Simple Estimation of Semiparametric Models with Measurement Errors

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Framework: EIV for general GMM models

- General moment condition model

$$\mathbb{E}\left[g(X_i^*, S_i, \theta)\right] = 0 \quad \text{iff} \quad \theta = \theta_0$$

- X_i^* is mismeasured:

$$X_i = X_i^* + \varepsilon_i,$$

  - For the talk: classical measurement error ε_i, i.e., $\varepsilon_i \perp (X_i^*, S_i)$, $\mathbb{E}[\varepsilon_i] = 0$
  - In the paper: non-classical, multivariate, and serially correlated measurement errors

- Leading example: nonlinear regression

$$Y_i = \rho(X_i^*, W_i, \theta_0) + U_i, \quad \mathbb{E}\left[U_i | X_i^*, W_i, Z_i\right] = 0$$
$$g(x, s, \theta) = (y - \rho(x, w, \theta))h(x, w, z)$$

- Logit/Probit/Tobit, CES production function ...
- and also multinomial choice, multi-equation models, structural models, ...

Framework: Moderate Measurement Errors

- Most of the literature treats $\sigma_\varepsilon^2 = \mathbb{E}\left[\varepsilon_i^2\right]$ as fixed
  - Hausman, Newey, Ichimura, and Powell (1991), Newey (2001), Li (2002), Schennach (2004, 2007), Hu and Schennach (2008), and many others
  - Estimation of an infinite dimensional nuisance parameter and/or numerical simulation are required
  - Specific models (typically, nonlinear regression) and rather restrictive identification conditions
  - Perhaps too pessimistic and not representative of most empirical settings

- In practice, noise-to-signal ratio $\tau = \sigma_\varepsilon / \sigma_{X^*}$ is typically small in applications
- We consider an alternative asymptotic approximation: $\tau \to 0$ as $n \to \infty$
  - Perhaps a better match for the problem
  - We construct a simple estimation procedure for the general moment condition model
  - For presentation, think of $\sigma_\varepsilon^2 \equiv \mathbb{E}\left[\varepsilon_i^2\right] \equiv \sigma_n^2 \to 0$

Estimation: Motivation

Taylor expansion of $g(X_i, S_i, \theta)$ (in the spirit of Chesher, 1991): $\sigma_n^2 \to 0$

$$\begin{aligned} g(X_i, S_i, \theta) &= \quad g(X_i^*, S_i, \theta) \ + \ \varepsilon_i g_x^{(1)}(X_i^*, S_i, \theta) \quad + \ \frac{\varepsilon_i^2}{2} g_x^{(2)}(X_i^*, S_i, \theta) + O(\sigma_n^3), \\ \mathbb{E}[g(X_i, S_i, \theta)] &= \mathbb{E}[g(X_i^*, S_i, \theta)] \qquad + \ \frac{\sigma_n^2}{2} \mathbb{E}[g_x^{(2)}(X_i^*, S_i, \theta)] + O(\sigma_n^3), \end{aligned}$$

where $g_x^{(k)}(x, s, \theta) \equiv \partial^k g(x, s, \theta)/\partial x^k$

Therefore, $\mathbb{E}\left[g(X_i, S_i, \theta_0)\right] = O(\sigma_n^2)$, and the standard (naive) estimator is:

- Asymptotically biased if $\sqrt{n}\sigma_n^2 \to C \in (0, +\infty)$
- Not $\sqrt{n}$-consistent if $\sqrt{n}\sigma_n^2 \to \infty$

MERM Estimator

- Define the Measurement Error Robust Moments (MERM):

$$\psi\left(X_i, S_i, \theta, \gamma\right) \equiv g\left(X_i, S_i, \theta\right) - \gamma g_x^{(2)}\left(X_i, S_i, \theta\right)$$

Assumption (Moderate Measurement Error, Special Case of $K = 2$)

$\sigma_n^2 = o(n^{-1/3})$

- $\mathbb{E}\left[\psi(X_i, S_i, \theta_0, \gamma_{0n})\right] = O(\sigma_n^3) = o(n^{-1/2})$, where $\gamma_{0n} \equiv \sigma_n^2/2$

- MERM estimator:

$$(\hat{\theta}, \hat{\gamma}) = \underset{\theta \in \Theta, \gamma \in \Gamma}{\text{argmin}}\, \hat{Q}(\theta, \gamma), \quad \hat{Q}(\theta, \gamma) = \overline{\psi}(\theta, \gamma)' \,\hat{\Xi}\, \overline{\psi}(\theta, \gamma)$$

- Higher order expansion is more delicate: one needs to bias correct the bias correction terms.

