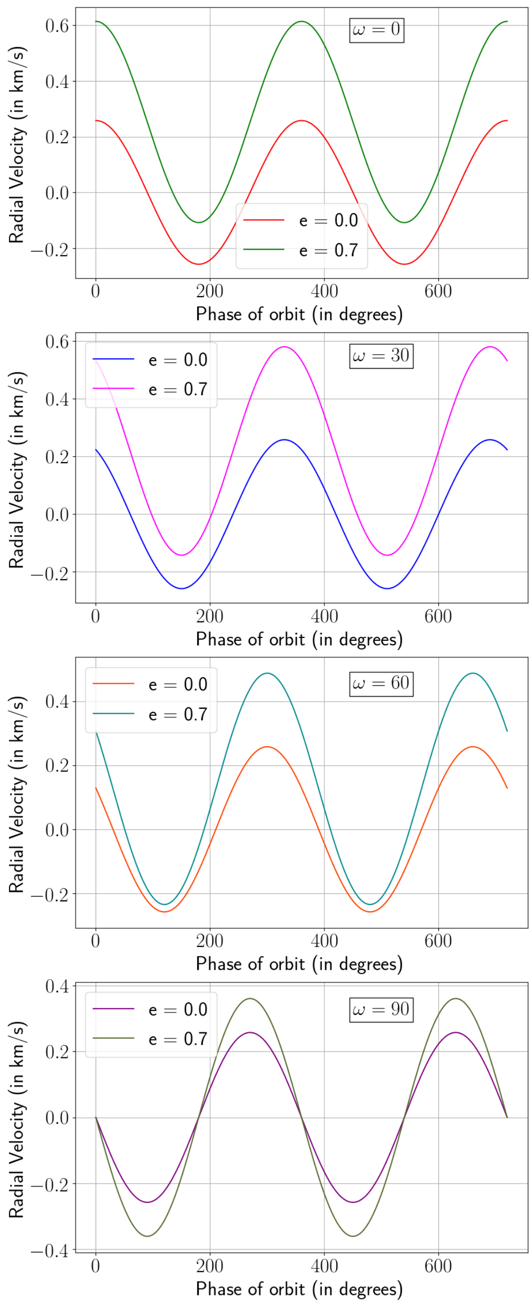

Figure 4: Radial Velocity Curves for Different values of eccentricity (e) for given argument of periastron (ω)

