set Operations

$\mathcal{P}$ $\mathcal{U}$ $\mathcal{Z}$ $\mathcal{Z}$ $\mathcal{L}$ $\mathcal{E}$

Are there sets A and B such that
A is both an **element** and a **subset** of B?

$A \in B$

$A \subseteq B$

Yes!

$\varnothing \subseteq$ any set

$A = \varnothing$

need $\varnothing \in B$

$B = \{\varnothing\}$

$\varnothing \in \{\varnothing\}$ and $\varnothing \subseteq \{\varnothing\}$

Also solutions with nonempty sets, but this is simplest

Plan

Again generalize numbers

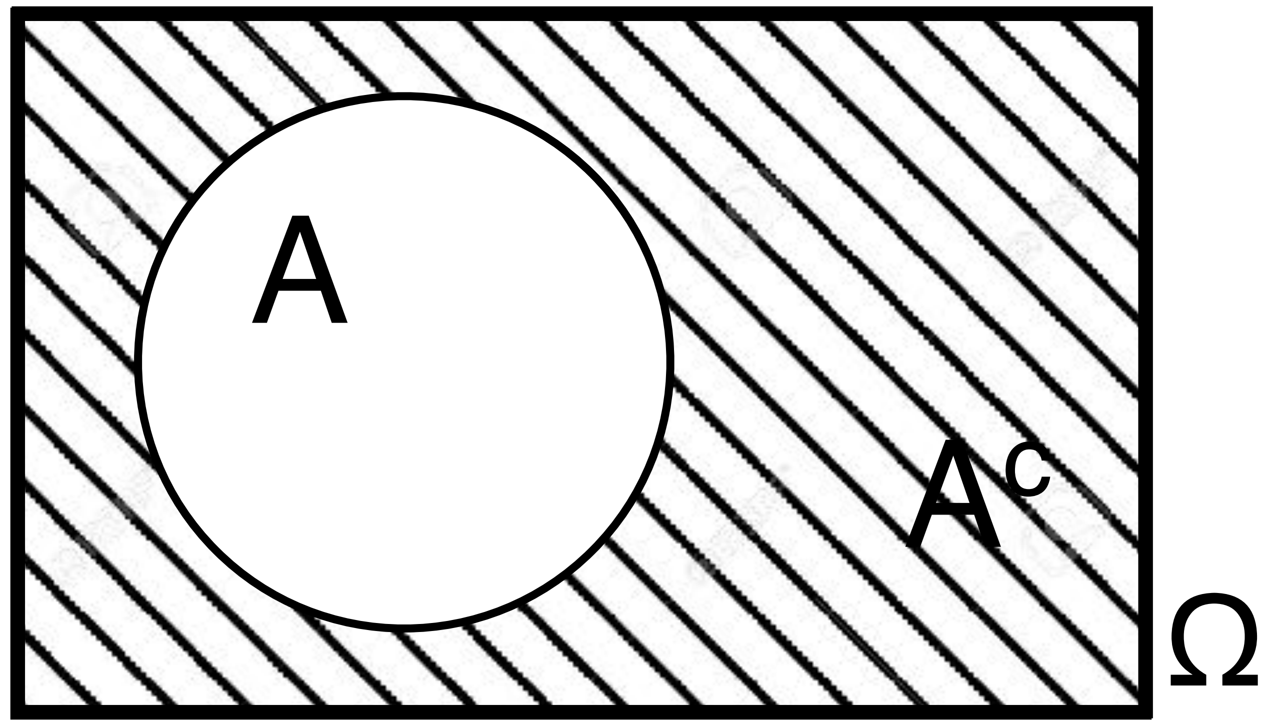
	Relations			Operations		
Number	$=$	$\leq$	$<$	$+$	$-$	$\times$
Set	$=$	$\subseteq$	$\subset$	$\cup$	$-$	$\times$
				This lecture		Next lecture
				$\cap$	$\cap$	

Complement

Recall: Universal set Ω contains all elements

The complement A^c of A is the set of Ω elements not in A

$A^c = \{ x \in \Omega : x \notin A \}$



$\Omega = \{0,1\}$ $\{0\}^c = \{1\}$ $\{0,1\}^c = \varnothing$ $\varnothing^c = \{0,1\}$

A^c sometimes denoted $\bar{A}$

$\Omega = \{0,1,2\}$ $\{0\}^c = \{1,2\}$ ← A^c depends on both A and Ω

$\Omega = \mathbb{Z}$ $\{\ldots,-2,-1\}^c = \mathbb{N}$

$\mathbb{E}$ - even $\mathbb{E}^c = \{\ldots, -3, -1, 1, 3, \ldots\} \stackrel{\text{def}}{=} \mathbb{O}$ Odd

Set Identities

Relations that hold for all sets

$\varnothing^c = \Omega$ $\Omega^c = \varnothing$

A and A^c are disjoint

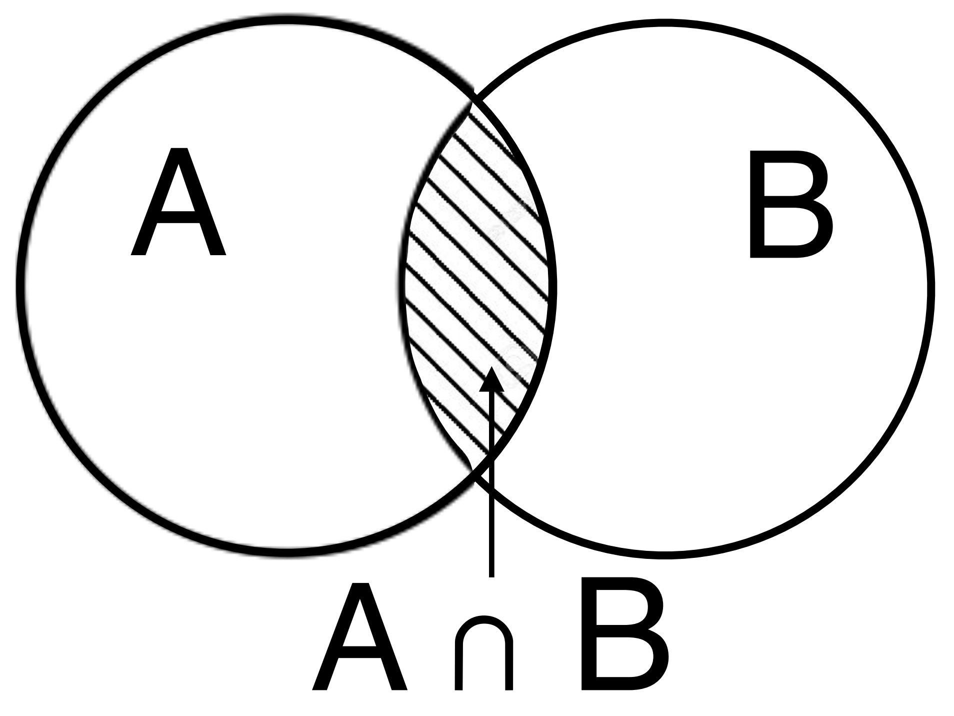
$(A^c)^c = A$ "involution"

