Cartesian products as trees

Trees generalize products

Systematic tree counts

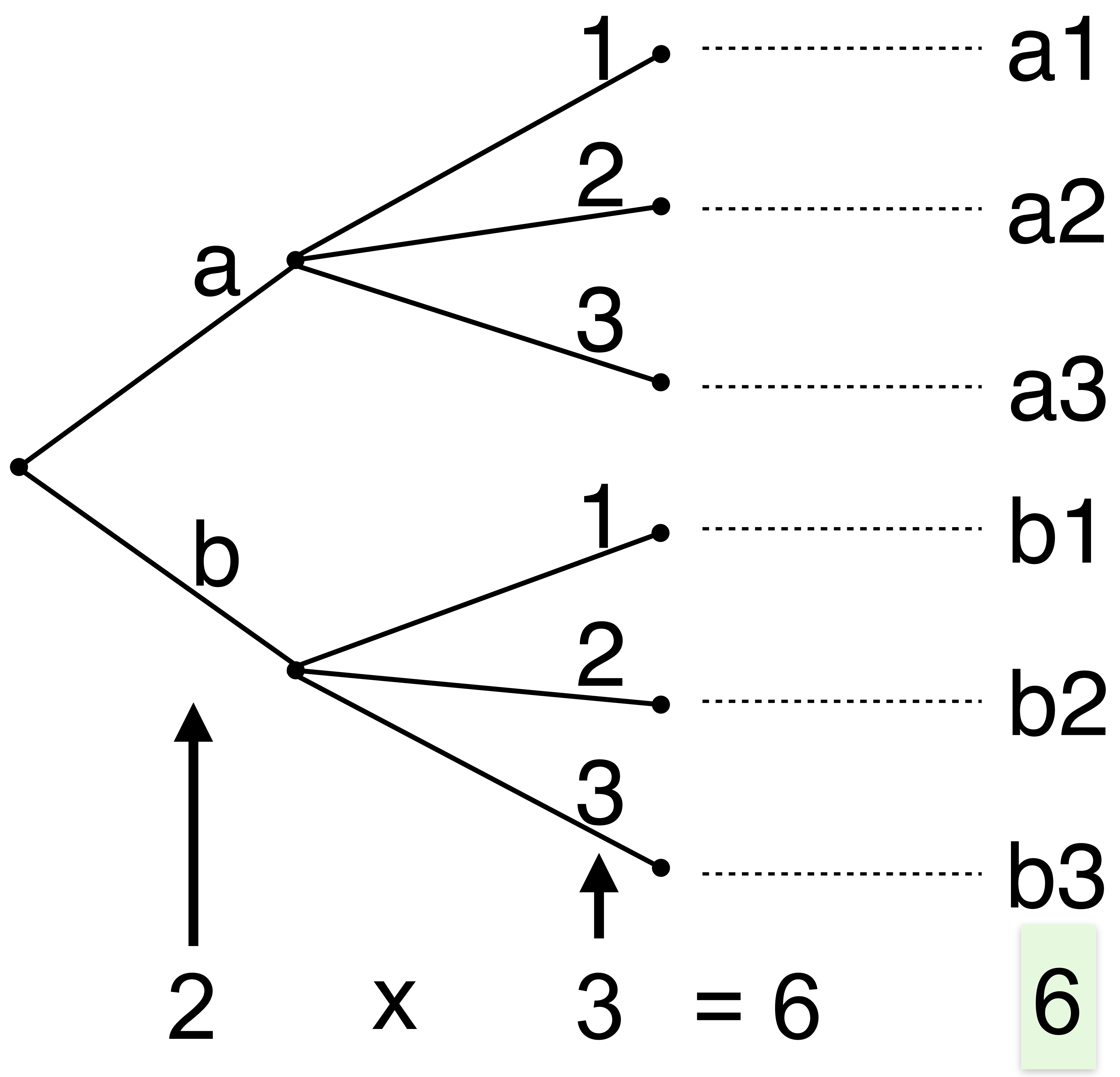
Paths in graphs

Useful for modeling randomness

Counting Trees

Cartesian Products as Trees

Tree ← Sequences ← Cartesian product



$\{a,b\} \times \{1,2,3\}$

$|\{a,b\} \times \{1,2,3\}| = 2\times3 = 6$

Used only

At any level, all nodes have same degree

Trees are More General

San-Diego University of Data Science (SUDS)

