Planetary Sciences Assignment 3
Exoplanet Transit Duration Curves

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1 Introduction

Most of the studies of the planets orbiting stars other than the Sun are possible only through the indirect methods, owing to the limitations of visibility of the planets due to their small size. One of the popular indirect methods is the Transit method, in which , various physical characteristics of the planet are determined using the photometric light curves of the planet transiting the star (at least 3 times for confirming that is truly corresponds to the transit of the planet). In this assignment, we explore the effect of orbital inclination angle ("i") and orbital separation ("a") on the duration of transit (t_T) by plotting the transit duration curves as a function of orbital separation for various values of orbital inclination angle.

For analysis, we consider two planetary systems - one comprising of Earth-like planet orbiting Sun-like star and another comprising of Jupiter-like planet orbiting Sun-like star. With the "i" and "a" being variable, we explore their effect on the duration of transit. The relation between the "t_T" and "i" and "a" is as follows:

$$t_T = \frac{P}{\pi} sin^{-1} \left(\frac{1}{sin(i)} \sqrt{\left(\frac{R_S + R_P}{a} \right)^2 - cos^2(i)} \right) \tag{1}$$

Observations and inferences drawn from the transit duration plots (plotted vs "a", for different values of orbital inclination) are listed in the sections below.

2 Observations

In this section we describe the observations made from the results obtained.

1. The transit duration for an edge on system is found to be monotonously increasing with increase in orbital separation (as shown in Figure 1).

2. With decrease in the orbital inclination angle, we observe that the transit duration is no longer a monotonous function of orbital separation. We observe that the curve peaks for a specific value of orbital separation, which is unique for a given orbital inclination angle for the same system (as can be seen from Figure 3).

3. Also, there is a minimum value of orbital separation below which no transit is observed. The minimum value of the orbital separation is the sum of radius of planet and the star, which corresponds to the case when the planet is in contact with the star.

4. The above case corresponds to an un-physical situation, since, it is more likely for the planet to be disrupted due to tidal forces of the star.

5. For large values of orbital separation, we observe that the transit duration is proportional to the square root of orbital separation (as shown in Figure 2).

6. For edge on view, the transit duration is an unbounded function of orbital separation, whereas it becomes bounded for all orbital inclination angles less than 90^o, as can be seen from Figure 3, where the transit duration curve peaks for some value of "a" for all cases where the orbital inclination is less than 90^o.

3 Inferences

In this section we enumerate the inferences drawn from the observations made (from the results) along with proper justification (reasoning).

1. From the plot in Figure 2, we infer that for most of the planetary systems found in the universe, the transit duration when viewed edge on increases monotonously with the orbital separation ($t_T \propto \sqrt{a}$). This is due to the contribution from two factors:

   (a) for large values of a and when viewing the planetary system edge on, the term $sin^{-1}(\frac{R_S+R_P}{a}) \approx \frac{R_S+R_P}{a}$

   (b) the period of orbit is proportional to $a^{\frac{3}{2}}$

   Thus, the product of the above two terms results in a term proportional to $\sqrt{a}$. Thus, for transits of planets with orbital inclination 90^2 the transit duration is proportional to $\sqrt{a}$.

2. For all cases other than edge on view transits, the transit duration increases, attains a peak value and decreases with increase in orbital separation. This is due to the following two opposing factors:

   (a) for (relatively) small values of a, the $sin^{-1}(\frac{R_S+R_P}{a})$ is approximately $\frac{\pi}{2}$ (because, $\frac{R_S+R_P}{a} \approx 1$, with values of a such that $a \geq (R_S + R_P)$). Thus, the transit duration is mainly dependent on the term $\frac{P}{\pi} \propto a^{\frac{3}{2}}$.

   (b) for (relatively) large values of a, the term $\frac{R_S+R_P}{a} \ll 1$ which implies $sin^{-1}(\frac{R_S+R_P}{a}) \approx \frac{R_S+R_P}{a}$. Thus, the transit duration, $t_T \propto \frac{P}{\pi}\frac{R_S+R_P}{a} \propto \frac{a^{\frac{3}{2}}}{a} = \sqrt{a}$.

   (c) since, the orbital inclination is less than 90^o, the cosine term restricts the value of a from being arbitrarily small.