Monte Carlo: Point Estimation

From Schennach (2007, Ecta):

$$\begin{aligned} Y_i &= \rho(X_i^*, \theta_0) + U_i \\ X_i^* &= Z_i + \eta_i \\ X_i &= X_i^* + \varepsilon_i \end{aligned} \qquad \begin{pmatrix} Z_i \\ \eta_i \\ U_i \\ \varepsilon_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4^{\dagger} & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix} \right)$$

(Original) moment function:

$$g(y, x, z, \theta) = (\rho(x, \theta) - y)h(x, z)$$

The noise-to-signal is $\tau = \sigma_\varepsilon / \sigma_{X^*} \approx 0.45$ – "fairly large" measurement errors

Specifications:

- Polynomial
- Rational Fraction
- Probit

Monte Carlo: Polynomial Specification

$$\rho(x,\theta) = \theta_1 + \theta_2 x + \theta_3 x^2 + \theta_4 x^3, \quad \theta_0 = (1,1,0,-0.5)'$$

	Bias				Std. Dev.				RMSE				
	θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_4	All
OLS	-0.00	-0.43	0.00	0.21	0.07	0.13	0.06	0.04	0.07	0.45	0.06	0.22	0.51
S07	-0.05	-0.07	-0.02	0.05	0.17	0.19	0.24	0.05	0.17	0.20	0.24	0.07	0.36
$K=2$	-0.00	0.10	0.00	0.00	0.10	0.23	0.10	0.08	0.10	0.25	0.10	0.08	0.29
$K=4$	-0.00	0.00	0.00	0.02	0.09	0.21	0.10	0.08	0.09	0.21	0.10	0.08	0.27

$$\sigma_\varepsilon/\sigma_{X^*} \approx 0.45, \;\; Y_i = \rho(X_i^*, \theta_0) + U_i, \;\; n = 1000$$

Monte Carlo: Polynomial Specification

$$\rho(x,\theta) = \theta_1 + \theta_2 x + \theta_3 x^2 + \theta_4 x^3, \quad \theta_0 = (1, 1, 0, -0.5)'$$

	Bias				Std. Dev.				RMSE				
	θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_4	All
OLS	-0.00	-0.43	0.00	0.21	0.07	0.13	0.06	0.04	0.07	0.45	0.06	0.22	0.51
S07	-0.05	-0.07	-0.02	0.05	0.17	0.19	0.24	0.05	0.17	0.20	0.24	0.07	0.36
$K = 2$	-0.00	0.10	0.00	0.00	0.10	0.23	0.10	0.08	0.10	0.25	0.10	0.08	0.29
$K = 4$	-0.00	0.00	0.00	0.02	0.09	0.21	0.10	0.08	0.09	0.21	0.10	0.08	0.27

$$\sigma_\varepsilon/\sigma_{X^*} \approx 0.45, \;\; Y_i = \rho(X_i^*, \theta_0) + U_i, \;\; n = 1000$$

Monte Carlo: Rational Fraction Specification

$$\rho(x,\theta) = \theta_1 + \theta_2 x + \frac{\theta_3}{(1+x^2)^2}, \qquad \theta_0 = (1,1,2)'$$

	Bias			Std. Dev.			RMSE			
	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3	All
OLS	0.339	-0.167	-0.644	0.040	0.020	0.076	0.341	0.168	0.648	0.752
S07	0.107	0.117	-0.150	0.146	0.139	0.328	0.181	0.182	0.361	0.443
$K=2$	-0.004	-0.018	0.014	0.062	0.026	0.139	0.062	0.032	0.139	0.156
$K=4$	0.014	-0.002	-0.024	0.062	0.031	0.154	0.063	0.031	0.156	0.171

