Pratical 1 - Group 01

Lodrik Adam, Alexandra Boado, Sophie Daya, Jeff Macaraeg, Julien Perini

November 14, 2024

Table of contents


```
# load the required packages and install them if they are not.
source(here::here("code","setup.R"))

# getiing the working directory
wd <- here::here()

# Loading the data
crypto_data <- read.csv(here("data", "crypto_data.csv"))

# scaling factor
scaling_factor <- 100000
```

Part 1: Financial returns and normality

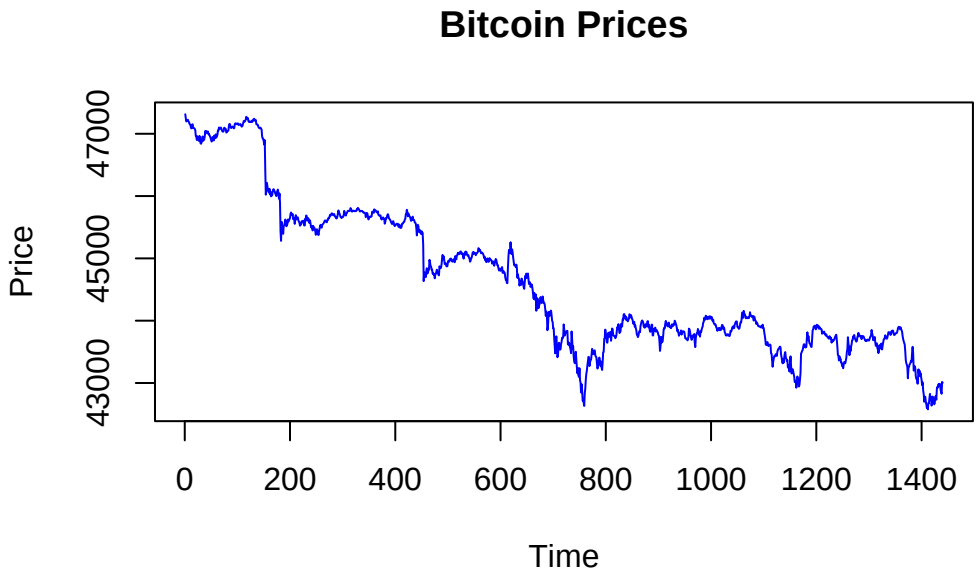
The working directory is set to: /Users/lodrik/Documents/GitHub/RA_Praticals

a) Load Bitcoin data and assess price stationarity

> Question
>
> Read in the Bitcoin data from file Crypto data.csv. Then, assess the stationarity of the (raw) Bitcoin prices.

First, let's take a look at the Bitcoin Prices on a plot.

```
## Step 1: Extract the Bitcoin prices
bitcoin_prices <- crypto_data$Bitcoin

## Step 2: Plot the Bitcoin prices
plot(bitcoin_prices,
     type="l",
     col="blue",
     main="Bitcoin Prices",
     xlab="Time",
     ylab="Price")
```



The graph of the raw Bitcoin prices suggest that the series might not be stationary.

Let's perform the Augmented Dickey-Fuller test to check if the raw Bitcoin prices are stationary.

```
## Step 3: test for stationarity
adf.test(crypto_data$Bitcoin)
```

```
    Augmented Dickey-Fuller Test

data:  crypto_data$Bitcoin
Dickey-Fuller = -2.4484, Lag order = 11, p-value = 0.3885
alternative hypothesis: stationary
```

Since the p-value is significantly bigger than 0.05, we can not reject the null hypothesis and therefore, we can conclude that the raw Bitcoin prices are non-stationary.

