Estimate a least-squares regression.

$$Y = X_1^T \tilde{\beta}_1 + \tilde{\mu}$$

Then regress the residuals on another set of regressors.

$$\tilde{\mu} = X_2^T \tilde{\beta}_2 + \tilde{e}$$

Then regress on both set of regressors.

$$Y = X_1^T \hat{\beta}_1 + X_2^T \hat{\beta}_2 + \hat{e}$$

Is it true that $\tilde{\beta}_2 = \hat{\beta}_2$?

Proof.
$\tilde{\beta}_2 = \hat{\beta}_2$ if and only if X_1 and X_2 are uncorrelated, i.e. $X_1^T X_2 = 0$
Define $M_i = I - X_i(X_i^T X_i)^{-1} X_i^T$
We obtain $\tilde{\mu} = M_1 Y$
$\tilde{\beta}_2 = (X_2^T X_2)^{-1} X_2^T \tilde{\mu} = (X_2^T X_2)^{-1} X_2^T M_1 Y$
$\hat{\beta}_2 = (X_2^T M_1 X_2)^{-1} X_2^T M_1 Y$
Suppose $\tilde{\beta}_2 = \hat{\beta}_2$, then

$$(X_2^T X_2)^{-1} X_2^T M_1 Y = (X_2^T M_1 X_2)^{-1} X_2^T M_1 Y$$

By algebra, we obtain $X_2^T X_2 = X_2^T M_1 X_2$
Thus,

$$X_2^T X_2 = X_2^T (I - X_1(X_1^T X_1)^{-1} X_1^T) X_2 = X_2^T X_2 - X_2^T X_1 (X_1^T X_1)^{-1} X_1^T X_2$$

Thus $X_1^T X_2 = 0$ since $(X_1^T X_1)^{-1}$ is invertible. $\square$