   (d) since, the term inside the square root needs to be positive for the equation to be physically valid, this condition restricts the value of a from growing arbitrarily large. As the value of a approaches its upper bound, the $sin^{-1}\left(\frac{1}{sin(i)}\sqrt{\left(\frac{R_S+R_P}{a}\right)^2 - cos^2(i)}\right)$ term becomes vanishingly small such that it dominates over the $a^{\frac{3}{2}}$ term, causing the whole expression to vanish.

3. The peak is obtained when the trade off between the two - at the value of orbital separation, a small value more than which the arcsine term dominates and a small value less than which the $a^{\frac{3}{2}}$ term dominates.

4. Also, physically the upper bound on a is because, for any value of inclination other than 90^o, there exists a value of a for which $(a \times cos(i)) \geq (R_P + R_S)$, which causes the observer to miss the transit.

5. We further infer that for a given planetary system, there exists a unique one-to-one relation between the value of orbital separation for which peak occurs and the orbital inclination angle.

6. Upon setting the first derivative of the transit duration expression with respect to orbital separation, we get a transcendental equation in a, which upon solving gives the value of a corresponding to peak transit duration as a function of orbital inclination angle.

7. Since, the orbital separation corresponding to peak transit duration decreases with decrease in orbital inclination angle, we infer that “a” and “i” are positively correlated and non-linearly related (since, for a small change in i from 89.9^o to 90^o results in the a_{peak} to become unbounded, though bounded for all values of $i < 90^o$).
   That is, the transit duration is an un-bounded function of orbital separation for orbital inclination of 90^o and is bounded for all values of orbital inclination less than 90^o.

4 Results

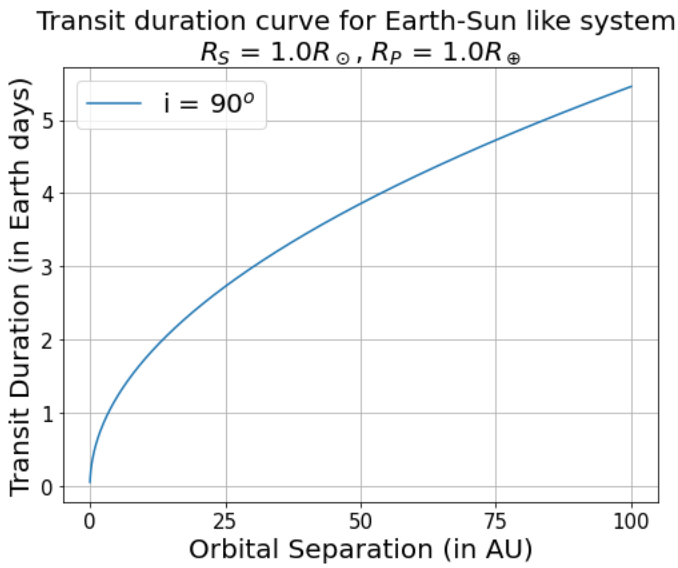

Figure 1: Transit duration curve for Earth-size planet in orbit around a Sun-like star, for i = 90^o

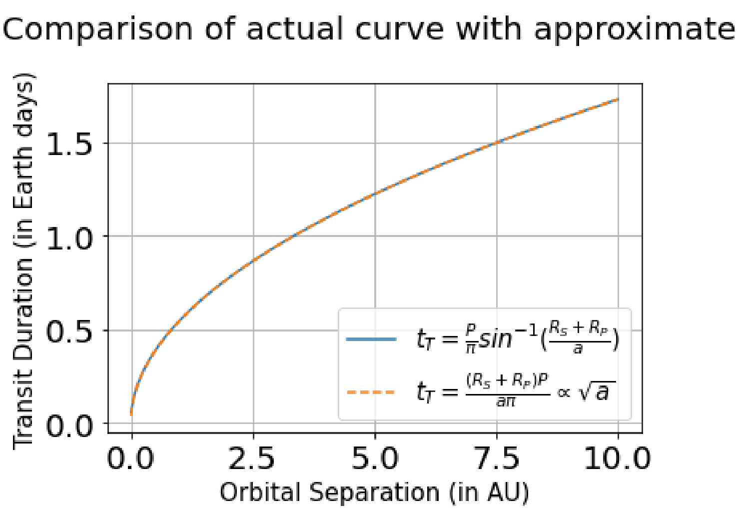

Figure 2: Transit duration curve for Earth-size planet in orbit around a Sun-like star (with i = 90^o) proportional to $\sqrt{a}$ for large values of a