b) Create and plot Bitcoin negative log returns, assess stationarity

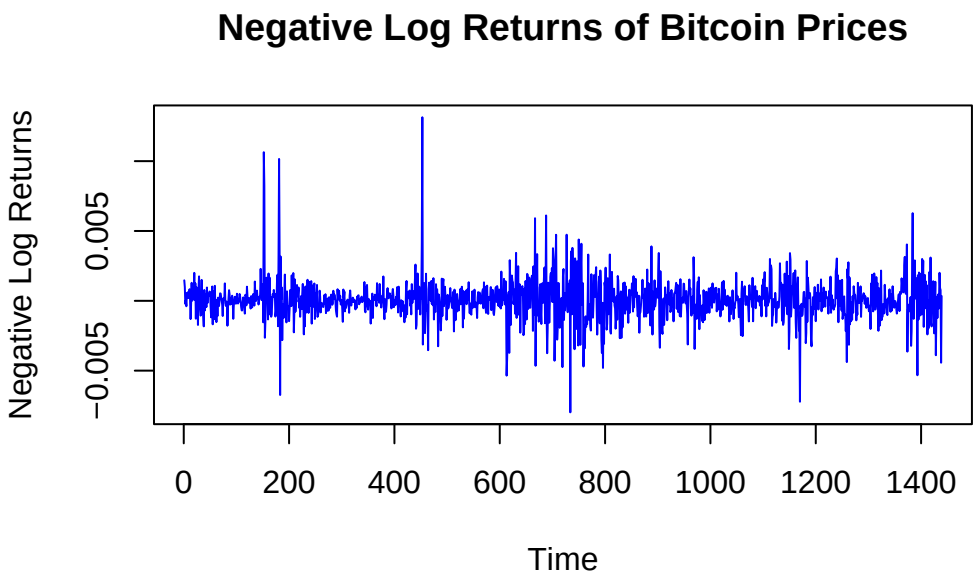
> Question
>
> Create a function to transform the Bitcoin prices into their negative log returns counterparts. Plot the latter series and assess their stationarity. To compare the series, also plot the negative log returns on a common scale.

Let's create a function to compute the negative log returns of a given price series. We will then apply this function to the Bitcoin prices to compute the negative log returns.

```
## Step 1: Create a function to compute negative log returns
negative_log_returns <- function(prices) {
  return(-diff(log(prices)))
}

## Step 2: Use the fucntion on Bitcoin prices
neg_log_returns_bitcoin <- negative_log_returns(bitcoin_prices)
```

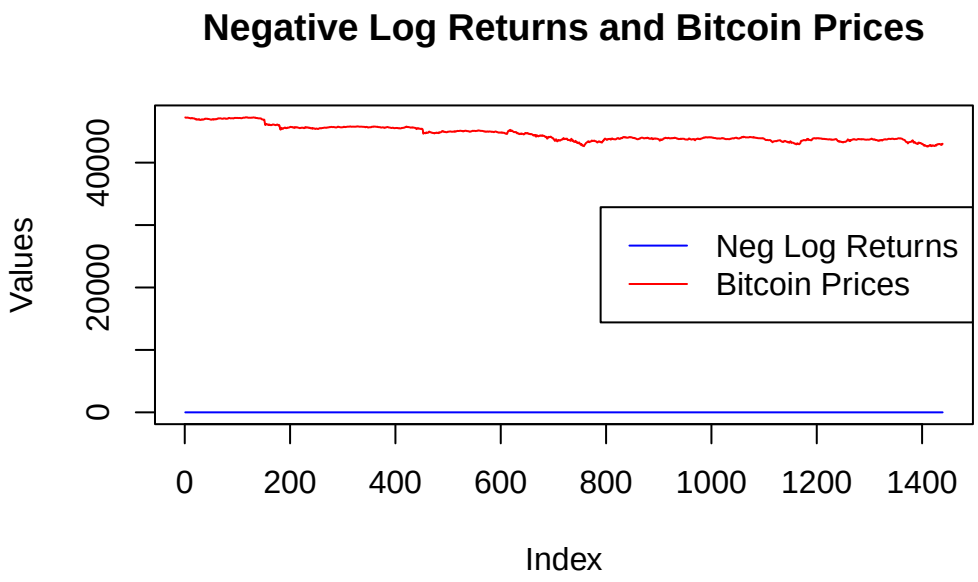
We can now plot the negative log returns series and the raw Bitcoin prices to compare.

```
## Step 3: Plot the negative log returns series
plot(neg_log_returns_bitcoin,
     type="l",
     col="blue",
     main="Negative Log Returns of Bitcoin Prices",
     xlab="Time",
     ylab="Negative Log Returns")
```



