LI 511: Computational Models of Sound Change

James Kirby and Morgan Sonderegger

16 July 2013

Learning from parents & grandparents (Sarah & Jevon)

- Single teacher: Selected randomly from G_{n-1} and G_{n-2}
- Multiple teacher: Draw examples from all of G_{n-1} and G_{n-2}

- Single teacher:
  - Mean evolves more slowly with grandparents
  - Variance evolves similarly `` ``

Learning from parents & grandparents (Sarah & Jevon)

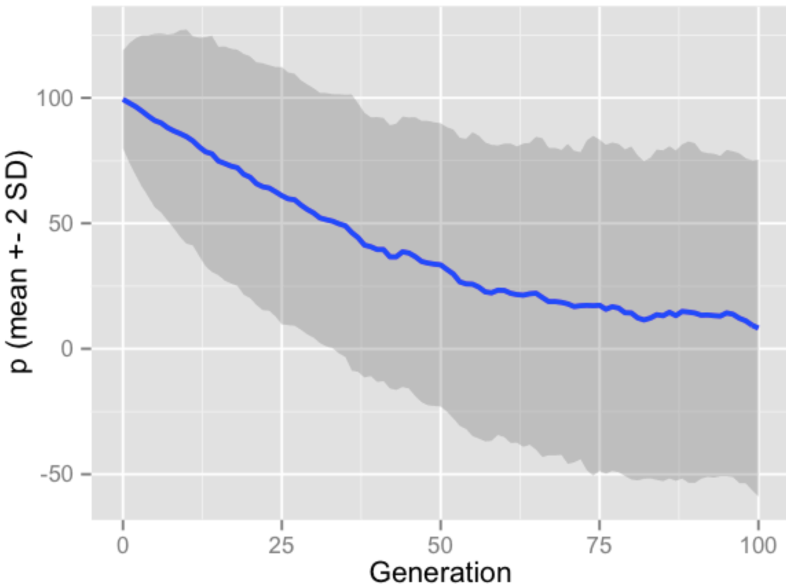
- Single teacher: Selected randomly from G_{n-1} and G_{n-2}
- Multiple teacher: Draw examples from all of G_{n-1} and G_{n-2}

- Single teacher:
  - Mean evolves more slowly with grandparents
  - Variance evolves similarly `` ``

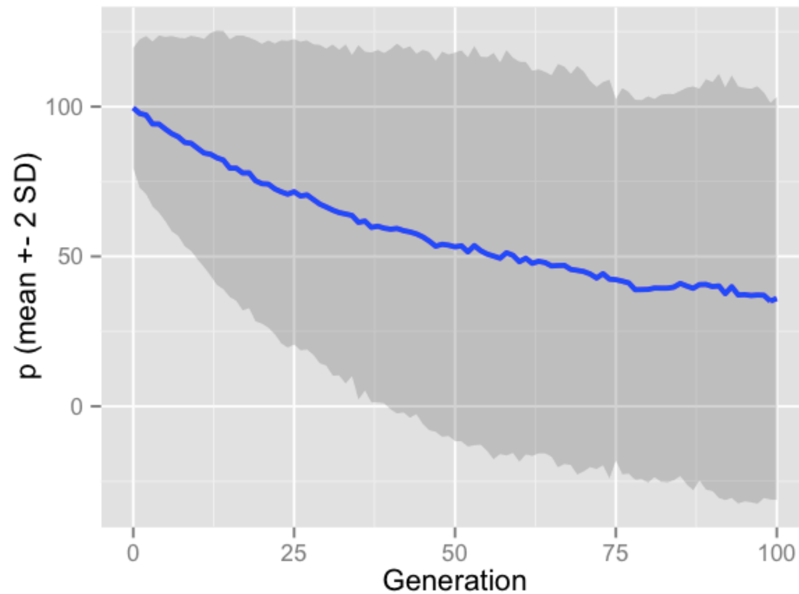
- "slows down" evolution.

- Gaussian prior, p_{start}=100 :

without grandparents



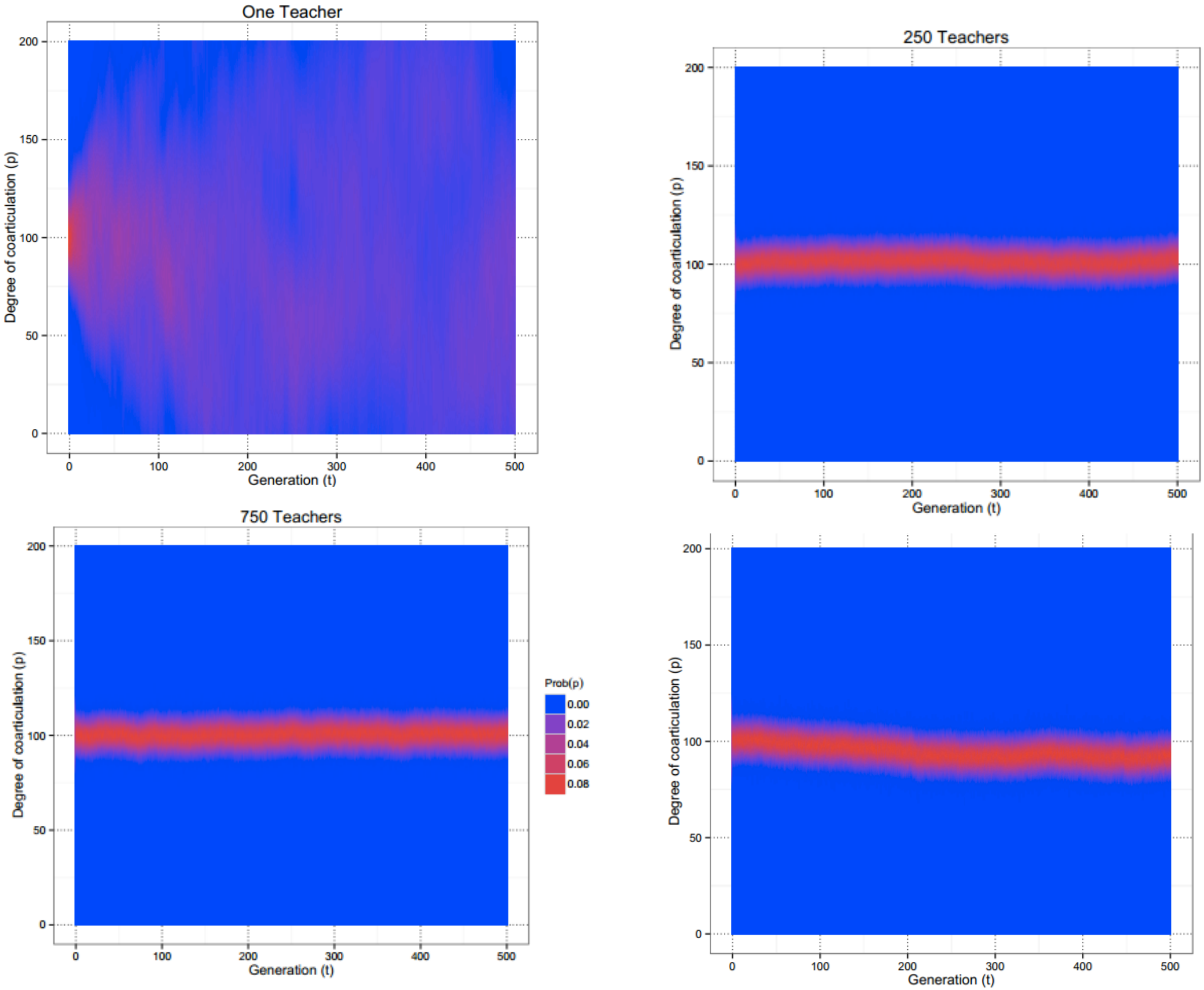
with grandparents



Varying $n_{teachers}$ (Megan and Jon F.)

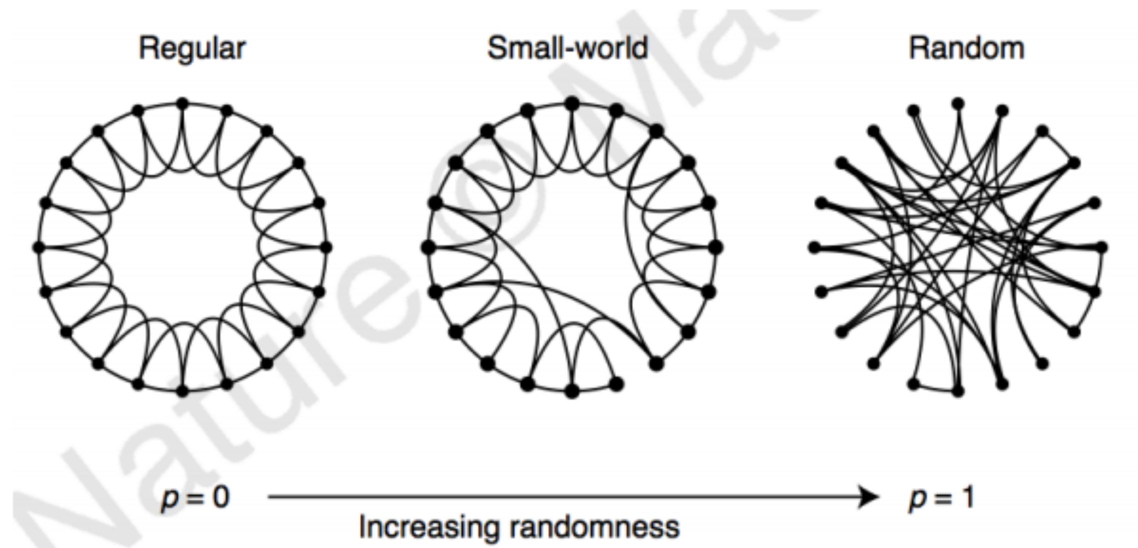
- $n_{teachers}$ = 1-$n_{learners}$, with A_{naive}
- Recall:
  - Single teacher: Variance increases
  - All teachers: Variance $\rightarrow$ fixed point

- Variance $\rightarrow$ fixed point for $n_{teachers} > 1$ (!)
- As $n_{teachers}$ increases:
  - Stable variance decreases
  - Looks like $n_{teachers} = n_{learners}$ quickly


One Teacher
250 Teachers
750 Teachers
Degree of coarticulation (p)
Generation (t)
Prob(p)
0.00
0.02
0.04
0.06
0.08
0
50
100
150
200
300
400
500

Network structure (Jon H. & Russell)

- Learners/teachers connected in a network, choose p based on distance from teacher
- 4 network types: Fully connected +



- $A_{complex}$, with lenition

- Single teacher (tentative)
  - Low shortest path length or high clustering coefficient → faster convergence
- Multiple teachers (note def'n)
  - Network structure doesn't have much effect
- Interpretation
- Overall: No effect of network structure on qualitative outcome

