$$\begin{bmatrix}\cos\left(q_{1}\right) & \sin\left(q_{1}\right) & 0\\-\sin\left(q_{1}\right) & \cos\left(q_{1}\right) & 0\\0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\cos\left(q_{2}\right) & \sin\left(q_{2}\right) & 0\\-\sin\left(q_{2}\right) & \cos\left(q_{2}\right) & 0\\0 & 0 & 1\end{bmatrix}$$

- Звено 1

$$\omega_1 =^0 R_1^T[\omega_0 + \dot{q}_1 z_0] = \begin{bmatrix}\cos\left(q_{1}\right) & \sin\left(q_{1}\right) & 0\\-\sin\left(q_{1}\right) & \cos\left(q_{1}\right) & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix} = \begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}$$

$$\dot{\omega}_1 =^0 R_1^T[\dot{\omega}_0 + \ddot{q}_1 z_0 + \dot{q}_1\omega_0 \times z_0] = \begin{bmatrix}\cos\left(q_{1}\right) & \sin\left(q_{1}\right) & 0\\-\sin\left(q_{1}\right) & \cos\left(q_{1}\right) & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\\ddot{q}_1\end{bmatrix} = \begin{bmatrix}0\\0\\\ddot{q}_1\end{bmatrix}$$

$$\begin{aligned}a_1 &=^0 R_1^T a_0 + \dot{\omega}_1 \times^1 r_{01} + \omega_1 \times (\omega_1 \times^1 r_{01}) = \\ &= \begin{bmatrix}g\sin\left(q_{1}\right)\\g\cos\left(q_{1}\right)\\0\end{bmatrix} + \begin{bmatrix}0\\\ddot{q}_1 l_1\\0\end{bmatrix} + \begin{bmatrix}-\dot{q}_1^2 l_1\\0\\0\end{bmatrix} = \begin{bmatrix}-\dot{q}_1^2 l_1 + g\sin\left(q_{1}\right)\\\ddot{q}_1 l_1 + g\cos\left(q_{1}\right)\\0\end{bmatrix}\end{aligned}$$

$$\begin{aligned}a_{c1} &= a_1 + \dot{\omega}_1 \times r_{1,c1} + \omega_1 \times (\omega_1 \times r_{1,c1}) = \\ &= \begin{bmatrix}-\dot{q}_1^2 l_1 + g\sin\left(q_{1}\right)\\\ddot{q}_1 l_1 + g\cos\left(q_{1}\right)\\0\end{bmatrix} + \begin{bmatrix}0\\\ddot{q}_1\left(-l_1 + r_1\right)\\0\end{bmatrix} + \begin{bmatrix}-\dot{q}_1^2\left(-l_1 + r_1\right)\\0\\0\end{bmatrix} = \\ &= \begin{bmatrix}-\dot{q}_1^2 l_1 - \dot{q}_1^2\left(-l_1 + r_1\right) + g\sin\left(q_{1}\right)\\\ddot{q}_1 l_1 + \ddot{q}_1\left(-l_1 + r_1\right) + g\cos\left(q_{1}\right)\\0\end{bmatrix}\end{aligned}$$

- Звено 2

$$\omega_2 =^1 R_2^T[\omega_1 + \dot{q}_2 z_1] = \begin{bmatrix} \cos(q_2) & \sin(q_2) & 0 \\ -\sin(q_2) & \cos(q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix}$$