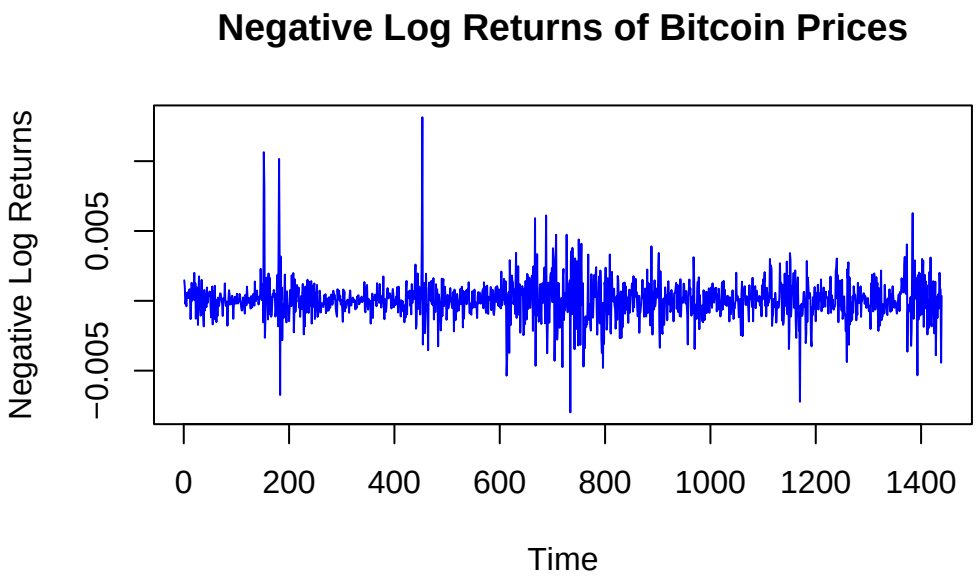
If we scale bitcoin prices and negative log returns, we can compare both time series on a plot with a common scale.

```
# Top Plot: Plot both time series on the same graph
trimmed_bitcoin_prices <- bitcoin_prices[-1]  # Make sure lengths match
plot(neg_log_returns_bitcoin,
     type = "l",
     col = "blue",
     ylab = "Values",
     xlab = "Index",
     main = "Negative Log Returns and Bitcoin Prices",
     ylim = range(c(neg_log_returns_bitcoin, trimmed_bitcoin_prices)))

# Add Bitcoin prices on the same plot
lines(trimmed_bitcoin_prices, col = "red")

# Add a legend
legend("right",
       legend = c("Neg Log Returns", "Bitcoin Prices"),
       col = c("blue", "red"),
       lty = 1)
```

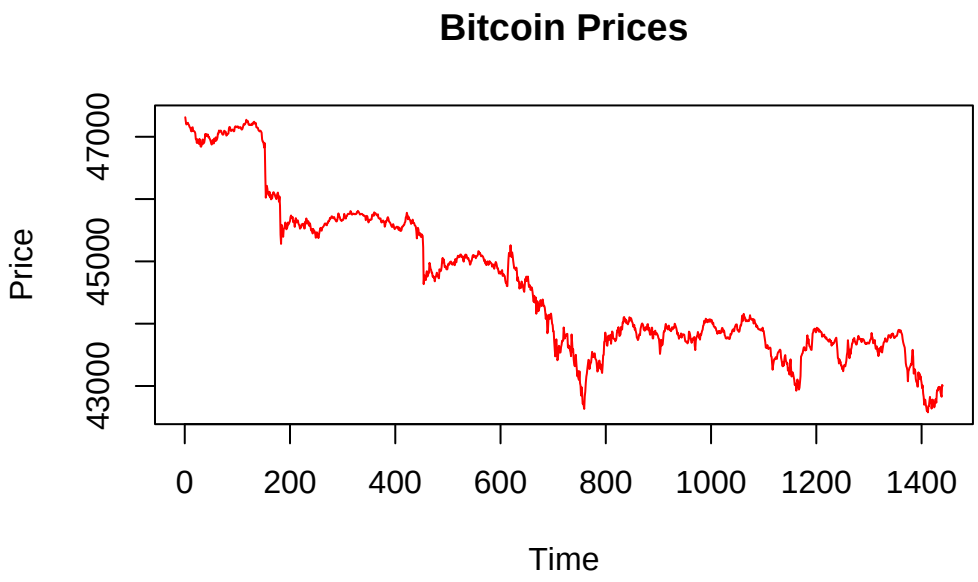



```
# Reset the plotting area to default settings (optional, for future plots)
par(mfrow=c(1,1))
```

```
# Set up the plotting area to have 3 rows and 1 column
par(mfrow=c(1,1))

# Bottom Left Plot: Plot the negative log returns series
plot(neg_log_returns_bitcoin,
     type = "l",
     col = "blue",
     main = "Negative Log Returns of Bitcoin Prices",
     xlab = "Time",
     ylab = "Negative Log Returns")
```




```r
# Bottom Right Plot: Plot the Bitcoin prices
plot(bitcoin_prices,
     type = "l",
     col = "red",
     main = "Bitcoin Prices",
     xlab = "Time",
     ylab = "Price")
```



