

CZ4041/CE4041: Machine Learning

Lesson 3: Naïve Bayes Classifiers

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Acknowledgements: some tables are adapted from the lecture notes of the book
“Introduction to Data Mining”

Bayesian Classifiers: Recall

- To learn a prediction function via $P(y|\mathbf{x})$ using Bayes rule

$$P(y = c|\mathbf{x}) = \frac{P(\mathbf{x}, y = c)}{P(\mathbf{x})} = \frac{P(\mathbf{x}|y = c)P(y = c)}{P(\mathbf{x})}$$

- Make predictions based on maximum posterior

$$y^* = c^* \text{ if } c^* = \arg \max_c \frac{P(\mathbf{x}|y = c)P(y = c)}{P(\mathbf{x})} \quad \text{the 0/1 loss}$$

$$y^* = c^* \text{ if } c^* = \arg \max_c \boxed{P(\mathbf{x}|y = c)} \boxed{P(y = c)}$$

Easy to estimate

Still difficult to estimate as \mathbf{x} contains many input variables, some are discrete, and others are continuous

Naïve Bayes Classifiers

- How to estimate $P(\mathbf{x}|y)$ from the training data
- Assume that the features are conditionally independent given the class label:

$$P(\mathbf{x}|y = c) = \prod_{i=1}^d P(x_i|y = c)$$

where $\mathbf{x} = [x_1, x_2, \dots, x_d]$

$$P(x_1, x_2, \dots, x_d|y = c) = \prod_{i=1}^d P(x_i|y = c)$$

Independence

- Let A and B be two random variables
- A is said to be independent of B , if the following condition holds:

$$P(A, B) = \underbrace{P(A|B)}_{\text{red arrow from } P(A|B)=P(A)} \times P(B) = \underbrace{P(A)}_{\text{red arrow from } P(A|B)=P(A)} \times P(B)$$

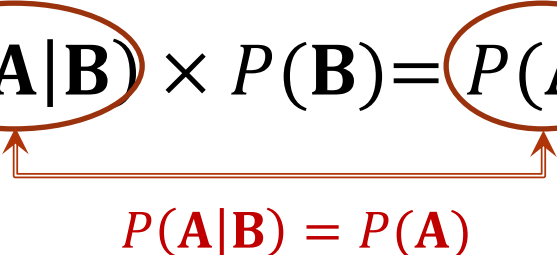
$$P(A, B) = \underbrace{P(B|A)}_{\text{red arrow from } P(B|A)=P(B)} \times P(A) = P(A) \times \underbrace{P(B)}_{\text{red arrow from } P(B|A)=P(B)}$$

- E.g., let A and B denote the results of two matches in different leagues, knowing the result of one match (e.g., the value of A) does not affect the possibility of result for the other match (i.e., the value of B)

Independence (cont.)

A more general case:

- Let **A** and **B** be two sets of random variables
- The variables in **A** are said to be independent of the variables in **B**, if the following condition holds:

$$P(\mathbf{A}, \mathbf{B}) = P(\mathbf{A}|\mathbf{B}) \times P(\mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$$


$P(\mathbf{A}|\mathbf{B}) = P(\mathbf{A})$

Conditional Independence

- Let A , B and C be three random variables
- A is said to be conditionally independent of B , given C , if the following condition holds:

$$P(A|B, C) = P(A|C)$$

Conditional Independence (cont.)

A more general case

- Let **A**, **B**, and **C** be three sets of random variables.
- The variables in **A** are said to be conditionally independent of the variables in **B**, given the variables in **C** are observed, if the following condition holds:

$$P(\mathbf{A}|\mathbf{B}, \mathbf{C}) = P(\mathbf{A}|\mathbf{C})$$

$$P(\mathbf{x}|y = c) = \prod_{i=1}^d P(x_i|y = c)$$



Conditional Independence (cont.)

