

## Explorations for Pierrehumbert (2001)

### Parameters of the Pierrehumbert model

| Parameter     | Description   | Pierrehumbert value |
|---------------|---|---------------------|
| $\tau$        | Decay rate of exemplars   | 2000                |
| $\mathcal{E}$ | Magnitude of uniform distribution from which the production noise $\epsilon$ is drawn | 0.1                 |
| $w$           | width of window used in classification  | 0.05                |
| $\lambda$     | lenition bias   | varies              |
| $n_{trench}$  | number of exemplars to average over for entrenchment                                  | 500                 |
| $p$           | probability of producing category 1, vs. category 2 (for Model 4)                     | 0.75                |

### Exploration topics

Your group should choose a topic from the list below, then post your choice to Piazza. Try to choose a topic which no other group has chosen (though this may not be possible, if there are enough groups). Topics are listed roughly in increasing order of complexity.

1. **Noise.** In the paper, noise is drawn from a uniform distribution on  $[-\mathcal{E}, \mathcal{E}]$ . What is the effect of changing  $\mathcal{E}$  on the evolution of the mean and variance, for the one-category case?
2. **Entrenchment: two categories.** Explore the evolution of the two category distributions in a two-category model with entrenchment only ( $\lambda = 0$ ). Do the category variances stabilize as for the single-category case? (Optional: What does the evolution look like when lenition is added ( $\lambda > 0$ )? How do the values of  $\lambda$  and  $n_{trench}$  interact in determining the evolution?)
3. **Window size.** How does changing the width of the classification window  $w$  impact the evolution in the two-category case, when other parameters ( $\lambda, \tau, p$ ) are held constant?
4. **Lenition.** What happens in the neutralization model as the amount of lenition ( $\lambda$ ) is varied? Does the speed at which the categories merge change? Does the location of the peak of the merged category change?
5. **Changes to  $\tau$ .** How does changing  $\tau$  effect the evolution of the mean and variance of a single category over time? Explore and compare two or more of the one-category models (noise only, noise + lenition, noise + entrenchment, noise + lenition + entrenchment) for several different values of  $\tau$ .

6. **Entrenchment: one category.** Examples in the paper and in class showed that the variance of a single category stabilizes with the implementation of entrenchment. Check if this is true for a range of values of  $n_{trench}$ . In addition, evaluate the claim that the size of the stable variance decreases as  $n_{trench}$  increases.
7. **Frequency without lenition.** The neutralization model presented in the paper includes entrenchment, a lenition bias for the ‘marked’ category, and a higher frequency ( $p = 0.75$ ) for the marked category. Pierrehumbert suggests that the difference in category frequency and the lenition bias drive neutralization of the marked category towards the unmarked category. What happens in the case where the categories have unequal frequencies, but there is *no* lenition bias? Are there initial values (e.g., by setting the starting means of the two categories very close together) which lead to the less frequent category neutralizing to the more frequent category?
8. **Frequency and lenition.** Consider the more general case of #7, where both the size of the lenition bias and the frequency of the ‘marked’ category are varied. Explore the consequences of varying  $p$  and  $\lambda$ . What is required for neutralization to obtain: differences in  $p$ ,  $\lambda$ , or both?

### *Suggestions*

- The number of values you try when varying a parameter is up to you, but you should make sure to report a range of values which demonstrate how the behavior of the model changes as the parameter is varied. (For example,  $n_{trench} = 100, 101, 102$  will probably give very similar behavior when other parameters are held constant, while  $n_{trench} = 5, 100, 250, 500$  will not.)
- You may wish to run your initial simulations for a relatively short number of iterations (e.g., 3000) to check that they are doing what you think they should be doing before running them for a more realistic number of iterations (e.g., 100000).

### **Advanced topics**

These topics involve modifications to the source code. They are listed more or less in order of the amount of modification required. These may also provide some inspiration for possible course projects.

9. **Perceptual entrenchment.** Pierrehumbert describes the possibility of implementing entrenchment over a fixed neighborhood of exemplars in perception, but notes that this introduce too much instability when there are very few exemplars in a category. Implement this type of entrenchment and describe its behavior. Is instability simply a function of the starting state?
10. **Related models.** Wedel (2004) asserts that his exemplar model doesn’t require explicit entrenchment because this behavior emerges from other properties of the architecture. What aspects of the model as it is implemented here would need to be modified or extended to produce this behavior? Are actual modifications necessary, or is Wedel’s model just a version of Pierrehumbert’s with particular parameter settings?

11. **3 categories.** Modify the architecture to add a third category label. What kinds of behaviors do you observe as the probabilities of producing each category label are varied? Are these behaviors modulated by the inclusion/exclusion of lenition, entrenchment, or both?
12. **2 dimensions.** Pierrehumbert's model implements a phonetic category using a single dimension. However, phonetic categories are in general *multidimensional*: stop consonants (P's example) are cued primarily by VOT, but also by  $F_0$ , formant transitions, and other cues depending on position; vowels are cued by at least  $F_1$  and  $F_2$ ; and so on. Modify the architecture to work for a phonetic category in two dimensions. (It may be helpful to keep  $F_1$  &  $F_2$  of vowels in mind as an example.) This will entail a number of decisions, including how to define a window function, production noise, and category variance.<sup>1</sup>

With a 2D exemplar model implemented, there are many questions you could explore, such as:

- Is the qualitative relationship between entrenchment and category variance the same as in the 1D case? (Without entrenchment, the category variance always spreads out; with entrenchment, it always stabilizes.)
- How do  $p$  and the amount/direction of lenition relate to whether neutralization occurs?
- What kind of behavior can occur if there is much more noise in one dimension than another? (For example, /u/ is subject to greatly variable degrees of fronting ( $F_2$ ) in English depending on context, while  $F_1$  varies much less.)

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<sup>1</sup>If you choose this topic, we recommend first getting a minimal 2D model to work: one category, no lenition or entrenchment. Then, add additional functionality (lenition, entrenchment, two categories) a little at a time, making sure your code gives sensible simulation results after each addition.