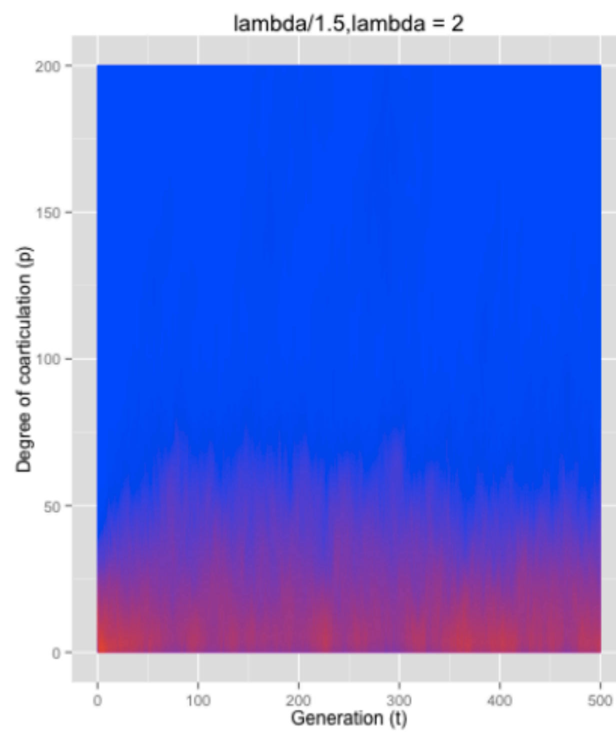
Horizontal transmission (Christina & Rebecca)

- Step 1: Draw half of examples from G_{n-1}, learn p
- Step 2: Draw other half of examples from other G_n learners, relearn p (?)
- Result: Noisier evolution of mean; variance grows faster
- Interpretation

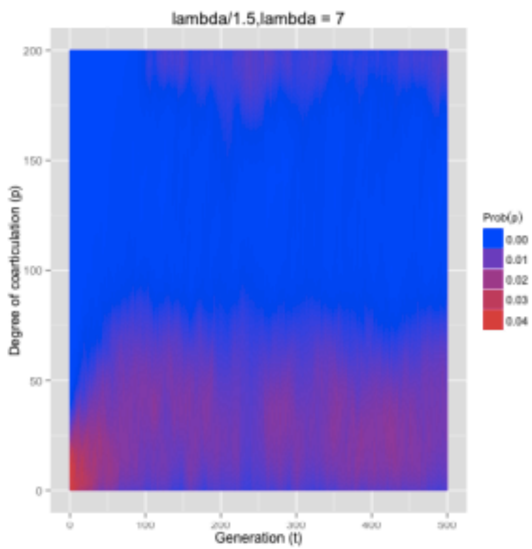
Changing the lenition distribution (Anthony, Emily)

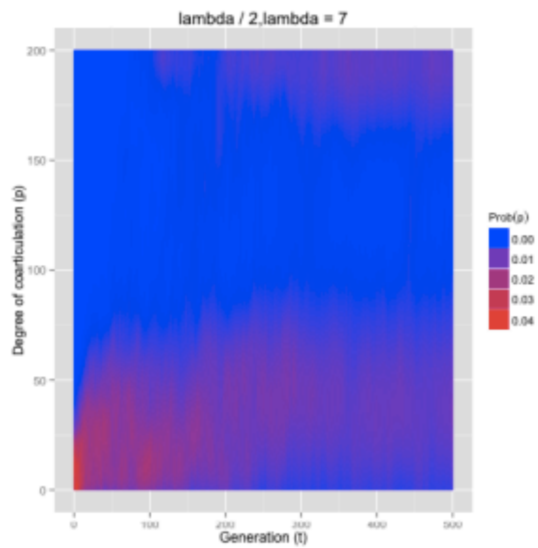
- $A_{complex}$, lenition ~ $N(\lambda, \lambda/k)$
  - $k \in$ 1.5, 2 (default), 4
  - $\lambda \in$ 2-10

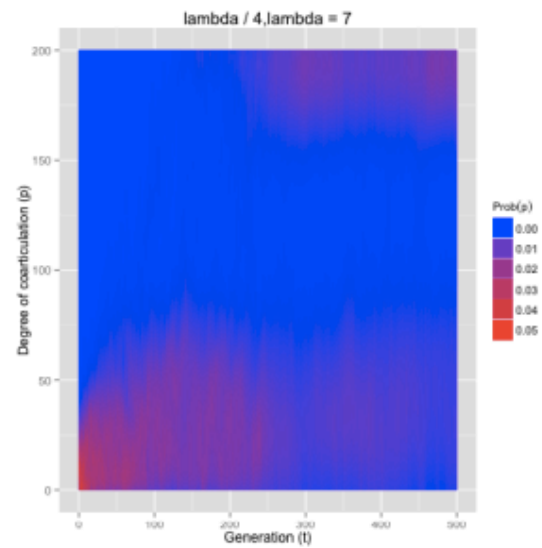
- $\lambda < 5$
  - No effect of k



- $\lambda > 5$: Affects rate of change (?)
  - Not qualitative outcome







- Interpretation

Varying $n_{examples}$ (Jiang, Mingxing, Korous)

- 100, 1000, 10000 examples, A_{naive}
- As $n_{examples}$ increases, variance of p decreases (multiple teachers), or increases more slowly (single teacher)
  - Expected from analytic results

Model extensions: Summary

- Messier evolution
  - Horizontal transmission
- Rate of change (in mean or variance)
  - Network structure, $n_{teachers}$, grandparents, lenition distribution (…)
- Equilibrium values of mean or variance
  - $n_{teachers}$, $n_{examples}$ (…)
- Qualitative outcome: $n_{teachers}$ (and prior, etc.)

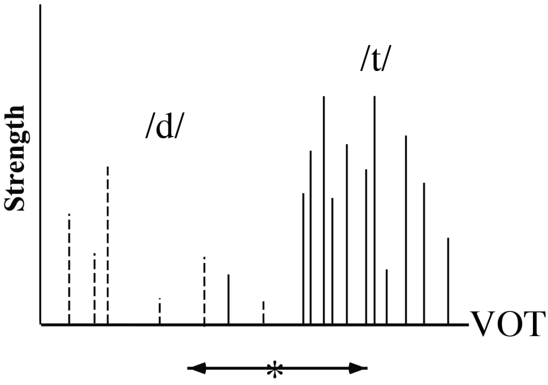
- Model components
  1. Knowledge state of individuals
  2. Network/social structure
  3. Assumptions about communication
  4. Learning algorithm

- What matters?

Multidimensional change case study 1: Kirby (2013)

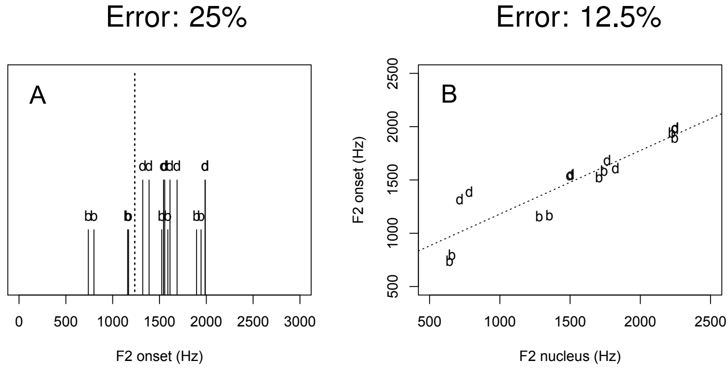
Exemplar models: extensions

- From unidimensionality to multidimensionality

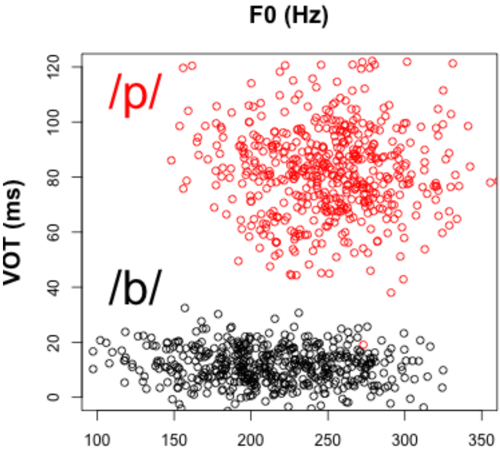


- Phonologization vs. neutralization

Contrasts are redundantly cued



Not all cues are created equal



Not all cues are created equal

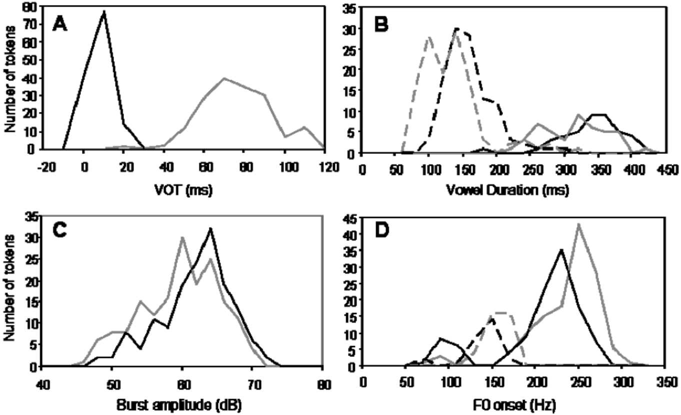


Figure 5.4: Word-initial productions. Histograms of the distribution of each cue for each category for all speakers. Black lines are "b" words, grey lines are "p" words. [A] VOT. [B] Vowel duration, solid lines voiced offsets, dashed lines voiceless offset. [C] Burst Amplitude. [D] F0 Onset, solid lines female speakers, dashed lines male speakers.