Visually, the negative log returns series does not appear to indicate a clear trend or seasonality. The variance, although it fluctuates in the middle, seems relatively constant. This observation suggests that the series may be stationary. To confirm this, we will perform the Augmented Dickey-Fuller test to assess the stationarity of the negative log returns.

```
## Step 5: Test the stationarity of the negative log returns with the Augmented Dickey-Ful
adf.test(neg_log_returns_bitcoin)
```

```
    Augmented Dickey-Fuller Test

data:  neg_log_returns_bitcoin
Dickey-Fuller = -11.035, Lag order = 11, p-value = 0.01
alternative hypothesis: stationary
```

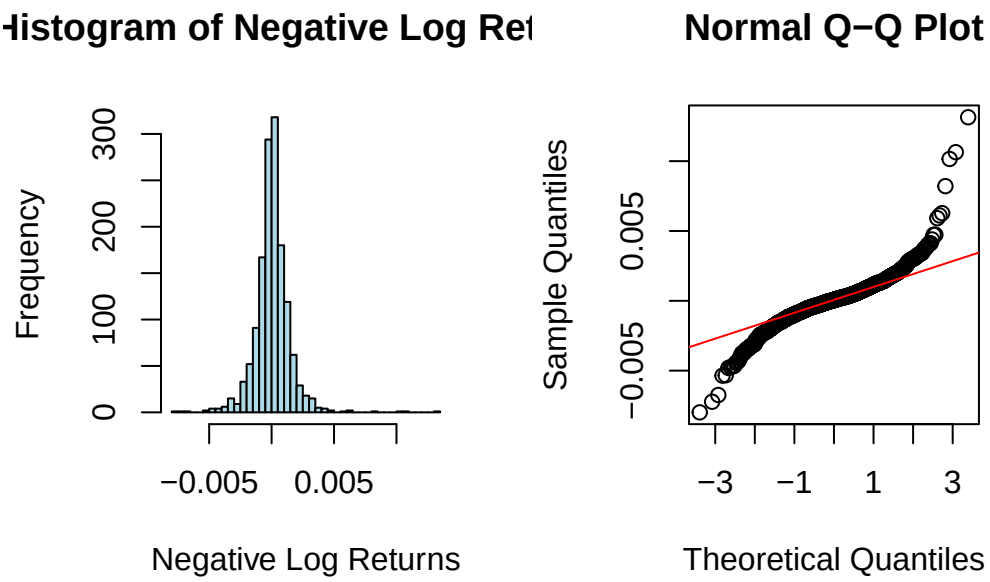
Since the p-value is significantly smaller than 0.05, we can reject the null hypothesis and conclude that the negative log returns series is stationary.

c) Check negative log returns normality with histograms, QQ-plots, Anderson-Darling

> Question
>
> Are the negative log returns normally distributed? Draw histograms, check QQ-plots and use an Anderson-Darling testing procedure to answer this question.

Let's first plot the histogram and QQ-plot of the negative log returns to visually assess the normality.

```
## Step 1: Plot the histogram and QQ-plot of the negative log returns
par(mfrow=c(1, 2))

# Plot the histogram of the negative log returns
hist(neg_log_returns_bitcoin,
     breaks=50,
     col="lightblue",
     main="Histogram of Negative Log Returns",
     xlab="Negative Log Returns")

# Plot the QQ-plot of the negative log returns
qqnorm(neg_log_returns_bitcoin)
qqline(neg_log_returns_bitcoin,
       col="red")
```




```
par(mfrow = c(1, 1))
```

The histogram of the negative log returns suggests that the data may follow a normal distribution. However, we need to perform a formal test to confirm this.

```
## Step 2: Perform Anderson-Darling test for normality
ad.test(neg_log_returns_bitcoin)
```

```
    Anderson-Darling normality test

data:  neg_log_returns_bitcoin
A = 26.277, p-value < 2.2e-16
```

Even though the Histogram suggest that the negative log returns follows a normal distribution, the p-value when performing the Andersen-Darling test is smaller than 5%. It indicates that the data does not follow a normal distribution. The Normal Q-Q plot suggest also that the data does not follow a normal distribution.

d) Fit t-distribution, compare with Normal via QQ-plot analysis

> Question
>
> Fit a t-distribution to the negative log returns using `fitdistr()`. Using a QQ-plot, decide whether the fit is better than with a Normal distribution, based on your answer in (c).

