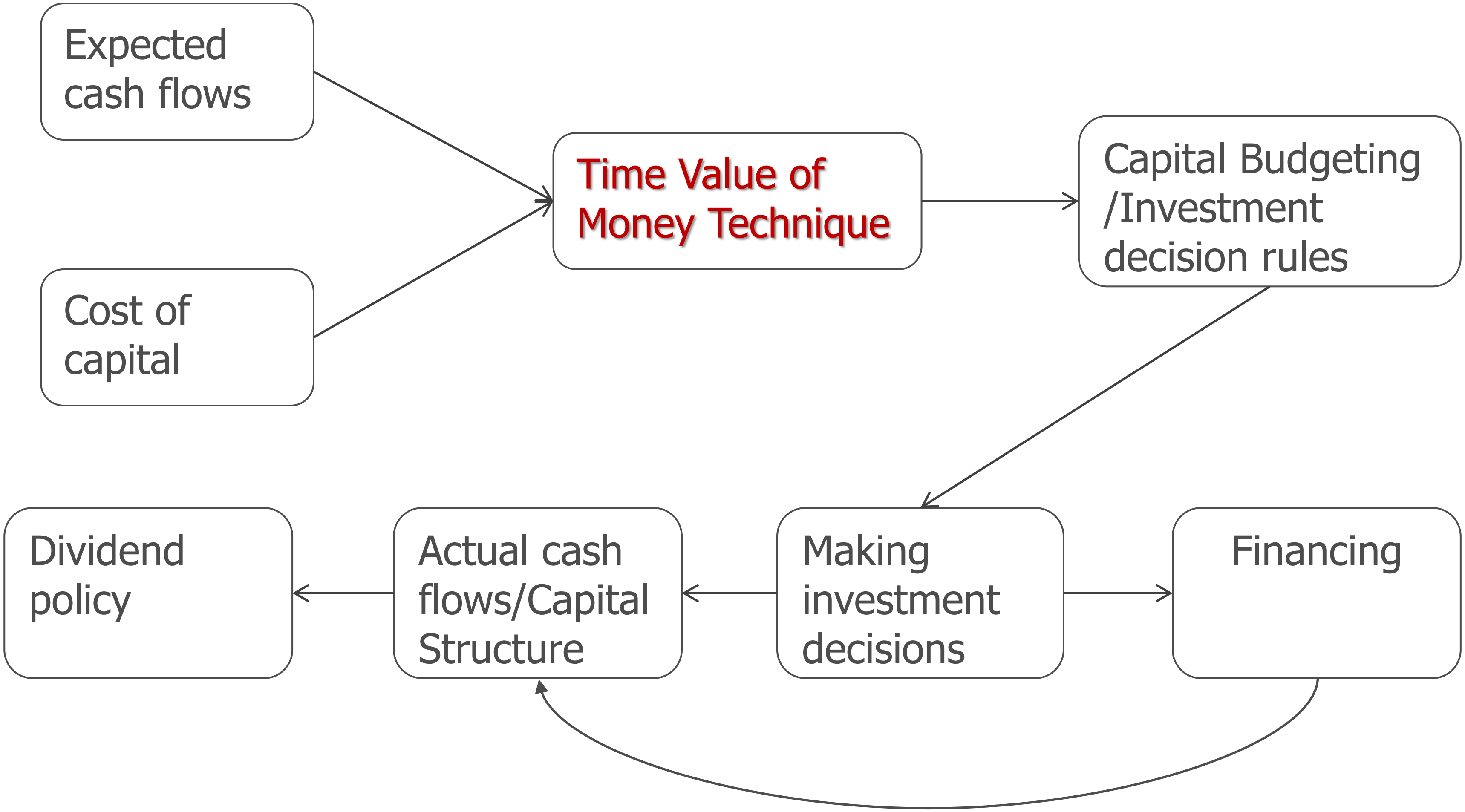
Financial Economics

Lecture 02. Time Value of Money

LIN, Mengyun SOE&WISE

AY22-23, SPRING

Corporate Financial Decision



Time Value of Money (TVM)

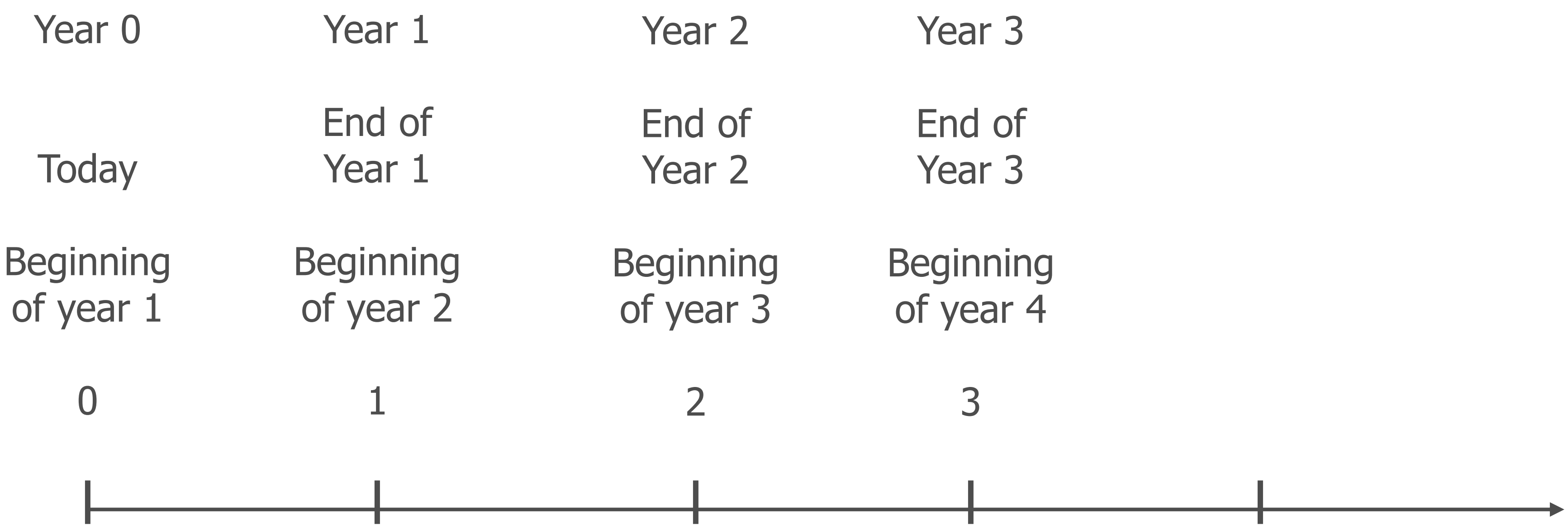
- One of the basic problems faced by the financial manager is how to determine the value today of cash flows expected in the future
- $20 today is worth more than the expectation of $20 tomorrow because:
  - **Interest**:
    - You can invest it and earn the interest, such as saving in the bank
  - **Inflation**
    - Tomorrow's $20 less valuable than today's $20
  - **Uncertainty**
    - You may not receive tomorrow's $20

Outline

- **Future Value and Compounding**
- Present Value and Discounting
- More about PV and FV
- Capital Budgeting Decision Rule

Timelines

- Drawing timelines can be helpful in understanding time value of money problems



Basic Definitions: PV and FV

- **Present Value (PV)**: the value of something today.
  - On a timeline $t = 0$.
  - Also referred to as the **market value** of a cash flow to be received in the future.
  - Translating a value that comes at some point in the future to its value in the present is referred to as **discounting**.
- **Future Value (FV):** the value of a cash flow sometime in the future.
  - On a timeline $t > 0$.
  - Translating a value to the future is referred to as **compounding**.
- All that present values and future values do is to put cash flows which come in at different times on a comparable basis!
- Once they are in the "same units", we will be able to compare and make decisions on which pattern of cash flows are preferable.

