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Amplitude analysis of $D_s^+ \to K^+ K^- \pi^+$

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Abstract

We have performed an amplitude analysis of $D_s^+ \to K^+ K^- \pi^+$ using a data sample of $3.19 fb^{-1}$ recorded with BESIII detector at 1.178GeV. 4265 events with about %0.5 background are used in this analysis. There are 6 intermediate resonances in the model which does the best fit to data. Especially, $a_0(980)$ and $f_0(980)$ are taken as a whole.

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1   Introduction

The decay $D_s^+ \to K^+ K^- \pi^+$ is a Cabibbo-favored (CF) channel and has a large branching fraction for the D_s meson. Thus, this decay channel is usually used to normalize measurements of decay chains involving charm quarks.

Knowledge of the decay amplitude allows us to properly account for interference effects when measuring absolute hadronic branching fractions of D_s mesons.

Below is the table of previous analyses of this decay channel.

Decay mode	Decay fraction (%)		
	BABAR	E687	CLEO-c
$\bar{K}^*(892)^0 K^+$	$47.9 \pm 0.5 \pm 0.5$	$47.8 \pm 4.6 \pm 4.0$	$47.4 \pm 1.5 \pm 0.4$
$\phi(1020)\pi^+$	$41.4 \pm 0.8 \pm 0.5$	$39.6 \pm 3.3 \pm 4.7$	$42.2 \pm 1.6 \pm 0.3$
$f_0(980)\pi^+$	$16.4 \pm 0.7 \pm 2.0$	$11.0 \pm 3.5 \pm 2.6$	$28.2 \pm 1.9 \pm 1.8$
$\bar{K}_0^*(1430)^0 K^+$	$2.4 \pm 0.3 \pm 1.0$	$9.3 \pm 3.2 \pm 3.2$	$3.9 \pm 0.5 \pm 0.5$
$f_0(1710)\pi^+$	$1.1 \pm 0.1 \pm 0.1$	$3.4 \pm 2.3 \pm 3.5$	$3.4 \pm 0.5 \pm 0.3$
$f_0(1370)\pi^+$	$1.1 \pm 0.1 \pm 0.2$	$\cdots$	$4.3 \pm 0.6 \pm 0.5$
Sum	$110.2 \pm 0.6 \pm 2.0$	111.1	$129.5 \pm 4.4 \pm 2.0$
χ^2/NDF	$\frac{2843}{(2305-14)} = 1.2$	$\frac{50.2}{33} = 1.5$	$\frac{178}{117} = 1.5$
Events	96307 ± 369	701 ± 36	12226 ± 22

Figure 1: previous analyses

From figure.**??** [1], we can see an obvious difference of decay fraction of $f_0(980)\pi^+$ between BARBAR and CLEO-c. E687 used about 700 events and the E687 model did not take $f_0(1370)\pi^+$ into account. For CLEO-c, about 14400 events with purity about 84.9% were selected with single tag method. The analysis of BARBAR used about 100000 events with purity about 95%. In this analysis with double tag method, we can get a nearly background free data sample, that will be good to perform the amplitude analysis.

2 Data Set and Monte Carlo Samples

We use 3.195 fb^{-1} data set collected at E_{cm} = 4.178GeV by BESIII detector in 2016. Both data sample and Monte Carlo samples are reconstructed under BOSS7.0.3. Totally 35 rounds of generic MC with each round equaling to data size are used for background study. They are avaulable at /besfs3/offline/data/703-1/4180/mc/.

For the Signal MC, we generate the signal events with one D_s decaying to signal mode using the generator "DIY", in which the parameters are obtained from the fit to data. PHSP MC and Signal MC are used in MC integration required for the amplitude fit. The Signal MC is also used in the input/output check.

Table 1: Component and corresponding size, assume luminosity = 3195/pb.

Component	cross section (pb)	Size(M)	directory
$D^0 D^0$	179	0.5719	D0D0
$D^+ D^-$	197	0.6294	DpDm
$D^{*0} D^0$	1211	3.8691	DST0D0
$D^{*+} D^-$	1296	4.1407	DSTpDm
$D^{*0} D^{*0}$	2173	6.9427	DST0DST0
$D^{*+} D^{*-}$	2145	6.8533	DSTpDSTm
$D_s^+ D_s^-$	7	0.0225	DsDs
$D_s^{*+} D_s^-$	961	3.0700	DsSTDs
$DD^* \pi^+$	383	1.2237	DDSTPIp
$DD^* \pi^0$	192	0.6134	DDSTPI0
$DD \pi^+$	50	0.1598	DDPIp
$DD \pi^0$	25	0.0799	DDPI0
Component	cross section (nb)	Size(M)	
$q\bar{q}$	13.8	44.0910	qq
$\gamma J/\psi$	0.40	1.2780	RR1S
$\gamma \psi(2S)$	0.42	1.3419	RR2S
$\gamma \psi(3770)$	0.06	0.1917	RR3770
$\tau\tau$	3.45	11.0228	tt
$\mu\mu$	5.24	16.7418	mm
ee	423.99	13.5465(0.01×)	ee
$\gamma\gamma$	1.7	5.4315	TwoGam
HCT	0.10178	0.3252	HCT

Table 2: Component and corresponding observed cross section (output from ConExc) for charmonium hadronic transition (HCT) processes.

