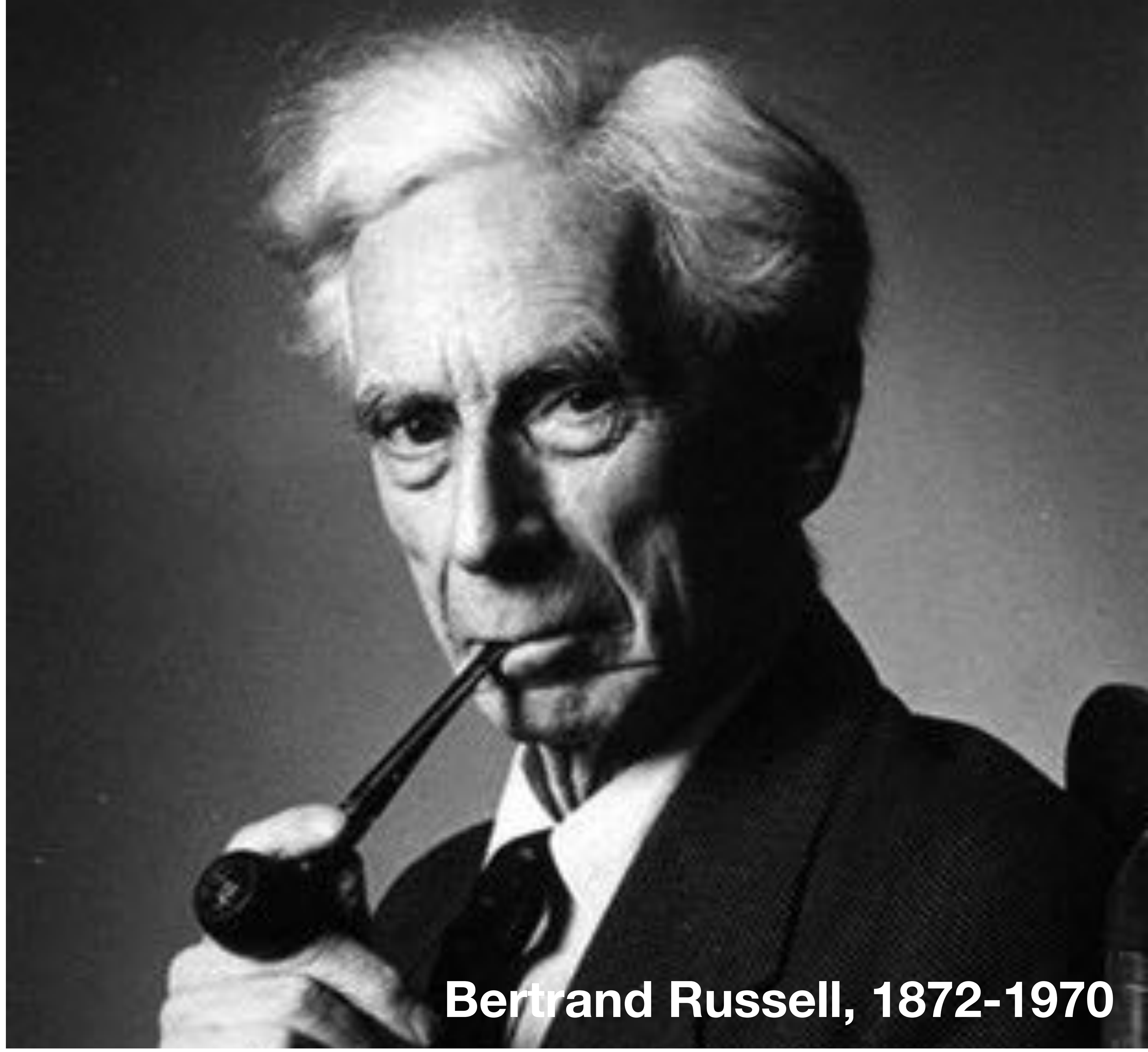
A Perplexing Paradox

Bertrand Russell

Mathematician, philosopher, author

1950 literature Nobel Laureate

Democracy: fools have a right to vote.
Dictatorship: fools have a right to rule.



