
Counting
Cartesian
Products

Cartesian Products

$|\{a,b\}| = 2$ $\quad$ $|\{1,2,3\}| = 3$

$$\{a,b\} \times \{1,2,3\} = \left\{\begin{matrix} (a,1) & (a,2) & (a,3) \\ (b,1) & (b,2) & (b,3) \end{matrix}\right\} \begin{matrix} \leftarrow 3 \\ \leftarrow 3 \end{matrix}$$

$$|\{a,b\} \times \{1,2,3\}| \overset{+}{=} 3+3 = 2\times3 = 6$$

3

2	(a,1)	(a,2)	(a,3)
	(b,1)	(b,2)	(b,3)

area 2x3 = 6

The size of a Cartesian Product is the product of the set sizes

$|A \times B| = |A| \times |B|$

Product Rule ×

Another application of + rule

Tables

Cartesian product

3 attributes

5 records

Adam	M	3.5
Eve	F	3.7
John	M	3.4
Lisa	F	3.2
Mary	F	3.9

X

5x3=15 cells

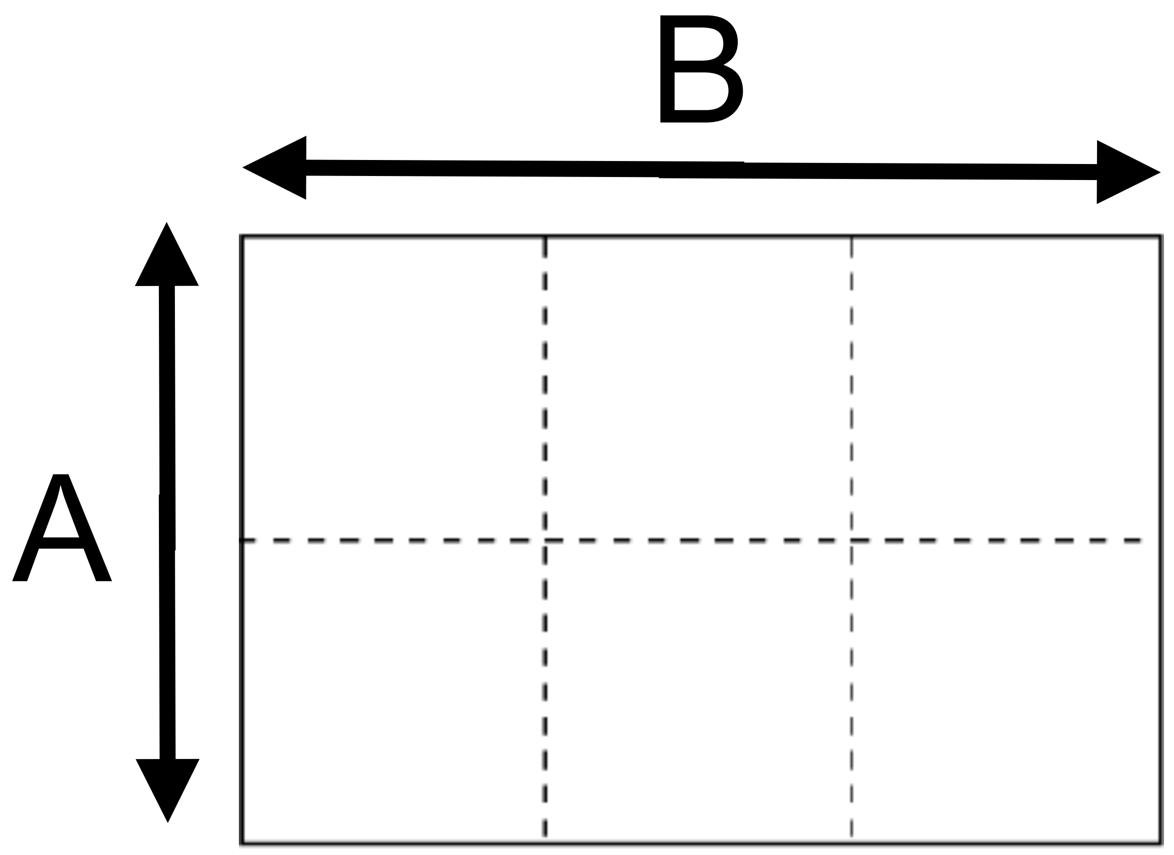
Three Sets

A x B

$\{(a,b): a \in A, b \in B\}$

rectangle

$|A \times B| = |A| \times |B|$

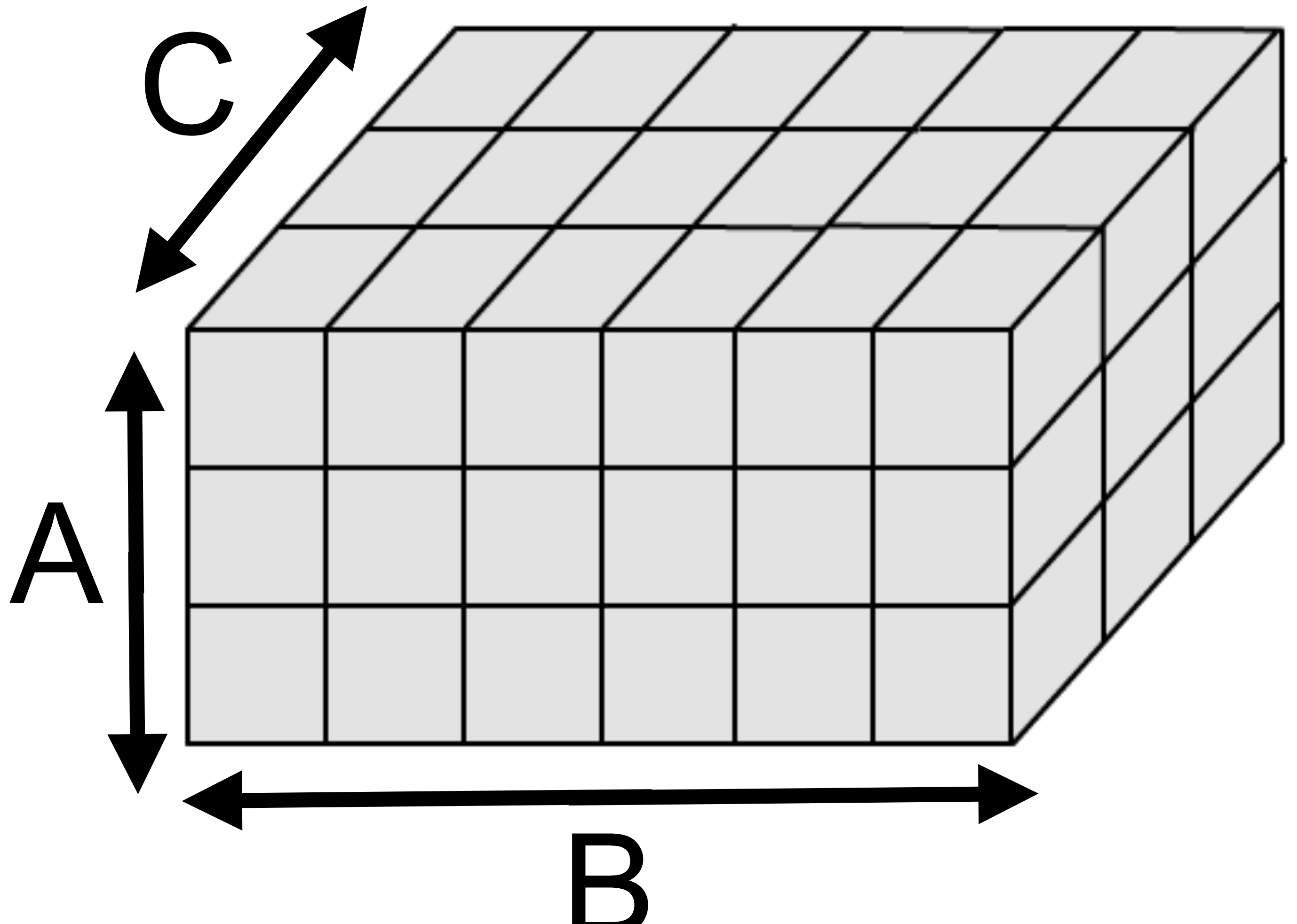


A x B x C

$\{(a,b,c): a \in A, b \in B, c \in C\}$

"cuboid"

$|A \times B \times C| = |A| \times |B| \times |C|$



Dandy Dresser