Mode index	Final state	Observed cross section @ 4180 MeV (nb)	Referee of input line shape
79	$\pi^0\pi^0\psi(2S)$	0.00342491	BELLE PRL99, 142002 (2007)
91	$\pi^+\pi^-\psi(2S)$	0.00684981	BELLE PRL99, 142002 (2007)
80	$\eta J/\psi$	0.0321958	BELLE PRD87, 051101(R) (2013)
81	$\pi^+\pi^- h_c$	0.0122136	BESIII PRL111,242001 (2013)
82	$\pi^0\pi^0 h_c$	0.00610681	BESIII PRL111,242001 (2013)
83	$K^+ K^- J/\psi$	0.000671349	BELLE PRD77, 011105(R) (2008)
84	$K_S^0 K_S^0 J/\psi$	0.000167837	BELLE PRD77, 011105(R) (2008)
90	$\pi^+\pi^- J/\psi$	0.026767	BELLE PRL99, 182004 (2007)
99	$\pi^0\pi^0 J/\psi$	0.0133835	BELLE PRL99, 182004 (2007)
sum		0.101780616	

3 Event Selection

At $E_{cm} = 4.178 GeV$, pairs of $D_s D_s^*$ are produced, and the D_s^* decays to either $D_s \gamma$ or $D_s \pi^0$. We use double tag method to select our signal events.

3.1 Tracking, PID, $\pi^0/\eta^{(')}$ and K_S^0 Rconstruction

D_s candicates are built from $K^\pm$, $\pi^\pm$, $\pi^0/\eta^{(0)}$ and K_S^0. The selections of the particles to build D_S candicates are performed with DTagAlg-00-01-05 package with the default setting, which are summarized below.

- Tracking:

    - The properties of charged tracks are determined based on the MDC information. Charged track candidates must satisfy:

        - $|cos\theta| < 0.93$.
        - $|dr| < 1cm$ and $|dz| < 10cm$,
          where $|dr|$ and $|dz|$ are defined as the one reconstructed minus the interaction point.

- Particle ID:

    - Charged tracks are identified as pion or kaon with Particle Identification (PID), which is implemented by combing the information of the energy loss (dE/dx) in MDC and the time-of-flight measured from the TOF system. Kaon and Pion are identified with the requirements that

        - $Prob(K) > 0.00$ and $Prob(K) > Prob(\pi)$ for K,
        - $Prob(\pi) > 0.00$ and $Prob(\pi) > Prob(K)$ for π, where $Prob(X)$ is the probability of hypothesis X, X can be π or K.

- π^0/η selection: π^0 candidates are reconstructed through $\pi^0 \to \gamma\gamma$ with package of PioEtaToGGRecAlg. The photons are reconstructed as energy showers on the EMC. We require:

    - Minimum energy for barrel showers($|cos\theta| < 0.8$): $E_{min} > 25 MeV/c^2$,
    - Minimum energy for endcap showers($0.86 < |cos\theta| < 0.92$): $E_{min} > 50 MeV/c^2$,
    - Shower within other $|cos\theta|$ regions are rejected.
    - EMC time requirements for events with at least one charged track detected: $0 \le t \ge 14(50ns)$,

Then we perform a constrained fit on the photon pairs to the nominal π^0/η mass and require:

- The unconstrained invariant mass for π^0: $0.115 < M(\gamma\gamma) < 0.015 GeV/c^2$,

- The unconstrained invariant mass for η : $0.490 < M(\eta) < 0.580 GeV/c^2$,

- Mass fit: $\chi^2_{1c} < 30$.

- η' selection: The η' candidates are reconstructed with $\pi^+\pi^-\eta$, the invariant mass for $\pi^+\pi^-\eta$ is required to fall into the range of $[0.938, 0.978] GeV^2$.

- K^0_S selection: K^0_S candidates are reconstructed using VeeVertexAlg package with two opposite charged tracks with requiring:

- $|cos\theta| < 0.93$

- $|dz| < 20cm$

For each pair of tracks, a constrained vertex fit is performed and the track parameters' results are used to get the invariant mass $M(K^0_S)$. Then the decay length of K^0_S is obtained with second vertex fit by the SecondVertexFit package. For K^0_S selection, we require:

- $0.487 GeV/c^2 < M(K^0_S) < 0.511 GeV/c^2$.

- Significance of the K^0_S decay lenght has two standard deviations.

- D_s selection: According to the MC study on D_s reconstruction (BESSIII-DocDB-380), the D_s candidates fall into the mass window of $1.90 < m_{D_s} < 2.03 GeV/c^2$ and the corresponding M_{rec} satisfied $2.051 < M_{rec} < 2.180 GeV/c^2$ are retained for further study. In which, the definition of M_{rec} is

$$M_{rec} = \sqrt{(E_{cm} - \sqrt{p^2_{D_s} + m^2_{D_s}})^2 - |\vec{p}_{cm} - \vec{p}_{D_s}|^2} \, , \tag{1}$$

where E_{cm} is the energy of initial state, calculated from the beam energy [2], $\vec{p}_{D_s}$ is the momentum of D_s candidate, m_{D_s} is D_s mass quoted from PDG [3] and $\vec{p}_{cm}$ and $\vec{p}_{D_s}$ are four-momentum of initial state and the decay products of a D_s candidate, respectively.