- The conditional independence between **A** and **B** given **C** can also be written as follows

The diagram illustrates the derivation of conditional independence from joint probability. It starts with the joint probability $P(\mathbf{A}, \mathbf{B} | \mathbf{C})$ and shows how it can be decomposed using the product rule and the conditional independence assumption. Arrows indicate the flow of the derivation and the application of rules.

$$\begin{aligned} P(\mathbf{A}, \mathbf{B} | \mathbf{C}) &= \frac{P(\mathbf{A}, \mathbf{B}, \mathbf{C})}{P(\mathbf{C})} \quad \text{Product rule} \\ &= \frac{P(\mathbf{A}, \mathbf{B}, \mathbf{C})}{P(\mathbf{B}, \mathbf{C})} \times \frac{P(\mathbf{B}, \mathbf{C})}{P(\mathbf{C})} \\ &= P(\mathbf{A} | \mathbf{B}, \mathbf{C}) \times P(\mathbf{B} | \mathbf{C}) \quad \text{Product rule} \\ &= P(\mathbf{A} | \mathbf{C}) \times P(\mathbf{B} | \mathbf{C}) \quad \text{Conditional independence} \end{aligned}$$

Side notes:

- $\frac{P(\mathbf{A}, \mathbf{U})}{P(\mathbf{U})} = P(\mathbf{A} | \mathbf{U})$ where $\mathbf{U} = \{\mathbf{B}, \mathbf{C}\}$
- $P(\mathbf{V} | \mathbf{C}) = \frac{P(\mathbf{V}, \mathbf{C})}{P(\mathbf{C})}$ where $\mathbf{V} = \{\mathbf{A}, \mathbf{B}\}$

Naïve Bayes – Induction

- The conditional independence between **A** and **B** given **C** can also be written as follows

$$P(\mathbf{A}, \mathbf{B}|\mathbf{C}) = P(\mathbf{A}|\mathbf{C}) \times P(\mathbf{B}|\mathbf{C})$$



Naïve Bayes Classifier: $P(\mathbf{x}|y = c) = \prod_{i=1}^d P(x_i|y = c)$

Assume that the features are conditionally independent given the class label

Naïve Bayes – Induction (cont.)

$$P(\mathbf{x}|y = c) = P(x_1, x_2, \dots, x_d|y = c)$$

$$\text{Define } \mathbf{X}^{(d-1)} = [x_1, \dots, x_{d-1}]$$

$$= P(\mathbf{X}^{(d-1)}, x_d|y = c)$$

Features are conditionally independent given class label

$$= P(\mathbf{X}^{(d-1)}|y = c)P(x_d|y = c)$$

$$\text{Define } \mathbf{X}^{(d-2)} = [x_1, \dots, x_{d-2}]$$

$$= P(\mathbf{X}^{(d-2)}, x_{d-1}|y = c)P(x_d|y = c)$$

$$= P(\mathbf{X}^{(d-2)}|y = c)P(x_{d-1}|y = c)P(x_d|y = c)$$

Features are conditionally independent given class label

Recursively apply conditional independence

$$= P(x_1|y = c)P(x_2|y = c) \cdots P(x_d|y = c)$$

$$= \prod_{i=1}^d P(x_i|y = c)$$

How Naïve Bayes Classifier Work

Naïve Bayes Classifier: $P(\mathbf{x}|y = c) = \prod_{i=1}^d P(x_i|y = c)$ Easier to estimate

- To classify a test record \mathbf{x}^* , we need to compute the posteriors for each class by using

$$P(y = c|\mathbf{x}^*) = \frac{(\prod_{i=1}^d P(x_i^*|y = c))P(y = c)}{P(\mathbf{x}^*)}$$

- $P(\mathbf{x}^*)$ is constant for each class c , it is sufficient to choose the class that maximizes the numerator term

$$\left(\prod_{i=1}^d P(x_i^*|y = c) \right) P(y = c)$$

Illustrative Example

- Consider the problem of predicting whether a loan applicant will repay his/her loan obligation (no cheat) or become delinquent (cheat).

Predefined categories

Example



Example (cont.)

<i>Tid</i>	Home Owner	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training data



$P(y|\mathbf{x})$

Home Owner	Marital Status	Taxable Income	Cheat
No	Married	120K	?

