

$$\begin{aligned}
& \tau_1 = \tau_2 - f_1 \times ({}^1r_{01} + r_{1,c1}) + f_2 \times r_{1c1} + J_1 \dot{\omega}_1 + \omega_1 \times (J_1 \omega_1) = \\
& = \begin{bmatrix} 0 \\ 0 \\ m_2 r_2 (\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + \\ + g \cos(q_1 + q_2)) + \frac{1}{12} (\ddot{q}_1 + \ddot{q}_2) (12 m_2 r_{l2}^2 + m_2 (2 l_2 + 3 r_{l2}^2)) \end{bmatrix} - \\
& - \begin{bmatrix} 0 \\ 0 \\ -r_1 (m_1 (\ddot{q}_1 r_1 + g \cos(q_1)) + m_2 (\ddot{q}_1 l_1 \cos(q_2) + \\ + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g \cos(q_1 + q_2))) \end{bmatrix} + \\
& + \begin{bmatrix} 0 \\ 0 \\ m_2 (l_1 - r_1) (\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g \cos(q_1 + q_2)) \end{bmatrix} + \\
& + \begin{bmatrix} 0 \\ 0 \\ \frac{\ddot{q}_1}{12} (12 m_1 r_{l1}^2 + m_1 (l_1^2 + 3 r_{l1}^2)) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \\
& = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 l_1^2 m_2 \cos(q_2) + \ddot{q}_1 l_1 m_2 r_2 \cos(q_2) + \ddot{q}_1 l_1 m_2 r_2 + \ddot{q}_1 m_1 r_1^2 + \\ + \ddot{q}_1 m_1 r_{l1}^2 + \ddot{q}_1 m_2 r_2^2 + \ddot{q}_1 m_2 r_{l2}^2 + \frac{\ddot{q}_1}{12} m_1 (l_1^2 + 3 r_{l1}^2) + \\ + \frac{\ddot{q}_1}{12} m_2 (2 l_2 + 3 r_{l2}^2) + \ddot{q}_2 l_1 m_2 r_2 + \\ + \ddot{q}_2 m_2 r_2^2 + \ddot{q}_2 m_2 r_{l2}^2 + \frac{\ddot{q}_2}{12} m_2 (2 l_2 + 3 r_{l2}^2) + \\ + \dot{q}_1^2 l_1^2 m_2 \sin(q_2) + \dot{q}_1^2 l_1 m_2 r_2 \sin(q_2) + \\ + g l_1 m_2 \cos(q_1 + q_2) + g m_1 r_1 \cos(q_1) + g m_2 r_2 \cos(q_1 + q_2) \end{bmatrix}
\end{aligned}$$