$\sigma_\varepsilon/\sigma_{X^*} \approx 0.45, \;\; Y_i = \rho(X_i^*, \theta_0) + U_i, \;\; n = 1000$

Monte Carlo: Probit Model

$$\rho(x,\theta) = \frac{1}{2}(1+\text{erf}(\theta_1+\theta_2 x)), \quad \theta_0 = (-1,2)'$$

	Bias		Std. Dev.		RMSE		
	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	All
NLLS	0.38	-0.97	0.06	0.08	0.39	0.98	1.05
S07	0.05	-0.06	0.39	0.53	0.39	0.53	0.69
$K=2$	0.11	-0.31	0.18	0.34	0.21	0.46	0.51
$K=4$	-0.01	-0.01	0.23	0.42	0.23	0.42	0.48

$\sigma_\varepsilon/\sigma_{X^*} \approx 0.45,\ \ Y_i = \rho(X_i^*,\theta_0)+U_i,\ \ n=1000$

Monte Carlo: Multinomial Choice Model

For an agent i the utility of option j is:

$$U_{ij} = \theta_{0j1}X_i^* + \theta_{0j2}W_{ij} + \theta_{0j3} + \epsilon_{ij} \qquad \text{for } j \in \{1,2\},$$
$$U_{i0} = \epsilon_{i0} \qquad \text{for } j = 0,$$
$$\epsilon_{ij} \sim_{iid} \text{type-I EV distribution}$$

The researcher observes (X_i, W_i, Y_i),

$$Y_i = \operatorname{argmax}_{j\in\{0,1,2\}} U_{ij}$$
$$X_i = X_i^* + \varepsilon_i, \quad W_{ij} = \rho X_i^*/\sigma_X^* + \sqrt{1-\rho^2}\nu_{ij},$$
$$X_i^* = V_{i1}Z_i + V_{i0}$$

$(\theta_{011}, \theta_{012}, \theta_{013}, \theta_{021}, \theta_{022}, \theta_{023}, \rho, \sigma_Z^2, \sigma_\nu^2) = (1, 0, 0, 0, 0, 0, 0.7, 1, 1)$ and $n = 2000$.

Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K=2$				$K=4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
						$\tau = 1/4$						
$\partial p_1/\partial x$	-3.24	1.36	3.51	66.98	0.74	2.63	2.74	4.30	1.13	2.73	2.95	7.86
$\partial p_1/\partial w_1$	2.32	1.64	2.84	30.74	-0.11	2.30	2.30	4.82	-0.31	2.29	2.31	6.54
$\partial p_1/\partial w_2$	0.48	0.75	0.90	9.40	-0.04	0.87	0.87	4.82	-0.08	0.87	0.87	5.36
$\partial p_2/\partial x$	1.96	1.17	2.28	39.44	-0.40	1.88	1.92	4.72	-0.63	1.93	2.03	6.74
$\partial p_2/\partial w_1$	-1.16	0.82	1.42	30.66	0.06	1.15	1.15	4.84	0.15	1.15	1.16	6.48
$\partial p_2/\partial w_2$	-0.96	1.50	1.78	9.48	0.09	1.74	1.74	4.98	0.17	1.74	1.75	5.44
$\partial p_0/\partial x$	1.28	1.01	1.63	25.28	-0.34	1.43	1.47	5.08	-0.50	1.48	1.56	7.36
$\partial p_0/\partial w_1$	-1.16	0.82	1.42	30.60	0.06	1.15	1.15	4.82	0.15	1.15	1.16	6.46
$\partial p_0/\partial w_2$	0.48	0.74	0.88	9.46	-0.05	0.87	0.87	4.90	-0.09	0.87	0.88	5.32

True marginal effects are $(\partial p_1/\partial x, \partial p_2/\partial x, \partial p_0/\partial x) = (0.222, -0.111, -0.111)$ and zeros for the rest. Bias, std, rmse are $\times 10^{-2}$.

Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K=2$				$K=4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
						$\tau = 1/4$						
$\partial p_1/\partial x$	-3.24	1.36	3.51	66.98	0.74	2.63	2.74	4.30	1.13	2.73	2.95	7.86
$\partial p_1/\partial w_1$	2.32	1.64	2.84	30.74	-0.11	2.30	2.30	4.82	-0.31	2.29	2.31	6.54
$\partial p_1/\partial w_2$	0.48	0.75	0.90	9.40	-0.04	0.87	0.87	4.82	-0.08	0.87	0.87	5.36
$\partial p_2/\partial x$	1.96	1.17	2.28	39.44	-0.40	1.88	1.92	4.72	-0.63	1.93	2.03	6.74
$\partial p_2/\partial w_1$	-1.16	0.82	1.42	30.66	0.06	1.15	1.15	4.84	0.15	1.15	1.16	6.48
$\partial p_2/\partial w_2$	-0.96	1.50	1.78	9.48	0.09	1.74	1.74	4.98	0.17	1.74	1.75	5.44
$\partial p_0/\partial x$	1.28	1.01	1.63	25.28	-0.34	1.43	1.47	5.08	-0.50	1.48	1.56	7.36
$\partial p_0/\partial w_1$	-1.16	0.82	1.42	30.60	0.06	1.15	1.15	4.82	0.15	1.15	1.16	6.46
$\partial p_0/\partial w_2$	0.48	0.74	0.88	9.46	-0.05	0.87	0.87	4.90	-0.09	0.87	0.88	5.32

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Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K=2$				$K=4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
						$\tau = 1/4$						
$\partial p_1/\partial x$	-3.24	1.36	3.51	66.98	0.74	2.63	2.74	4.30	1.13	2.73	2.95	7.86
$\partial p_1/\partial w_1$	2.32	1.64	2.84	30.74	-0.11	2.30	2.30	4.82	-0.31	2.29	2.31	6.54
$\partial p_1/\partial w_2$	0.48	0.75	0.90	9.40	-0.04	0.87	0.87	4.82	-0.08	0.87	0.87	5.36
$\partial p_2/\partial x$	1.96	1.17	2.28	39.44	-0.40	1.88	1.92	4.72	-0.63	1.93	2.03	6.74
$\partial p_2/\partial w_1$	-1.16	0.82	1.42	30.66	0.06	1.15	1.15	4.84	0.15	1.15	1.16	6.48
$\partial p_2/\partial w_2$	-0.96	1.50	1.78	9.48	0.09	1.74	1.74	4.98	0.17	1.74	1.75	5.44
$\partial p_0/\partial x$	1.28	1.01	1.63	25.28	-0.34	1.43	1.47	5.08	-0.50	1.48	1.56	7.36
$\partial p_0/\partial w_1$	-1.16	0.82	1.42	30.60	0.06	1.15	1.15	4.82	0.15	1.15	1.16	6.46
$\partial p_0/\partial w_2$	0.48	0.74	0.88	9.46	-0.05	0.87	0.87	4.90	-0.09	0.87	0.88	5.32

True marginal effects are $(\partial p_1/\partial x, \partial p_2/\partial x, \partial p_0/\partial x) = (0.222, -0.111, -0.111)$ and zeros for the rest. Bias, std, rmse are $\times 10^{-2}$.

Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K=2$				$K=4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
$\tau = 1/2$												
$\partial p_1/\partial x$	-8.97	1.09	9.04	100.00	-1.69	2.60	3.10	9.84	0.97	2.89	3.05	6.04
$\partial p_1/\partial w_1$	6.44	1.53	6.62	98.96	1.39	2.44	2.81	12.14	-0.21	2.41	2.42	5.54
$\partial p_1/\partial w_2$	1.28	0.72	1.47	42.54	0.29	0.90	0.94	7.00	-0.06	0.92	0.92	5.00
$\partial p_2/\partial x$	5.22	0.97	5.31	99.98	1.05	1.89	2.16	10.16	-0.53	2.08	2.15	6.14
$\partial p_2/\partial w_1$	-3.21	0.77	3.30	98.96	-0.69	1.22	1.40	12.10	0.10	1.21	1.21	5.52
$\partial p_2/\partial w_2$	-2.52	1.41	2.88	42.82	-0.56	1.79	1.87	7.20	0.13	1.84	1.84	4.98
$\partial p_0/\partial x$	3.75	0.86	3.85	98.78	0.64	1.45	1.59	8.18	-0.44	1.59	1.65	6.50
$\partial p_0/\partial w_1$	-3.23	0.78	3.32	98.96	-0.70	1.22	1.41	12.06	0.10	1.21	1.21	5.48
$\partial p_0/\partial w_2$	1.23	0.69	1.41	42.90	0.28	0.89	0.93	7.20	-0.07	0.92	0.92	4.94