Compounding

- Assume that the interest rate is 10% p.a. (per annual)
- If you invest \$100 for **one year**, how much will you have in **one year**?
- In general, if you invest for one period at an interest rate of i your investment will grow to $(1 + i)$ per dollar invested
  - $i = 10\%$
  - $1 + 10\% = 1.1$ dollars per dollar invested
  - $\$100 \times 1.1 = \$110$
- If you invest \$100 for **two years**, how much will you have in **two years**?
- It is equivalent to after you save for one year and earn interest of \$10, importantly, not take the interest out, and save the total of \$110 for another year
  - $\$110 \times 1.1 = \$121$

Compounding

- The process of accumulating interest on an investment over time to earn more interest
- Compounding the interest means earning interest on interest, so we call the result **compound interest**
- With **simple interest**, interest is earned only on the original principal amount invested
  - Back to the $100 investment case, it is equivalent to withdrawing the $10 interest earned in year one, and only saving the principal $100 for the second year, you get $10 by the end of year 1 and $110 by the end of year 2. The total interest received is $20.

Future Value

- The future value of \$1 invested for n periods at a rate of i per period is

$$Future\ Value(FV) = \$1 \times \boldsymbol{(1+i)^n}$$

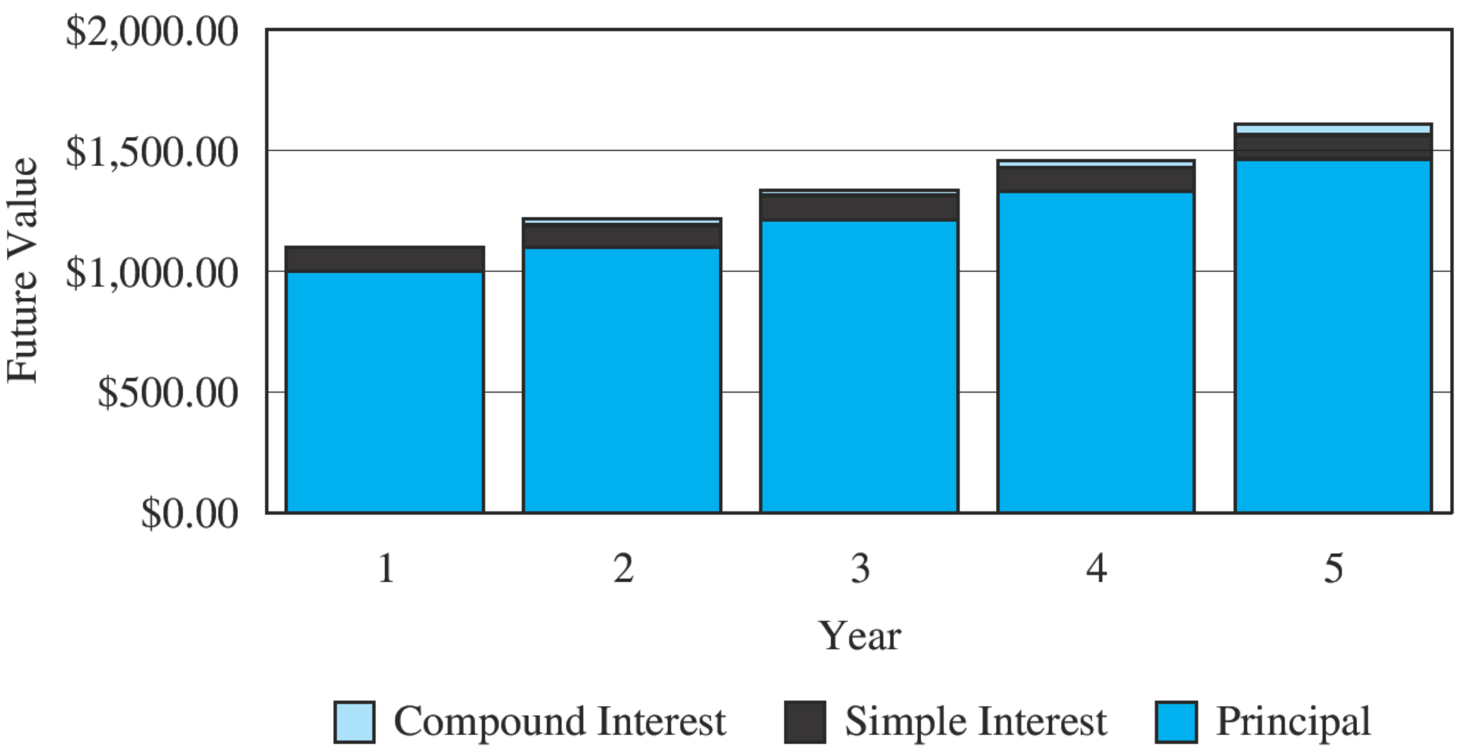
Future value factor for \$1 dollar invested at a rate of i for n periods

- Going back to our \$100 investment, what will you have after five years, assuming the interest rate does not change?
  - With simple interest: $\$100 + \$100 \times 10\% \times 5\ years = \$150$
  - With compound interest:
    - $Future\ value\ factor = (1 + 10\%)^5 = 1.1^5 = 1.6106$
    - Your \$100 will thus grow to $\$100 \times 1.6105 = \$161.05$