5 Program

```python

##################################
###--- Importing Libraries ---####
##################################

import numpy as np
import matplotlib
import matplotlib.pyplot as plt
from astropy import units as u
from matplotlib.pyplot import cm

##################################
###--- Function Defintion ---####
##################################

def rv(theta, e=0, w=0, i=60, a=0.05, P=5):  # angles -> degrees
    theta = np.radians(theta)
    w = np.radians(w)
    i = np.radians([i])
    k = ((2*np.pi*np.sin(i)*a.to(u.km))/(P.to(u.s)*np.sqrt(1-e**2))).value
    return k*( e*np.cos(w) + np.cos(w+theta) ).squeeze()

##################################
#####--- Input Variables ---######
##################################

a = 0.05 * u.au
P = 5 * u.yr
i = 60
w = np.array([0,30,60,90])
e= np.array([0,0.7])
theta = np.arange(0,720,0.01)

n_row, n_col = len(w),len(e)

matplotlib.rc('text', usetex=True) #use latex for text

# add amsmath to the preamble
matplotlib.rcParams['text.latex.preamble']=[r"\usepackage{amsmath}"]

##################################
#########--- Grid Plot---#########
##################################

plt.figure(figsize=(8 * n_col, 6 * n_row))
plt.subplots_adjust(bottom=0, left=.01, right=.99, top=.90, hspace=.20)

colors_list = ["red","green","blue","magenta","orangered","darkcyan","purple"
    ,"darkolivegreen"]
color=iter(colors_list)
ctr = 1
for k in range(len(w)):
    for j in range(len(e)):
        c = next(color)
        vr = rv(theta=theta,e=e[j],w=w[k],i=i,a=a,P=P)
        t='e = ' +str(e[j])+'\n $\omega$ = {}$^o$'.format(w[k])
        plt.subplot(n_row, n_col, ctr)
        plt.xlabel("Phase of orbit (in degrees)",fontsize = 25)
        plt.ylabel("Radial Velocity (in km/s)",fontsize = 25)
        plt.rcParams['xtick.labelsize']=25
        plt.rcParams['ytick.labelsize']=25
        plt.ylim(np.amin(vr)-0.05, np.amax(vr)+0.05)
        plt.plot(theta, vr, c=c,label=t)
        plt.grid()
```

```
        leg = plt.legend(handlelength=0, handletextpad=0, fancybox=True, fontsize=25,loc="lower right")
        for item in leg.legendHandles:
            item.set_visible(False)
        ctr += 1

plt.savefig("rvc.png", bbox_inches='tight')
plt.close()

###################################
#####--- Plots with fixed ---######
##--- angle of periastron and ---##
###--- variable eccentricity ---###
###################################

plt.figure(figsize=(8 * 1, 6 * n_row))
plt.subplots_adjust(bottom=0, left=.01, right=.99, top=.90, hspace=.20)

colors_list = ["red","green","blue","magenta","orangered","darkcyan","purple","darkolivegreen"]
color=iter(colors_list)
ctr = 1
for k in range(len(w)):
    plt.subplot(n_row, 1, ctr)
    min_lim = np.inf
    max_lim = -np.inf
    for j in range(len(e)):
        c = next(color)
        vr = rv(theta=theta,e=e[j],w=w[k],i=i,a=a,P=P)
        t='e = ' +str(e[j])
        plt.plot(theta, vr, c=c,label=t)
        plt.legend(fontsize=25)
        if(np.amin(vr)-0.05 < min_lim):
            min_lim = np.amin(vr)-0.05
        if(np.amax(vr)+0.05 > max_lim):
            max_lim = np.amax(vr)+0.05
        plt.legend(fontsize = 25)
    plt.text(450,max_lim-0.1,r"$\omega = {}$".format(w[k]),fontsize=25, bbox=dict(facecolor='none', edgecolor='black'))
    plt.xlabel("Phase of orbit (in degrees)",fontsize = 25)
    plt.ylabel("Radial Velocity (in km/s)",fontsize = 25)
    plt.grid()
    plt.rcParams['xtick.labelsize']=25
    plt.rcParams['ytick.labelsize']=25
    plt.ylim(min_lim,max_lim)
    ctr += 1

plt.savefig("rvc_e.png", bbox_inches='tight')
plt.close()

###################################
#####--- Plots with fixed ---######
#--- eccentricity and variable ---#
####--- angle of periastron ---####
###################################

plt.figure(figsize=(8 * n_col, 6 * 1))
plt.subplots_adjust(bottom=0, left=.01, right=.99, top=.90, hspace=.20)

colors_list = ["red","green","blue","magenta","orangered","darkcyan","purple","darkolivegreen"]
color=iter(colors_list)
ctr = 1
for j in range(len(e)):
    plt.subplot(1, n_col, ctr)
    min_lim = np.inf
    max_lim = -np.inf
    for k in range(len(w)):
```

```
            c = next(color)
            vr = rv(theta=theta,e=e[j],w=w[k],i=i,a=a,P=P)
            t='$\omega$ = {}$^o$'.format(w[k])
            plt.plot(theta, vr, c=c,label=t)
            plt.grid()
            plt.legend(fontsize=25)
            if(np.amin(vr)-0.05 < min_lim):
                min_lim = np.amin(vr)-0.05
            if(np.amax(vr)+0.05 > max_lim):
                max_lim = np.amax(vr)+0.05
            plt.legend(fontsize=20)
        plt.text(450,max_lim-0.1,r"e = {}".format(e[j]),fontsize=25, bbox=dict(
         facecolor='none', edgecolor='black'))
        plt.xlabel("Phase of orbit (in degrees)",fontsize = 25)
        plt.ylabel("Radial Velocity (in km/s)",fontsize = 25)
        plt.rcParams['xtick.labelsize']=25
        plt.rcParams['ytick.labelsize']=25
        plt.ylim(min_lim,max_lim)
        plt.grid()
        ctr += 1

plt.savefig("rvc_w.png", bbox_inches='tight')
plt.close()

##################################
####--- Plotting variation ---#####
####--- of degree of offset ---####
#####--- with eccentricity ---#####
##################################

e = np.arange(0,1,0.01)
plt.plot(e,e/np.sqrt(1-e**2))
plt.xlabel("Eccentricity (e)",fontsize = 15)
plt.ylabel(r"$f(e) = \frac{e}{\sqrt{1-e^2}}$",fontsize = 15)
plt.title("Variation of offset in RV curve with eccentricity", fontsize=25)
plt.grid()
plt.savefig("offset.png")
plt.close()

##################################
#######--- End of Code ---########
##################################
```

6 Conclusion

Through this analysis, we have been able to determine the variation of the radial velocity curves with changes in argument of periastron (ω) and eccentricity (e) of the orbit of star (hence, that of planet, under the assumption of dynamical stability). We conclude that the effect of ω is to produce a phase shift and an (positive) offset to the radial velocity curve. The effect of eccentricity is to vary the semi-amplitude of the radial velocity curve (due to $\frac{1}{\sqrt{1-e^2}}$ dependence) as well as determine the amount of offset in the radial velocity curve (along with ω since, the offset is proportional to $\frac{ecos(\omega)}{\sqrt{1-e^2}}$).

Thus, a good understanding of the effect of each of the above parameters explored is important for obtaining a good synthetic radial velocity curve fit to the observational data.