Design and Optimization of an Axial Compressor Turbojet Engine

Trevor Taylor and Blake Lipofsky
San Diego State University, San Diego, CA, 92115

Kyle Kline
San Diego State University, San Diego, CA, 92115

Kiril Yampolsky
San Diego State University, San Diego, CA, 92115

and
Logan Resendez
San Diego State University, San Diego, CA, 92115

Within the realm of advanced propulsion systems, this paper will discuss the design and optimization of an axial compressor turbojet engine that develops 350 kN of thrust at a flight speed of 250 m/s. The aim of this design is to provide the necessary specifications of the turbojet that meet the performance metrics while operating within a web of constraints detailed in the forthcoming sections. Said constraints will ensure safe and efficient operation of the turbojet. Beyond meeting stringent performance and safety standards, this turbojet engine design will optimize cost based on overall size and efficiency values. The preliminary design process for the turbojet will be followed by avenues for improvement aimed at reducing cost without compromising performance or safety.

I. Introduction

In this report, the design and analysis of an axial compressor turbojet will be presented. The goal of this engine is to minimize the cost function displayed below:

$$f_c = \frac{A_{max,engine}^{0.5} l^{0.5}}{\eta_0^2}(3 + N_{spools}) \tag{1}$$

Where η_0 represents the engines overall efficiency that depends on the thrust and the $A_{max,engine}$ is the maximum area. The thrust is identified in (Eq.2):

$$F = F_{uninst} - D_{add} + F_{lip} \tag{2}$$

D_{add} is the additive drag, F_{uninst} is the uninstalled thrust, and the F_{lip} is the lip suction force. The entire engine length:

$$l = l_{id} + l_c + l_{bd} + l_b + l_t + l_n \tag{3}$$

Where the length of the inlet diffuser is l_{id}, the compressor is l_c, the burner diffuser is l_{bd}, the burner is l_b, the turbine is l_t, and the length of the nozzle is l_n. Each portion of the engine is defined in a certain location which is displayed in Table 1. Table 2 identifies all the design constraints of the engine.

Table 1: Engine Labels and Components

Location Number	Name
0	Capture Stream tube
1	Start of Inlet
2	Start of Compressor
3	Start of Burner
4	Start of Turbine
5-9	Nozzle

Table 2: Engine Labels and Components

Variable	**Location**	**Value/Constraint**
T_0	Inlet	=220K
P_0	Inlet	=23000 Pa
ω, N_{Spools}		Only allowed N_{spools} different ω
γ_{0-3}	Inlet/Compressor/Beginning of Burner	=1.4
$C_{p,0-3}$	Inlet/Compressor/Beginning of Burner	$=1000\ \frac{J}{kgK}$
γ_{4-9}	End of Burner/Turbine/Nozzle	=1.3
$C_{p,4-9}$	End of Burner/Turbine/Nozzle	$=1250\frac{J}{kgK}$
$\frac{\varepsilon}{g}$	Compressor	=0.55
$\frac{\theta^*}{c}$	Compressor	$=0.02(1+2D^2)$ cascade aerodynamics
$r_h,\ r_m,\ r_t$	Compressor	Compressor blades cannot stall
$rh,\ rm,\ rt$	Compressor	$0 \leq x \leq 1$
X_{CO}	Burner	≤ 0.0002
$\tau_{resident}$	Burner	$\geq 0.005s$
π_b	Burner	=0.98
$C_{12}H_{24}$-1	Burner	fuel
c	Compressor/Turbine	=0.085m
σ_c	Compressor/Turbine	$\leq 70kPa$
C_z	Compressor/Turbine	$\approx c\ \times \cos\ (\cos\left(\frac{\beta_1+\beta_2}{2}\right)$ for the rotator blades
C_z	Compressor/Turbine	$\approx c\ \times \cos\ (\cos\left(\frac{\alpha_1+\alpha_2}{2}\right)$ for stator blades
η_m	Compressor/Turbine	=0.98
$M_{r,tip}$	Compressor/Turbine	≤ 1.2
C_z	Compressor/Turbine	Constant
P_{blades}	Compressor/Turbine	$=8\frac{kg}{m^3}$