$A \subseteq B \rightarrow A^c \supseteq B^c$

Intersection

The intersection A ∩ B is the set of elements in both A and B

$$A \cap B = \{\, x : x \in A \wedge x \in B \,\}$$

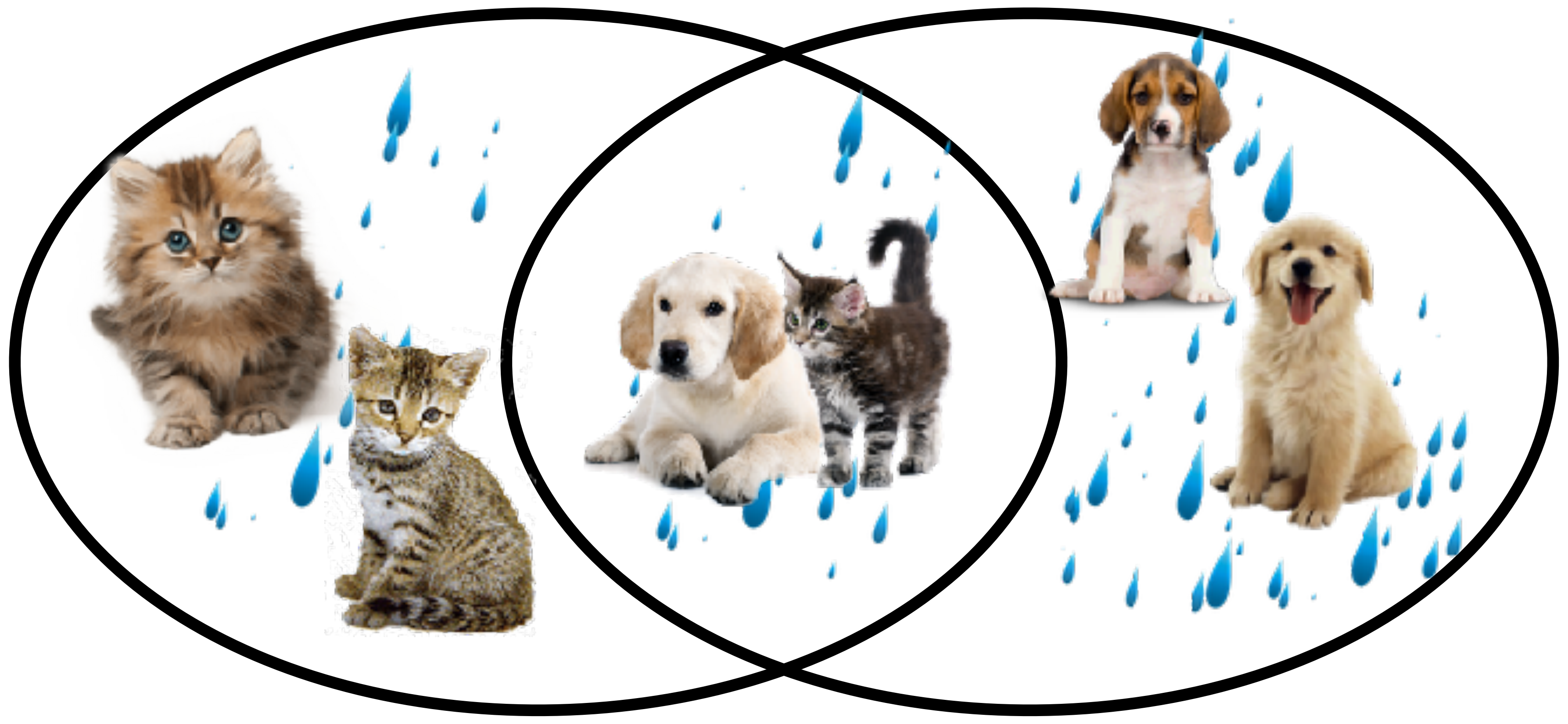


$\{0,1\} \cap \{1,3\} = \{1\}$

$\{0\} \cap \{1\} = \varnothing$

$[0,4) \cap [3,6] = [3,4)$

$[0,2] \cap (2,5] = \varnothing$

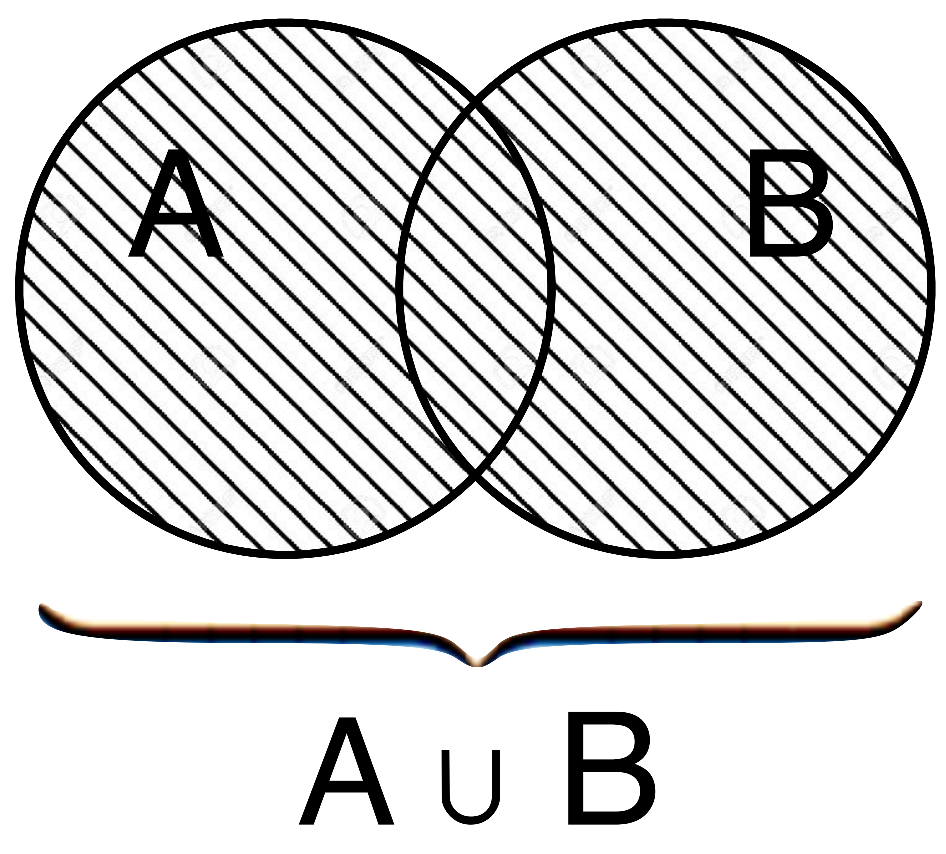


Raining cats and dogs

Union

The union $A \cup B$ is the collection of elements in A, B, or both

