

The measure of a countable set is 0.

Proof.

A countable set E can be expressed as $\{x_1, x_2, x_3, \dots, x_n, \dots\}$

Let $I_k = (x_k - \varepsilon \frac{2^{-k}}{4}, x_k + \varepsilon \frac{2^{-k}}{4}), \forall \varepsilon$

$$m(\bigcup_{k=1}^{\infty} I_k) \leq \sum_{k=1}^{\infty} m(I_k) = \sum_{k=1}^{\infty} 2\varepsilon \frac{2^{-k}}{4} = \varepsilon \sum_{k=1}^{\infty} 2^{-k-1} = \frac{\varepsilon}{2} < \varepsilon$$

Thus, $m(E) = 0$

□