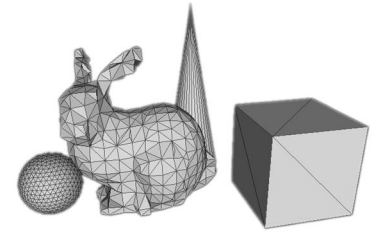


# Introduction to Computer Graphics

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## Ray Tracing 光线跟踪

# 第七章 光线跟踪

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光线跟踪基本原理

Whitted-Style 光线跟踪

光线跟踪实现细节

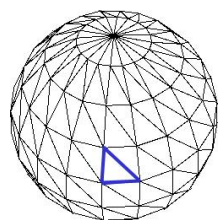
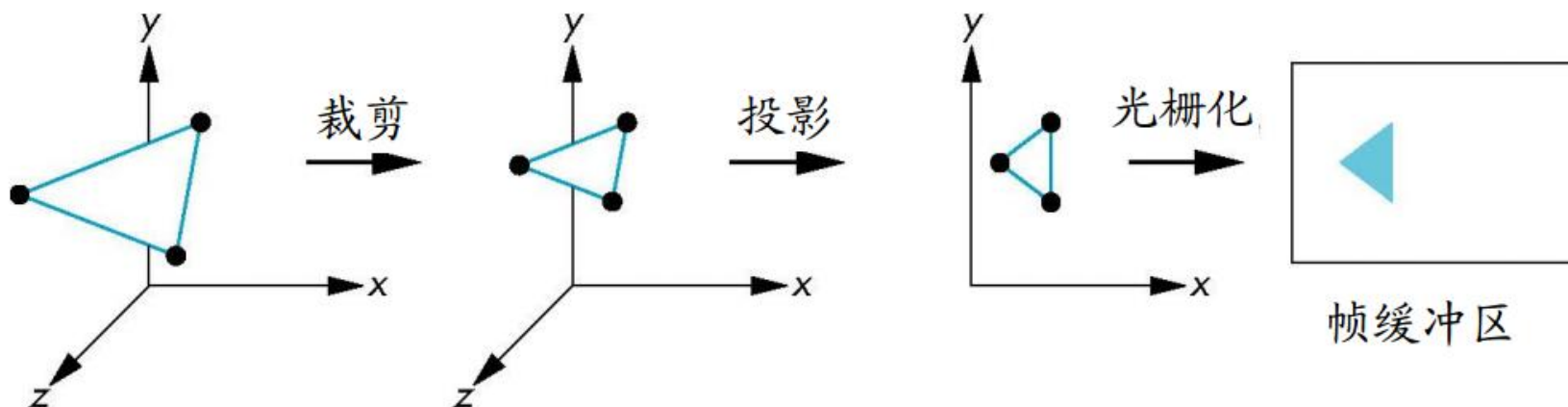
光线跟踪进阶话题

# 两类绘制方式

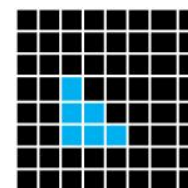
- 对于每个对象，确定它所覆盖的像素，并用对象的状态确定像素的明暗值

- 流水线方法
- 必须跟踪深度值

```
for (each_object)  
    render(object)
```



(triangle rendered to screen)

