Probability Inequalities

Three Axioms

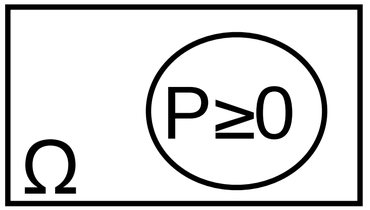
Inequalities

Union bound

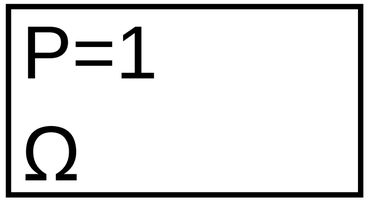
$1.4M question

Three Axioms

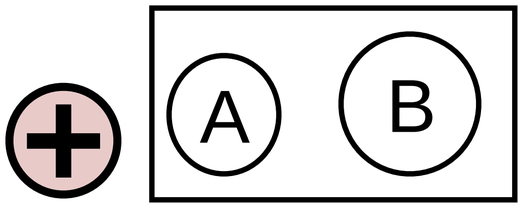
Non-negativity $P(A) \geq 0$



Unitarity $P(\Omega) = 1$



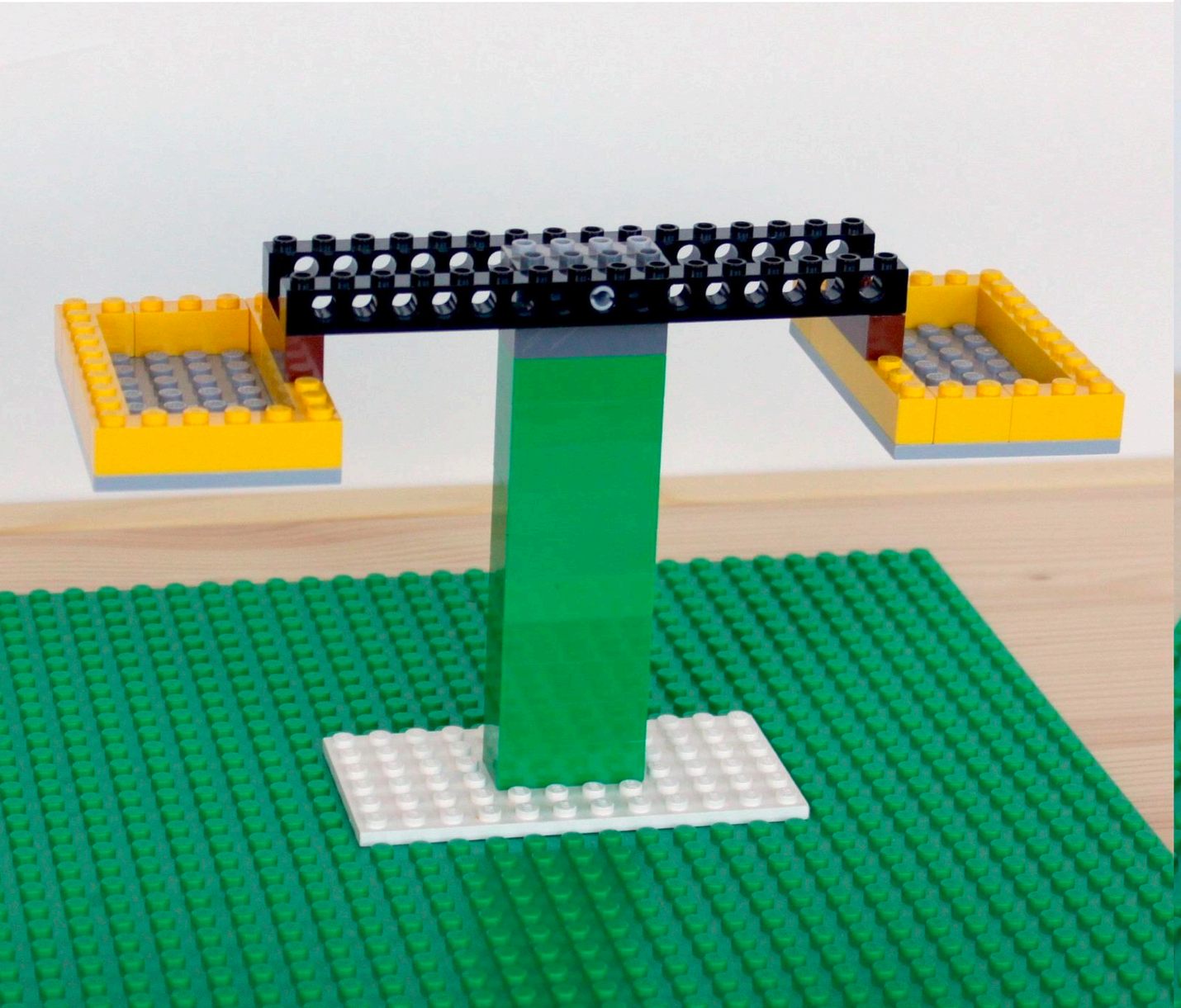
Addition rule A, B disjoint → $P(A \cup B) = P(A) + P(B)$