T_{t4}	Turbine	$\leq 1900K$
T_{wg}	Turbine	$\leq 1300K$
$\gamma_{0\text{-}3}$ $C_{p,0\text{-}3}$	Turbine	Turbine coolant equal to 0-3 states
coolanttemperature	Turbine	$\leq 1000k$
k_w	Turbine	$=50\frac{W}{m^2K}$
t_w	Turbine	=0.002m
Pr	Turbine	=0.7 for turbine coolant
μ	Turbine	$=4\text{x}10^{-5}\ \frac{kg}{ms}$ for turbine coolant
P_{t3}, T_{t3}	Turbine	Use for turbine coolant
$A_g = A_w = A_c$	Turbine	$=2c(r_t\text{-}r_h)$
h_g	Turbine	$=7000\frac{W}{m^2K}$
Stanton Number	Turbine	Given by $x=\frac{r_t-r_h}{2}$
$\dot{Q}_c$	Turbine	$=c_{pc}\dot{m}_c(T_{c,max}\text{-}T_{c,initial})$
$T_{c,Initial}$	Turbine	$=T_{t3}$
η_t	Turbine	Drops by 3.2% for each 1% coolant flow rate
η_n	Nozzle	=0.95

II. Inlet design

To begin the design of the inlet and understand the equations, it is important to note which area of the inlet each subscript denotes. Table 3 shows matches the subscripts to physical locations on the subsonic nacelle/diffuser. Many assumptions were also made in the design of the diffuser, such as an inlet efficiency of 0.95, a Mach at the throat of 0.60 and a mass flow rate of 350 kg/s.

Table 3: Diffuser

Location number	Physical Location
0	Capture Stream tube
1	Beginning of Inlet
Th	Throat
Max	Maximum Area of Nacelle
2	End of Diffuser

The inlet begins with the flight conditions at location zero, or the capture stream tube. Table 2 shows the temperature, pressure, and flight speed at this location. Using equation 4, the density of the inlet air is calculated.

$$\rho_0 = \frac{P_0}{RT_0} \tag{4}$$

Other simple calculations such as the speed of sound, Mach, and total temperature at point zero can be found in the attached inlet.m code. Since our inlet is adiabatic, the total temperature at locations 0-2 remains the same throughout each of these locations. Equation 5 is used to find the area of our capture stream tube, which, along with known Mach numbers, will drive the rest of the area relations of the inlet and diffuser section of the subsonic nacelle.

$$A_0 = \frac{\dot{m}_0}{\rho_0 * V_0} \tag{5}$$

The area of the throat of the nacelle can be found using the area ratio relation in equation 6 and then multiplying by the area of the capture stream tube as seen in equation 7.

$$\frac{A_{th}}{A_0} = \frac{M_0}{M_{th}} \left(\frac{1 + \frac{\gamma - 1}{2} M_{th}^2}{1 + \frac{\gamma - 1}{2} M_0^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 1.16 \tag{6}$$

$$A_{th} = \frac{A_{th}}{A_0} \times A_0 = 4.46 m^2 \tag{7}$$

Similarly, the area of the inlet of the nacelle is found using area ratios and known area values. The ratio of the area at location 1 to hat of the throat is found in the same manner as equation 6. The area at 1 can then be found using equation .

$$A_1 = \frac{A_1}{A_{th}} \frac{A_{th}}{A_0} \times A_0 = 4.71 m^2 \tag{8}$$

The next area of the diffuser to find is the maximum area, which will occur just before location 2 where the compressor begins. To begin, the Mach number at the throat is designed to be Mach 0.60, which is used in equation 9 to find the Cp value.

$$C_{p,crit} = \overline{C_p} = \frac{2}{\gamma M_{th}^2} \left[\left(\frac{1 + \frac{\gamma - 1}{2} M_{th}^2}{\frac{\gamma + 1}{2}} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right] = -0.3247 \tag{9}$$

Many parameters at location 1 were calculated using a calculated Mach 1 shown in the inlet.m code attached. The temperature, speed of sound, and pressure were calculated in equations 10,11, and 12 respectively.

$$T_1 = \frac{T_{t1}}{1 + (M_1^2 \frac{\gamma_1 - 1}{2})} = 236.8 K \tag{10}$$

$$a_1 = \sqrt{\gamma_1 R_1 T_1} = 308.4 m/s \tag{11}$$

$$p_1 = p_0 \times \left[\frac{1+\frac{\gamma_1 - 1}{2}M_0^2}{1+\frac{\gamma_1 - 1}{2}M_1^2}\right]^{\frac{\gamma_1}{\gamma_1 - 1}} = 29750\ Pa \tag{12}$$

With Mach at location 1 and previously calculated areas, the maximum area of the diffuser can now be calculated as shown in equation 13.

$$A_{max} = 1 + \frac{\left[2\frac{A_0}{A_1}\left(\frac{M_1}{M_0}\sqrt{\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_1^2}} - 1\right) + \frac{2}{\gamma M_0^2}\left(\left(\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)\right]}{-\bar{C}_p} * A_1 \tag{13}$$

$$= 5.71m^2$$

To size the length of the inlet diffuser, a Klomp chart was used with approximated and known values, all of which can be found in the inlet.m attached code. The final length of the inlet diffuser came out to 1.36 meters.

