Design and Analysis
of Algorithms I

Introduction

Karatsuba Multiplication

Example

$x = 5678$ ($a = 56$, $b = 78$)

$y = 1234$ ($c = 12$, $d = 34$)

Step 1: compute $a \cdot c = 672$

Step 2: Compute $b \cdot d = 2652$

Step 3: Compute $(a+b)(c+d) = 134 \cdot 46 = 6164$

Step 4: Compute ③ − ② − ① $= 2840$

Step 5:

$$\begin{array}{r} 6720000 \\ 2652 \\ 284000 \\ \hline 7006652 \end{array} = (1234)(5678)$$

A Recursive Algorithm

Write $x = 10^{n/2}a + b$ and $y = 10^{n/2}c + d$

Where a,b,c,d are n/2-digit numbers.

[example: a=56, b=78, c=12, d=34]

Then $x.y = (10^{n/2}a + b)(10^{n/2}c + d)$

$$= (10^n ac + 10^{n/2}(ad + bc) + bd \qquad (*)$$

Idea : recursively compute ac, ad, bc, bd, then compute (*) in the obvious way

Simple Base Case Omitted

Karatsuba Multiplication

$$x.y = (10^n ac + 10^{n/2}(ad + bc) + bd$$

1. Recursively compute ac
2. Recursively compute bd
3. Recursively compute (a+b)(c+d) = ac+bd+ad+bc

Gauss' Trick : (3) – (1) – (2) = ad + bc

Upshot : Only need 3 recursive multiplications (and some additions)

Q : which is the fastest algorithm ?

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