Test application \mathbf{x}^*

- To classify the application, we need to compute the posterior probabilities $P(\text{Yes}|\mathbf{x}^*)$ and $P(\text{No}|\mathbf{x}^*)$
- If $P(\text{Yes}|\mathbf{x}^*) > P(\text{No}|\mathbf{x}^*)$ then classified as Yes
- Otherwise classified No

Estimate Priors

- Class:

#instances in class c

\downarrow

$$P(y = c) = \frac{|y = c|}{N}$$

\uparrow

- e.g.,

#training instances

$$P(\text{No}) = 0.7$$

$$P(\text{Yes}) = 0.3$$

<i>Tid</i>	Home Owner	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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9	No	Married	75K	No
10	No	Single	90K	Yes

Estimate Conditional Probabilities for Discrete Features

#instances in class c , whose values of feature x_i are k

$$P(x_i = k | y = c) = \frac{|(x_i = k) \wedge (y = c)|}{|y = c|}$$

A specific value k of the feature x_i

$$\begin{aligned} &P(\text{Status} = \text{Married} | \text{Cheat} = \text{No}) \\ &= \frac{\#(\text{Status} = \text{Married} \wedge \text{Cheat} = \text{No})}{\#(\text{Cheat} = \text{No})} = \frac{4}{7} \end{aligned}$$

$$\begin{aligned} &P(\text{Home Owner} = \text{Yes} | \text{Cheat} = \text{Yes}) \\ &= \frac{\#(\text{Home Owner} = \text{Yes} \wedge \text{Cheat} = \text{Yes})}{\#(\text{Cheat} = \text{Yes})} = \frac{0}{3} = 0 \end{aligned}$$

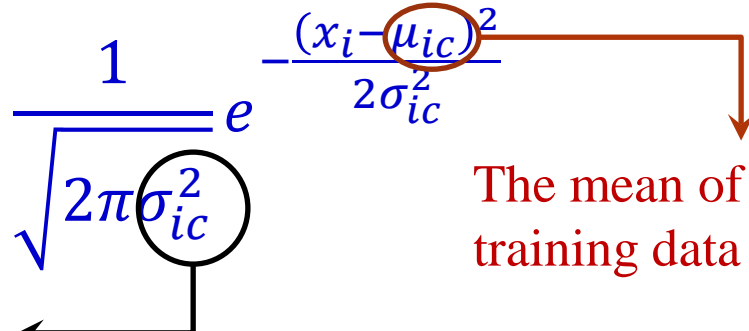
<i>Tid</i>	Home Owner	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
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5	No	Divorced	95K	Yes
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Estimate Conditional Probabilities for Continuous Features

- For continuous features:
 - Probability density estimation (more details will be introduced in the 2nd half of the semester):
 - Assume the values of a feature given a class label follows a Gaussian distribution, i.e., assume $P(x_i|y = c)$ is a Gaussian distribution
 - Use training data in the class c to estimate parameters of distribution (e.g., mean μ and variance σ^2)
 - Once probability density function is known, we can use it to estimate the conditional probability

Estimate Conditional Probabilities for Continuous Features (cont.)

- For each class c , assume values of the feature x_i follow a Gaussian distribution:

$$P(x_i | y = c) = \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(x_i - \mu_{ic})^2}{2\sigma_{ic}^2}}$$


The variance of x_i of the training data in class c

The mean of x_i of the training data in class c

- Suppose there are N_c instances in class c , then

$$\mu_{ic} = \frac{1}{N_c} \sum_{j=1}^{N_c} x_{ij}$$

$$\sigma_{ic}^2 = \frac{1}{N_c - 1} \sum_{j=1}^{N_c} (x_{ij} - \mu_{ic})^2$$

Value of feature x_i of the j -th training data in class c

Estimate Conditional Probabilities for Continuous Features (cont.)

$$P(x_i|y = c) = \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(x_i - \mu_{ic})^2}{2\sigma_{ic}^2}}$$

- For {Income, Cheat = No}:
 - sample mean = 110
 - sample variance = 2975
(standard deviation = 54.54)

Tid	Home Owner	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

$$P(\text{Income}|\text{Cheat} = \text{No}) = \frac{1}{\sqrt{2\pi} \times 54.54} e^{-\frac{(x_i - 110)^2}{2 \times 2975}}$$

Income

Estimate Conditional Probabilities for Continuous Features (cont.)