Most people would rather die than think.
In fact, most do.

Men are born ignorant, not stupid.
They are made stupid by education.

Sets in Sets

Sets can be elements $\{\ \{0\},\ \{1,2\}\ \}$

Every set is a subset of itself $\{0\} \subseteq \{0\}$

Can a set belong to (be an element of) itself? $S \in S$?

Typically, sets do not belong to themselves $\{0\} \notin \{0\}$ $\varnothing \notin \varnothing$

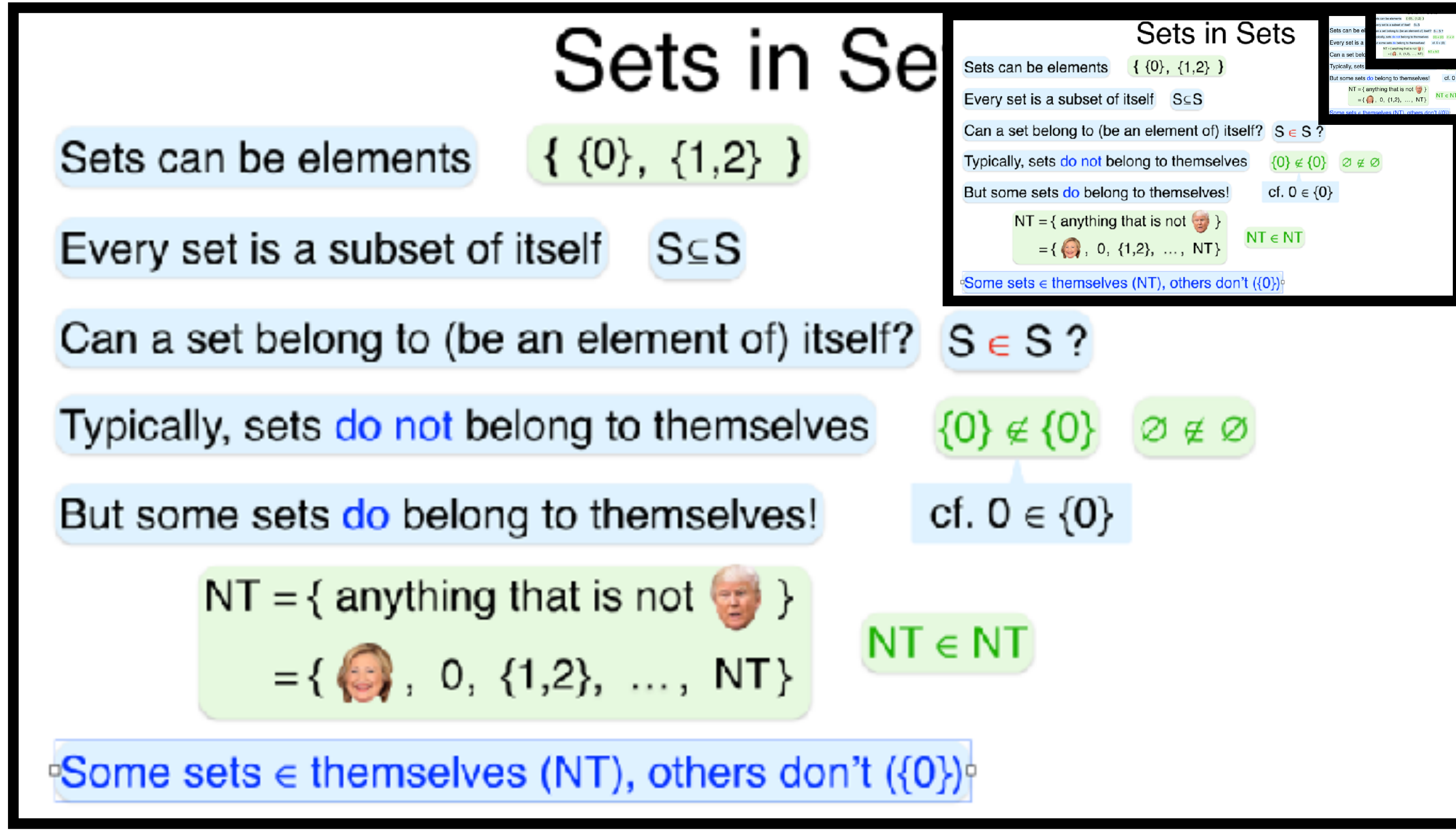
But some sets do belong to themselves! $0 \in \{0\}$

