Basic Sets

ol

New toys

Intervals

Divisors/multiples

Useful later

Sets within Sets

Specify a set within a universe, or any other set

$\{ x \in A \mid \ldots \} = \{$ elements x in A such that $\ldots \}$ or $\{ x \in A : \ldots \}$

$\mathbb{N} = \{ x \in \mathbb{Z} \mid x \geq 0 \}$ $\mathbb{P} = \{ x \in \mathbb{N} \mid x > 0 \}$

Solutions to equations

$\{ x \in \mathbb{R} \mid x^2 \geq 0 \} = \mathbb{R}$ $\{ x \in \mathbb{R} : x^2 = 1 \} = \{ -1, 1 \}$

$\{ x \in \mathbb{R} \mid x^2 = 0 \} = \{ 0 \}$

a single-element set is a singleton

$\{ x \in \mathbb{R} \mid x^2 = -1 \} = \varnothing$ $\{ x \in \mathbb{C} \mid x^2 = -1 \} = \{ i, -i \}$

Integer Intervals

$\{m,\ldots, n\} = \{i \in \mathbb{Z} \mid m \leq i \leq n\}$

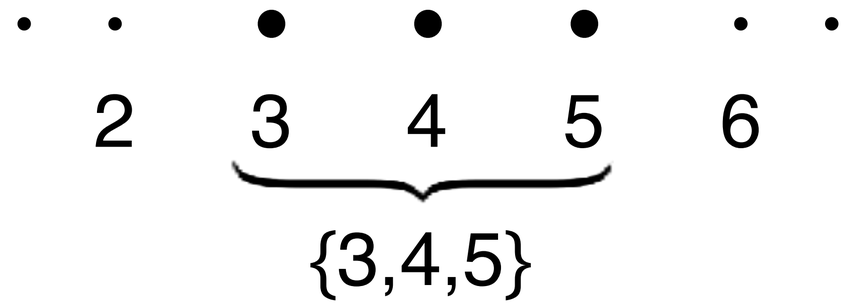
Integers from m to n, inclusive

$\{3,\ldots,5\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 5\} = \{3,4,5\}$

$\{3,\ldots,4\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 4\} = \{3,4\}$

$\{3,\ldots,3\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 3\} = \{3\}$

$\{3,\ldots,2\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 2\} = \varnothing$



CONVENIENTTION

$[n] = \{1\ldots,n\}$

Real Intervals

In set

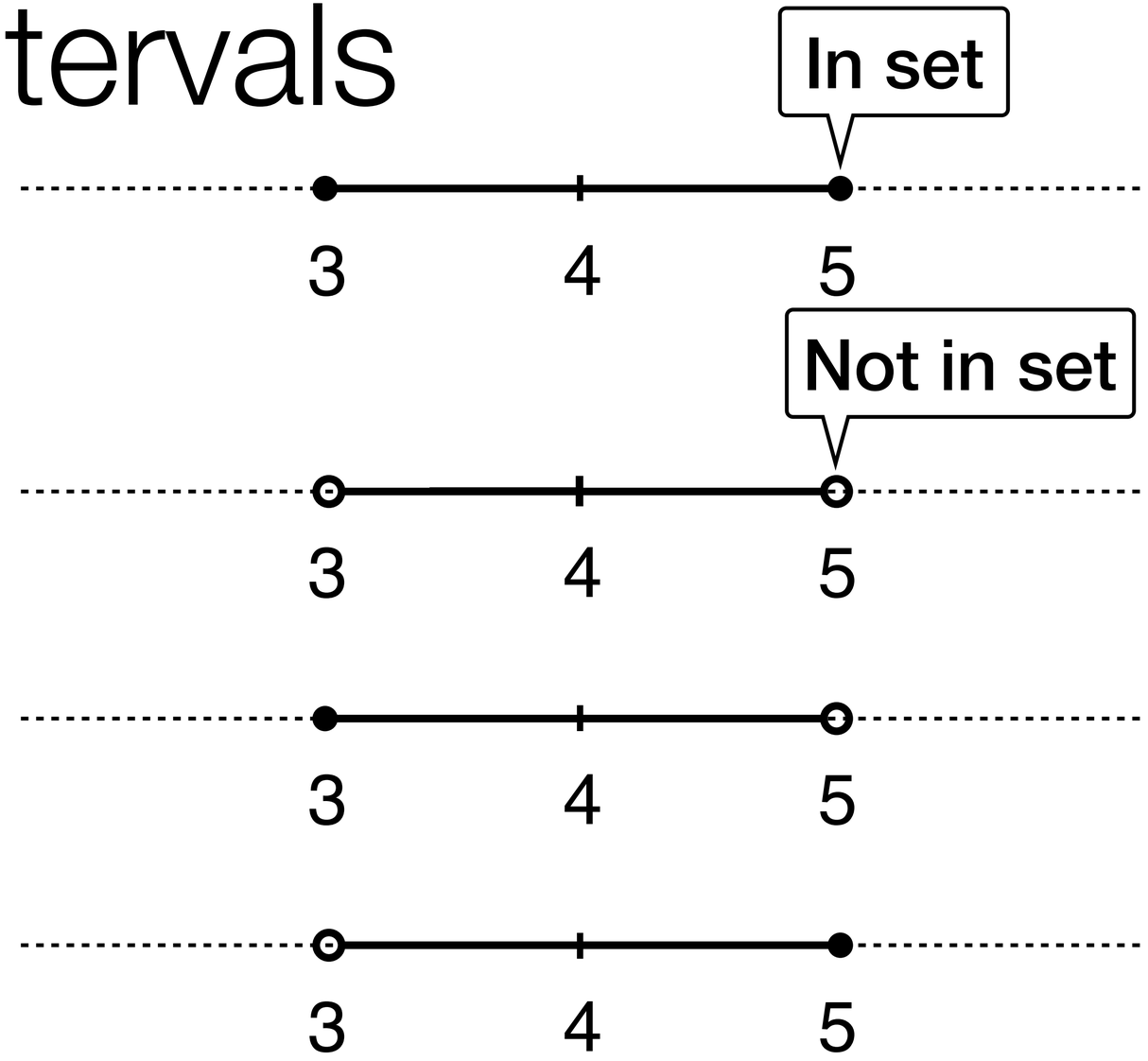
$[a,b]$ $\{x \in \mathbb{R} \mid a \leq x \leq b\}$ $[3,5]$