3.2 Single Tag Selection

As D^+_s and D^-_s should appear in pairs, so we can use double tag method to select signal events. After $K^\pm$, K^0_S, $\pi^\pm$ and γ are identified, hadronic D_s decays can be reconstructed with the DTag package. 8 tag modes are used:

$D_s^- \to K^+K^-\pi^-$, $D_s^- \to K_S^0 K^-$, $D_s^- \to K_S^0 K^-\pi^+\pi^-$, $D_s^- \to K^-\pi^+\pi^-$, $D_s^- \to K_S^0 K^+\pi^-\pi^-$, $D_s^- \to \pi^+\pi^-\pi^-$, $D_s^- \to \eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$, $D_s^- \to K^+K^-\pi^-\pi^0$.

Before selecting the best candidate, we vote the candidates with $\pi^\pm(\pi^0)$ whose momentum is less than $0.1 GeV$ to remove soft $\pi^\pm(\pi^0)$ from D^* decays.

For every candidate of D_s decays, all tracks at signal side and tag side as well as gamma from D_s^* are added to apply kinematic fitting. 5 constrains are added in kinematic fitting: four-momentum of $D_s D_s^*$, mass of D_s^*. Then we select the candidate with minmimum χ_{5c}^2.

The candidates satisfy:

- $1.95GeV < m_{sig} < 1.985GeV$

- $1.95GeV < m_{tag} < 1.985GeV$

- $\chi_{5c}^2 < 50$

are retained for amplitude analysis, where m_{sig} and m_{tag} refer to mass of D_s at signal side and tag side respectively.

3.3 Background Analysis

We use generic MC to estimate the background. The background and signal shape of generic MC is shown in Fig.2. According to the luminosities of the data and the generic MC, after scaling the background sample to the data size, the background yiels in Signal region is 11.8. Then the fit to the signal D_s invariant mass (m_{sig}) spectrum gives the background yield in Signal region is 11.3 ± 3.9, shown as in Fig.3. The background level in MC is then consistent with the data. In the fit, the signal shape is the MC shape convoluted with a Gaussian function and the background is the MC shape.
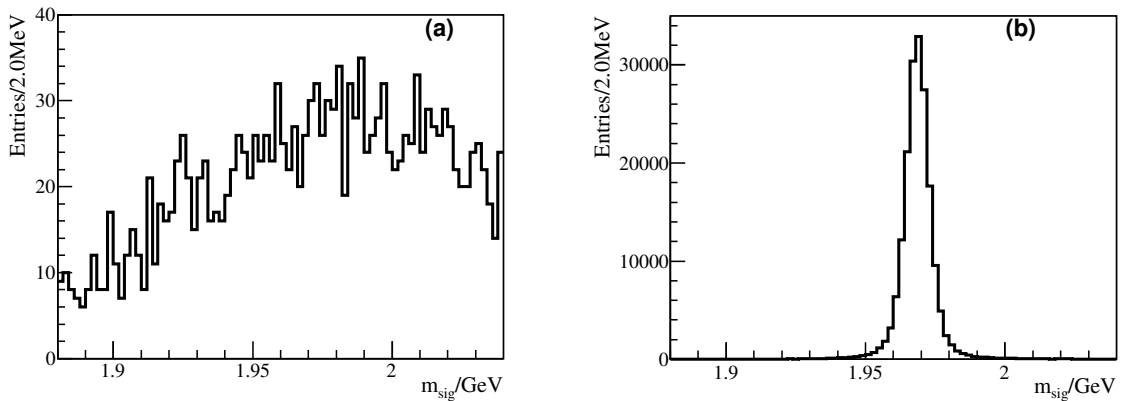

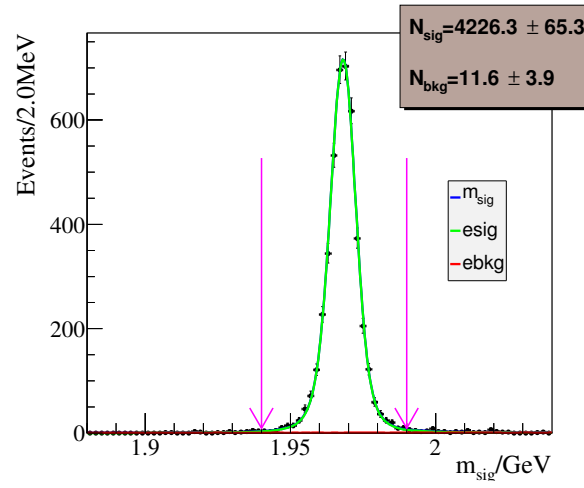Figure 2: The background (a) and signal (b) distributions from generic MC after selection

Figure 3: The fit to m_{sig} for data after selections, the area between the purple arrows is the signal area of the sample for the amplitude analysis.