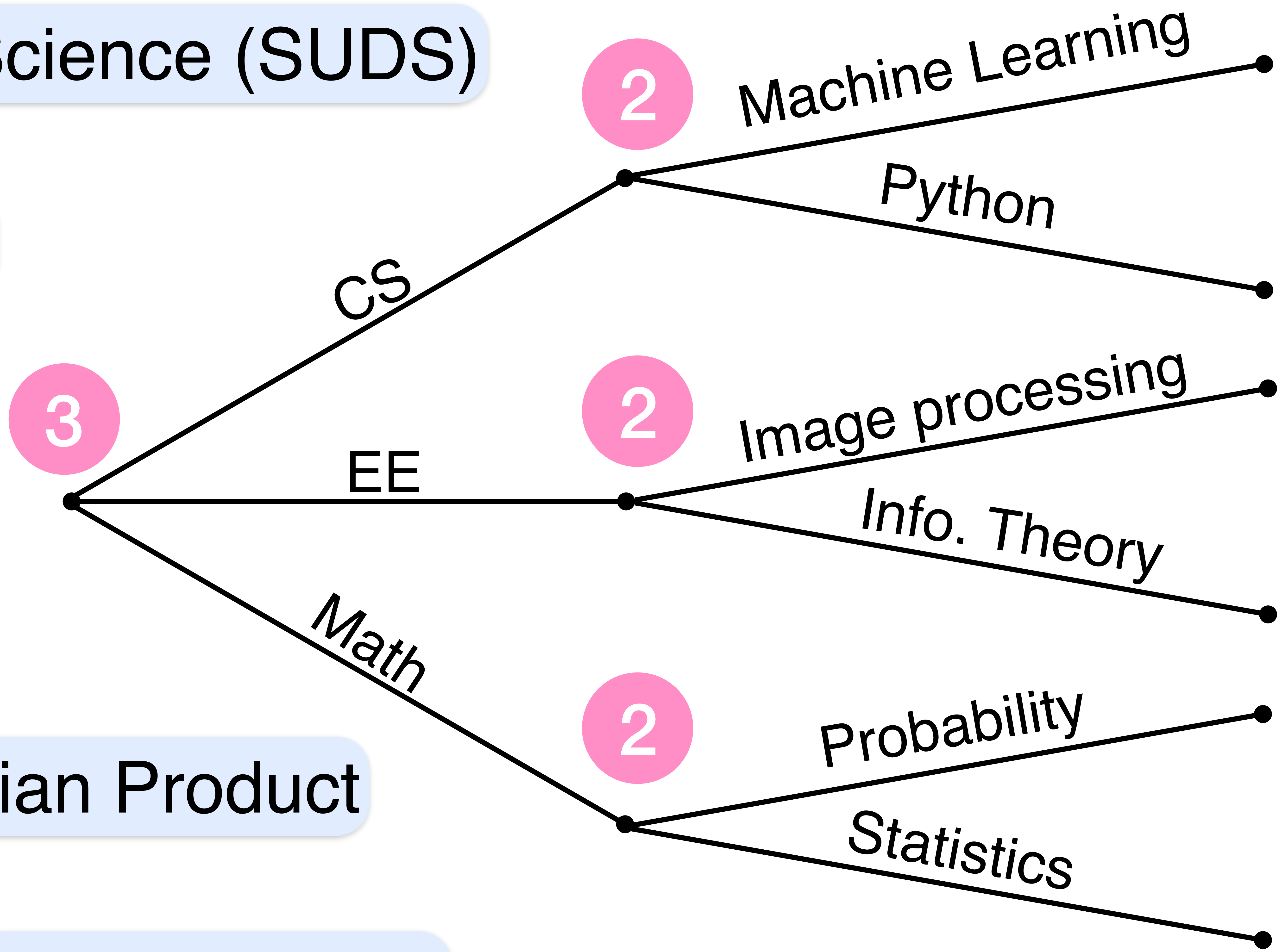
3 departments: CS, EE, Math

Each offers two courses

courses = ?

Departments offer *different* courses

Not Cartesian Product



Still

2 courses / department

Each level, all degrees equal

courses = 3 x 2 = 6

"X"

