Cartesian Products

Philosopher, scientist, mathematician

Considered father of western philisophy

Cogito ergo sum

I think therefore I am

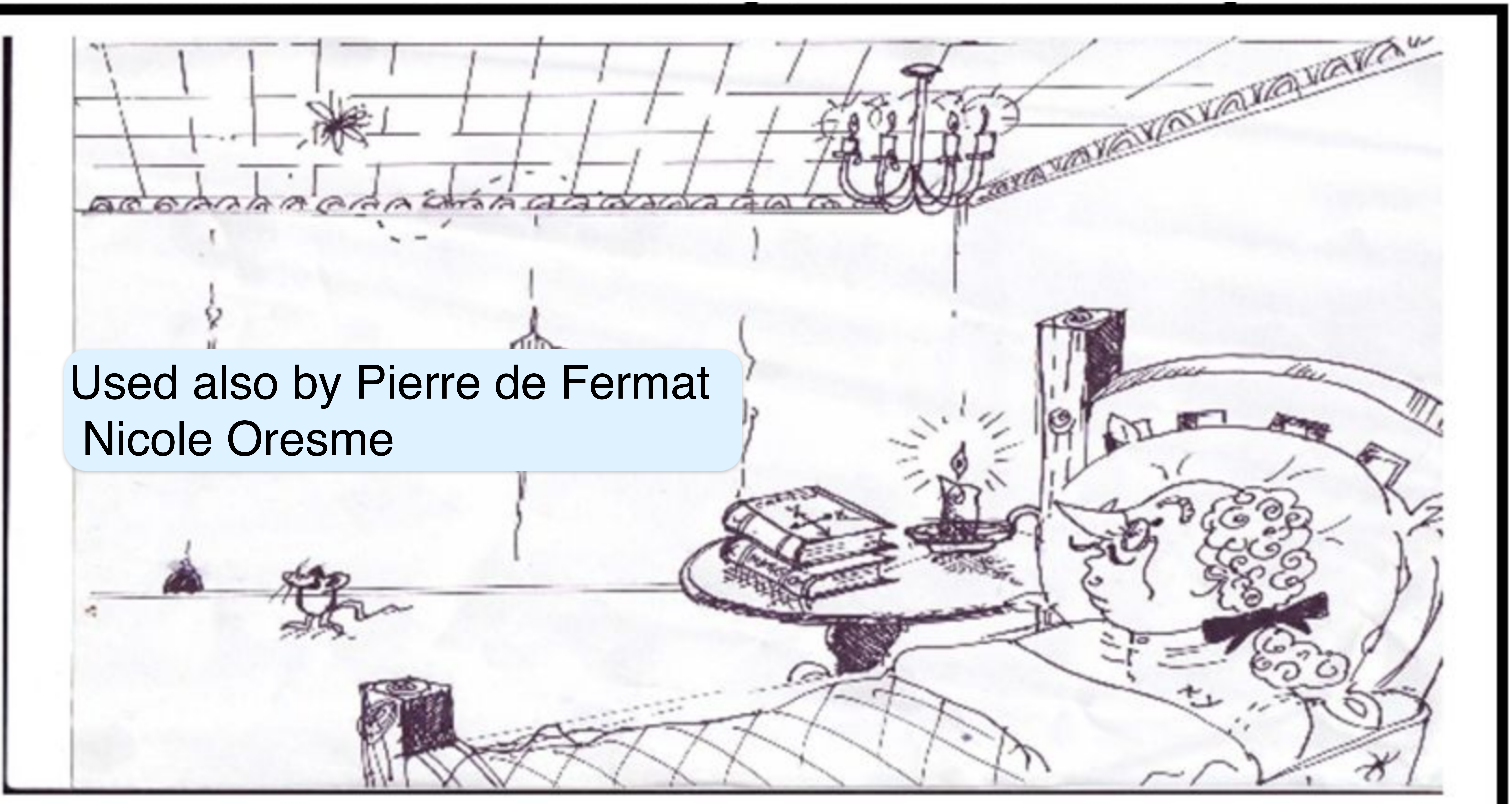
Divide each difficulty into as many parts as necessary to solve it

