

Simple Regression Models

Introduction

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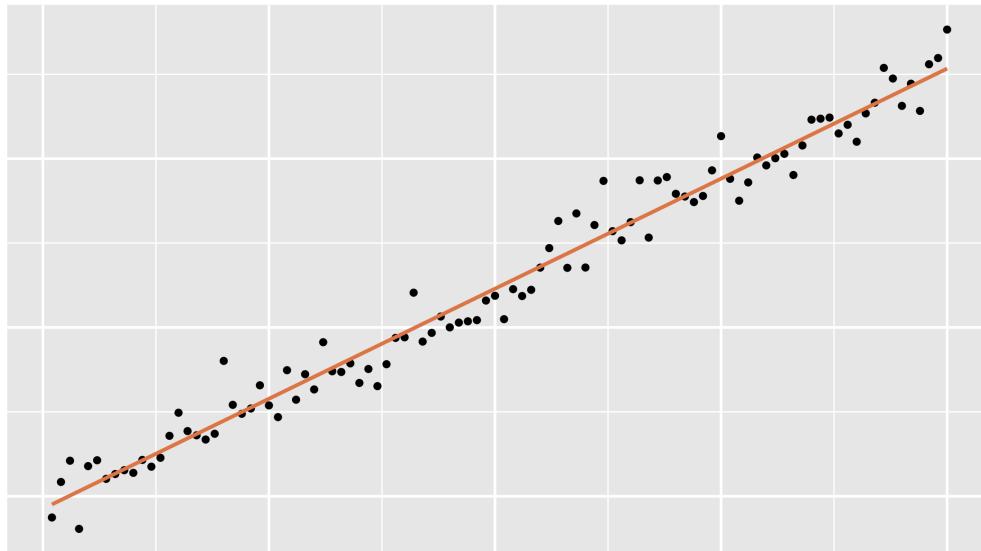
The following content is based on Mine Çetinkaya-Rundel's excellent book Data Science in a Box

What is a model?

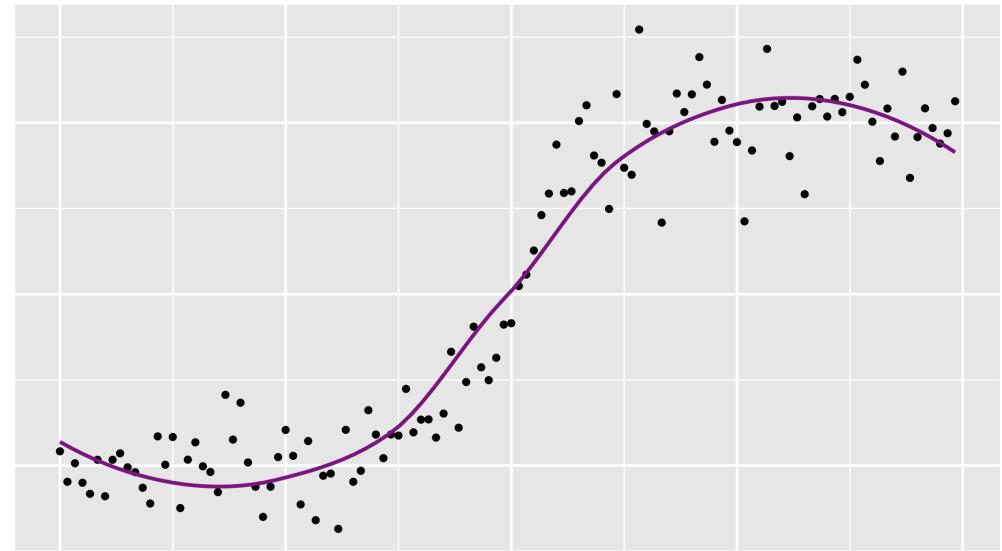
Modelling

- Use models to explain the relationship between variables and to make predictions
- For now we will focus on **linear** models (but remember there are *many many* other types of models too!)

Linear



Non-linear



Data: Paris Paintings

Paris Paintings

```
pp <- read_csv("data/paris-paintings.csv", na = c("n/a", "", "NA"))
```

- Source: Printed catalogues of 28 auction sales in Paris, 1764 - 1780
- Data curators Sandra van Ginhoven and Hilary Coe Cronheim (who were PhD students in the Duke Art, Law, and Markets Initiative at the time of putting together this dataset) translated and tabulated the catalogues
- 3393 paintings, their prices, and descriptive details from sales catalogues over 60 variables