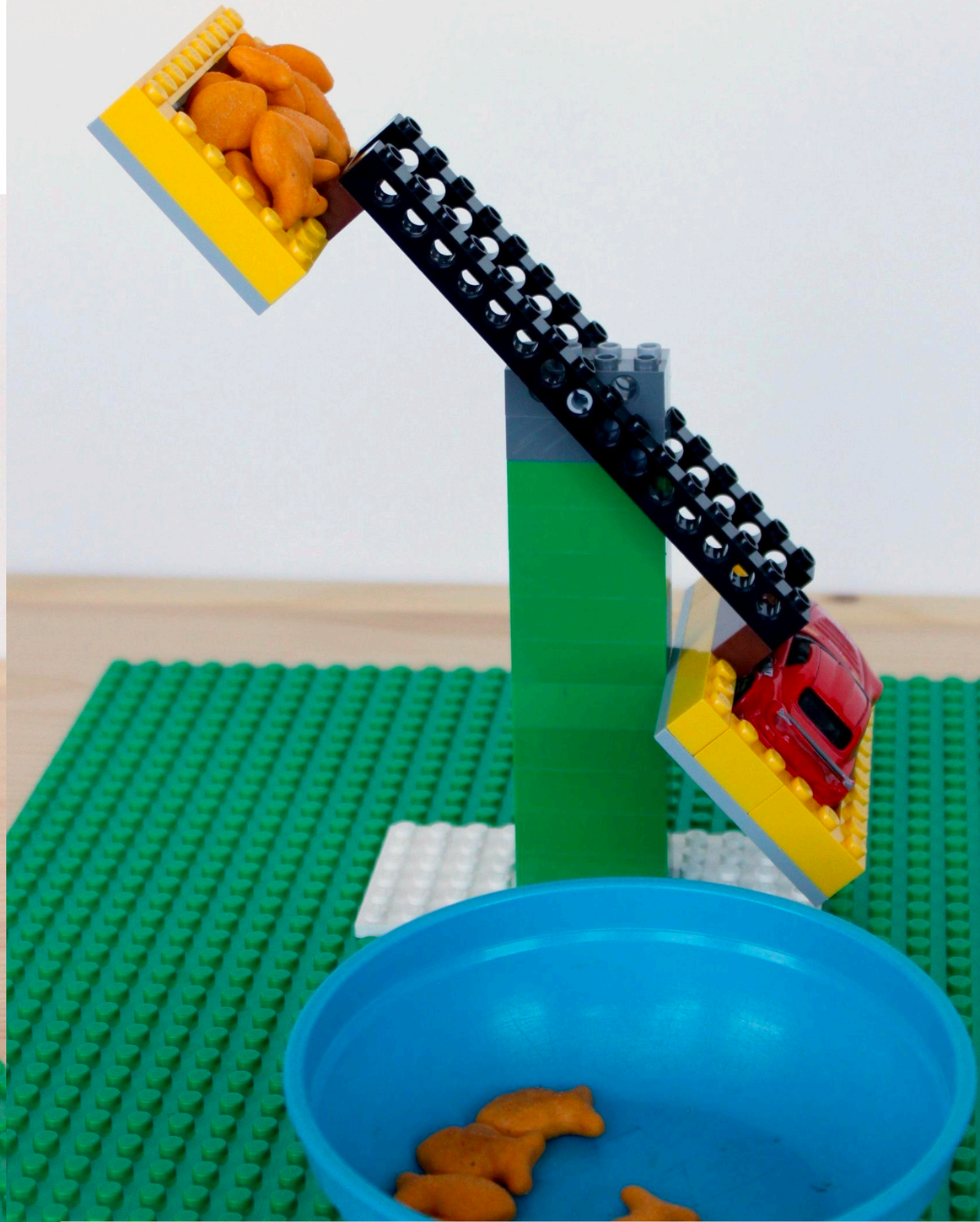


$A_1, A_2, \ldots$ disjoint → $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$

Equalities → Inequalities

Inequalities





Probability of Null Event

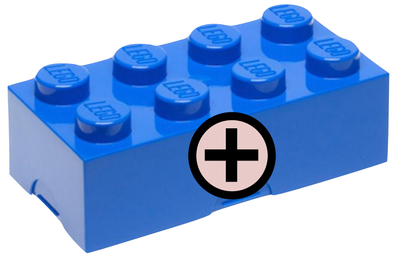
① $P(\Omega) = 1$

What about $P(\varnothing)$?