$A \cup B = \{ x : x \in A \lor x \in B \}$



$\{0,1\} \cup \{1,2\} = \{0,1,2\}$

$\{0,1\} \cup \{2\} = \{0,1,2\}$

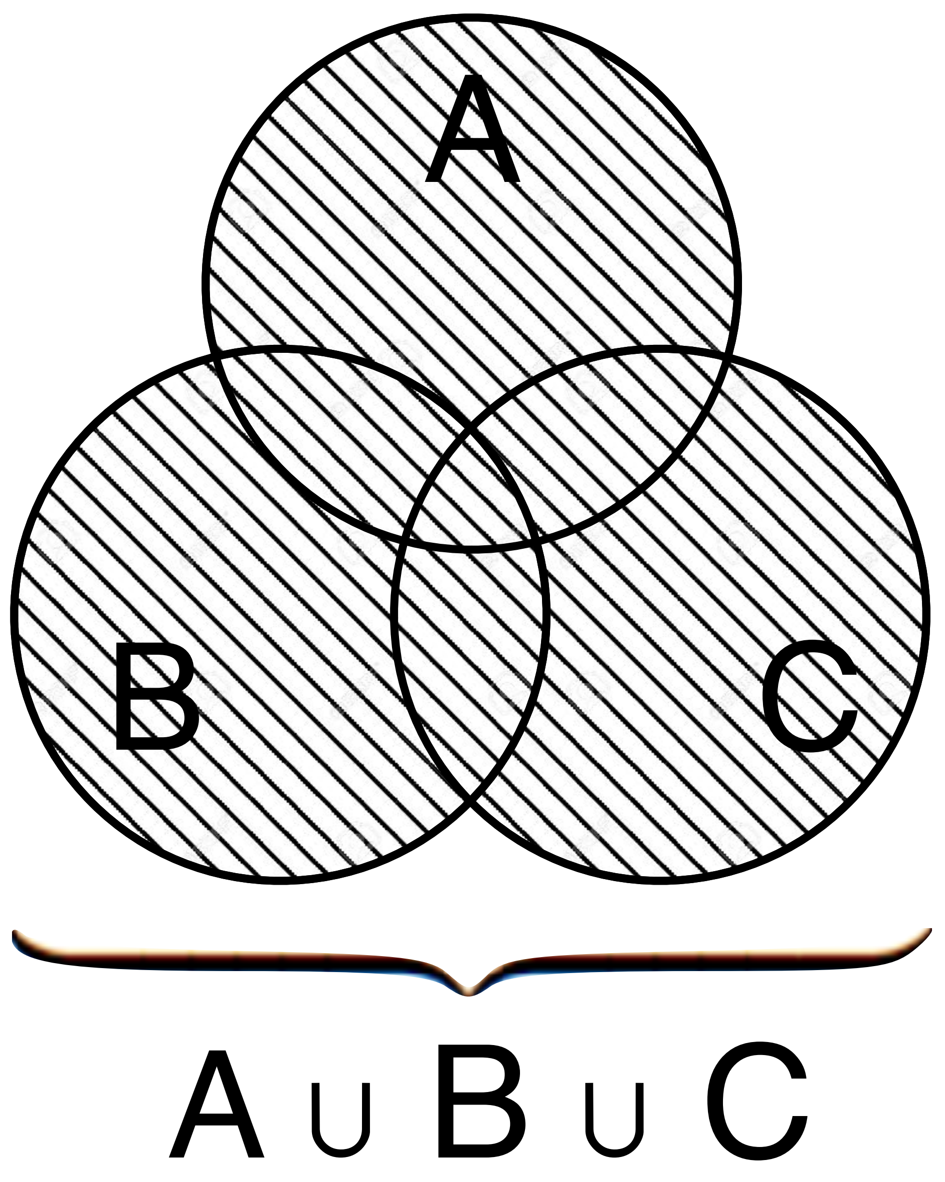
$[0,2] \cup [1,3] = [0,3]$

$(0,1) \cup \{1\} = (0,1]$

$\mathbb{E} \cup \mathbb{O} = \mathbb{Z}$

Multiple Sets

$A \cup B \cup C = \{ x \in \Omega : x \in A \lor x \in B \lor x \in C \}$

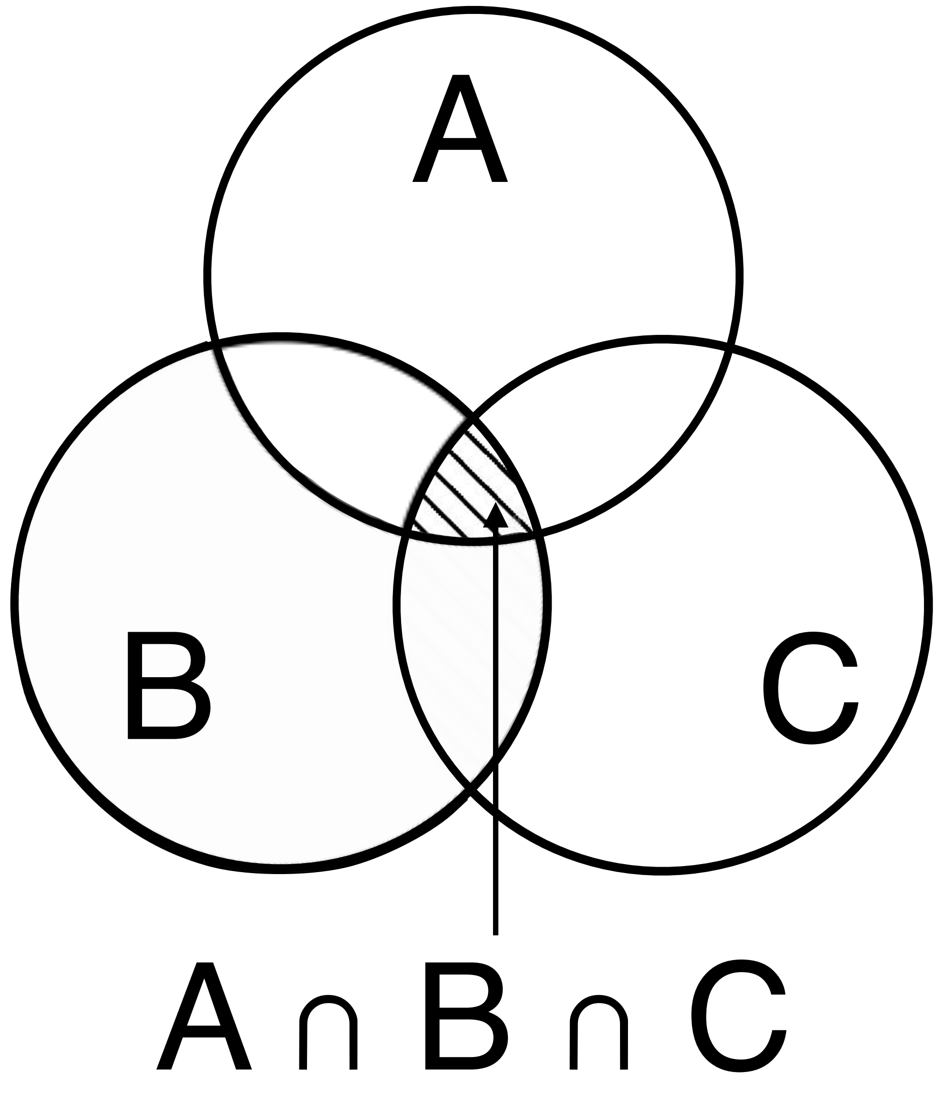


$\{0,1\} \cup \{1,2\} \cup \{2,3\} = \{0,1,2,3\}$