Auctions today

Old Master & British Paintings Evening Sale Soars over Estimate

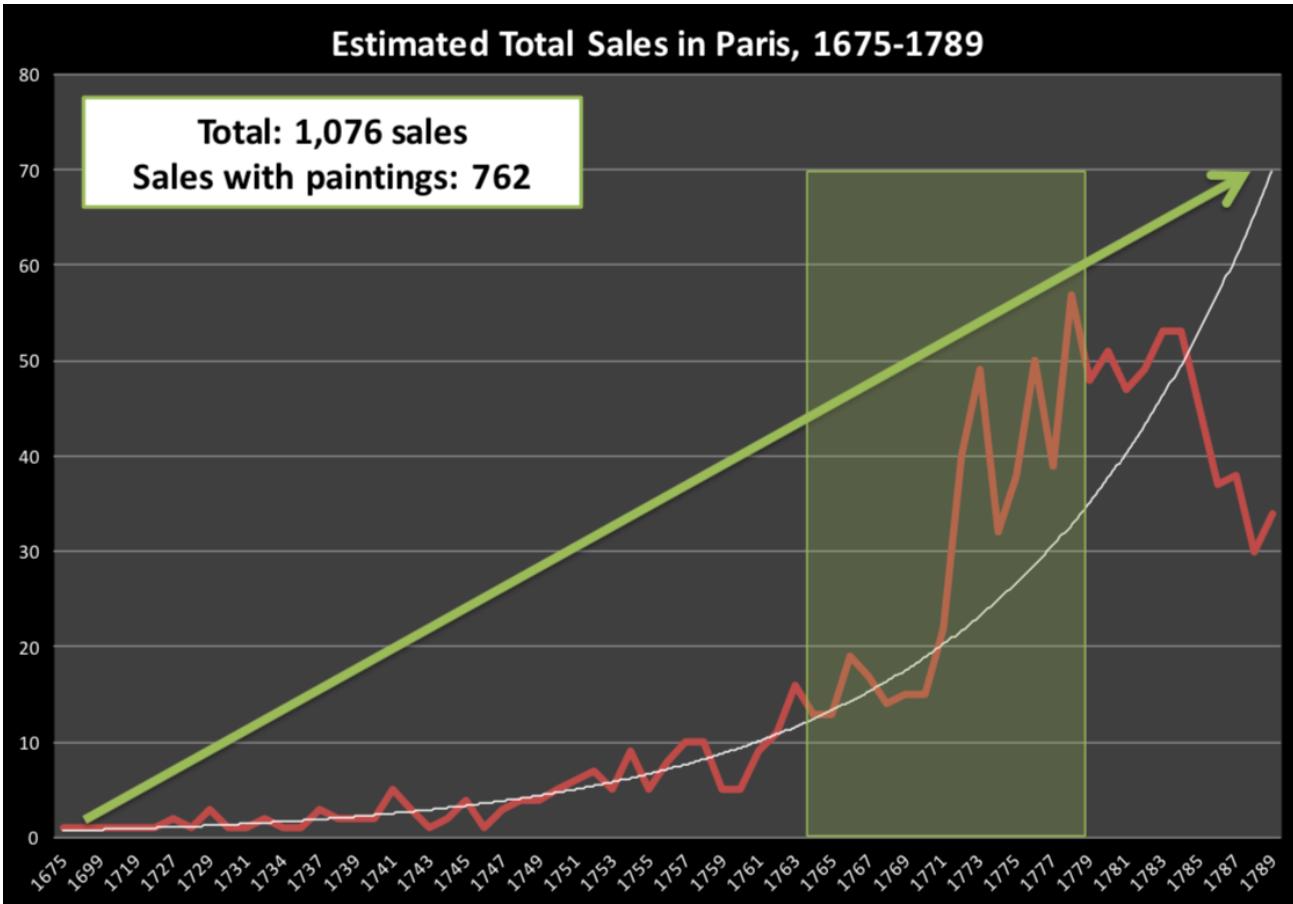


Auctions back in the day



Pierre-Antoine de Machy, Public Sale at the Hôtel Bullion, Musée Carnavalet, Paris (18th century)

Paris auction market

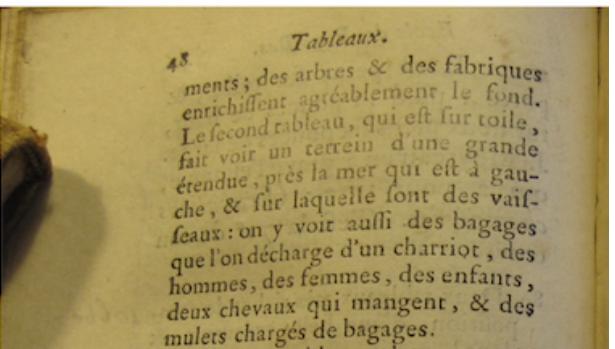
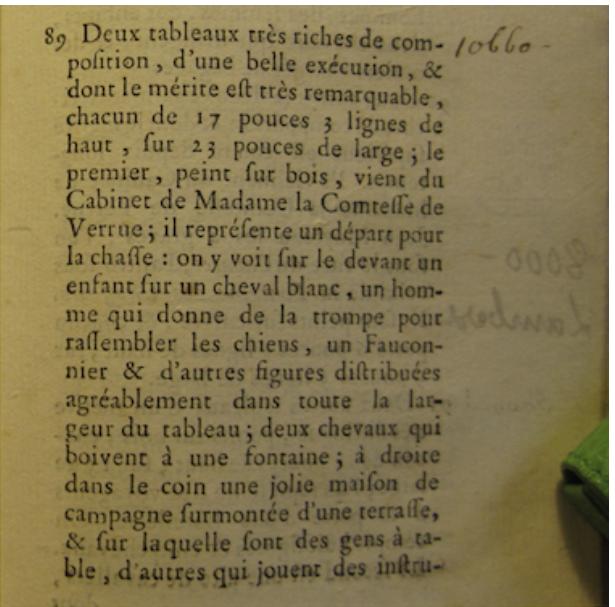


Plot credit: Sandra van Ginhoven

Départ pour la chasse



Auction catalog text



Two paintings very rich in composition, of a beautiful execution, and whose merit is very remarkable, each 17 inches 3 lines high, 23 inches wide; the first, painted on wood, comes from the Cabinet of Madame la Comtesse de Verrue; it represents a departure for the hunt: it shows in the front a child on a white horse, a man who gives the horn to gather the dogs, a falconer and other figures nicely distributed across the width of the painting; two horses drinking from a fountain; on the right in the corner a lovely country house topped by a terrace, on which people are at the table, others who play instruments; trees and fabriques pleasantly enrich the background.