True marginal effects are $(\partial p_1/\partial x, \partial p_2/\partial x, \partial p_0/\partial x) = (0.222, -0.111, -0.111)$ and zeros for the rest. Bias, std, rmse are $\times 10^{-2}$.

Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K = 2$				$K = 4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
$\tau = 1/2$												
$\partial p_1/\partial x$	-8.97	1.09	9.04	100.00	-1.69	2.60	3.10	9.84	0.97	2.89	3.05	6.04
$\partial p_1/\partial w_1$	6.44	1.53	6.62	98.96	1.39	2.44	2.81	12.14	-0.21	2.41	2.42	5.54
$\partial p_1/\partial w_2$	1.28	0.72	1.47	42.54	0.29	0.90	0.94	7.00	-0.06	0.92	0.92	5.00
$\partial p_2/\partial x$	5.22	0.97	5.31	99.98	1.05	1.89	2.16	10.16	-0.53	2.08	2.15	6.14
$\partial p_2/\partial w_1$	-3.21	0.77	3.30	98.96	-0.69	1.22	1.40	12.10	0.10	1.21	1.21	5.52
$\partial p_2/\partial w_2$	-2.52	1.41	2.88	42.82	-0.56	1.79	1.87	7.20	0.13	1.84	1.84	4.98
$\partial p_0/\partial x$	3.75	0.86	3.85	98.78	0.64	1.45	1.59	8.18	-0.44	1.59	1.65	6.50
$\partial p_0/\partial w_1$	-3.23	0.78	3.32	98.96	-0.70	1.22	1.41	12.06	0.10	1.21	1.21	5.48
$\partial p_0/\partial w_2$	1.23	0.69	1.41	42.90	0.28	0.89	0.93	7.20	-0.07	0.92	0.92	4.94

True marginal effects are $(\partial p_1/\partial x, \partial p_2/\partial x, \partial p_0/\partial x) = (0.222, -0.111, -0.111)$ and zeros for the rest. Bias, std, rmse are $\times 10^{-2}$.

Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K = 2$				$K = 4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
$\tau = 1/2$												
$\partial p_1/\partial x$	-8.97	1.09	9.04	100.00	-1.69	2.60	3.10	9.84	0.97	2.89	3.05	6.04
$\partial p_1/\partial w_1$	6.44	1.53	6.62	98.96	1.39	2.44	2.81	12.14	-0.21	2.41	2.42	5.54
$\partial p_1/\partial w_2$	1.28	0.72	1.47	42.54	0.29	0.90	0.94	7.00	-0.06	0.92	0.92	5.00
$\partial p_2/\partial x$	5.22	0.97	5.31	99.98	1.05	1.89	2.16	10.16	-0.53	2.08	2.15	6.14
$\partial p_2/\partial w_1$	-3.21	0.77	3.30	98.96	-0.69	1.22	1.40	12.10	0.10	1.21	1.21	5.52
$\partial p_2/\partial w_2$	-2.52	1.41	2.88	42.82	-0.56	1.79	1.87	7.20	0.13	1.84	1.84	4.98
$\partial p_0/\partial x$	3.75	0.86	3.85	98.78	0.64	1.45	1.59	8.18	-0.44	1.59	1.65	6.50
$\partial p_0/\partial w_1$	-3.23	0.78	3.32	98.96	-0.70	1.22	1.41	12.06	0.10	1.21	1.21	5.48
$\partial p_0/\partial w_2$	1.23	0.69	1.41	42.90	0.28	0.89	0.93	7.20	-0.07	0.92	0.92	4.94

True marginal effects are $(\partial p_1/\partial x, \partial p_2/\partial x, \partial p_0/\partial x) = (0.222, -0.111, -0.111)$ and zeros for the rest. Bias, std, rmse are $\times 10^{-2}$.

Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K=2$				$K=4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
$\tau = 3/4$												
$\partial p_1/\partial x$	-13.35	0.86	13.38	100.00	-6.83	2.64	7.32	80.32	0.71	3.22	3.29	4.74
$\partial p_1/\partial w_1$	9.69	1.45	9.80	100.00	4.95	2.65	5.61	65.52	0.01	2.62	2.62	5.34
$\partial p_1/\partial w_2$	1.81	0.69	1.94	75.30	1.01	0.89	1.35	26.08	-0.01	0.98	0.98	5.24
$\partial p_2/\partial x$	7.48	0.79	7.52	100.00	4.06	1.83	4.45	68.82	-0.37	2.32	2.35	5.74
$\partial p_2/\partial w_1$	-4.83	0.73	4.88	100.00	-2.47	1.32	2.81	65.46	-0.01	1.31	1.31	5.28
$\partial p_2/\partial w_2$	-3.51	1.33	3.76	75.60	-1.99	1.76	2.66	26.38	0.03	1.97	1.97	5.32
$\partial p_0/\partial x$	5.87	0.73	5.92	100.00	2.77	1.47	3.14	56.28	-0.34	1.77	1.80	5.82
$\partial p_0/\partial w_1$	-4.87	0.75	4.93	100.00	-2.48	1.33	2.81	65.40	-0.01	1.31	1.31	5.32
$\partial p_0/\partial w_2$	1.70	0.64	1.82	75.76	0.98	0.87	1.31	26.50	-0.02	0.99	0.99	5.30

True marginal effects are $(\partial p_1/\partial x, \partial p_2/\partial x, \partial p_0/\partial x) = (0.222, -0.111, -0.111)$ and zeros for the rest. Bias, std, rmse are $\times 10^{-2}$.

Monte Carlo: Multinomial Choice Model, Case RC

	MLE				$K = 2$				$K = 4$			
	bias	std	rmse	size	bias	std	rmse	size	bias	std	rmse	size
						$\tau = 3/4$						
$\partial p_1/\partial x$	-13.35	0.86	13.38	100.00	-6.83	2.64	7.32	80.32	0.71	3.22	3.29	4.74
$\partial p_1/\partial w_1$	9.69	1.45	9.80	100.00	4.95	2.65	5.61	65.52	0.01	2.62	2.62	5.34
$\partial p_1/\partial w_2$	1.81	0.69	1.94	75.30	1.01	0.89	1.35	26.08	-0.01	0.98	0.98	5.24
$\partial p_2/\partial x$	7.48	0.79	7.52	100.00	4.06	1.83	4.45	68.82	-0.37	2.32	2.35	5.74
$\partial p_2/\partial w_1$	-4.83	0.73	4.88	100.00	-2.47	1.32	2.81	65.46	-0.01	1.31	1.31	5.28
$\partial p_2/\partial w_2$	-3.51	1.33	3.76	75.60	-1.99	1.76	2.66	26.38	0.03	1.97	1.97	5.32
$\partial p_0/\partial x$	5.87	0.73	5.92	100.00	2.77	1.47	3.14	56.28	-0.34	1.77	1.80	5.82
$\partial p_0/\partial w_1$	-4.87	0.75	4.93	100.00	-2.48	1.33	2.81	65.40	-0.01	1.31	1.31	5.32
$\partial p_0/\partial w_2$	1.70	0.64	1.82	75.76	0.98	0.87	1.31	26.50	-0.02	0.99	0.99	5.30

True marginal effects are $(\partial p_1/\partial x, \partial p_2/\partial x, \partial p_0/\partial x) = (0.222, -0.111, -0.111)$ and zeros for the rest. Bias, std, rmse are $\times 10^{-2}$.

