

# Distribution Types

Uniform sample spaces

Coin, die, cards

Non-uniform spaces

Tetrahedral die



# Uniform Probability Spaces

Generally, outcomes may have different probabilities

Rain

$P(\text{rain}) = 10\%$

$P(\text{no rain}) = 90\%$

Uniform (equiprobable) spaces

Uniform distribution

All outcomes are equally likely

Fair coin

$P(h) = P(t) = \frac{1}{2}$



Drastically simplifies probability specification

# Uniform Probability Spaces

All outcomes are equally likely

$$\sum_{x \in \{3,5\}} x^2 = 3^2 + 5^2 = 34$$

$$\forall x \in \Omega \quad P(x) = p$$

$$\sum_{x \in \{3,5\}} x = 3 + 5 = 8$$

$$1 = \sum_{x \in \Omega} P(x) = \sum_{x \in \Omega} p = |\Omega| \cdot p$$

$$\sum_{x \in \{3,5\}} p = p + p = 2p$$

$$p = 1 / |\Omega|$$

$$\sum_{x \in \Omega} p = p + \dots + p = |\Omega| \cdot p$$

Fair coin

$$P(h) = P(t) = p$$

$$1 = P(h) + P(t) = 2 \cdot p$$

$$p = \frac{1}{2}$$

Uniform spaces

Every outcome has probability  $1 / |\Omega|$

U

All you need to know is  $|\Omega|$  !