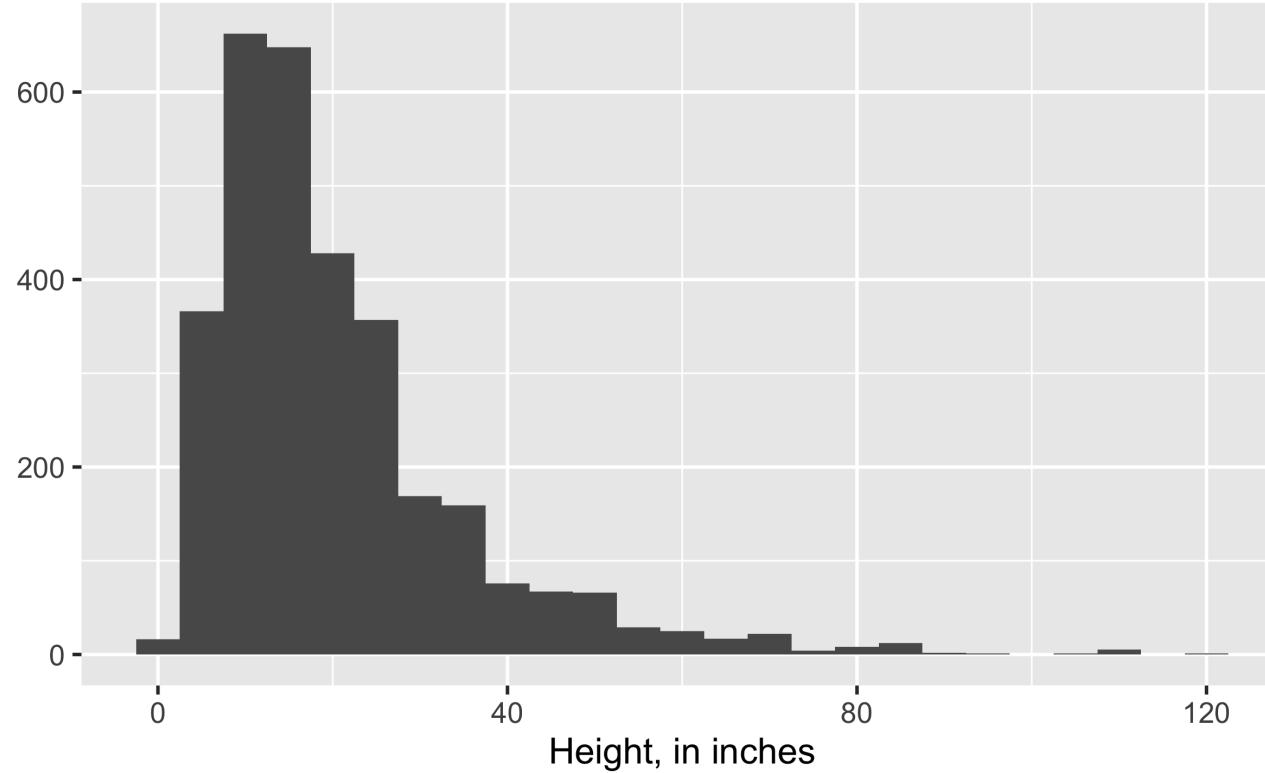

```
pp %>%
  filter(name == "R1777-89a") %>%
  glimpse()
```

```
## Rows: 1
## Columns: 61
## $ name      <chr> "R1777-89a"
## $ sale       <chr> "R1777"
## $ lot        <chr> "89"
## $ position   <dbl> 0.3755274
## $ dealer     <chr> "R"
## $ year       <dbl> 1777
## $ origin_author <chr> "D/FL"
## $ origin_cat  <chr> "D/FL"
## $ school_pntg <chr> "D/FL"
## $ diff_origin <dbl> 0
## $ logprice    <dbl> 8.575462
## $ price       <dbl> 5300
## $ count       <dbl> 1
## $ subject     <chr> "D\u00e9part pour la chasse"
## $ authorstandard <chr> "Wouwerman, Philips"
## $ artistliving  <dbl> 0
## $ authorstyle   <chr> NA
## $ author      <chr> "Philippe Wouwermans"
## $ winningbidder <chr> "Langlier, Jacques for Poullain, Ant..."
## $ winningbiddertype <chr> "DC"
## $ endbuyer    <chr> "C"
...
## $ Interm      <dbl> 1
## $ type_intermed <chr> "D"
## $ Height_in    <dbl> 17.25
## $ Width_in     <dbl> 23
## $ Surface_Rect <dbl> 396.75
## $ Diam_in      <dbl> NA
## $ Surface_Rnd  <dbl> NA
## $ Shape        <chr> "squ_rect"
## $ Surface       <dbl> 396.75
## $ material     <chr> "bois"
## $ mat          <chr> "b"
## $ materialCat   <chr> "wood"
## $ quantity     <dbl> 1
## $ nfigures     <dbl> 0
## $ engraved      <dbl> 0
## $ original      <dbl> 0
## $ prevcoll     <dbl> 1
## $ othartist    <dbl> 0
## $ paired        <dbl> 1
## $ figures       <dbl> 0
## $ finished      <dbl> 0
```

Modeling the relationship between variables

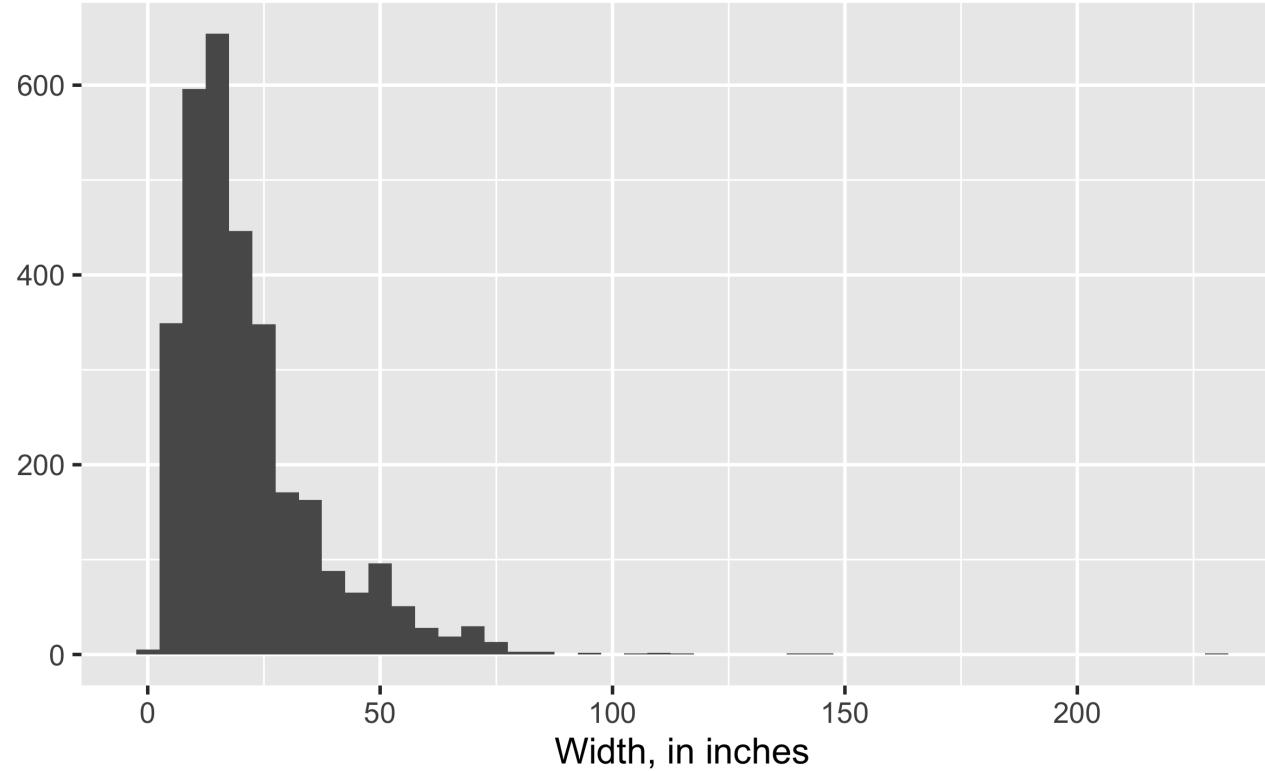
Heights

```
ggplot(data = pp, aes(x = Height_in)) +  
  geom_histogram(binwidth = 5) +  
  labs(x = "Height, in inches", y = NULL)
```