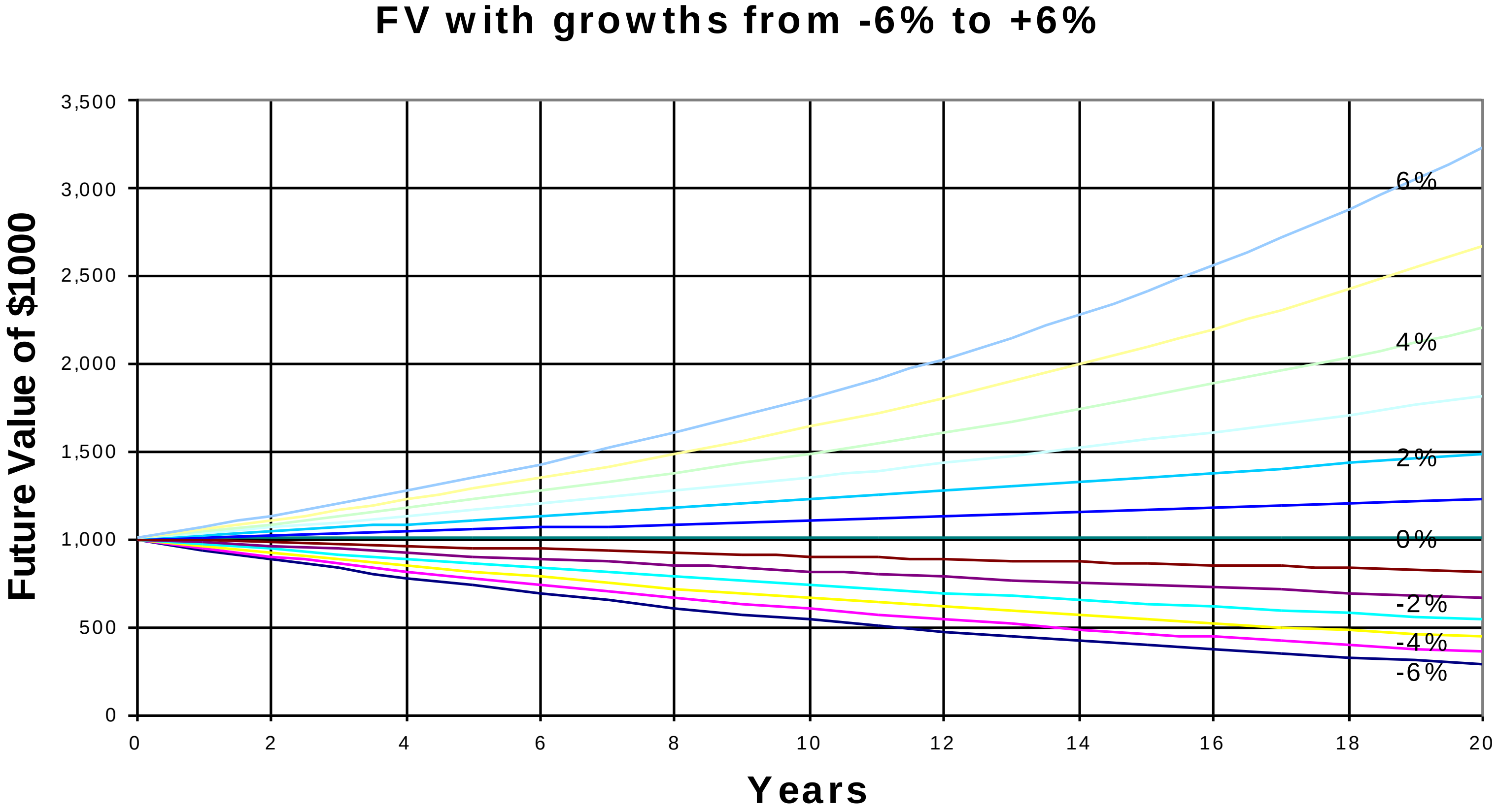
Future Value and Compound Interest

Year	Beginning Amount	Simple Interest	Compound Interest	Total Interest Earned	Ending Amount
1	$100.00	$10	$ .00	$10.00	$110.00
2	110.00	10	1.00	11.00	121.00
3	121.00	10	2.10	12.10	133.10
4	133.10	10	3.31	13.31	146.41
5	146.41	10	4.64	14.64	161.05
Total		$50	$11.05	$61.05	

Future Value and Compound Interest



Future Value of a Lump Sum



$$FV = PV \times (1 + i)^n$$

Example: Future Value of a Lump Sum

- Your bank offers a CD with an interest rate of 3% for a 5-year investment.
- You wish to invest $1,500 for 5 years, how much will your investment be worth?
- $FV = PV \times (1 + i)^n = \$1500 \times (1 + 0.03)^5 = \$1738.11$

Outline

- Future Value and Compounding
- **Present Value and Discounting**
- More about PV and FV
- Capital Budgeting Decision Rule

Present Value and Discounting

- **Present value** is the current value of future cash flows discounted at the appropriate **discount rate**
  - **Discounting**: Calculating the present value
  - **Discount rate**: The interest rate used in discounting

- Present value is the reverse of future value
  - $FV = PV \times (1+i)^n$
  - Divide both sides by $(1+i)^n$ to obtain
  - $PV = \frac{FV}{(1+i)^n} = FV \times (1+i)^{-n}$

Discounting factor /Present value factor for $1 dollar invested at a rate of i for n periods

Present Value and Discounting

- Example:
- You have been offered $40,000 for your printing business, payable in 2 years.
- Given the risk, you require a return of 8%.
- What is the present value of the offer?

- $PV = \frac{FV}{(1+i)^n}$

$$= \frac{40,000}{(1+0.08)^2}$$

$$= 34293.55$$

Lump Sums Formula

- You have solved a **present value** and a **future value** of a <u>lump sum</u>. There remains two other variables that may be solved for
  - interest, i

$$PV = \frac{FV}{(1+i)^n} \Rightarrow (1+i) = \sqrt[n]{\frac{FV}{PV}} \Rightarrow i = \sqrt[n]{\frac{FV}{PV}} - 1$$

  - number of periods, n

$$PV = \frac{FV}{(1+i)^n} \Rightarrow \ln\left(\frac{FV}{PV}\right) = \ln\left((1+i)^n\right) = n * \ln(1+i) \Rightarrow n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)} = \frac{\ln(FV) - \ln(PV)}{\ln(1+i)}$$

Interest (i)

- Example
- If you invest $15,000 for ten years
- You receive $30,000.
- What is your annual return?

$$i = \sqrt[n]{\frac{FV}{PV}} - 1$$

$$= \sqrt[10]{\frac{30000}{15000}} - 1 = \sqrt[10]{2} - 1 = 2^{\frac{1}{10}} - 1$$

$$= 0.071773463$$

$$= 7.18\% \text{ (to the nearest basis point)}$$

Number of Periods (n)

- How long will it take for $75 to grow to $100?
- The interest rate is 8%

$$n = \frac{\ln(FV) - \ln(PV)}{\ln(1+i)}$$

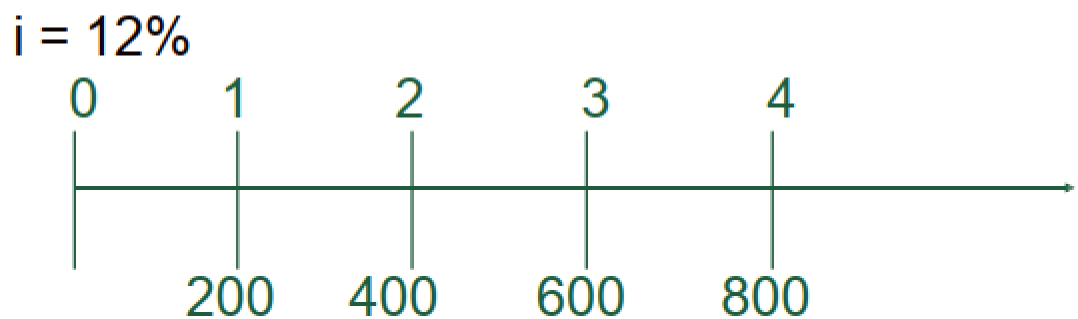
$$n = \frac{\ln(100) - \ln(75)}{\ln(1+0.08)} = 3.74$$

Multiple Cash Flows

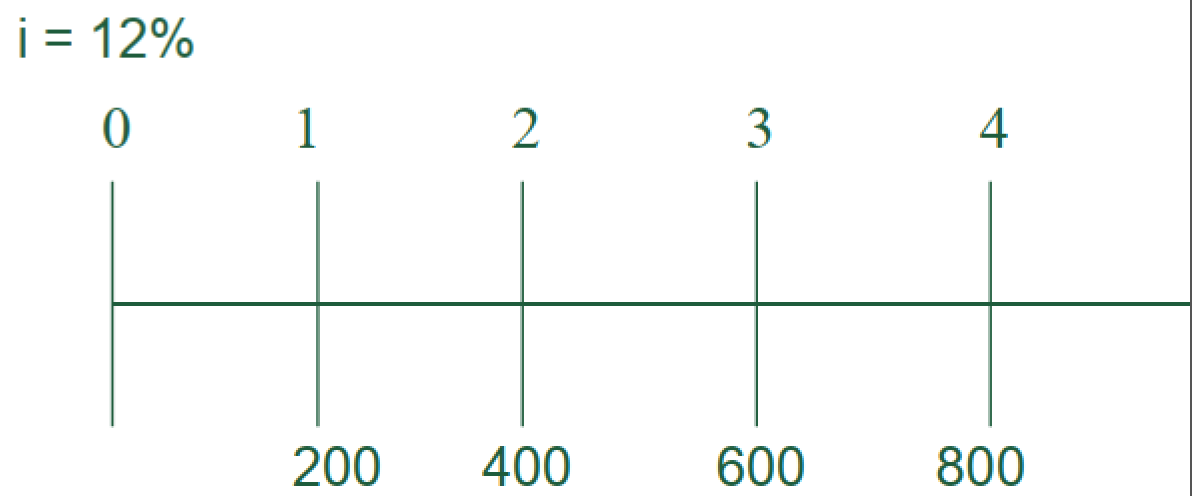
Time	0	1	2	3
Cash Flow	−100	20	50	60

Present Value of Multiple Cash Flows

- Suppose you plan to deposit $200 into an account in one year’s time and $400 into the account in two years from now, $600 in 3 years, and $800 in four years. If the interest rate is 12%
- What is the present value of the entire cash flow?
- What is the future value of the entire cash flow in 5 years?