Why Trees

A tree can represent any set of sequences, not just Cartesian Products

Enable systematic counting technique

Useful in modeling random phenomena

Best of n

Many sports

Two teams or players compete to determine stronger

Single competition too random

Play odd # games

n

NBA Playoffs

n = 7 games

Tennis matches

n = 3 or 5 sets

Goal

Win majority of n games

Once someone wins > n/2

Stop

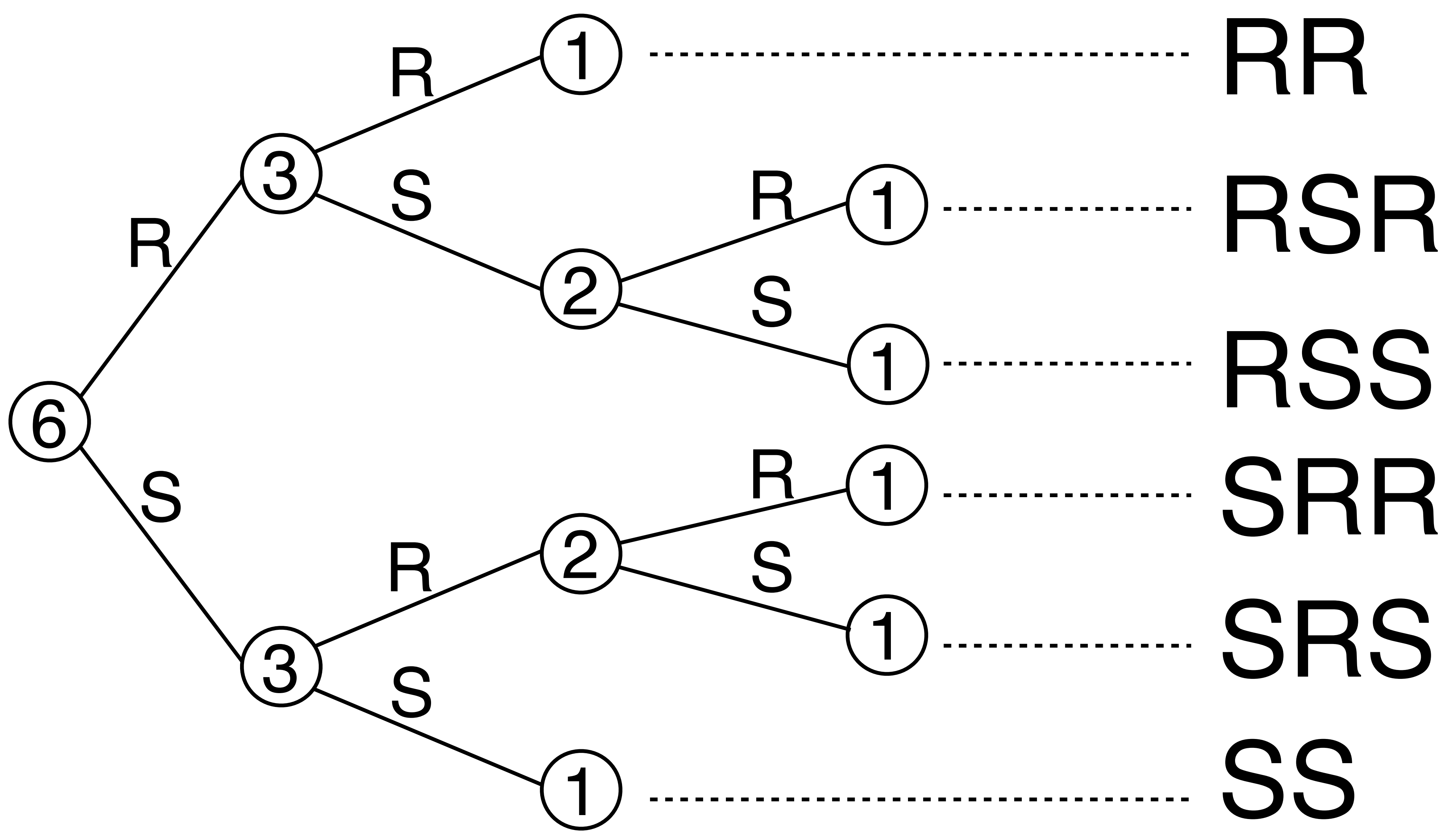
Paths to Victory

Roger and Serena

3-set match

Stop when one wins two sets

\# win sequences = ?



