Lecture 11

Shuo Jiang

The Wang Yanan Institute for Studies in Economics, Xiamen University

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Potential Outcome and Causal Effect

- Let Y_1^* be the potential income for a guy if he graduates from college, and Y_0^* be the potential income for a guy.
- The causal effect (treatment effect) of college graduation on income for this guy is $Y_1^* - Y_0^*$.
- Y_d^* for $d \in \{0, 1\}$ is called potential income because for each individual only one realized outcome is observed: if an individual graduates, we observe Y_1^* for him but not Y_0^*; if he does not graduate, we observe Y_0^* but not Y_1^*.
- Usually we researchers only have access to a dataset of individual's treatment status D and actual outcome Y: $\{Y_i, D_i\}_{i=1}^n$, **one important task of econometrics is to connect those observed data to some mathematical characteristics of $Y_1^* - Y_0^*$.**

Potential Outcome and Causal Effect

- For each individual i, his observed income Y_i and potential income $(Y^*_{1,i}, Y^*_{0,i})$ are related in the following way: $Y_i = Y^*_{0,i} + (Y^*_{1,i} - Y^*_{0,i})D_i$.
- Since the causal effect for different individuals will be different in general, we usually focus on the following average effects:
  - Average treatment effect: $\alpha_{ATE} \equiv \mathbb{E}(Y^*_1 - Y^*_0) = \mathbb{E}(Y^*_{1,i} - Y^*_{0,i})$
  - Average treatment effect on the treated: $\alpha_{ATT} \equiv \mathbb{E}(Y^*_1 - Y^*_0 | D = 1) = \mathbb{E}(Y^*_{1,i} - Y^*_{0,i} | D_i = 1)$

Selection, Confounding and RCT

- In general, the observed difference between those who graduate from college and those who do not does not equal to the causal effect:

$$\begin{aligned} & \mathbb{E}(Y|D=1) - \mathbb{E}(Y|D=0) \\ = \ & \mathbb{E}(Y_1^* - Y_0^*|D=1) + \color{red}{\mathbb{E}(Y_0^*|D=1) - \mathbb{E}(Y_0^*|D=0)} \end{aligned}$$

- The red part in the above equation is called "selection bias", which means that people who graduate from college and people who do not are systematically different, and that their untreated potential outcome will be different anyway.
- For example, those who graduate from college might have higher ability than those who do not. Thus those who graduate from college will be more likely to get high-paying jobs anyway. Here, ability is called a confounding variable.

Selection, Confounding and RCT

- Whenever selection bias is not zero, comparison between the treatment and control is not comparing “apples” to “apples”, but comparing “apples” to “oranges”.
- The ideal and classic solution to this problem is RCT: randomly assign individuals to treatment and control groups, which implies that the treatment status will be independent of all potential outcomes: $(Y_1^*, Y_0^*) \perp D$.
- Intuitively, under randomization, there will be no systematic difference between those who are treated and those who are not and selection bias dissapears:

$$\mathbb{E}(Y_0^*|D=1) - \mathbb{E}(Y_0^*|D=0) = \mathbb{E}(Y_0^*) - \mathbb{E}(Y_0^*) = 0$$

- Observed group difference reveals the average causal effect: $\alpha_{ATE} = \alpha_{ATT} = \mathbb{E}(Y|D=1) - \mathbb{E}(Y|D=0)$

Regression and Causality

- Sometimes, we have that $(Y_1^*, Y_0^*) \perp D|X$: conditional on X the treatment is randomly assigned.
- In such case, we could identify the average treatment effect for different subgroups: $\alpha_{ATE}(x) = \alpha_{ATT}(x) = \mathbb{E}(Y|D=1, X=x) - \mathbb{E}(Y|D=0, X=x)$

Now lets get to the final question: if I regress Y on D and X using OLS, when does the coefficient in front of D have some causal interpretation?

Regression and Causality

Let's look at this problem using the example of causal effect of college graduation on income. Below we give some sufficient conditions:

- **Firstly, we need a linear constant effect causal model for the potential outcome:**

  $$Y^*_{D,i} = a + \rho D + \eta_i,$$

  where η_i captures factors that affect the potential income beyond college graduation status D (note that here D is used instead of D_i because this causal model tells us what person i's income for any value of D and not just the realized D_i in the dataset).
- Note that each individual has different D_i and η_i, and given the realized D_i for individual i, its potential outcome $Y^*_{D,i}$ simply becomes the observed outcome Y_i:

  $$Y_i = a + \rho D_i + \eta_i, \tag{1.1}$$

Regression and Causality

- Equation (1.1) looks like the predictive linear regression we have seen before, except that it is the realized version of a causal model which gives ρ a causal interpretation.
- **Secondly, we need conditional independence** $(Y^*_{1,i}, Y^*_{0,i}) \perp D_i | X_i$ **and the following linear model for** η**:**

$$\eta_i = X_i'\gamma + v_i,$$

where $\mathbb{E}[\eta|X_i] = X_i'\gamma$.

- These conditions imply:

$$\begin{aligned} E(Y^*_{D,i}|X_i, D_i) &= E(Y^*_{D,i}|X_i) \\ &= \alpha + \rho D + X_i'\gamma \end{aligned}$$

and $E(Y_i|X_i, D_i) = \alpha + \rho D_i + X_i'\gamma$

- The full regression could be written as

$$Y_i = a + \rho D_i + X_i'\gamma + v_i$$

Regression and Causality

- v_i is uncorrelated with both regressors, which implies that ρ could be consistently estimated.
- X_i is the only reason why other factor η_i and treatment D_i are correlated (or equivalently D_i and $Y^*_{D,i}$). In our example, X_i might include GPA, family income and so on. Given X_i, college graduation is uncorrelated with other factors that affect income (or college graduation is uncorrelated with potential earnings).
- The same reasoning holds when D_i is not binary.

Regression and Causality

Summary:

- If regression is used for predictive modeling, we summarize the statistical dependence between Y and D (given other variables X). The regression result can be used to answer questions like: given that we observe the education status of some people, what value can we expect their income to take? how are the variation in education correlated to variation in income?
- If regression is used for causal analysis, we give causal interpretation to the regression coefficient. The regression result can be used to answer questions like: if we make some students graduate from college, how much more will they earn on average compared to the world in which they do not graduate.
- Prediction vs Intervention: $E(Y|D)$ vs $E(Y|do(D))$