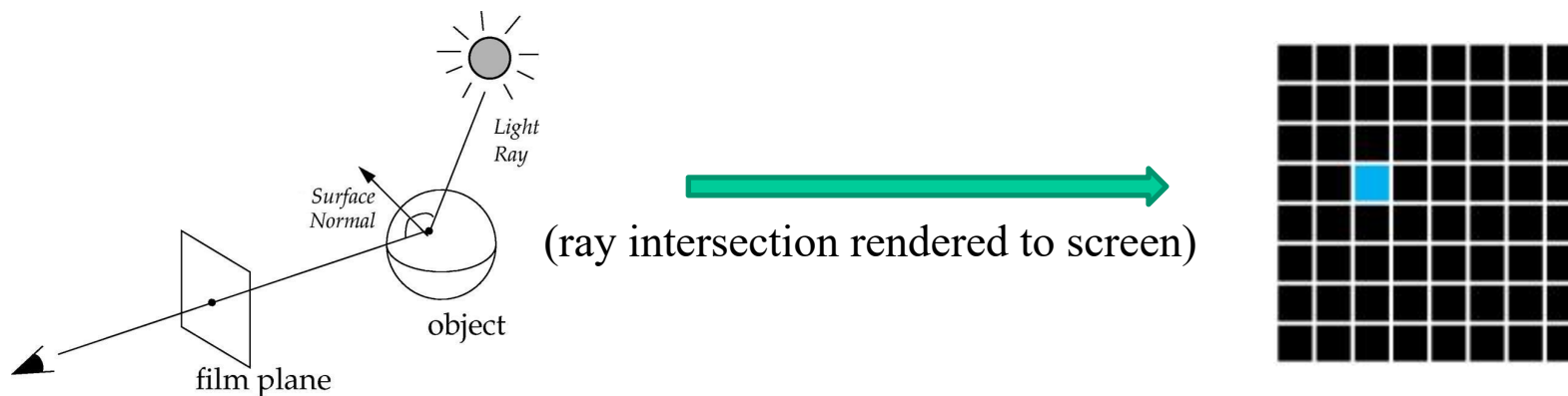


# 两类绘制方式

- 对于每个像素，确定投影到这个像素的离观察者最近的那个对象，从而基于该对象计算像素的明暗值

- 光线跟踪框架

```
for (each_pixel)  
    assign_a_color(pixel)
```



# 两类绘制方式

---

## Rasterization

```
for (each triangle)
  for (each pixel)
    if (triangle covers pixel)
      keep closest hit
```

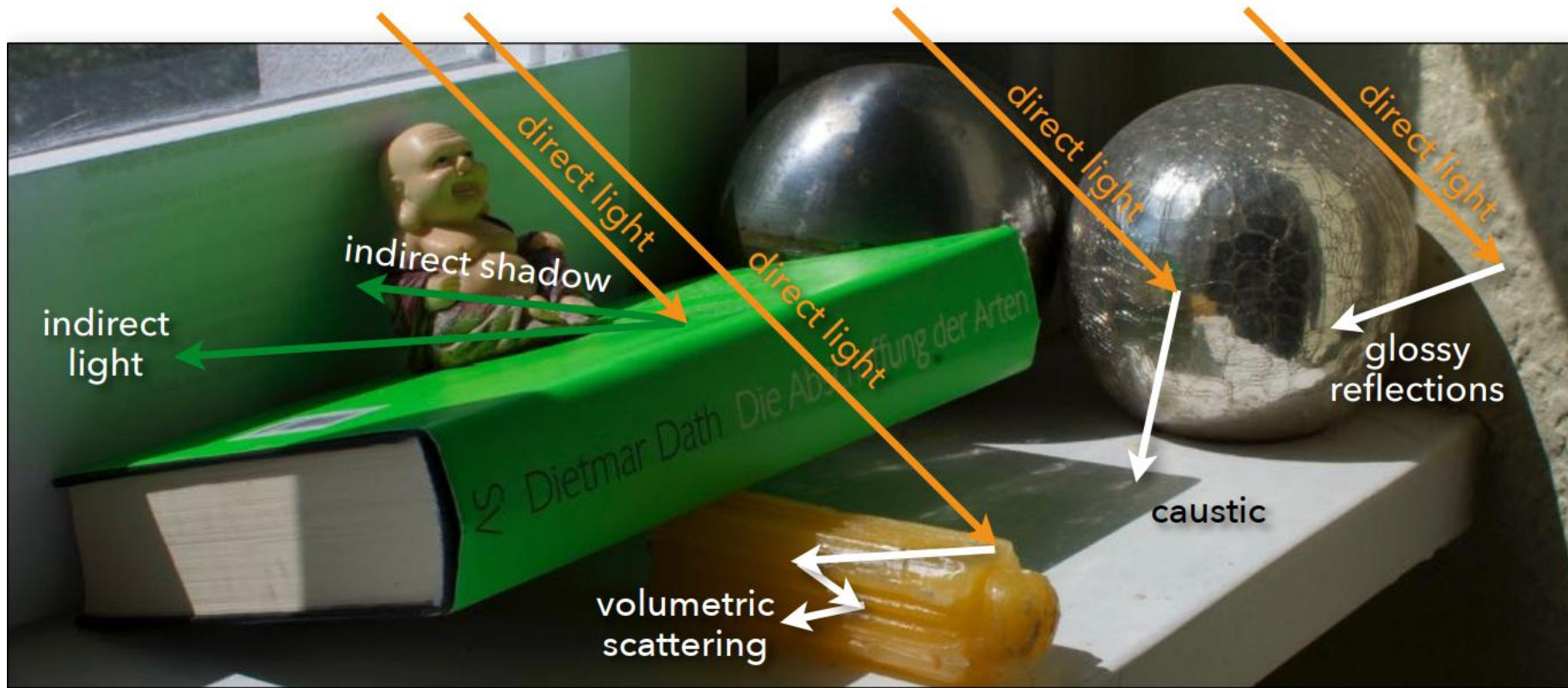
**Triangle-centric**

## Ray tracing

```
for (each pixel or ray)
  for (each triangle)
    if (ray hits triangle)
      keep closest hit
```

**Ray-centric**

# 真实世界中的光线传播

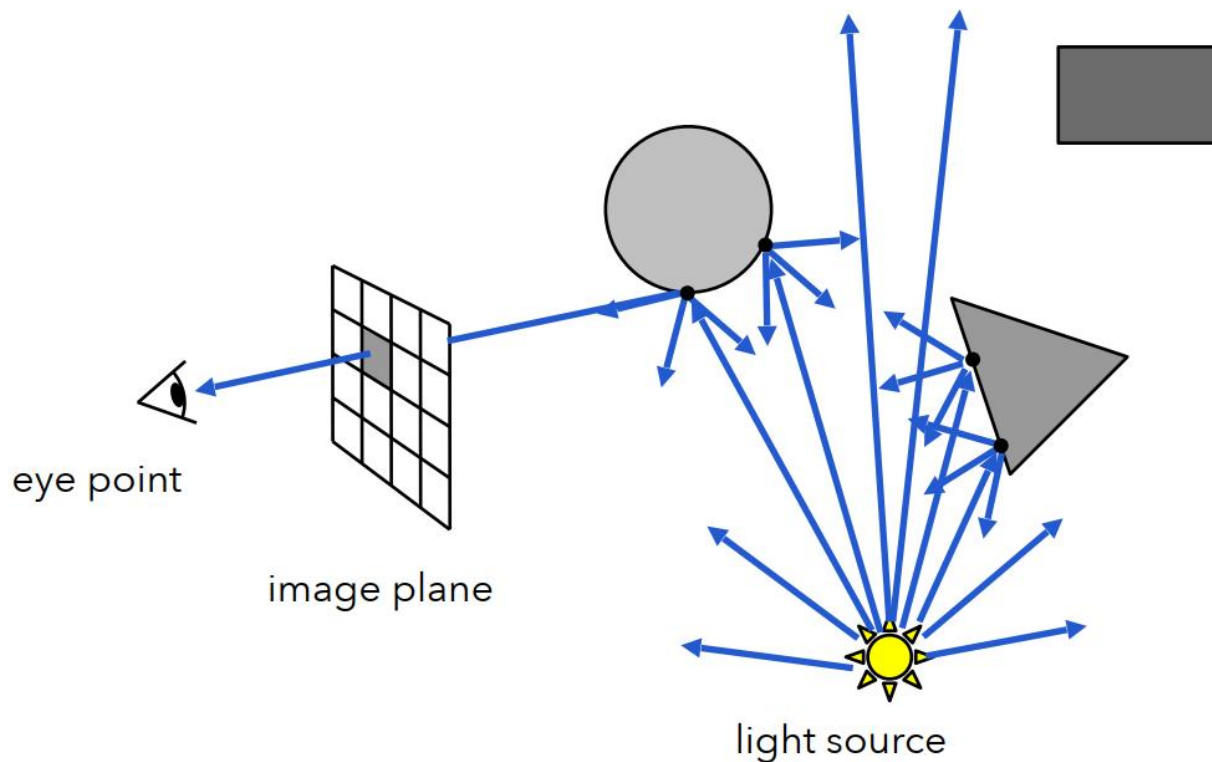


After [Ritschel et al 2011]