Let's fit a t-distribution to the negative log returns and compare it with the normal distribution using a QQ-plot.

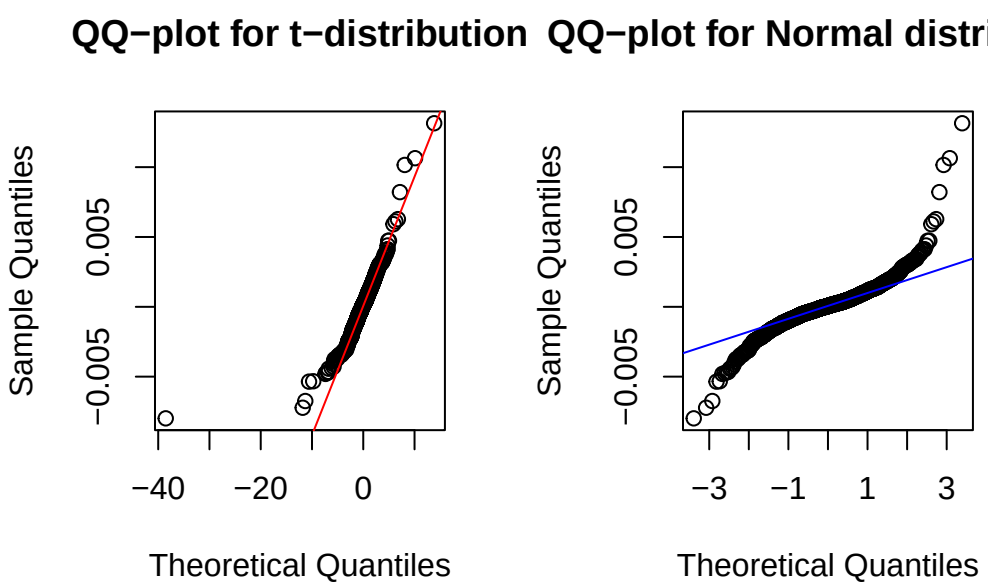
```
## Step 1: Fit a t-distribution to the negative log returns
fit_t <- fitdistr(scaling_factor * neg_log_returns_bitcoin, "t") # Multiply by 100000 to a
```

The t-distribution fit parameters are:

- mean: 5.65
- standard deviation: 84.15
- degrees of freedom: 2.77

We can now compare the QQ-plot of the t-distribution with the QQ-plot of the normal distribution of question c).

```
## Step 2: Create a QQ-plot for the t-distribution and the normal distribution
par(mfrow = c(1, 2))

# Generate QQ-plot for t-distribution
df_t <- fit_t$estimate[3]  # Degrees of freedom from the fit
qqplot(rt(length(neg_log_returns_bitcoin),
          df=df_t),
       neg_log_returns_bitcoin,
       main="QQ-plot for t-distribution",
       xlab="Theoretical Quantiles",
       ylab="Sample Quantiles")
qqline(neg_log_returns_bitcoin,
       col="red")

# Generate QQ-plot for normal distribution
qqnorm(neg_log_returns_bitcoin,
       main="QQ-plot for Normal distribution")
qqline(neg_log_returns_bitcoin,
       col="blue")
```




```
par(mfrow = c(1, 1))
```

As we can see, the QQ-plot for the t-distribution is closer to the 45-degree line than the QQ-plot for the normal distribution. This suggests that the t-distribution is a better fit for the negative log returns than the normal distribution.