More later

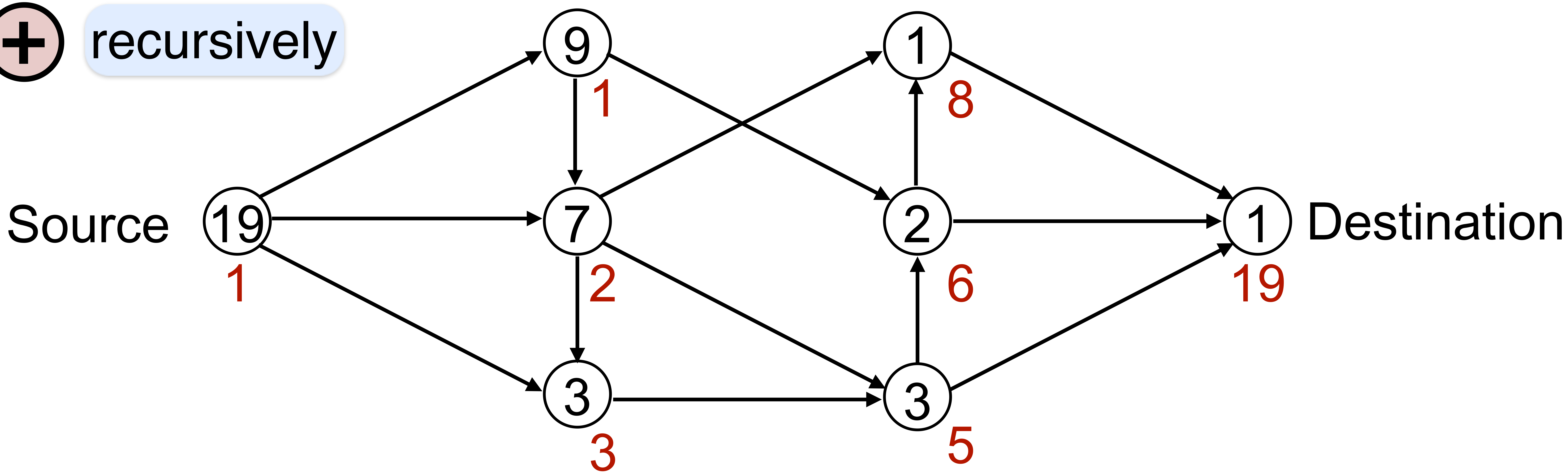
Paths from Source to Destination

Generalize to directed acyclic graph

paths from source to destination