Each problem I solved became a rule I later used to solve other problems

A good mind is not enough, the main thing is to use it well

To improve the mind, learn less and contemplate more

Cartesian Coordinates



Used also by Pierre de Fermat
Nicole Oresme

Tuples and Ordered Pairs

Set	Order and repetition do not matter	$\{a, b, c\} = \{b, c, a\}$
Tuple	Both order and repetition matter	$(a, b, c) \neq (b, c, a)$
		$(a, a, a) \neq (a)$
n-tuple	Tuple with n elements	$(a_1, a_2, \ldots, a_n)$
2-tuple	Ordered pair	$(3, 7)$

Also real interval

Tell from context

Cartesian Products

The cartesian product of A and B is the set $A \times B$ of ordered pairs (a, b) where $a \in A$ and $b \in B$

$A \times B = \{(a,b): a \in A,\ b \in B\}$

$A \times A$ denoted A^2

$\mathbb{R}^2 = \{ (x,y): x,y \in \mathbb{R} \}$

Cartesian Plane

$A, B \subseteq \mathbb{R}$

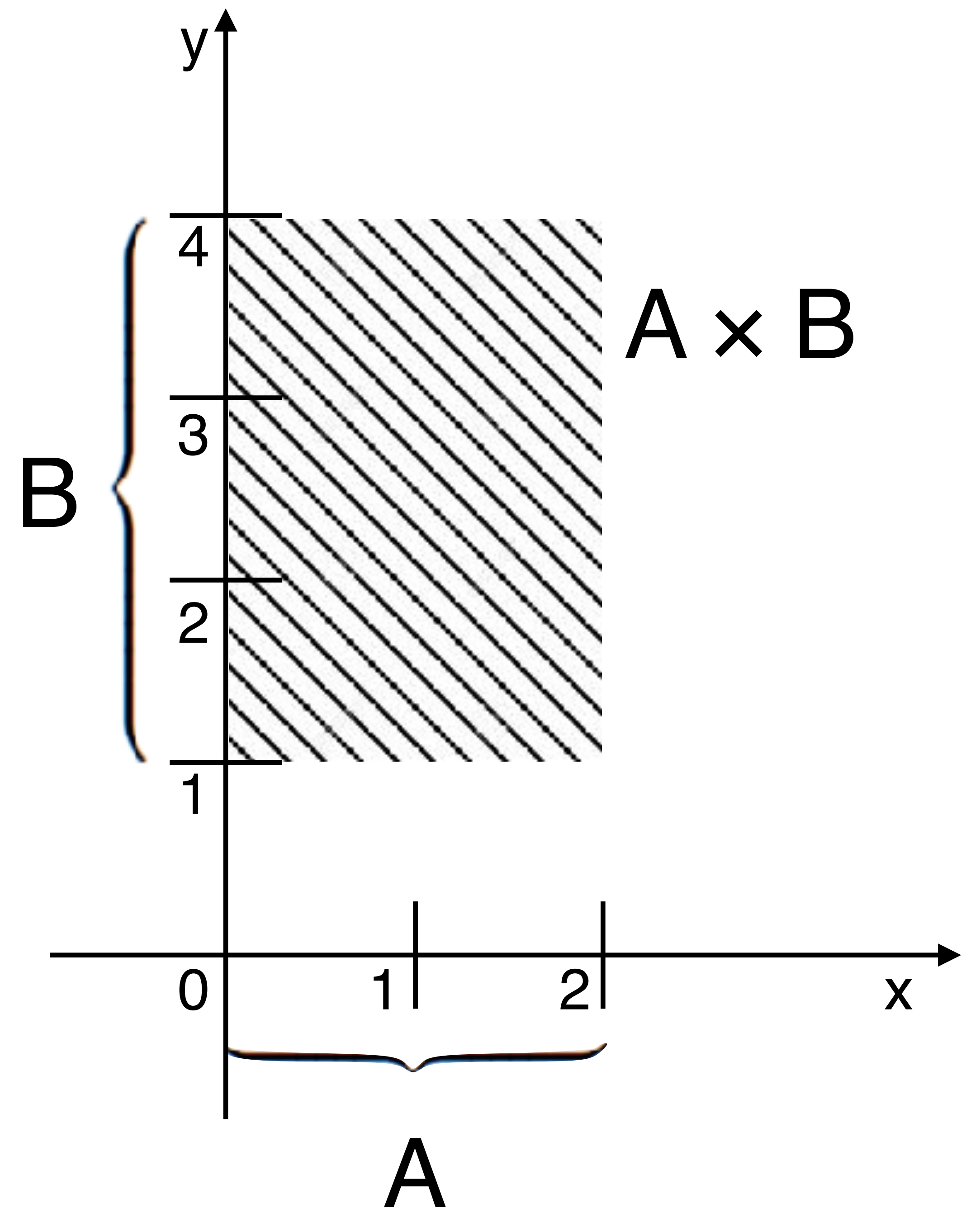
$A \times B \subseteq \mathbb{R}^2$

Rectangle

$A = [0, 2]$

$B = [1, 4]$

