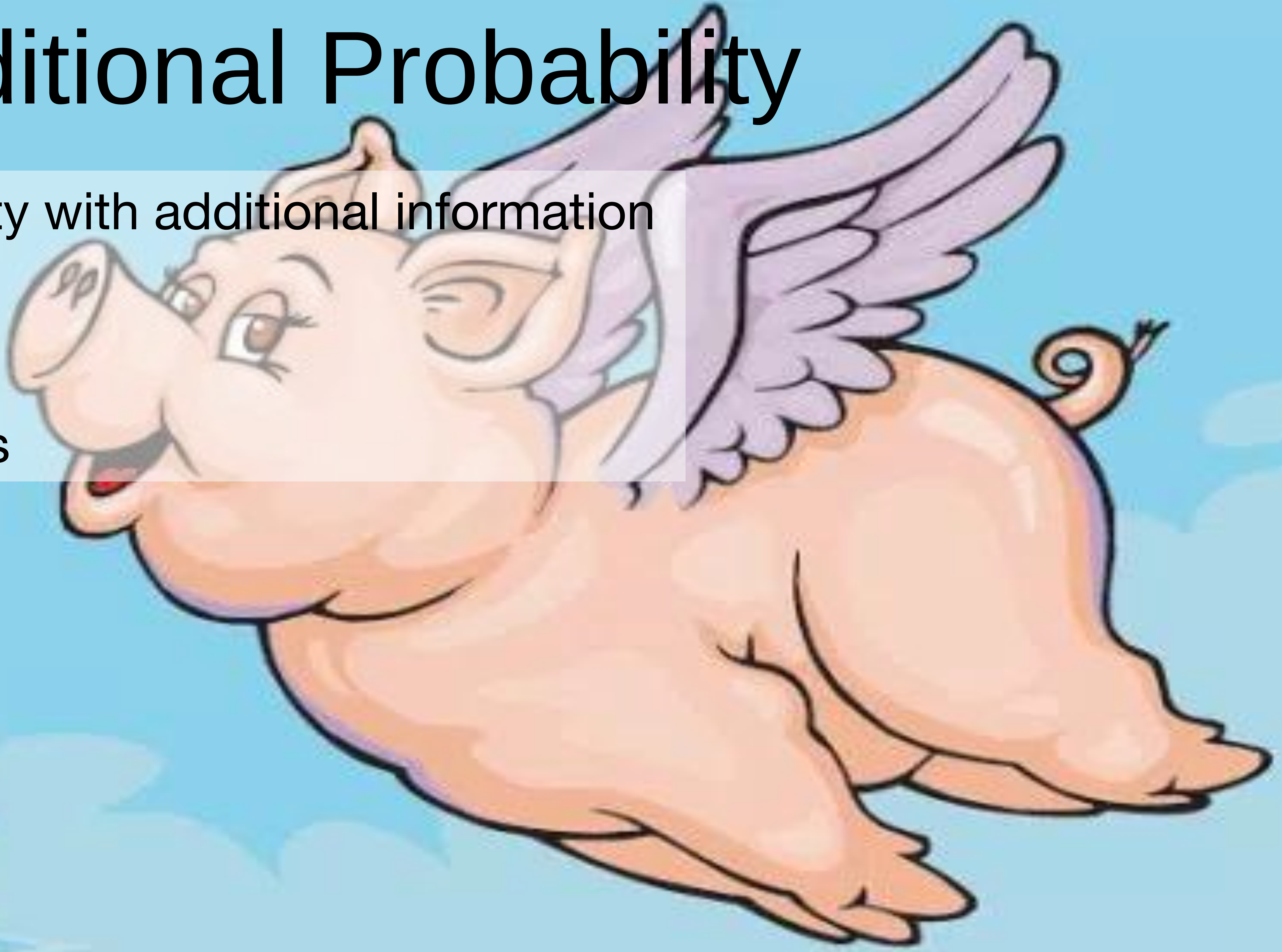
Conditional Probability

Probability with additional information

Definition

Examples



Why Condition

Often have partial information about the world

Modifies event probabilities

- Unemployment numbers → Stock prices
- LeBron James injured → Cavaliers game result
- Sunny weekend → Beach traffic