Show $P(\varnothing) = 0$

$\Omega = \varnothing \cup \Omega$

$P(\Omega) = P(\varnothing \cup \Omega) = P(\varnothing) + P(\Omega)$



$P(\varnothing) = 0$

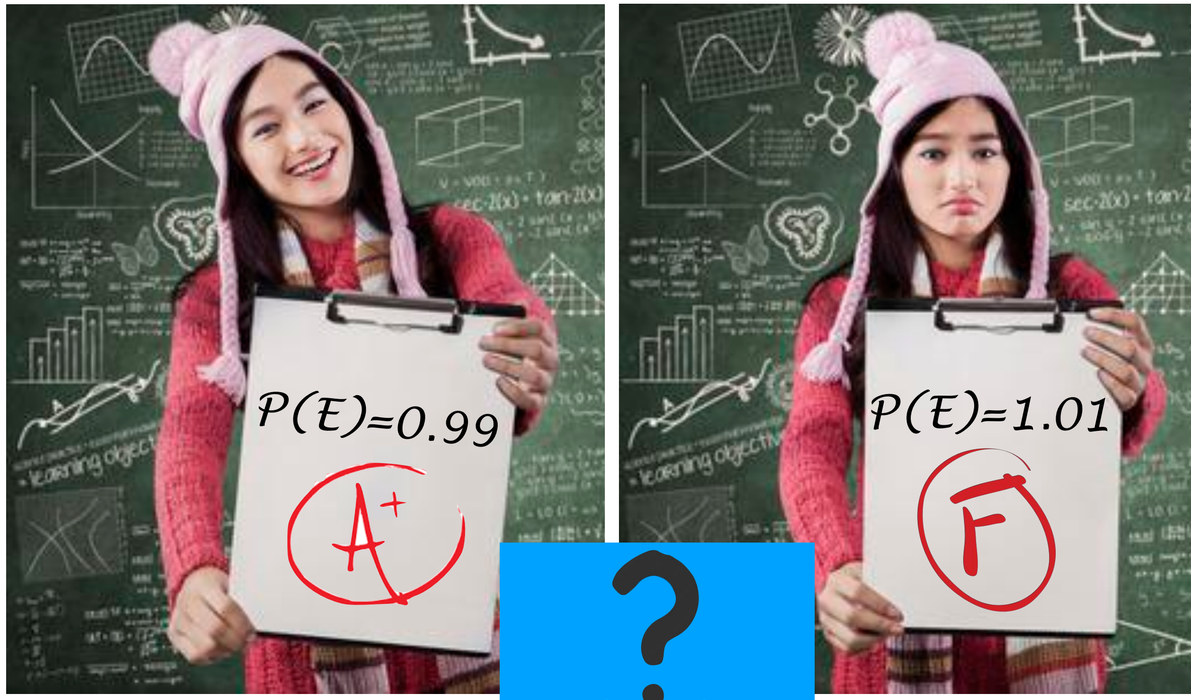
P(null) = nil

0

$0 \leq P(A) \leq 1$

≥ $P(A) \geq 0$

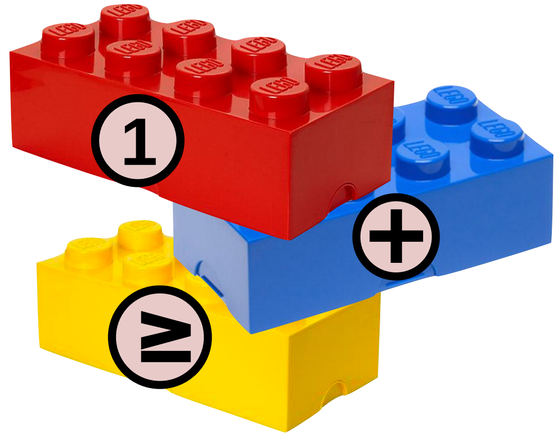
Show $P(A) \leq 1$



$A \cup A^c = \Omega$

≥ ⊕ ●● 1

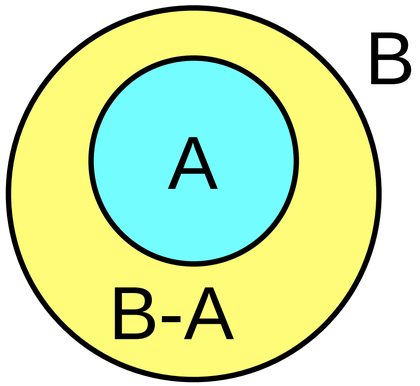
$P(A) \leq P(A) + P(A^c) = P(A \cup A^c) = P(\Omega) = 1$



