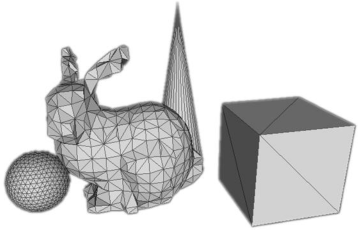
Introduction to Computer Graphics



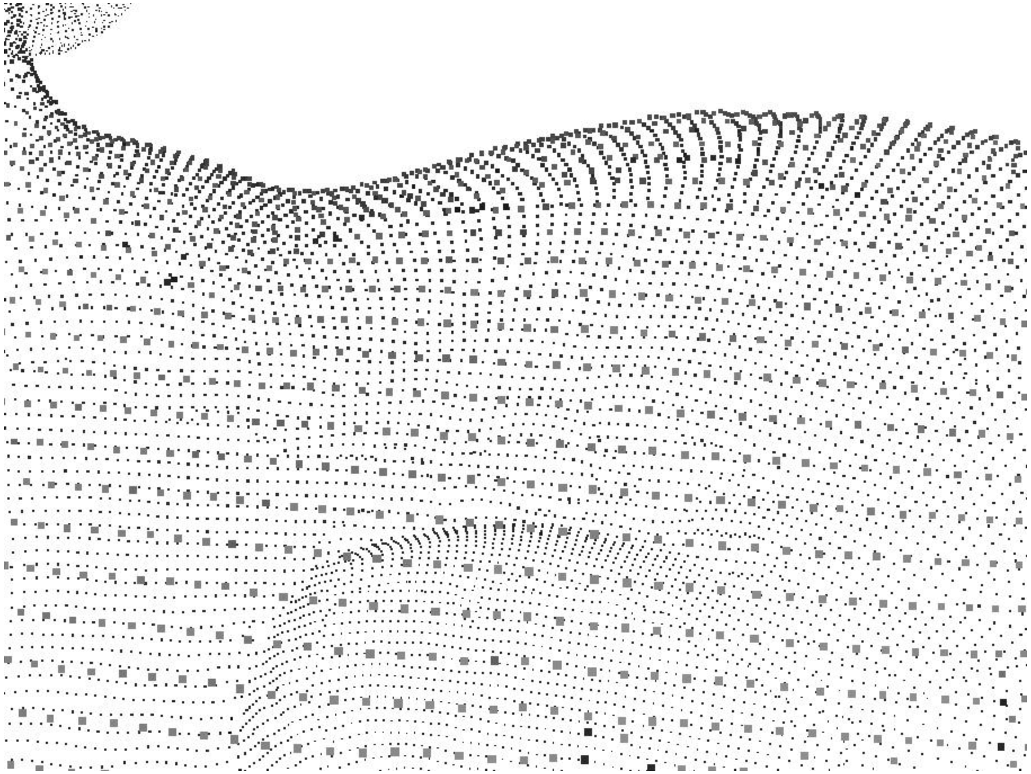
Geometry 2
几何 2

第八章 几何

- 几何的表示形式
  - 隐式表示
  - 显式表示
- 参数化曲线曲面
  - 贝塞尔曲线
  - 贝塞尔曲面
- 点云表示
- 体表示
- 网格表示
- 网格处理

点云表示

- 点的集合P={P1, P2, P3, …, Pn}
- Pi={xi, yi, zi}



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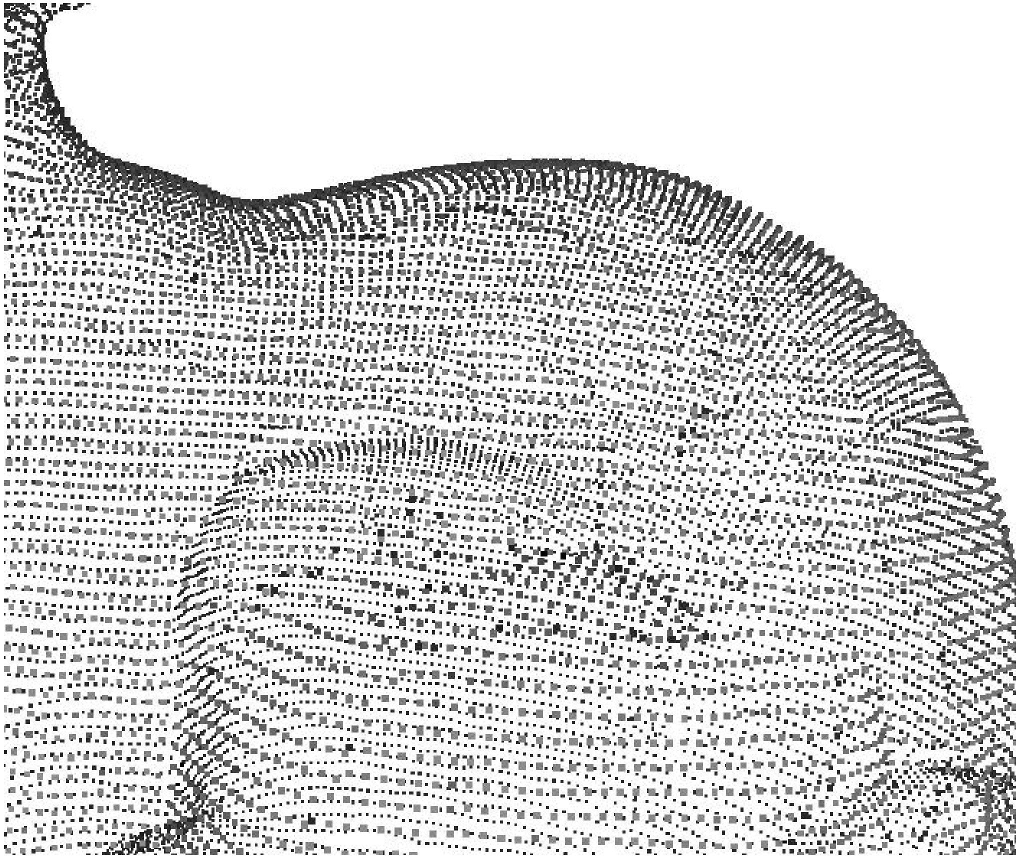

点云表示

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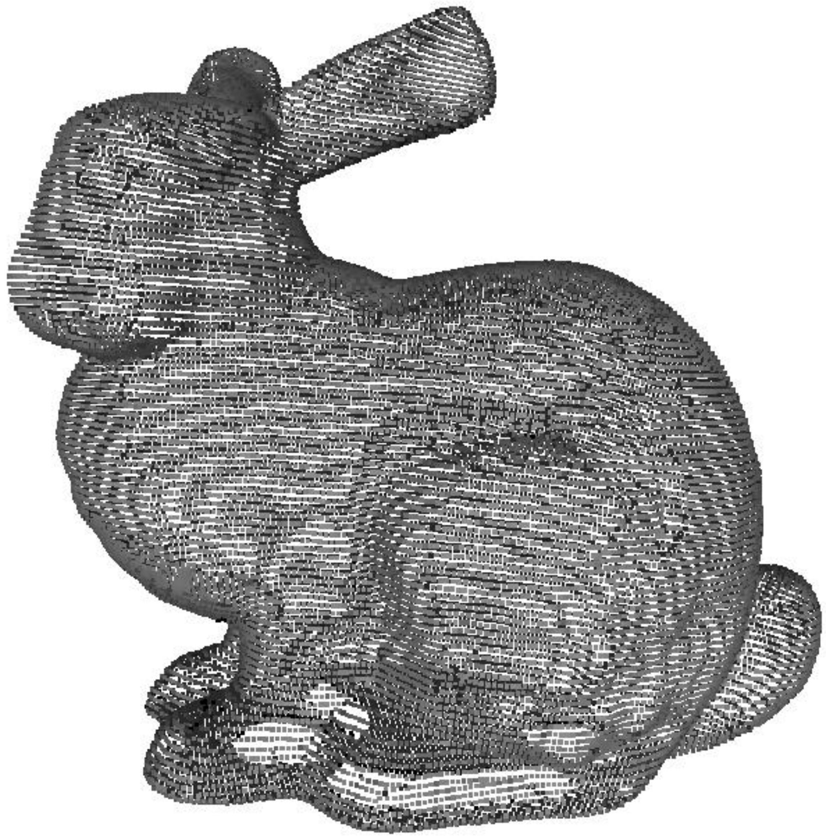


点云表示

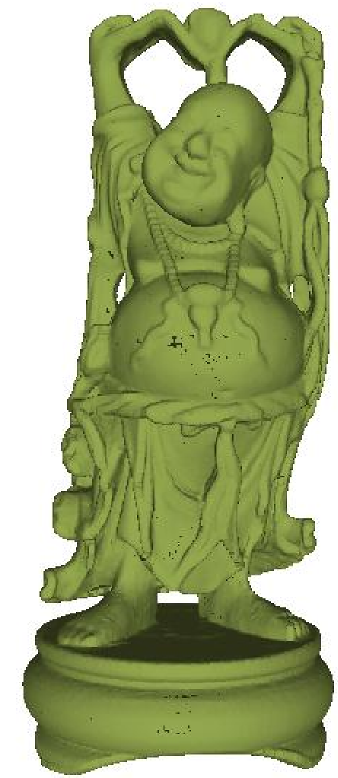
- 点的集合P={P1, P2, P3, …， Pn}
- Pi={xi, yi, zi}

