Probability introduction

Randomness motivation

Outcomes

Sample space

Intuitive definition of Probability

Probability distribution

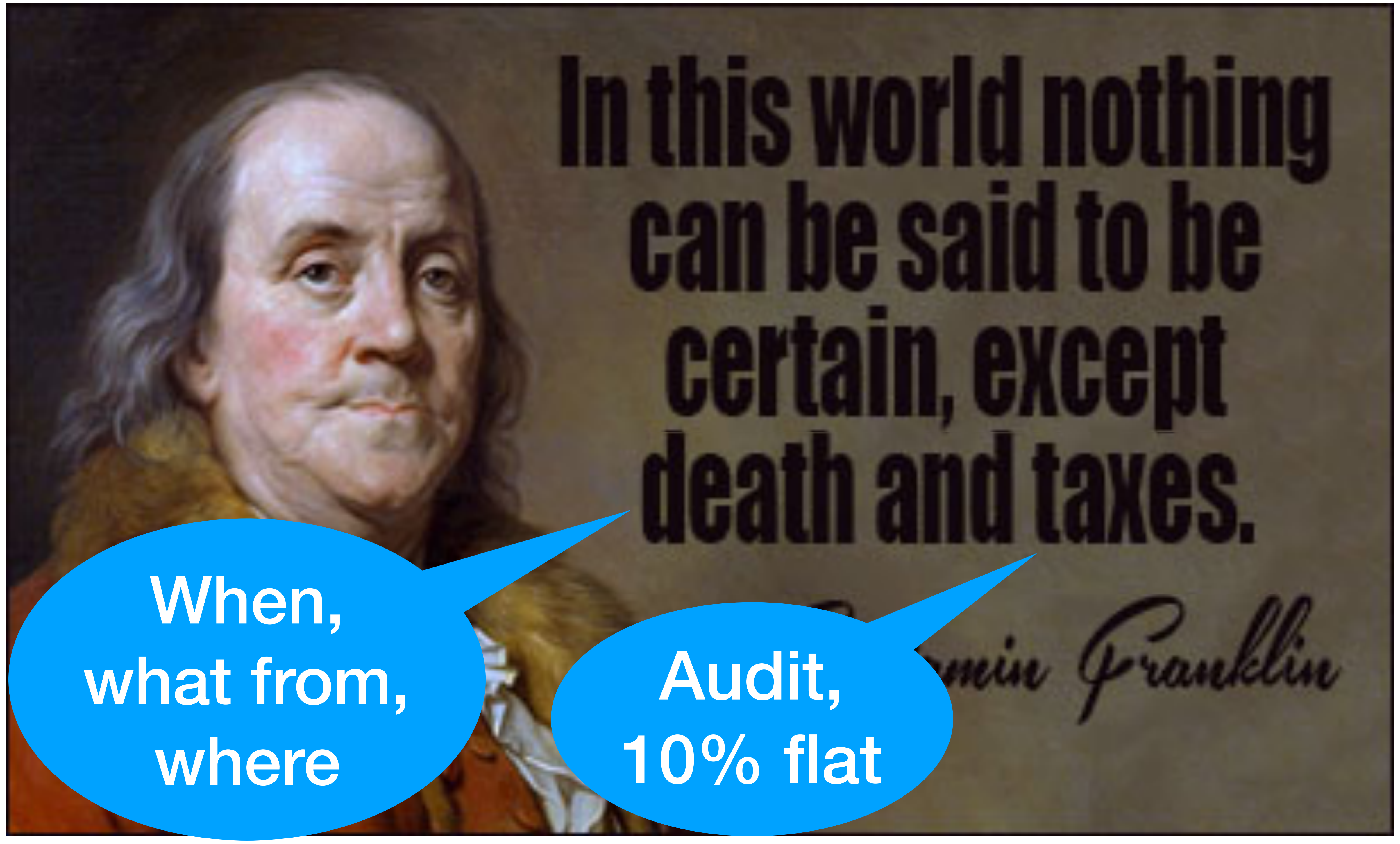
Probability space

Why Probability?