Present Value of Multiple Cash Flows



Year 1 CF: $200 / (1.12)^1 =$ 178.57

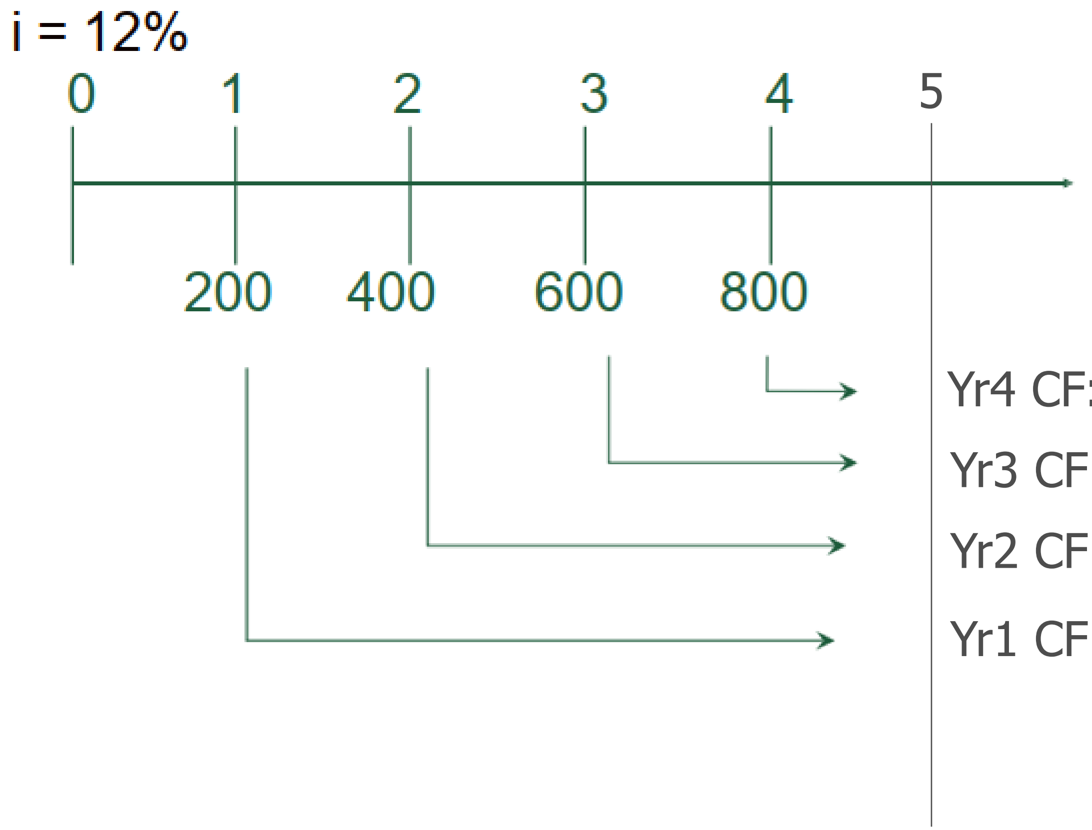
Year 2 CF: $400 / (1.12)^2 =$ 318.88

Year 3 CF: $600 / (1.12)^3 =$ 427.07

Year 4 CF: $800 / (1.12)^4 =$ 508.41

1432.93 = PV

Future Value of Multiple Cash Flows



Yr4 CF: 800*(1+0.12)^1=986

Yr3 CF: 600*(1+0.12)^2=752.64

Yr2 CF: 400*(1+0.12)^3=561.97

Yr1 CF: 200*(1+0.12)^4=314.70

2525.32

Reference Table for FV (and the inverse of PV)

Future Value of 1 Table (FV of 1 Table)
FV Factors for a Single Amount of 1.000
(rounded to three decimal places).

Note: This table begins with row n = 0, which is different from most future value of 1 tables.

	i=1%	i=2%	i=3%	i=4%	i=5%	i=6%	i=8%	i=10%	i=12%
n = 0 →	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 1 →	1.010	1.020	1.030	1.040	1.050	1.060	1.080	1.100	1.120
n = 2 →	1.020	1.040	1.061	1.082	1.103	1.124	1.166	1.210	1.254
n = 3 →	1.030	1.061	1.093	1.125	1.158	1.191	1.260	1.331	1.405
n = 4 →	1.041	1.082	1.126	1.170	1.216	1.262	1.360	1.464	1.574
n = 5 →	1.051	1.104	1.159	1.217	1.276	1.338	1.469	1.611	1.762
n = 6 →	1.062	1.126	1.194	1.265	1.340	1.419	1.587	1.772	1.974
n = 7 →	1.072	1.149	1.230	1.316	1.407	1.504	1.714	1.949	2.211
n = 8 →	1.083	**1.172**	1.267	1.369	1.477	1.594	1.851	2.144	2.476
n = 9 →	1.094	1.195	1.305	1.423	1.551	1.689	1.999	2.358	2.773
n = 10 →	1.105	1.219	1.344	1.480	1.629	1.791	2.159	2.594	3.106
n = 11 →	1.116	1.243	1.384	1.539	1.710	1.898	2.332	2.853	3.479
n = 12 →	1.127	1.268	1.426	1.601	1.796	2.012	2.518	3.138	3.896

n = the number of time periods in which the interest is compounded
i = the interest rate per period with the interest added and compounded at the end of each period

Outline

- Future Value and Compounding
- Present Value and Discounting
- More about PV and FV
  - The frequency of compounding
  - Inflation
- Capital Budgeting Decision Rule

More about PV and FV

- The frequency of compounding
  - You have a credit card that carries a rate of interest of **18% per year compounded monthly.** What is the **interest rate compounded annually**?
- Inflation
  - Real interest/discount rate vs. nominal interest/discount rate

The Frequency of Compounding

- **Annual Percentage Rate (APR)**
  - Nominal Annual Rate or Quoted Rate or Stated Rate
  - The nominal rate stated with a certain frequency of compounding
  - E.g. 18% per year compounded monthly **vs.** 20% per year compounded annually
  - APRs with different compounding frequency are not comparable
- **Effective Annual Rate (EAR** or EFF in the textbook**)**
  - The actual rate after taking into consideration any compounding that may occur *during* the year.
  - EARs are comparable

- If interest is compounded ***more than*** *once* a year, then $APR < EAR$.
- If interest is compounded ***exactly*** *once a year*, then $APR = EAR$.
- If you want to compare two alternative investments with different compounding periods, you need to compute the EAR and use that for comparison.

The Frequency of Compounding

- APR = period rate * the number of periods per year
- Period rate = APR / number of periods per year

  - What is the APR if the monthly rate is 0.5%?

    0.5% * (12) = 6%

  - What is the APR if the semiannual rate is 4%?

    4% * (2) = 8%

  - What is the monthly rate if the APR (based on the monthly rate) is 12%?