点云表示

- 点的集合P={P1, P2, P3, …， Pn}
- Pi={xi, yi, zi}

Armadillo

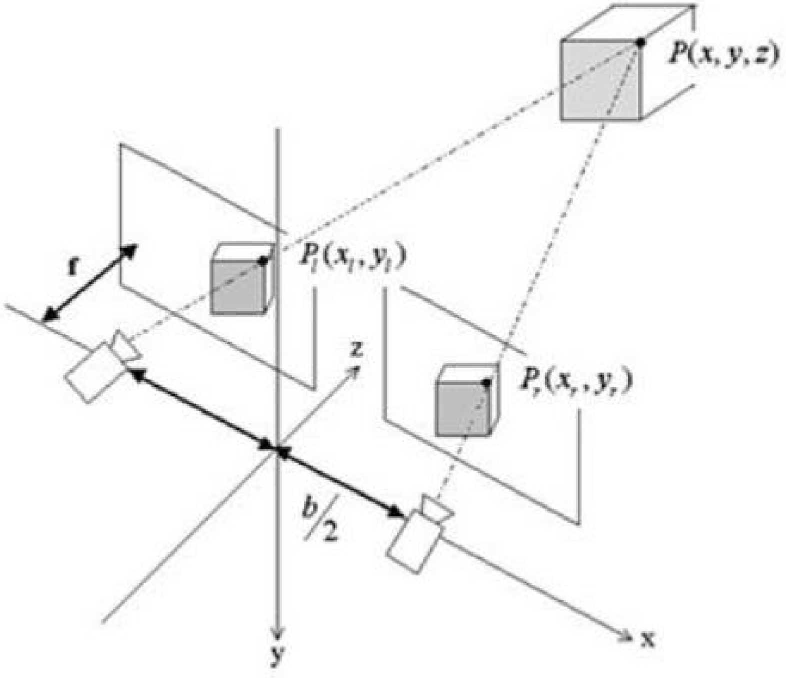
dragon



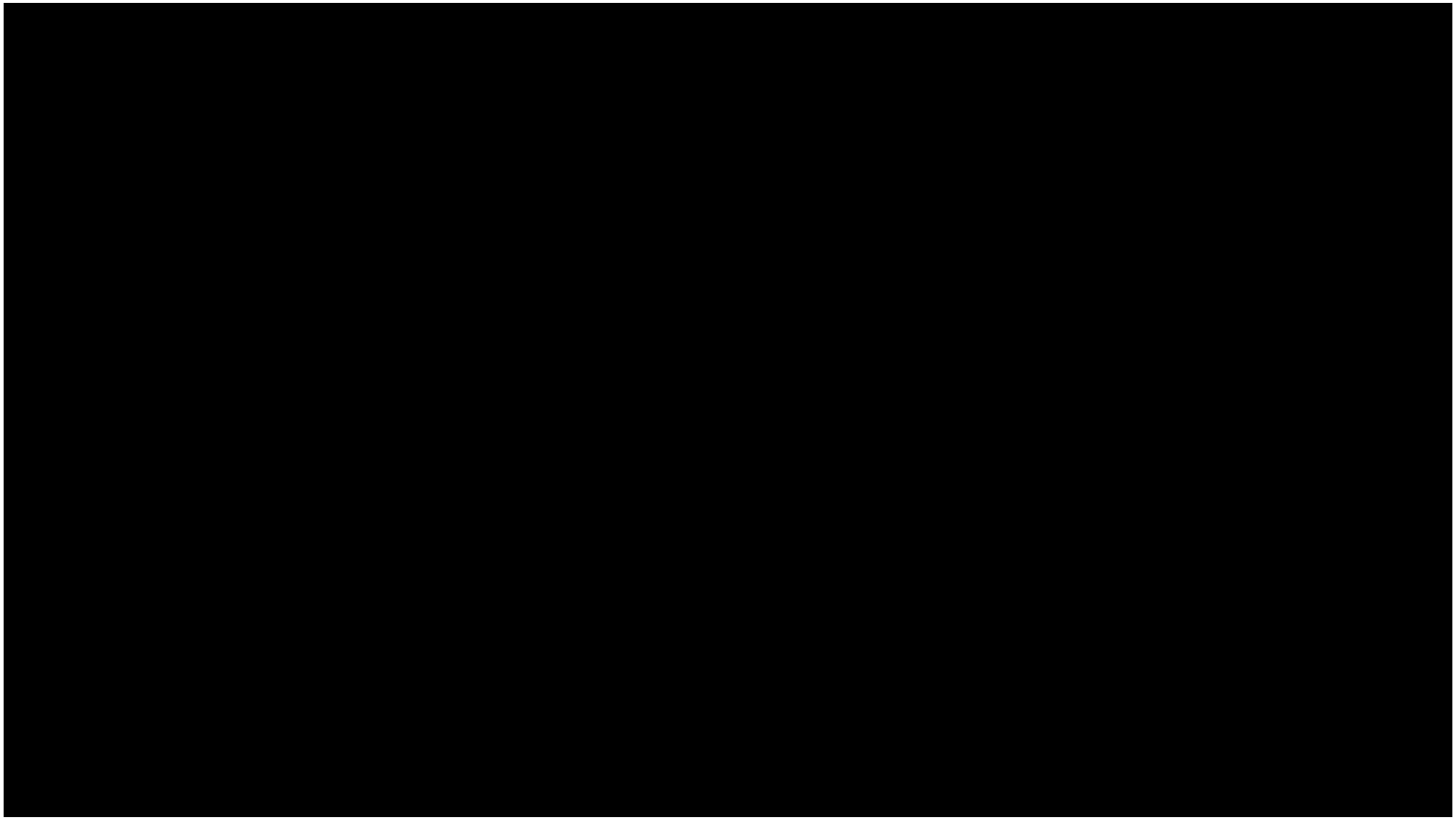
Happy buddha

点云的获取

- 三维重建
  - 主要思想：双目视觉三角化

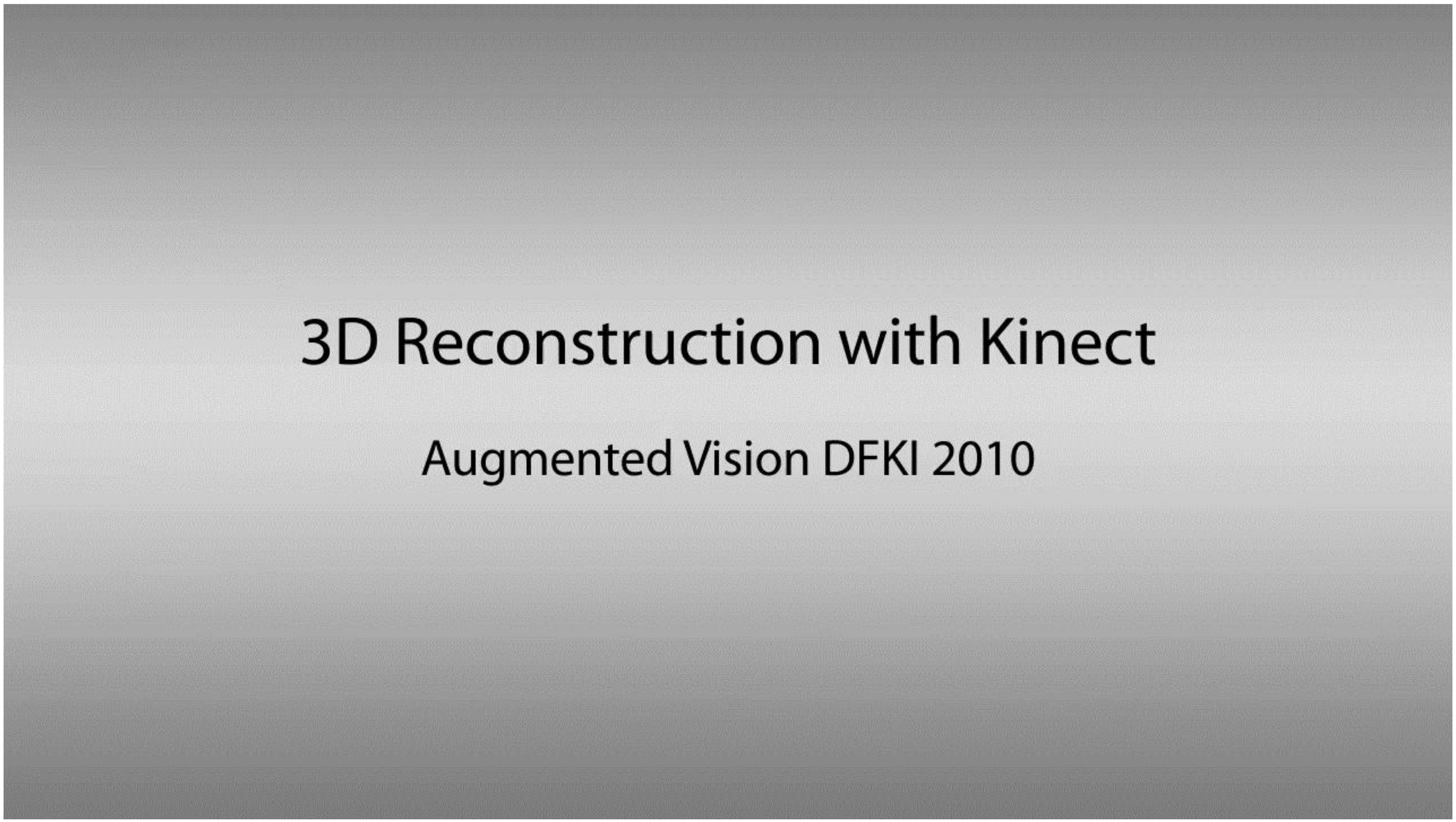


激光条带

