Probability Events

← Single outcome

→ Set of outcomes - events

Occurrence

Probability

Uniform-space probability

Single outcomes

Uninteresting

Event ful

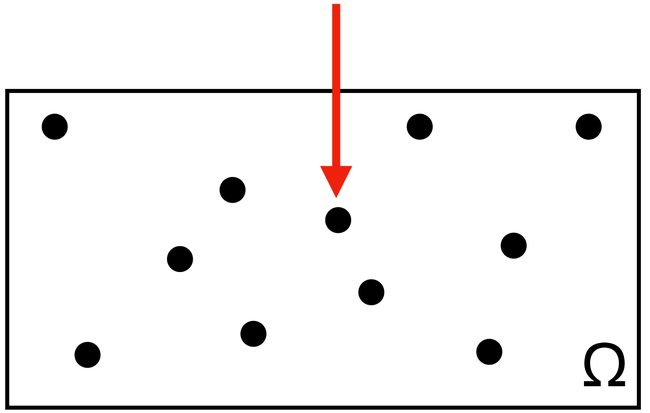
Events

Sometimes Care about one particular outcome

Temperature 98.6 °F

Dow Jones will close at $18,040

Get exactly B+ in class



Single element of Ω

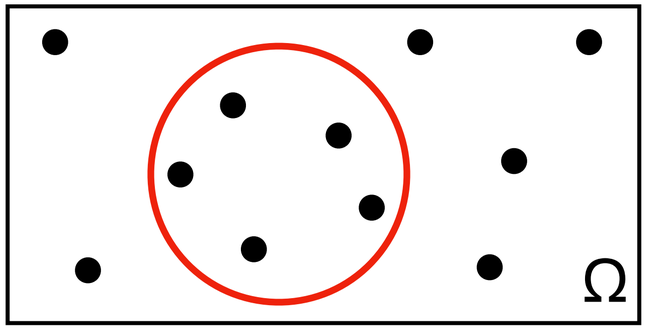
Outcome

Usually Interested in a set of possible outcomes

Temperature ≠ 98.6 °F

Stock will close higher

Pass the course



Subset of the space Ω

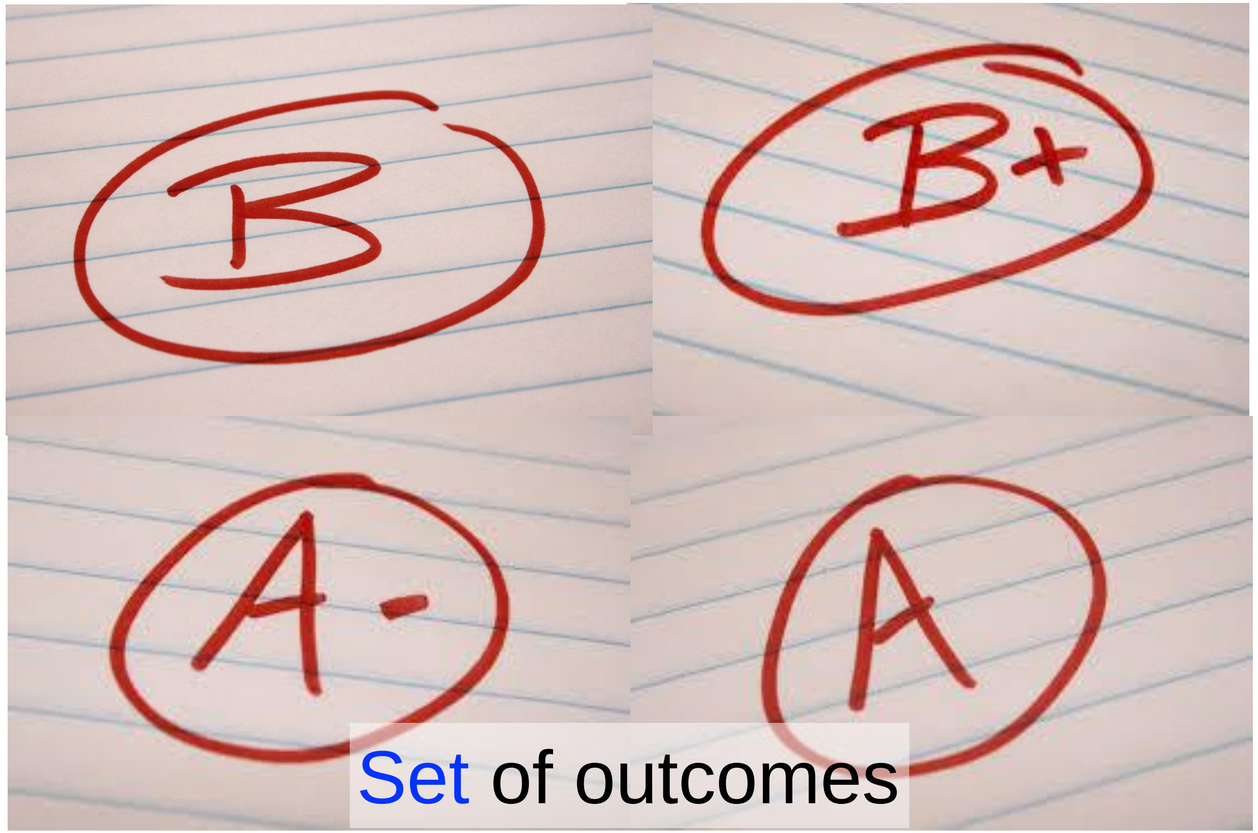
Event

Event?

Social event

Set of People

Probabilistic event



Set of outcomes

Die Events

Event — Subset of sample space Ω

Die — Ω = { 1, …, 6 } ⊇ Event