III. Compressor

A. Geometry

The compressor was designed using a high-pressure section and a low-pressure section making two spools. There will be a total of 22 stages (11 for each spool). The number of stages multiplied by two equals the number of hubs (n).

$$n = 2 \times Stages \tag{14}$$

β_1 and β_2 are the angles of attack for the leading and trailing edges of the rotator blades. These were chosen to be -20 and -30 respectively. A repeated-stage and repeated-row were integrated and designed into the compressor.

$$\propto_1 = -\beta_2 = 20 \tag{15}$$

$$\propto_2 = -\beta_1 = 30 \tag{16}$$

A design parameter was used for each chord length (meters) of the blades.

$$c = 0.085 \text{ m}$$

For the rotator and stator blades, the axial chord lengths are found from the equations below. The chord length multiplied by the cosine of the angles of attack average for their respective blades.

$$c_{rot} = c \times \cos\left(\frac{\beta_1+\beta_2}{2}\right) = 0.0770m \tag{17}$$

$$c_{stat} = c \times \cos\left(\frac{\alpha_1+\alpha_2}{2}\right) = 0.0770m \tag{18}$$

s_z is the distance between each hub.

$$s_z = 0.04$$

The total length of the compressor is displayed below. This is found from combining the parameters that were discovered above.

$$l_c = \frac{n}{2}(c_{rot} + c_{stat}) + s_2(n-1) = 2.53m \tag{19}$$

To find Mach numbers, the speed of sound needs to be calculated. The equation to find the speed of sound is used to then solve for Mach numbers.

$$r_{compressor} = \sqrt{\frac{A_2}{\pi}} = 1.32m \tag{20}$$

$$r_{tip} = \frac{a}{\omega}\sqrt{M_{tip}^2 - M_2^2} = 0.652m \tag{21}$$

$$r_{hub} = r_{comp} - r_{tip} = 0.648m \tag{22}$$

$$r_m = \frac{(r_{tip} + (r_{comp} - r_{tip}))}{2} = 0.663m \tag{23}$$

The pitch line radius is defined by r_m. The equation for degree of reaction follows.

$$°R = 1 - \frac{a}{2\omega r^2} = 0.28m \tag{24}$$

The speed of sound is defined as a and r is the pitchline, hub radius or tip. A 0 to 1 degree of reaction is needed so the minimum rotational velocity possible was used. The low and high rotational velocities are then identified.

$$\omega_{low} = 450\ rad/s$$

$$\omega_{high} = 520\ rad/s$$

The compressor geometry is depicted in this table below that identifies all the important parameters for the compressor section.

Table 5: Compressor Geometry

Parameter	Units	Value
Stages		
β_1	degrees	-30
β_2	degrees	-20
α_1	degrees	20
α_2	degrees	30
c	m	0.0850
c_{rot}	m	0.0770
c_{stat}	m	0.0770
s_z	m	0.04
l_c	m	2.53
r_{comp}	m	1.325
$r_{tip,low}$	m	0.711
$r_{tip,high}$	m	0.615

$r_{hub,low}$	m	0.614
$r_{hub,high}$	m	0.7197
$r_{m,low}$	m	0.6625
$r_{m,high}$	m	0.6625
$°R_{m,low}$		0.2178
$°R_{tip,low}$		0.3211
$°R_{hub,low}$		0.0891
$°R_{hub,high}$		0.4101

B. Stage Pressure Ratios

In the rotating frame of reference, P_{t1r} and P_{t2r} are the total pressures. The static pressure is the same in both these cases. The total pressure in the axial frame can be connected to the total pressure in the rotating frame that is illustrated below.