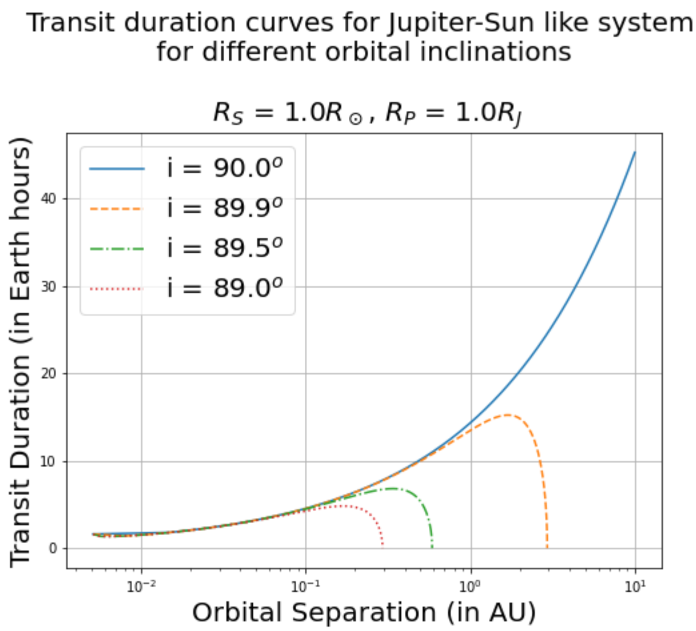

Figure 3: Transit duration curve for Jupiter-size planet in orbit around a Sun-like star, for different values of orbital inclination

5 Program

```

##################################
###--- Importing Libraries ---####
##################################

import matplotlib.pyplot as plt
import numpy as np
from astropy import units as u
from astropy import constants as const
from itertools import cycle

##################################
###--- Function Defintion ---####
##################################

def rv(theta, e=0, w=0, i=60, a=0.05*u.au, P=5*u.yr, theta_radians=True):  # angles -> degrees, theta -> radians
    #theta = np.radians(theta)
    w = np.radians(w)
    i = np.radians([i])
    k = ((2*np.pi*np.sin(i)*a.to(u.km))/(P.to(u.s)*np.sqrt(1-e**2))).value
    return k*( e*np.cos(w) + np.cos(w+theta) ).squeeze(), k*( e*np.cos(w)).squeeze()

##################################
#####--- Class Defintion ---######
##################################