Phonologization (Hyman, 1976)

‘The historical development of tones (tonogenesis) can result from the reinterpretation by listeners of a previously intrinsic cue after recession and disappearance of the main cue’ (Hombert, Ohala, and Ewan, 1979)

Stage 1	*Stage 2*	*Stage 3*
pá [ ]	pá [ ]	pá [ ]
bá [ ]	bǎ [ ]	pǎ [ ]
universal	allophonic	contrastive

Phonologization of F_0 in Seoul Korean (Kang and Guion, 2008)

manner	*1960s*	*2000s*	*gloss*
fortis 뿔	p*ul		'horn'
lenis 불	pul		'fire'
aspirated 풀	p^hul		'grass'

Recordings courtesy of UCLA Phonetics Lab Archive

(`http://archive.phonetics.ucla.edu/`)

Phonologization of F_0 in Seoul Korean (Kang and Guion, 2008)

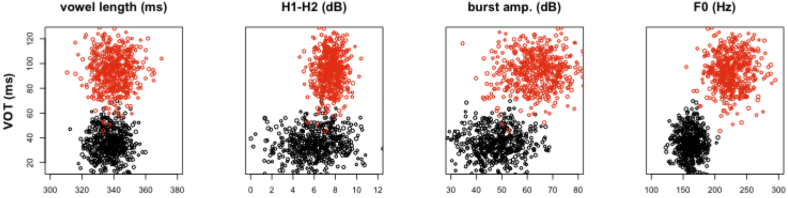
manner	*1960s*	*2000s*	*gloss*
fortis 뿔	p*ul	púl	‘horn’
lenis 불	pul	p^{h}ùl	‘fire’
aspirated 풀	p^{h}ul	p^{h}úl	‘grass’

Recordings courtesy of UCLA Phonetics Lab Archive

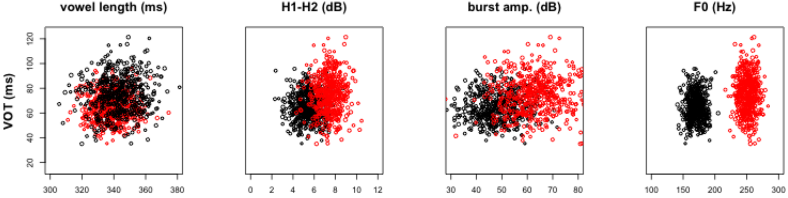
(`http://archive.phonetics.ucla.edu/`)

Phonologization of F_0 in Seoul Korean (Kang and Guion, 2008)

Lenis and aspirated stops, Seoul Korean,1960s



Lenis and aspirated stops, Seoul Korean, 2000s

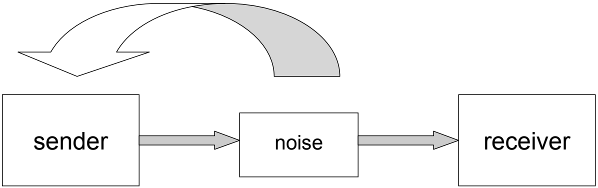


Questions

- Can we predict which cue?
- Can we predict when a contrast is transphonologized? (as opposed to merged)
- How might talkers respond to noise?

Dealing with noise

- One idea: talkers enhance portion(s) of the signal



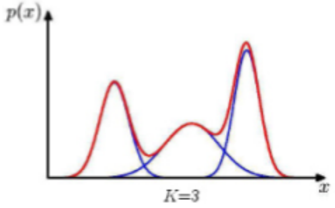
- Adaptive strategy: enhance $\propto$ informativeness

Probabilistic enhancement hypothesis

- Cues are enhanced probabilistically, in proportion to their contribution to successful perception and categorization of a phonetic contrast (cf. Diehl et al., 1990; Kingston & Diehl, 1994)

- Based on cue reliability and contrast precision
  - Reliability inversely proportional to variance (Clayards, 2008)
  - Precision = error rate of an optimal classifier
  - Degree of enhancement $\propto$ reliability & precision

Finite mixture models (McLachland and Peel, 2000)

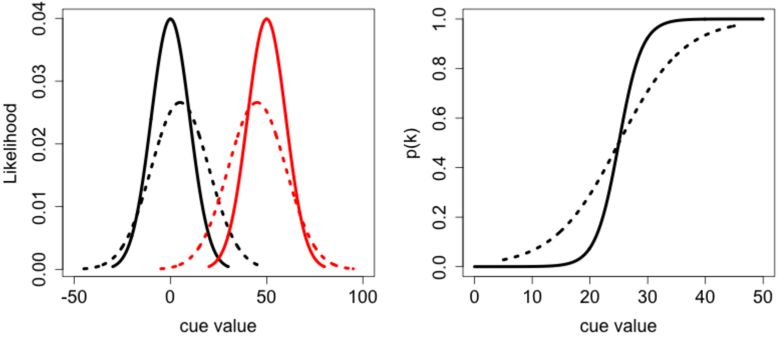


$$p(\mathbf{x}) = \sum_{k=1}^{K} \underbrace{\pi_k}_{\text{Mixing coefficient}} \underbrace{\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)}_{\text{Component}}$$

$$\forall k : \pi_k \geqslant 0 \qquad \sum_{k=1}^{K} \pi_k = 1$$

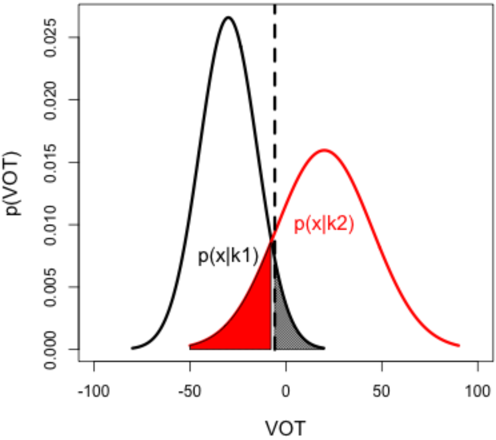
Mixture models have K **components** and D **dimensions**

Reliability is proportional to variance



(Adapted from Clayards et al., 2008)

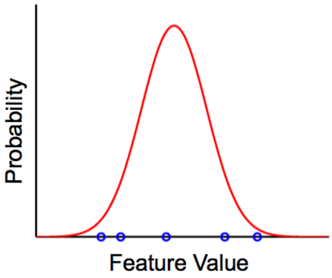
Precision measured by classification error

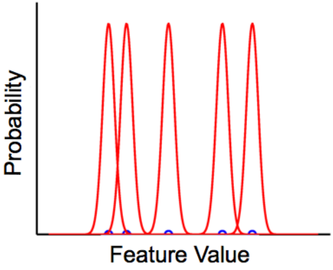


$$\epsilon = 1 - \sum_{i=1}^{K} \int p(\mathbf{x}|k)p(k)d\mathbf{x}$$

Prototypes & exemplars

- **'Prototypes':** $P(k|\mathbf{x}) = \arg\max_{k_i} P(k_i|\mathbf{x})$
- **'Exemplars':** $P(k|\mathbf{x}) = p(\mathbf{x}|k)P(k) / \sum_{k=1}^{K} p(\mathbf{x}|k_i)P(k_i)$



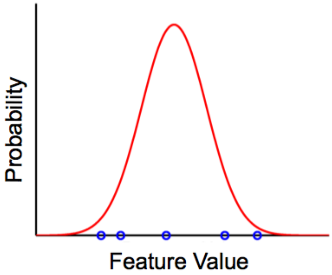


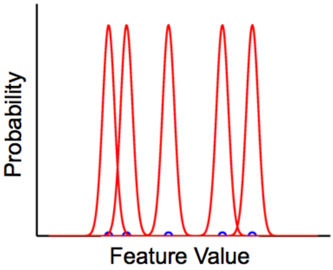
(Ashby & Alfonso-Reese, 1995; Rosseel, 2002; Jäkel et al, 2009)

Prototypes & exemplars

- **'Prototypes':** $P(k|\mathbf{x}) = \arg\max_{k_i} P(k_i|\mathbf{x})$

- **'Exemplars':** $P(k|\mathbf{x}) = \frac{p(x_1|k)p(x_2|k),\ldots,p(x_D|k)p(k)}{\sum_{i=1}^{K} p(x_1|k_i)p(x_2|k_i),\ldots,p(x_D|k_i)p(k_i)}$

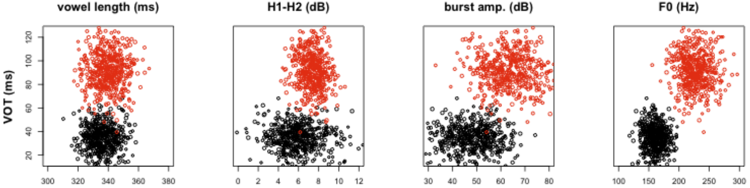




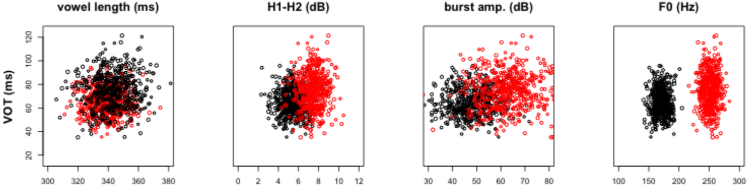
(Ashby & Alfonso-Reese, 1995; Rosseel, 2002; Jäkel et al, 2009)

Case study: Seoul Korean

Lenis and aspirated stops, Seoul Korean, 1960s

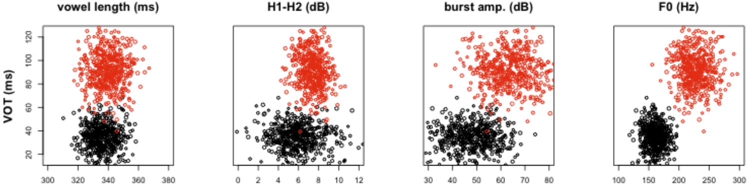


Lenis and aspirated stops, Seoul Korean, 2000s

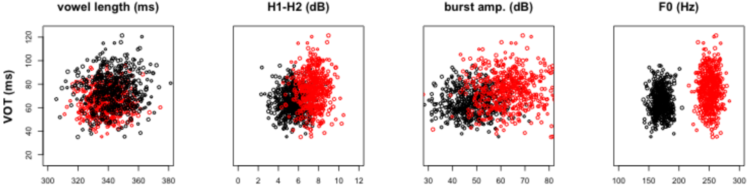


Case study: Seoul Korean

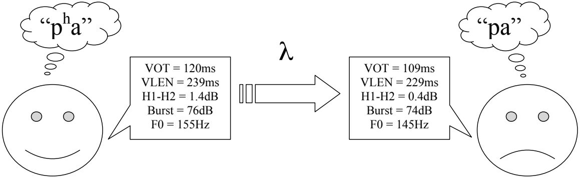
Lenis and aspirated stops, Seoul Korean, 1960s



Lenis and aspirated stops, Seoul Korean, 2000s



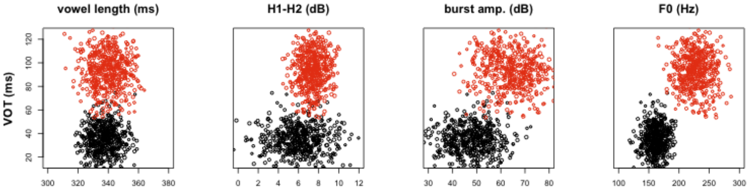
Agent-based simulations



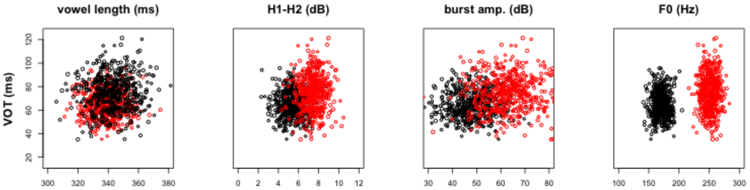
1. Agent 1 selects a phonetic category target k
2. ....samples from each conditional density: $x_d \sim \mathcal{N}(d|k; \mu_d, \sigma_d)$
3. ....enhances dimension d with some probability
4. ....produces $\mathbf{x}$, which is further modified by bias λ
5. Agent 2 assigns a label to $\mathbf{x}$, re-estimates parameters
6. Wash, rinse, repeat until a stable state is reached