# 光线跟踪基本原理

- 光线跟踪思路

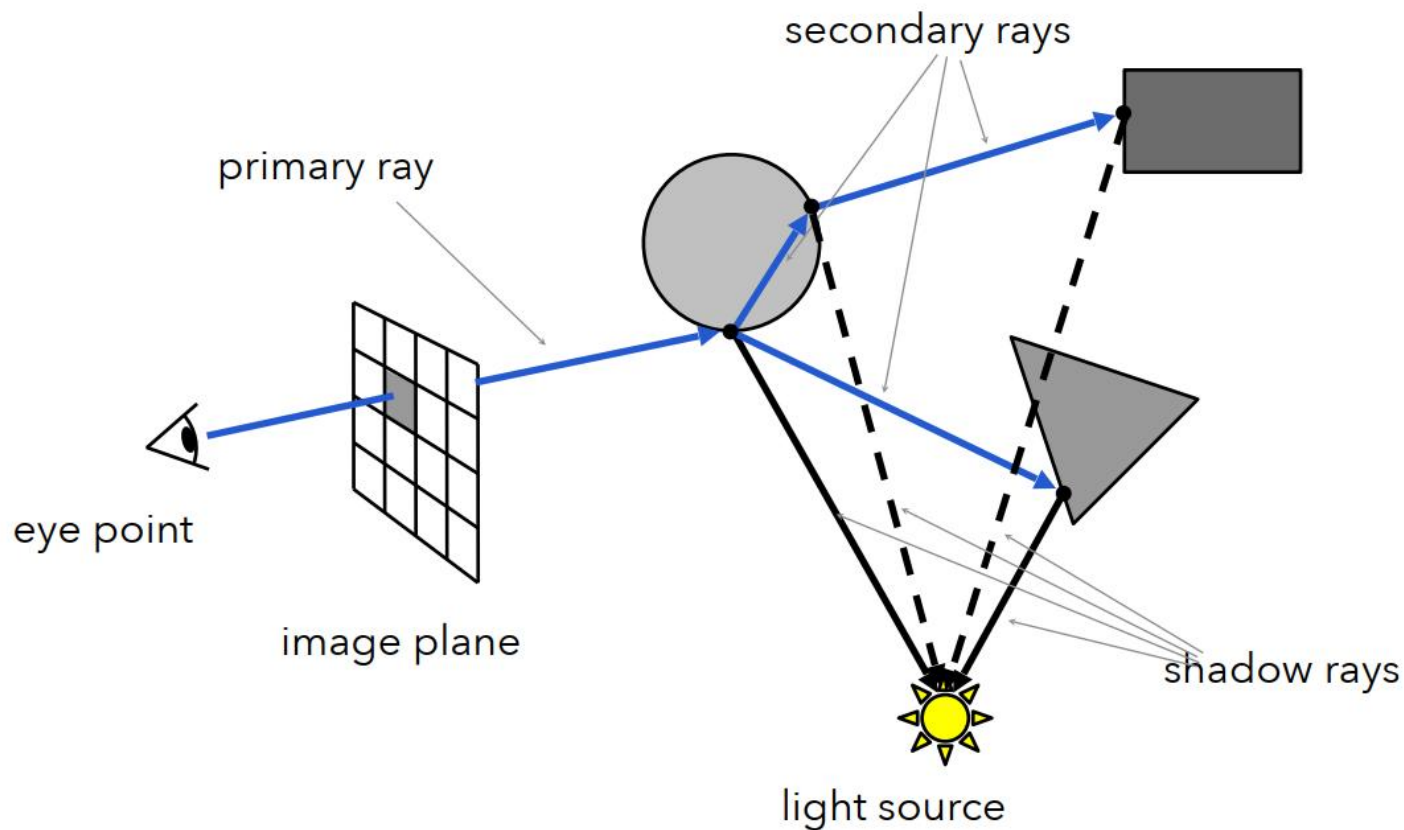
## “Forward” Raytracing



# 光线跟踪基本原理

- 光线跟踪思路

## "Backward" Raytracing





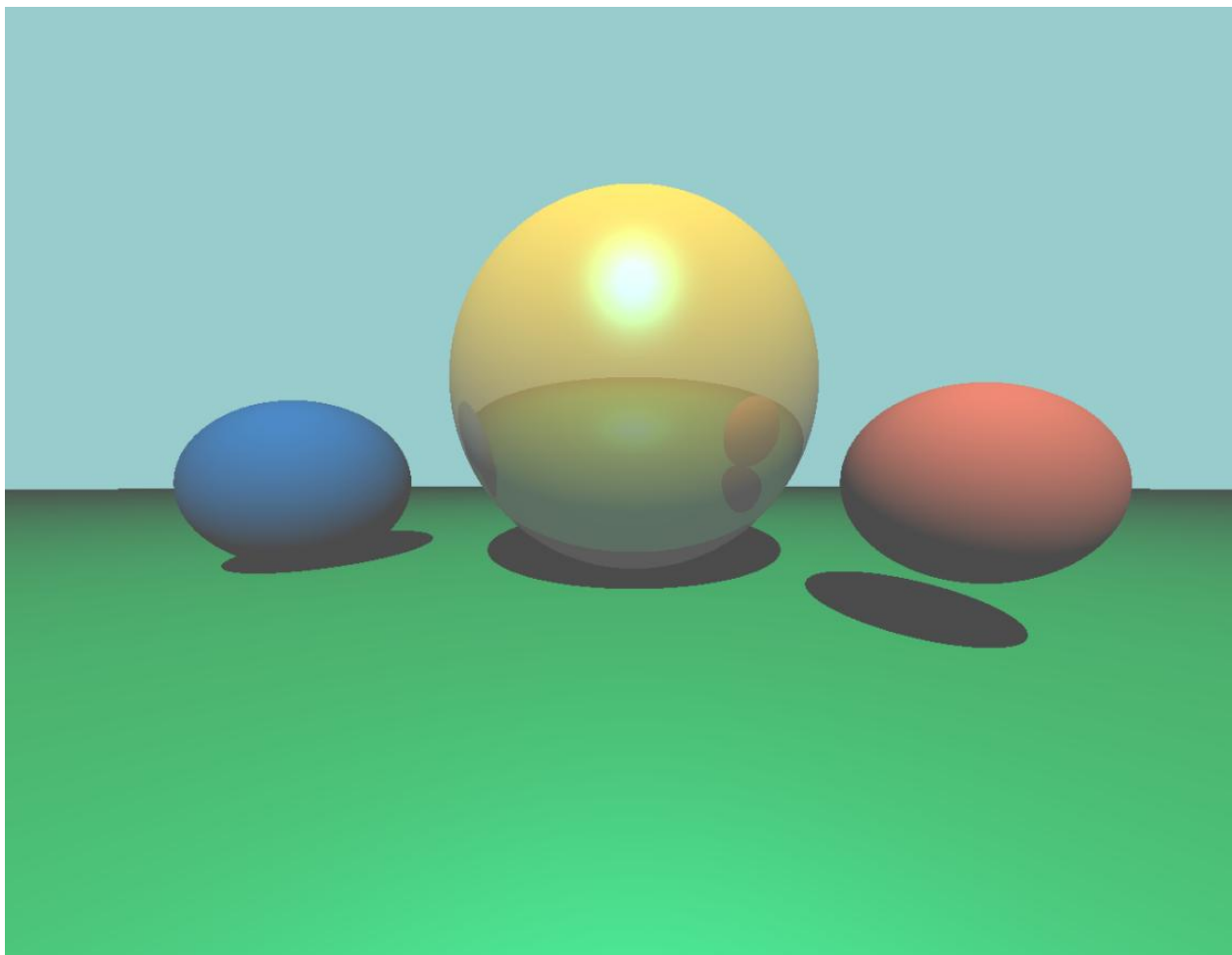
# 光线跟踪效果

- Whitted 1980



# 光线跟踪效果

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# 光线跟踪效果