Some things in life are certain

Most are a less predictable

Physicians	illness, medication
Farmers	rain, diet trends
Investors	stock price, economy
Advertisers	views, competition
Consumers	availability, sale
Students	food line, grade, parents, job, date, game



When, what from, where

Audit, 10% flat

Randomness is everyone's business

Random Phenomena

Give up?	Reason intelligently?		
Learn	Range	Average	Variability
Infer	Relations	Structure	Change
Predict	Future	Likelihood	Guarantees
Benefit	Plan	Create	Compete

Coming to Terms

As with sets

Need terminology

Discuss

Concisely

Precisely

Effectively

Process of generating and observing data

Individual and collection of observations

Meaning of probability

Several approaches

Intuitive

Axioms

Data

Experiments

Probability developed in part to aid science

Process

Generate random data

Observe outcome

Experiment

Unified approach

Applies generally

Biology

Engineering

Business

…

Sociology

Understand

Analyze

generalize

Our experiments

Start simple

Get very complex

Outcomes and Sample Space

Potential experiment results are called (possible) outcomes

Set of possible outcomes is the sample space, denoted Ω

Notetation

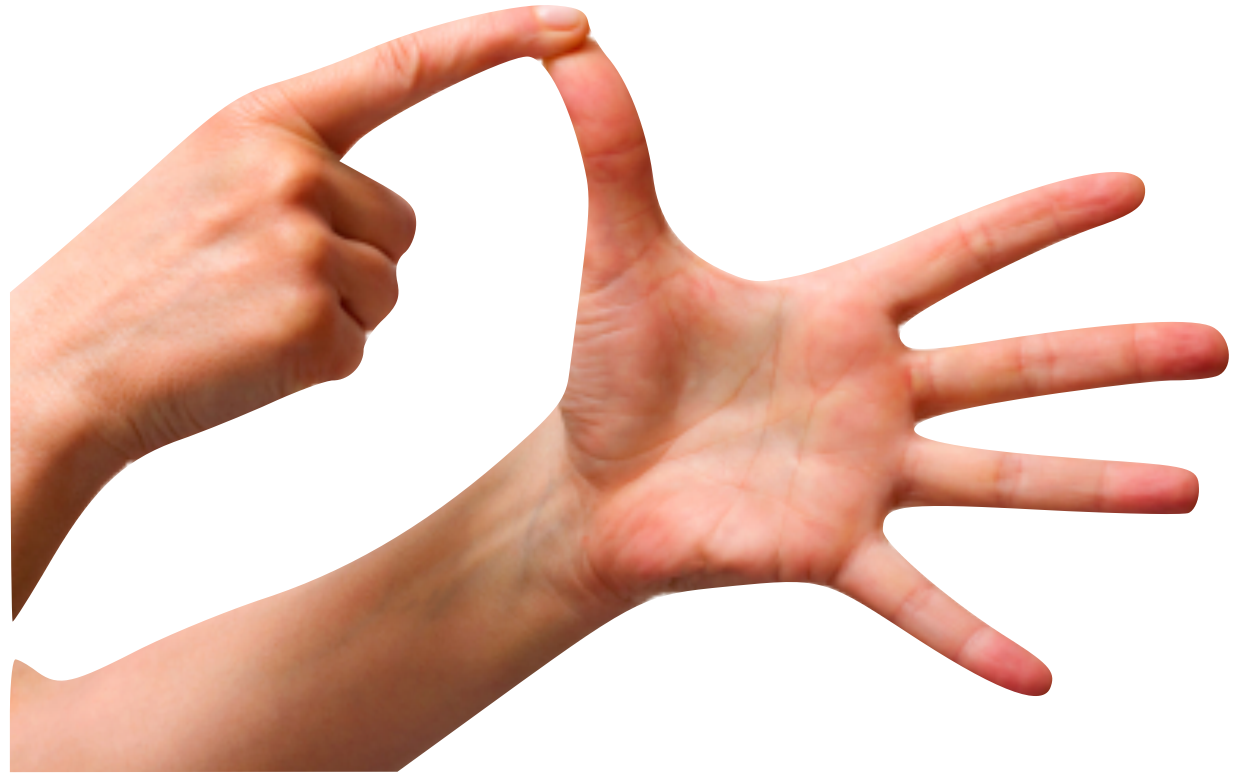
Some use
S or U

Experiment	Ω
Coin	{ h, t }
Die	{ 1, 2, ..., 6 }
Gender	{ m, f }
Age	$\mathbb{N}$
Temperature	$\mathbb{R}$

