

A photograph of a warehouse interior. In the background, there are tall stacks of white cardboard boxes on red pallets. In the foreground, a forklift is moving a red pallet loaded with many green and red cans. A worker in a blue uniform and yellow hard hat is operating the forklift. The scene is well-lit with overhead industrial lights.

**Counting**

**Cartesian**

**Products**



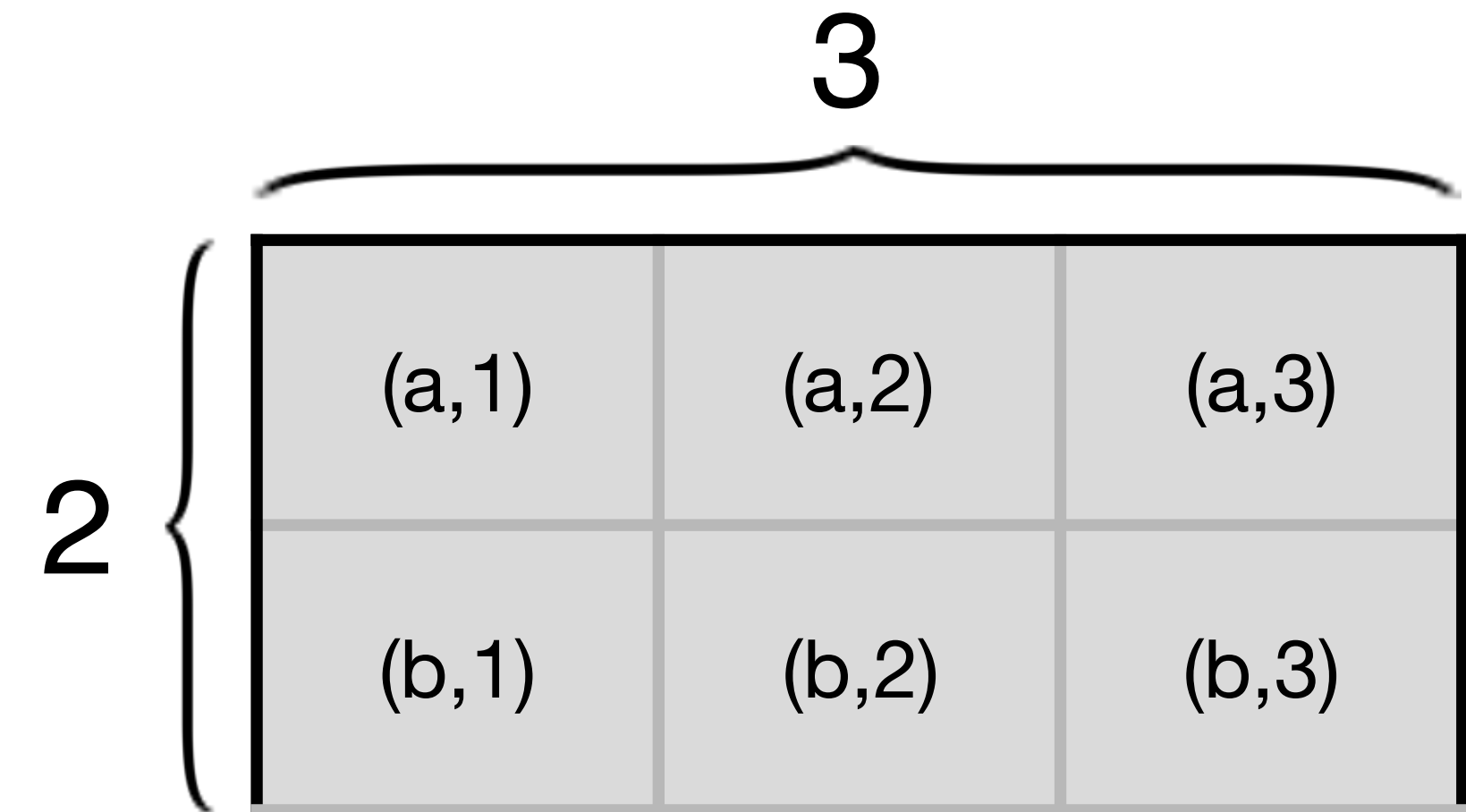
# Cartesian Products

$$|\{a,b\}| = 2$$

$$|\{1,2,3\}| = 3$$

$$\{a,b\} \times \{1,2,3\} = \left\{ \begin{array}{ccc} (a,1) & (a,2) & (a,3) \\ (b,1) & (b,2) & (b,3) \end{array} \right\}$$