Generally $\bigcup_{i=1}^{t} A_i = \{x : \exists 1 \leq i \leq t, x \in A_i\}$

$\bigcup_{i=-\infty}^{\infty} \{i\} = \mathbb{Z}$

Similarly for intersection



Identities - One Set

Also called laws

Relations that hold for all sets

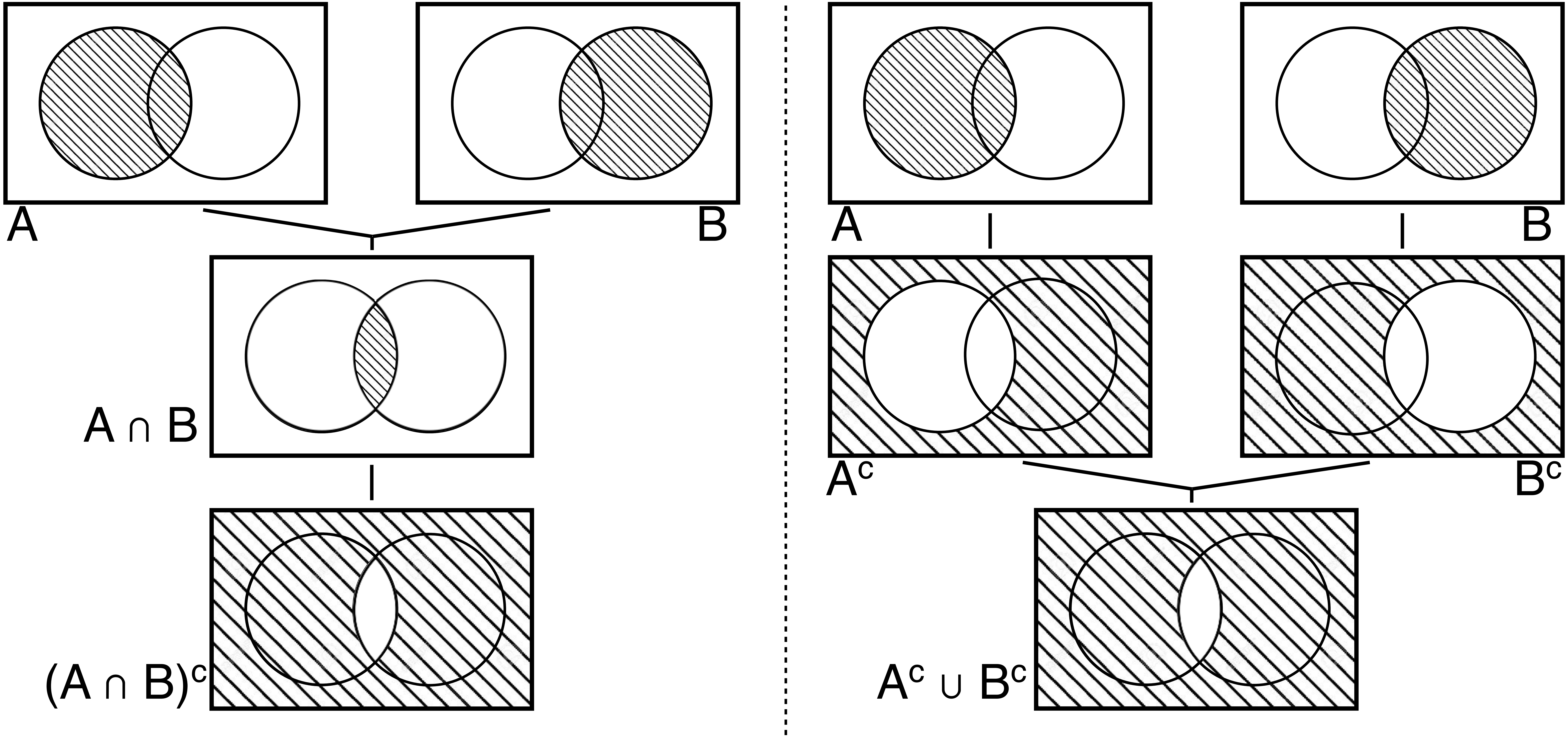
Identity	$A \cap \Omega = A$	$A \cup \Omega = \Omega$
Universal bound	$A \cap \varnothing = \varnothing$	$A \cup \varnothing = A$
Idempotent	$A \cap A = A$	$A \cup A = A$
Complement	$A \cap A^c = \varnothing$	$A \cup A^c = \Omega$