e) Compare t-distribution and normal tails

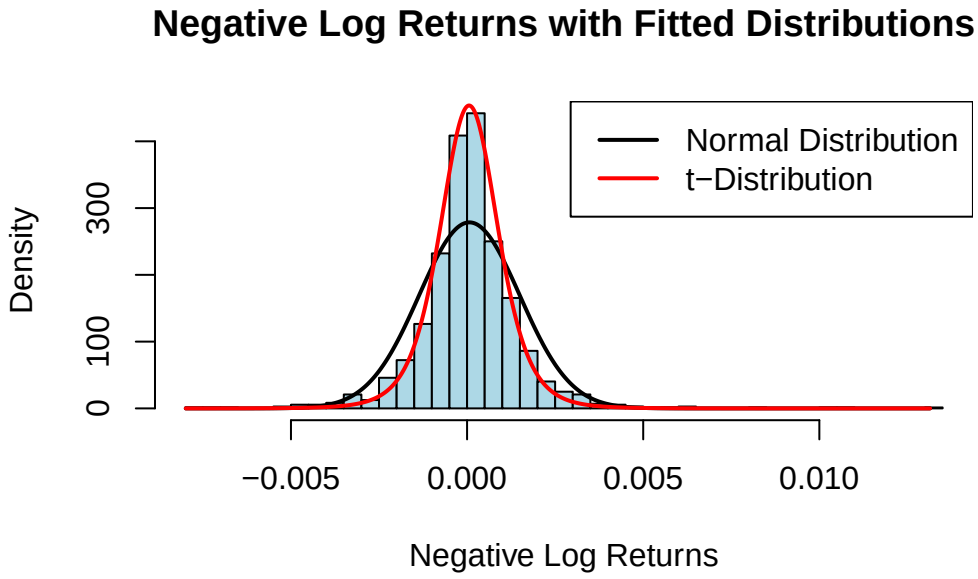
> Question
>
> Compare the tails of the density of the t-distribution and the normal distribution. Can we expect more extreme, unexpected events in t-distribution or in normal distribution? What can you conclude about the extreme events of our bitcoin data?

To compare the tails of the t-distribution and the normal distribution, we will plot the density functions of both distributions and visually assess the differences.

```
## Step 1: Fit the normal distribution to the negative log returns
fit_norm <- fitdistr(scaling_factor * neg_log_returns_bitcoin, "normal")

# Generate a sequence of values for the x-axis (log returns)
```

```r
x <- seq(min(neg_log_returns_bitcoin), max(neg_log_returns_bitcoin), length = 1000)

## Step 2: Scale back the mean and sd for plotting (for both normal and t-distributions)
# For normal distribution
scaled_mean_norm <- fit_norm$estimate[1] / scaling_factor
scaled_sd_norm <- fit_norm$estimate[2] / scaling_factor

# For t-distribution
scaled_mean_t <- fit_t$estimate[1] / scaling_factor
scaled_sd_t <- fit_t$estimate[2] / scaling_factor

# Density for the normal distribution using the scaled mean and sd
dens_norm <- dnorm(x, mean = scaled_mean_norm, sd = scaled_sd_norm)

# Density for the t-distribution using the scaled parameters
dens_t <- dt((x - scaled_mean_t) / scaled_sd_t, df = fit_t$estimate[1]) / scaled_sd_t

## Step 3 : Plot the histogram of negative log returns
hist(neg_log_returns_bitcoin,
     breaks = 50,
     col = "lightblue",
     freq = FALSE,  # For density plot
     main = "Negative Log Returns with Fitted Distributions",
     xlab = "Negative Log Returns")

# Add the normal distribution curve
lines(x,
      dens_norm,
      col = "black",
      lwd = 2,
      lty = 1)

# Add the t-distribution curve
lines(x,
      dens_t,
      col = "red",
      lwd = 2,
      lty = 1)

# Add a legend
legend("topright",
```

```
        legend = c("Normal Distribution", "t-Distribution"),
        col = c("black", "red"),
        lty = c(1, 1),
        lwd = 2)
```



Visually, the tails of the t-distribution are heavier than those of the normal distribution. This means that the t-distribution assigns more probability to extreme events than the normal distribution. Therefore, we can expect more extreme, unexpected events in the t-distribution than in the normal distribution. This observation is consistent with the QQ-plot analysis in question d), where the t-distribution was a better fit for the negative log returns than the normal distribution.

```
# load the required packages and install them if they are not.
source(here::here("code","setup.R"))

# Loading the data
crypto_data <- read.csv(here("data", "crypto_data.csv"))
```