4 Model Independent Partial Wave Analysis

In the K^+K^- threshold both $a_0(980)$ and $f_0(980)$ can be present, and both resonances have very similar parameters which suffer from large uncertainties. In this section we obtain model-independent information on the K^+K^-S wave by performing a MIPPPWA(model indendent partial wave analysis) in the K^+K^- threshold region.

4.1 Signal Candidate Selection

As MIPWA need more data events, we do not requre D_s^+ and D_s^- appear in pairs. Before selecting the best candidate, we vote the candidates with $\pi^\pm(\pi^0)$ whose momentum is less than $0.1GeV$ to remove soft $\pi^\pm(\pi^0)$ from D^* decays. For every candidates of D_s decays, all tracks at signal side are added to apply kinematric fitting. Only mass of D_s from signal side is constrained.Then we select the candidate with minimum χ^2_{1c}.

4.2 Background Analysis

In order to further suppress the background, the multiple analysis method (MVA) is used. We train MVA methos separately with different sets of variables for the two event categories depending on the D_s^+ origin. These two categories of events are selected in a $M_{rec} - \Delta M$ 2D plane shown in Fig.4:
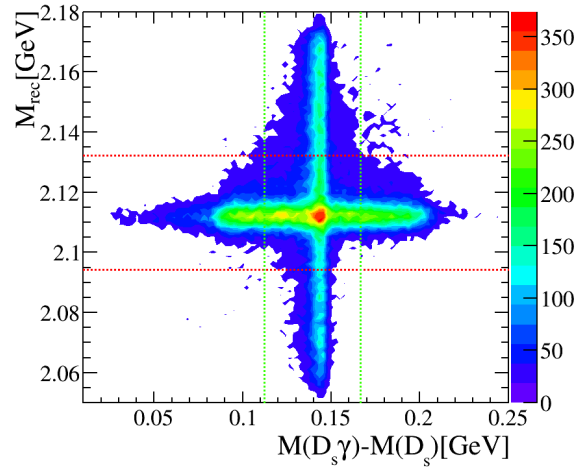

Figure 4: Two dimensional plane of $\Delta M \equiv M(D_s^+\gamma) - M(D_s^+)$ versus M_{rec} from the simulated $D_s^+ \to K^+K^-\pi^+$ decyas. The red(green) dashed lines mark the mass window for the D_s^+ Cat. #0(Cat. #1) around the $M_{rec}(\Delta M)$ peak.

- Cat. #0: Direct D_s^+. We use the following variables whose distributions for signal and background are shown in Fig.5,

1. M_{rec},

2. P_{rest}, defined as the total momentum of the tracks and neutrals in the rest of event (not part of the $D_s^+ \rightarrow K^+ K^- \pi^+$ candidate),

3. N_{tracks}, defined as the total number of tracks and neutrals in an event.

- Cat. #1: Indirect D_s^+. We use the following variables whose distributions for signal and background are shown in Fig.6,

1. ΔM,

2. M'_{rec}, defined as $M'_{rec} = \sqrt{\left(E_{cm} - \sqrt{p_{D_s\gamma}^2 + m_{D_s^*}^2}\right)^2 - p_{D_s\gamma}^2}$, with $p_{D_s\gamma}$ as the momentum of the $D_s\gamma$ combination, $m_{D_s^*}$ as the nominal D_s^* mass,

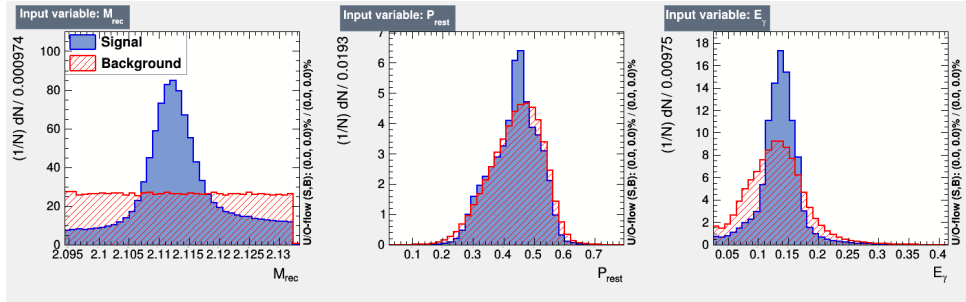3. E_γ, defined as the energy of gamma from D_s^*.

Figure 5: For event Cat. #0, distributions of MVA variables from simulated signal decays and background events.
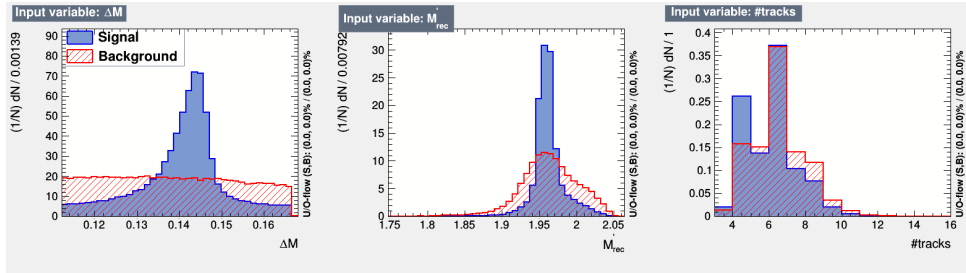

Figure 6: For event Cat. #1, distributions of MVA variables from simulated signal decays and background events.