    12% / 12 = 1%

- Note that you should NEVER divide the effective rate by the number of periods per year. It will NOT give you the period rate

The Frequency of Compounding

- $EAR = \left[1 + \frac{APR}{m}\right]^m - 1$
  - m: compounding frequency per year
- Example: EAR for a nominal rate of 10% compounded semiannually?
  - $EAR = \left[1 + \frac{APR}{m}\right]^m - 1 = \left(1 + \frac{0.1}{2}\right)^2 - 1 = 10.25\%$
- Example: You have a credit card that carries a rate of interest of **18% per year compounded monthly.** What is the **interest rate compounded annually**?
  - The APR 18% per year compounded monthly
  - The (real) monthly rate is 18%/12 = 1.5%
  - The $EAR = (1 + 0.015)^{12} - 1 = 19.56\%$

The Frequency of Compounding

Annual Percentage rate	Frequency of Compounding	Annual Effective Rate
18	1	18.00
18	2	18.81
18	4	19.25
18	12	19.56
18	52	19.68
18	365	19.72

The Frequency of Compounding

- Note that as the frequency of compounding increases, so does the annual effective rate
- What occurs as the frequency of compounding rises to infinity?

$$EAR = \lim_{m \to \infty} \left(\left(1 + \frac{k}{m} \right)^m \right) - 1 = e^k - 1$$

  - e is the constant 2.71828 (rounded off to the fifth decimal place)

The Frequency of Compounding

Annual Percentage rate	Frequency of Compounding	Annual Effective Rate
18	1	18.00
18	2	18.81
18	4	19.25
18	12	19.56
18	52	19.68
18	365	19.72
18	Infinity	19.72

Outline

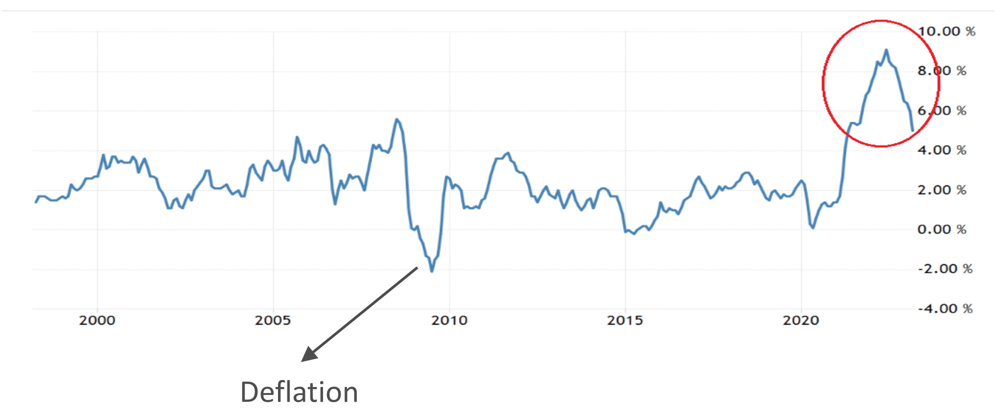
- Future Value and Compounding
- Present Value and Discounting
- **More about PV and FV**
  - The frequency of compounding
  - **Inflation**
- Capital Budgeting Decision Rule

Inflation and Real Interest Rates

- Money is only valuable if it can be used to buy things.
- **Nominal** rate of return: expressed in terms of currency (the actual money you receive)
  - With a $100 bond with an interest rate of 5%, you will receive $105 at maturity
- **Real** rate of return: expressed in terms of purchasing power
  - With a $100 bond with an interest rate of 5%, you will receive $105 at maturity
  - but the price of goods goes up by 5% (**Inflation rate**)
  - With $105 could buy things only worth $100 at the beginning. You could not buy more things.
  - Real rate of return is 0%, in terms of purchasing power.

Inflation

- Price increases -- Money depreciates
- Hyperinflation
  - Germany 1923: $1.5 \times 10^{56}\%$
  - Hungary 1946: $7.5 \times 10^{170}\%$
  - Zimbabwe 2008: $7.3 \times 10^{108}\%$
- US 2022



Nominal to Real

$$(1+NominalRate)=(1+RealRate)*(1+InflationRate)$$
$$\Rightarrow$$
$$RealRate=\frac{NominalRate-InflationRate}{1+InflationRate}$$

- Inflation rate: the change rate of price of a basket of consumption goods.
- Example:
  - With a \$100 bond with an interest rate of 5%, you will receive \$105 at maturity
  - The inflation rate is 8%, indicating that, a basket of good worth \$100 now, will be worth \$108 in one year
  - For every basket you give up now, you can only buy \$105/\$108=0.9722 basket
  - Real rate of return = (0.9722-1)/1*100%=-2.78%
- An easy approximation: $RealRate = NominalRate - InflationRate$ (Not used in this course unless stated explicitly)

Inflation and Future Value

- A common planning situation is determining how long it takes to save for something, but the problem is that the thing being saved for increases in (nominal) price due to inflation
- Example: If you save \$100 at age 20 and do not take out until age 65, given the nominal interest rate of 8% per year, how much will you have?
  - Nominal FV in 45 years $= \$100 \times 1.08^{45} = \$3{,}192$
- But what matters is how much you will have in terms of purchasing power, given inflation rate of 5% per year
  - Price level in 45 years $= 1.05^{45} = 8.985$
  - Real FV in 45 years $= \frac{Nominal\ FV}{Future\ price\ level} = \frac{\$3{,}192}{8.985} = \$355$
- You can also compute the real interest rate first, with which to compute the real future value
  - Real interest rate $= \frac{Nominal\ interest\ rate - Inflation\ rate}{1 + Inflation\ rate} = \frac{0.08-0.05}{1+0.05} = 0.02875$
  - Real FV in 45 years $= \$100 \times 1.02875^{45} = 355$

Inflation and Present Value

- Example: You plan to buy a car four years from now and want to save enough money. The care costs \$10,000 now and the interest rate is 8% per year. How much will you save now
  - Nominal PV $= \frac{\$10{,}000}{1.08^4} = \$7{,}350$
- What if the car costs more than \$10,000 due to inflation of 5% per year?
  - Car cost in 4 years$= \$10{,}000 \times 1.05^4 = \$12{,}155$
  - Real PV $= \frac{\$12{,}155}{1.08^4} = \$8{,}943$
- You can also compute the real interest rate first, with which to compute the real present value
  - Real interest rate $= \frac{Nominal\ interest\ rate - Inflation\ rate}{1 + Inflation\ rate} = \frac{0.08 - 0.05}{1 + 0.05} = 0.02875$
  - Real PV $= \$10{,}000/1.02875^4 = \$8{,}943$

Some exercises

Problem 1

- You've received a $40,000 legal settlement. Your great-uncle recommends investing it for retirement in 27-years by “rolling over” one-year certificates of deposit (CDs)
- Your local bank has 3% 1-year CDs
- How much will your investment be worth?

Data Extraction

- PV = $40,000
- i = 3%
- n = 27-years
- FV = ?

Solution

$$
\begin{aligned}
F &= P(1+i)^n \\
&= 40{,}000(1+0.03)^{27} \\
&= \$88{,}851.56
\end{aligned}
$$

Problem 2

- If you have five years to increase your money from $3,287 to $4,583, at what interest rate should you invest?

Solution

$$F = P(1+i)^n \Leftrightarrow i = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1$$

$$i = \left(\frac{4583}{3287}\right)^{\frac{1}{5}} - 1 = 0.0687 = 6.87\%$$

Problem 3

- How many years would it take for an investment of \$9,284 to grow to \$22,450 if the interest rate is 7% p.a. ?
- p.a. = per annum = per year

Data Extraction

- PV = $9,284
- FV = $22,450
- i = 7% p.a.
- n = ?

Solution by Equation

$$FV = PV\left(1+i\right)^{n} \Leftrightarrow n = \frac{\ln\left(\dfrac{FV}{PV}\right)}{\ln\left(1+i\right)}$$

$$n = \frac{\ln\left(\dfrac{22,450}{9,284}\right)}{\ln\left(1+0.07\right)} = 13.05 years$$

Problem 4

- If investment rates are 1% per month, and you have an investment that will produce $6,000 one hundred months from now, how much is your investment worth today?