$$\frac{p_{t1r}}{p_{t1}} = \frac{p_{t1r}}{p_1}\frac{p_1}{p_{t1}} = \left(\frac{1+\frac{\gamma-1}{2}M_{1r}^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}} = 0.4454 \quad (25)$$

$$\frac{p_{t2}}{p_{t2r}} = \frac{p_{t2}}{p_2}\frac{p_2}{p_{t2r}} = \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_{2r}^2}\right)^{\frac{\gamma}{\gamma-1}} = 2.4162 \quad (26)$$

Additionally, $C_\theta \; and \; W_\theta$ are swirling velocities (in their respective frames) and are defined below.

$$U = \omega \times r_m = 344.5 \quad (27)$$

$$W_\theta = W_z \times \tan(\beta) = 960.8 \quad (28)$$

$$C_\theta = W_\theta + U = 335 \quad (29)$$

The pitchline radius is identified as r. M_1 and M_{1r} can be found from plugging in values above and substituting them into the equations for M and M_r.

$$M1 = 1.36$$

$$M_{1r} = 0.6807$$

Since the total pressure at the beginning of the compressor (station 1) is equal to the total pressure at station 2, P_{t1r} can be found.

$$p_{t1r} = \frac{p_{t1r}}{p_{t1}} p_{t1} = 32562 \; Pa \quad (30)$$

The boundary layer thickness is defined below and is non-denationalized:

$$\frac{\theta^*}{c} = 0.02 \times (1 + 2D^2) \quad (31)$$

$$D = 1 - \frac{W_2}{W_1} \times (|W_{\theta 2} - W_{\theta 1}|) \quad (32)$$

We can solve for σ by dividing the cord length of the blade by the spacing between the blades.

$$\sigma = \frac{c}{s} \tag{33}$$

$\overline{\omega}$ can now be solved by combining the equations displayed above.

$$\overline{\omega} = \frac{\theta^*}{c} \times \frac{\sigma}{\cos{(\beta_2)}} \times \frac{\cos{(\beta_1)}^2}{\cos{(\beta_2)}^2} \tag{34}$$

The total pressure in over the stator in a rotating frame can be found from P_{t1r} and $\overline{\omega}$ which are identified above.

$$p_{t2r} = p_{t1r} - \left(\overline{\omega} \times \rho \times \left(\frac{W_1^2}{2}\right)\right) \tag{35}$$

The ratio $\frac{p_{t2r}}{p_{t1r}}$ can the be highlighted:

$$\frac{p_{t2r}}{p_{t1r}} =$$

Using a similar process as displayed above, the total pressure change over the stator can be found. This results in:

$$M_2 =$$

$$M_{2r} =$$

$$\frac{p_{t2}}{p_{t2r}} =$$

To calculate $P_{t2,}$ this next equation is utilized.

$$p_{t2} = \frac{p_{t2}}{p_{t2r}} p_{t2r} = \tag{36}$$

C and C_θ are then calculated because we are at the stator. Plugging in C_3, $C_{2.}$ $C_{\theta 3}, C_{\theta 2}, and\ \ \sigma_s$ we can find $D_{s.}$

$$D_s = 1 - \frac{C_3}{C_2} \frac{|C_{\theta 3} - C_{\theta 2}|}{2\sigma_s C_2} \tag{37}$$

The expression for $\overline{\omega_s}$ over the stator blade can be defined with these new values that are found above.

$$\overline{\omega_s} = \frac{\theta^*}{c} \times \frac{\sigma}{\cos{(\alpha_1)}} \times \frac{\cos{(\alpha_2)}^2}{\cos{(\alpha_1)}^2} \tag{38}$$

D. Polytropic Efficiency and Ratios

The final ratios of the and values for the compressor including the total pressure, compressor ratio, temperature ratio, total temperature, and the overall efficiency are listed below.

$$p_{t3} = p_{t2} - \overline{\omega_s} \times \rho \left(\frac{W_z^2 + C_{\theta 2}^2}{2}\right) \tag{39}$$

$$p_{t3} = 1430160\ Pa$$

$$\pi_c = \frac{p_{t3}}{p_{t2}} = 10 \tag{40}$$

$$\tau_c = \pi_c^{\left(\frac{\gamma-1}{\gamma e_c}\right)} = 1.99 \tag{41}$$

$$T_{t3} = T_{t2} \times \tau_c = 501.7\text{ K} \tag{42}$$

$$\eta_c = \frac{\pi_c^{\gamma-1} - 1}{\tau_c - 1} = 0.9319 \tag{43}$$

As expected, the total temperature and pressure rises as a result of the compressor stage, where the air is slowed down and squeezed tightly to prepare it for efficient combustion in the burner.