```

```python
class transit:

    def __init__(self,M = 1*const.M_sun,R = 1*const.R_sun,m = 1*const.M_earth,r = 1*const.R_earth,inc = [90],radians=False):
        self.M = M.to(u.kg)
        self.R = R.to(u.au)
        self.m = m.to(u.kg)
        self.r = r.to(u.au)
        if(radians==True):
            self.inc = np.array(inc)
            self.inc_deg = self.inc * (180/np.pi)
        else:
            self.inc_deg = np.array(inc)
            self.inc = self.inc_deg*(np.pi/180)

    def get_t(self,i,N=1000,max_max_a = 10*u.au,yscale="day"):
        min_a = (self.R+self.r)
        max_a = np.abs((self.R+self.r)/np.cos(i))
        if(max_a > max_max_a):
            max_a = max_max_a
        a = np.linspace(min_a,max_a,N)
        P = np.sqrt(((a**3)*(4*np.pi**2))/(const.G*(self.m+self.M)))
        if(yscale=="day"):
            P = P.to(u.day)
        if(yscale=="hr"):
            P = P.to(u.hr)
        t = ((P.value/np.pi)*np.arcsin( (1/np.sin(i))*np.sqrt(((self.r+self.R)/a)**2 - np.cos(i)**2)) )
        return a,P,t

    def plot_and_save(self,savename,N=1000,max_max_a = 10*u.au,ft = 20,log=False,yscale="day",p_ref="earth"):
        lines = ["-","--","-.",":"]
        linecycler = cycle(lines)
        plt.figure(figsize=(8,6))
        for i,j in zip(self.inc,self.inc_deg):
            a,_,t = self.get_t(i,N,max_max_a,yscale)
            plt.plot(a,t,label="i = ${}^o$".format(j),ls=next(linecycler))
        plt.grid()
        plt.legend(fontsize=ft)
        plt.xlabel("Orbital Separation (in AU)",fontsize=ft)
        if(yscale=="day"):
            plt.ylabel("Transit Duration (in Earth days)",fontsize=ft)
        elif(yscale=="hr"):
            plt.ylabel("Transit Duration (in Earth hours)",fontsize=ft)
        if(log==True):
            plt.gca().set_xscale("log")
        plt.rcParams['xtick.labelsize']=ft+5
        plt.rcParams['ytick.labelsize']=ft+5
        if(p_ref=="earth"):
            text = "Transit duration curve for Earth-Sun like system \n"
            plt.title(text+"$R_S$ = {}$R_\odot$, $R_P$ = {}$R_\oplus$".format(np.round(self.R.to(u.R_sun).value,2),np.round(self.r.to(u.R_earth).value,2)), fontsize=ft)
        elif(p_ref == "jupiter"):
            text = "Transit duration curves for Jupiter-Sun like system \n for different orbital inclinations \n"
            plt.title(text+"\n $R_S$ = {}$R_\odot$, $R_P$ = {}$R_J$".format(np.round(self.R.to(u.R_sun).value,2),np.round(self.r.to(u.R_jup).value,2)), fontsize=ft)

        plt.savefig(savename,bbox_inches="tight")

####################################
#--- Transit duration vs Orbital ---#
#####--- Separation Plot for ---#####
####--- Earth-Sun like system ---####
####################################
```

```python

es = transit()

# Plotting approximate curve

a,P,t = es.get_t(es.inc)
ax = plt.gca()
ax.plot(a,t,'-', label=r"$t_T = \frac{P}{\pi}sin^{-1}(\frac{R_S + R_P}{a})$")
tmp = (P*(es.r+es.R))/(np.pi*a)
ax.plot(a,tmp,'--',label=r"$t_T = \frac{(R_S+R_P)P}{a \pi} \propto \sqrt{a}$")#"")
plt.legend(fontsize=15)
plt.xlabel("Orbital Separation (in AU)",fontsize=15)
plt.ylabel("Transit Duration (in Earth days)",fontsize=15)
plt.rcParams['xtick.labelsize']=20
plt.rcParams['ytick.labelsize']=20
plt.grid()
plt.title("Comparison of actual curve with approximate \n", fontsize=20)
plt.savefig("sqrt_a.png", bbox_inches="tight")

# plotting accurate curve

es.plot_and_save("3_a.png",10000,100*u.au)

# Verification

a = 1*u.au
R = 1*const.R_sun
r = 1*const.R_earth
P = 1*u.yr
inc = np.pi/2.
t = (P.to(u.day)/np.pi)*np.arcsin( (1/np.sin(inc))*np.sqrt( ((R+r)/a)**2 - np.cos(inc)**2 ) )
print("Transit Duration for Earth-Sun system in edge on configuration and circular orbit: {} Earth days".format(np.round(t.value,3)))

####################################
#--- Transit duration vs Orbital ---#
#####--- Separation Plot for ---#####
###--- Jupiter-Sun like system ---###
####################################

js = transit(m=const.M_jup,r=const.R_jup,inc=[90,89.9,89.5,89])
js.plot_and_save("3_b.png",max_max_a=10*u.au,log=True,yscale="hr",p_ref="jupiter")

##################################
#######--- End of Code ---########
##################################
```

6 Conclusion

Through this analysis, we have been able to determine the variation of the transit duration with orbital separation for different values of orbital inclinations. For edge on view, the transit duration is found to grow as $\sqrt{a}$ for large a and hence, it is always possible to observe the transit of the planet in front of the star for all values of orbital separation. Wheres, even a slight decrease in the inclination angle results in the transit duration to be a bounded function of the orbital separation. The "a_{peak}" and "i" are related by a transcendental equation, which requires iterative procedures for solving for "a_{peak}" given "i".