Recursively determine # paths from a node to destination

⊕ recursively



Look for Structure

Don’t lose sight of the forest

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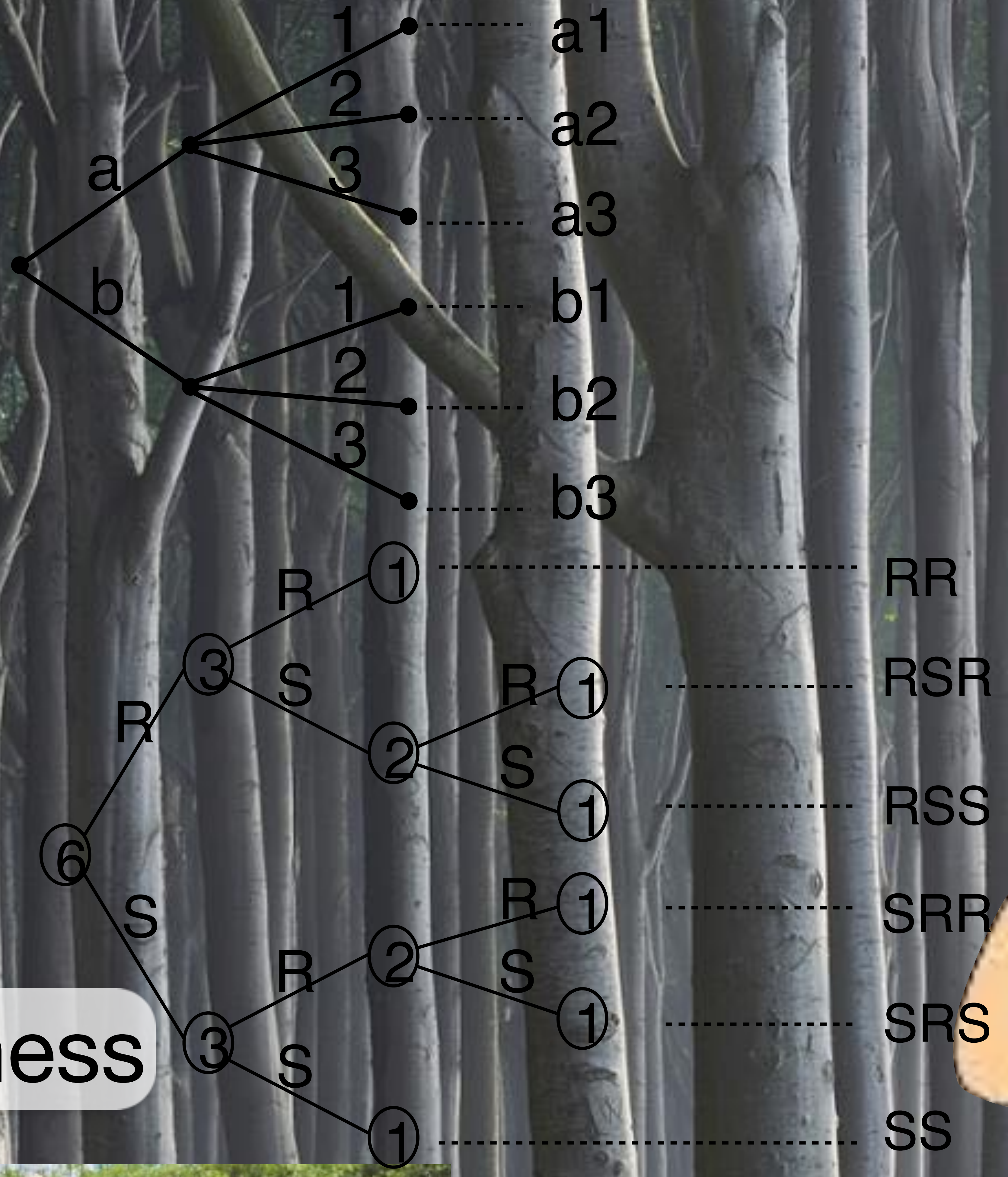