- The estimated Gaussian distribution for {Income, Cheat = No}:

A function of x_i

$$P(\text{Income} | \text{Cheat} = \text{No}) = \frac{1}{\sqrt{2\pi} \times 54.54} e^{-\frac{(x_i - 110)^2}{2 \times 2975}}$$

Tid	Home Owner	Marital Status	Taxable Income	Cheat
4	Yes	Married	120K	No

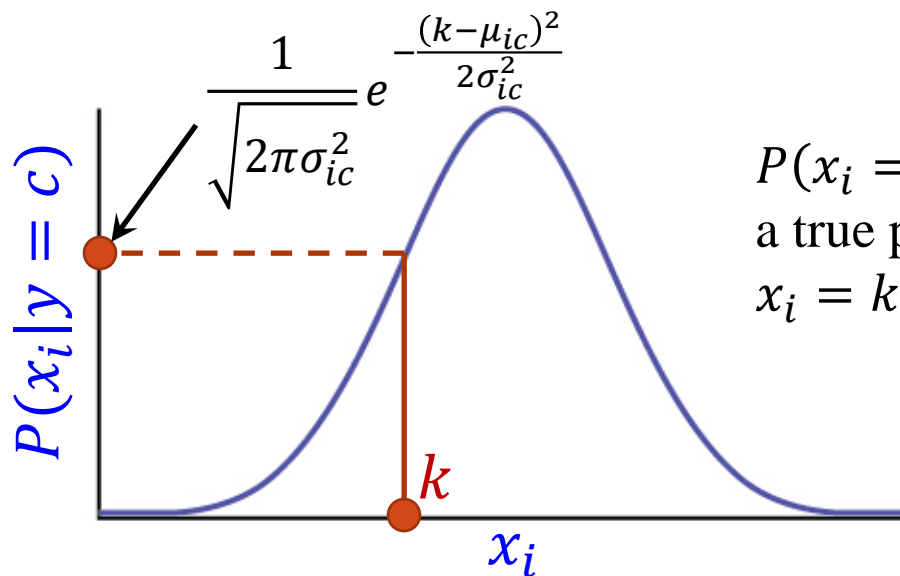
$$P(\text{Income} = 120 | \text{No}) = \frac{1}{\sqrt{2\pi} \times 54.54} e^{-\frac{(120 - 110)^2}{2 \times 2975}} = 0.0072$$

Note: in practice, 0.0072 can be used as to approximate the conditional probability, but in theory it is not a true probability

Additional Notes

Probability density function $P(x_i|y = c) = \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(x_i - \mu_{ic})^2}{2\sigma_{ic}^2}}$

- The probability density function is continuous, the probability is defined as the area under the curve of the probability density function

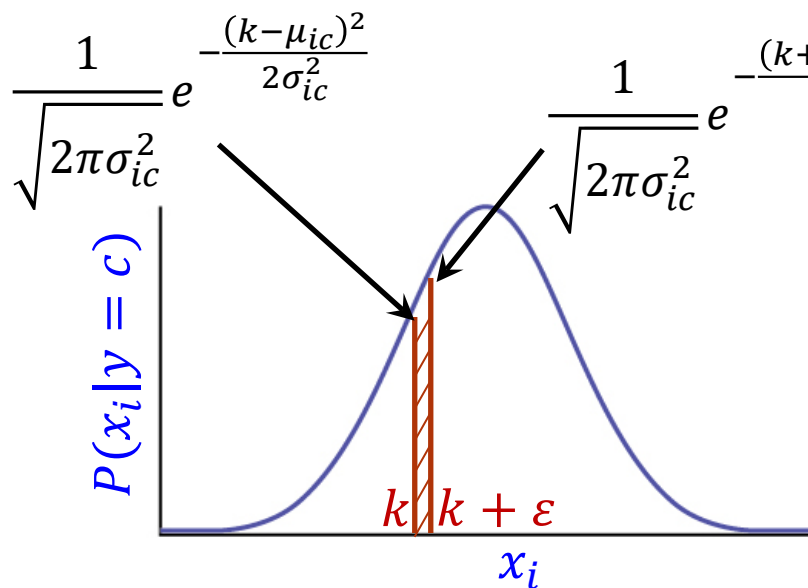


$P(x_i = k|y = c)$ is not a true probability that $x_i = k$ for class c

Additional Notes (cont.)