$A \times B = \{ (x, y) : x \in [0, 2], y \in [1, 4] \}$



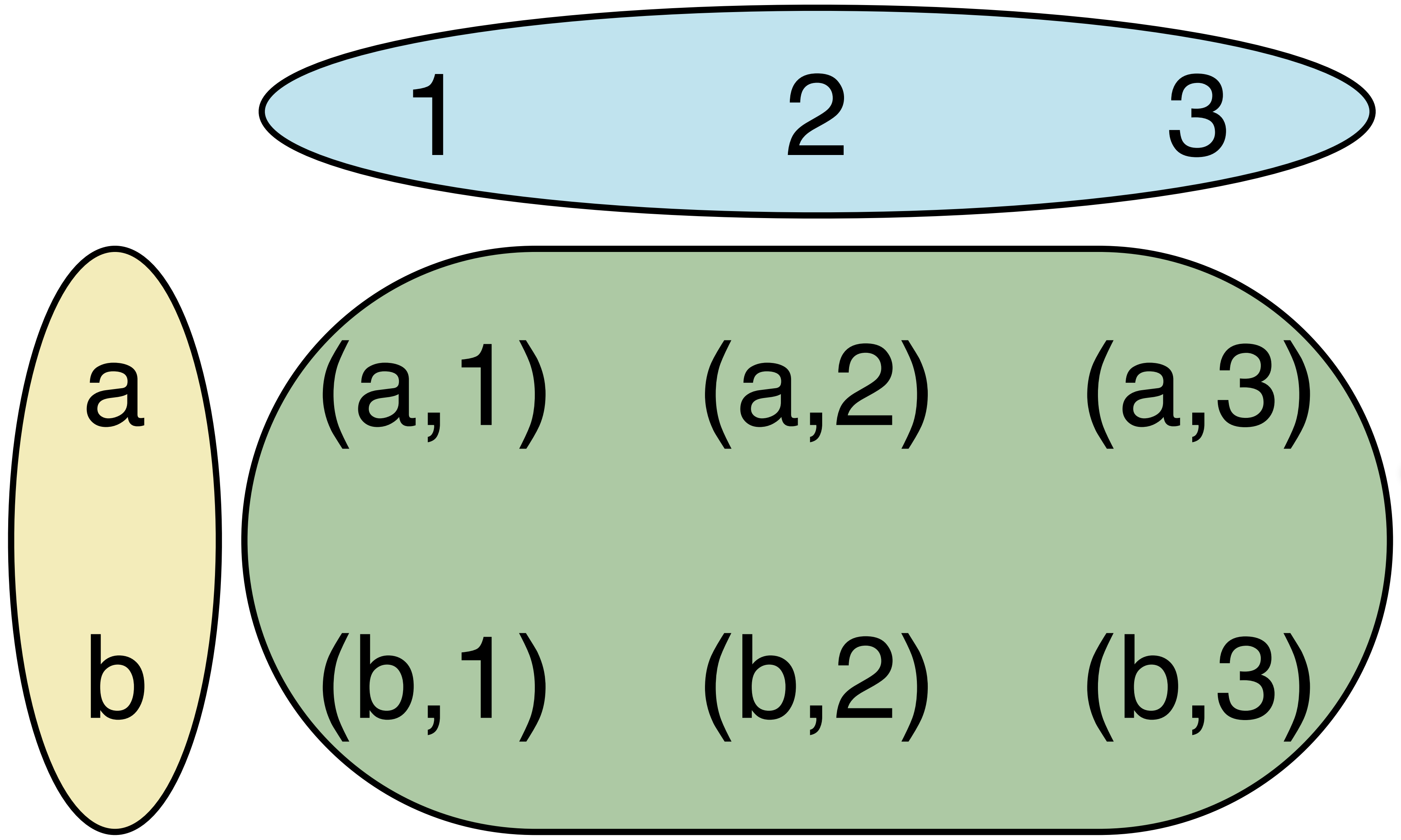
Discrete Sets

Similar

simpler

$\{a,b\} \times \{1,2,3\} = \{\ (x, y):\ x \in \{a,b\},\ y \in \{1,2,3\}\ \}$

$= \{\ (a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\ \}$



1st coordinate vertically,
2nd horizontally

Identities

$$A \times \varnothing = \varnothing \times A = \varnothing$$

$$A \times (B \cup C) = A \times B \cup A \times C$$

$$A \times (B \cap C) = A \times B \cap A \times C$$

$$A \times (B - C) = A \times B - A \times C$$

$$\begin{array}{ccccc} & & & B & \\ & & 1 & 2 & 3 \\ A & a & a1 & a2 & a3 \\ & b & b1 & b2 & b3 \end{array} \quad A \times B$$

$$\begin{array}{ccccc} & & & C & \\ & & 2 & 3 & 4 \\ A & a & a2 & a3 & a4 \\ & b & b2 & b3 & b4 \end{array} \quad A \times C$$

$$\begin{array}{cccccc} & & & B \cup C & & \\ & & 1 & 2 & 3 & 4 \\ A & a & a1 & a2 & a3 & a4 \\ & b & b1 & b2 & b3 & b4 \end{array} \quad A \times (B \cup C)$$