Transmission bias, no enhancement

Simulated

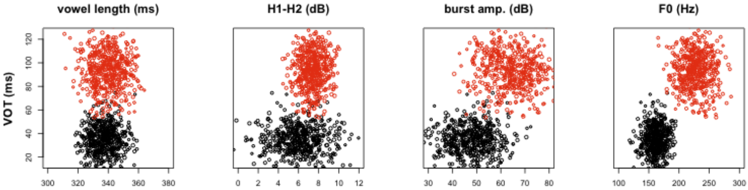


Empirical target

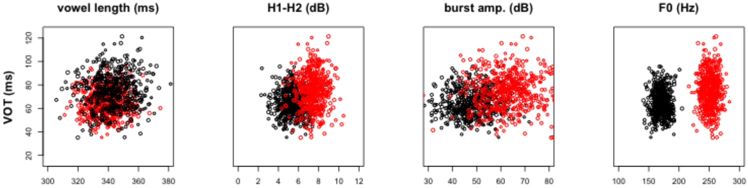


Transmission bias, no enhancement

Simulated

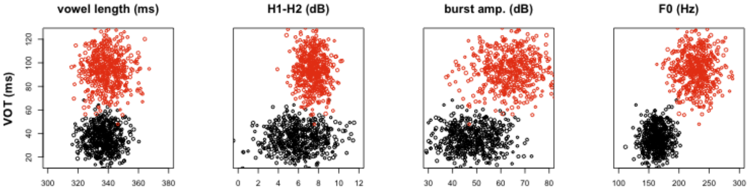


Empirical target

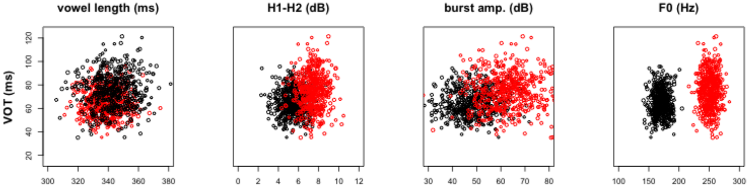


Enhancement, no transmission bias

Simulated

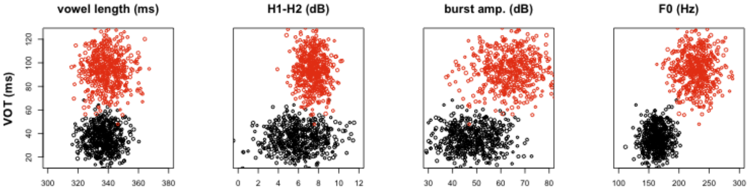


Empirical target

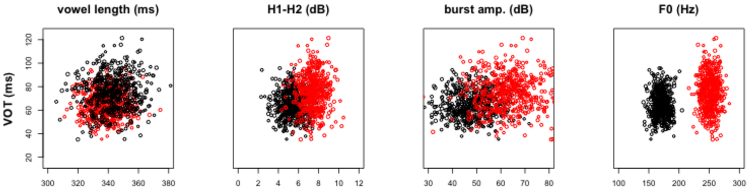


Enhancement, no transmission bias

Simulated

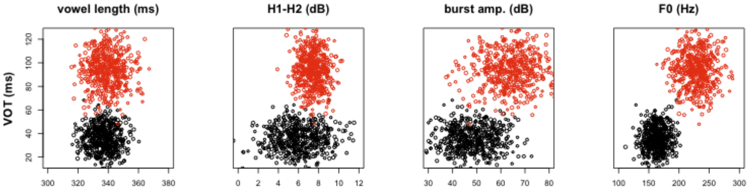


Empirical target

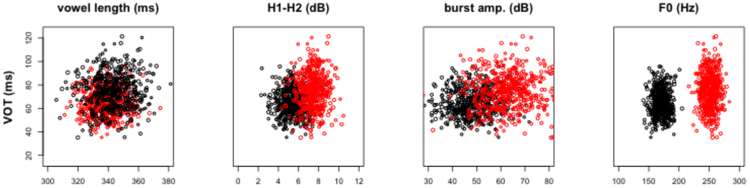


Transmission bias plus enhancement

Simulated

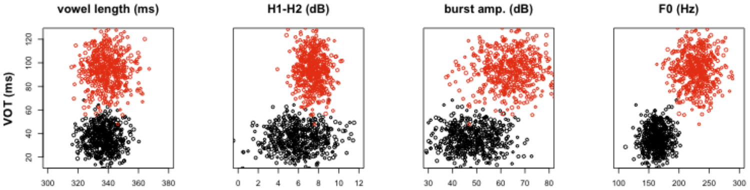


Empirical target

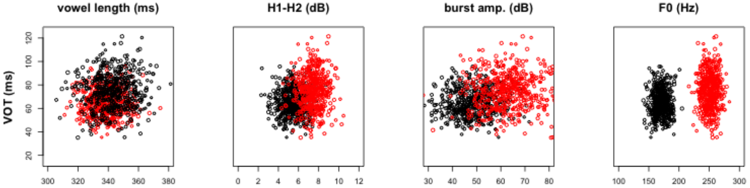


Transmission bias plus enhancement

Simulated

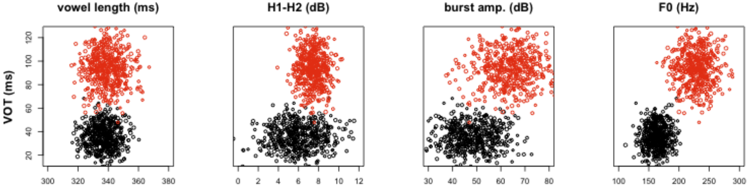


Empirical target

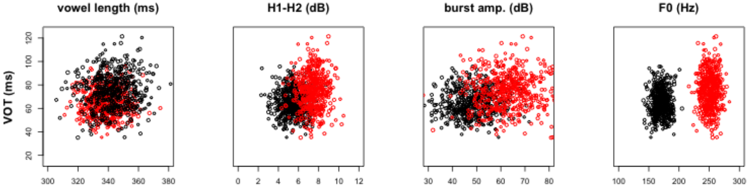


Equilibrium 1: channel noise matters

Simulated

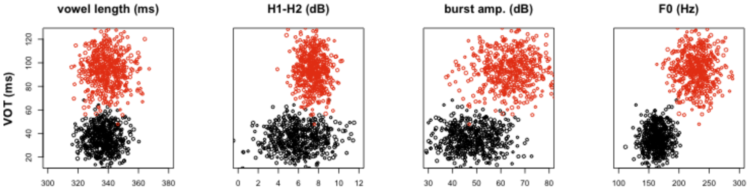


Empirical target

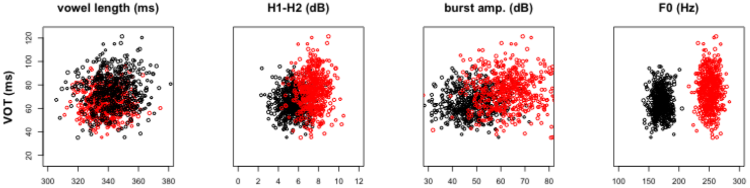


Equilibrium 1: channel noise matters

Simulated

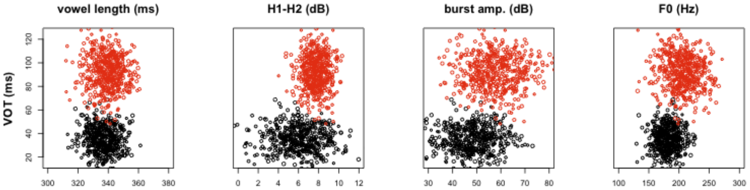


Empirical target

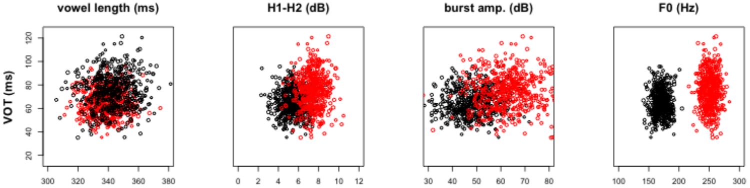


Equilibrium 2: initial state matters

Simulated

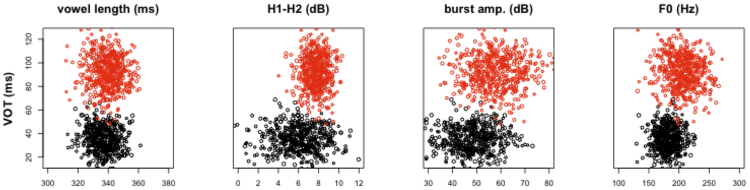


Empirical target

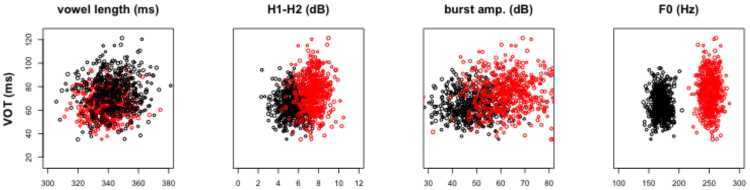


Equilibrium 2: initial state matters

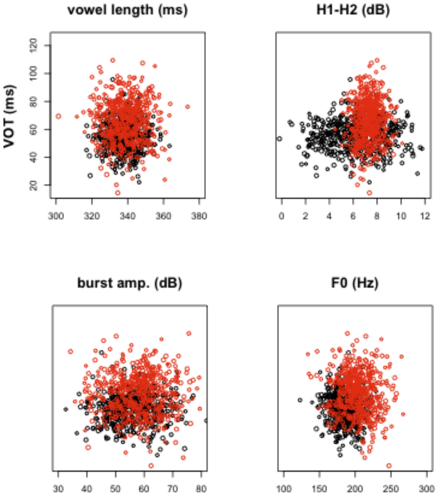
Simulated



Empirical target

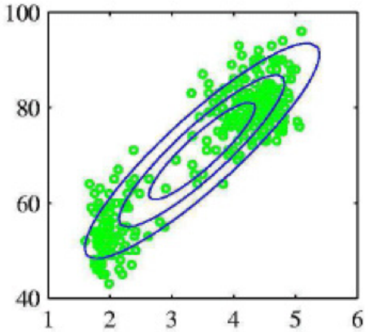


Category restructuring

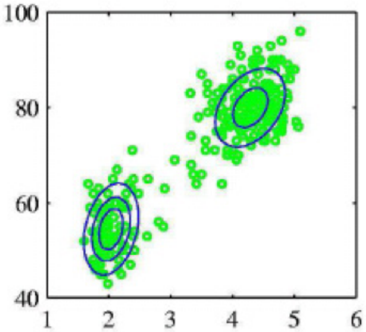


Model selection

How do we choose K?