Laws - Two and Three Sets

Commutative	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associative	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
De Morgan	$(A \cap B)^c = A^c \cup B^c$	$(A \cup B)^c = A^c \cap B^c$

De Morgan's Law

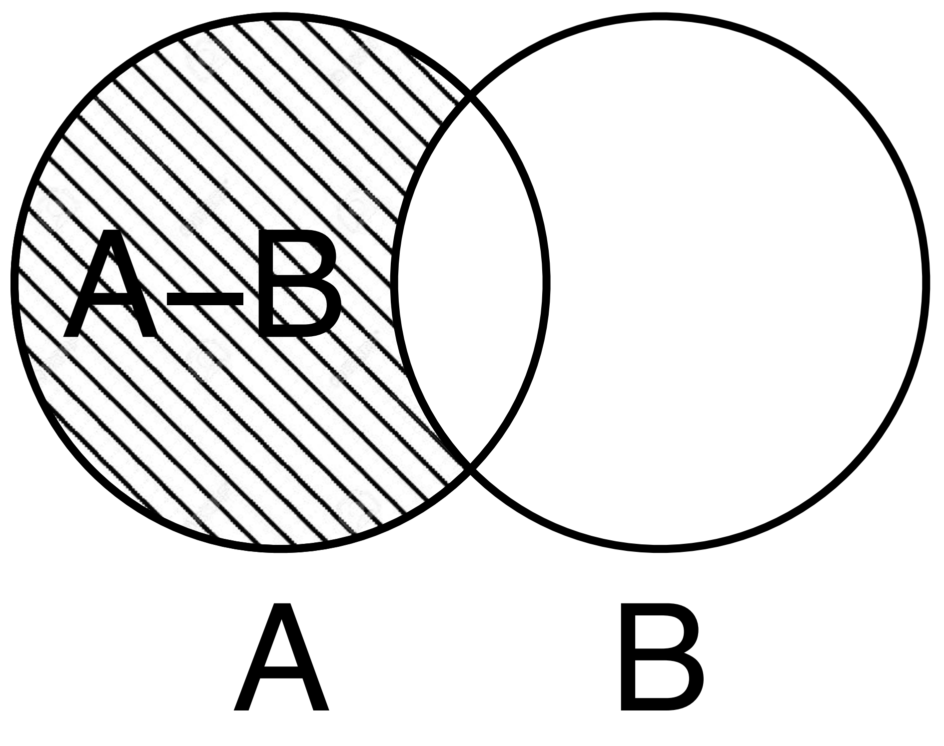
$(A \cap B)^c = A^c \cup B^c$



Set Difference

The difference A–B is the set of elements in A but not in B

$A-B = \{ x : x \in A \land x \notin B \}$



$\{0,1\} - \{1\} = \{0\}$

$\{0,1\} - \{0,1,2\} = \varnothing$

Notetation

Also

A \ B

$[1,3] - [2,4] = [1,2)$

$[1,3] - (1,3) = \{1,3\}$

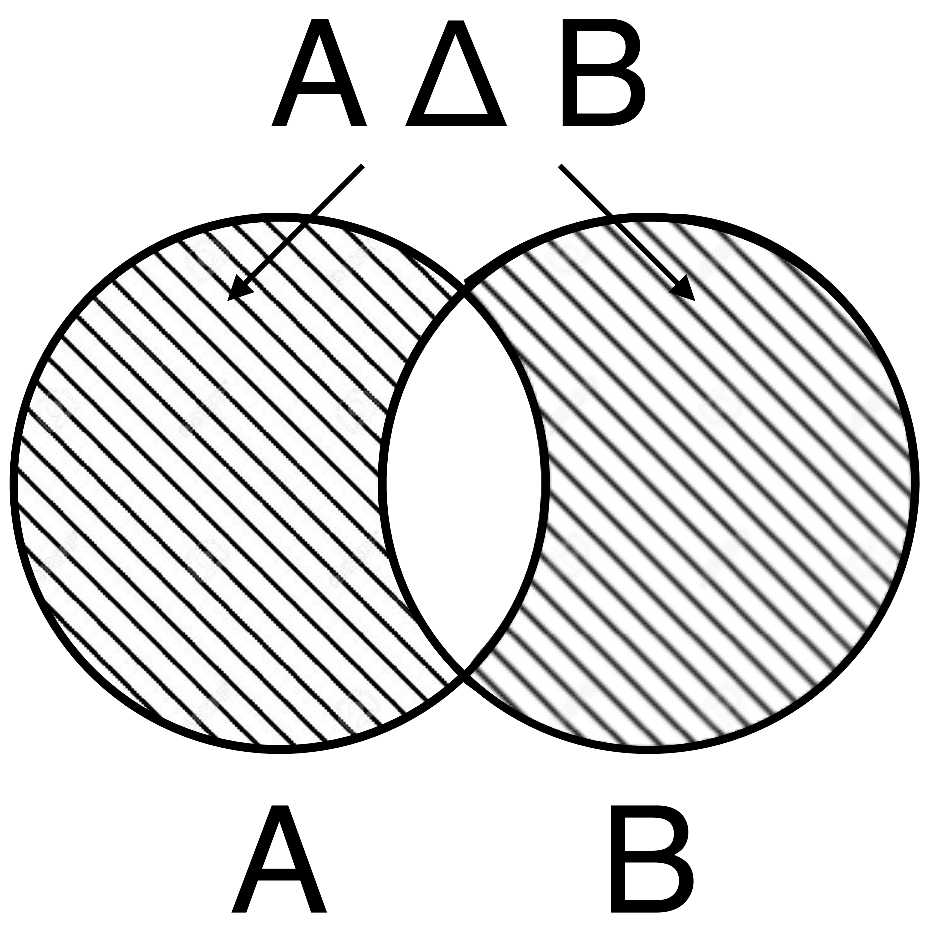
$A - B = A \cap B^c$

Symmetric Difference

The symmetric difference of two sets is the set of elements in exactly one set

$A \,\Delta\, B =$
$\{x\text{: } x \in A \wedge x \notin B \;\vee\; x \notin A \wedge x \in B\}$