Widths

```
ggplot(data = pp, aes(x = Width_in)) +  
  geom_histogram(binwidth = 5) +  
  labs(x = "Width, in inches", y = NULL)
```



Models as functions

- We can represent relationships between variables using **functions**
- A function is a mathematical concept: the relationship between an output and one or more inputs
 - Plug in the inputs and receive back the output
 - Example: The formula

$$y = 3x + 7$$

- is a function with input x and output y .
- If x is 5, y is 22,

$$y = 3 \times 5 + 7 = 22$$

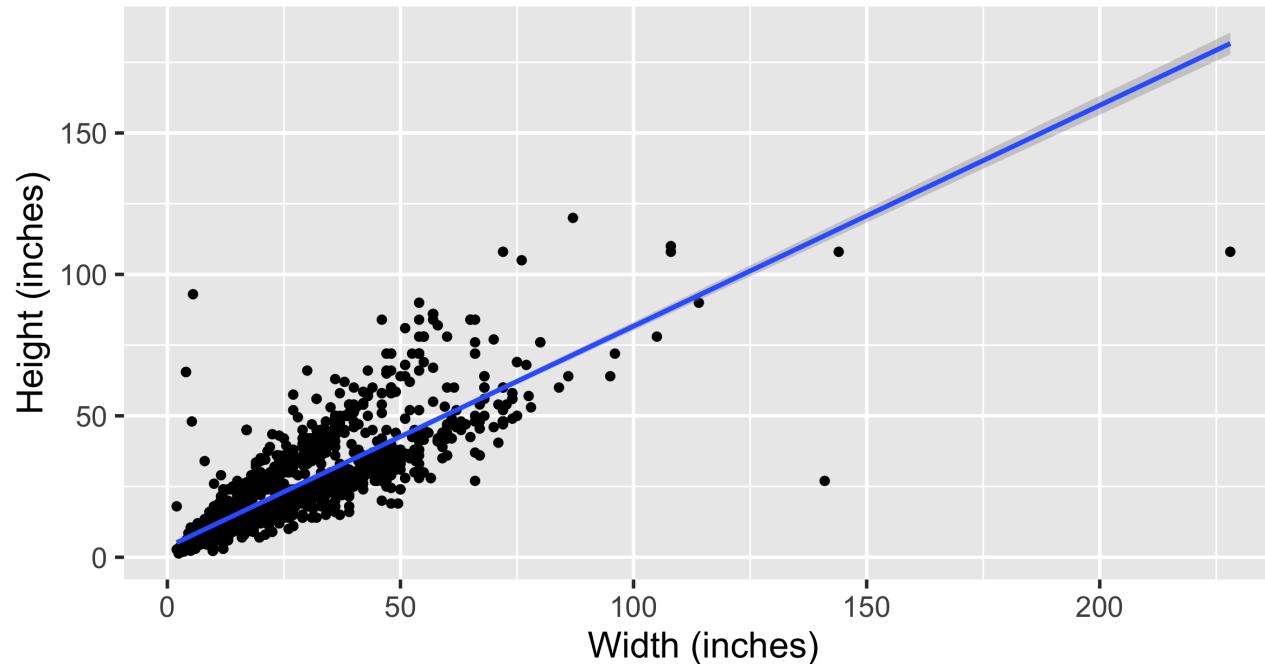