← 3  
← 3



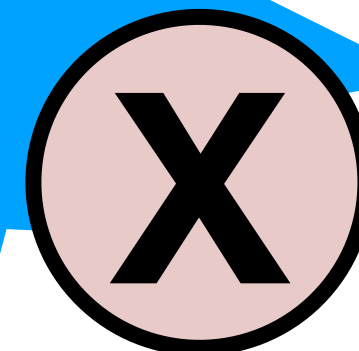
area  $2 \times 3 = 6$

$$|\{a,b\} \times \{1,2,3\}| = 3 + 3 = 2 \times 3 = 6$$

The size of a Cartesian Product is the product of the set sizes

$$|A \times B| = |A| \times |B|$$

**Product  
Rule**



Another  
application  
of  $\oplus$  rule

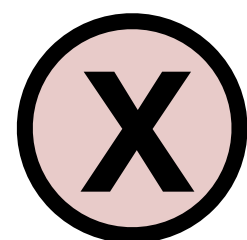
# Tables

Cartesian product

3 attributes

5 records

Adam	M	3.5
Eve	F	3.7
John	M	3.4
Lisa	F	3.2
Mary	F	3.9



$5 \times 3 = 15$  cells

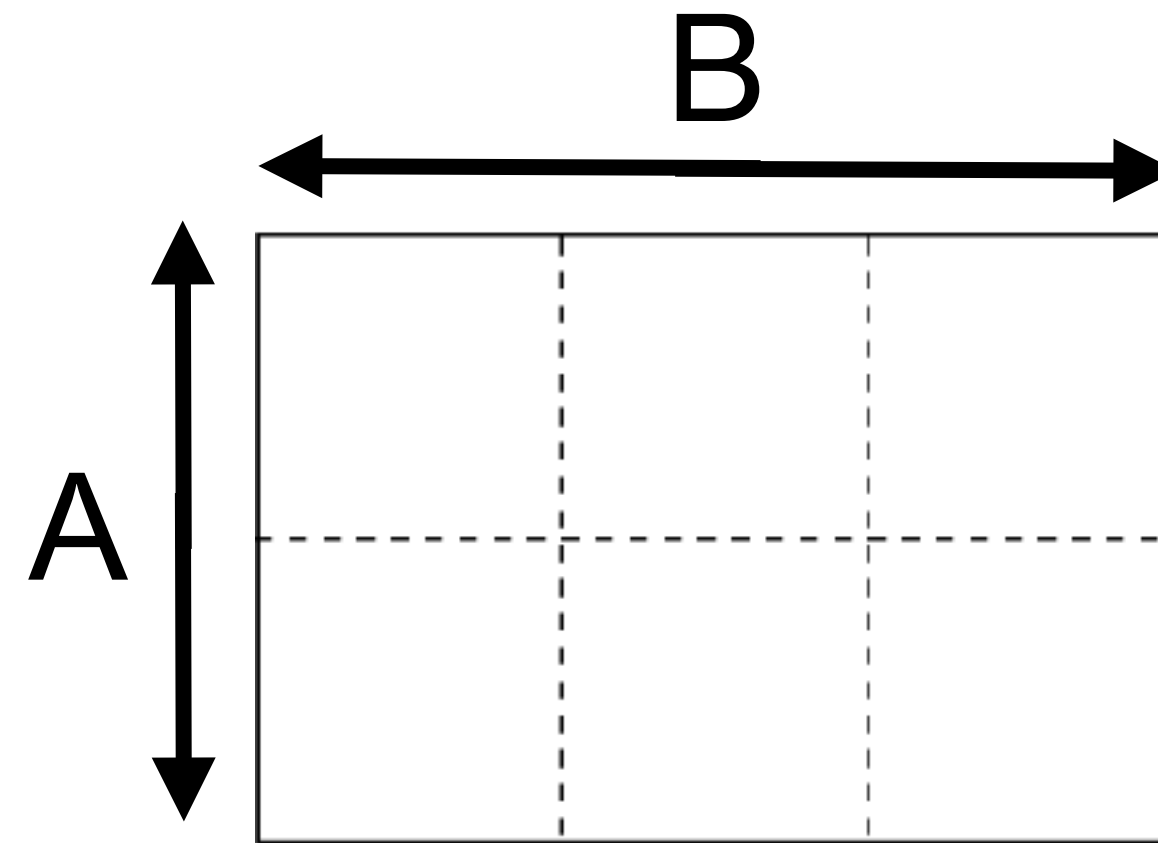
# Three Sets

$A \times B$

$\{(a,b): a \in A, b \in B\}$

rectangle

$|A \times B| = |A| \times |B|$

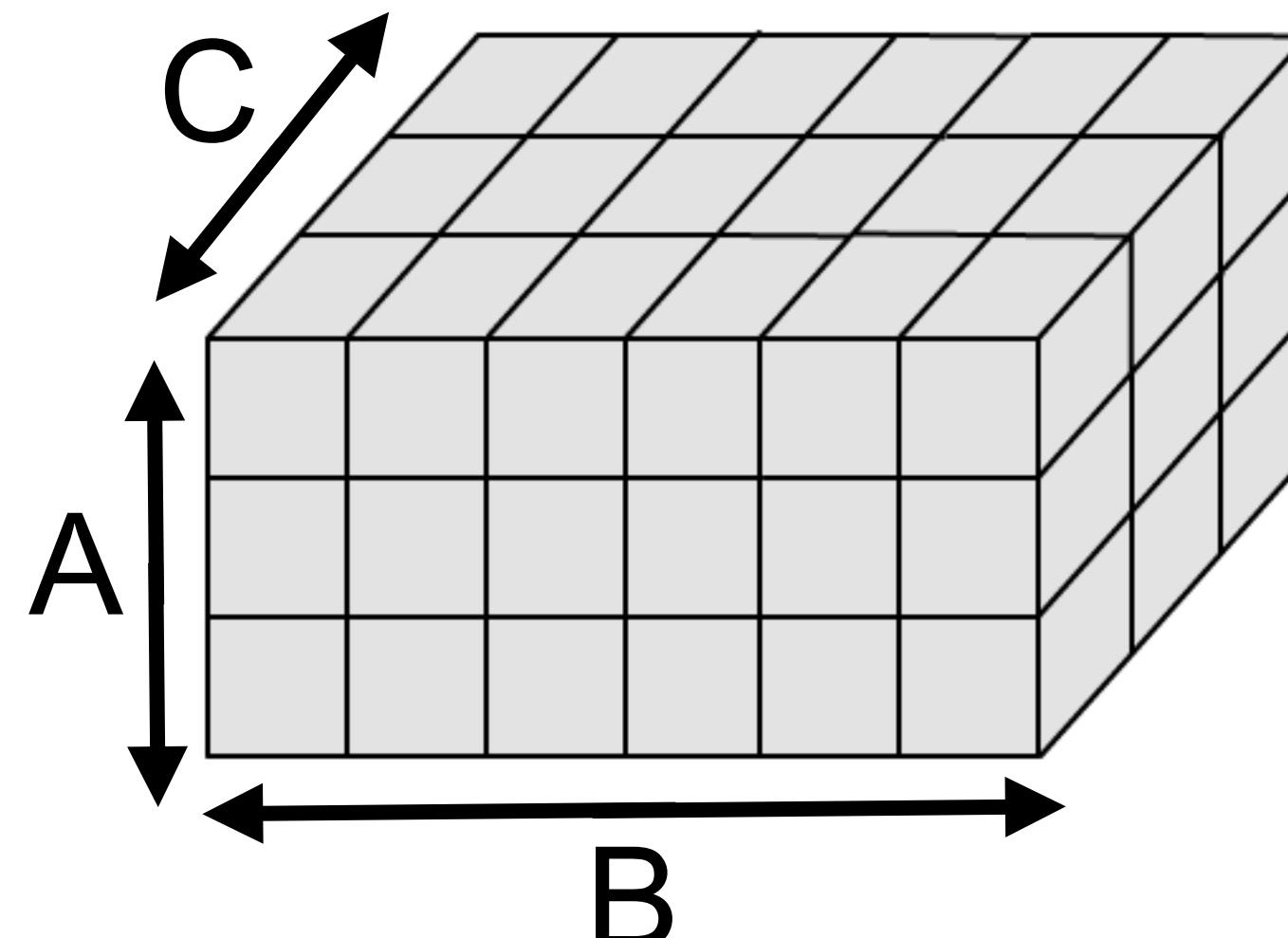


$A \times B \times C$