Data Extraction

- FV = $6000
- PV = ?
- n = 100 months
- i = 1%

Solution by Equation

$$FV = PV(1+i)^n \Leftrightarrow PV = \frac{FV}{(1+i)^n} = FV*(1+i)^{-n}$$

$$PV = \frac{6,000}{(1+0.01)^{100}} = \$2,218.27$$

Problem 5

- You have been offered a video business, and estimate that video rental technology will be obsolete in 8 years when cable bandwidths and video compression will permit “movie-on-demand.”
- The cash flows, starting 1-year from now, are: 90, 110, 140, 140, 130, 90, 70, 30 (thousands of $s)
- Would you accept the offer if it costs $500 now?

Solution

- The present value is $554.97
- The initial investment is $500
- A good investment

year_	Flow_	Discounted
0	$0.00	$0.00
1	$90.00	$81.82
2	$110.00	$90.91
3	$140.00	$105.18
4	$140.00	$95.62
5	$130.00	$80.72
6	$90.00	$50.80
7	$70.00	$35.92
8	$30.00	$14.00
Sum		**$554.97**

Outline

- Future Value and Compounding
- Present Value and Discounting
- More about PV and FV
- Capital Budgeting Decision Rule
  - Net Present Value Rule
  - The Payback Rule
  - The Internal Rate of Return

Capital Budgeting Decision Rules

- An investment is worth undertaking if it creates value for its owners
- Capital budgeting: Determine whether a proposed investment or project will be worth more, once it is in place, than it costs
  - What services will we offer or what will we sell?
  - In what markets will we compete?
  - What new products will we introduce?
- We need to ask ourselves the following questions when evaluating capital budgeting decision rules
  - Does the decision rule adjust for the **time value of money**?
  - Does the decision rule adjust for **risk**?
  - Does the decision rule provide information on whether we are **creating value for the firm**?

Rule 1: Net Present Value (NPV)

- The difference between an investment's present value of all future cash inflows minus the present value of all current and future cash outflows

NPV=PV of all cash inflows – PV of all cash outflows

- Capital budgeting process can be viewed as a search for investments with **positive net present values**
  - Accept any project with a present value of future cash flows that exceed the initial investment
  - An investment should be accepted if the net present value is positive and rejected if it is negative.

Rule 1: Net Present Value (NPV)

- **Discounted cash flow (DCF) valuation:** How much value is created from undertaking an investment?
  1. Estimate **future cash flows** we expect the business to produce
  2. Estimate the **required return** for projects of this risk level
  3. Apply basic discounted cash flow procedure to estimate the **PV of those cash flows**
  4. Estimate **NPV as the difference between the PV of the future cash flows and the cost of the investment**
- The task of coming up with the cash flows and the discount rate is much more challenging than the calculations themselves
- We use **the opportunity cost of capital** (also called the **market capitalization rate**) as the discount rate
  - The rate that we could earn somewhere else if we did not invest in the project under evaluation.
  - E.g., the interest rate of savings in the bank

Example: NPV rule

Using the NPV Rule

Suppose we are asked to decide whether a new consumer product should be launched. Based on projected sales and costs, we expect that the cash flows over the five-year life of the project will be \$2,000 in the first two years, \$4,000 in the next two, and \$5,000 in the last year. It will cost about \$10,000 to begin production. We use a 10 percent discount rate to evaluate new products. What should we do here?

Given the cash flows and discount rate, we can calculate the total value of the product by discounting the cash flows back to the present:

$$
\begin{aligned}
\text{Present value} &= \$2{,}000/1.1 + \$2{,}000/1.1^2 + \$4{,}000/1.1^3 \\
&\quad + \$4{,}000/1.1^4 + \$5{,}000/1.1^5 \\
&= \$1{,}818 + 1{,}653 + 3{,}005 + 2{,}732 + 3{,}105 \\
&= \$12{,}313
\end{aligned}
$$

The present value of the expected cash flows is \$12,313, but the cost of getting those cash flows is only \$10,000, so the NPV is \$12,313 – 10,000 = \$2,313. This is positive; so, based on the net present value rule, we should take on the project.

Rule 1: Net Present Value (NPV)

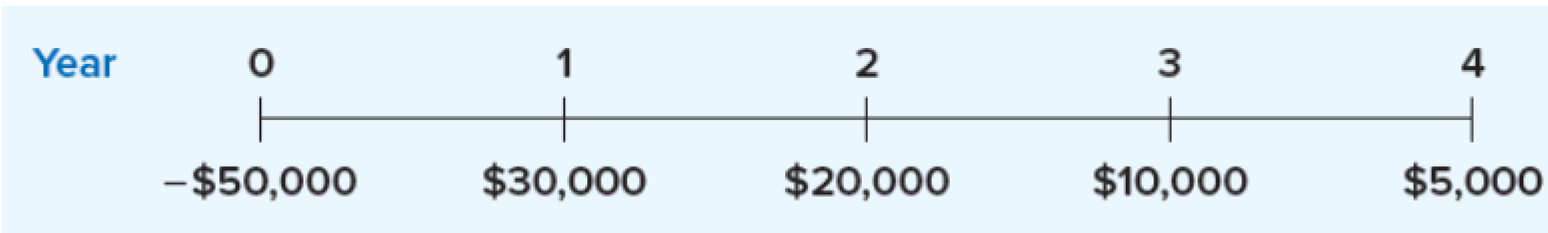
- **The NPV rule is the primary decision rule**
  - Accounts for time value of money: discounting
  - Accounts for risk of the cash flow: discount rate
  - Indicates the increase in value

Rule 2: the Payback Rule

- **Payback period**: the amount of time required for an investment to generate cash flows sufficient to recover its initial cost
  - Estimate the cash flows
  - Subtract the future cash flows from the initial cost until the initial investment has been recovered
- An investment is acceptable if its calculated payback period is less than some prespecified number of years.

Example: the Payback Rule

- Below, find the cash flows from a proposed investment. How many years do we have to wait until the accumulated cash flows from this investment equal or exceed the cost of the investment?



- Initial investment is $50,000
- After the first year, the firm has recovered $30,000, leaving $20,000.
- Cash flow in the second year is exactly $20,000, so payback period for this investment is **exactly two years**

Example: Payback period

- Figures shown as Year 0 cash flows are costs of the investments
- What are payback periods for each project?

Year	A	B	C	D	E
0	–$100	–$200	–$200	–$200	–$ 50
1	30	40	40	100	100
2	40	20	20	100	–50,000,000
3	50	10	10	–200	
4	60		130	200	

Example: Payback period

- Payback for project A is easily calculated as 2.6 years
- Project B never pays back
- Project C has a payback of exactly 4 years
- Project D has two different payback periods ± 2 years and 4 years ± both of which are correct
- Project E pays back in 6 months

Year	A	B	C	D	E
0	–$100	–$200	–$200	–$200	–$ 50
1	30	40	40	100	100
2	40	20	20	100	–50,000,000
3	50	10	10	–200	
4	60		130	200	

Comparing NPV rule to Payback rule

- When compared to the NPV rule, the payback period rule has some rather severe shortcomings:
    1. Time value of money is completely ignored
    2. Payback rule fails to consider any risk differences
    3. No economic rationale for looking at payback in the first place, so we have no guide for how to pick the cut-off

Comparing NPV rule to Payback rule

- Consider the two investments, Long and Short below. Both projects cost $250.

Year	Long	Short
0	–$250	–$250
1	100	100
2	100	200
3	100	0
4	100	0

- **What is the payback period on the two projects?**
  - Long: 2 + ($50/$100) = 2.5 years
  - Short: 1 + ($150/$200) = 1.75 years
- **Is the payback period rule guiding us to the right decisions?**
  - The payback period rule dictates that Short is acceptable and Long is not
- **Suppose we require a 15% return on this type of investment**
  - NPV (Short) = - $250 + $100/1.15 + $200/1 .15^2 = -$11.81
  - NPV (Long) = - $250 + $100/(1.15 + 1 .15^2+1.15^3+1.15^4) = $35.50
  - The NPV of Short is negative, while the NPV for Long is positive
  - Using a payback period rule will tend to bias us toward shorter-term investments

Rule 2: the Payback Rule

- Biggest drawback to the payback period rule is that it doesn't ask the right question
  - Relevant issue is the impact an investment will have on the value of the stock, not how long it takes to recover the initial investment
- But because it is so simple, companies often use it as a screen for investment decisions.