Single Gaussian



Mixture of two Gaussians

Model selection

- More components $\rightarrow$ more parameters
- MODEL SELECTION: penalize models with many parameters
- One family of approaches: AIC, BIC, DIC...
- (Bayesian interpretation: prior over # of model parameters)

Model selection for GMMs

Let $\mathcal{L}$ be the maximized log-likelihood of a Gaussian with K D-dimensional components characterized by parameters θ:

$$\mathcal{L} = \ln P(\mathbf{X}|\theta_{\max}) \tag{1}$$

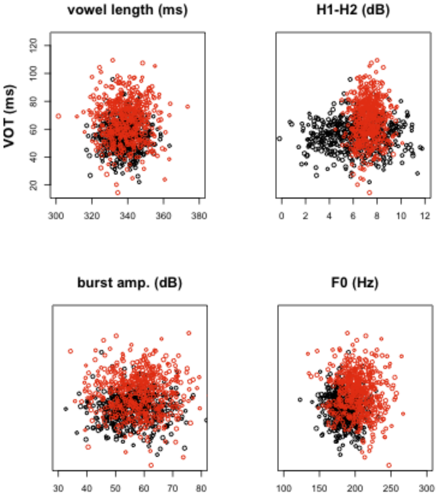
Let $\mathcal{Q}$ be the number of independent parameters in that model:

$$\mathcal{Q} = K(D + D(D+1)/2) + K - 1 \tag{2}$$

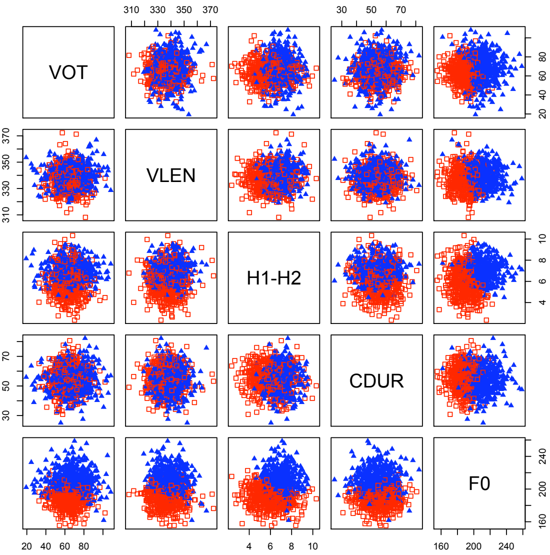
Then choose K that minimizes one of e.g.:

$$\begin{aligned} AIC &= -2\mathcal{L} + 2\mathcal{Q} \\ BIC &= -2\mathcal{L} + \ln(N)\mathcal{Q} \\ MDL &= \ln(\mathcal{L}) + \ell(\theta) \\ &\cdots \end{aligned} \tag{3}$$

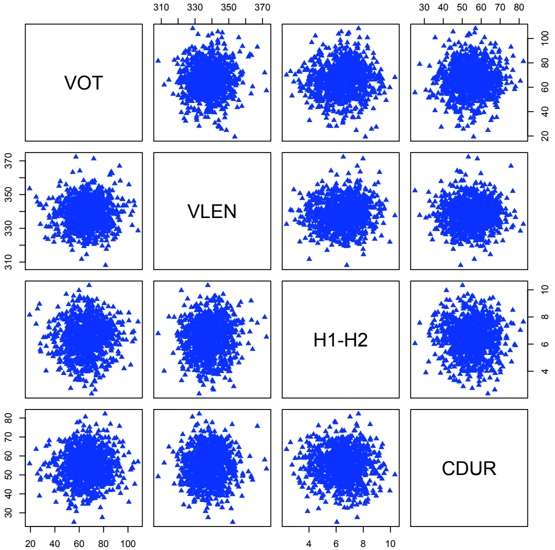
Does overlap entail neutralization?



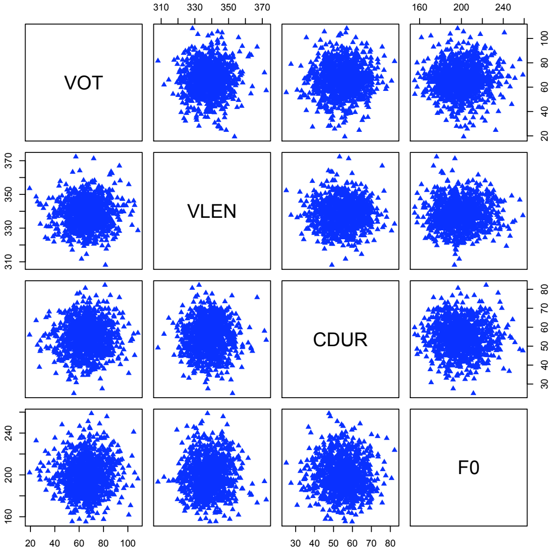
Not necessarily...



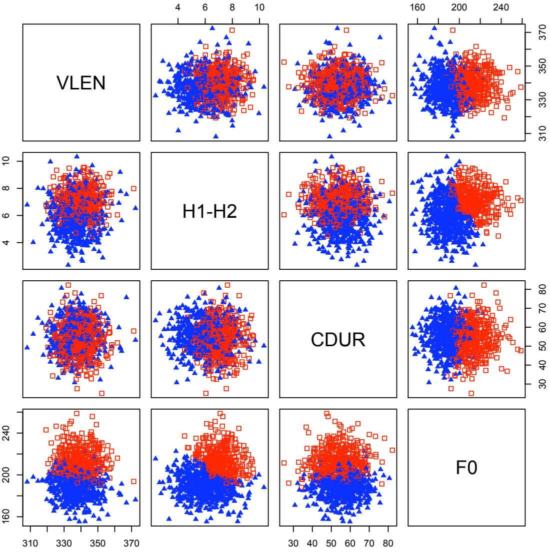
But if there is difference in attention to dimensions...



But if there is difference in attention to dimensions...



But if there is difference in attention to dimensions...

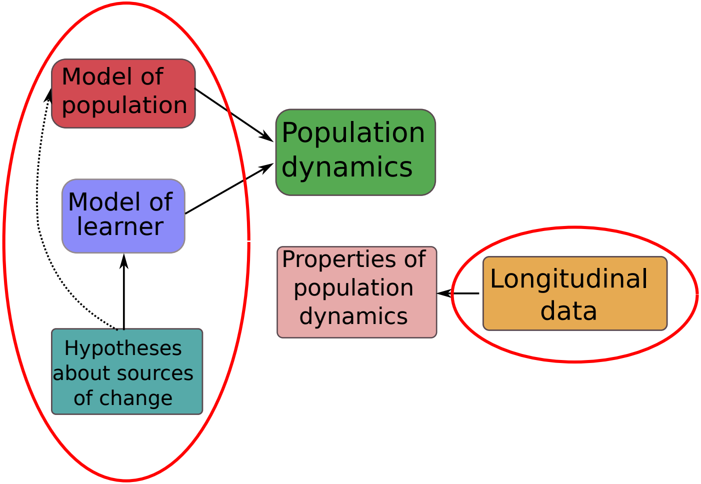


Multidimensional change case study 2: Sonderegger & Niyogi (2010)

- Weinreich et al. (1968):
  - Actuation: Why does change ever occur?
  - Constraints: What determines the set of possible changes, conditions for changes?
- Focus: Relationship between population-level change and assumptions about individuals.
- Approach combines:
  - Longitudinal data (English)
  - Mathematical models (dynamical systems)

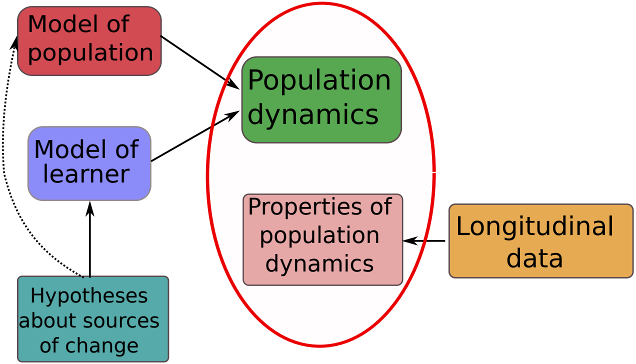
  in a concrete case

Approach



Model of population
Population dynamics
Model of learner
Properties of population dynamics
Longitudinal data
Hypotheses about sources of change

Approach



Compare using language of dynamical systems

Background: English disyllabic N/V pair stress

- Variable stress:

	N	V	
{1,1}	$\acute{\sigma}\sigma$	$\acute{\sigma}\sigma$	*anchor*, *fracture*
{1,2}	$\acute{\sigma}\sigma$	$\sigma\acute{\sigma}$	*consort*, *contest*
{2,2}	$\sigma\acute{\sigma}$	$\sigma\acute{\sigma}$	*police*, *review*

- Never {2,1}
- Variation, change since at least 1570 (Minkova, 1997)
- Present-day variation: *research*, *perfume*, *address*..

Previous work

- Sherman (1975), Phillips (1984, 2006)
  - Most common change {2,2}$\rightarrow${1,2}: "diatonic stress shift"
  - Less frequent N/V pairs change first
- Hock (1991), Kiparsky (1995)
  - Analogical change

Previous work

- Sherman (1975), Phillips (1984, 2006)
  - Most common change {2,2}$\rightarrow${1,2}: “diatonic stress shift”
  - Less frequent N/V pairs change first
- Hock (1991), Kiparsky (1995)
  - Analogical change
- Literature on English stress
  - Chomsky & Halle (1968), Ross (1973), Liberman & Prince (1977), Hayes (1982), Fudge (1984), Poldauf (1984), Burzio (1994), Halle (1997)...

Longitudinal dataset

- Overall stability of {1,1}, {1,2}, {2,2}
  - Random sample of 110 N/V pairs: 11.8% change 1700–2007.
- 149 N/V pairs ($\mathcal{L}$) have shown variation and change.

  (Sherman 1975)
- Most are “prefixed”
  - *aC*, *com/con*, *de*, *dis*, *eC*, *ex*, *in/im*, *mis*, *per*, *pre*, *pro*, *re*, *sub*, *sur*...