Height as a function of width

Plot

Code

Height vs. width of paintings

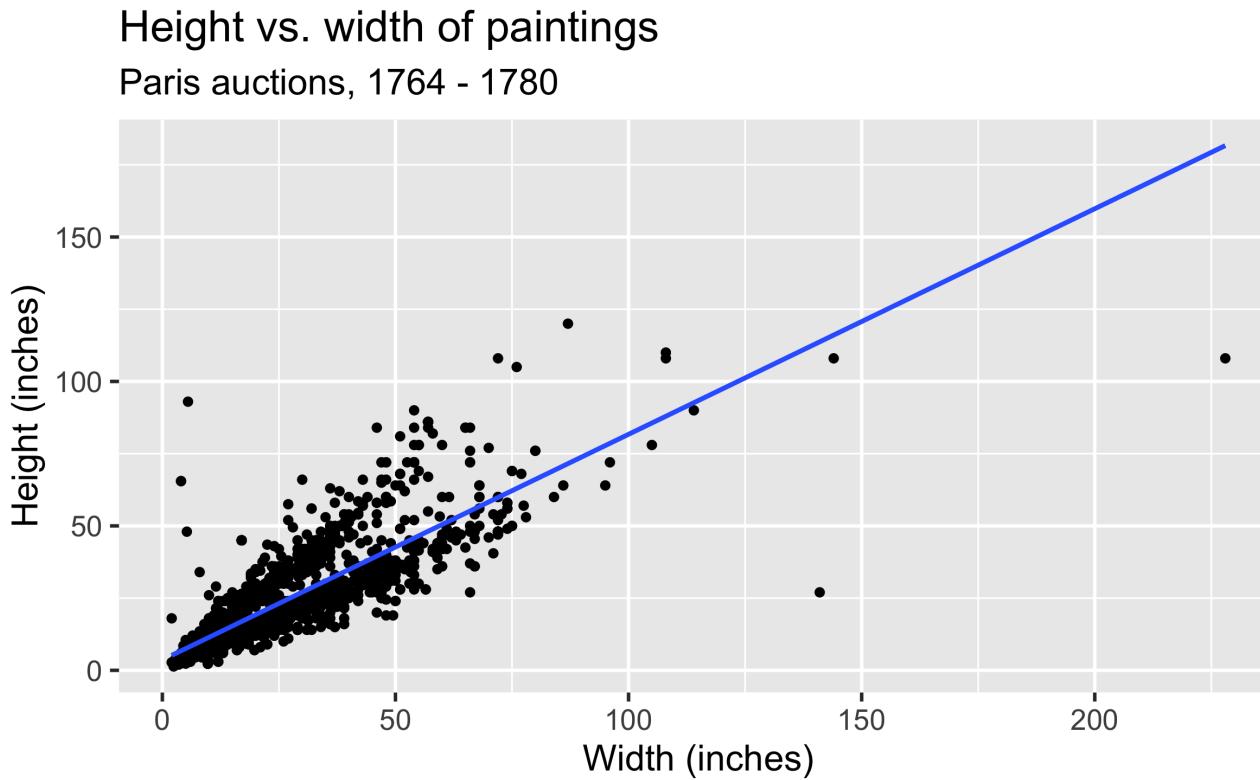
Paris auctions, 1764 - 1780



... without the measure of uncertainty

Plot

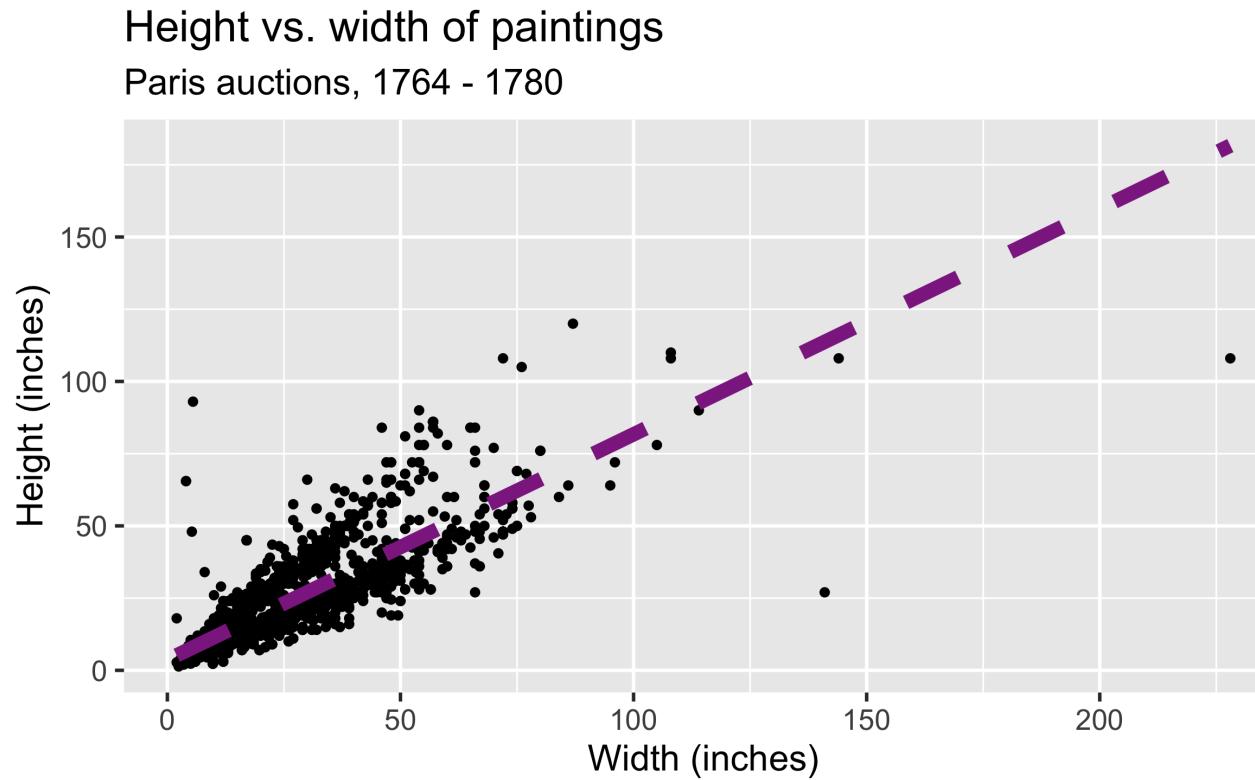
Code



... with different cosmetic choices

Plot

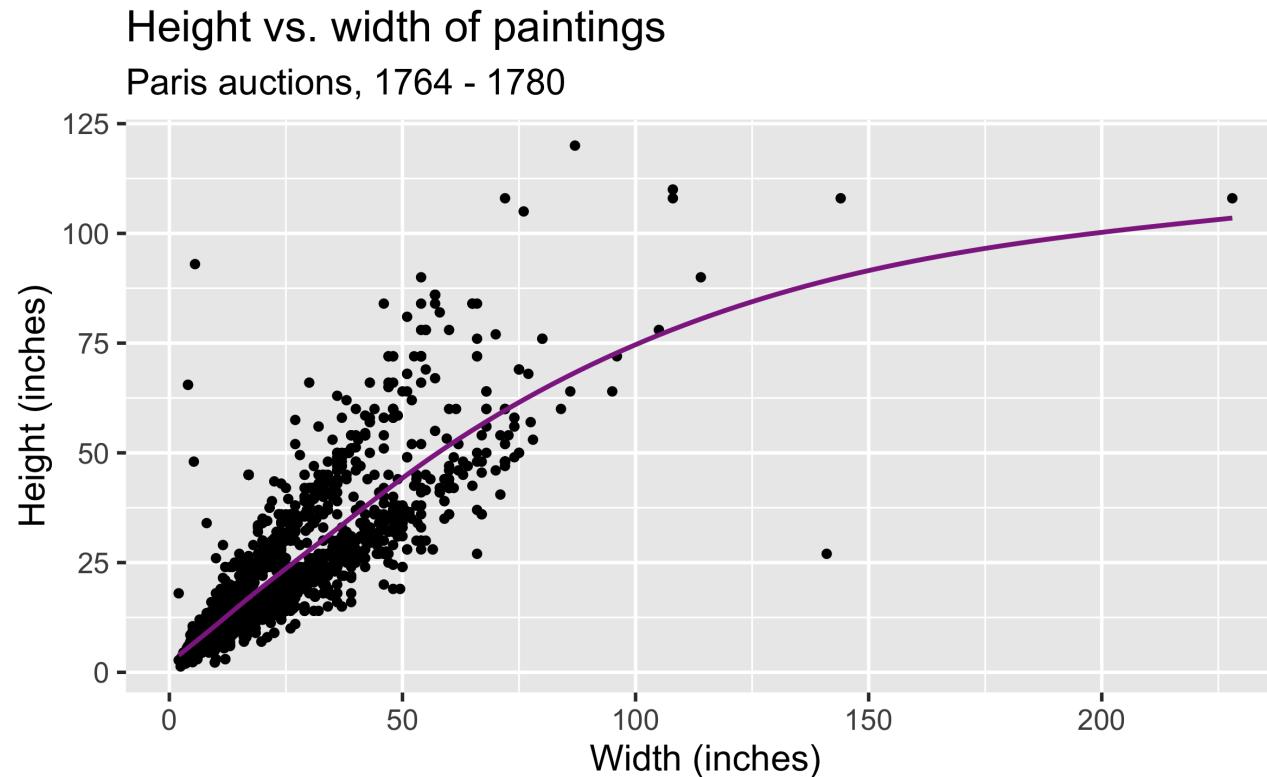
Code



Other smoothing methods: gam

Plot

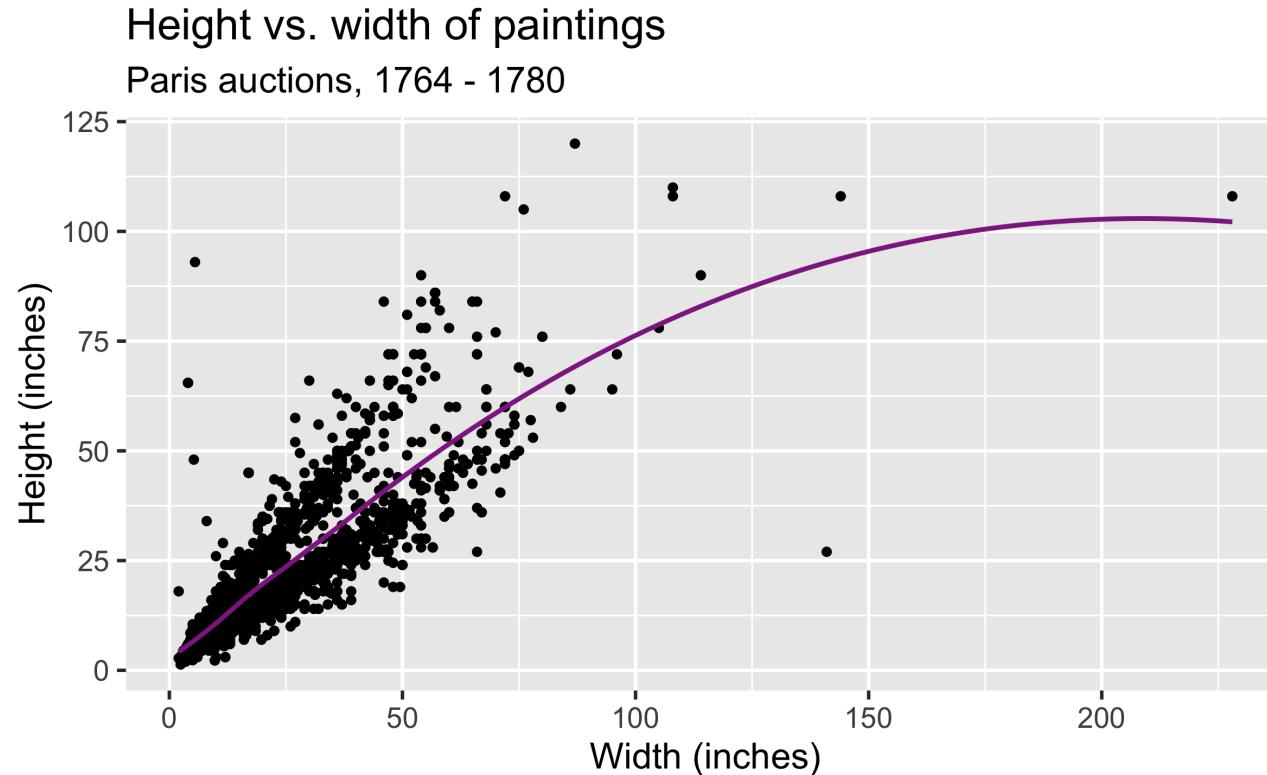
Code



Other smoothing methods: loess

Plot

Code



Vocabulary

- **Response variable:** Variable whose behavior or variation you are trying to understand, on the y-axis
- **Explanatory variables:** Other variables that you want to use to explain the variation in the response, on the x-axis
- **Predicted value:** Output of the **model function**
 - The model function gives the typical (expected) value of the response variable *conditioning* on the explanatory variables
 - **Residuals:** A measure of how far each case is from its predicted value (based on a particular model)
 - Residual = Observed value - Predicted value
 - Tells how far above/below the expected value each case is