- Instead, we should compute

$$P(k \leq x_i \leq k + \underbrace{\varepsilon}_{\text{Small positive constant}} | y = c) = \int_k^{k+\varepsilon} \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(x_i - \mu_{ic})^2}{2\sigma_{ic}^2}} dx_i$$



$$\text{Assume } \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(k - \mu_{ic})^2}{2\sigma_{ic}^2}} \approx \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(k + \varepsilon - \mu_{ic})^2}{2\sigma_{ic}^2}}$$

$$\approx \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(k - \mu_{ic})^2}{2\sigma_{ic}^2}} \times \varepsilon$$

Additional Notes (cont.)

- Since ε appears as a constant multiplicative factor for each class, it cancels out when comparing posterior probabilities $P(y = c|\mathbf{x})$ for each class
- E.g., consider binary classification and instance is represented by a single feature of continuous values

$$P(y = 0|x = k) \quad \text{vs.} \quad P(y = 1|x = k)$$



$$P(x = k|y = 0)P(y = 0) \quad \text{vs.} \quad P(x = k|y = 1)P(y = 1)$$



$$\frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{(k-\mu_0)^2}{2\sigma_0^2}} \times \cancel{\varepsilon} \times P(y = 0) \quad \text{vs.} \quad \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(k-\mu_1)^2}{2\sigma_1^2}} \times \cancel{\varepsilon} \times P(y = 1)$$

Additional Notes (cont.)

- Therefore, we can still apply the following equation to approximate the probability of $x_i = k$ for class c

$$P(x_i = k|y = c) = \frac{1}{\sqrt{2\pi\sigma_{ic}^2}} e^{-\frac{(k-\mu_{ic})^2}{2\sigma_{ic}^2}}$$

Example of Naïve Bayes Classifier

Naïve Bayes Classifier:

$$P(\text{HomO}=\text{Yes}|\text{No}) = 3/7$$

$$P(\text{HomO}=\text{No}|\text{No}) = 4/7$$

$$P(\text{HomO}=\text{Yes}|\text{Yes}) = 0$$

$$P(\text{HomO}=\text{No}|\text{Yes}) = 1$$

$$P(\text{Marital Status} = \text{Single}|\text{No}) = 2/7$$

$$P(\text{Marital Status} = \text{Divorced}|\text{No}) = 1/7$$

$$P(\text{Marital Status} = \text{Married}|\text{No}) = 4/7$$

$$P(\text{Marital Status} = \text{Single}|\text{Yes}) = 2/3$$

$$P(\text{Marital Status} = \text{Divorced}|\text{Yes}) = 1/3$$

$$P(\text{Marital Status} = \text{Married}|\text{Yes}) = 0$$

For taxable income:

If class=No: sample mean=110

sample variance=2975

If class=Yes: sample mean=90

sample variance=25

$$P(\text{Class} = \text{No}) = 7/10$$

$$P(\text{Class} = \text{Yes}) = 3/10$$

<i>Tid</i>	Home Owner	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

$$P(\mathbf{x}|y = c) = \prod_{i=1}^d P(x_i|y = c)$$

Test example:

Home Owner	Marital Status	Taxable Income	Cheat
No	Married	120K	?