$P(A) \leq 1$ Probability always between 0 and 1

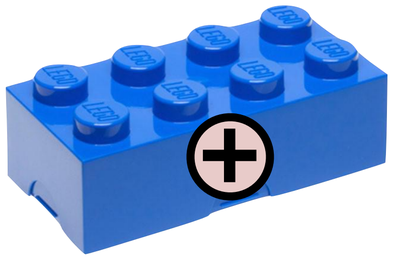
Subset

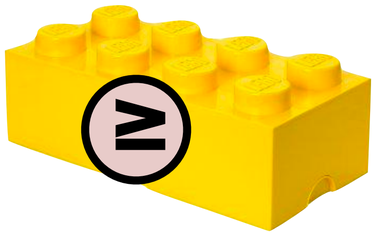
$A \subseteq B$ $P(A) \leq P(B)$



$A \subseteq B$ $B = A \cup (B-A)$

$$P(B) = P(A \cup (B-A)) = P(A) + P(B-A) \geq P(A)$$

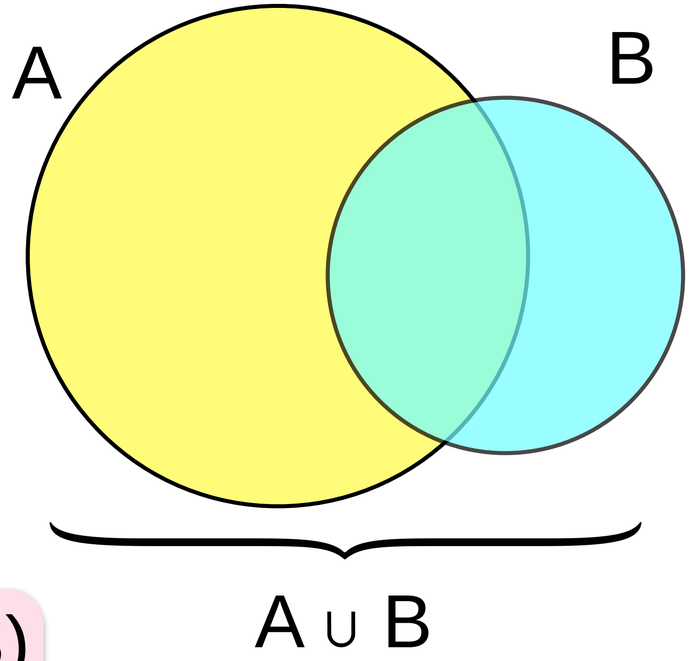




Obvious?

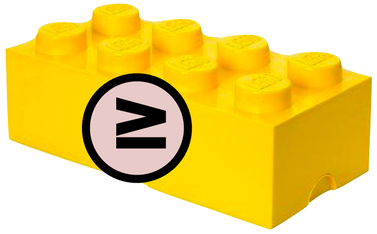
Union

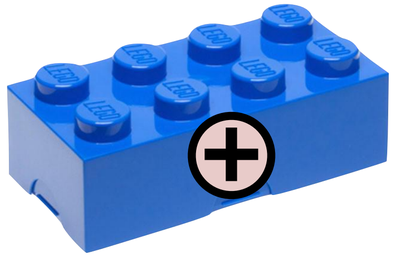
$\max(P(A),P(B)) \leq P(A \cup B) \leq P(A) + P(B)$

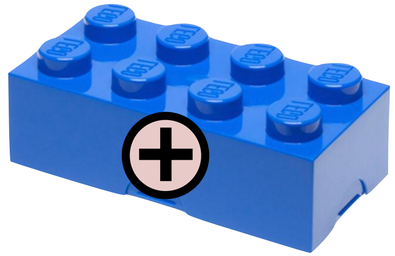


Left ≤ $A, B \subseteq A \cup B$ $P(A), P(B) \leq P(A \cup B)$

Right ≤  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$







$P(A \cup B) \leq P(A) + P(B)$ Union bound Very useful

$1,400,000 Question

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1 Linda is a bank teller

2 Linda is a bank teller and is active in the feminist movement

Please answer in the poll below

The Linda Problem

Psychological Review

VOLUME 90 NUMBER 4 OCTOBER 1983

Extensional Versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment

Amos Tversky
Stanford University

Daniel Kahneman
University of British Columbia, Vancouver, British Columbia, Canada

88 UBC Students

85% Bank teller **&** activist more likely

B - Bank teller

A - Active in feminist movement

$P(B)$ vs. $P(B \cap A)$

$B \supseteq B \cap A$

$P(B) \geq P(B \cap A)$

Irrational concept of probability

Related Questions

Several conjunction fallacy problems

Björn Borg — Preeminent tennis player of late ’70’s

6 French opens — Wimbledon ’76-‘80

1980 — Suppose Borg reaches the ’81 Wimbledon Finals

More likely?