Notetation

Draw  
Uniformly,  
Randomly

# Fair Coin

$\Omega = \{ \text{heads, tails} \} = \{ h, t \}$

$$|\Omega| = 2$$

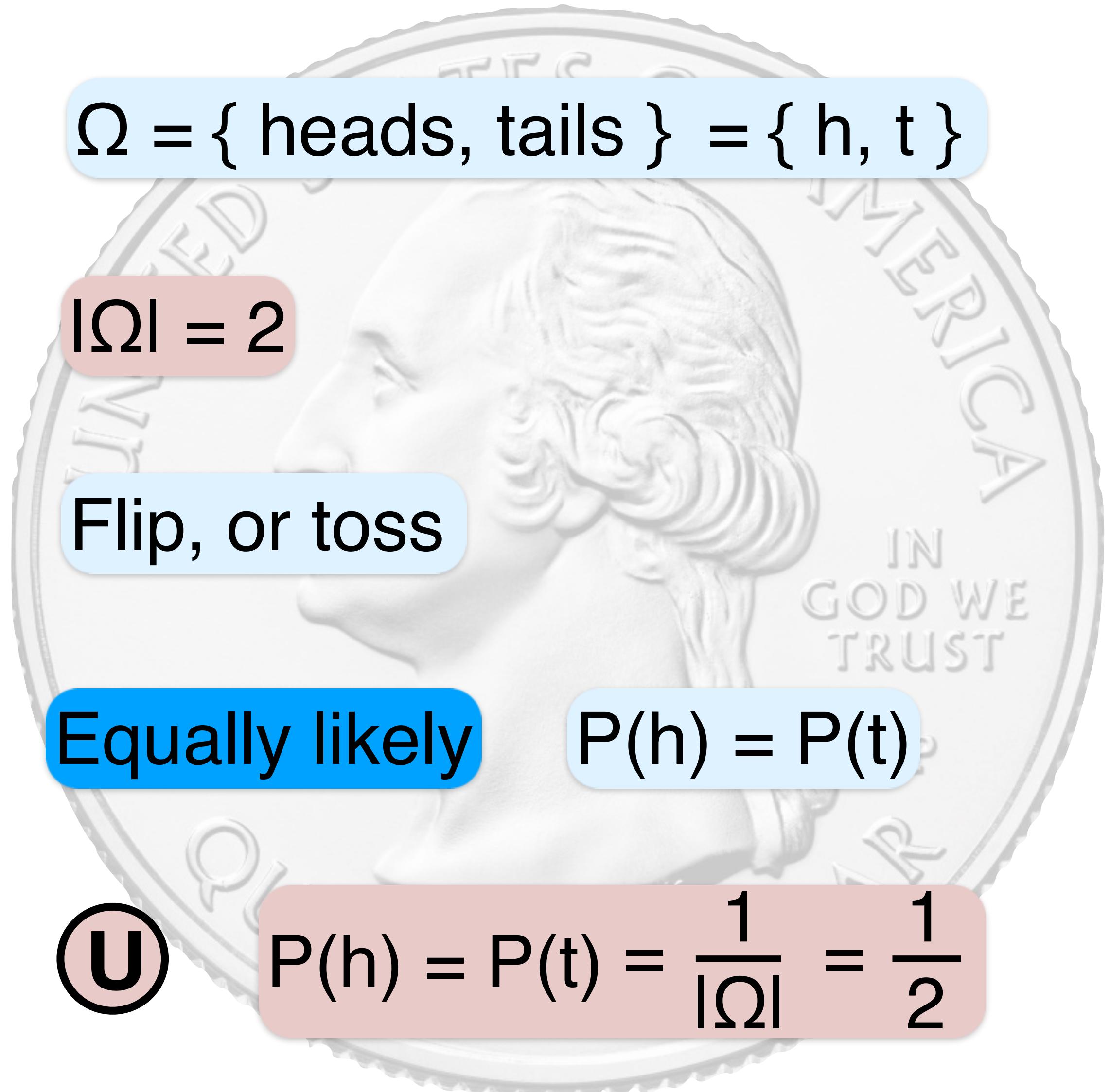
Flip, or toss

Equally likely

$$P(h) = P(t)$$

(U)

$$P(h) = P(t) = \frac{1}{|\Omega|} = \frac{1}{2}$$



# Fair Die

$$\Omega = \{ 1, 2, 3, 4, 5, 6 \}$$

$$|\Omega| = 6$$

Roll

Equally likely

$$P(1) = \dots = P(6)$$

(U)

$$P(1) = \dots = P(6) = \frac{1}{|\Omega|} = \frac{1}{6}$$



# Deck of Cards

$\Omega = \{ \text{cards} \}$

$|\Omega| = 52$

Draw a card

Equally likely

U

$$P(\begin{matrix} 3 \\ \clubsuit \\ \clubsuit \\ \clubsuit \end{matrix}) = \dots = P(\begin{matrix} Q \\ \heartsuit \\ \spadesuit \\ \diamondsuit \end{matrix}) = \frac{1}{|\Omega|} = \frac{1}{52}$$