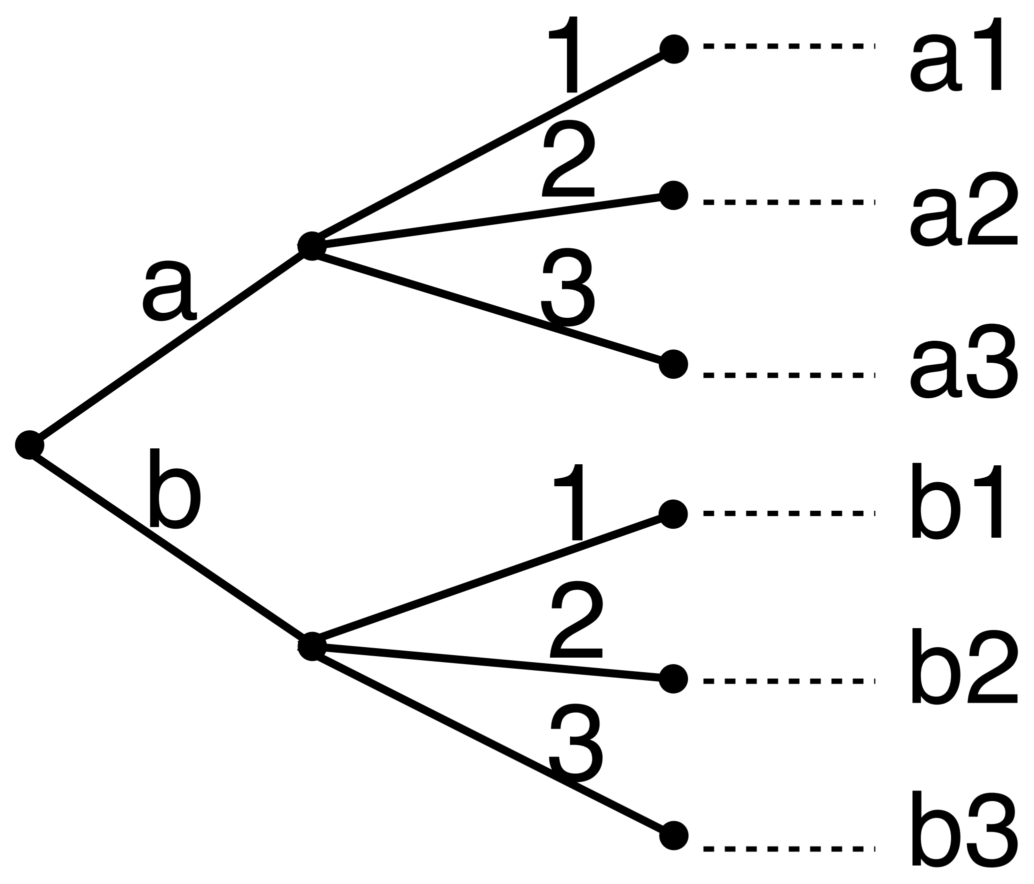
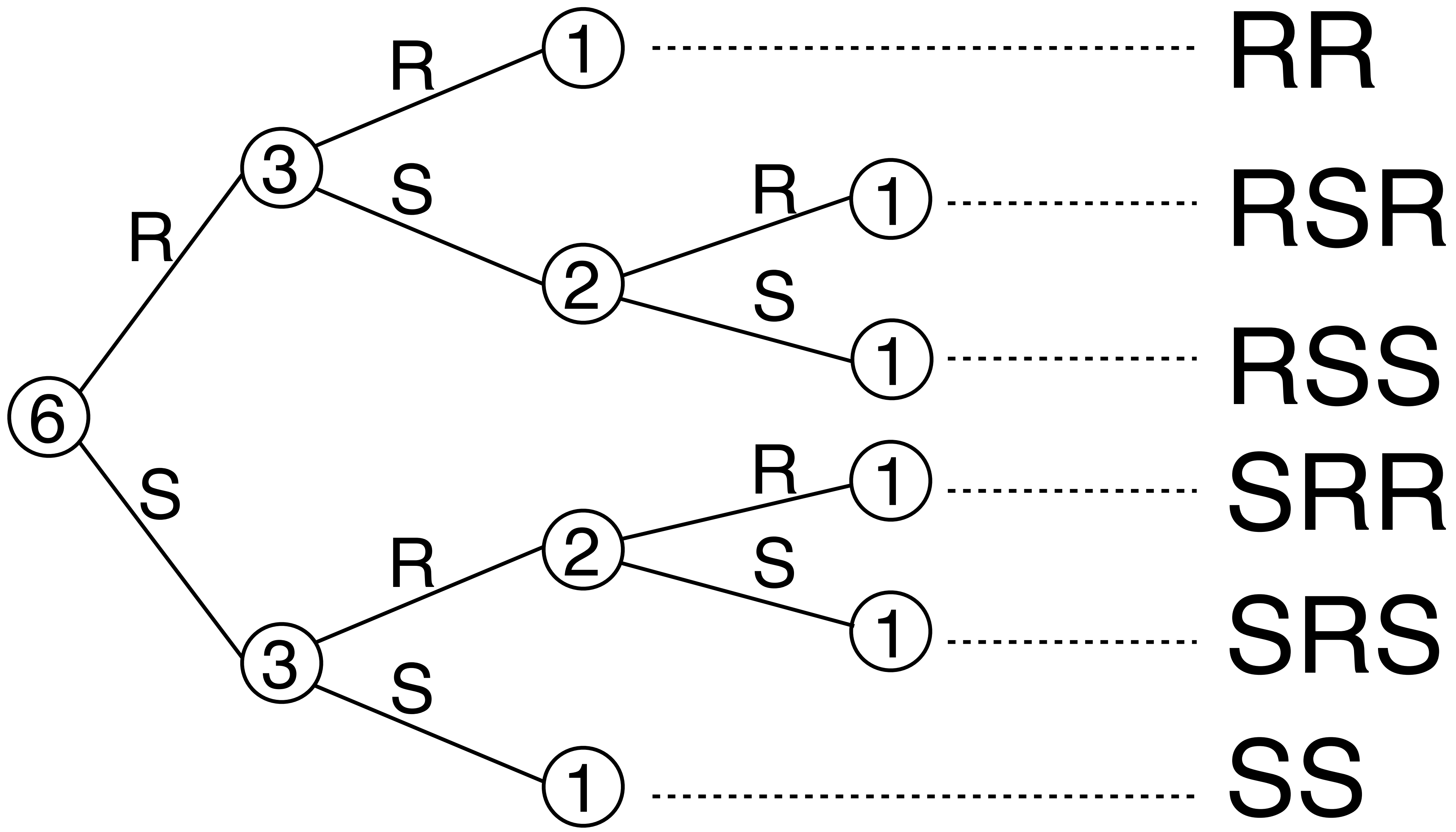
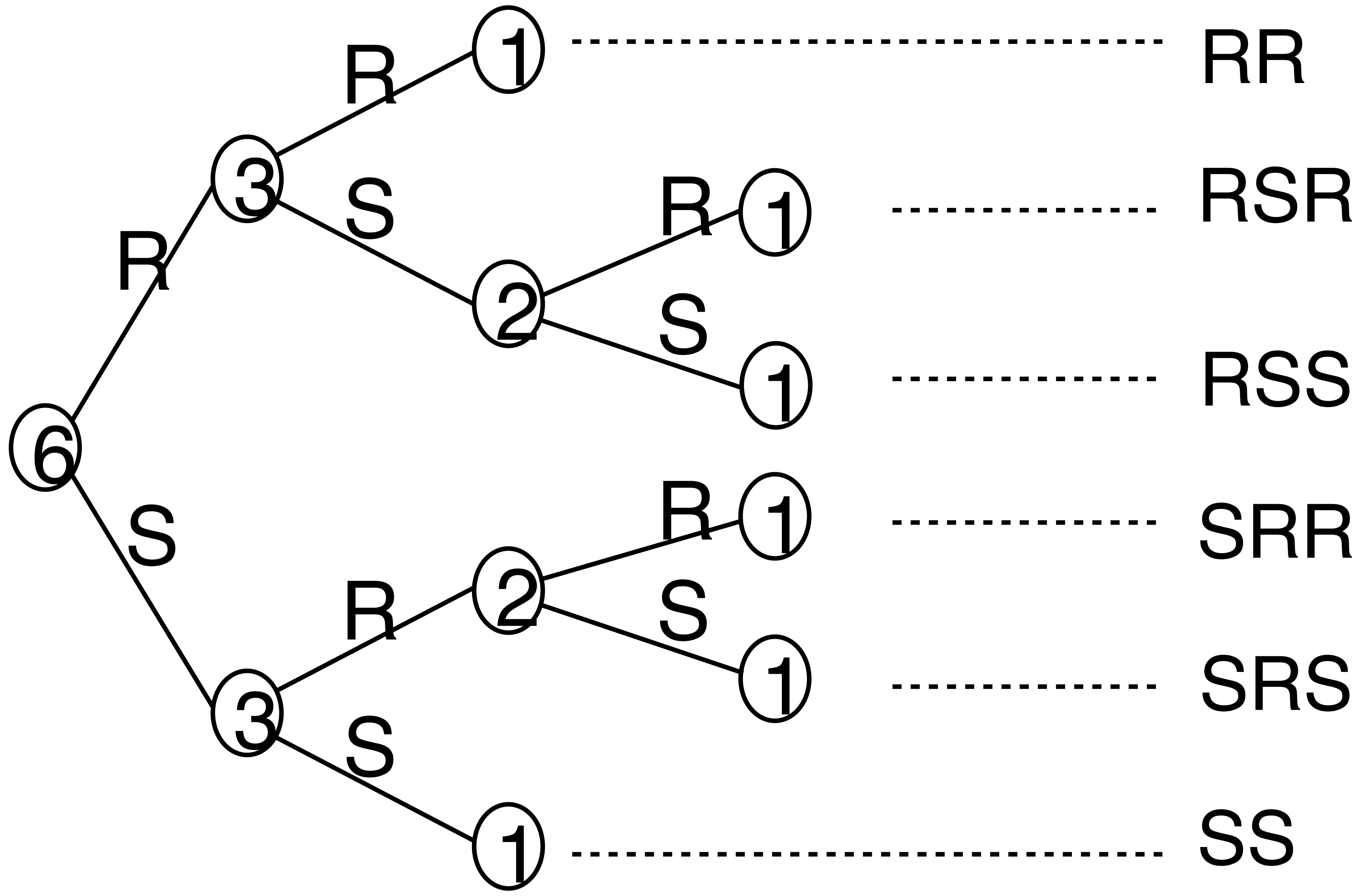


Counting **Trees**


a
b
1
2
3
1
2
3
a1
a2
a3
b1
b2
b3


6
R
S
3
3
R
S
R
S
1
2
2
1
R
S
R
S
1
1
1
1
RR
RSR
RSS
SRR
SRS
SS


6
R
S
3
3
R
S
R
S
1
2
2
1
R
S
R
S
1
1
1
1
RR
RSR
RSS
SRR
SRS
SS

Counting Sets

Disjoint unions

Complements

General unions

Cartesian products

Cartesian powers

Variations

Sequences

Trees

Graphs

Combinatorics

“Advanced counting”

Useful for determining probabilities