As the results shown in Fig.7, this BDTG training and test samples are well matched. For event Cat. #0 (Cat. #1), the sample with BDTG value larger than 0.33(0.65) is retained for further study.

After applying the BDTG requirement, the background shows no obviously peak around the region of [1.95, 1.982] GeV (Signal region), which are shown in Fig.8. The fit to the signal D_s invariant mass
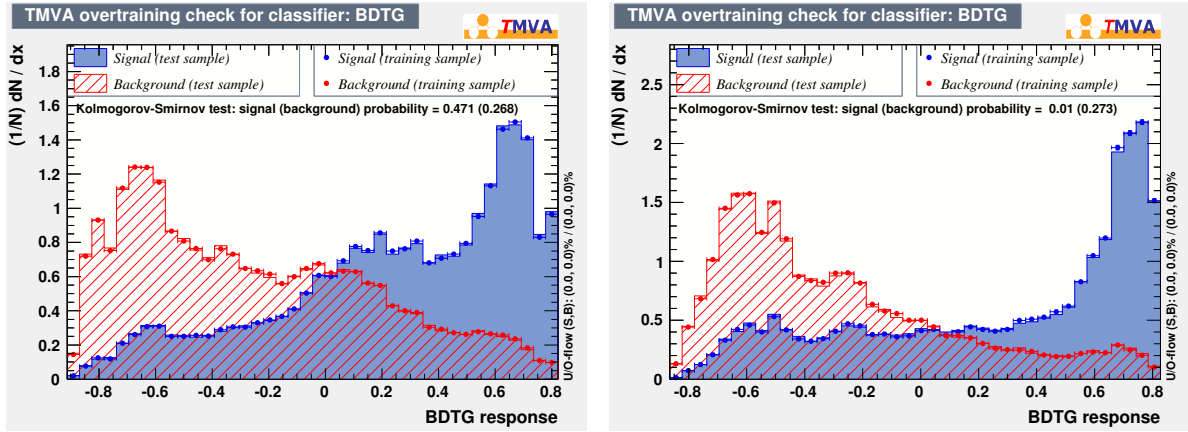
Figure 7: The comparisions between the training and test samples.

1 ($M_{D_s^2}$) spectrum gives the background yield in Signal region is 73.6 ± 18.7, shown as in Fig.9. In the fit,

2 the signal shape is the MC shape convoluted with a Gaussian function and the background is described
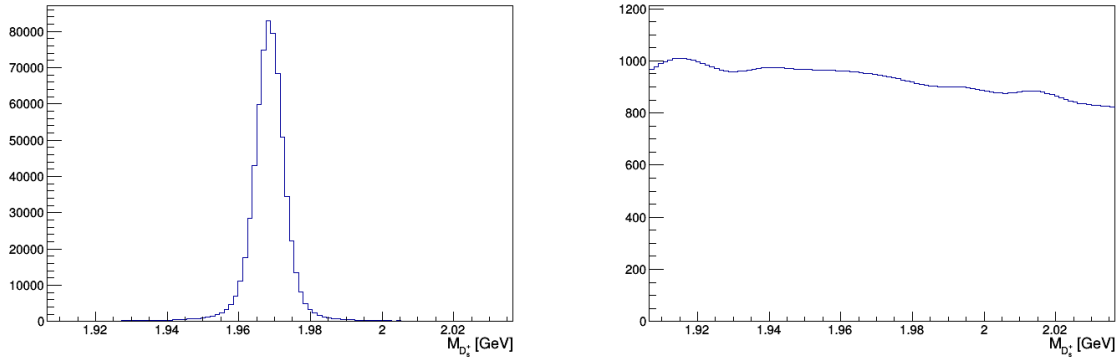
3 with 1st-order Chebychev polynomial.

Figure 8: The signal and background distributions from generic MC after BDTG requirement.

4 The projections of the "Sideband"($1.90 < M(D_s) < 1.95 GeV$ and $1.985 < M(D_s) < 2.03 GeV$) from

5 data and generic MC after signal events removed are shown in Fig.10. The corresponding plots agree

6 well.