# Uniform → Non

Uniform, equiprobable, spaces

Coin

Die

Cards

In nature, nonuniform spaces abound

rain

grades

words

illnesses

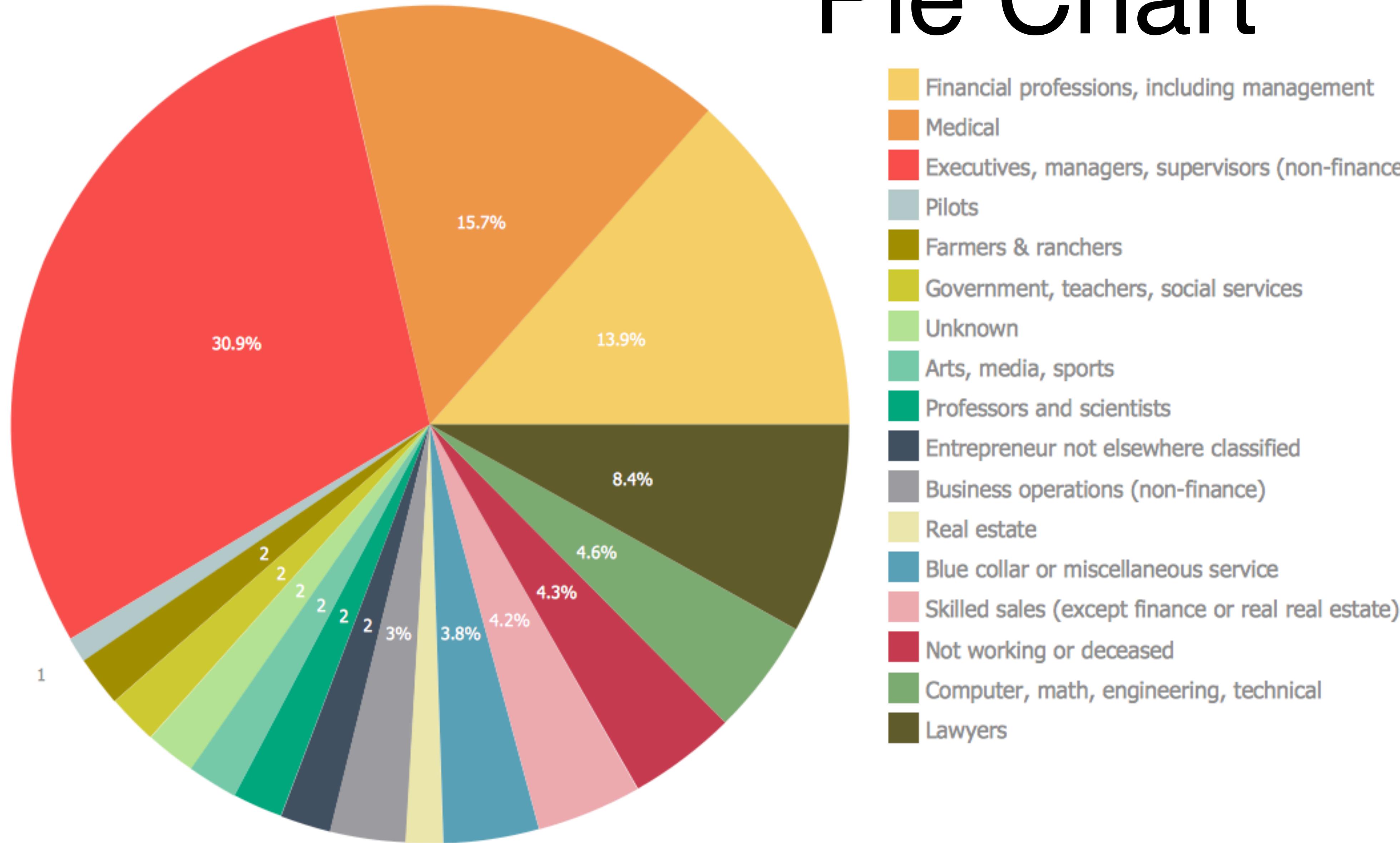
web pages

people

....



# Pie Chart



Challenge

Non-uniform distribution we can remember

# Tetrahedral Die

4-sided, pyramid die

Used in games, D&D

In games die equiprobable

We assume different probabilities

Easy to remember

Face	1	2	3	4
Probability	.1	.2	.3	.4

Conveniently, add to 1

Probability distribution



# Do's and Don'ts

Random notation may be confusing at first

Which expressions are valid?