$\{(a,b,c): a \in A, b \in B, c \in C\}$

“cuboid”

$|A \times B \times C| = |A| \times |B| \times |C|$



# Dandy Dresser

3 shirts



2 pants

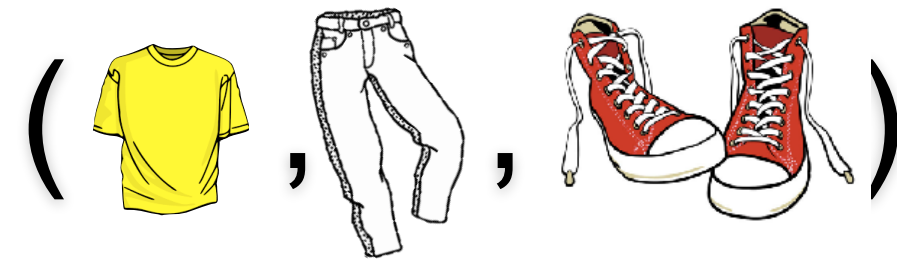


5 pairs of shoes



How many outfits can he have?

Outfit = (shirt, pants, shoes)

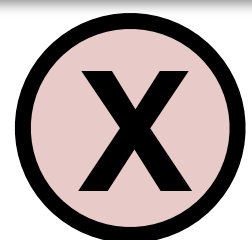


3-tuple

$\{\text{Outfits}\} = \{(\text{shirt, pants, shoes})\} = \{\text{shirts}\} \times \{\text{pants}\} \times \{\text{shoes}\}$

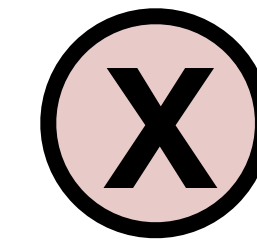
Cartesian  
Product

$|\{\text{Outfits}\}| = |\{\text{shirts}\}| \times |\{\text{pants}\}| \times |\{\text{shoes}\}| = 3 \times 2 \times 5 = 30$





# Useful?



3 x 3 x 4

36

# n Sets

For n sets, by **X** and induction

$$|A_1 \times \dots \times A_n| = |A_1| \times \dots \times |A_n|$$



# Subway

How many sandwiches can Subway make?

Bread = {Wheat, Italian}

Meat = {Turkey, Ham, Chicken, Bacon, Beef}

Cheese = {American, Monterey, Cheddar}

Veggie = {Cucumbers, Lettuce, Spinach, Onions}

Sauce = {Ranch, Mustard, Mayonnaise}

Sandwiches = Bread x Meat x Cheese x Veggie x Sauce

|Sand's| = |Bread| x |Meat| x |Cheese| x |Veggie| x |Sauce|

$$\textcircled{\mathbf{X}} = 2 \times 5 \times 3 \times 4 \times 3 = 360$$





# Cartesian Products

Product rule

$$|A \times B| = |A| \times |B|$$



Multiple sets

$$|A_1 \times \dots \times A_n| = |A_1| \times \dots \times |A_n|$$



Cartesian Powers