$$P(\mathbf{x}^* | y = c) = \prod_{i=1}^d P(x_i^* | y = c)$$

$$P(\text{HomO}=\text{Yes}|\text{No}) = 3/7$$

$$P(\text{HomO}=\text{No}|\text{No}) = 4/7$$

$$P(\text{HomO}=\text{Yes}|\text{Yes}) = 0$$

$$P(\text{HomO}=\text{No}|\text{Yes}) = 1$$

$$P(\text{Marital Status} = \text{Single}|\text{No}) = 2/7$$

$$P(\text{Marital Status} = \text{Divorced}|\text{No}) = 1/7$$

$$P(\text{Marital Status} = \text{Married}|\text{No}) = 4/7$$

$$P(\text{Marital Status} = \text{Single}|\text{Yes}) = 2/3$$

$$P(\text{Marital Status} = \text{Divorced}|\text{Yes}) = 1/3$$

$$P(\text{Marital Status} = \text{Married}|\text{Yes}) = 0$$

For taxable income:

If class=No: sample mean=110

sample variance=2975

If class=Yes: sample mean=90

sample variance=25

$$P(\text{Class} = \text{No}) = 7/10$$

$$P(\text{Class} = \text{Yes}) = 3/10$$

$$\begin{aligned} P(\mathbf{x}^* | \text{Class}=\text{No}) &= P(\text{HomO}=\text{No} | \text{Class}=\text{No}) \\ &\quad \times P(\text{Status}=\text{Married} | \text{Class}=\text{No}) \\ &\quad \times P(\text{Income}=120\text{K} | \text{Class}=\text{No}) \\ &= \frac{4}{7} \times \frac{4}{7} \times 0.0072 = 0.0024 \end{aligned}$$

$$\begin{aligned} P(\mathbf{x}^* | \text{Class}=\text{Yes}) &= P(\text{HomO}=\text{No} | \text{Class}=\text{Yes}) \\ &\quad \times P(\text{Status}=\text{Married} | \text{Class}=\text{Yes}) \\ &\quad \times P(\text{Income}=120\text{K} | \text{Class}=\text{Yes}) \\ &= 1 \times 0 \times (1.2 \times 10^{-9}) = 0 \end{aligned}$$

one of the conditional probability is 0, the entire expression is 0

$$P(\mathbf{x}^* | \text{No}) \times P(\text{No}) = 0.0024 \times 0.7 = 0.00168$$

$$> P(\mathbf{x}^* | \text{Yes}) \times P(\text{Yes}) = 0 \times 0.3 = 0$$

Therefore $P(\text{No} | \mathbf{x}^*) > P(\text{Yes} | \mathbf{x}^*)$ **Class = No**

Laplace Estimate

- Alternative probability estimation (discrete features):

Original: $P(x_i = k|y = c) = \frac{|(x_i = k) \wedge (y = c)|}{|y = c|}$

Laplace: $P(x_i = k|y = c) = \frac{|(x_i = k) \wedge (y = c)| + 1}{|y = c| + n_i}$

#distinct values of x_i

$$P(\text{Married}|\text{Yes}) = \frac{\#(\text{Married} \wedge \text{Yes})}{\#(\text{Yes})} = \frac{0}{3}$$

$$P(\text{Married}|\text{Yes}) = \frac{\#(\text{Married} \wedge \text{Yes}) + 1}{\#(\text{Yes}) + 3} = \frac{1}{6}$$

The same to $P(\text{Single}|\text{Yes})$ and $P(\text{Divorced}|\text{Yes})$

Extreme case - no training data:

$$P(\text{Single}|\text{Yes}) = P(\text{Married}|\text{Yes}) = P(\text{Divorced}|\text{Yes}) = \frac{1}{3}$$

Tid	Home Owner	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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8	No	Single	85K	Yes
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10	No	Single	90K	Yes

M-estimate

- A more general estimation:

Original: $P(x_i = k|y = c) = \frac{|(x_i = k) \wedge (y = c)|}{|y = c|}$

M-estimate: $P(x_i = k|y = c) = \frac{|(x_i = k) \wedge (y = c)| + m \times p}{|y = c| + m}$

e.g., if prior information of $P(x_i = k|y = c)$ is available, then we can set p as the prior

User-specified parameters

For example, based on domain knowledge, you have the prior information:

Domain knowledge,
not learned from data

$$\tilde{P}(\text{Single}|\text{Yes}) = \frac{1}{2} \quad \tilde{P}(\text{Divorced}|\text{Yes}) = \frac{1}{3} \quad \tilde{P}(\text{Married}|\text{Yes}) = \frac{1}{6}$$

Extreme case - no training data:

$$P(\text{Single}|\text{Yes}) = \frac{\#(\text{Single} \wedge \text{Yes}) + m \times \tilde{P}(\text{Single}|\text{Yes})}{\#(\text{Yes}) + m} = \frac{m \times \tilde{P}(\text{Single}|\text{Yes})}{m} = \tilde{P}(\text{Single}|\text{Yes})$$

Home Owner	Marital Status	Taxable Income	Cheat
No	Married	120K	?

$$m = 3$$

$p = 1/3$ for all discrete features of class **Yes**

$p = 2/3$ for all discrete features of class **No**

$$P(\text{HomO}=\text{Yes}|\text{No}) = 3/7$$

$$P(\text{HomO}=\text{No}|\text{No}) = 4/7$$

$$P(\text{HomO}=\text{Yes}|\text{Yes}) = 0/3$$

$$P(\text{HomO}=\text{No}|\text{Yes}) = 1$$

$$P(\text{Marital Status} = \text{Single}|\text{No}) = 2/7$$

$$P(\text{Marital Status} = \text{Divorced}|\text{No}) = 1/7$$

$$P(\text{Marital Status} = \text{Married}|\text{No}) = 4/7$$

$$P(\text{Marital Status} = \text{Single}|\text{Yes}) = 2/3$$

$$P(\text{Marital Status} = \text{Divorced}|\text{Yes}) = 1/3$$

$$P(\text{Marital Status} = \text{Married}|\text{Yes}) = 0/3$$

For taxable income:

If class=No: sample mean=110
sample variance=2975

If class=Yes: sample mean=90
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$$P(\text{Class} = \text{No}) = 7/10$$

$$P(\text{Class} = \text{Yes}) = 3/10$$

$$P(\text{HomO}=\text{Yes}|\text{No}) = ?$$

$$P(\text{HomO}=\text{No}|\text{No}) = ?$$

$$P(\text{HomO}=\text{Yes}|\text{Yes}) = ?$$

$$P(\text{HomO}=\text{No}|\text{Yes}) = ?$$

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$$P(\text{Marital Status} = \text{Divorced}|\text{Yes}) = ?$$

$$P(\text{Marital Status} = \text{Married}|\text{Yes}) = ?$$

M-estimate

$$P(x_i = k|y = c) = \frac{|(x_i = k) \wedge (y = c)| + m \times p}{|y = c| + m}$$

$$P(\mathbf{x}^*|\text{Class} = \text{No}) = ? \quad P(\mathbf{x}^*|\text{Class} = \text{Yes}) = ?$$



Tutorial

Implementation using scikit-learn

- API: `sklearn.naive_bayes`: Naive Bayes

https://scikit-learn.org/stable/modules/classes.html#module-sklearn.naive_bayes

`sklearn.naive_bayes`: Naive Bayes

The `sklearn.naive_bayes` module implements Naive Bayes algorithms. These are supervised learning methods based on applying Bayes' theorem with strong (naive) feature independence assumptions.