IV. Burner

The goal of the burner is to slow down the airflow so that the fuel can pass through a swirler. The initial fuel is liquid and must start by atomizing and evaporating. Once the fuel turns into a gas, the combustion can begin. The combustion chamber is the main source of energy for the gas turbine so it is imperative the chamber is efficient in regulating airflow.

The first step in calculating the length of our burner is the first obtain p_{t4}. By multiplying the total pressure at 3 by the pressure ratio of the burner which was given, p_{t4} is found.

$$\pi_b = \frac{p_{t4}}{p_{t3}} = .98 \tag{44}$$

After that, creating an equation that demonstrates the burning of $C_{12}H_{24}$-1 fuel and balancing it, the stoichiometric fuel to air ratio can be calculated.

$$C_{12}H_{24} + 18O_2 \rightarrow 12CO_2 + 12H_2O \tag{45}$$

For our engine, the mass of the fuel is the molar weight of our fuel, and the mass of the oxidizer is the molar weight of the oxygen times the 18 moles we calculated would balance the reaction.

$$f_{Stoich} = \frac{m_{fuel@stoich}}{m_{oxidizer@stoich}} \tag{46}$$

After calculating the stoichiometric fuel to air ratio, we could calculate the fuel to air ratio using equation X below. We assumed about our equivalence ration, ϕ, and made our $\phi < 1$ due to that making the fuel mixture lean.

$$f = f_{Stoich} \cdot \phi \tag{47}$$

Table 6 Start of Burner Values

Variable	Value
f_{Stoich}	0.2917
p_{t3}	715080 Pa
T_{t3}	502.68 K
T_3	498.68 K
V_3	$89.5\,\frac{m}{s}$
M_3	0.2
ϕ	0.13
ϵ	1.5

In order to get $Q_{R,actual}$ and $Q_{R,ideal}$, Professor Popov's Cantera scripts dieselEquilibrium.py and dieselEquilibriumMach.py were used. The first script, dieselEquilibrium.py could be used to get an ideal value of for the heat of the reaction by assuming the fuel will burn for a long time. $Q_{R,actual}$ can then be calculated by applying our parameters into the python script. In order to get M_4, plugging in calculated values into dieselEquilibriumMach.py outputs the Mach at the end of the burner. Both values can be found in table X. The burner efficiency can be calculated by the ratio of actual heat energy divided by the ideal heat energy.

$$\eta_b = \frac{Q_{R,actual}}{Q_{R,ideal}} = .90 \tag{48}$$

The sheer stress resident value parameter was $\tau_{resident} \geq .005$, so we assumed it to be 0.006. The sheer stress can further be used to solve the length, and by plugging in equation X into equation X and simplifying we get equation X.

$$\tau_{resident} \approx \frac{\rho_3 A_3 l_b}{\dot{m}_3} \tag{49}$$

$$\dot{m}_3 = \rho_3 * A_3 * V_3 \tag{50}$$

The length of the burner was found using equation X below.

$$l_b \approx V_3 \tau_{resident} \approx 0.5372 \tag{51}$$

To solve the area ratio between the end of the compressor and beginning of the burner, the burner Mach number needed to be calculated. Equation X below solves for the burner Mach number. The value of ϵ is between 1 and 2, so by averaging, 1.5 was used for it. As this relates to the start of the burner, γ_{1-3} was used.

$$\pi_b \approx 1 - \epsilon \frac{\gamma}{2} M_b^2 \tag{52}$$

Equation X below solves for the area ratio of the end of compressor and beginning of the burner.

$$\frac{A_{3,c}}{A_{3,b}} = \frac{M_b}{M_3} \left(\frac{1+\frac{\gamma-1}{2} M_3^2}{1+\frac{\gamma-1}{2} M_b^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{53}$$

Table X End of Burner Values

Variable	Value
M_b	0.207
M_4	0.378
V_4	294.62
T_4	1665.1
T_{t4}	1700
f	0.0379
$Q_{R,actual}$	446517763.93
$Q_{R,ideal}$	$4.2 * 10^7$
$\tau_{resident}$	0.006
l_b	0.5372

$\frac{A_{3,c}}{A_{3,b}}$	1.033
X_{CO}	$3.97391 * 10^{-8}$

V. Turbine

The turbine is a pivotal component in the turbojet engine system serving as the counterpart to the compressor, working in reverse to harness power from the fluid. In this phase, the fluid undergoes expansion and acceleration, facilitating the conversion of thermal energy to kinetic energy. The initial segment of the turbine, known as the stationary blade row or nozzle, marks the first stage where fluid acceleration occurs. This is followed by the second stage, known as the rotating blade row or rotor. The stator introduces swirl to the fluid flow and the rotor works to extract this energy from the high velocity fluid stream, ultimately driving the engine's power output. Throughout this energy extraction process there is a notable decrease in both static and total pressure.