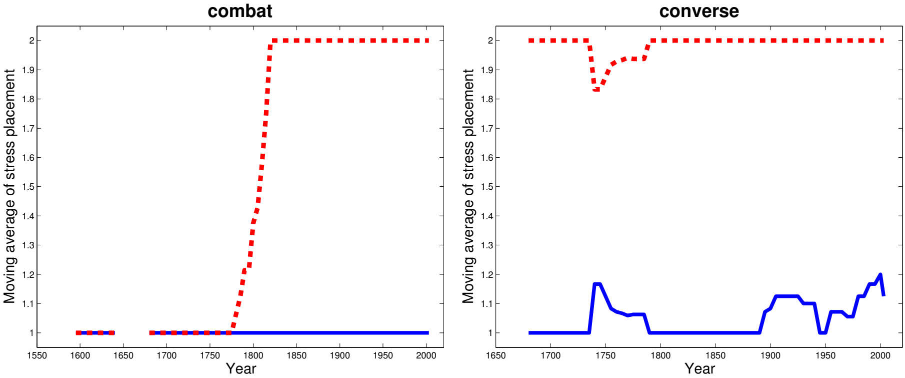
    (Phillips, 1984)

Sonderegger & Niyogi (in press), in A.C.L. Yu (Ed.), *Origins of Sound Change*

- Dataset: $\mathcal{L}$ stresses in 62 British dictionaries.
  - Sherman (1975): 1550–1800
  - Sonderegger (2009): 1800–present
- Variation often recorded, but never:
  - N=$\sigma\acute{\sigma}$, variation in V
  - V=$\acute{\sigma}\sigma$, variation in N

Stress trajectories

Moving average of N (blue), V (red) stress:



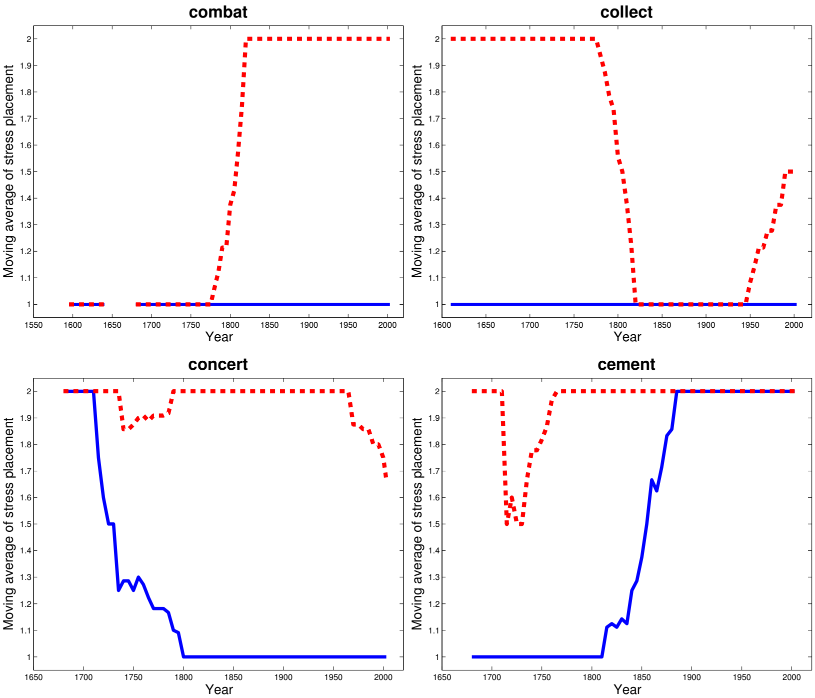
Observed dynamics

Common changes:

$$
\begin{tikzcd}
\{1,1\} \arrow[r, bend left, "\begin{array}{c}\textit{dictate}\\ \textit{combat}\end{array}"] & \{1,2\} & \{2,2\} \arrow[l, bend right, "\begin{array}{c}\textit{escort}\\ \textit{digest}\\ \textit{concert}\end{array}"']
\end{tikzcd}
$$

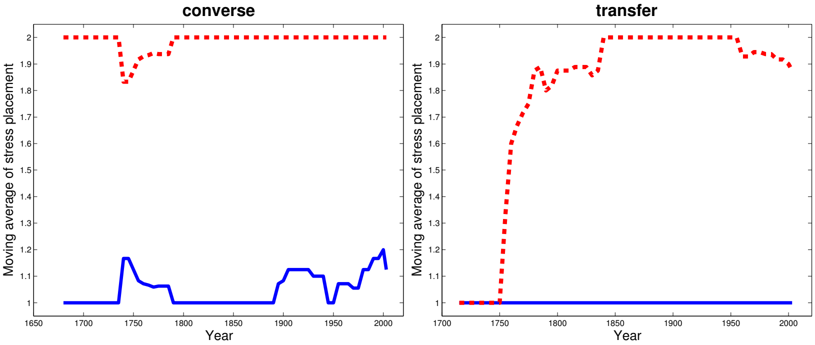
Uncommon changes:

$$
\begin{tikzcd}
\{1,1\} & \{1,2\} \arrow[l, bend right, "\textit{collect}"'] \arrow[r, bend left, "\textit{cement}"] & \{2,2\}
\end{tikzcd}
$$

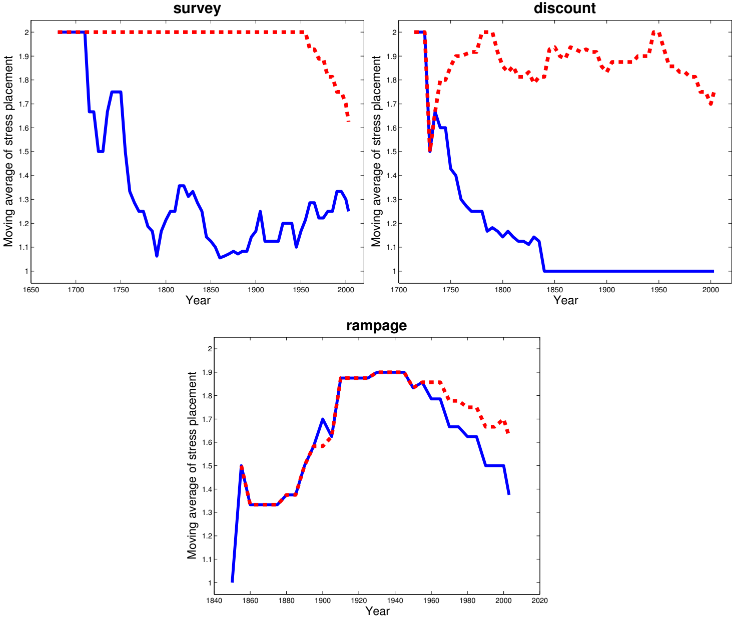

combat
collect
concert
cement
Moving average of stress placement
Year
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
1550
1600
1650
1700
1750
1800
1850
1900
1950
2000

Variation

- Short-term variation common



- Long-term variation (“stable variation”)
  - N or V, rarely both

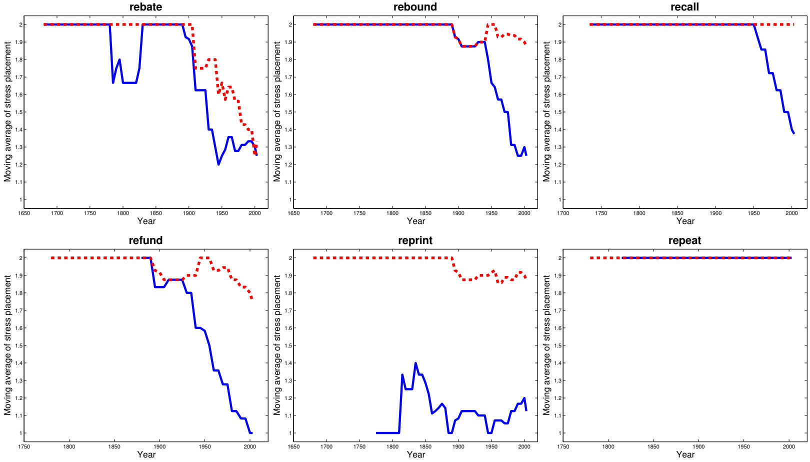


Individual variation?

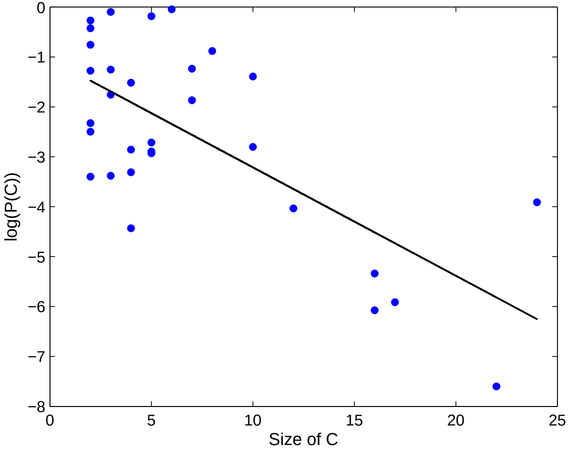
- Dictionaries report population-level variation.
- Important for modeling: intraspeaker variation?
- Evidence:
  - Radio speech for *research*, *perfume*, *address*
  - Metrical evidence from Middle & Early Modern English (Tamson, 1898)

Measuring coupling between pairs

- Do N/V pairs with same prefix “change like” each other?



- Define distance between any two pairs' trajectories
- For each prefix class $C \subset \mathcal{L}$:
  1. $R(C)$: measure of how different trajectories of $w \in C$ are (c.f. Newman & Girvan, 2004)
  2. $p(C)$: $R(C)$ position on distribution of $R(X)$ over *all* $X \subset \mathcal{L}$ s.t. $|X| = |C|$
- $p(\text{'re'}) = 0.05$, $p(\text{'pre'}) = 0.1$...



- Larger prefix classes ⇒ more similar trajectories.

Longitudinal data: Discussion

- Observed changes
  - Multidirectional
  - Majority {2,2}$\rightarrow${1,2}, but many other changes
- Unobserved changes
  - {1,1}$\leftrightarrow${2,2}
- Short-term variation
- Long-term variation in N or V
- Prefix classes
  - Analogical effect?