7 Thus the generic MC sample with signal events removed is used to subtract the background in
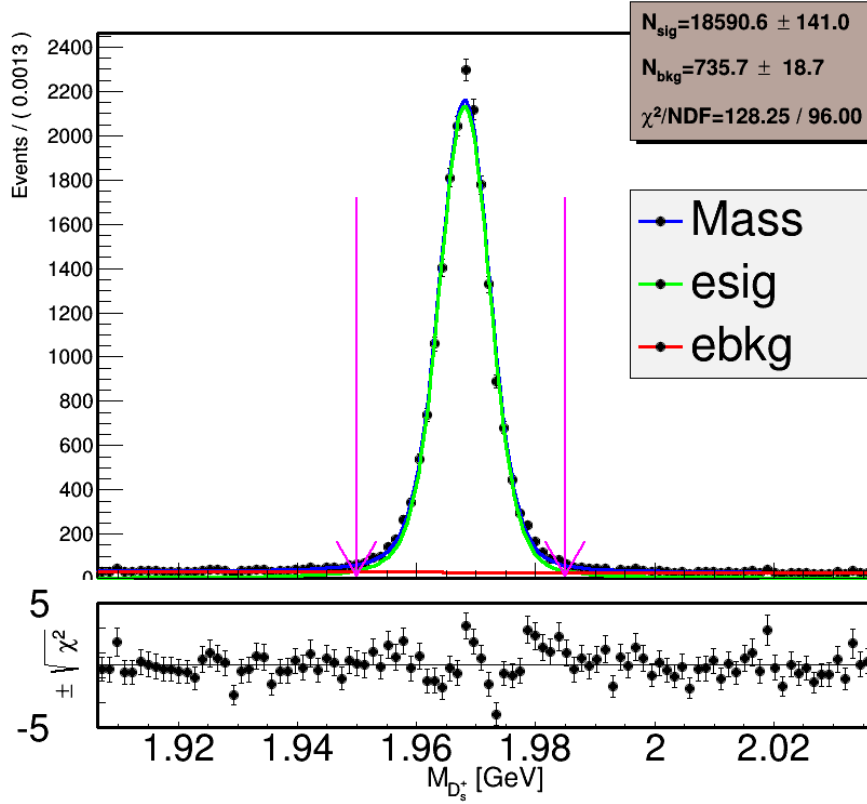
8 MIPWA.

Figure 9: The fit to the signal D_s invariant mass ($M_{D_s^2}$) spectrum after BDTG requirement, the area between the pink arrows is the signal area of the sample for MIPWA.

4.3 MIPWA method

Let N be the number of events for a given mass interval $I = [m_{K^+K^-}; m_{K^+K^-} + dm_{K^+K^-}]$. We write the corresponding angular distributions in terms of the appropriate spherical harmonic functions as

$$\frac{dN}{d\cos\theta} = 2\pi \sum_{k=0}^{L} \left\langle Y_k^0 \right\rangle Y_k^0(\cos\theta), \tag{2}$$

where $L - 2\ell_{max}$, and ℓ_{max} is the maximum orbital angular momentum quantum number required to describe the K^+K^- system at $m_{K^+K^-}$ (e.g. $\ell_{max} = 1$ for S-, P-wave description); θ is the angle between the K^+ direction in the K^+K^- system in the D_s^+ rest frame. The normalizations are such that

$$\int_{-1}^{1} Y_k^0(\cos\theta) Y_j^0(\cos\theta), d\cos\theta = \frac{\delta_{kj}}{2\pi}, \tag{3}$$

and it is assumed that the distribution $\frac{dN}{d\cos\theta}$ has been efficiency corrected and background subtracted. Using this orthogonality condition, the coefficiencies in the expansion are obtained from

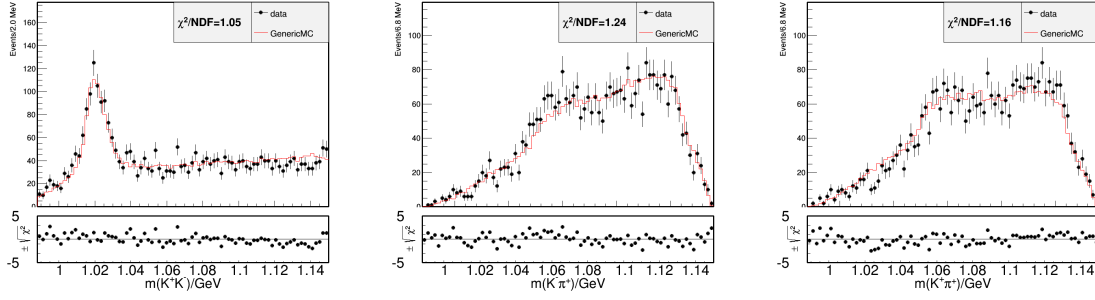$$\left\langle Y_k^0 \right\rangle = \int_{-1}^{1} \frac{dN}{d\cos\theta} d\cos\theta, \tag{4}$$

Figure 10: The projections of $m(K^+K^-)$, $m(K^-\pi^+)$, $m(K^+\pi^+)$ from "Sideband" for data(dots with error bars) and generic MC after signal events removed (red histogram) after BDTG requirement.

¹ where the intergral is given, to a good approximimation, by $\sum_{n=1}^{N} Y_k^0(\cos \theta_n)$, where θ_n is the value of θ for

² the n-th event.

³ Fig.11 shows the K^+K^- mass spectrum up to 1.15 GeV weighted by $Y_k^0(\cos \theta) = \sqrt{(2k+1)/(4\pi)}P_k(\cos \theta)$

⁴ for k=0, 1, and 2, where P_k is the Legendre ploynomial function of orkder k. These distributions are cor-

⁵ rected for efficiency and phase space, and background is subtracted using background from generic MC

⁶ after BDTG requirement.