Tables

Tables are Cartesian products

Column

attribute

name, gender, GPA

Row

record

student

Adam	M	3.5
Eve	F	3.7
John	M	3.4
Lisa	F	3.2
Mary	F	3.9

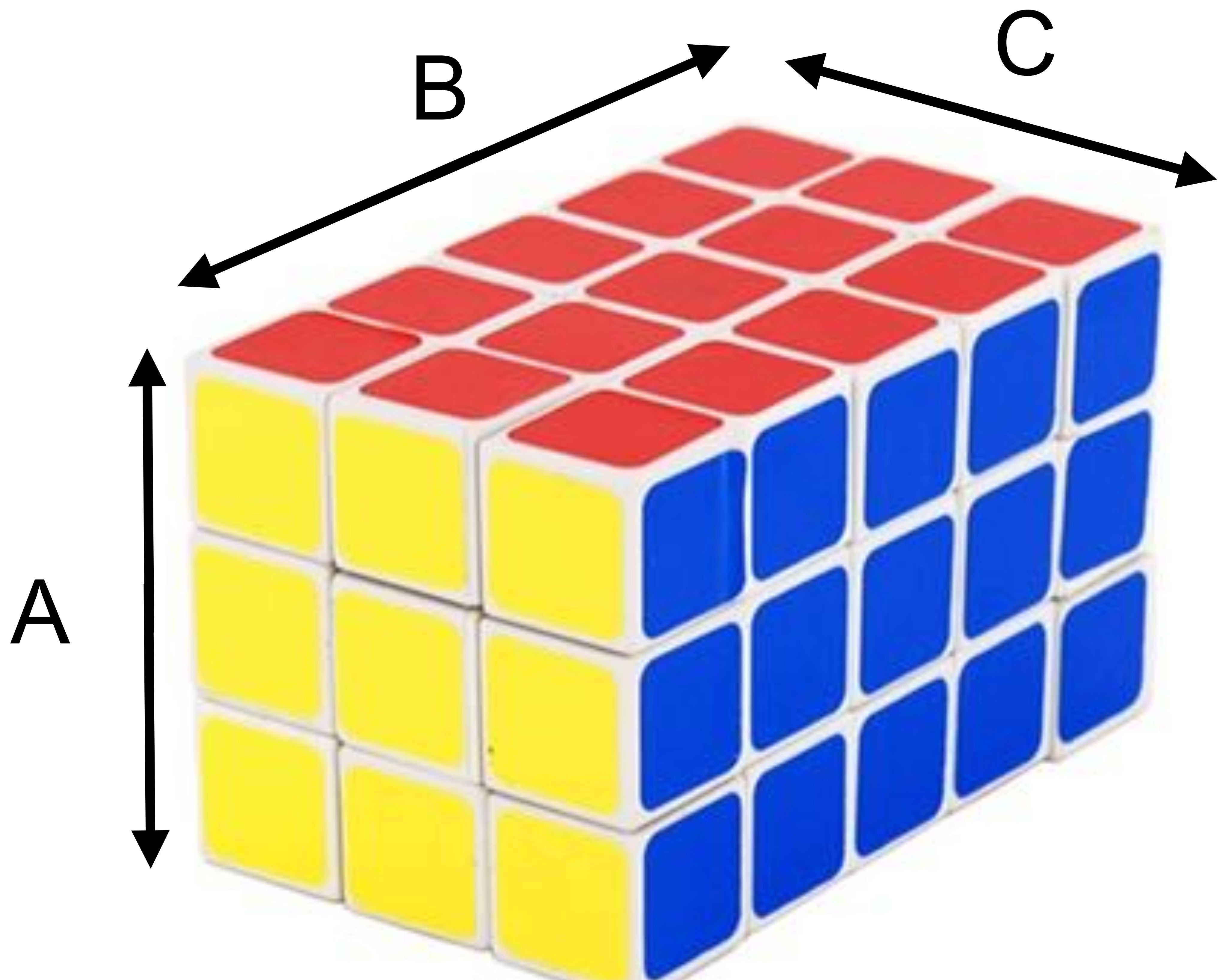
Cell

entry

Cartesian Product of 3 Sets

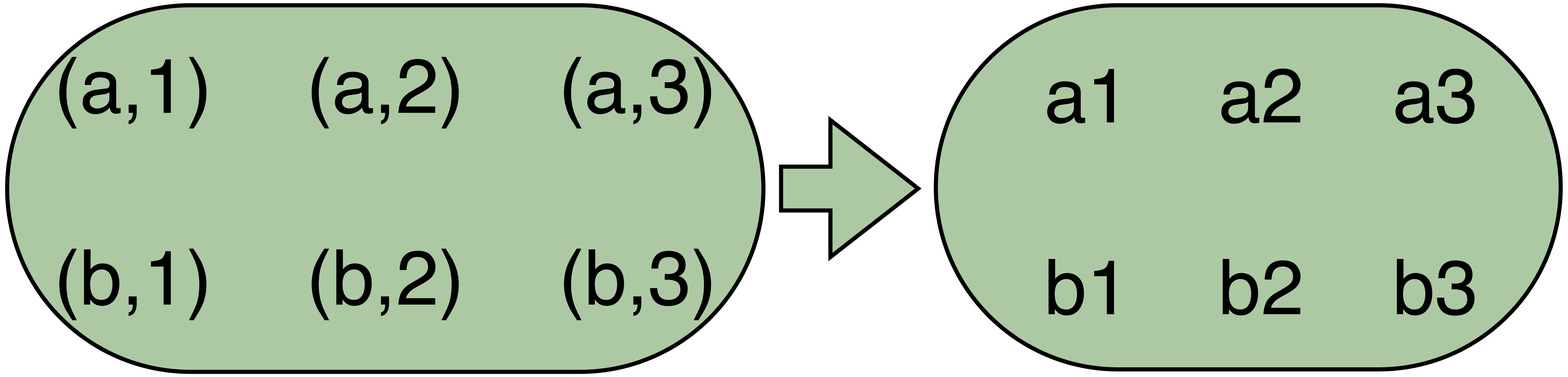
A x B — 2-dimensional rectangle

A x B x C — 3-dimensional “cuboid”



Sequences

Tuples, just without ( ) and sometimes without ,



$\{0,1\}^2 = \{\, xy:\ x, y \in \{0,1\} \,\} = \{\, 00, 01, 10, 11 \,\}$

$\{0,1\}^3 = \{\, 000, 001, 010, 011, 100, 101, \ldots, 111 \}$

Cartesian Products

Cartesian Product

Use **product** function in **itertools** library

```
from itertools import product
Faces = set({‘J’,‘Q’,‘K’})
Suits = {‘♦’,‘♠’}
for pair in product(Faces,Suits):
  print(pair)
(‘J’, ‘♦’)
(‘J’, ‘♠’)
(‘Q’, ‘♦’)
(‘Q’, ‘♠’)
(‘K’, ‘♦’)
(‘K’, ‘♠’)
```

Cartesian Products

Tuples $(a_1,\ldots,a_n)$

Ordered pairs (a,b)

Sequences

Python `product` function in `itertools` library