Can help

- Improve estimates
- Determine original unconditional probabilities

Back to Basics

Empirical frequency interpretation of probability

Probability P(E) of event E is the fraction of experiments where E occurs as # experiments → ∞

To estimate P(E) repeat the experiment many times, find fraction of experiments where E occurs

Fair Die

$$P(2) = \frac{2}{12} = \frac{1}{6}$$

Estimate

$$\underbrace{\boxed{2}\ \ 1\ \ 3\ \ 6\ \ 4\ \ \boxed{2}\ \ 5\ \ 4\ \ 3\ \ 6\ \ 5\ \ 1}_{12}$$

Conditional Probability

Let E and F be events. The conditional probability $P(F \mid E)$ of F given E is the fraction of times F occurs in experiments where E occurs

To estimate $P(F \mid E)$ take many samples, consider only experiments where E occurs, and calculate the fraction therein where F occurs too

Even = {2, 4, 6}

$$P(2 \mid \text{Even}) = \frac{2}{6} = \frac{1}{3}$$

1 2

2 1 3 6 4 2 5 4 3 6 5 1

1 2 3 4 5 6

Die

P({2}) = P(2) = ⅙

P(2) = ?

$$P(2 \mid \text{Odd}) = P(2 \mid \{1,3,5\}) = \frac{0}{6} = 0$$

2 1 3 6 4 2 5 4 3 6 5 1

P(≤ 2) = P({1,2})= ⅓

P(≤ 2) = ?

$$P(\leq 2 \mid \geq 2) = P(\{1,2\} \mid \{2,3,4,5,6\}) = \frac{2}{10} = \frac{1}{5}$$

2 1 3 6 4 2 5 4 3 6 5 1

General Events - Uniform Spaces

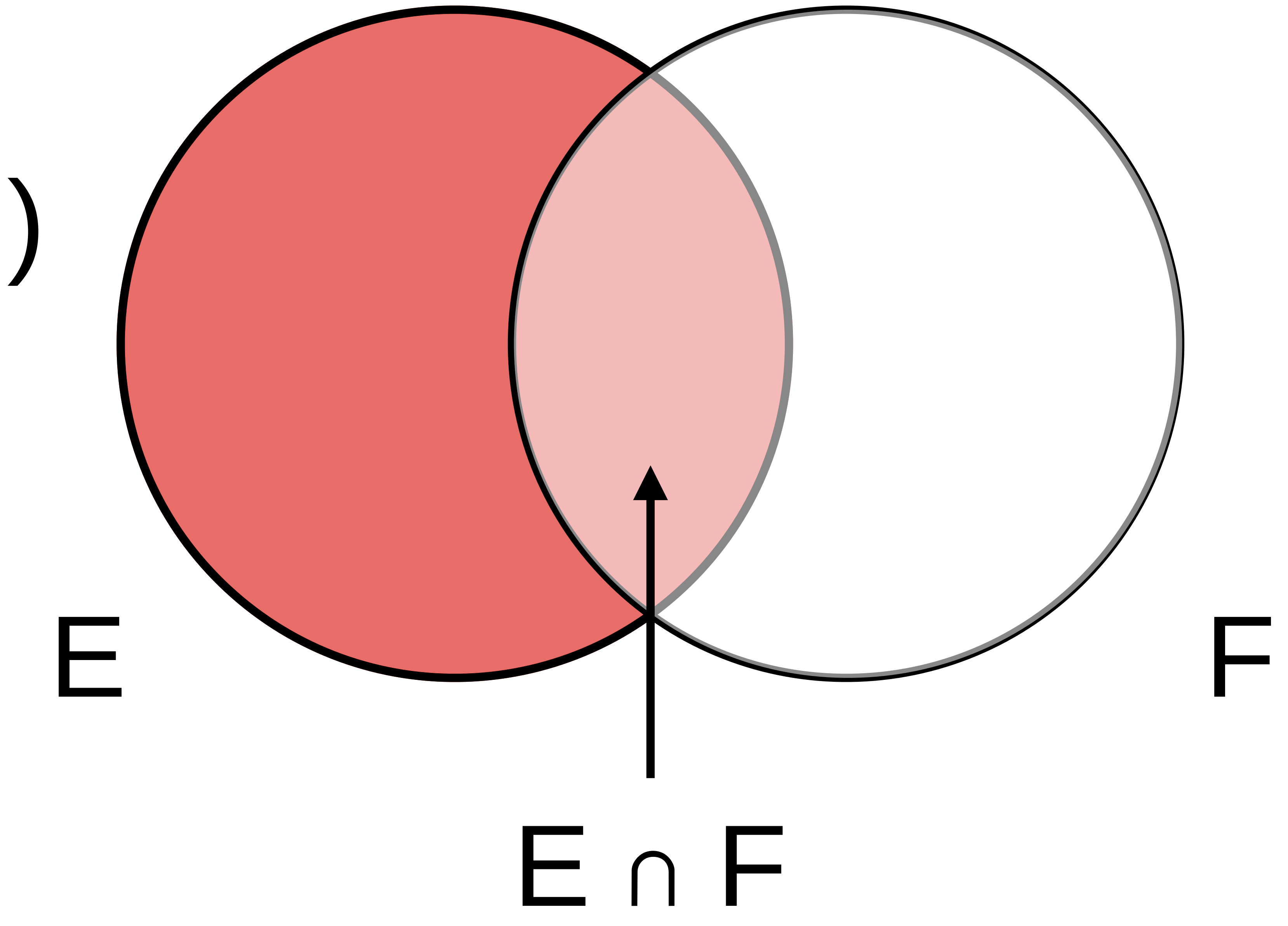
$$P(F \mid E) = P(\, X{\in}F \mid X{\in}E \,)$$