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- PBRT



# 光线跟踪效果

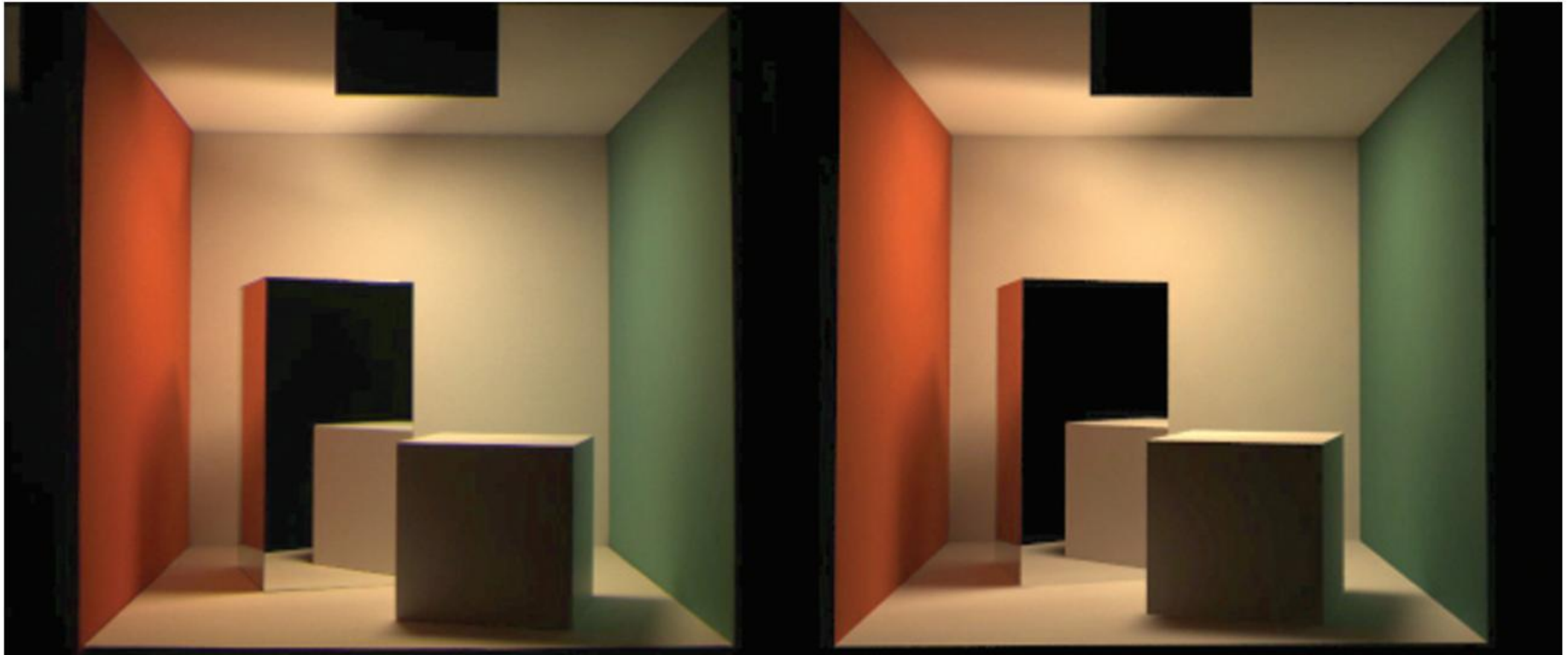
---

- PBRT



# 光线跟踪效果

- 渲染？照片？

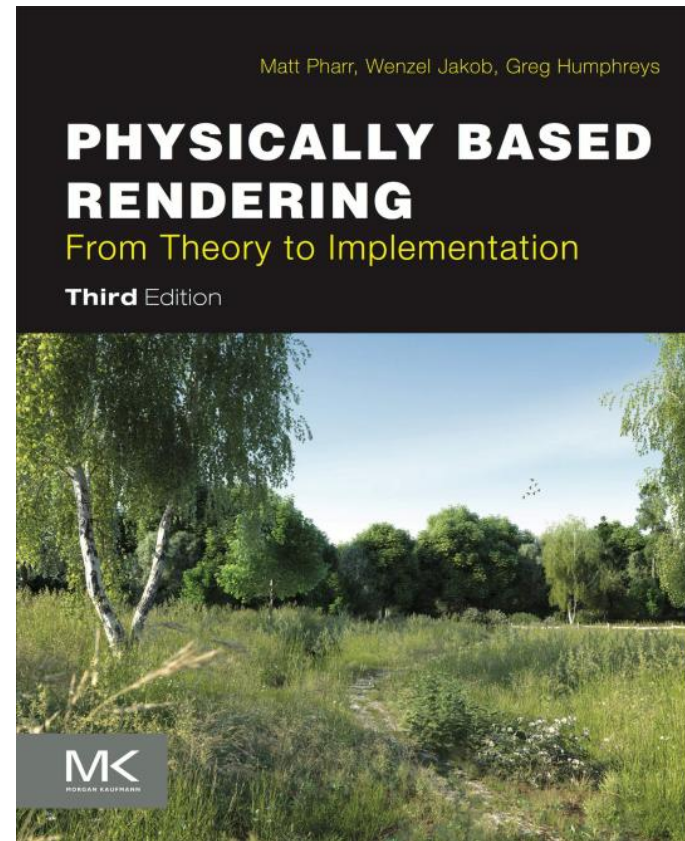
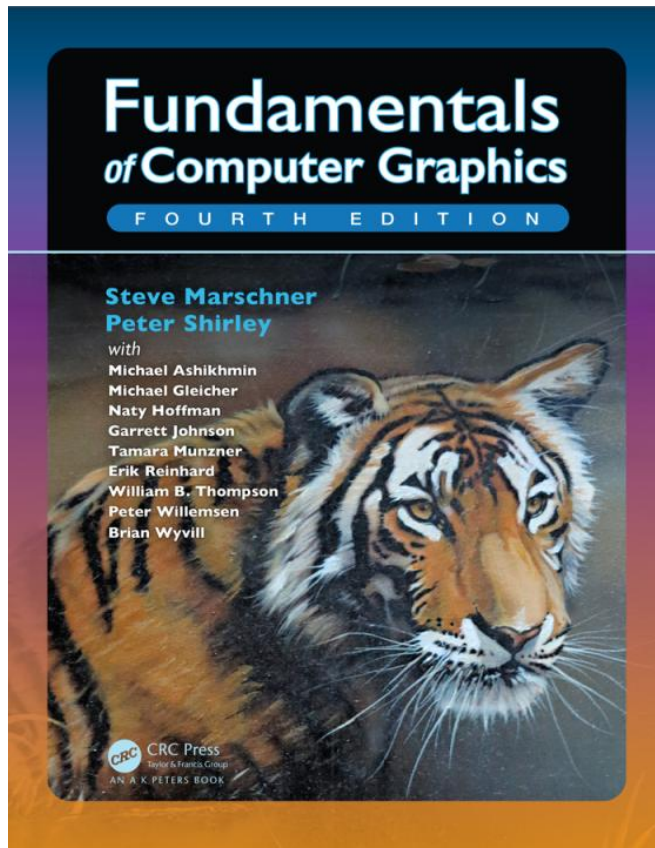