Estimator: General Case

MERM in the General Case:

$$\psi(X_i, S_i, \theta, \gamma) \equiv g(X_i, S_i, \theta) - \sum_{k=2}^{K} \gamma_k g_x^{(k)}(X_i, S_i, \theta), \quad \gamma = (\gamma_2, \ldots, \gamma_K)'$$

Assumption (Moderate Measurement Error)

$$\sigma_n^2 = o\left(n^{-1/(K+1)}\right)$$

- Under smoothness conditions, $\mathbb{E}[\psi(X_i, S_i, \theta_0, \gamma_{0n})] = o(n^{-1/2})$
- γ_{0n} is determined by the moments of ε_i
- Need to bias correct the bias correction terms
  - the implementation is still simple
- Theory: need to ensure that the Taylor's expansion remainder is negligible Details
- We establish asymptotic normality of the MERM estimator

Summary

- $\tau = \sigma_\varepsilon / \sigma_{X^*} \to 0$ is a better approximation
- we construct Measurement Error Robust Moments
  $\Rightarrow$ Simple estimation via GMM

More in the paper:

- Feasible to handle multivariate X
- Dependence, Panels
- Establish nonparametric identification in regression models
- Non-classical measurement errors

Thank You!

... and

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Assumption: Moment Function

Assumption (Lipschitz-Polynomial). For some functions $b_j(x, r, \theta)$ for $j \in \{1, \ldots, J\}$ s.t., $\forall x, x' \in \mathcal{X}$ and $\forall (r, \theta) \in \mathcal{R} \times \Theta$,

$$\left\| g_x^{(K)}(x', s, \theta) - g_x^{(K)}(x, s, \theta) \right\| \leqslant \sum_{j=1}^{J} b_j(x, s, \theta) |x' - x|^j,$$

and $\mathbb{E}\left[\sup_{\theta \in \Theta} b_j(X_i^*, S_i, \theta)\right] < C$ for $j \in \{1, \ldots, J\}$

- Key to show $\mathbb{E}\left[\psi(X_i, S_i, \theta_0, \gamma_{0n})\right] = O(\sigma_n^{K+1}) = o(n^{-1/2})$
- Satisfied in most models
- When $|\varepsilon_i / \sigma_n|$ has M bounded moments, can allow $J = M - K$
- A similar condition is imposed on $\nabla_\theta g_x^{(K)}(x, r, \theta)$

Asymptotic Normality

- Denote $\hat{\beta} \equiv (\hat{\theta}', \hat{\gamma}')'$, $\beta_{0n} \equiv (\theta_0', \gamma_{0n}')'$

Theorem (Asymptotic Normality)

Under standard assumptions,

$$n^{1/2}\Sigma^{-1/2}(\hat{\beta} - \beta_{0n}) \xrightarrow{d} N(0, I_{p+K-1}),$$

$$\Sigma = (\Psi^{*\prime}\Xi\Psi^{*})^{-1}\Psi^{*\prime}\Xi\Omega^{*}\Xi\Psi^{*}(\Psi^{*\prime}\Xi\Psi^{*})^{-1}$$

$$\Omega^{*} \equiv \mathbb{E}\left[g(X_i^{*}, S_i, \theta_0)g(X_i^{*}, S_i, \theta_0)'\right], \quad \Psi^{*} \equiv \mathbb{E}\left[\nabla_{\beta}\psi(X_i^{*}, S_i, \theta_0, 0)\right]$$

- For asymptotic normality, $\gamma_{0n} \to \gamma_0 = 0 \in \text{int}\,(\Gamma)$ or $n^{1/2}\gamma_{0n} \to \infty$
- Σ can be consistently estimated, the standard inference tools apply

$$n^{-1}\sum_{i=1}^{n}\psi_i(\hat{\beta})\psi_i(\hat{\beta})' = \Omega^{*} + o_{p,n}(1), \quad n^{-1}\sum_{i=1}^{n}\nabla_{\beta}\psi_i(\hat{\beta}) = \Psi^{*} + o_{p,n}(1)$$

back