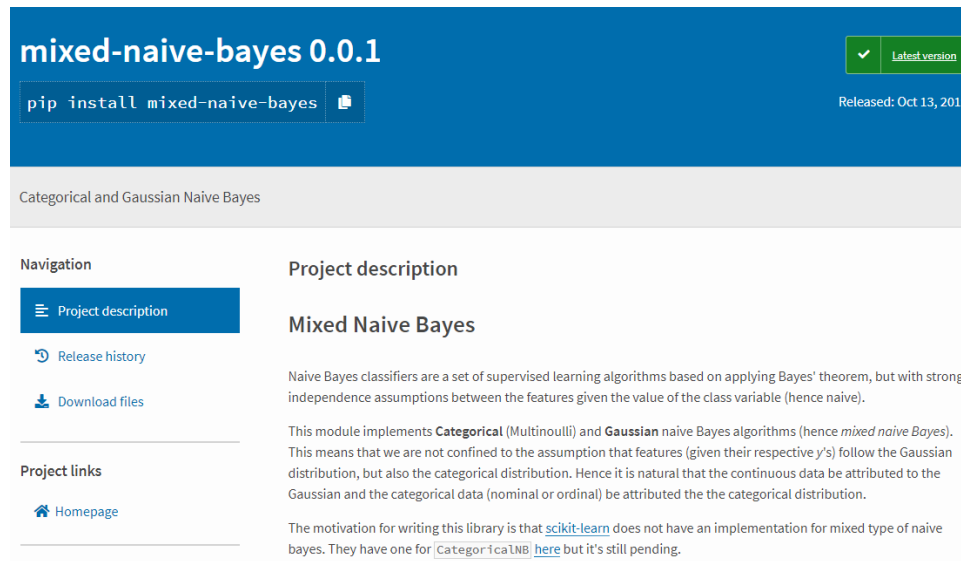
User guide: See the [Naive Bayes](#) section for further details.

<code>naive_bayes.BernoulliNB(* [, alpha, ...])</code>	Naive Bayes classifier for multivariate Bernoulli models.
<code>naive_bayes.CategoricalNB(* [, alpha, ...])</code>	Naive Bayes classifier for categorical features
<code>naive_bayes.ComplementNB(* [, alpha, ...])</code>	The Complement Naive Bayes classifier described in Rennie et al.
<code>naive_bayes.GaussianNB(* [, priors, ...])</code>	Gaussian Naive Bayes (GaussianNB)
<code>naive_bayes.MultinomialNB(* [, alpha, ...])</code>	Naive Bayes classifier for multinomial models

Documentation: https://scikit-learn.org/stable/modules/naive_bayes.html

Mixed Naïve Bayes Implementation

<https://pypi.org/project/mixed-naive-bayes/#installation>



The screenshot shows the PyPI page for the 'mixed-naive-bayes' package, version 0.0.1. The header is blue with the package name and version. Below it, there's a green button with a checkmark and the text 'Latest version'. To the right, it says 'Released: Oct 13, 2019'. A code box contains the command 'pip install mixed-naive-bayes'. The main content area has a navigation sidebar on the left with links for 'Project description', 'Release history', and 'Download files'. The main text area is titled 'Mixed Naive Bayes' and contains a description of the package, its features, and a link to the source code.

mixed-naive-bayes 0.0.1 ✓ Latest version
Released: Oct 13, 2019

`pip install mixed-naive-bayes`

Categorical and Gaussian Naive Bayes

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Project description

Mixed Naive Bayes

Naive Bayes classifiers are a set of supervised learning algorithms based on applying Bayes' theorem, but with strong independence assumptions between the features given the value of the class variable (hence naive).

This module implements **Categorical** (Multinoulli) and **Gaussian** naive Bayes algorithms (hence *mixed naive Bayes*). This means that we are not confined to the assumption that features (given their respective y's) follow the Gaussian distribution, but also the categorical distribution. Hence it is natural that the continuous data be attributed to the Gaussian and the categorical data (nominal or ordinal) be attributed the the categorical distribution.

The motivation for writing this library is that [scikit-learn](#) does not have an implementation for mixed type of naive bayes. They have one for [CategoricalNB](#) [here](#) but it's still pending.

```
>>> from mixed_naive_bayes import MixedNB
```

```
>>> nbC = MixedNB(categorical_features=[0,1,3])
```

```
>>> nbC.fit(X, y)
```

```
>>> nbC.predict(X)
```

Specify which columns
are categorical features

Naïve Bayes Classifier: Summary

- Based on a very strong assumption on conditional independence: all the input features are independent to each other given a class label

$$P(\mathbf{x}|y = c) = \prod_{i=1}^d P(x_i|y = c)$$

- Computationally efficient
- Independence assumption may not hold in practice (for most of time), that is why it is called “naïve”
 - Correlated features can degrade the performance
 - To be continued ...

Thank you!