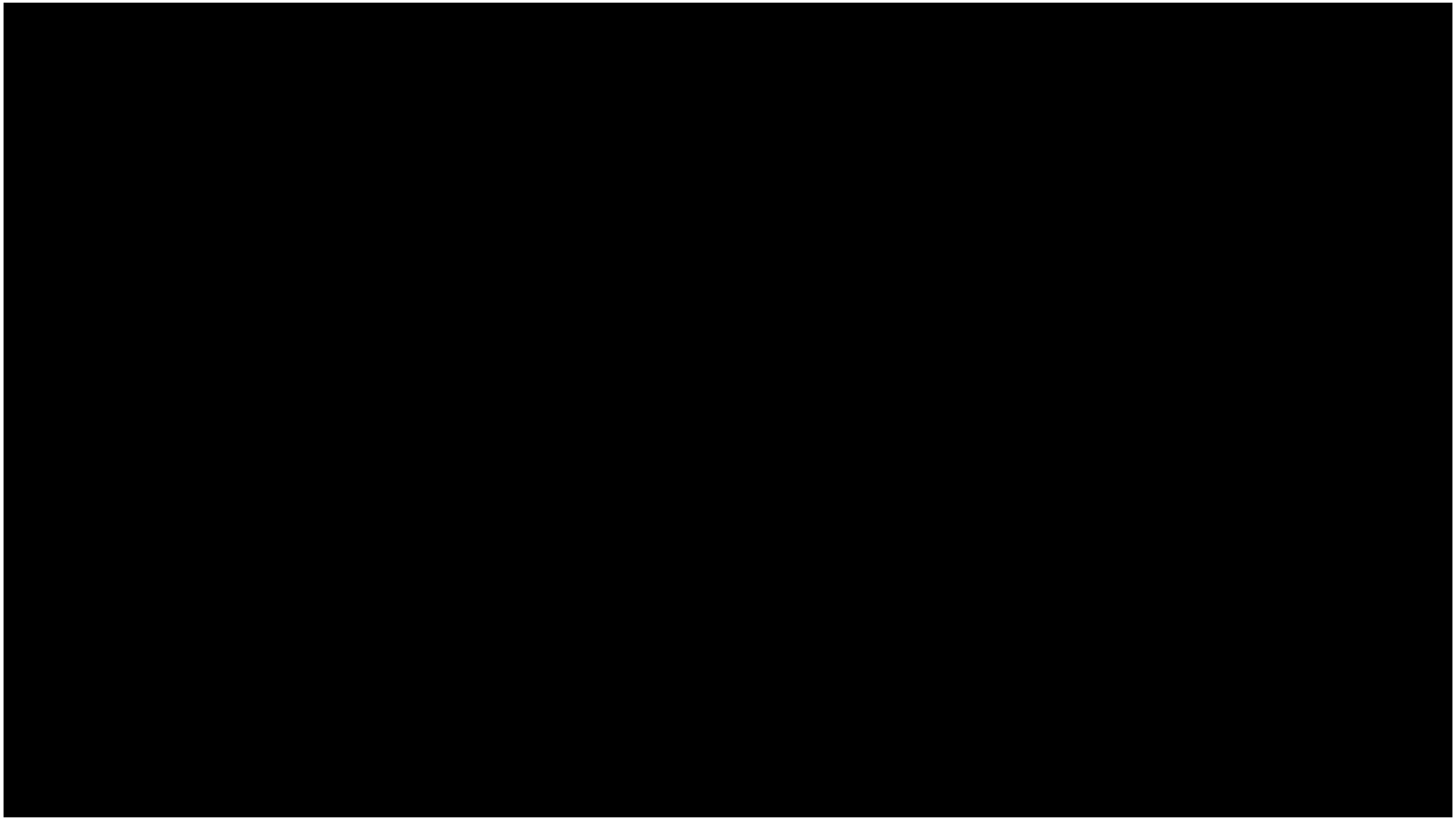


结构光

Kinect

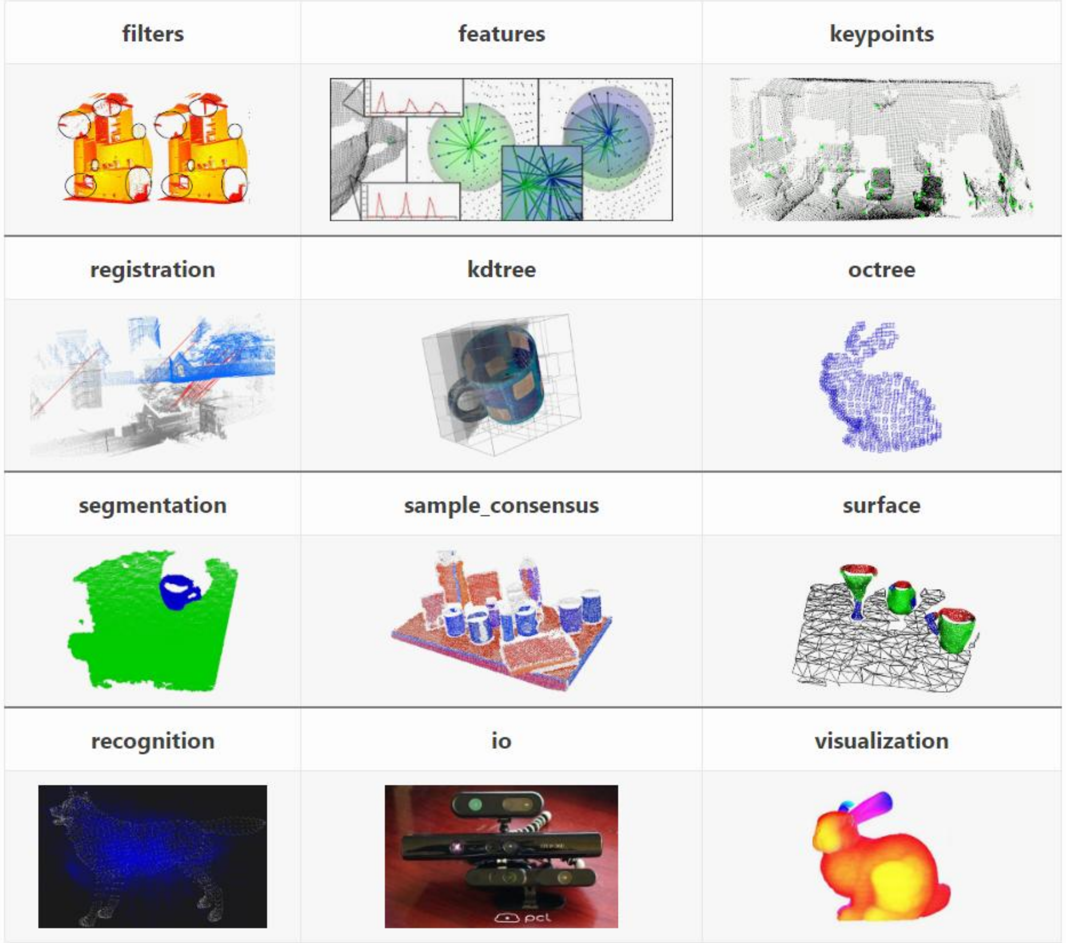


多视图重建



点云的处理与分析

- 点云库 PCL
- 点云去噪
- 点云下采样
- 估计法向
- 点云分割
- …



点云的注册

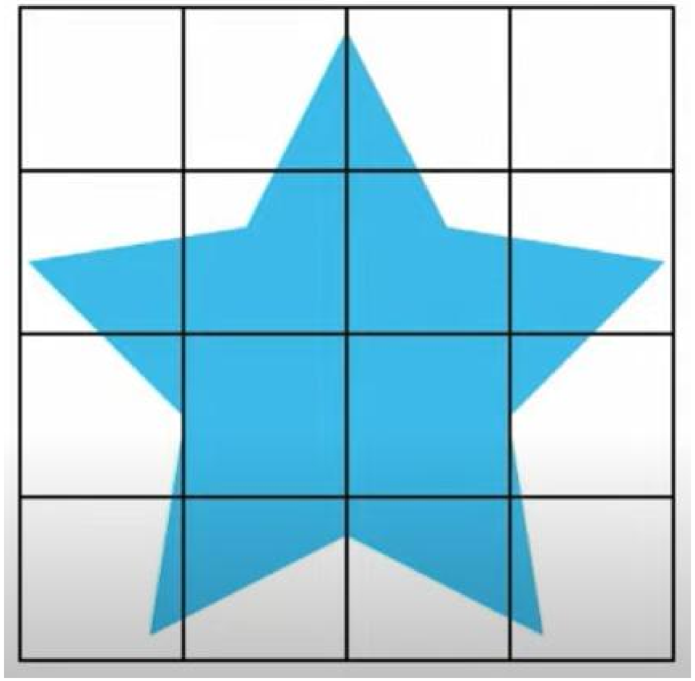
- Registration
- 将多个点云拼接成完整的点云
- 求最优的刚体变换，使得两个点云可以重合

