Matematiskt korrekt beräkning av komplexitet

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1.

$$
\begin{aligned}
\sum_{i=0}^{n}\sum_{j=i}^{n}\sum_{k=i}^{j}1 &= \sum_{i=0}^{n}\sum_{j=i}^{n}\sum_{k=1}^{j-i+1} 1 \\
&= \sum_{i=0}^{n}\sum_{j=i}^{n}(j-i+1) \\
&= \sum_{i=0}^{n}\sum_{j=0}^{n-i}(j+1) \\
&= \sum_{i=0}^{n}\sum_{j=1}^{n-i+1} j \\
&= \sum_{i=0}^{n}\frac{(n-i+1)(n-i+2)}{2} \\
&= \frac{1}{2}\sum_{i=0}^{n}(n^2+i^2-2ni+3n-3i+2) \\
&= \frac{1}{2}\sum_{i=1}^{n+1}(n^2+(i-1)^2-2n(i-1)+3n-3(i-1)+2) \\
&= \frac{1}{2}\sum_{i=1}^{n+1}(n^2+5n+i^2-5i-2ni+6) \\
&= \frac{1}{2}((n+1)(n^2+5n+6)-\frac{1}{2}(n^2+3n+2)(2n+5)+\sum_{i=1}^{n+1}i^2) \\
&= \frac{1}{2}(\frac{n^2}{2}+\frac{3n}{2}+n+1+\frac{n^3}{3}+\frac{3n^2}{2}+\frac{13n}{6}+1) \\
&= \frac{1}{6}n^3+n^2+\frac{11}{6}n+1 \\
&= O(n^3)
\end{aligned}
$$

2.

$$\begin{aligned}
\sum_{i=0}^{n}\sum_{j=i}^{n}1 &= \sum_{i=0}^{n}\sum_{j=1}^{n-i+1}1 \\
&= \sum_{i=0}^{n}(n-i+1) \\
&= (n+1)(n+1)-\sum_{i=0}^{n}i \\
&= n^2+2n+1-\frac{n^2}{2}-\frac{n}{2} \\
&= \frac{n^2}{2}+\frac{3n}{2}+1 \\
&= O(n^2)
\end{aligned}$$

3.

$$\begin{aligned}
\sum_{i=0}^{n}1 &= \sum_{i=1}^{n+1}1 \\
&= n+1 \\
&= O(n)
\end{aligned}$$

Referens

Wikipedia (`http://en.wikipedia.org/wiki/Sums#Some_summations_of_polynomial_expressions`) för hur man löser en kvadratisk summa.