$$= P(\, X{\in}E \text{ and } X{\in}F \mid X \in E \,)$$

$$= P(\, X \in E{\cap}F \mid X{\in}E \,)$$

$$= P(\, E{\cap}F \mid E \,)$$

$$= \frac{|E \cap F|}{|E|}$$



Fair Die Again

$$P(\text{Prime} \mid \text{Odd}) = P(\{2,3,5\} \mid \{1,3,5\})$$

$$= \frac{|\{2,3,5\} \cap \{1,3,5\}|}{|\{1,3,5\}|} = \frac{|\{3,5\}|}{|\{1,3,5\}|} = \frac{2}{3}$$

$$P(\{4\} \mid \text{Prime}) = P(\{4\} \mid \{2,3,5\})$$

$$= \frac{|\{4\} \cap \{2,3,5\}|}{|\{2,3,5\}|} = \frac{|\emptyset|}{|\{2,3,5\}|} = 0$$

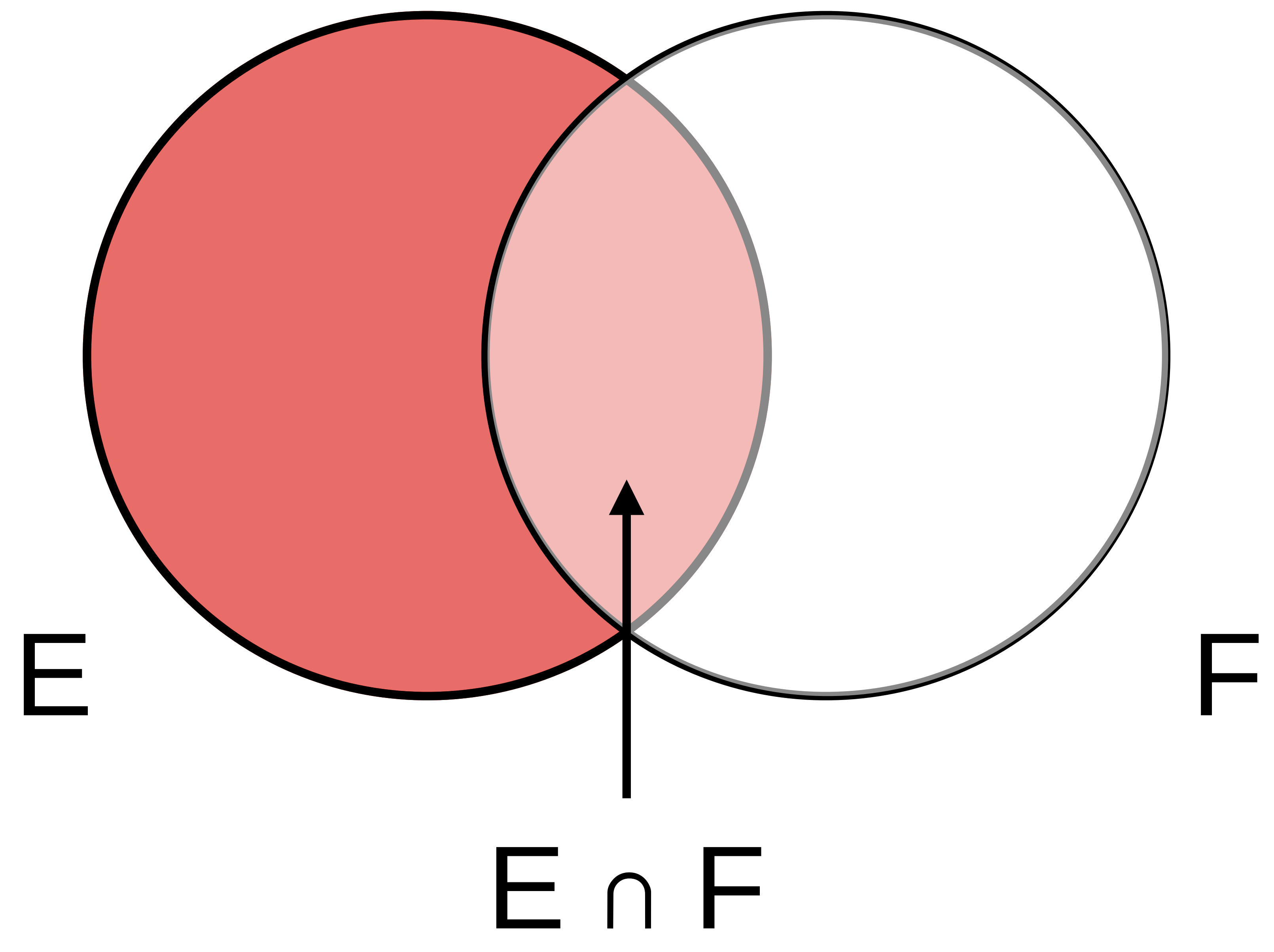
General Spaces

$$P(F \mid E) = P(X \in F \mid X \in E)$$

$$= P[X \in E \cap X \in F \mid X \in E]$$

$$= P[X \in E \cap F \mid X \in E]$$

$$= \frac{P(E \cap F)}{P(E)}$$



4-Sided Die

Face	1	2	3	4
Probability	.1	.2	.3	.4

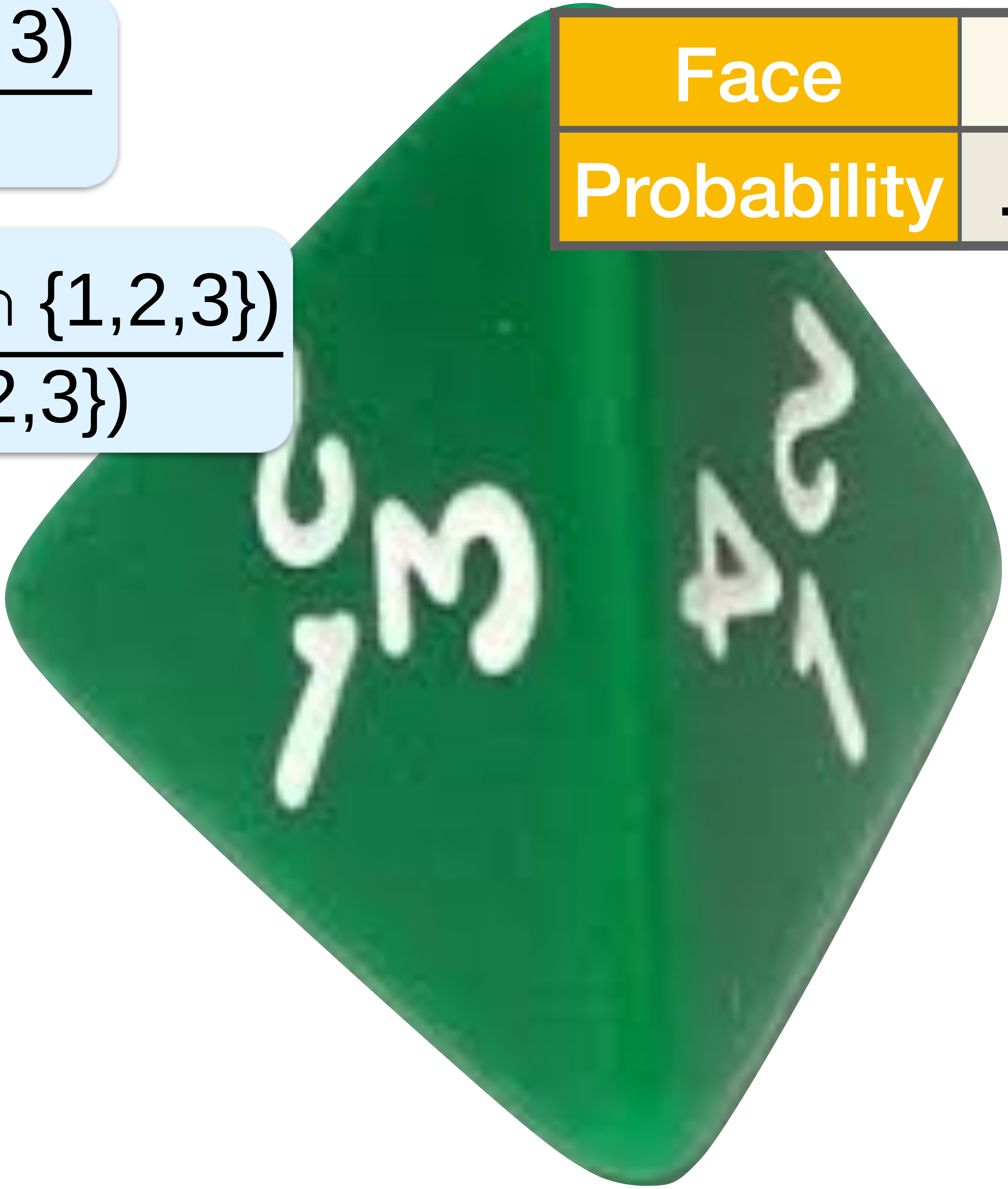
$$P(\geq 2 \mid \leq 3)$$

$$= \frac{P(\geq 2 \cap \leq 3)}{P(\leq 3)}$$

$$= \frac{P(\{2,3,4\} \cap \{1,2,3\})}{P(\{1,2,3\})}$$

$$= \frac{P(\{2,3\})}{P(\{1,2,3\})}$$

$$= \frac{.5}{.6} = \frac{5}{6}$$




KIDS
ARE PEOPLE TOO

Conditionals are Probabilities Too

≥ Non-negativity $P(B|A) \geq 0$

1 Unitarity $P(\Omega|A) = 1$

+ Addition B, C disjoint $\rightarrow$ $P(B \cup C \mid A) = P(B|A) + P(C|A)$

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Examples