Sources of change

- Why is change (mostly) $\rightarrow \{1,2\}$, and no $\{2,1\}$?
- Two explanations in individual perception/production

Explanation 1: Mistransmission

Kelly (1988 *et seq*), Guion et al. (2003):

- Biased production & perception of stress.
- N biased $\rightarrow \acute{\sigma}\sigma$, V biased $\rightarrow \sigma\acute{\sigma}$.
- Prosodic context, syllabic structure

Explanation 2: Analogy/lexicon

Ross (1973), Sereno & Jongman (1995), Phillips (2006):

- English lexicon: N stress earlier than V stress.
  - Productive
- Disyllables:

	Nouns	Verbs
$\acute{\sigma}\sigma$	94%	6%
$\sigma\acute{\sigma}$	24%	76%

- Role of prefix classes

Modeling framework: Discrete dynamical systems

- Used to analyze evolutionary dynamics of systems.
  (Hirsch et al., 2004; Hofbauer & Sigmund, 1988)

- System state evolves by $\alpha_{t+1} = f(\alpha_t)$:

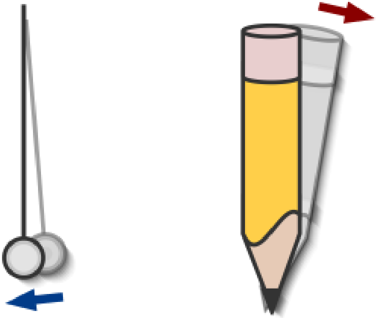
$$\alpha_0 \xrightarrow{f} \alpha_1 \xrightarrow{f} \alpha_2 \xrightarrow{f} \cdots$$

- Dynamical systems viewpoint: Examine limiting ($t \to \infty$) behavior.

- Linguistic populations: Niyogi & Berwick (1995 *et seq.*), Niyogi (2006)

Fixed points

- $f(\alpha^*) = \alpha^*$
- As t increases, $\alpha_t \to$ a FP.
- *stable* or *unstable* under small perturbations:



Stable Unstable

Bifurcations

- Change in number or stability of FPs as system parameter passes a critical value.
- $\Rightarrow$ Qualitative change in dynamics
- Change from α_*: Bifurcation where FP α_* loses stability.

Bifurcations

- Change in number or stability of FPs as system parameter passes a critical value.
- $\Rightarrow$ Qualitative change in dynamics
- Change from α_*: Bifurcation where FP α_* loses stability.
- Example:
  - *research* pronounced {2,2}, but {1,2} and {2,2} are stable states.
  - System parameter: Analogical support for {2,2}
  - Bifurcation: {2,2} becomes unstable $\Rightarrow$ change to {1,2}

Goal of DS analysis of f

- Find FPs and stabilities.
- Given system parameters, find bifurcations.

  Bifurcation structure $\Rightarrow$ possible/impossible changes

N/V trajectory dynamics as DS desired properties

1. {1,1}, {1,2}, {2,2}: Stable states
   - For some system parameter values
2. Stable variation: Stable state for var in N or V, not both.
   - ″
3. *{2,1}: Unstable state
4. Patterns of change: Bifurcation structure

N/V trajectory dynamics as DS desired properties

1. {1,1}, {1,2}, {2,2}: Stable states
   - For some system parameter values
2. Stable variation: Stable state for var in N or V, not both.
   - ″
3. *{2,1}: Unstable state
4. Patterns of change: Bifurcation structure

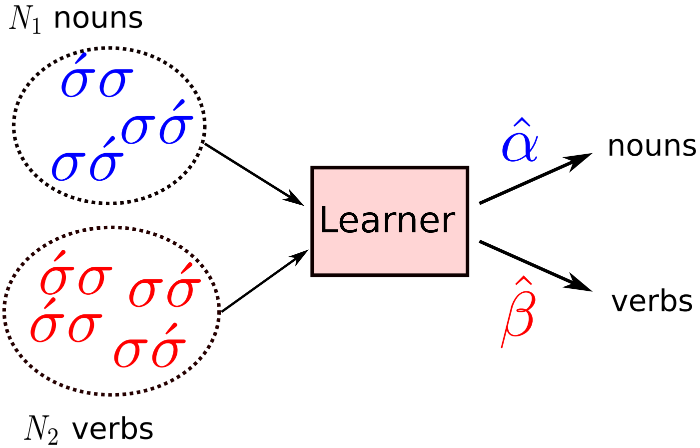
Want models of learning by individuals resulting in a DS with these population-level properties.

Models: Outline

- Describe model structure
- Consider 6 models of learning (individual)
- For each, dynamics of resulting DS (population)

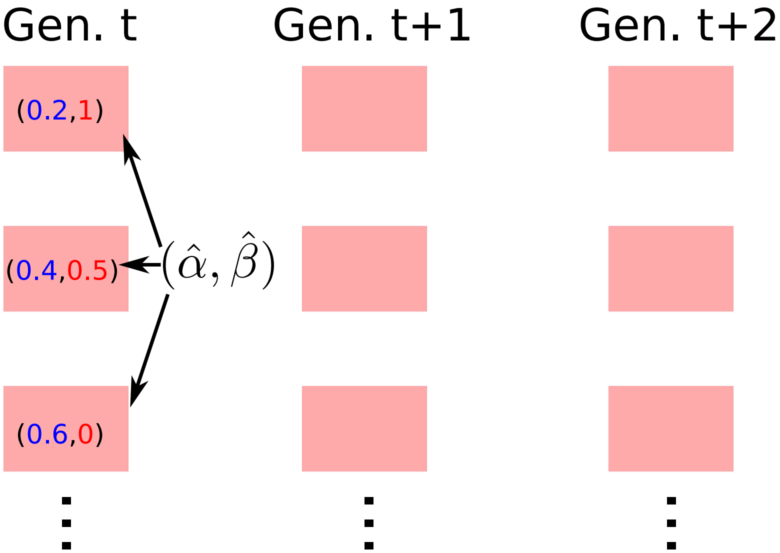
Sonderegger & Niyogi (2010), *ACL 48*

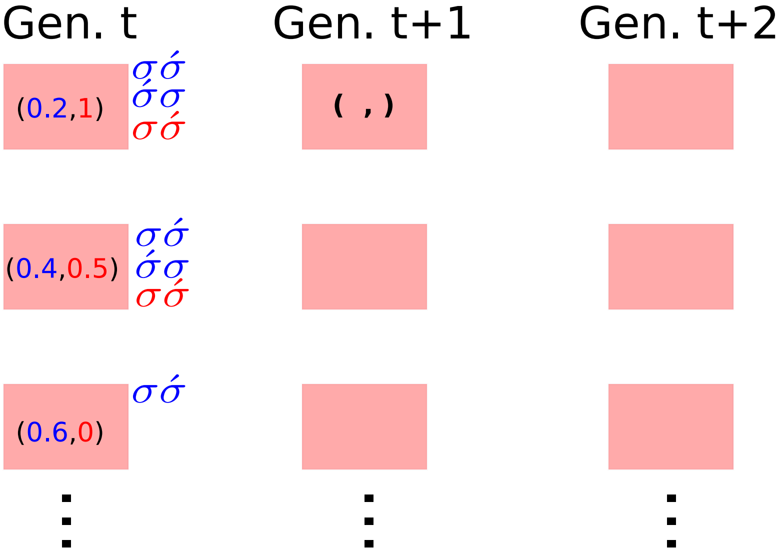
One learner



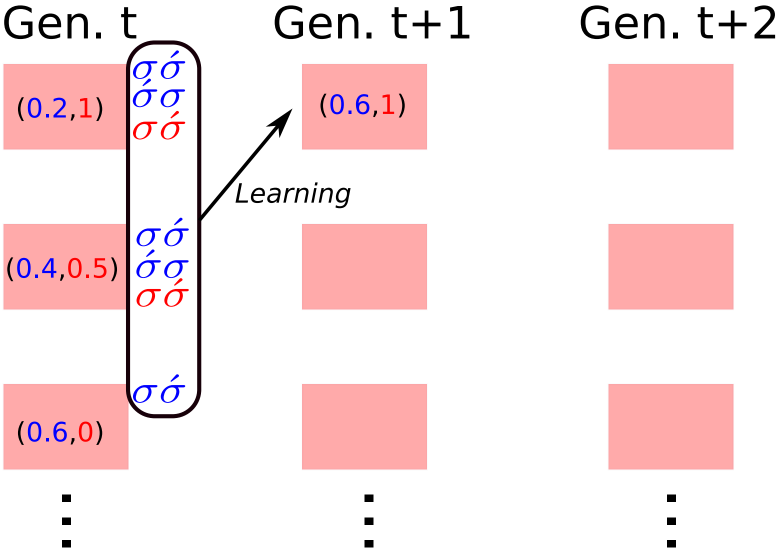
$$\hat{\alpha} = P(\sigma\acute{\sigma}|\text{noun}), \quad \hat{\beta} = P(\sigma\acute{\sigma}|\text{verb})$$