⁷ The number of events N for the mass interval I can be expressed also in terms of the partial-wave

⁸ amplitudes describing the K^+K^- system. Assuming that only S- and P-wave amplitudes are necessary in

⁹ this limited region, we can write:

$$\frac{dN}{d\cos\theta} = 2\pi \left| S Y_0^0(\cos \theta) + PY_1^0(\cos \theta) \right|^2 . \tag{5}$$

¹⁰ By comparing Eqa. 2 and 5, we obtain

$$\sqrt{4\pi} \left\langle Y_0^0 \right\rangle = |S|^2 + |P|^2 , \\ \sqrt{4\pi} \left\langle Y_2^0 \right\rangle = \frac{2}{\sqrt{5}} |P|^2 , \tag{6}$$
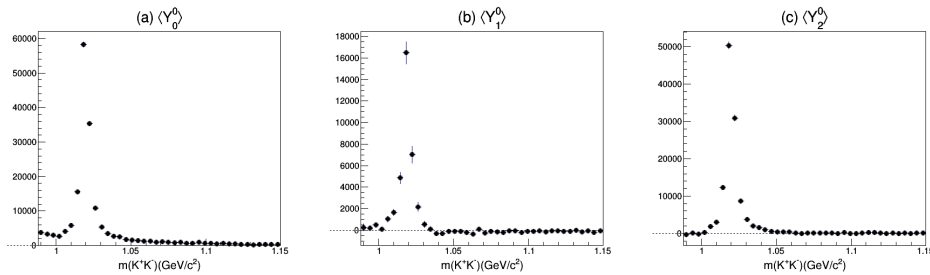

Figure 11: K^+K^- mass spectrum in the threshold region weighted by (a) Y_0^0, (b) Y_1^0 and (c) Y_2^0, corrected for efficiency and phase space, and background subtracted.

¹¹ The above system of equations can be solved in each interval of K^+K^- invariant mass for $|S|$ and $|P|$

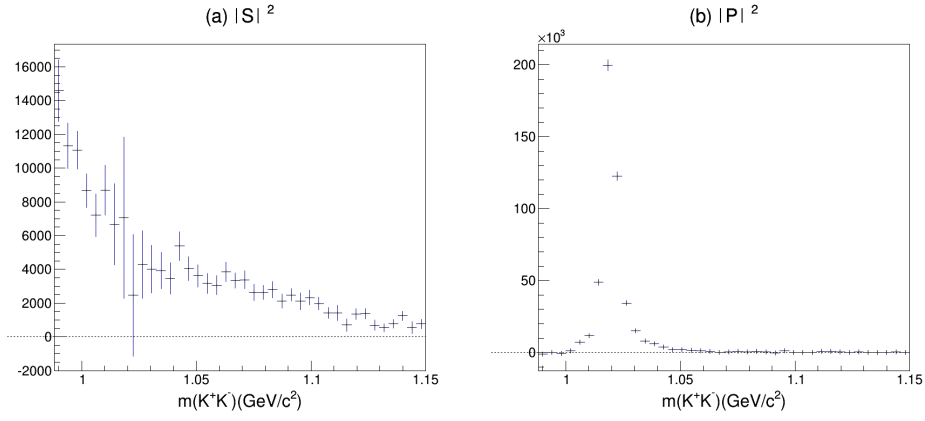¹² and the resulting distributions are shown in Fig.12.

Figure 12: Squared (a) S- and (b) P-wave amplitudes

4.4   S-wave parameterization at the K^+K^- threshold

We empirically parameterize the $f_0(980)$ with the following function:

$$A_{f_0(980)/a_0(980)} = \frac{1}{m_0^2 - m^2 - im_0\Gamma_0\rho_{KK}}, \tag{7}$$

where $\rho_{KK} = 2p/m$, and obtain the flollowing parameter values:

$$m_0 = (0.919 \pm 0.006_{stat})GeV,$$
$$\Gamma_0 = (0.272 \pm 0.020_{stat})GeV. \tag{8}$$

The errors are statistical only. The fit result are shown in Fig.13.
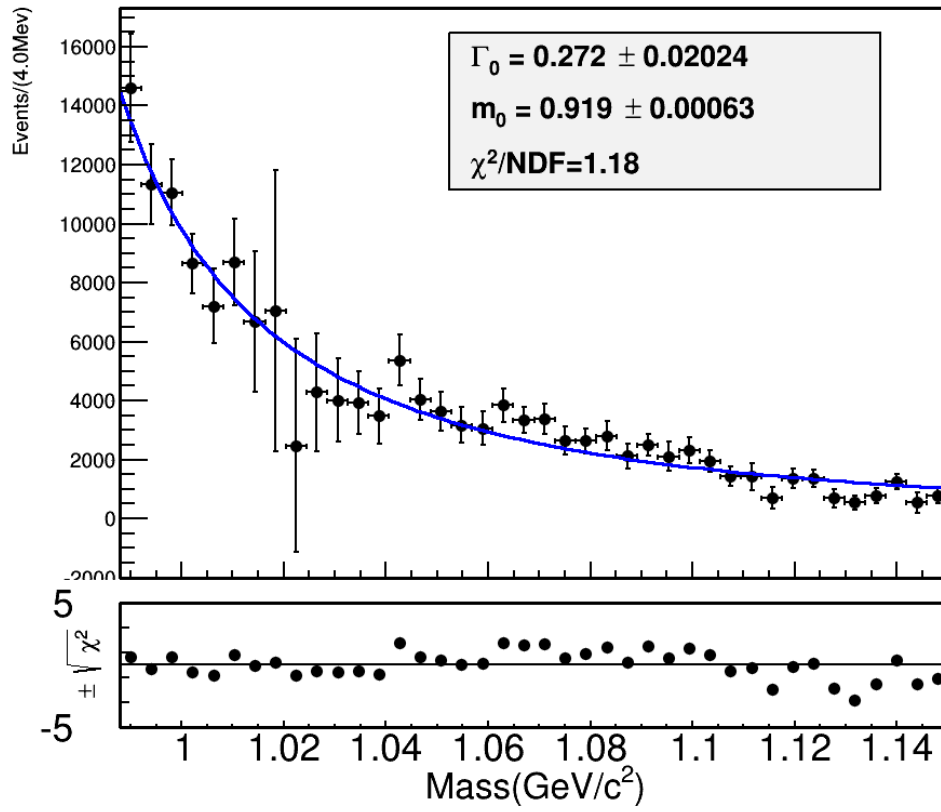
Figure 13: Fit of squared S-wave amplitudes. the curves result from the fit described in the text.