$$\begin{aligned} \dot{\omega}_2 &=^1 R_2^T[\dot{\omega}_1 + \ddot{q}_2 z_1 + \dot{q}_2 \omega_1 \times z_1] = \\ &= \begin{bmatrix} \cos(q_2) & \sin(q_2) & 0 \\ -\sin(q_2) & \cos(q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + \ddot{q}_2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} a_2 &=^1 R_2^T a_1 + \dot{\omega}_2 \times^2 r_{12} + \omega_2 \times (\omega_2 \times^2 r_{11}) = \\ &= \begin{bmatrix} (\ddot{q}_1 l_1 + g\cos(q_1))\sin(q_2) + \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\cos(q_2) \\ (\ddot{q}_1 l_1 + g\cos(q_1))\cos(q_2) - \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\sin(q_2) \\ 0 \end{bmatrix} + \\ &+ \begin{bmatrix} 0 \\ l_2(\ddot{q}_1 + \ddot{q}_2) \\ 0 \end{bmatrix} + \begin{bmatrix} -l_2(\dot{q}_1 + \dot{q}_2)^2 \\ 0 \\ 0 \end{bmatrix} = \\ &= \begin{bmatrix} -l_2(\dot{q}_1 + \dot{q}_2)^2 + (\ddot{q}_1 l_1 + g\cos(q_1))\sin(q_2) + \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\cos(q_2) \\ l_2(\ddot{q}_1 + \ddot{q}_2) + (\ddot{q}_1 l_1 + g\cos(q_1))\cos(q_2) - \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\sin(q_2) \\ 0 \end{bmatrix} \end{aligned}$$

$$a_{c2} = a_2 + \dot{\omega}_2 \times r_{2,c2} + \omega_2 \times (\omega_2 \times r_{2,c2}) =$$

$$= \begin{bmatrix} -l_2 (\dot{q}_1 + \dot{q}_2)^2 + (\ddot{q}_1 l_1 + g\cos(q_1))\sin(q_2) + \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\cos(q_2) \\ l_2(\ddot{q}_1 + \ddot{q}_2) + (\ddot{q}_1 l_1 + g\cos(q_1))\cos(q_2) - \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\sin(q_2) \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ (\ddot{q}_1 + \ddot{q}_2)(-l_2 + r_2) \\ 0 \end{bmatrix} + \begin{bmatrix} -(\dot{q}_1 + \dot{q}_2)^2 (-l_2 + r_2) \\ 0 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -l_2(\dot{q}_1 + \dot{q}_2)^2 - (\dot{q}_1 + \dot{q}_2)^2(-l_2 + r_2) + (\ddot{q}_1 l_1 + \\ +g\cos(q_1)\sin(q_2) + \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\cos(q_2) \\ l_2(\ddot{q}_1 + \ddot{q}_2) + (\ddot{q}_1 + \ddot{q}_2)(-l_2 + r_2) + (\ddot{q}_1 l_1 + \\ +g\cos(q_1)\cos(q_2) - \left(-\dot{q}_1^2 l_1 + g\sin(q_1)\right)\sin(q_2) \\ 0 \end{bmatrix}$$

- Звено 2

$$f_2 = f_3 + m_2 a_{c2} =$$

$$= \begin{bmatrix} -m_2\left(-\ddot{q}_1 l_1 \sin(q_2) + \dot{q}_1^2 l_1 \cos(q_2) + \dot{q}_1^2 r_2 + 2\dot{q}_1\dot{q}_2 r_2 + \dot{q}_2^2 r_2 - g\sin(q_1 + q_2)\right) \\ m_2\left(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g\cos(q_1 + q_2)\right) \\ 0 \end{bmatrix}$$

$$\tau_2 = \tau_3 - f_2 \times ({}^2r_{12} + r_{2,c2}) + f_3 \times r_{2c2} + J_2\dot{\omega}_2 + \omega_2 \times (J_2\omega_2) =$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -m_2 r_2 \left(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g\cos(q_1 + q_2)\right) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{12}(\ddot{q}_1 + \ddot{q}_2)\left(12 m_2 r_2^{l\,2} + \mathrm{m}_2\left(2l_2 + 3r_2^{l\,2}\right)\right) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 0 \\ m_2 r_2 \left(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g\cos(q_1 + q_2)\right) + \\ + \frac{1}{12}(\ddot{q}_1 + \ddot{q}_2)\left(12 m_2 r_2^{l\,2} + \mathrm{m}_2\left(2l_2 + 3r_2^{l\,2}\right)\right) \end{bmatrix}$$