Part 3: Financial returns and normality

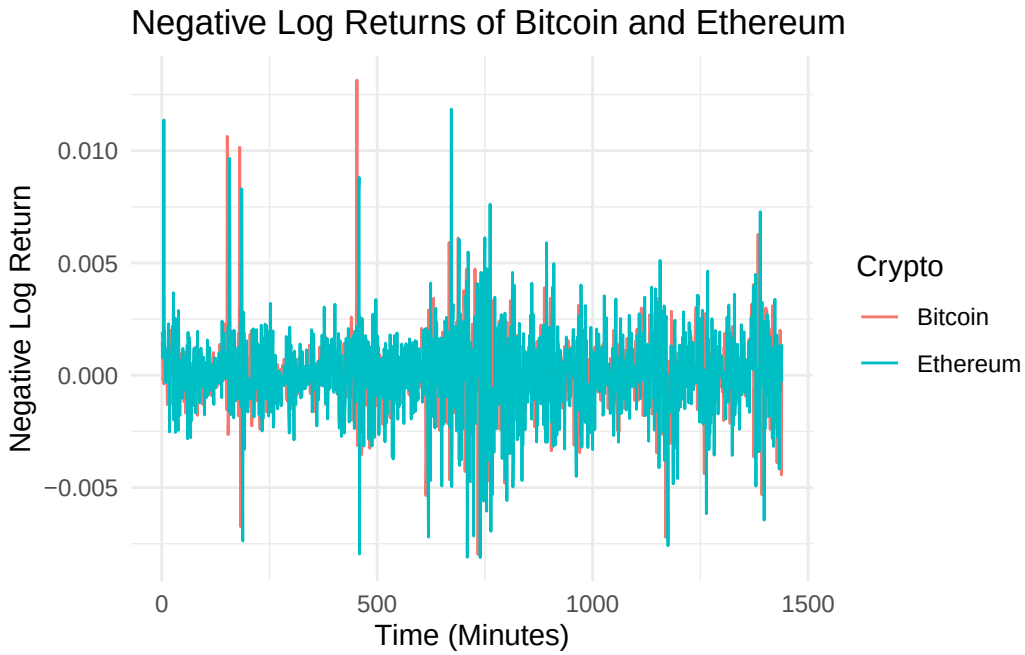
a) Negative log returns of Bitcoin and ETH independent?

Question

Are the negative log returns of Bitcoin and ETH dependent? Compute the correlation using `cor.test()` function. Can we conclude that these series are independent?

```
# Negative log returns function
negative_log_returns <- function(prices) {
  -diff(log(prices))
}

# Compute negative log returns for Bitcoin and Ethereum
neglogret_eth <- negative_log_returns(crypto_data$Ethereum)
neglogret_btc <- negative_log_returns(crypto_data$Bitcoin)

# Create a sequence for time indices matching the length of negative log returns
time_indices <- seq_along(neglogret_eth)

# Create a data frame with time indices and negative log returns
nlr_df <- data.frame(
  Time = time_indices,
  Ethereum = neglogret_eth,
  Bitcoin = neglogret_btc
)

# Reshape data to long format for plotting
nlr_long_df <- nlr_df %>%
  pivot_longer(
    cols = c(Ethereum, Bitcoin),
    names_to = "Crypto",
    values_to = "NegativeLogReturn"
  )

# Plot the negative log returns of Bitcoin and Ethereum
ggplot(nlr_long_df, aes(x = Time, y = NegativeLogReturn, color = Crypto)) +
  geom_line() +
  labs(
    title = "Negative Log Returns of Bitcoin and Ethereum",
```

```
    x = "Time (Minutes)",
    y = "Negative Log Return"
  ) +
  theme_minimal()
```




```
# Perform correlation test between Ethereum and Bitcoin negative log returns
cor_test_result <- cor.test(nlr_df$Ethereum, nlr_df$Bitcoin)
print(cor_test_result)
```