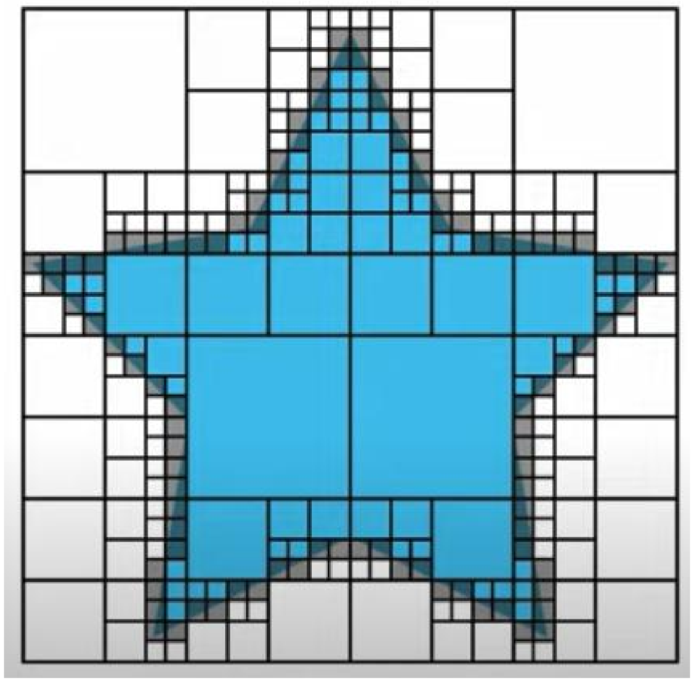
体表示

- 像素的扩展

体表示及稀疏结构

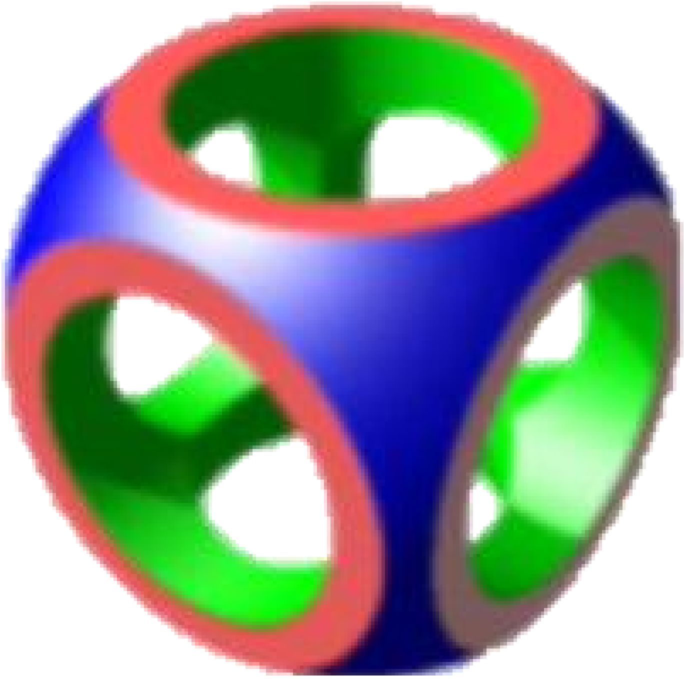
- 存储方式：1表示体内，0表示体外
  - 蓝色表示1，白色为0（既有白色也有蓝色可以任取一个）





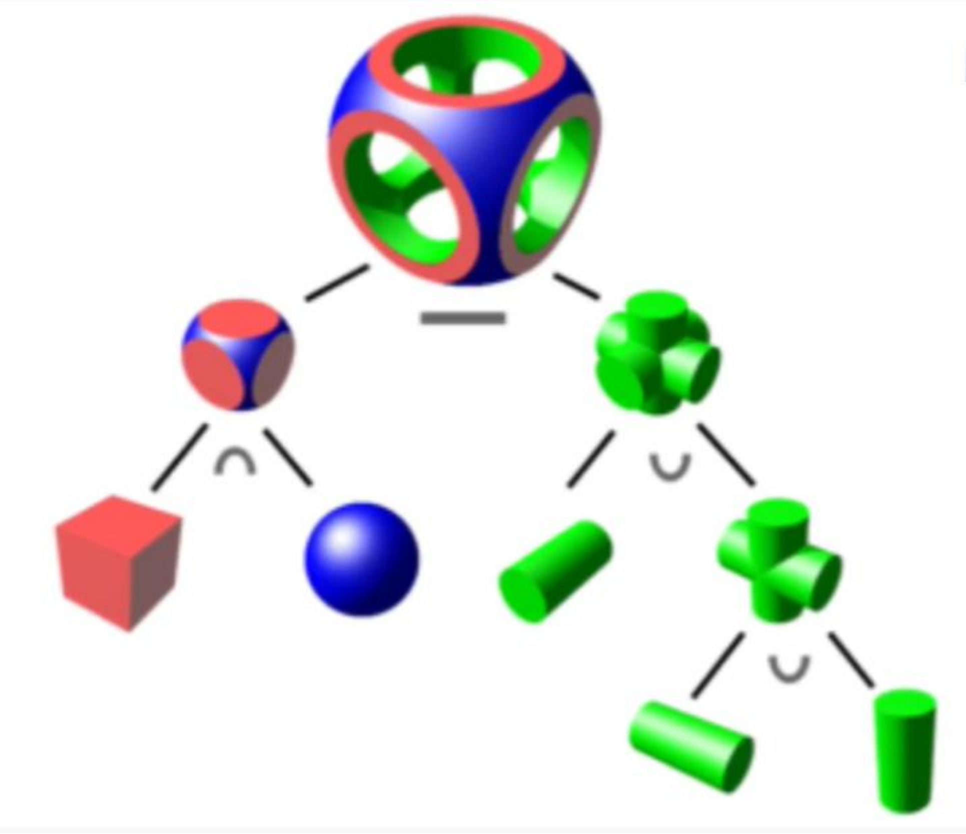
CSG

- Constructive Solid Geometry



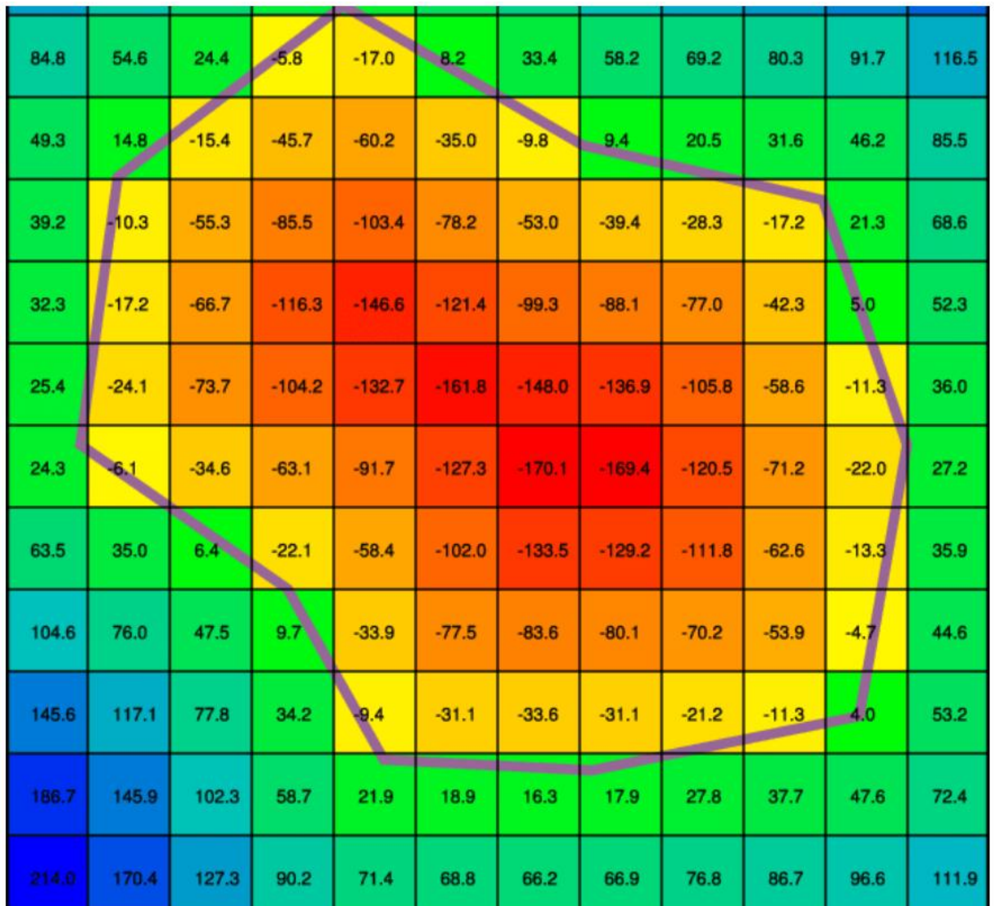
CSG

- Constructive Solid Geometry
- 体表示+布尔运算



符号距离场

- 符号距离场 (Signed Distance Field)



符号距离场

- 截断符号距离场 (Truncated Signed Distance Field)

-0.9	-0.3	0.0	0.2	1	1	1	1	1
-1	-0.9	-0.2	0.0	0.2	1	1	1	1
-1	-0.9	-0.3	0.0	0.1	0.9	1	1	1
-1	-0.8	-0.3	0.0	0.2	0.8	1	1	1
-1	-0.9	-0.4	-0.1	0.1	0.8	0.9	1	1
-1	-0.7	-0.3	0.0	0.3	0.6	1	1	1
-1	-0.7	-0.4	0.0	0.2	0.7	0.8	1	1
-0.9	-0.7	-0.2	0.0	0.2	0.8	0.9	1	1
-0.1	0.0	0.0	0.1	0.3	1	1	1	1
0.5	0.3	0.2	0.4	0.8	1	1	1	1