- Звено 1

$$f_1 = f_2 + m_1 a_{c1} =$$

$$= \begin{bmatrix} -m_2\left(-\ddot{q}_1 l_1 \sin(q_2) + \dot{q}_1^2 l_1 \cos(q_2) + \dot{q}_1^2 r_2 + 2\dot{q}_1\dot{q}_2 r_2 + \dot{q}_2^2 r_2 - g\sin(q_1 + q_2)\right) \\ m_2\left(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g\cos(q_1 + q_2)\right) \\ 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} m_1\left(-\dot{q}_1^2 r_1 + g\sin(q_1)\right) \\ m_1(\ddot{q}_1 r_1 + g\cos(q_1)) \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} m_1\left(-\dot{q}_1^2 r_1 + g\sin(q_1)\right) - m_2(-\ddot{q}_1 l_1 \sin(q_2) + \\ +\dot{q}_1^2 l_1 \cos(q_2) + \dot{q}_1^2 r_2 + 2\dot{q}_1\dot{q}_2 r_2 + \dot{q}_2^2 r_2 - g\sin(q_1 + q_2) \\ m_1(\ddot{q}_1 r_1 + g\cos(q_1)) + m_2(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \\ +\ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g\cos(q_1 + q_2) \\ 0 \end{bmatrix}$$

$$\tau_1 = \tau_2 - f_1 \times ({}^1 r_{01} + r_{1,c1}) + f_2 \times r_{1c1} + J_1 \dot{\omega}_1 + \omega_1 \times (J_1 \omega_1) =$$

$$= \begin{bmatrix} 0 \\ 0 \\ m_2 r_2 \left(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g \cos(q_1 + q_2)\right) + \frac{1}{12}\left(\ddot{q}_1 + \ddot{q}_2\right)\left(12n \right.\end{bmatrix}$$

$$- \begin{bmatrix} 0 \\ 0 \\ -r_1 \left(m_1 \left(\ddot{q}_1 r_1 + g \cos(q_1)\right) + m_2 \left(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g \cos(q \right.\right.\end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ m_2 (l_1 - r_1)\left(\ddot{q}_1 l_1 \cos(q_2) + \ddot{q}_1 r_2 + \ddot{q}_2 r_2 + \dot{q}_1^2 l_1 \sin(q_2) + g \cos(q_1 + q_2)\right) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{\ddot{q}_1}{12}\left(12 m_1 r_{l1}^2 + \mathrm{m}_1\left(l_1^2 + 3 r_{l1}^2\right)\right) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 l_1^2 m_2 \cos(q_2) + \ddot{q}_1 l_1 m_2 r_2 \cos(q_2) + \ddot{q}_1 l_1 m_2 r_2 + \ddot{q}_1 m_1 r_1^2 + \\ + \ddot{q}_1 m_1 r_1^{l\,2} + \ddot{q}_1 m_2 r_2^2 + \ddot{q}_1 m_2 r_2^{l\,2} + \frac{\ddot{q}_1}{12}\mathrm{m}_1\left(l_1^2 + 3 r_1^{l\,2}\right) + \\ + \frac{\ddot{q}_1}{12}\mathrm{m}_2\left(2 l_2 + 3 r_2^{l\,2}\right) + \ddot{q}_2 l_1 m_2 r_2 + \ddot{q}_2 m_2 r_2^2 + \\ + \ddot{q}_2 m_2 r_2^{l\,2} + \frac{\ddot{q}_2}{12}\mathrm{m}_2\left(2 l_2 + 3 r_2^{l\,2}\right) + \dot{q}_1^2 l_1^2 m_2 \sin(q_2) + \dot{q}_1^2 l_1 m_2 r_2 \sin(q_2) + g l_1 m_2 \cos(q_1 - \\ + q_2 + g m_1 r_1 \cos(q_1) + g m_2 r_2 \cos(q_1 + q_2) \end{bmatrix}$$