Partial Permutations

Partial Permutations

orders of n objects = n!

orders of some of the n objects = ?

2-digit PINs

any digits

11 45

10 x 10

distinct digits

05 32 ~~33~~

10 x 9

1st digit 2nd digit

3-letter words

any letters

mom xyz

26 x 26 x 26

distinct letters

abc ~~dad~~

26 x 25 x 24

1st 2nd 3rd

Sequences without Repetition

k-permutation of [n]	Length-k sequences over [n]
with repetition	n^k
without repetition	$n^{\underline{k}}$

k-Permutations

An ordering of k elements in a set S is a k-permutation of S

2-permutations of {a,b,c} ab, ac, ba, bc, ca, cb

n-permutation of an n-set is just a permutation of the set

k-permutations of an n-set $n \cdot (n-1) \cdot \ldots \cdot (n-k+1) \stackrel{\text{def}}{=} n^{\underline{k}}$

k	$n^{\underline{k}}$
1	n
2	n(n-1)
3	n(n-1)(n-2)
…	…
k	n(n-1)…(n-k+1)

k^{th} falling power of n

Also denoted P(n,k) or $(n)_k$

Falling Powers and Factorials

Falling powers are simply related to factorials

$$n^{\underline{k}} \qquad n \cdot (n-1) \cdot \ldots \cdot (n-k+1) \quad = \frac{n!}{(n-k)!}$$

Factorials and Permutations

4 programming, 5 probability, 6 machine-learning books

ordered lists with 2 books from each subject where same subject books are listed consecutively = ?

Prob 3, Prob1, ML 5, ML 2, Prog 1, Prog 4

ML 2, ML 6, Prog 1, Prog 2, Prob 5, Prob 1

$3! \cdot 4^{\underline{2}} \cdot 5^{\underline{2}} \cdot 6^{\underline{2}}$ = 6 x (4x3) x (5x4) x (6x5) = 43,200

Subject Order

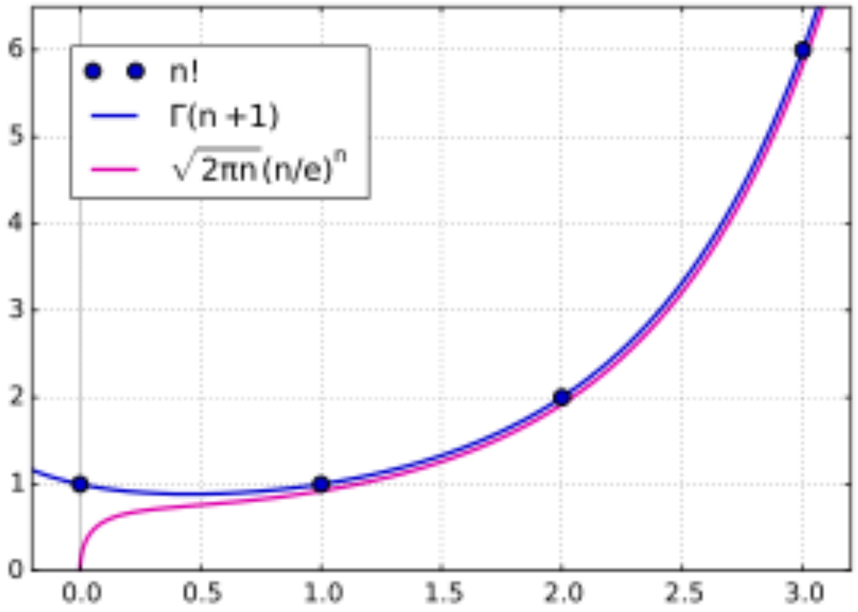
Progra-mming

Proba-bility

Machine Learning

Stirling's Approximation

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$



N	N!	Stirling approximation	Error %
1	1	1.00	0.227445%
2	2	2.00	0.032602%
3	6	6.00	0.009986%
4	24	24.00	0.004266%
5	120	120.00	0.002198%
6	720	720.01	0.001276%
7	5040	5040.04	0.000805%
8	40320	40320.22	0.000540%
9	362880	362881.38	0.000380%
10	3628800	3628810.05	0.000277%
11	39916800	39916883.11	0.000208%
12	479001600	479002368.48	0.000160%
13	6227020800	6227028659.89	0.000126%
14	87178291200	87178379323.32	0.000101%
15	1307674368000	1307675442913.47	0.000082%
16	20922789888000	20922804061389.80	0.000068%
17	355687428096000	355687629001078.00	0.000056%
18	6402373705728000	6402376752492220.00	0.000048%
19	121645100408832000	121645149634119000.00	0.000040%
20	2432902008176640000	2432902852332160000.00	0.000035%