NT = { anything that is not  }

= {  , 0, {1,2}, … , NT}

$NT \in NT$ 

Some sets $\in$ themselves (NT), others don't ({0}) 



Russell's Paradox

Define a set that cannot exist

R = { sets that don't belong to themselves } = { S: S $\notin$ S }

{0} $\notin$ {0}, hence {0} $\in$ R   NT $\in$ NT, hence NT $\notin$ R

Must have R $\in$ R → R $\notin$ R

or

R $\notin$ R → R $\in$ R

Both lead to contradiction!

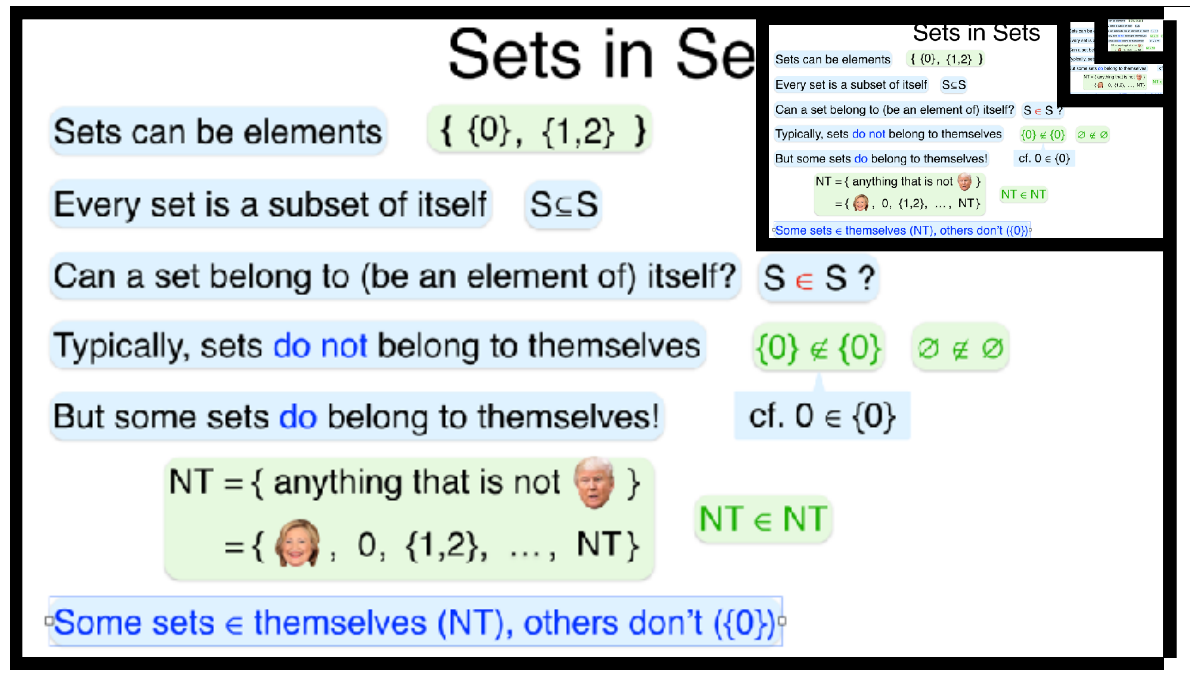
If R existed then both R $\in$ R and R $\notin$ R would hold

R defined but cannot exist!

What Happened?

Self-referential definition

$\{ S : S \in S \}$



$\{ S : S \notin S \}$

Will avoid

Not needed for exam

Variations

René Magritte

Who sh