$P( X = 3 )$

fair die:  $\frac{1}{6}$

$P( 3 )$

$\stackrel{\text{def}}{=} P( X = 3 )$

$P( x )$

specify  $x$ , e.g., for  $\forall x$ ,  $P(x)=\frac{1}{4}$



$P( 1 = 3 )$

0

$P( X )$

random value

Possible, but less common  
Make sure it's what you mean



$P( x = 3 )$

Even less likely, probably wrong

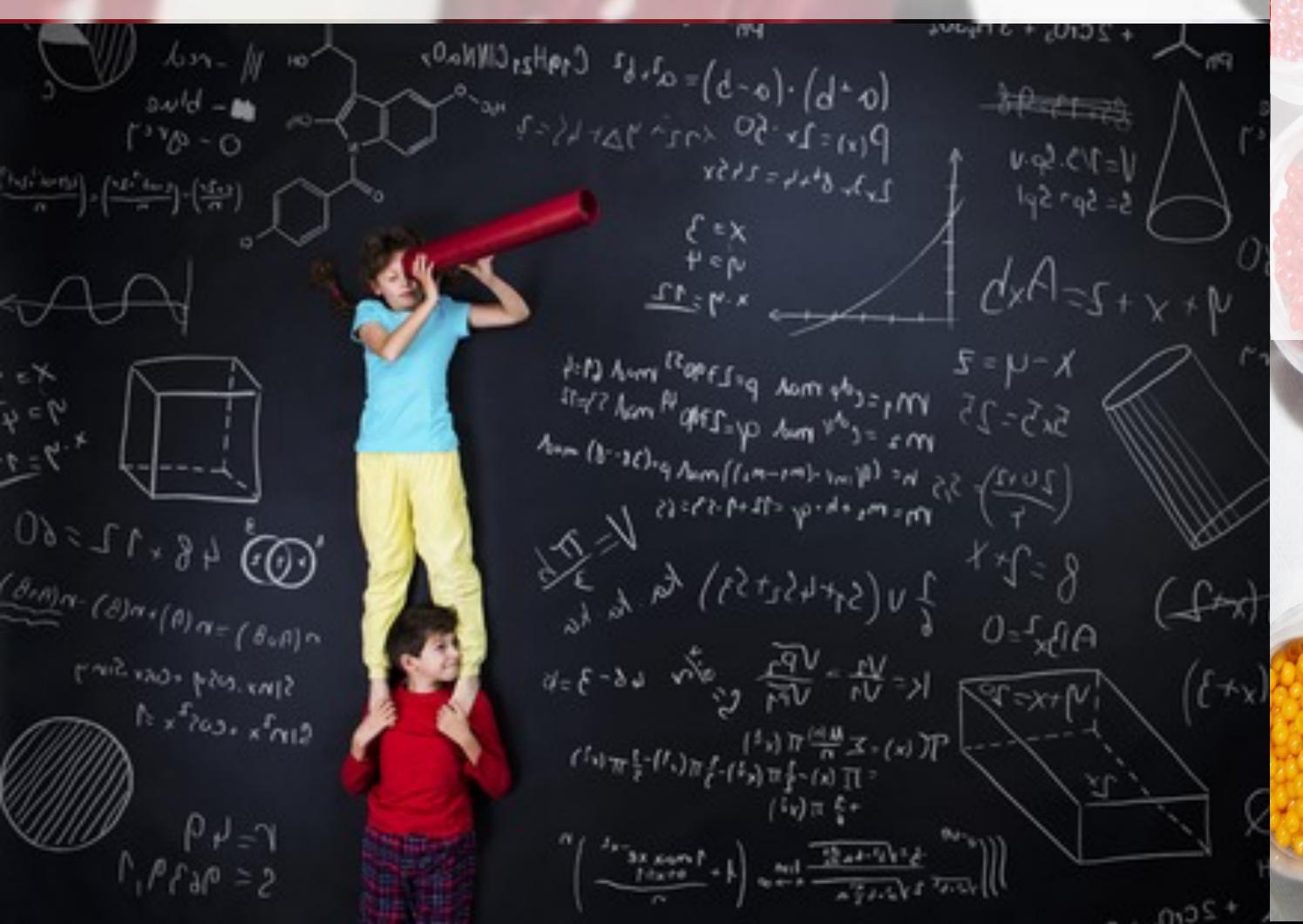
# Distribution Types

Uniform sample spaces

Coin, die, cards

Non-uniform spaces

Tetrahedral die



# Events