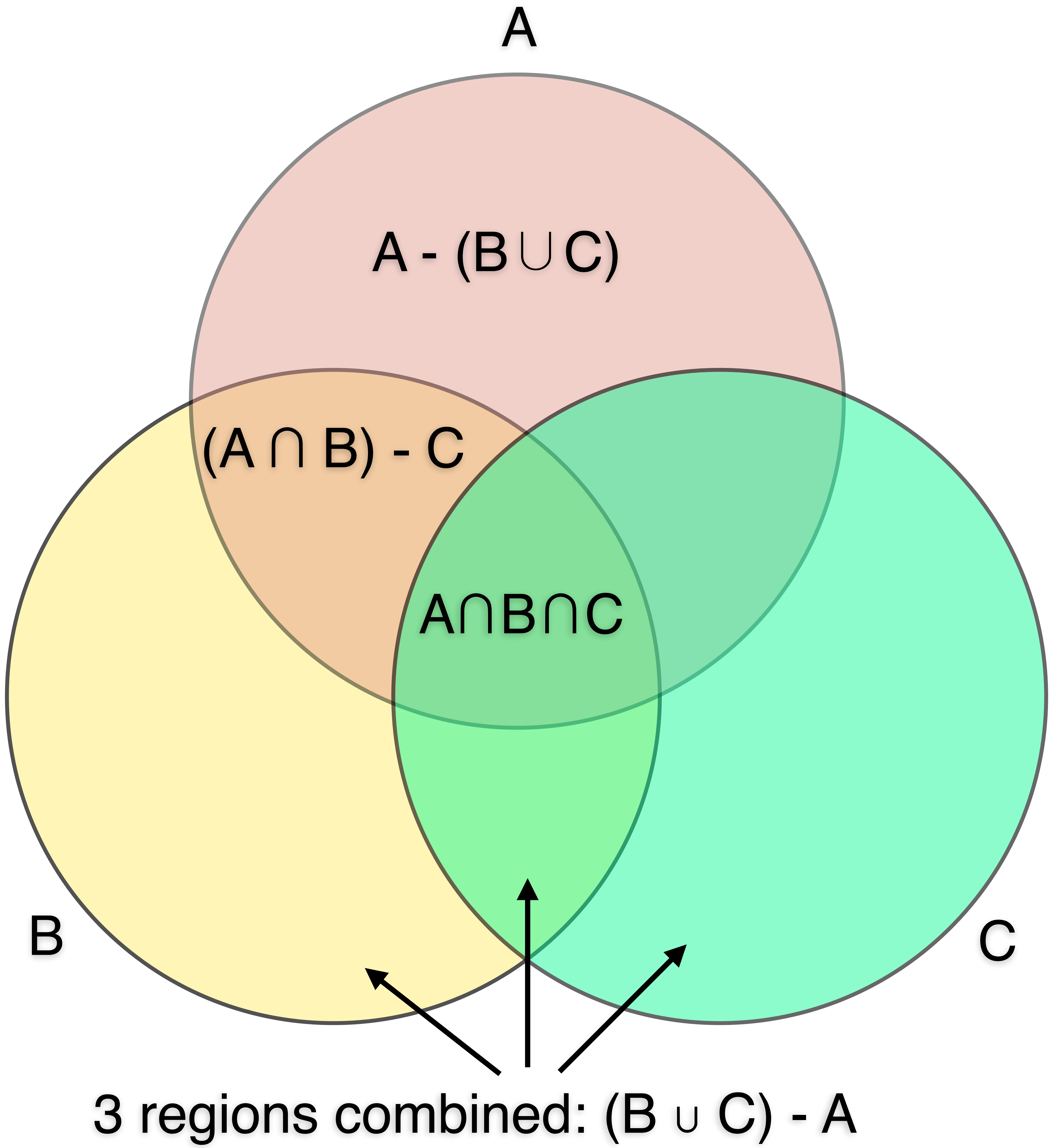
$\{0,1\} \,\Delta\, \{1,2\} = \{0,2\}$



$[0,2] \,\Delta\, [1,4] = [0,1) \cup (2,4]$

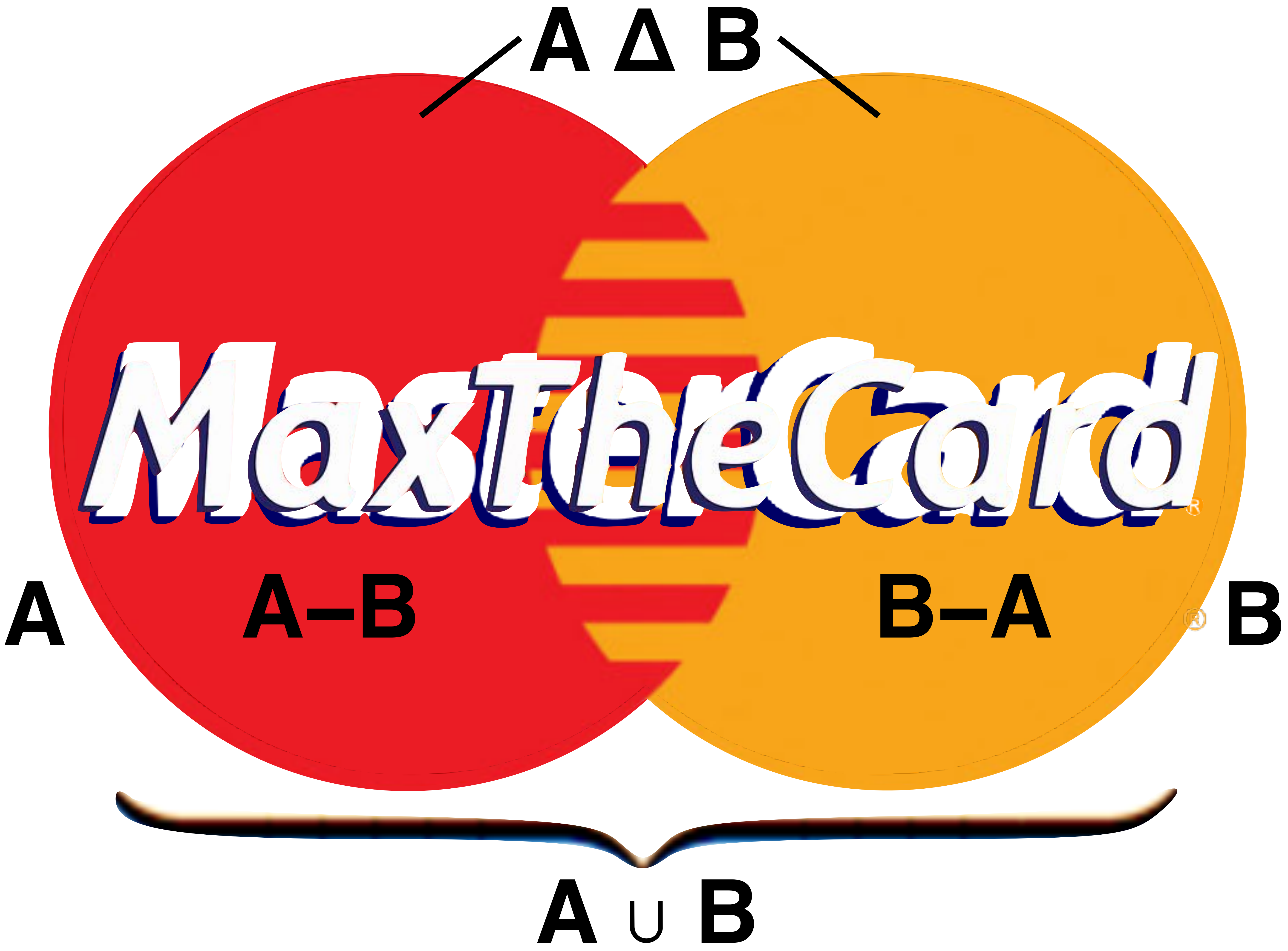
$A \,\Delta\, B = (A-B) \cup (B-A)$

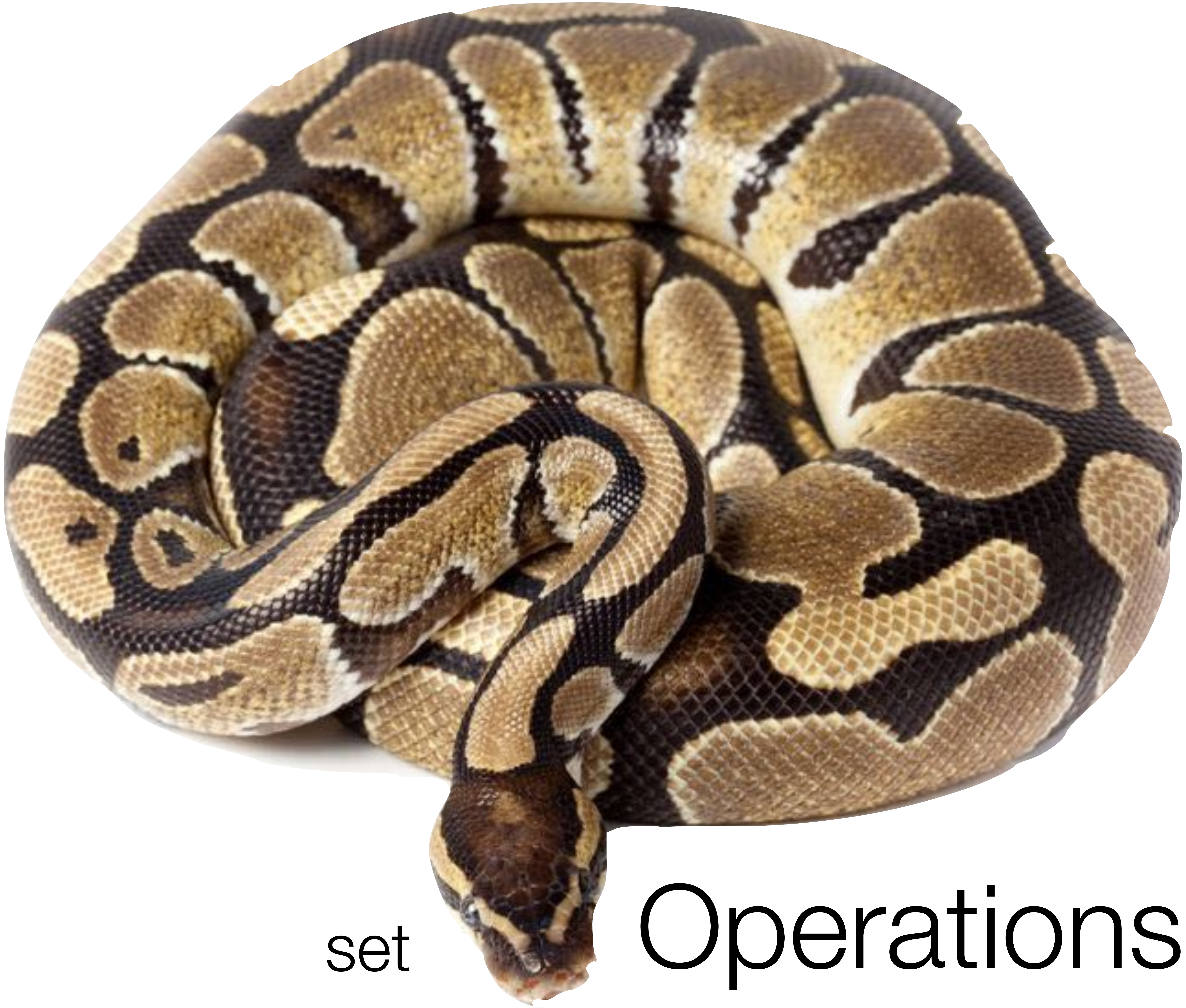
3 Sets



Venn's Master Chart

QUIZ





set Operations

Union and Intersection

```
A = {1,2}
B = {2,3}
```

∪ | or **union**

∩ & or **intersection**

```
A | B
{1, 2, 3}
C = A.union(B)
print(C)
{2, 1, 3}
```

```
A & B
{2}
C = A.intersection(B)
print(C)
{2}
```