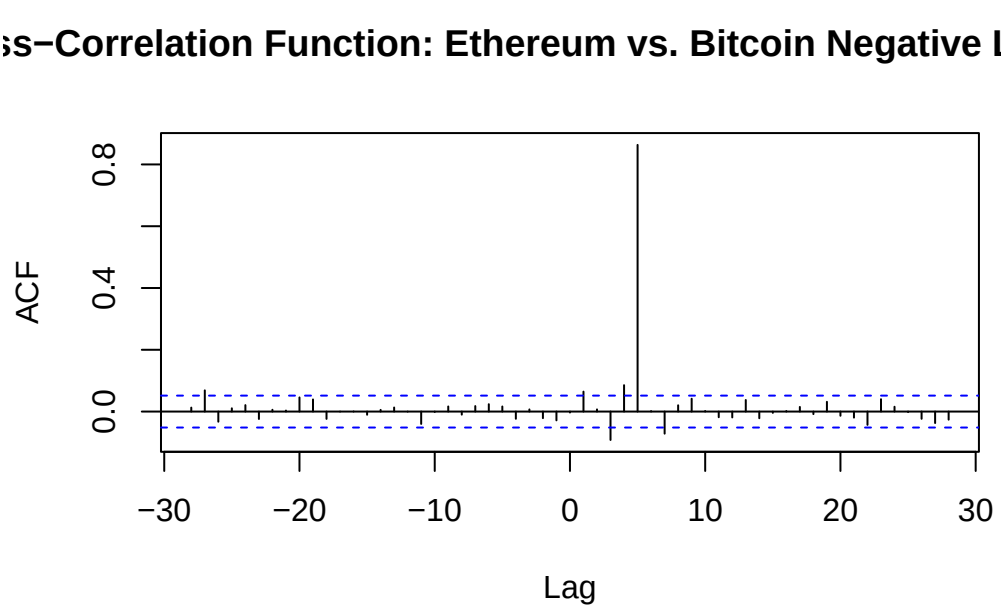
```
    Pearson's product-moment correlation

data:  nlr_df$Ethereum and nlr_df$Bitcoin
t = -0.11935, df = 1437, p-value = 0.905
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.05481486  0.04853492
sample estimates:
        cor
-0.00314838
```

```
#No significant linear relationship between the variables (p-value = 1). However,
#We cannot conclude that they are independant only based on the correlation test.
#There could be, for example, a non-linear relationship between the 2.
```

b)

> Question
>
> Calculate the cross-correlation function (CCF) between the negative log returns of Bitcoin and ETH. What do you observe?

```
# Calculate and plot the cross-correlation function (CCF)
ccf(nlr_df$Ethereum, nlr_df$Bitcoin, main = "Cross-Correlation Function: Ethereum vs. Bitc
```




```
#We can see a very significant spike in cross correlation at around lag 5.
```

c)

> Question
>
> Is one of the time series good predictor of the second? Assess whether there is any predictive power between the negative log returns of Bitcoin and ETH. You can use `grangertest()` in the `lmtest` package with carefully chosen hyperparameter `order`. What is your conclusion?

```
# Testing if Ethereum Granger-causes Bitcoin
granger_test_eth_to_btc <- grangertest(Bitcoin ~ Ethereum, order = 5, data = nlr_df)
print(granger_test_eth_to_btc)
```

```
Granger causality test

Model 1: Bitcoin ~ Lags(Bitcoin, 1:5) + Lags(Ethereum, 1:5)
Model 2: Bitcoin ~ Lags(Bitcoin, 1:5)
  Res.Df Df      F Pr(>F)
1   1423
2   1428 -5 0.5828 0.7132
```

```
# Interpretation: A very tiny p-value suggests Ethereum Granger-causes Bitcoin

# Testing if Bitcoin Granger-causes Ethereum
granger_test_btc_to_eth <- grangertest(Ethereum ~ Bitcoin, order = 5, data = nlr_df)
print(granger_test_btc_to_eth)
```

```
Granger causality test

Model 1: Ethereum ~ Lags(Ethereum, 1:5) + Lags(Bitcoin, 1:5)
Model 2: Ethereum ~ Lags(Ethereum, 1:5)
  Res.Df Df      F    Pr(>F)
1   1423
2   1428 -5 974.03 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Interpretation: A large p-value suggests Bitcoin does not Granger-cause Ethereum
```

d)

> Question
>
> Based on your answer in (c), answer the following questions:

d.1)

> Question
>
> We observe an extreme sudden drop in Bitcoin stocks. What should we expect that will happen with ETH stocks?

d.2)

> Question
>
> We observe an extreme sudden drop in ETH stocks. What should we expect that will happen with Bitcoin stocks?