A — Borg will lose first set

B — Borg will lose first set but win match

72% chose B — Again B ⊆ A — P(B) ≤ P(A)

1981 — Reached final — Won first set — Lost match — to 

1983 — Age 26 — Retired

The Ultimate Reward

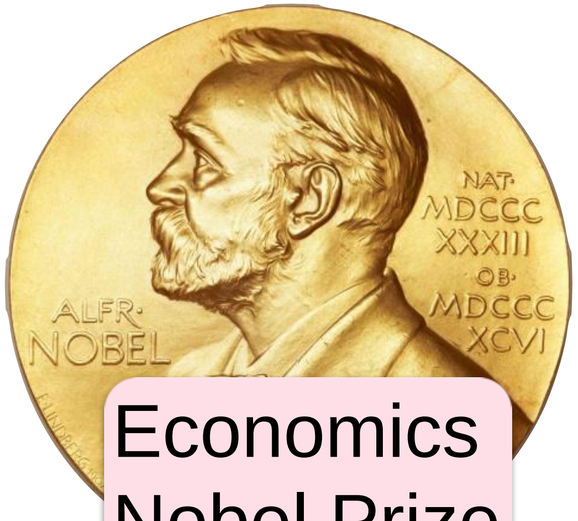
T & K — Many probability-perception studies

1996 — Tversky passed away

2002 — Kahneman

best known

“for integrating insights from psychological research into economic science, especially regarding human judgment and decision-making under uncertainty”



Economics Nobel Prize

Criticism

Multiple choice and ranking questions often disjoint

Coffee or Tea

Tea hot or cold

Rank Mon, Tue, or Wed

Often, when not explicitly disjoint, we still interpret them as such

With milk or with milk and sugar → Milk only or Milk and sugar

Meet Monday or Monday and Tuesday → Mon only or Mon and Tue

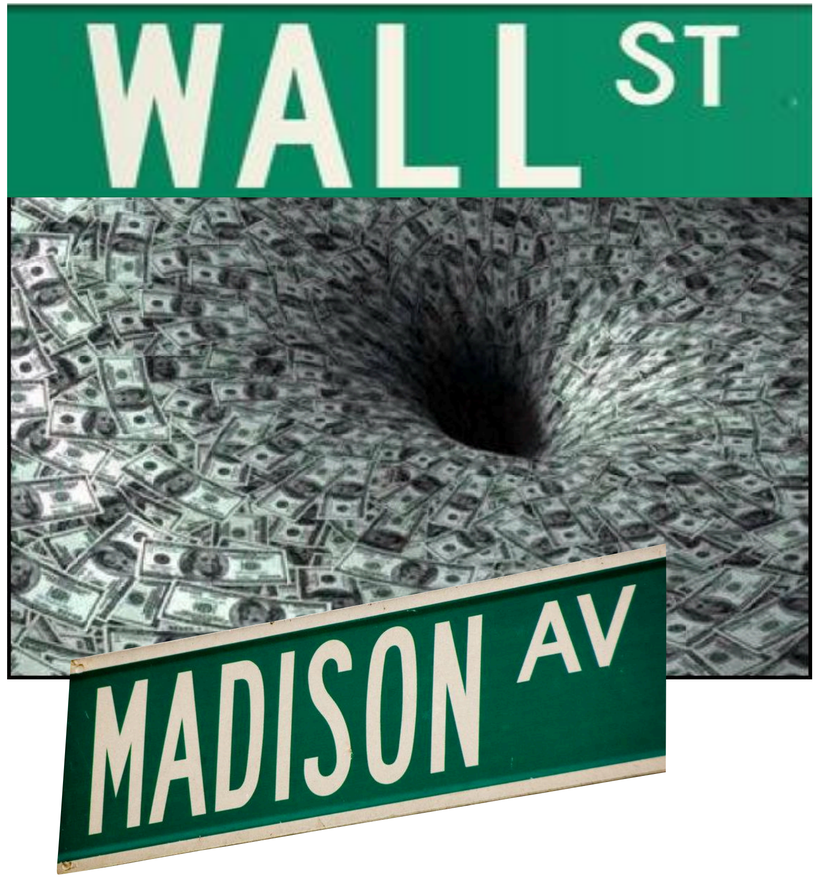
Students may have similarly interpreted the Linda question

Teller or Teller and activist → Teller only or Teller and activist

Simply thought Linda more likely active in movement than not

Bottom Line

Humans only moderately good at estimating probability



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$1.4M question

Conditional Probability