Set	Name
{1, …, 6}	Ω (certain)
{2, 4, 6}	Even
{1, 4}	Square
{5, 6}	≥ 5, > 4
{1, 2, 5}	{1, 2, 5}

Complement

Set	Name
{ }	∅ (null)
{1, 3, 5}	Odd
{2, 3, 5, 6}	Non square
{1, 2, 3, 4}	≤ 4, < 5
{3, 4, 6}	{3, 4, 6}

Tetrahedral Die

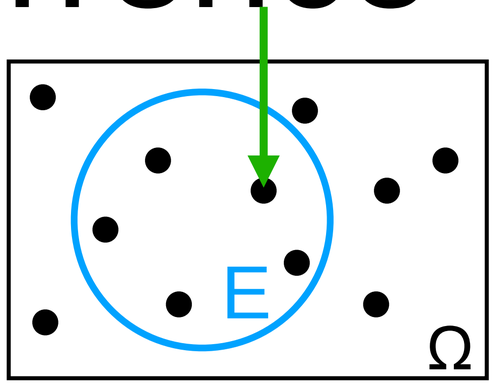
Face	1	2	3	4
Probability	.1	.2	.3	.4

Name	Set
Ω (certain)	{1,2,3,4}
Even	{2, 4}
Prime	{2, 3}
∅ (null)	{ }

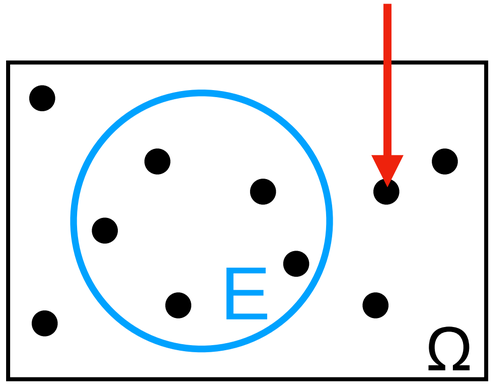
Event Occurrence

An event occurs, or happens, if it contains the observed outcome

E occurs if $X \in E$



E Occurred ✅



E did not occur ❌

Event	Subset	Outcome			
		1	2	3	4
Ω (certain)	{1,2,3,4}	✅	✅	✅	✅
Even	{2, 4}	❌	✅	❌	✅
Prime	{2, 3}	❌	✅	✅	❌
∅ (null)	{ }	❌	❌	❌	❌

Difference between the following events?