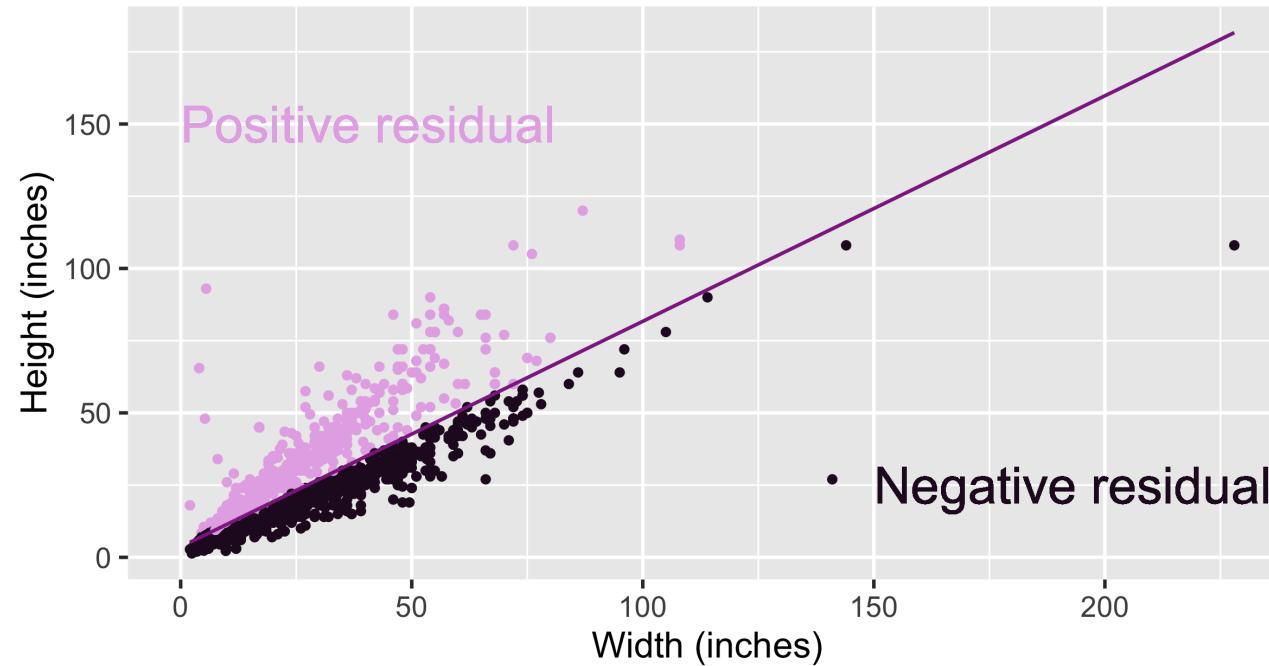
Residuals

Plot

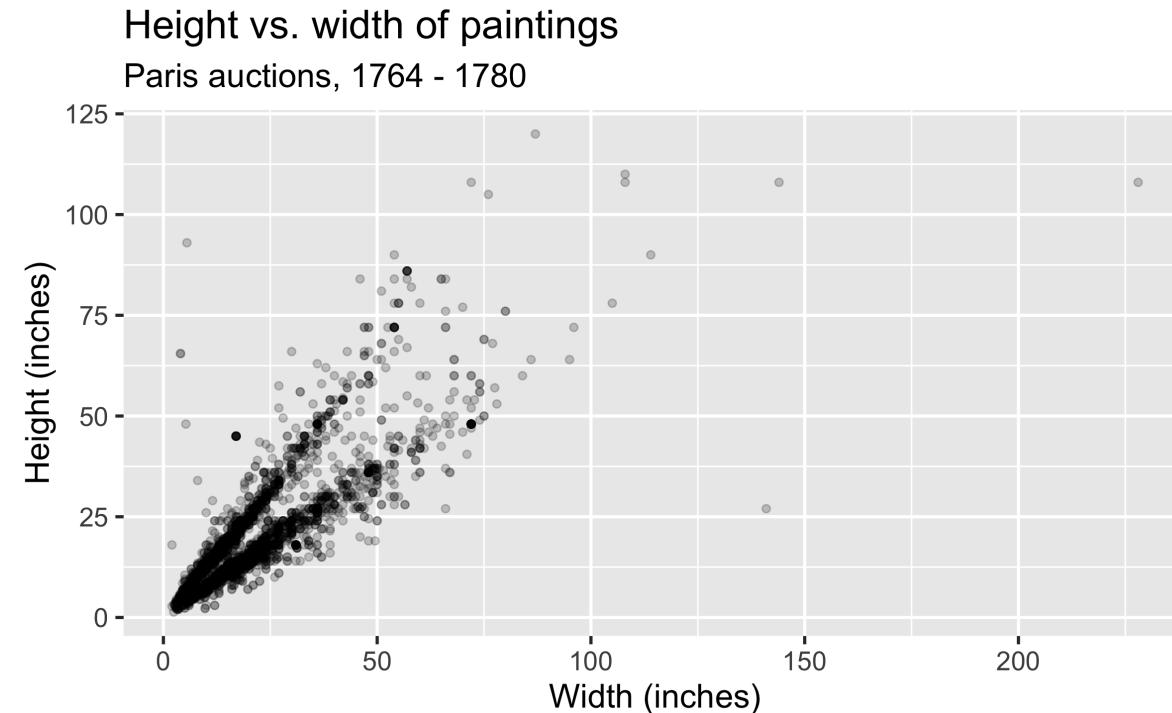
Code

Height vs. width of paintings

Paris auctions, 1764 - 1780



The plot below displays the relationship between height and width of paintings. The only difference from the previous plots is that it uses a smaller alpha value, making the points somewhat transparent. What feature is apparent in this plot that was not (as) apparent in the previous plots? What might be the reason for this feature?



Landscape paintings

- Landscape painting is the depiction in art of landscapes – natural scenery such as mountains, valleys, trees, rivers, and forests, especially where the main subject is a wide view – with its elements arranged into a coherent composition.¹
 - Landscape paintings tend to be wider than they are long.
- Portrait painting is a genre in painting, where the intent is to depict a human subject.²
 - Portrait paintings tend to be longer than they are wide.