An essential aspect of turbine design involves the expansion of the turbine's cross-sectional area from stages 4 to 5. This expansion is directly tied to the parameters that govern the turbine, including pressure ratio and Mach numbers between these stages. The first step in calculating the area ratio is to use the conservation of energy from the compressor to the turbine. We are then able to calculate the total temperature at stage 5 which can then be used to find the pressure ratio between stages 4 and 5.

$$\eta_m(1+f)\left(c_{pt}T_{t4} - c_{pt}T_{t5}\right) = c_{p0-3}T_{t3} - c_{p0-3}T_{t2} \tag{54}$$

$$\tau_t = \frac{T_{t5}}{T_{t4}} \tag{55}$$

$$\pi_t = \tau_t^{\frac{\gamma_t}{(\gamma_t-1)e_t}} \tag{56}$$

$$P_{t5} = \pi_t P_{t4} \tag{57}$$

$$\frac{A_5}{A_4} = \frac{M_4}{M_5}\frac{\sqrt{\tau_t}}{\pi_t}\frac{(1+\frac{\gamma-1}{2}M_5^2)^{\frac{\gamma+1}{2(\gamma-1)}}}{(1+\frac{\gamma-1}{2}M_4^2)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{58}$$

The length of the turbine can be calculated using the same method that was used to determine the length of the compressor, denoted by Eq. () below.

$$l_t = \frac{n}{2}(c_{rot} + c_{stat}) + s_z(n-1) \tag{59}$$

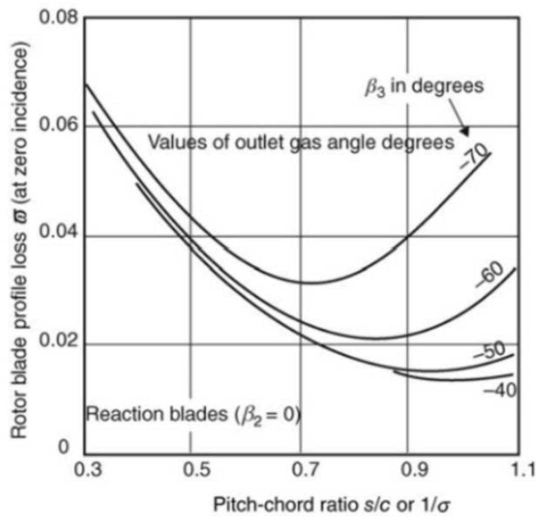


Fig X Profile Loss Coefficient

The high energy fluid that leaves the compression chamber causes the turbine blades to turn. The airfoil shape of each of the turbine blades generates a lift force aiding the rotation and producing more power. The power that is generated by the turbine is also used to power the compressor as they share the same shaft. The power produced by the turbine can be calculated using Eq () and the power of the compressor can be determined using the given value for the turbine-to-compressor mechanical efficiency ($n_m = 0.98$).

$$\dot{m}_0 = \rho_0 V_0 A_0 \tag{60}$$

$$Power_t = \dot{m}_0(1+f)c_{pt}(T_{t4} - T_{t5}) \tag{61}$$

$$Power_c = Power_t n_m \tag{62}$$

The losses in the uncooled turbine can be attributed to the blade profile loss which is found using Fig. X, which illustrates the pitch-chord ratio against the rotor blade profile loss.

$$\sigma = \frac{s}{c} = 0.5882 \tag{63}$$

$$\omega = \frac{p_{t2r} - p_{t3r}}{\rho_t\left(\frac{C_1}{2}\right)} \tag{64}$$

The high temperature of the outflow from the combustor requires the application of coolant in the initial turbine stage in order to protect the structural integrity of the blades. For this particular case we needed to ensure the coolant value did not exceed a temperature of 1000K. Coolant can be defined as the air that is bled from the tail end of the compressor. The implementation of turbine cooling is integral to the design, as it also enhances the engine's thrust output and overall efficiency. It is worth noting, however, that this cooling approach introduces additional losses in the turbine which can be calculated through determining the mass flow rate of the coolant shown by Eq() below.