Set- and Symmetric-Difference

```
A = {1,2}
B = {2,3}
```

Set difference

- or **difference**

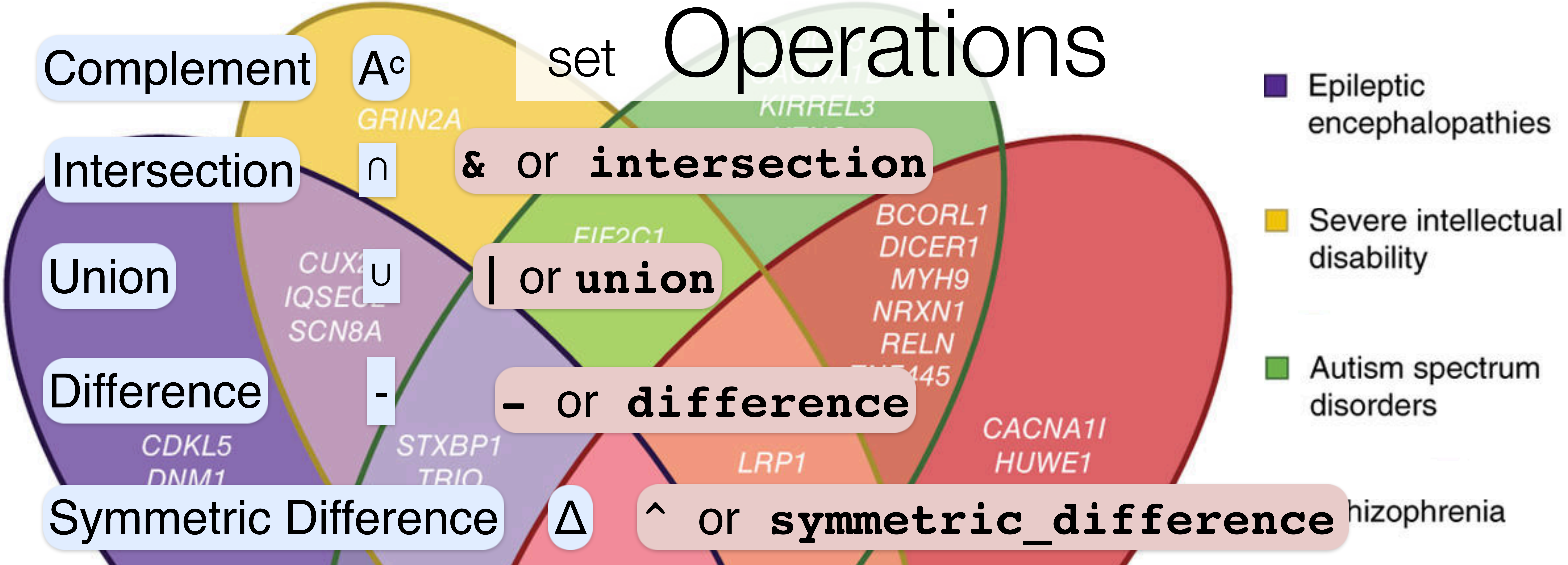
```
A - B
{1}
B.difference(A)
{3}
```

Symmetric difference

^ or **symmetric_difference**

```
A ^ B
{3,1}
B.symmetric_difference(A)
{3,1}
```

set Operations



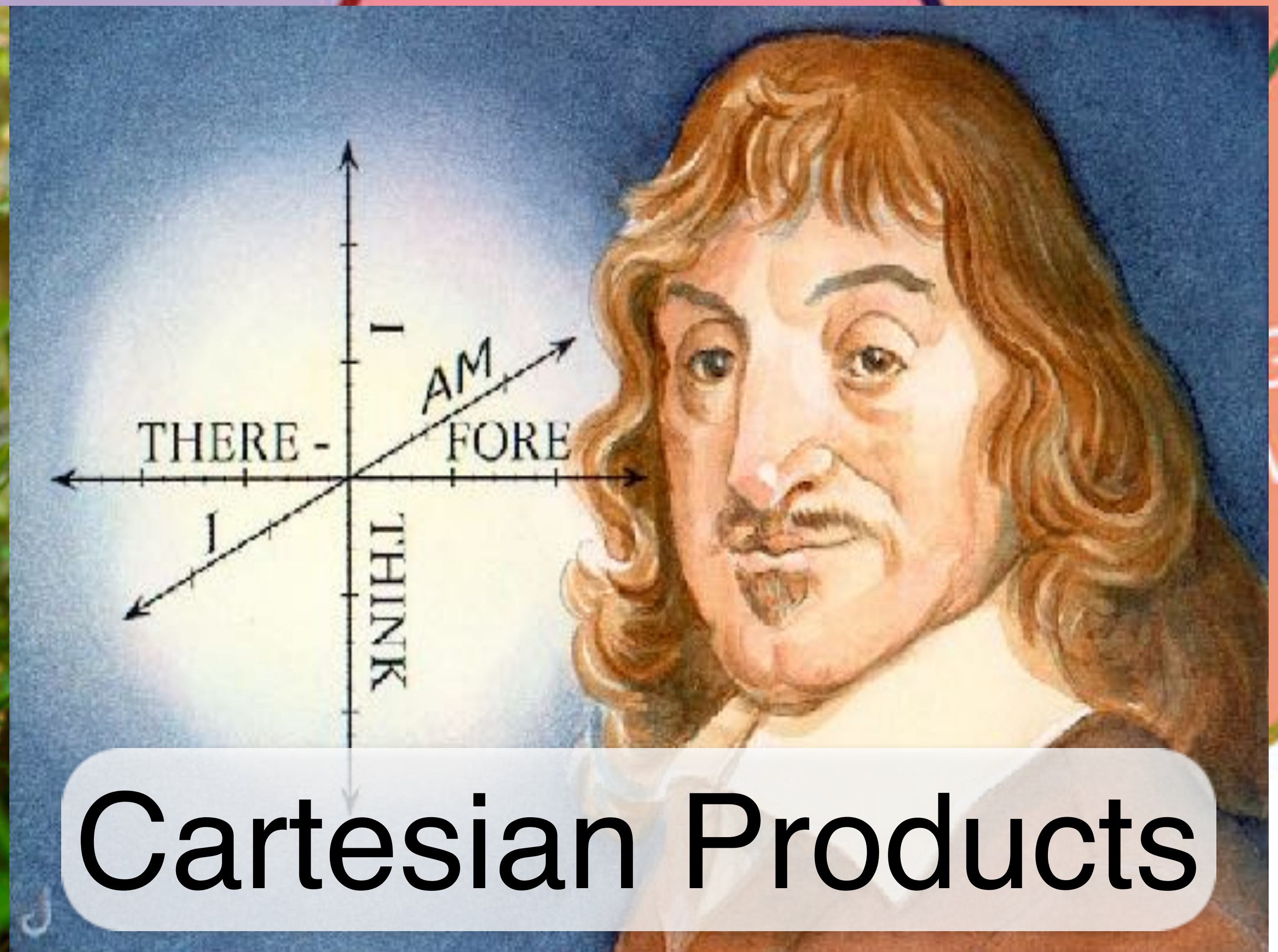
Complement A^c

Intersection $\cap$ **& or intersection**

Union $\cup$ **| or union**

Difference - **- or difference**

Symmetric Difference Δ **^ or symmetric_difference**



Cartesian Products