[1] Source: Wikipedia, [Landscape painting](#)

[2] Source: Wikipedia, [Portrait painting](#)

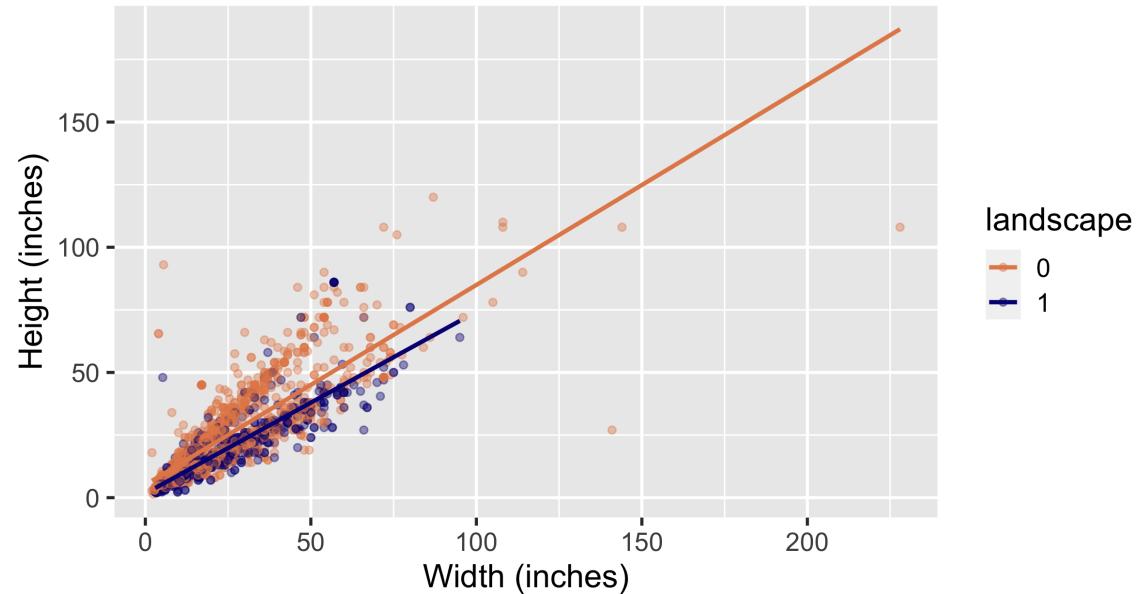
Multiple explanatory variables

Plot

Code

How, if at all, does the relationship between width and height of paintings vary by whether or not they have any landscape elements?

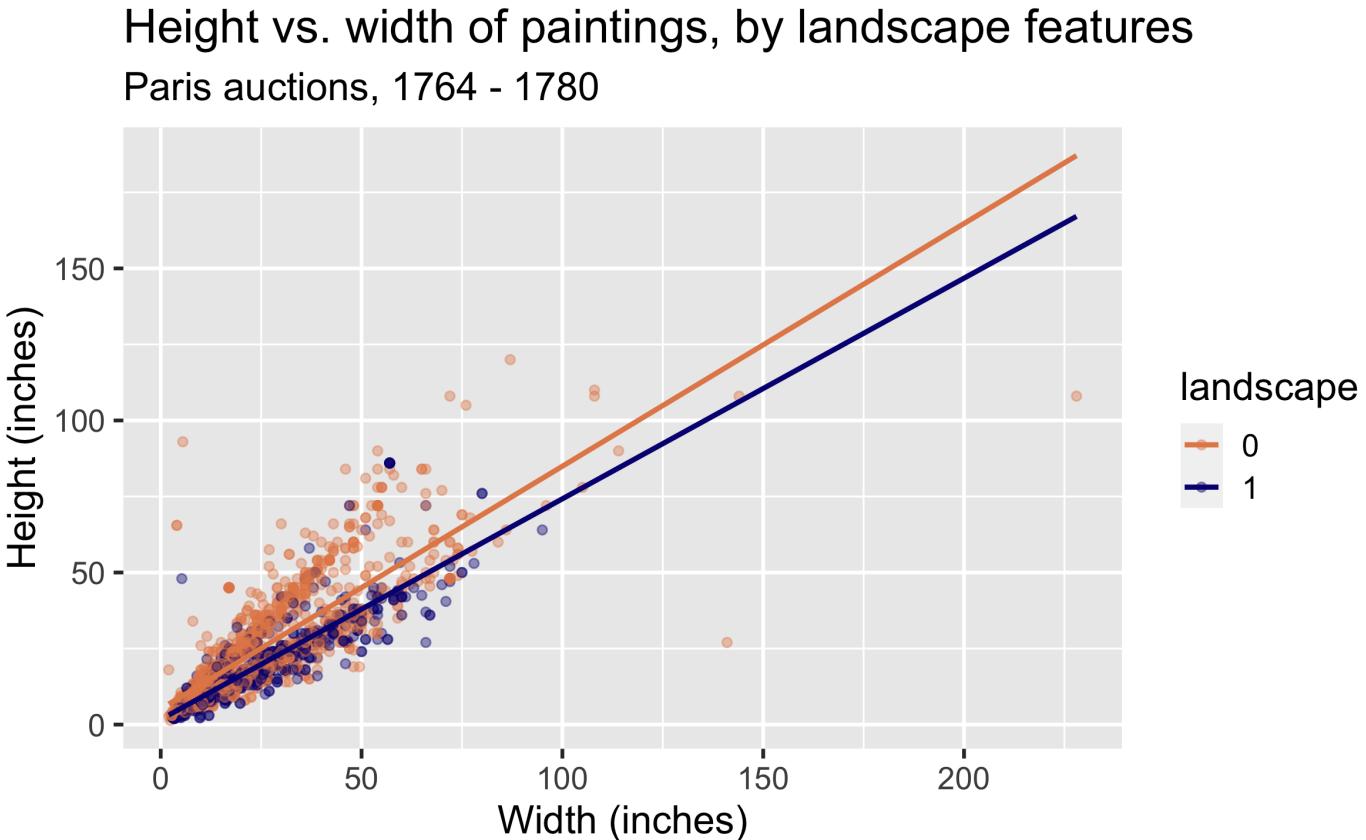
Height vs. width of paintings, by landscape features
Paris auctions, 1764 - 1780



Extending regression lines

Plot

Code



Models - upsides and downsides

- Models can sometimes reveal patterns that are not evident in a graph of the data. This is a great advantage of modeling over simple visual inspection of data.
- There is a real risk, however, that a model is imposing structure that is not really there on the scatter of data, just as people imagine animal shapes in the stars. A skeptical approach is always warranted.

Variation around the model...

is just as important as the model, if not more!

Statistics is the explanation of variation in the context of what remains unexplained.

- The scatter suggests that there might be other factors that account for large parts of painting-to-painting variability, or perhaps just that randomness plays a big role.
- Adding more explanatory variables to a model can sometimes usefully reduce the size of the scatter around the model. (We'll talk more about this later.)

How do we use models?

- Explanation: Characterize the relationship between y and x via *slopes* for numerical explanatory variables or *differences* for categorical explanatory variables
- Prediction: Plug in x , get the predicted y