$$T_4 = T_{t4} - \frac{{C_1}^2}{2c_{p4-9}} \tag{65}$$

$$r = \sqrt[3]{P_r} \tag{66}$$

$$T_{aw} = T_4 + r\frac{C_1^{\,2}}{2c_{p4-9}} \tag{67}$$

$$A_g = 2c(r_t - r_h) \tag{68}$$

$$\dot{Q}_c = A_g h_g (T_{aw} - T_{wg}) \tag{69}$$

$$\dot{m}_c = \frac{\dot{Q}_c}{c_{p0-3}(T_{c,max} - T_{c,initial})} \tag{70}$$

Once the mass flow rate of the coolant is determined, the total adiabatic efficiency of the turbine with implemented cooling can be calculated using Eq. (). First, some preliminary values are needed which include the total temperature of the coolant as well as the ratio of coolant to gas mass flow rate.

$$\dot{m}_4 = (1+f)\dot{m}_0 \tag{71}$$

$$\varepsilon = \frac{\dot{m}_c}{\dot{m}_g} = \frac{\dot{m}_c}{\dot{m}_4} \tag{72}$$

$$T_{tc} = T_3 + \frac{c_1^{\,2}}{2c_{p0-3}} \tag{73}$$

$$\eta_t = \frac{(1-\varepsilon)c_{pt}(T_{t4} - T_{t5}) + \varepsilon c_{p0-3}(T_{tc} - T_{t5})}{\left[(1-\varepsilon)c_{pt}T_{t4} + \varepsilon c_{p0-3}T_{tc}\right]\left[1 - \left(\frac{p_{t5}}{p_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{74}$$

Table X Turbine Values

Variable	Value	Units
T_{t5}	1498.663	K
P_{t5}	3.943e+05	Pa
τ_t	0.881	-
π_t	0.562	-
A_5/A_4	1.991	-
$Power_t$	8.984e+07	W
$Power_c$	8.804e+07	W
$\dot{Q}_c$	1.654e+06	kJ/kg
$\dot{m}_c$	1.3816	kg/s
T_{tc}	513.079	K
l_t	0.7120	m
η_t	0.9597	-

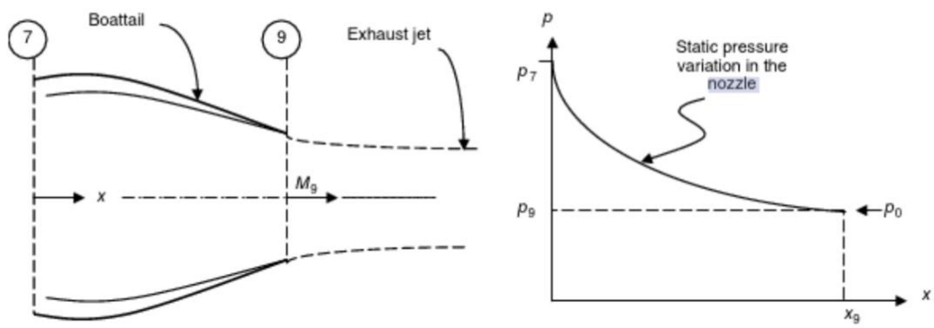
VI. Nozzle

The primary objective of an aircraft engine exhaust system is to efficiently accelerate the gas. The crucial nozzle parameter for propulsion is the gross thrust, denoted as Fg. The previously derived expression for gross thrust is represented as:

$$\dot{m}_9 \cdot V_9 + (p_9 - p_0) \cdot A_9$$

In this equation, the initial term on the right-hand side refers to the momentum thrust, while the second term is known as the pressure thrust. To maximize the nozzle's gross thrust, it is essential to achieve perfect expansion in the nozzle flow. This means pressure thrust will be equal to zero. Below is a figure displaying what the perfectly expanded converging nozzle will appear as.