Happened

Did not happen

Event Probability

Probability of event E P(E) Probability that E occurs P(X∈E)

Fraction of experiments where E occurs, as # experiments grows

Even

✅		✅		✅			✅	✅	✅		✅	7
④	③	②	①	④	③	③	④	②	④	③	④	12

$$P(\text{Even}) \approx \text{fraction} = \frac{7}{12} \approx 0.583$$

Want P(E) = Fraction # experiments 12 → ∞

General distributions and events

P(x) → P(E)

Relate probability of event to probability of its elements (outcomes)

times Even occurs = sum of # times 2 and 4 occur

P(Even) = fraction of times Even occurs
= sum of fraction of times 2 and 4 occur = P(2) + P(4)

General Event E

times E occurs = sum of # times its elements occur

P(E) = fraction of times E occurs
= sum of fraction of times its elements occur
= sum of its element probabilities

$$P(E) = P(X \in E) = \sum_{x \in E} P(x)$$

Tetrahedral Die

Probability	.1	.2	.3	.4

Event	Set	1	2	3	4	P(E)
Ω (certain)	{1,2,3,4}	✅	✅	✅	✅	.1+.2+.3+.4 = 1
Even	{2, 4}	❌	✅	❌	✅	.2 + .4 = .6
Prime	{2, 3}	❌	✅	✅	❌	.2 + .3 = .5
∅ (null)	{ }	❌	❌	❌	❌	0

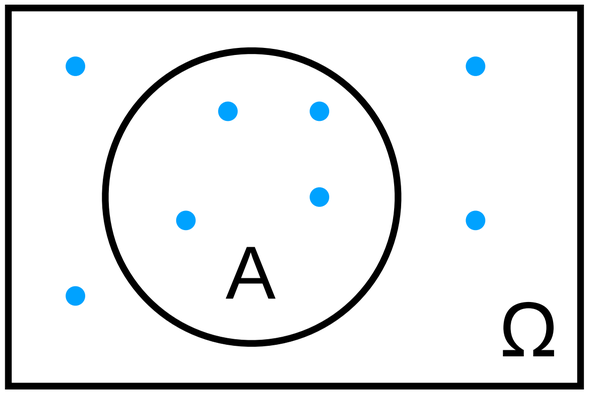
Uniform Spaces

Equiprobable spaces

Simple formula for probability of outcome

$$P(x) = \frac{1}{|\Omega|}$$

events?



Uniform

$$P(E) = \sum_{x \in E} P(x) = \sum_{x \in E} \frac{1}{|\Omega|} = \frac{\sum_{x \in E} 1}{|\Omega|} = \frac{|E|}{|\Omega|}$$

Die 

Ω = {1,2,3,4,5,6}

|Ω| = 6

Event	Set	$\lvert Event \rvert$	$P(Event) = \frac{\lvert Event \rvert}{6}$
Even	{2,4,6}	3	$\frac{3}{6} = \frac{1}{2}$
Square	{1,4}	2	$\frac{2}{6} = \frac{1}{3}$

Probability that X=2, 4 ,or 6

Terminology

A, B Events

Intersect

Disjoint

Mutually exclusive

Do’s and Don’ts

Capital

$P(X \in \text{Even})$

die: $P(X \in \{2,4,6\}) = 3/6 = 1/2$

$P(\text{Even})$

$\stackrel{\text{def}}{=} P(X \in \text{Even})$

$P(3 \in \text{Even})$

0

3 is a (constant) random variable, so possible
The "r.v." 3 is never in Even, so probability is 0

$P(4 \in \text{Even})$

1

Less common

Check that you meant it

$P(x \in \text{Even})$

Small

Previous anomaly on steroids

What is x?

$\forall\, x \in \{3,7\}$, $P(x \in \text{Even})=0$

Double check meant it

Probability Events

Single outcomes to sets

Events: subsets of Ω

Occurrence: outcome in event

Probability $$P(E) = \sum_{x \in E} P(x)$$

Repeated Experiments