CSS 415: Methods of Mathematical Statistics I

Fall 2024

Midterm Examination II

Name: _____ Student ID: _____

Notes:

1. The exam is online, no proctoring needed, but must be your independent work. Course policies are enforced.
2. All problems carry equal marks.
3. Neat handwriting is required.
4. Except stated clearly otherwise, each answer must be justified and NO credits will be given for final answer ONLY.
5. Each final answer should be emphasized (boxed or underlined).
6. Only pertinent solutions should be written on the paper.
7. Submission should be in one PDF file uploaded to Canvas.

Problem 1:	/10
Problem 2:	/10
Problem 3:	/10
Problem 4:	/10
Problem 5:	/10
Problem 6:	/10
Total:	/60 = /200

1. Question:

(Section 3.2, Q10 on page 177)

In a Couette flow, two large flat plates lie one on top of another, separated by a thin layer of fluid. If a shear stress is applied to the top plate, the viscosity of the fluid produces motion in the bottom plate as well. The velocity V in the top plate relative to the bottom plate is given by

$$V = \frac{\tau h}{\mu},$$

where τ is the shear stress applied to the top plate, h is the thickness of the fluid layer, and μ is the viscosity of the fluid. Assume that $\mu = 1.49$ Pa·s and $h = 10$ mm, both with negligible uncertainty.

(a) Suppose that $\tau = 30.0 \pm 0.1$Pa is known. Estimate V, and find the uncertainty in the estimate.

(b) If it is desired to estimate V with an uncertainty of 0.2 mm/s, what must be the uncertainty in τ?

Notes: This problem tests

- Compute uncertainties in linear combinations of independent measurements

2. Question:

(Section 3.3, Q4 on page 184) The velocity V of sound in air at temperature T is given by the equation:

$$V = 20.04\sqrt{T},$$

where T is measured in kelvins (K) and V is in meters per second (m/s). Assume that $T = 300 \pm 0.4$ K. Estimate V and find the uncertainty in the estimate.

Notes: This problem tests

- to estimate the uncertainty in a nonlinear function (one variable) of a measurement

3. Question:

(Section 3.4, Q2 on page 192)

The volume of a cone is given by $V = \frac{\pi r^2 h}{3}$, where r is the radius of the base and h is the height. Assume the height is measured to be $h = 6.00 \pm 0.01$ cm and the radius is $r = 5.00 \pm 0.02$ cm.

(a) Estimate the volume of the cone, and find the uncertainty in the estimate.

(b) Which would provide a greater reduction in the uncertainty in V: reducing the uncertainty in h to 0.005 cm or reducing the uncertainty in r to 0.01 cm?

Notes: This problem tests

- to estimate the uncertainty in a nonlinear function (two variables) of a measurement

4. Question:

(Section 4.2, Q4 on page 212) At a certain airport, 75% of the flights arrive on time. A sample of 10 flights is studied.

(a) Find the probability that all 10 of the flights were on time.

(b) Find the probability that exactly eight of the flights were on time.

(c) Find the probability that eight or more of the flights were on time.

Notes: This problem tests

- Binomial distribution

5. Question:

(Section 4.2, Q24 on page 255) The molarity of a solute in solution is defined as the number of moles of solute per liter of solution (1 mole = 6.02×10^{23} molecules).

If X is the molarity of a solution of sodium chloride (NaCl), and Y is the molarity of a solution of sodium carbonate (Na_2CO_3), the molarity of sodium ion (Na^+) in a solution made of equal parts NaCl and Na_2CO_3 is given by:

$$M = 0.5X + Y.$$

Assume X and Y are independent and normally distributed, with X having a mean of 0.450 and a standard deviation of 0.050, and Y having a mean of 0.250 and a standard deviation of 0.025.

(a) What is the distribution of M?

(b) Find $P(M > 0.5)$.

Notes: This problem tests

- Normal distribution
- Linear combination of independent random variables

6. Question:

(Section 4.11, Q4 on page 301) Among all monthly bills from a certain credit card company, the mean amount billed was \$485 and the standard deviation was \$300. In addition, for 15% of the bills, the amount billed was greater than \$1000. A sample of 900 bills is drawn.

(a) What is the probability that the average amount billed on the sample bills is greater than \$500?

(b) What is the probability that more than 150 of the sampled bills are for amounts greater than \$1000?

Notes: This problem tests

- Central limit theorem
- Continuity correction