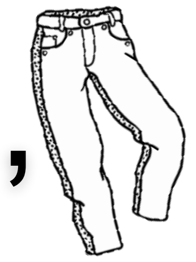
3 shirts

2 pants

5 pairs of shoes

How many outfits can he have?

Outfit = (shirt, pants, shoes)

( ,  , )

3-tuple

{Outfits} = {(shirt, pants, shoes)} = {shirts} x {pants} x {shoes}

Cartesian Product

| {Outfits} | = | {shirts} | x | {pants} | x | {shoes} | = 3 x 2 x 5 = 30

X

Useful?

COSTCO® WHOLESALE

X 3 x 3 x 4 36

n Sets

For n sets, by X and induction

$$|A_1 \times \ldots \times A_n| = |A_1| \times \ldots \times |A_n|$$

Subway

How many sandwiches can Subway make?

Bread = {Wheat, Italian}

Meat = {Turkey, Ham, Chicken, Bacon, Beef}

Cheese = {American, Monterey, Cheddar}

Veggie = {Cucumbers, Lettuce, Spinach, Onions}

Sauce = {Ranch, Mustard, Mayonnaise}

Sandwiches = Bread x Meat x Cheese x Veggie x Sauce

|Sand’s| = |Bread| x |Meat| x |Cheese| x |Veggie| x |Sauce|

X = 2 x 5 x 3 x 4 x 3 = 360

Cartesian Products

Product rule $|A \times B| = |A| \times |B|$ X

Multiple sets $|A_1 \times \dots \times A_n| = |A_1| \times \dots \times |A_n|$