5 Amplitude Analysis

5.1 Fit Method

The method used in amplitude analysis is the same as the Ref. [4]. In this section, we briefly review the amplitude analysis method used int this analysis.

The relative magnitudes and phases of the partial waves and the mass and width of intermediate resonances are determined by a maximum-likelihood fit to the data selected. The formulas are constructed with covariant tensors [5].

Since there are three final state particles, only one possible resonant state is allowed in any intermediate process. Thus the amplitude of the nth intermediate state (A_n) is,

$$A_n = P_n S_n F_n^r F_n^D, \tag{9}$$

where S_n and $F_n^{r(D)}$ are the spin factor and the Blatt-Weisskopf barriers of the intermediate state (the D_s meson), respectively. P_n is the propagator of the intermediate resonance. For a decay process $a \rightarrow bc$, $s_{a/b/c}$ is denoted to be the invariant mass square of the particle a/b/c, and q is denoted as the magnitude of the momentum daughter particle in the rest system of a

$$q = \sqrt{\frac{(s_a + s_b + s_c)^2}{4s_a} - s_b}. \tag{10}$$

Then for $K^*(892)^0$, $\Phi(1020)^0$, $K_0^*(1430)^0$ and $f_0(1710)$, they are parameterized as a RBW,

$$P = \frac{1}{(m_0^2 - s_a) - im_0\Gamma(m)},$$
$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^3 \left(\frac{m_0}{m}\right) (F^r)^2, \tag{11}$$

where m_0 and Γ_0 are the mass and the width of the intermediate resonances, and are fixed to the PDG values [6] except the mass and the widtch of $f_0(1370)$. The mass and width of $f_0(1370)$ are fixed to Ref. [7]. $K_0^*(1430)^0$ is parameterized with Flatte formula:

$$P_{K_0^*(1430)^0} = \frac{1}{M^2 - s - i(g_1\rho_{K\pi}(s) + g_2\rho_{\eta'K}(s))}, \tag{12}$$

where s is the $K^-\pi^+$ invariant mass squared, $\rho_{K\pi}(s)$ and $\rho_{\eta'K}(s)$ are Lorentz invariant PHSP factor, and $g_{1,2}$ are coupling constants to the corresponding final state. The parameters of $K_0^*(1430)^0$ are fixed to values measured by CLEO [8]. For resonance $f_0(980)$ & $a_0(980)$, the RBW form 7 is used to describe the propagator and the values of parameters are fixed to the values in 8.

The Blatt-Weisskopf barriers are given by

$$F_n = 1, (S\,wave),$$
$$F_n = \sqrt{\frac{z_0^2 + 1}{z^2 + 1}}, (P\,wave),$$
$$F_n = \sqrt{\frac{z_0^4 + 3z_0^2 + 9}{z^4 + 3z^2 + 9}}, (D\,wave), \tag{13}$$

1 where z and z_0 are qR and q_0R, respectively. The R is the effective radius of the intermediate state or D_s

2 meson and set to $3.0 GeV^{-1}$ for intermediate states and $5.0 GeV^{-1}$ for D_s meson [4], respectively.

6 Branching Fraction Measurements

Branching Fraction Measurement

7 Summary

Summary

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Appendices

A   Substract of the Secondary proton

A sizable of (anti-)proton candidates are produced from the electron/positron beam or particles in final states interacting with the MDC inner wall or residual gas inside beam-pipe, especially for proton. Here, we take the decay mode $\Lambda_c^+ \rightarrow p\eta(\gamma\gamma)$ as an example. Fig. **??** shows the compared V_r distributions of proton and anti-proton in different samples. In this analysis, we use the scaled inclusive MC to optimize the tight requirement with the $/\sqrt{S+B}$ curve. Fig. 1 shows the efficiency distribution and $S/\sqrt{S+B}$ distribution with different V_r requirement. Where, S represents signal events, and B is scaled inclusive MC sample.