Advantages and Disadvantages of the Payback Period Rule	
Advantages	**Disadvantages**
1. Easy to understand. 2. Adjusts for uncertainty of later cash flows. 3. Biased toward liquidity.	1. Ignores the time value of money. 2. Requires an arbitrary cutoff point. 3. Ignores cash flows beyond the cutoff date. 4. Biased against long-term projects, such as research and development, and new projects.

Rule 2.1: the Discounted Payback Rule

- **Discounted payback:** the length of time required for an investment's discounted cash flows to equal its initial cost
  - Figure out n, given PV, FV, and i.
- An investment is acceptable if its discounted payback is less than some prespecified number of years
  - If a project ever pays back on a discounted basis, then it must have a positive NPV

Rule 2.1: the Discounted Payback Rule

- **Example:**
- Suppose we require a 12.5% return on new investments. We have an investment that costs $300 and has cash flows of $100 per year for five years. To get the discounted payback, we have to discount each cash flow at 12.5% and then start adding them.

	Cash Flow		Accumulated Cash Flow	
Year	Undiscounted	Discounted	Undiscounted	Discounted
1	$100	$89	$100	$ 89
2	100	79	200	168
3	100	70	300	238
4	100	62	400	301
5	100	55	500	356

Rule 2.1: the Discounted Payback Rule

- All things considered, the discounted payback is a compromise between a regular payback and NPV, lacking simplicity of the first and conceptual rigor of the second
  - Rarely used in practice

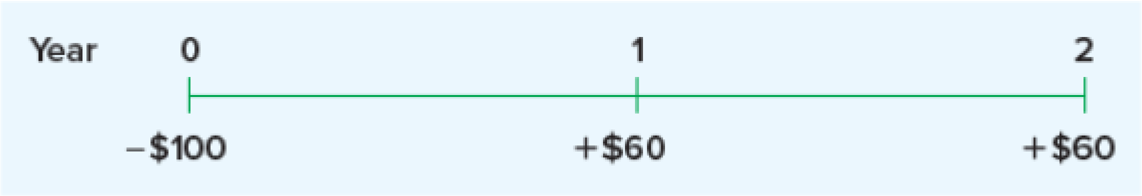
Advantages and Disadvantages of the Discounted Payback Period Rule	
Advantages	**Disadvantages**
1. Includes time value of money.	1. May reject positive NPV investments.
2. Easy to understand.	2. Requires an arbitrary cutoff point.
3. Does not accept negative estimated NPV investments.	3. Ignores cash flows beyond the cutoff date.
4. Biased toward liquidity.	4. Biased against long-term investments, such as research and development, and new projects.

Rule 3: The Internal Rate of Return Rule

- **Internal rate of return (IRR):** the discount rate that makes the NPV of an investment zero
  - Figure out i, given PV, FV, and n.
  - Determine single rate of return summarizing the merits of a project
  - Often used in practice and intuitively appealing (a percentage return)
  - Most important alternative to NPV
- An investment is acceptable if the IRR exceeds the required return (e.g., the opportunity cost of capital) and rejected otherwise.
  - If IRR is higher than the opportunity cost of capital, then the NPV at the opportunity cost of capital itself must be positive.

Rule 3: The Internal Rate of Return Rule

- Example
- Suppose you were now looking at an investment that costs \$100 and has a cash flow of \$60 per year for two years. What is the return on this investment?



- Set the NPV equal to zero and solve for the discount rate:
  – NPV = 0 = -\$100 + \$60/(1 + IRR) + \$60/(1 + IRR)^2
- In multi-period case, the only way to find IRR in general is by trial and error, either by hand or by calculator
  – At 0% rate, NPV = \$120-100 = \$20
  – At 10% discount rate, NPV = -\$100 + \$60/1.1 + \$60/1.1^2 = \$4.13
  – At 15% discount rate, …

Rule 3: The Internal Rate of Return Rule

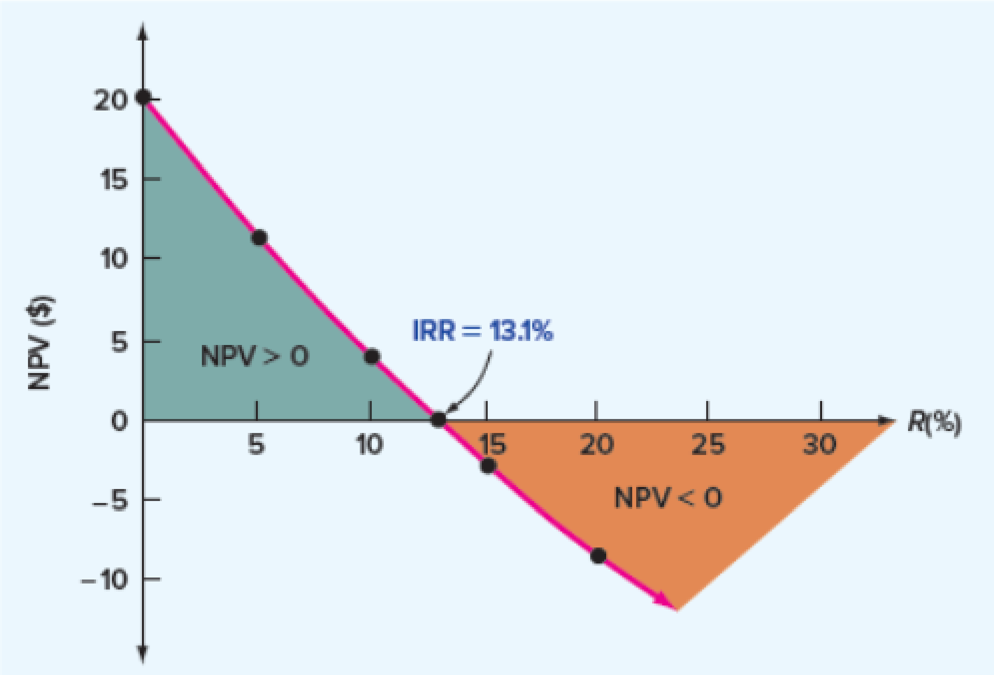
- NPV appears to be zero with a discount rate between 10% and 15%, so the IRR is somewhere in that range

Discount Rate	NPV
0%	$20.00
5	11.56
10	4.13
15	– 2.46
20	– 8.33

- IRR is about 13.1%
- If our required return were less than 13.1%, we would take this investment, but if required return exceeded 13.1%, we would reject it

IRR and NPV

- IRR rule and NPV rule are identical if
  - Cash flows are non-negative
  - Projects are independent