*Image courtesy Sumant Pattanaik and the Cornell Program of Computer Graphics.*



# 光线跟踪参考资料

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# 光线跟踪算法

- 光线跟踪算法框架
  - Viewing (Primary) Ray
  - Secondary Ray
  - Shadow Ray

**for** each pixel **do**

    compute viewing ray

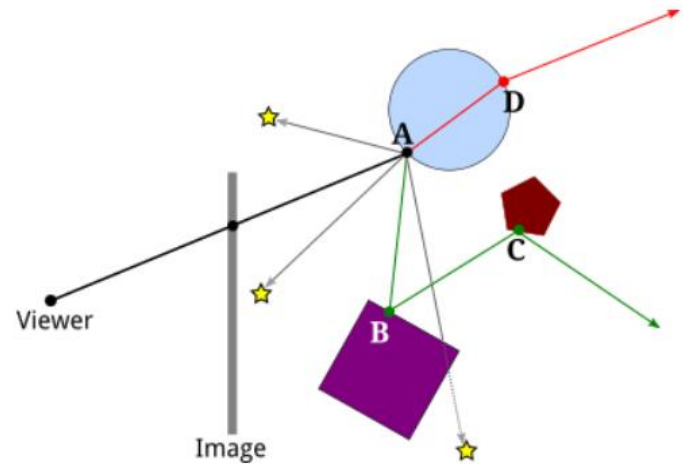
**if** (ray hits an object with  $t \in [0, \infty)$ ) **then**