Generations of learners

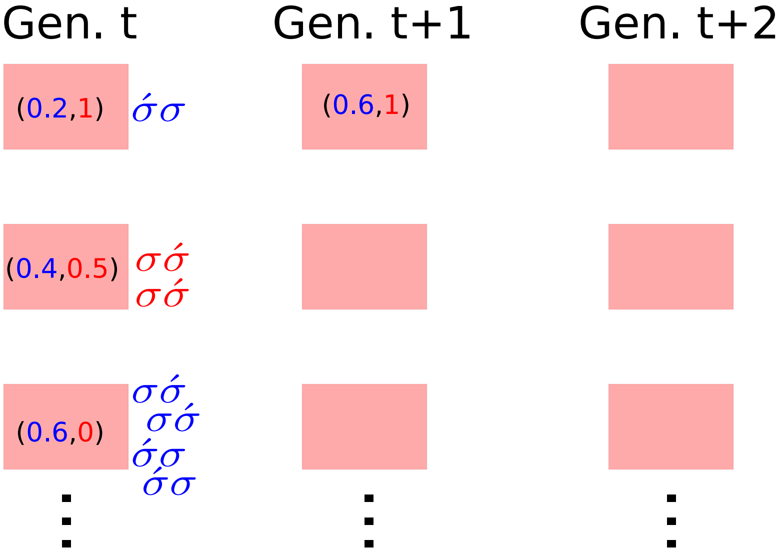


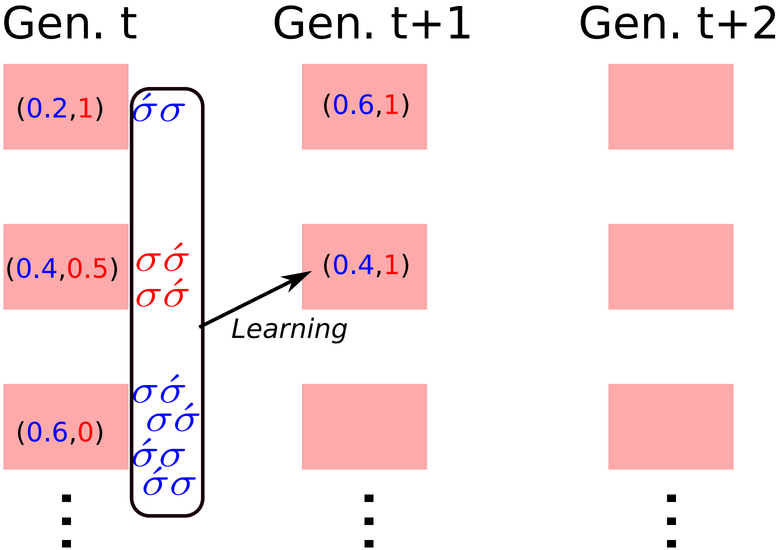


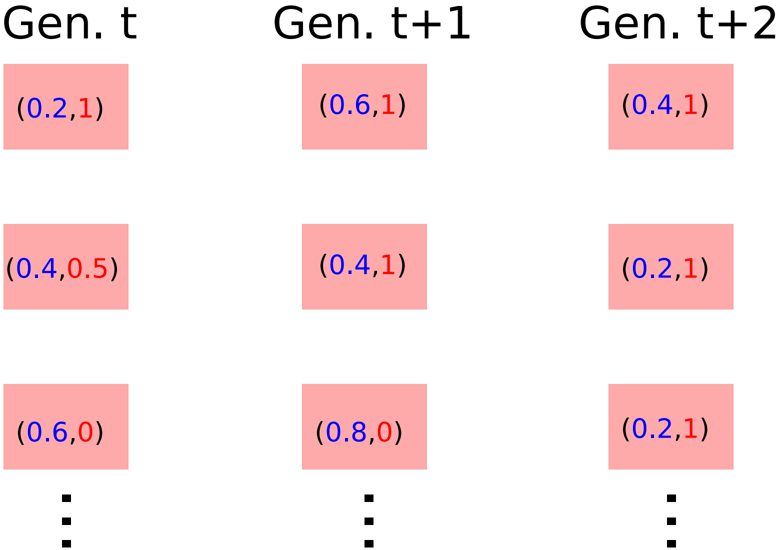
Gen. t
Gen. t+1
Gen. t+2
(0.2,1)
σσ́
σ́σ
σσ́
(,)
(0.4,0.5)
σσ́
σ́σ
σσ́
(0.6,0)
σσ́



Gen. t
Gen. t+1
Gen. t+2
(0.2,1)
(0.4,0.5)
(0.6,0)
(0.6,1)
σ σ́
σ́ σ
σ σ́
σ σ́
σ́ σ
σ σ́
σ σ́
Learning


Gen. t
Gen. t+1
Gen. t+2
(0.2,1)
$\acute{\sigma}\sigma$
(0.6,1)
(0.4,0.5)
$\sigma\acute{\sigma}$
$\sigma\acute{\sigma}$
(0.6,0)
$\sigma\acute{\sigma}$
$\sigma\acute{\sigma}$
$\acute{\sigma}\sigma$
$\acute{\sigma}\sigma$


Gen. t
Gen. t+1
Gen. t+2
(0.2,1)
(0.4,0.5)
(0.6,0)
(0.6,1)
(0.4,1)
Learning



Gen. t
Gen. t+1
Gen. t+2
(0.2,1)
(0.6,1)
(0.4,1)
(0.4,0.5)
(0.4,1)
(0.2,1)
(0.6,0)
(0.8,0)
(0.2,1)

Assumptions

- Each generation ∞ ($\Rightarrow$ deterministic)
- Perfect mixing
- N_1, N_2 fixed

Notation

- System state at t:

$$\alpha_t = E(\hat{\alpha}|\alpha_{t-1}, \beta_{t-1}), \quad \beta_t = E(\hat{\beta}|\alpha_{t-1}, \beta_{t-1})$$

  ensemble averages over generation t

- Dynamical system:

$$(\alpha_t, \beta_t) = f(\alpha_{t-1}, \beta_{t-1})$$

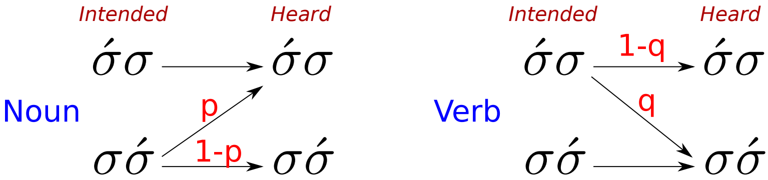
From hypothesized sources of change, two dimensions of assumptions about learner:

1. Mistransmission
   - Yes
   - No
2. Analogy
   - None
   - Coupling by constraint
   - Coupling by prior probabilities

$\Rightarrow$ 6 models

Model 1: Mistransmission

- Asymmetric mistransmission possible for individual examples:



- Prob. example heard as $\sigma\acute{\sigma}$, generation t:

$$p_{N,t} = \alpha_{t-1}(1-p), \quad p_{V,t} = \beta_{t-1} + (1-\beta_{t-1})q$$

- One learner: Hear k_1^t nouns, k_2^t verbs as $\sigma\acute{\sigma}$:

$$k_1^t \sim \text{Bin}(N_1, p_{N,t}), \quad k_2^t \sim \text{Bin}(N_2, p_{V,t}),$$

- Probability match:

$$\hat{\alpha} = \frac{k_1^t}{N_1}, \quad \hat{\beta} = \frac{k_2^t}{N_2}$$

- One learner: Hear k_1^t nouns, k_2^t verbs as $\sigma\acute{\sigma}$:

$$k_1^t \sim \mathrm{Bin}(N_1, p_{N,t}), \quad k_2^t \sim \mathrm{Bin}(N_2, p_{V,t}),$$

- Probability match:

$$\hat{\alpha} = \frac{k_1^t}{N_1}, \quad \hat{\beta} = \frac{k_2^t}{N_2}$$

$\vdots$ (derive population dynamics)

- Unique fixed point:

$$(\alpha_*, \beta_*) = (0, 1)$$

$\Rightarrow$ use {1,2} exclusively

Model 1: Properties

- {1,1}, {1,2}, {2,2}: ✗
- Stable var: ✗
- *{2,1}: ✓
- Patterns of change: ✗

Aside: Model 0

- Model 1 with $p = q = 0$
- No change between generations: All (α^*, β^*) stable FPs.

Model 2: Coupling by constraint

- Motivation: Knowledge N stress earlier than V stress.
- Learner constraint: $\hat{\alpha} < \hat{\beta}$

Learner

- Get examples, no mistransmission
- If $\frac{k_1}{N_1} < \frac{k_2}{N_2}$,

$$\hat{\alpha} = \frac{k_1}{N_1}, \quad \hat{\beta} = \frac{k_2}{N_2}$$

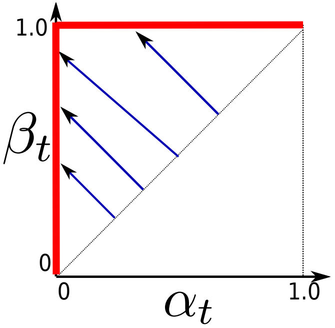
- Otherwise:

$$\hat{\alpha} = \hat{\beta} = \frac{1}{2}(\frac{k_1}{N_1} + \frac{k_2}{N_2})$$

$\vdots$ (derive population dynamics)

Fixed points

- Stable FPs: $(0, x)$, $(x, 1)$ $(x \in [0, 1])$
- Means:
  - N $\acute{\sigma}\sigma$, with V variation
  - V $\sigma\acute{\sigma}$ with N variation



Model 2: Summary

- {1,1}, {1,2}, {2,2}: ✓
- Stable var: ✓
- *{2,1}: ✓
- Patterns of change: ✗

Model 3: Coupling by constraint + mistransmission

- Model 2, but with asymmetric mistransmission.
- Unique fixed point:

$$(\alpha_*, \beta_*) = (0, 1)$$

  $\Rightarrow$ use {1,2} exclusively

- From mistransmission, not asymmetry.

Model 3: Properties

- {1,1}, {1,2}, {2,2}: ✗
- Stable var: ✗
- *{2,1}: ✓
- Patterns of change: ✗

Model 4: Coupling by priors

- Learner estimates probabilities of N and V final stress:

$$P_N = \frac{k_1}{N_1}, \quad P_V = \frac{k_2}{N_2}$$

- which give probabilities of patterns:

$$P(\{1,1\}) = (1 - P_N)(1 - P_V) \quad P(\{1,2\}) = (1 - P_N)P_V$$

$$P(\{2,2\}) = P_N P_V \qquad P(\{2,1\}) = P_N(1 - P_V)$$

- Prior probs on patterns: λ_{11}, λ_{12}, λ_{22}, λ_{21}
- Assume $\lambda_{21} = 0$

$$\hat{\alpha} = \frac{\lambda_{22} P_{22}}{\lambda_{11} P_{11} + \lambda_{12} P_{12} + \lambda_{22} P_{22}}, \quad \hat{\beta} = \frac{\lambda_{12} P_{12} + \lambda_{22} P_{22}}{\lambda_{11} P_{11} + \lambda_{12} P_{12} + \lambda_{22} P_{22}}$$

Fixed points

- (1,1), (0,0), (0,1)

Bifurcations

- Corresponding to observed changes:

1. {1,1} $\rightarrow$ {1,2}
2. {2,2} $\rightarrow$ {1,2}
3. {1,2} $\rightarrow$ {1,1}
4. {1,2} $\rightarrow$ {2,2}

Model 4: Summary

- {1,1}, {1,2}, {2,2}: ✓
- Stable var: ✗
- *{2,1}: ✓
- Patterns of change: ✓

Model 5: Coupling by priors + mistransmission

- Model 4, but with asymmetric mistransmission
- Same dynamics, except:
- Stable variation in *one* of N or V
  - Can't have 100% {1,1} or {2,2}
- All bifurcations frequency-dependent.
  - change to {1,2} triggered by falling frequency.

Model 5: Summary

- {1,1}, {1,2}, {2,2}: ✓
- Stable var: ✓
- *{2,1}: ✓
- Patterns of change: ✓

Model comparison

Property	Model 1	Model 2	Model 3	Model 4	Model 5
*{2,1}	✓	✓	✓	✓	✓
{1,1}, {1,2}, {2,2}	✗	✓	✗	✓	✓
Obs. stable variation	✗	✓	✗	✗	✓
Observed changes	✗	✗	✗	✓	✓

Models: observations

- Mistransmission (Models 1, 3, 5)
  - No 100% {1,1}, {1,2} stable states
  - c.f. proposed misperception, misproduction sources of change (Ohala, Blevins)
- Analogy via priors (Models 4, 5)
  - Bifurcations corresponding to observed changes
- Mistransmission + analogy (Model 5)
  - Captures much of observed dynamics
  - Stable variation emergent: not present in Model 1 or 4
  - Prefix classes? (via λ_{ij})

Discussion

- Actuation: Why does stress ever change?
  - Bifurcation where a fixed point becomes unstable.
- Constraints: Why these patterns of change?
  - Bifurcation structure