IRR and NPV

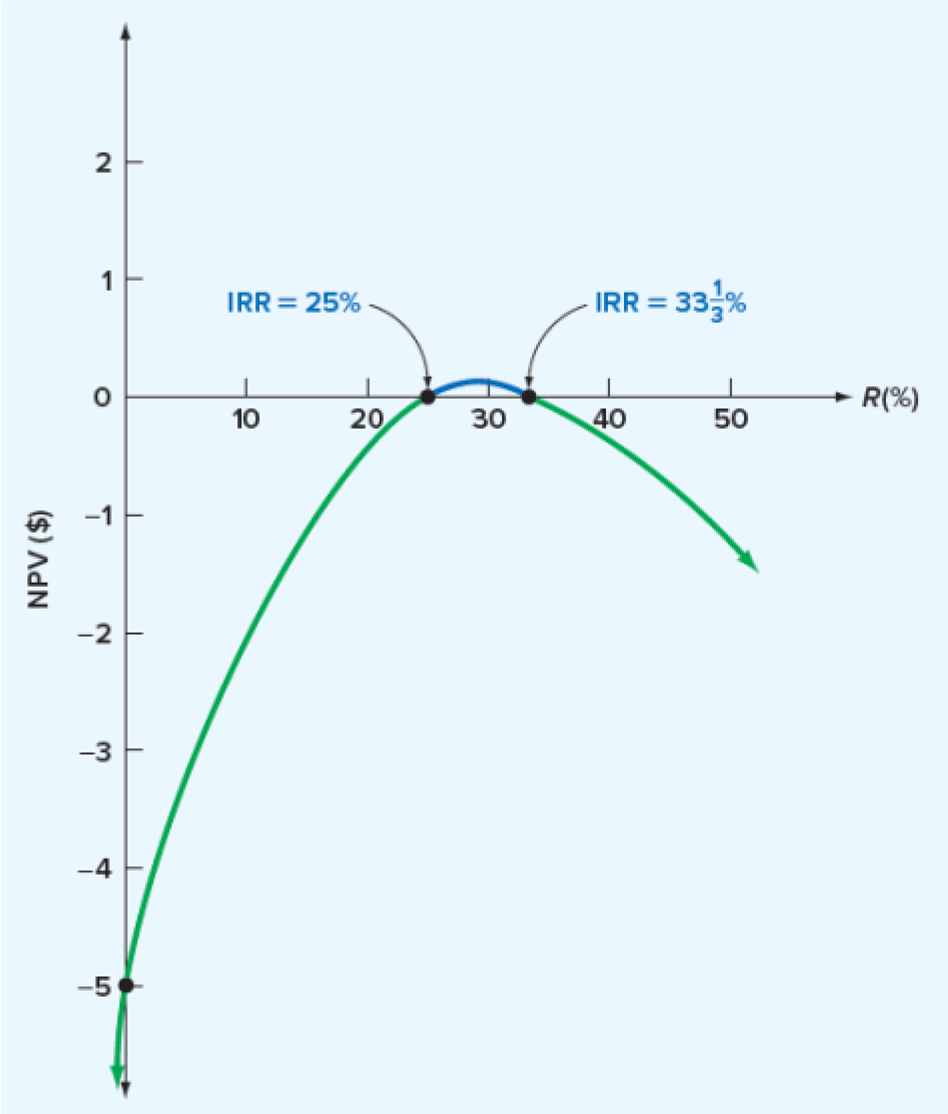
- Negative cash flows
- Suppose we have a strip-mining project that requires a $60 investment. Our cash flow in the first year will be $155. In the second year, the mine will be depleted, but we will have to spend $100 to restore the terrain.

- To find the IRR on this project, calculate the NPV at various rates:

Discount Rate	NPV
0%	−$5.00
10	− 1.74
20	− .28
30	.06
40	− .31

IRR and NPV

- Notice the NPV is zero when the discount rate is 25%, as well as when it is 33.33%
- We should take this investment only if our required rate of return is between 25% and 33.33%



IRR and NPV

- Mutually Exclusive Investments: choose from several projects
  - Two projects that are not mutually exclusive are independent
  - You need a rank.
- Given two or more mutually exclusive investments, which one is the best? Can we also say that the best one has the highest return?
  - The best one is the one with the largest NPV
  - No, we cannot say the best one has the highest return
- Consider the following cash flows from two mutually exclusive investments, where the IRR for A is 24% and the IRR for B is 21%:

Year	Investment A	Investment B
0	–$100	–$100
1	50	20
2	40	40
3	40	50
4	30	60

IRR and NPV

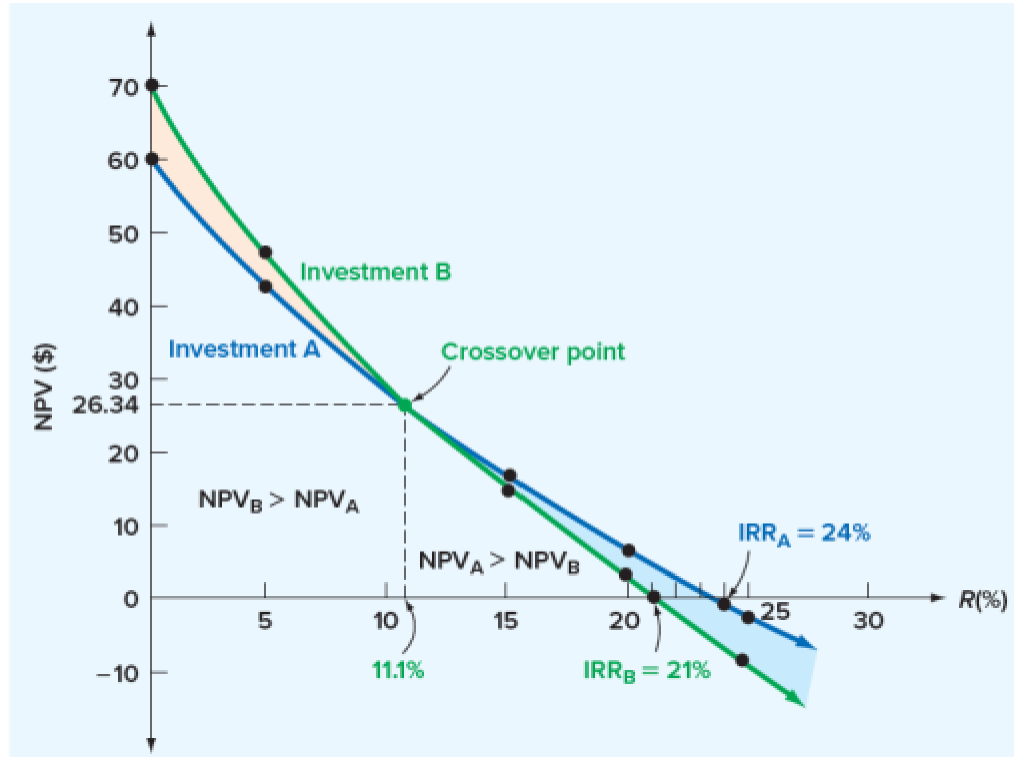
- The NPV:

Discount Rate	NPV(A)	NPV(B)
0%	$60.00	$70.00
5	43.13	47.88
10	29.06	29.79
15	17.18	14.82
20	7.06	2.31
25	− 1.63	− 8.22

- IRR for A (24%) is larger than the IRR for B (21%), but in comparing the NPVs, the higher NPV investment depends on our required return

IRR and NPV

- If our required return is 10%, B has the higher NPV, but if our required return is 15%, A has the higher NPV
- When we have mutually exclusive projects, we should rank them based on their returns



Rule 3: The Internal Rate of Return Rule

- Despite its flaws, the IRR is very popular in practice more so than even the NPV
  - Prefer rates of return rather than dollar values
  - IRR may have a practical advantage over the NPV: can estimate the IRR if we do not know the appropriate discount rate

Advantages and Disadvantages of the Internal Rate of Return	
Advantages	**Disadvantages**
1. Closely related to NPV, often leading to identical decisions.	1. May result in multiple answers or not deal with nonconventional cash flows.
2. Easy to understand and communicate.	2. May lead to incorrect decisions in comparisons of mutually exclusive investments.

To summarize

- Future Value and Compounding
- Present Value and Discounting
- More about PV and FV
  - The frequency of compounding
  - Inflation rate
- Capital Budgeting Decision Rule
  - Net Present Value Rule
  - The Payback Rule
  - The Internal Rate of Return

Reference: Chapter 3 Allocating resources over time in BMC testbook.

- Next lecture (May 13)
  - Corporate finance