typically
lower-case
h, t, x

Next lecture:
subsets

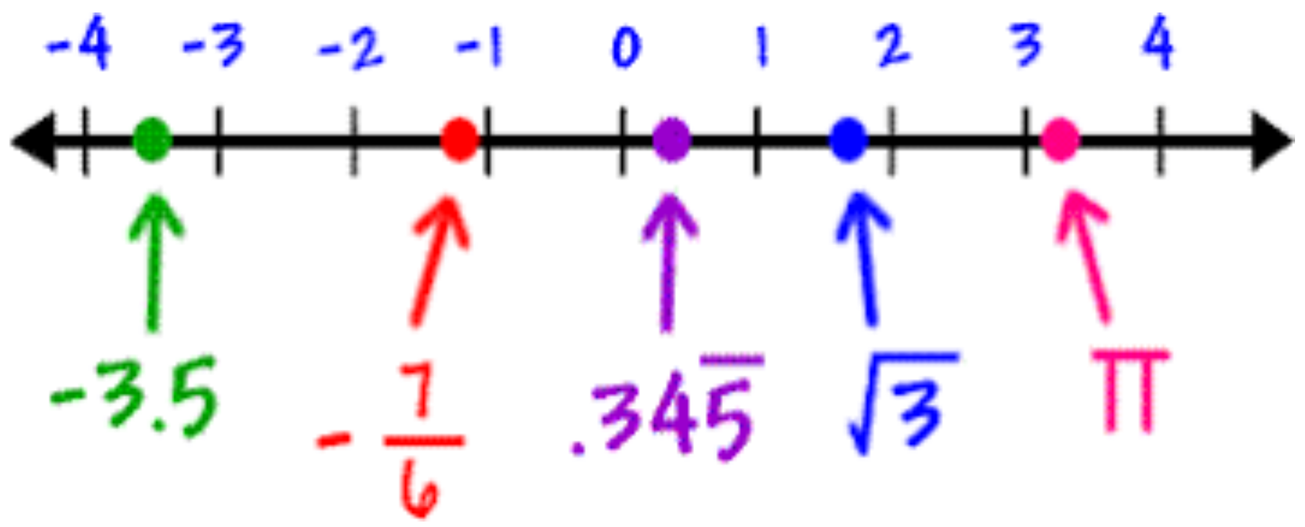
Two Sample-Space Types



Finite or countably infinite sample space is discrete

{ h, t }　{ 1, 2, …, 6 }　$\mathbb{N}$　$\mathbb{Z}$　{ words }　{ cites }　{ people }

Uncountably infinite sample space - continuous



$\mathbb{R}$　[0,1]　{ temperatures }　$\underbrace{\text{\{ salaries \}}\quad\text{\{ prices \}}}_{\text{upgraded}}$