        Compute **n**

        Evaluate shading model and set pixel to that color

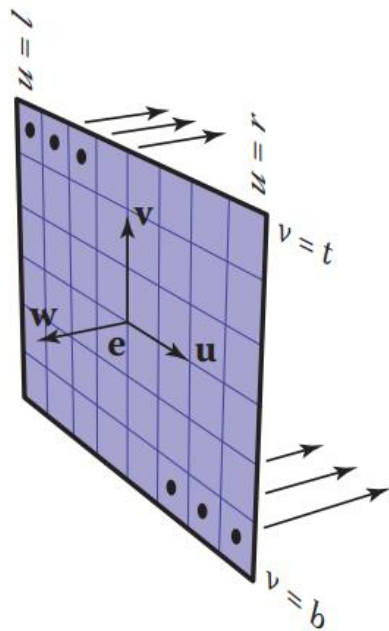
**else**

        set pixel color to background color

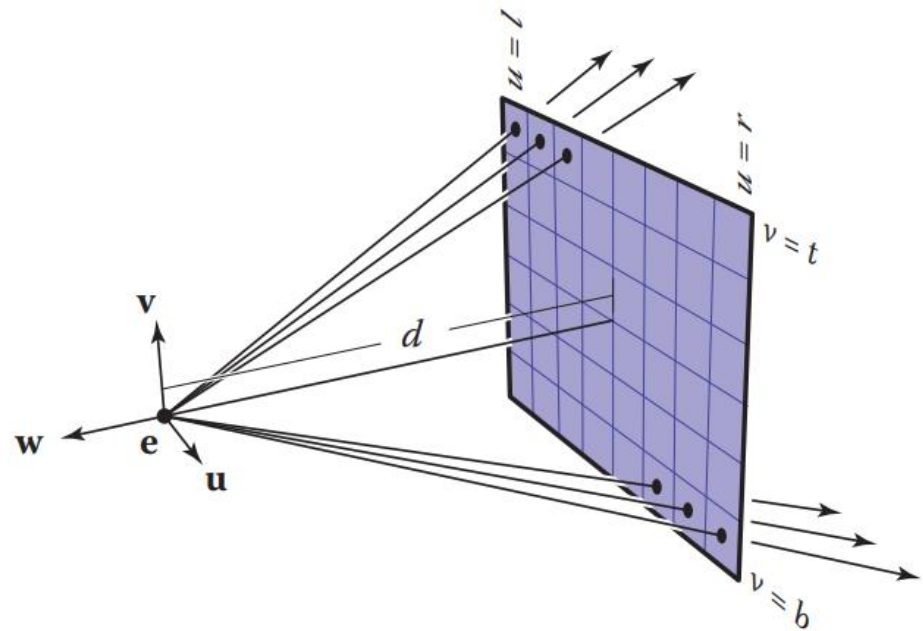


# 光线的生成

- 给定相机中心，成像平面左下角，成像平面竖直方向，成像平面水平方向



**Parallel projection**  
same direction, different origins

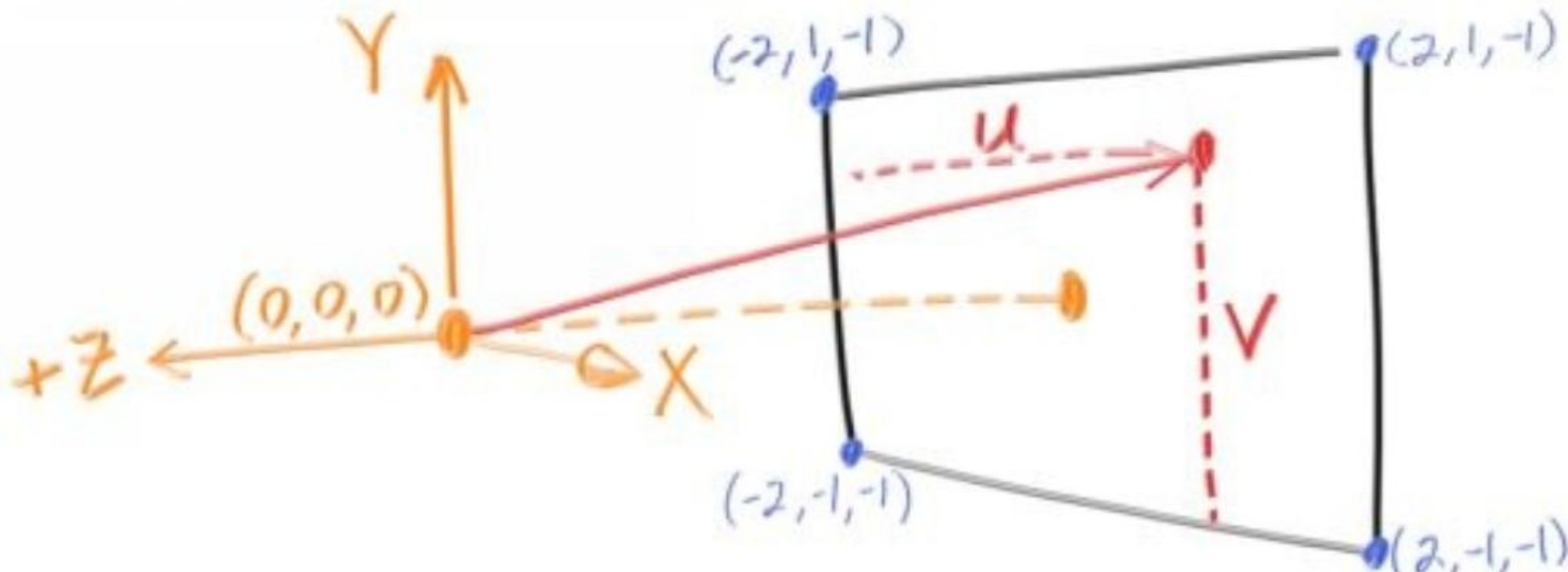


**Perspective projection**  
same origin, different directions

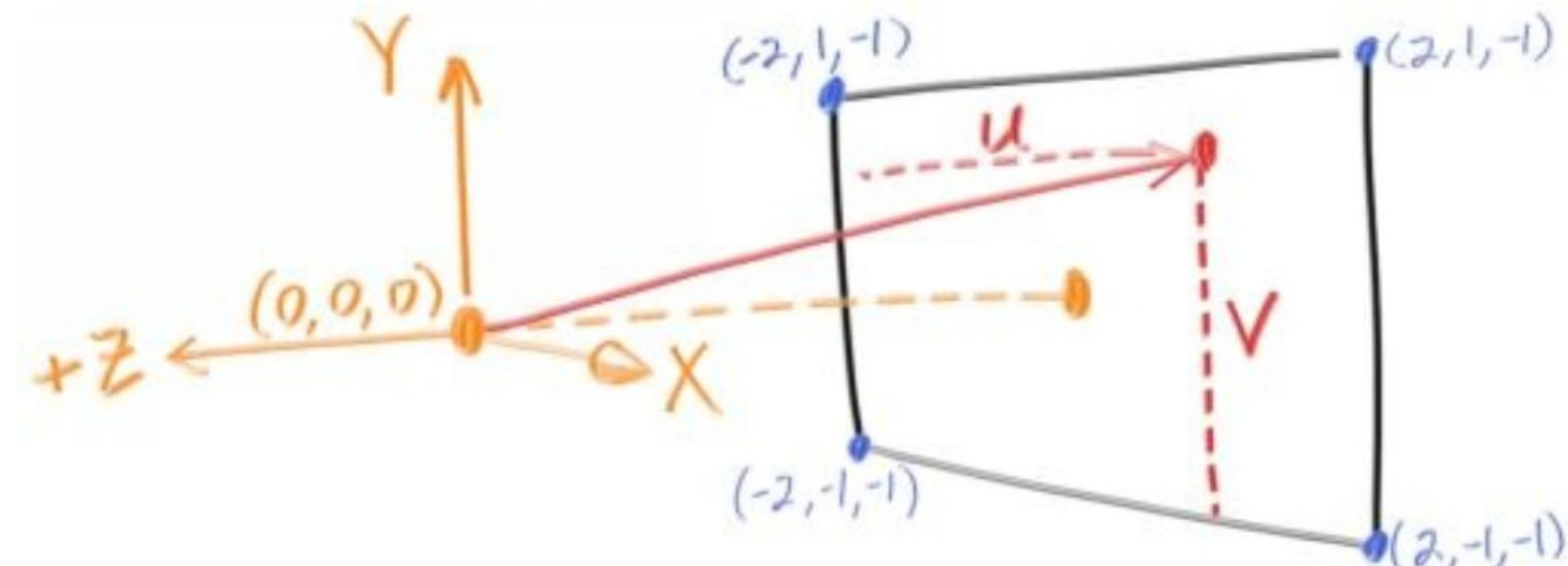


# 光线的生成

- 如何与像素坐标联系起来？



# 光线的生成



# 光线与场景求交

---

- 光线与球面求交

- 光线方程:

$$\mathbf{p}(t) = \mathbf{e} + t\mathbf{d};$$

- 球面方程:

$$(\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$$

- 带入求解:

$$(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - R^2 = 0.$$

# 光线与场景求交

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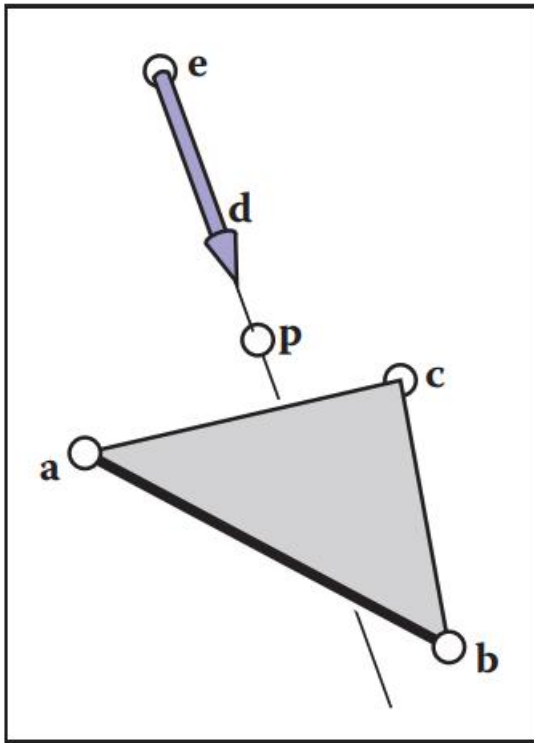
- 光线与球面求交

$$(\mathbf{d} \cdot \mathbf{d})t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - R^2 = 0$$

$$t = \frac{-\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}) \pm \sqrt{(\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}))^2 - (\mathbf{d} \cdot \mathbf{d})((\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - R^2)}}{(\mathbf{d} \cdot \mathbf{d})}$$