■ FIGURE 4.18 Schematic drawing of a subsonic nozzle with its static pressure distribution



Considering compressible duct flows in aerodynamics, it is evident that the exit pressure is directly influenced by the nozzle area ratio and the nozzle pressure ratio (NPR). Referring to the NPR definition:

$$NPR = \frac{p_{t7}}{p_0} = 17.2$$

From the provided parameters for the nozzle's hot air post-turbine, the total nozzle pressure ratio can be determined using the formula:

$$\pi_n = \left[NPR^{\frac{y-1}{y}} - \eta_n\left(NPR^{\frac{y-1}{y}} - 1\right)\right]^{\frac{-y}{y-1}}$$

The local Mach number can be computed by simultaneously knowing the total and static pressures. Utilizing NPR and the total pressure ratio, the relationship is expressed as:

$$\frac{p_{t9}}{p_9} = \pi_n \cdot NPR \cdot \frac{p_0}{p_9} = 14.12$$

By employing the general expression for total pressure and Mach number and isolating the Mach number as follows:

$$M_9 = \sqrt{\frac{2}{y-1}\left[\left(\frac{p_{t9}}{p_9}\right)^{\frac{y-1}{y}} - 1\right]}$$

Using continuity, the calculation of area ratio becomes:

$$\frac{A_9}{A_8} = \epsilon = \frac{1}{M_9}\sqrt{\left(\frac{2 + M_9^2(y-1)}{y+1}\right)^{\frac{y+1}{y-1}}}$$

For the gross thrust, the mass flow rate at the core nozzle along with the velocity at the exit. For the velocity, the equation becomes:

$$V_9 = \sqrt{2c_p T_{t9}\left(\frac{y-1}{y+1}\right)}$$

In the design phase, a mass flow rate was chosen for the core nozzle. This is the value that can be manipulated to try and maximize gross thrust.

$$F_{g-conv} = \dot{m}V_9$$

Below is a table with all calculated nozzle values.

Variable	Value	Units
NPR	17.2	-
π_n	0.8216	-
$\frac{p_{t9}}{p_9}$	14.12	-
M_9	2.37	-
$\frac{A_9}{A_8}$	2.57	-
V_9	699	m/s
$\dot{m}_9$	50	kg/s
F_{g-conv}	34.95	kN

VI. Optimizing Cost

Now that the turbojet has been designed, the cost can be estimated and solutions for improvement can be explored. Using equation 2, shown below for convenience, the cost estimate comes out to $133,000,000.

$$f_c = \frac{A_{max,engine}^{0.5}\, l^{0.5}}{\eta_0^2}\left(3 + N_{spools}\right) = \$133M$$

To improve on this cost, there are many avenues to take. Shrinking the overall size and length of the engine is the most straightforward way of bringing costs down. This starts by defining a lower mass flow rate at the inlet, which in turn would shrink the maximum area of the nacelle architecture. Another approach would be to maximize the overall efficiency of the engine or lessen the number of spools in the compressor and turbine regions. This may, however, compromise performance and was therefore not considered.

VI. Conclusion

The objective of this report, given the parameters, was to optimize the cost function stated in the beginning of the report. This technical report calculated and analyzed a two-spool axial turbojet engine to complete our intended goal. Ideally the length of the engine would be at a minimum while the area would be at maximum to maximize the efficiency of the engine.

Some assumptions were made when designing the inlet and diffuser. These assumptions were made to create appropriate and functional dimensions. In the Inlet, our Mach number was used to prevent supersonic flow pockets. Using our assumptions, we were able to calculate the area for all parts of the engine including compressor, throat and inlet entrances. The inlet design is finished after finding the coefficient of pressure, conical diffuser and the length of the inlet.

Since the cost function would change drastically depending on how many spools for the compressor we used, a fair assumption needed to take place to optimize our engine. This needed to be done to keep our cost function and length to a minimum.

The Nozzle parameters are found from the values at the end of the turbine. Perfectly expanded flow was found through pressure leaving the nozzle and ambient pressure being the same value. To increase overall produced thrust and exit velocity, the converging-diverging nozzle was to be sonic. The nozzle was calculated very similar to the inlet with a few different equations like the angle of the conical nozzle. The designed turbojet met the requirements provided.

VI. Appendix

A. Matlab Codes
   - a. inlet.m
   - b. compressor.m
   - c. burner.m
   - d. burner.py
   - e. turbine.m
   - c. Nozzle.m

VI. Acknowledgements

Thank you to San Diego State University and Dr. Pavel Popov for his teaching and instructions throughout to conduct aircraft propulsion systems. All calculations and problems in this report were found through Dr. Popov’s guidance.

VI. References

Farokhi, S., Aircraft Propulsion, Wiley, Chichester, West Sussex, United Kingdom, 2014