Discrete spaces　Easier to understand, visualize, analyze

Important　First

Next few topics: Discrete

Continuous　Important　Conceptually harder　Later

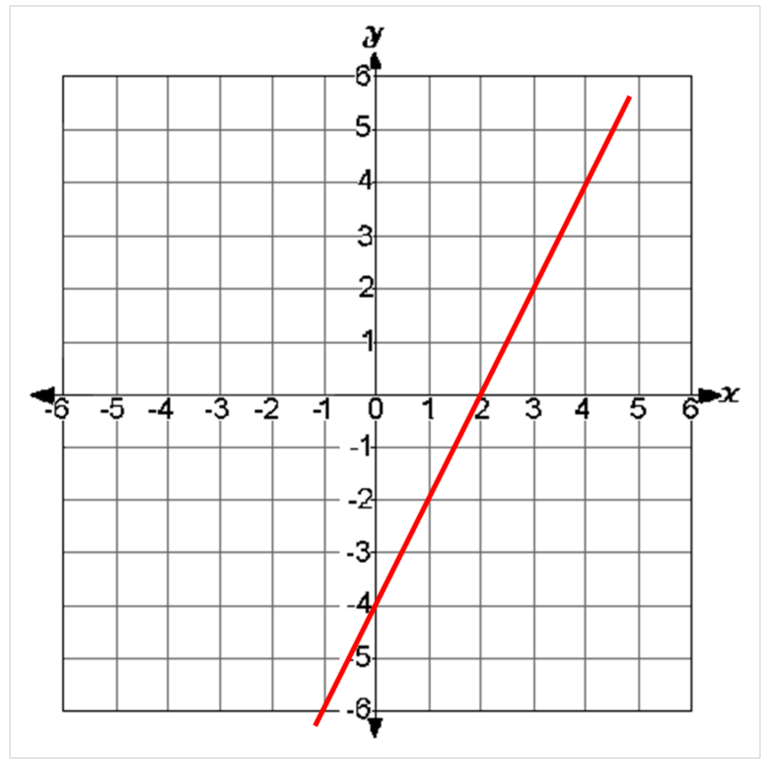
Random Outcomes

Algebra

Unknown value denoted by x

$2x - 4 = 0$

lower case



Solve | Before | After

$x \in \mathbb{R}$ | $x = 2$ | Solution

Notetation

If needed use y, z, …

UPPER CASE

Probability

Random outcome denoted by X

X - coin flip outcome

Experiment | Before | After

$X \in \Omega$ | get h | $X = h$

get t | $X = t$

Outcome seen called observation

Notetation

If needed use Y, Z, …

Probability of an Outcome

The probability, or likelihood, of an outcome $x \in \Omega$, denoted $P(x)$, or $P(X=x)$, is the fraction of times x will occur when experiment is repeated many times

Fair coin

As # experiments $\to \infty$, fraction of heads (or tails) $\to$ ½

heads has probability ½ $\quad P(h) = ½ \quad P(X=h) = ½$

tails has probability ½ $\quad P(t) = ½ \quad P(X=t) = ½$

Fair die

As # experiments $\to \infty$, fraction of 1's (or 2,…,6) $\to$ ⅙

1 has probability ⅙ $\quad P(1) = ⅙ \quad P(X=1) = ⅙$

$P(x)$ $\quad P(X=x)$ $\quad$ probability of x $\quad$ fraction of times x will occur

Probability Portrait

n experiments

$x \in \Omega$ n_x - # times x appeared

$$P(x) = \lim_{n \to \infty} \frac{n_x}{n}$$

$0 \le n_x \le n$ $$0 \leq \frac{n_x}{n} \leq 1$$ $0 \le p(x) \le 1$

$$\sum_{x \in \Omega} n_x = n \qquad \sum_{x \in \Omega} \frac{n_x}{n} = 1 \qquad \sum_{x \in \Omega} p(x) = 1$$

Sum of Probabilities

P(x) is the fraction of times outcome x occurs

P(h) = ½

P(1) = ⅙

Viewed over the whole sample space…

a pattern emerges

Coin	P(h) = ½	P(t) = ½	
Die	P(1) = ⅙	…	P(6) = ⅙
Rain	P(rain) = 10%	P(no rain) = 90%	

Probability Distribution

P(x) is the fraction of times outcome x occurs

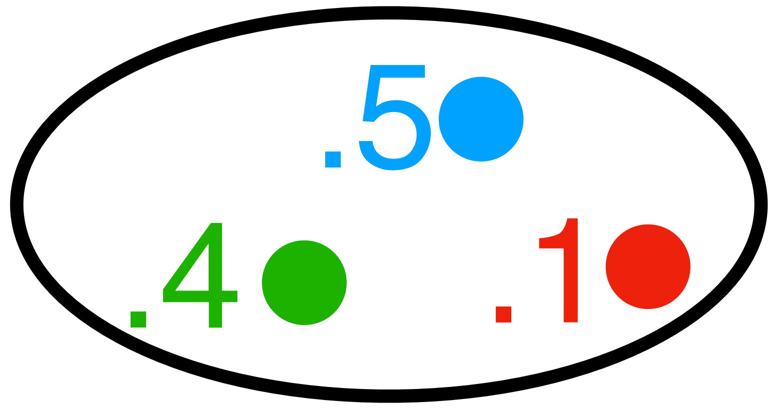
P(h) = ½ P(1) = ⅙

Viewed over the whole sample space…

a pattern emerges

Coin	P(h) = ½	P(t) = ½	
Die	P(1) = ⅙	…	P(6) = ⅙
Rain	P(rain) = 10%	P(no rain) = 90%	

P maps outcomes in Ω to nonnegative values that sum to 1



P function: $\Omega \rightarrow \mathbb{R}$ $P(x) \geq 0$ $\sum_{x \in \Omega} P(x) = 1$ Probability distribution function

Sample space Ω + Distribution P = Probability space

Determines everything!

Probability introduction

Randomness motivation

Outcomes x & Sample space Ω

Probability P(x) of x - fraction of times x will appear as # experiments → ∞

Probability distribution   P: Ω→ℝ   P(x) ≥0   ∑P(x)=1

Ω + P = Probability space