符号距离场

- 如何从点云表示转成SDF表示呢？
  - 对每个voxel计算其与点云的最近距离
  - 根据点云的法向判断内外

- 如何从SDF转成点云
  - 找到临界点并进行插值

-0.9	-0.3	0.0	0.2	1	1	1	1	1
-1	-0.9	-0.2	0.0	0.2	1	1	1	1
-1	-0.9	-0.3	0.0	0.1	0.9	1	1	1
-1	-0.8	-0.3	0.0	0.2	0.8	1	1	1
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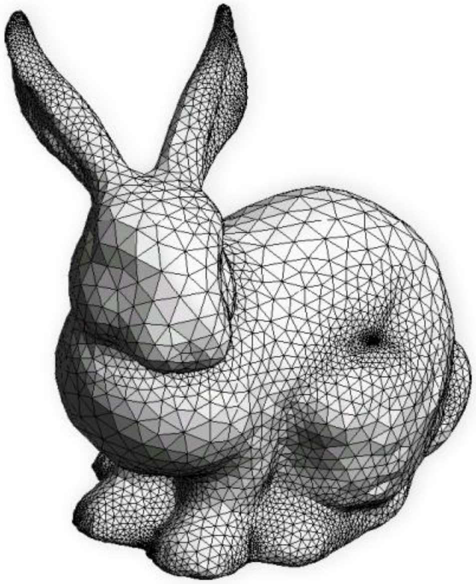
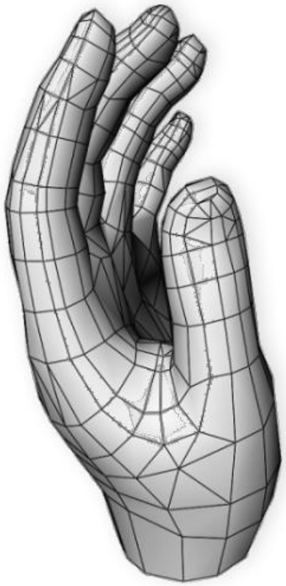
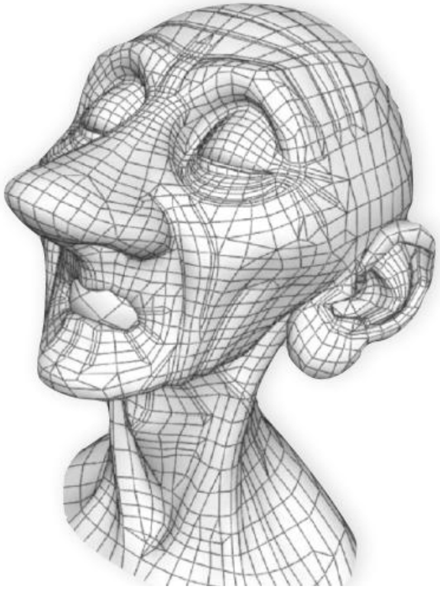
-0.9	-0.7	-0.2	0.0	0.2	0.8	0.9	1	1
-0.1	0.0	0.0	0.1	0.3	1	1	1	1
0.5	0.3	0.2	0.4	0.8	1	1	1	1

符号距离场

- 优点
  - 数据表示规整，与像素类似，便于做卷积等操作
  - 数据整体性强，不容易受噪声干扰
  - 适合用于数据的融合
- 缺点:
  - 数据量大
  - 往往需要稀疏表示

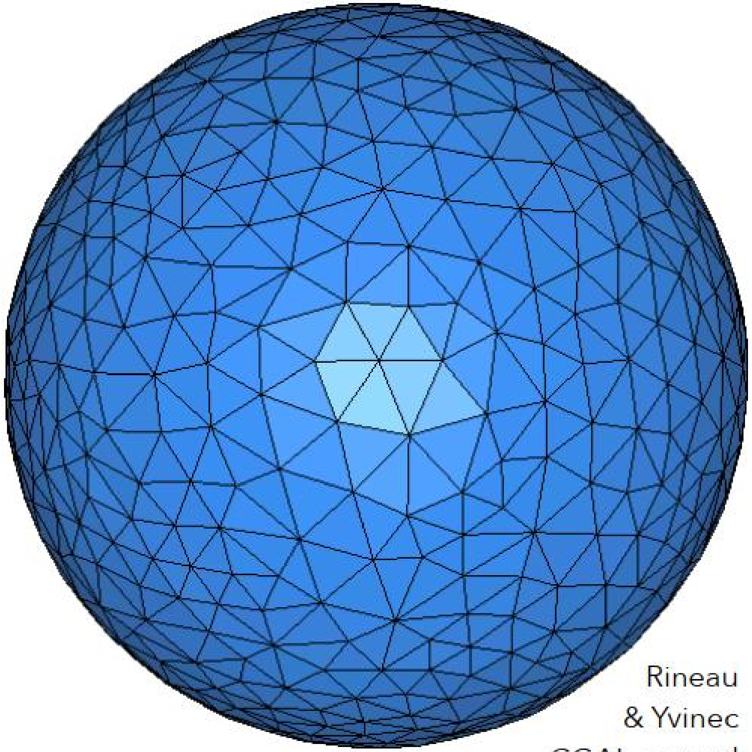
网格表示



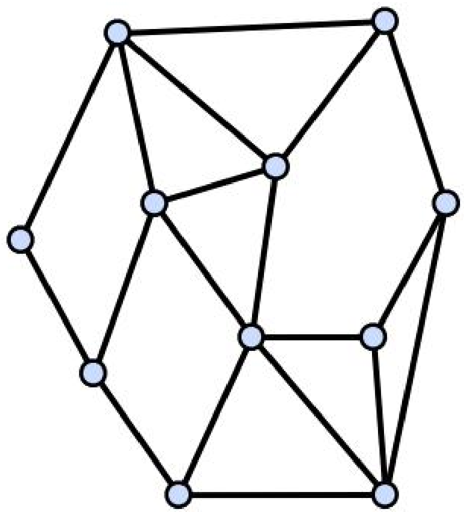
网格表示

spheres



approximate sphere

网格表示



$$M = \langle V, E, F \rangle$$

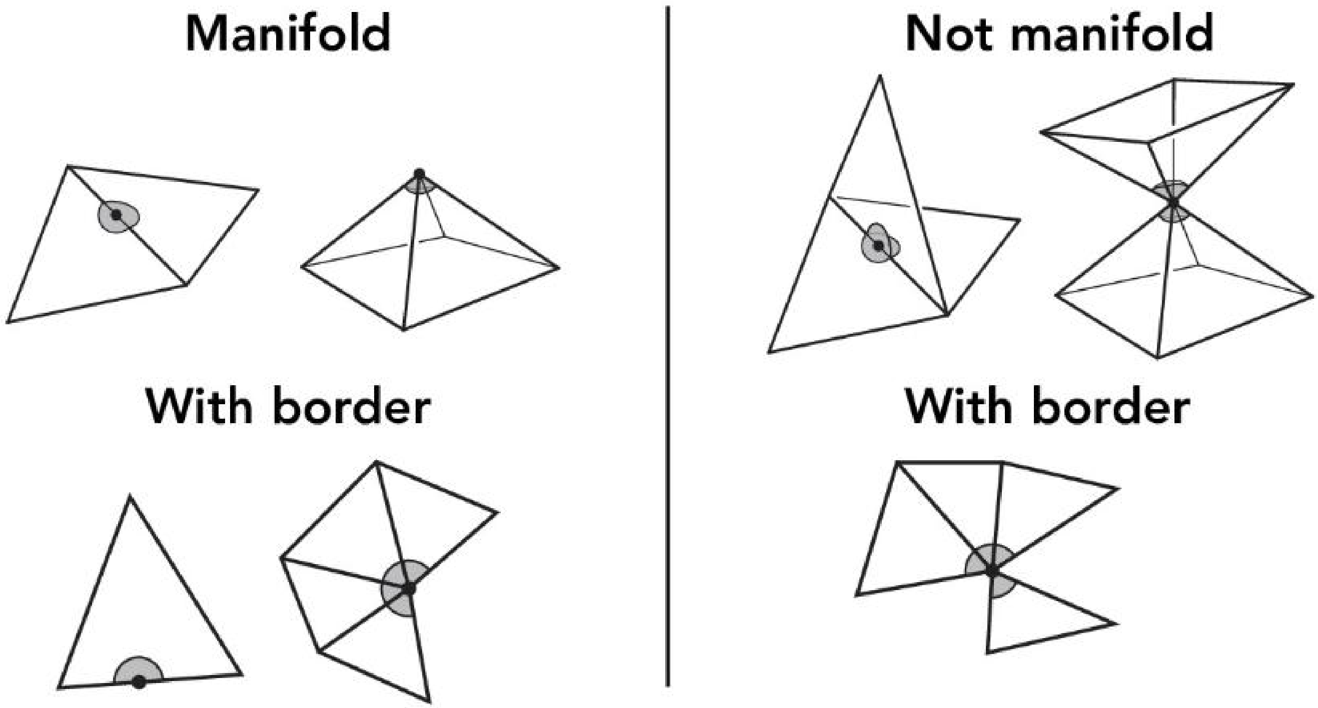
vertices edges faces

拓扑与几何

- Topology and Geometry
- 拓扑是什么？顶点的连接关系
- Manifold (流形):流形是局部具有欧几里得空间性质的空间
- 二维流形：一个表面，当用一个小球去切割时，得到一个二维的圆盘。