Not in set

(a,b) $\{x \in \mathbb{R} \mid a < x < b\}$ $(3,5)$

$[a,b)$ $\{x \in \mathbb{R} \mid a \leq x < b\}$ $[3,5)$

$(a,b]$ $\{x \in \mathbb{R} \mid a < x \leq b\}$ $(3,5]$



$[3,3] = \{3\}$ singleton

$[3,2] = [3,3) = (3,3] = \varnothing$

Divisibility

$m, n \in \mathbb{Z}$

if $n = c \cdot m$ for some $c \in \mathbb{Z}$, we say that n is a multiple of m, or m divides n, and write $m \mid n$

$6 = 2 \cdot 3$ → $3 \mid 6$

n c m — m n

$-8 = (-2) \cdot 4$ → $4 \mid -8$

n c m — m n

$0 = 0 \cdot (-2)$ → $-2 \mid 0$

If no such c exists, m does not divide n, or n is not a multiple of m, denoted $m \nmid n$

There is no $c \in \mathbb{Z}$ such that $4 = c \cdot 3$ → $3 \nmid 4$

$0 \nmid n$ for any $n \neq 0$

Multiples

3 | ? {…, -6, -3, 0, 3, 6,…}

1 | ? $\mathbb{Z}$

0 | ? 0

Divisors

? | 4 ±1, ±2, ±4

? | 0 $\mathbb{Z}$

? | any n≠0 ±1 ±n

Set of Multiples

$m \in \mathbb{Z}$

$_m\mathbb{Z} \stackrel{\text{def}}{=} \{ i \in \mathbb{Z} : m \mid i \}$

integer multiples of m

$_2\mathbb{Z} = \{\ldots, -4, -2, 0, 2, 4, \ldots\} \stackrel{\text{def}}{=} \mathbb{E}$

Even numbers

$_1\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} = \mathbb{Z}$

$_0\mathbb{Z} = \{0\}$

$m \in \mathbb{Z}, n \in \mathbb{P}$

$_m[n] \stackrel{\text{def}}{=} \{ i \in [n] : m \mid i \}$

multiples of m in {1..n}

$_3[13] = \{ i \in \{1,\ldots,13\} : 3 \mid i \} = \{3, 6, 9, 12\}$

$_7[13] = \{7\}$

$_1[13] = [13]$

$_{14}[13] = {}_0[13] = \varnothing$

Will use shortly

Intervals, Multiples

{0,..,n-1}

`range(n)`

note: [n] = {1…n}

```
print(set(range(3)))
{0,1,2}
```

Returns type **`range`**

{m,..,n-1}

`range(m, n)`

```
print(set(range(2, 5)))
{2,3,4}
```

To print, convert to set

{m, m+d, m+2d, … } < n-1

`range(m, n, d)`

```
print(set(range(2, 12, 3)))
{8,2,11,5}
```

Basic Sets

ol

New toys

Intervals {m,..,n} [n] [a,b)

Divisors/multiples mln

Sets of multiples $_d$[n]