# 光线与场景求交

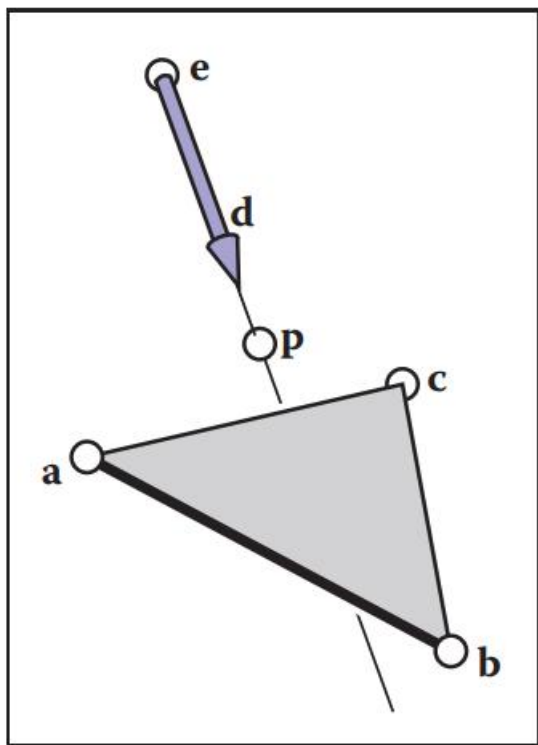
- 光线与三角形求交



$$\mathbf{e} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

# 光线与场景求交

- 光线与三角形求交



$$\begin{aligned}x_e + tx_d &= x_a + \beta(x_b - x_a) + \gamma(x_c - x_a), \\y_e + ty_d &= y_a + \beta(y_b - y_a) + \gamma(y_c - y_a), \\z_e + tz_d &= z_a + \beta(z_b - z_a) + \gamma(z_c - z_a).\end{aligned}$$

$$\begin{bmatrix}x_a - x_b & x_a - x_c & x_d \\y_a - y_b & y_a - y_c & y_d \\z_a - z_b & z_a - z_c & z_d\end{bmatrix} \begin{bmatrix}\beta \\ \gamma \\ t\end{bmatrix} = \begin{bmatrix}x_a - x_e \\y_a - y_e \\z_a - z_e\end{bmatrix}$$

# 光线与场景求交

---

- 光线与三角形求交

$$\beta = \frac{\begin{vmatrix} x_a - x_e & x_a - x_c & x_d \\ y_a - y_e & y_a - y_c & y_d \\ z_a - z_e & z_a - z_c & z_d \end{vmatrix}}{|\mathbf{A}|},$$

$$\gamma = \frac{\begin{vmatrix} x_a - x_b & x_a - x_e & x_d \\ y_a - y_b & y_a - y_e & y_d \\ z_a - z_b & z_a - z_e & z_d \end{vmatrix}}{|\mathbf{A}|},$$

$$t = \frac{\begin{vmatrix} x_a - x_b & x_a - x_c & x_a - x_e \\ y_a - y_b & y_a - y_c & y_a - y_e \\ z_a - z_b & z_a - z_c & z_a - z_e \end{vmatrix}}{|\mathbf{A}|},$$

$$\mathbf{A} = \begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix}$$

# 像素着色

## • Whitted着色模型

$$I_{\lambda} = L_{a\lambda}k_a + \sum_{lights} f_{att}L_{p\lambda}[k_d(\vec{n} \cdot \vec{l}) + k_s(\vec{r} \cdot \vec{v})^n] + k_r I_{r\lambda} + k_t I_{t\lambda}$$

- 前三部分为直接光照
- 后两部分为间接光照

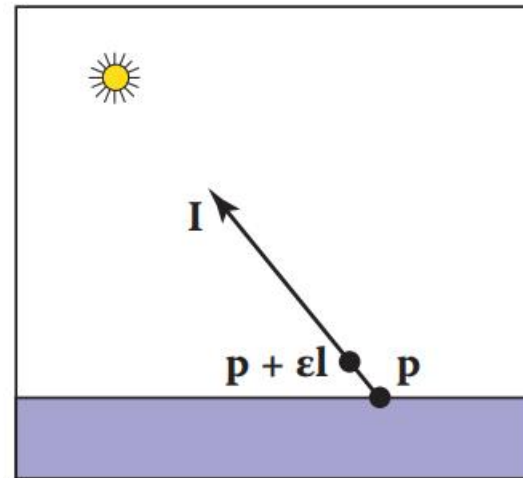
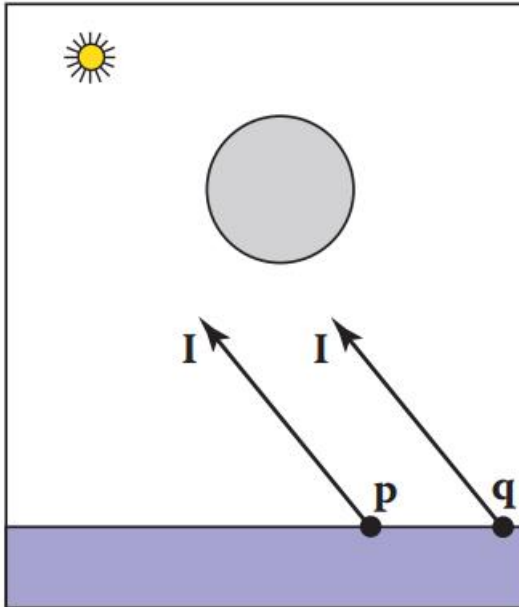
递归反射项

递归折射项



# 阴影

- Shadow ray



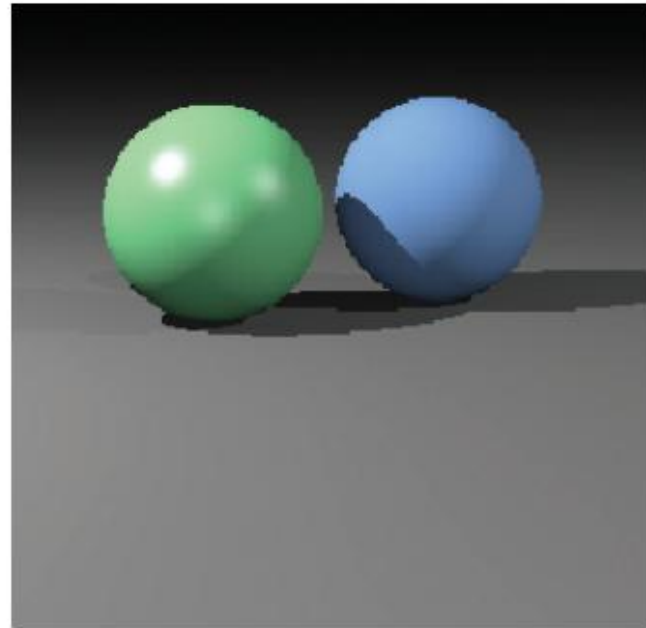
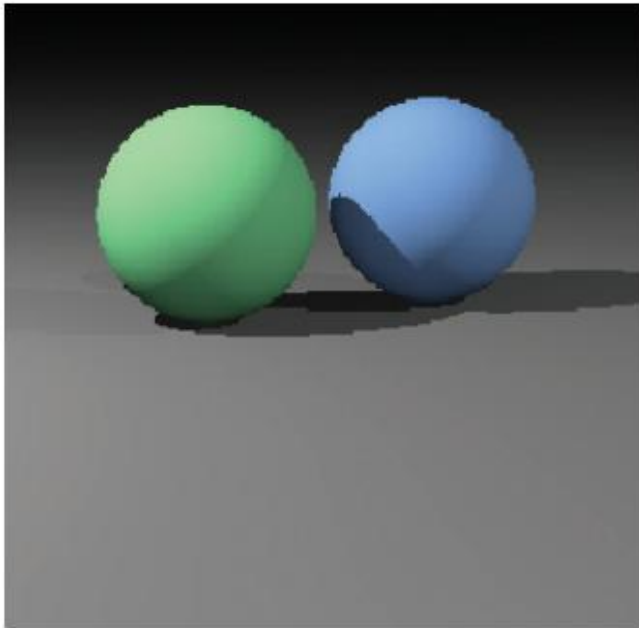
# 阴影