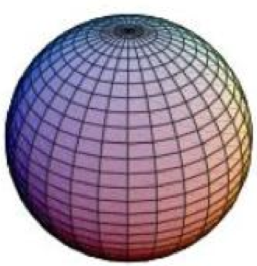
拓扑与几何

- 二维流形：一个表面，当用一个小球去切割时，得到一个二维的圆盘。

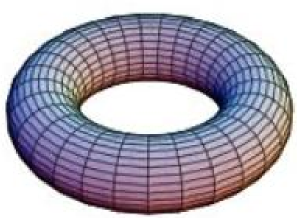


拓扑与几何

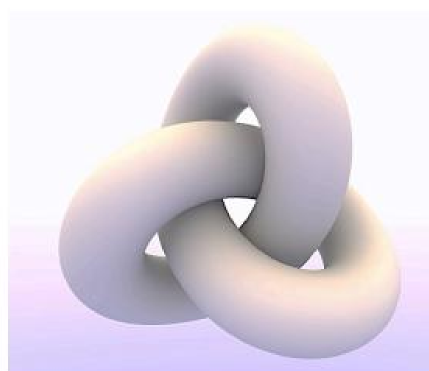
- 哪些是流形？



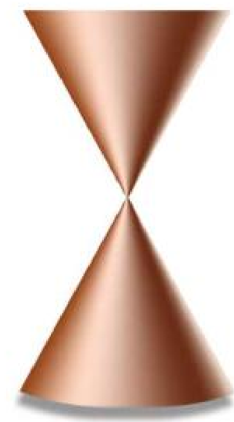
Case 1



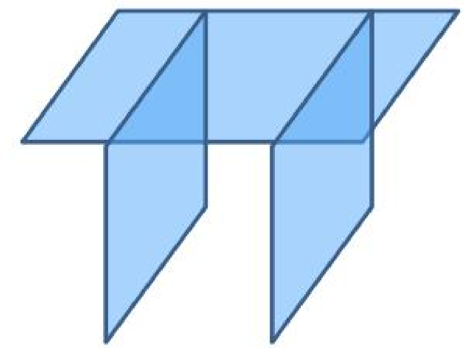
Case 2



Case 3



Case 4



Case 5

网格存储格式

- PLY, OBJ, STL, OFF等

Triangles			
0	x0	y0	z0
1	x1	y1	z1
2	x2	y2	z2
3	x3	y3	z3
4	x4	y4	z4
5	x5	y5	z5
6	x6	y6	z6
...	...	...	...

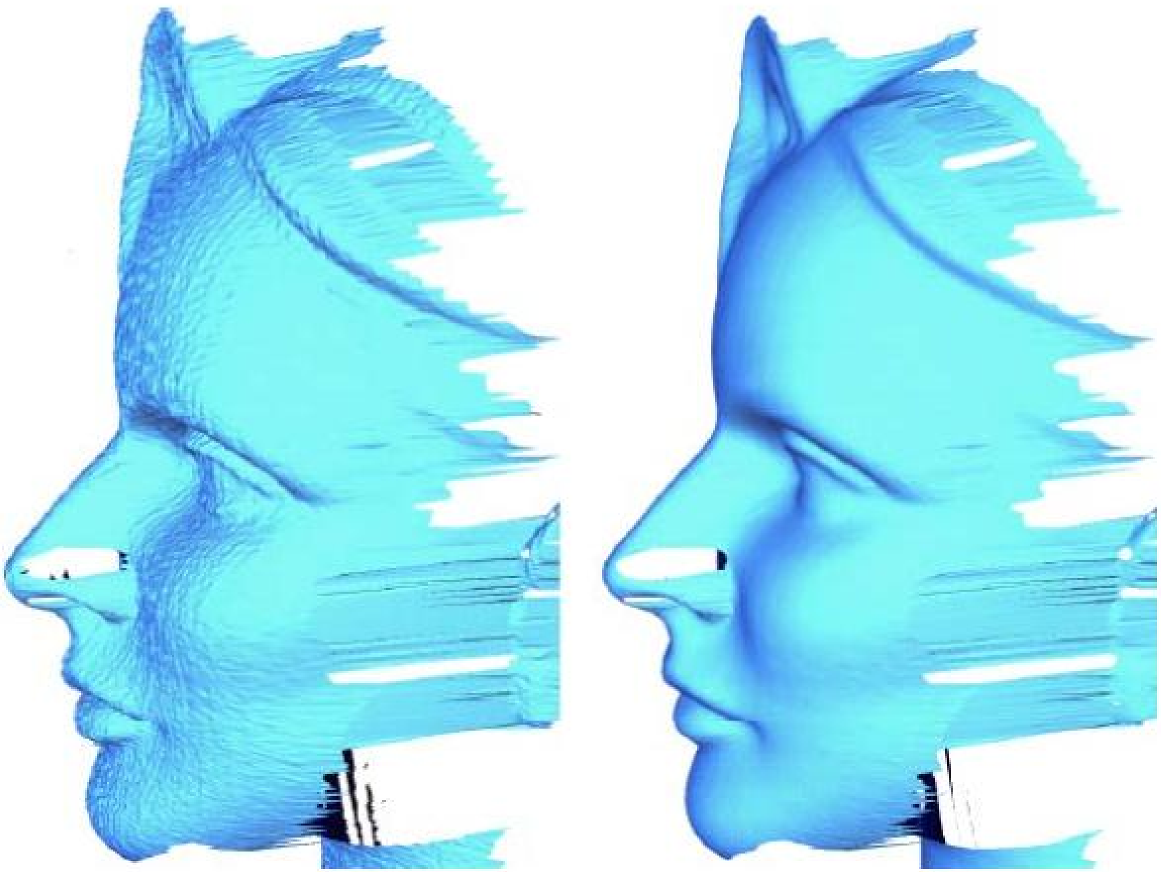
三角形直接存顶点

Triangles			
t0	v0	v1	v2
t1	v0	v1	v3
t2	v2	v4	v3
t3	v5	v2	v6
...	...	...	...

Vertices			
v0	x0	y0	z0
v1	x1	x1	z1
v2	x2	y2	z2
v3	x3	y3	z3
v4	x4	y4	z4
v5	x5	y5	z5
v6	x6	y6	z6
...	...	...	...