---

```
function raycolor( ray  $\mathbf{e} + t\mathbf{d}$ , real  $t_0$ , real  $t_1$  )  
hit-record rec, srec  
if (scene→hit( $\mathbf{e} + t\mathbf{d}$ ,  $t_0$ ,  $t_1$ , rec)) then  
     $\mathbf{p} = \mathbf{e} + (\text{rec}.t) \mathbf{d}$   
    color  $c = \text{rec}.k_a I_a$   
    if (not scene→hit( $\mathbf{p} + s\mathbf{l}$ ,  $\epsilon$ ,  $\infty$ , srec)) then  
        vector3  $\mathbf{h} = \text{normalized}(\text{normalized}(\mathbf{l}) + \text{normalized}(-\mathbf{d}))$   
         $c = c + \text{rec}.k_d I \max(0, \text{rec}.\mathbf{n} \cdot \mathbf{l}) + (\text{rec}.k_s) I (\text{rec}.\mathbf{n} \cdot \mathbf{h})^{\text{rec}.p}$   
    return  $c$   
else  
    return background-color
```

# 阴影

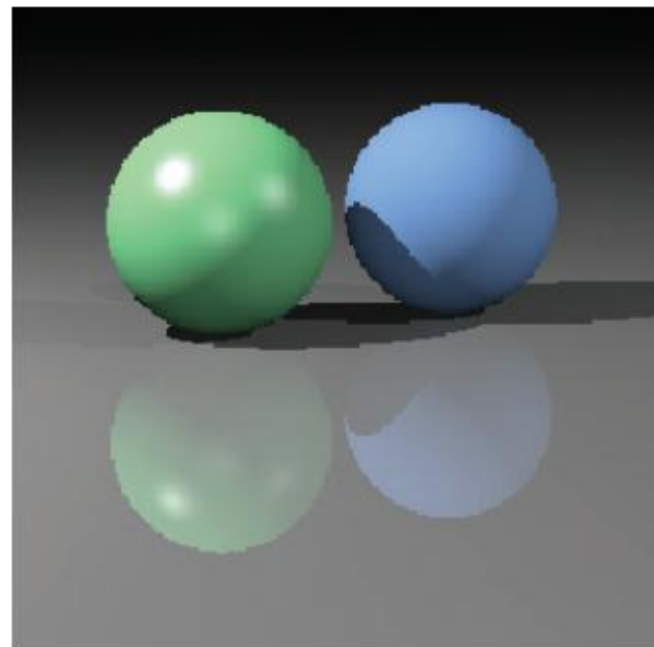
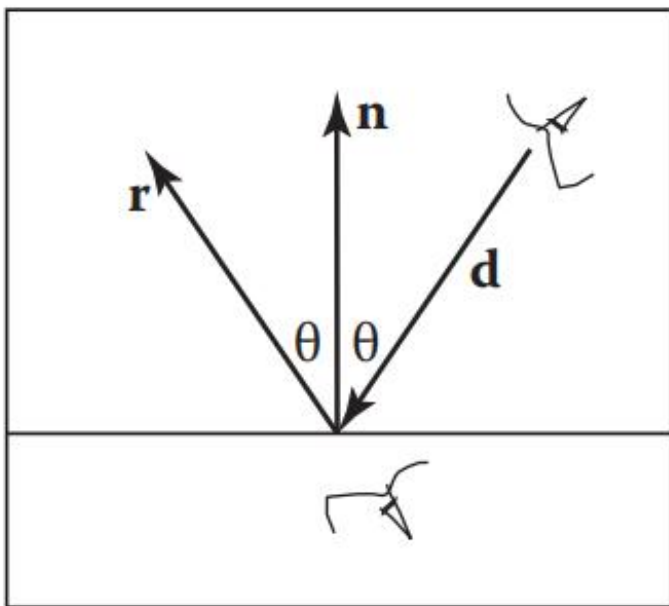
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# 反射

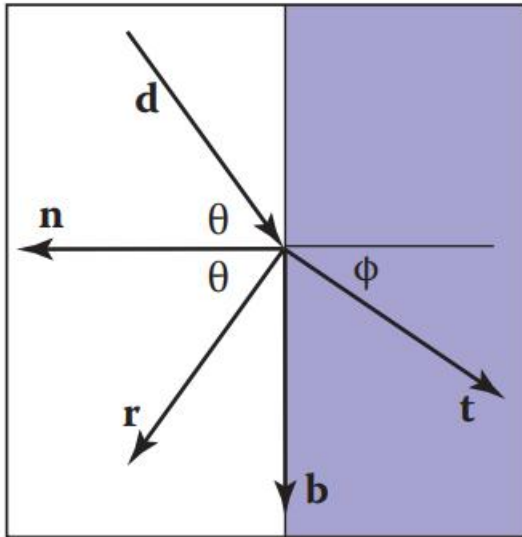
$$\mathbf{r} = \mathbf{d} - 2(\mathbf{d} \cdot \mathbf{n})\mathbf{n}$$

$$\text{color } c = c + k_m \text{raycolor}(\mathbf{p} + s\mathbf{r}, \epsilon, \infty)$$



# 折射

- Snell's Law



$$n \sin \theta = n_t \sin \phi.$$

$$\begin{aligned} \mathbf{t} &= \frac{n (\mathbf{d} + \mathbf{n} \cos \theta)}{n_t} - \mathbf{n} \cos \phi \\ &= \frac{n (\mathbf{d} - \mathbf{n} (\mathbf{d} \cdot \mathbf{n}))}{n_t} - \mathbf{n} \sqrt{1 - \frac{n^2 (1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_t^2}} \end{aligned}$$

- 根号内小于零?