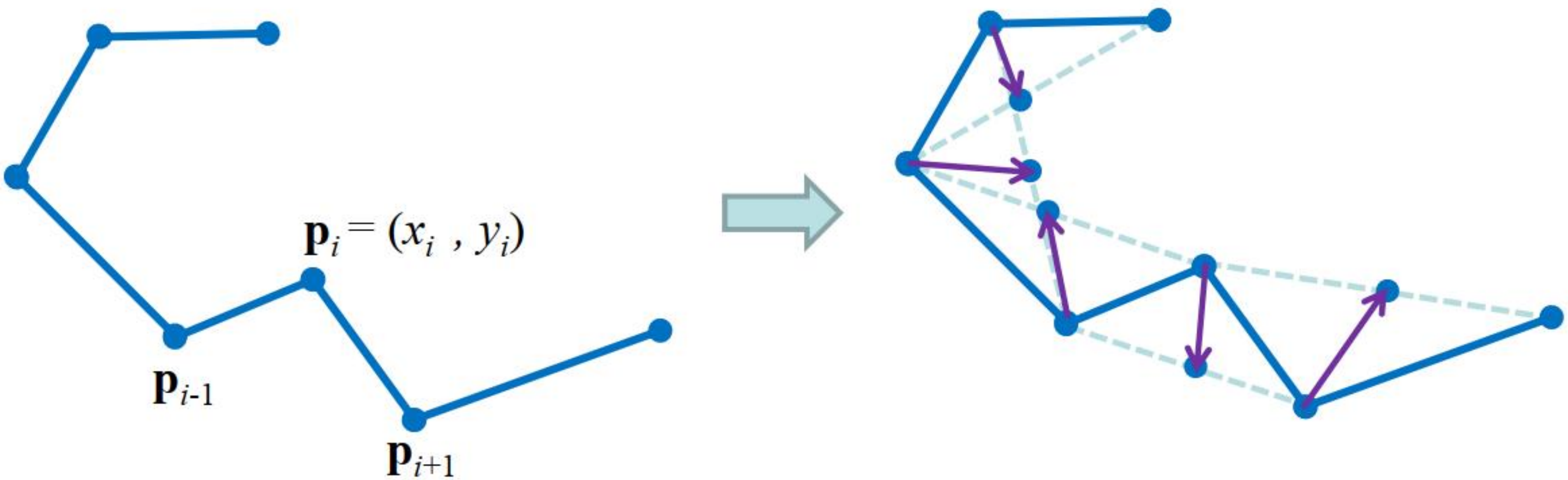
三角形存顶点的索引

网格去噪



网格去噪

- 先以曲线为例，定义Laplacian算子

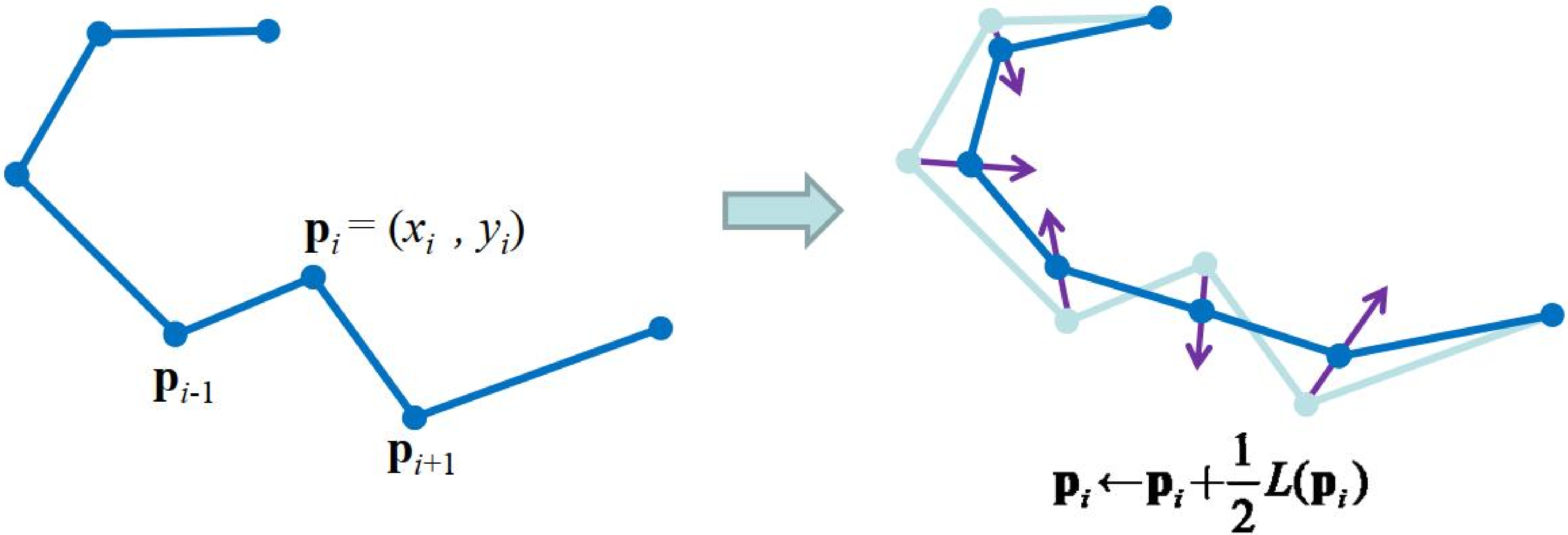


$(\mathbf{p}_{i\text{-}1} + \mathbf{p}_{i+1})/2 - \mathbf{p}_i$

$$L(\mathbf{p}_i)=\frac{1}{2}(\mathbf{p}_{i+1}-\mathbf{p}_i)+\frac{1}{2}(\mathbf{p}_{i-1}-\mathbf{p}_i)$$

网格去噪

- 每个顶点沿着Laplacian算子方向移动

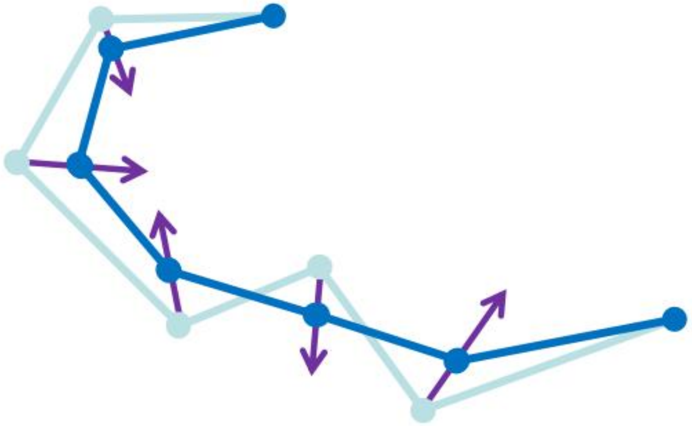


网格去噪

Algorithm:

Repeat for *m* iterations (for non boundary points):

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$



For which λ?

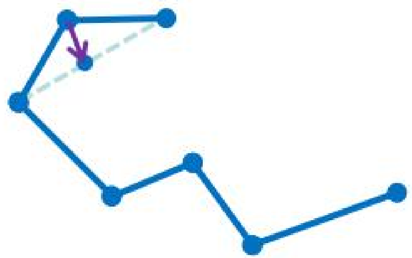
$$0 < \lambda < 1$$

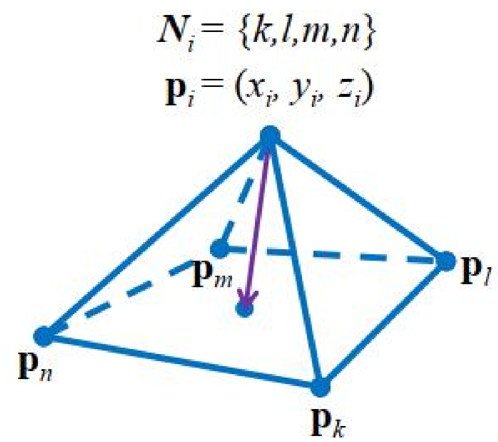
网格去噪

Same as for curves:

$$\mathbf{p}_i^{(t+1)} = \mathbf{p}_i^{(t)} + \lambda \Delta \mathbf{p}_i^{(t)}$$

What is $\Delta\mathbf{p}_i$?

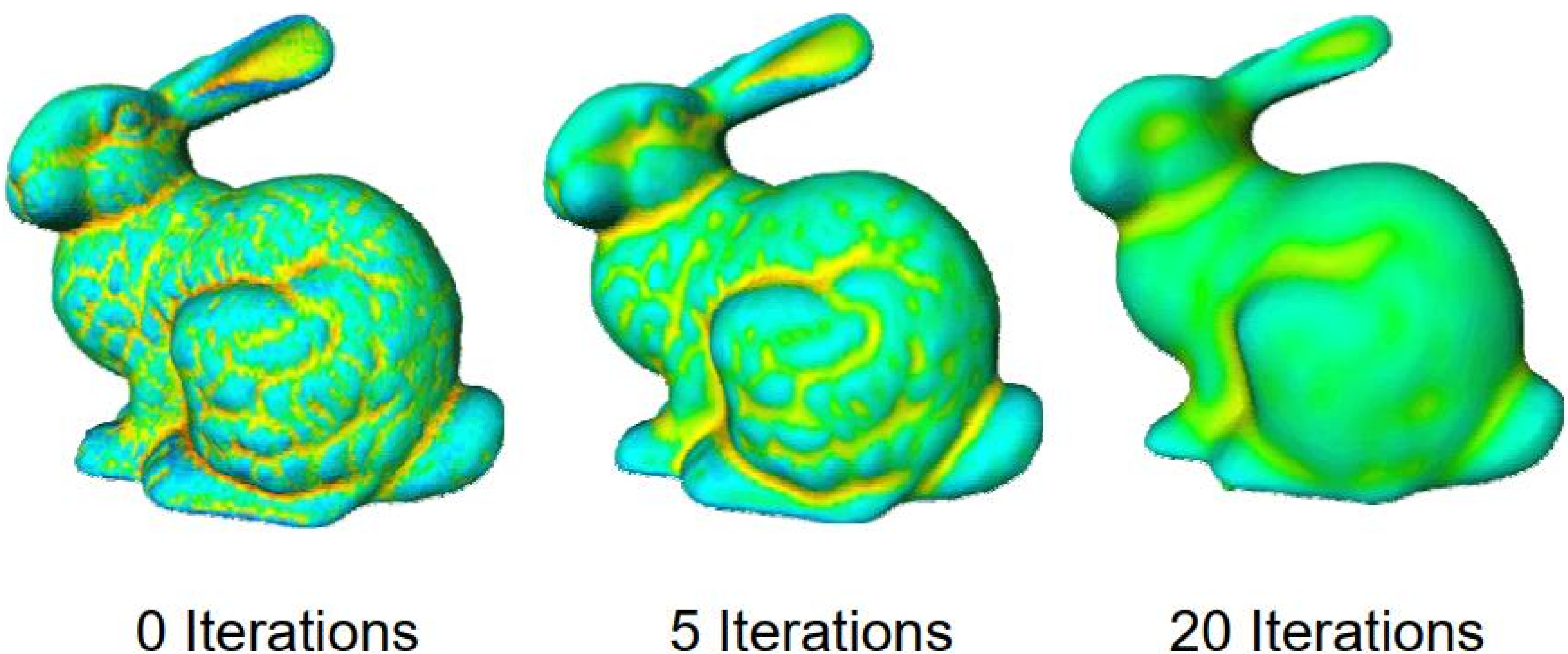




$$\frac{1}{2}(\mathbf{p}_{i+1} + \mathbf{p}_{i-1}) - \mathbf{p}_i \qquad \frac{1}{|N_i|}\left(\sum_{j\in N_i} \mathbf{p}_j\right) - \mathbf{p}_i$$

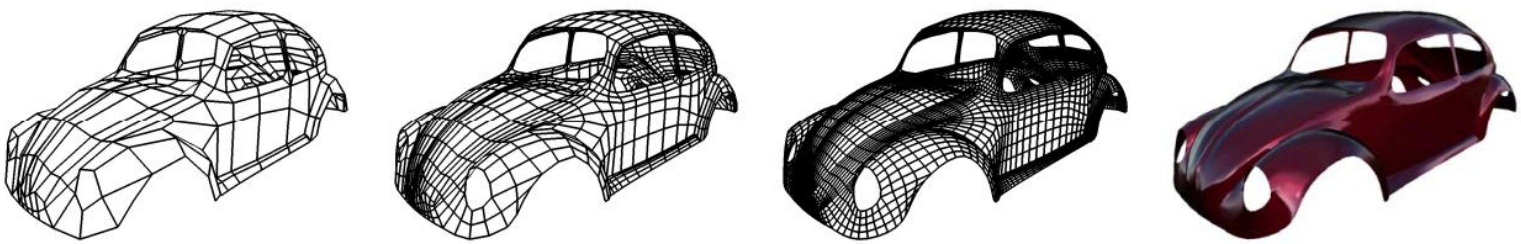
网格去噪

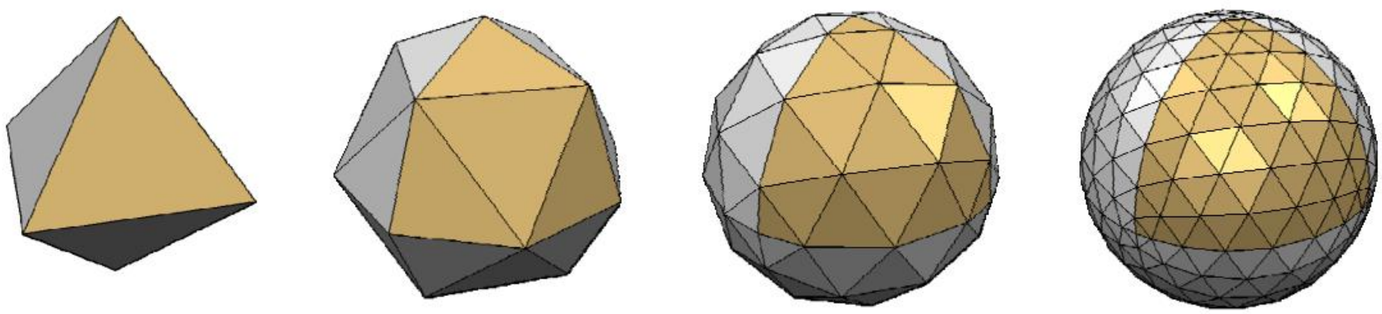
- Result of Laplacian Smoothing



网格细分 (Subdivision)

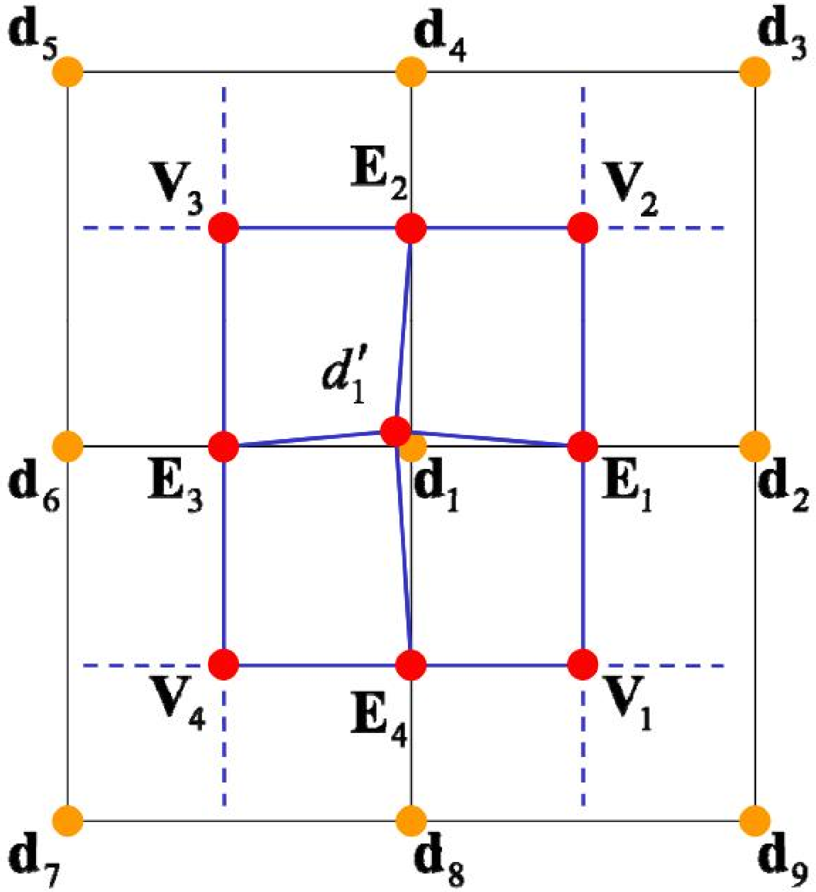
- 增加更多控制点,引入更多细节
- 往往会改变形状





网格细分

- **Catmull-Clark Subdivision**



$$\mathbf{V}_2 = \frac{1}{n} \times \sum_{j=1}^{n} \mathbf{d}_j$$

$$\mathbf{E}_i = \frac{1}{4}\left(\mathbf{d}_1 + \mathbf{d}_{2i} + \mathbf{V}_i + \mathbf{V}_{i+1}\right)$$

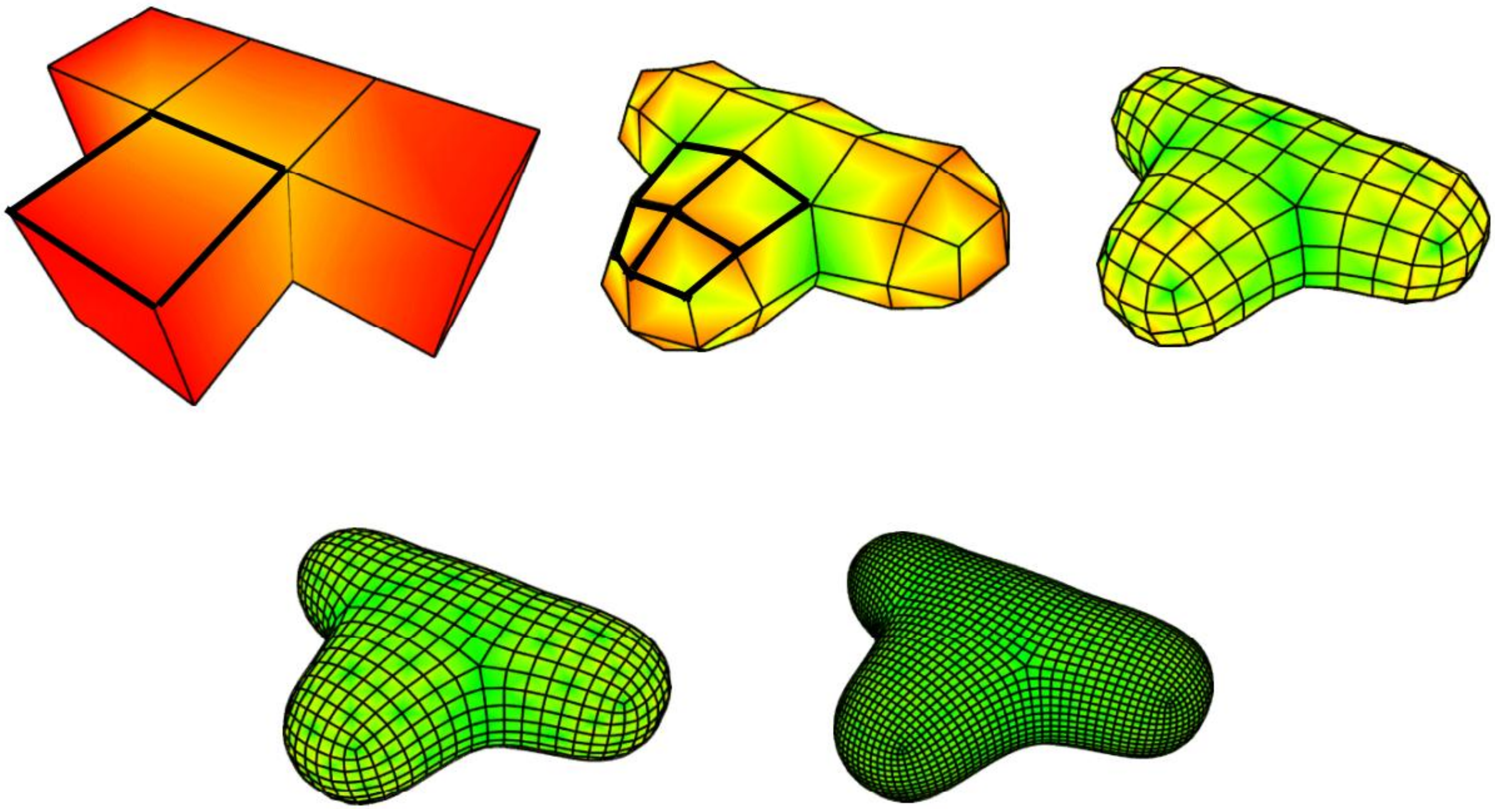
$$\mathbf{d}_1' = \frac{(n-3)}{n}\mathbf{d}_1 + \frac{2}{n}\mathbf{R} + \frac{1}{n}\mathbf{S}$$

$$\mathbf{R} = \frac{1}{m}\sum_{i=1}^{m}\mathbf{E}_i \quad \mathbf{S} = \frac{1}{m}\sum_{i=1}^{m}\mathbf{V}_i$$

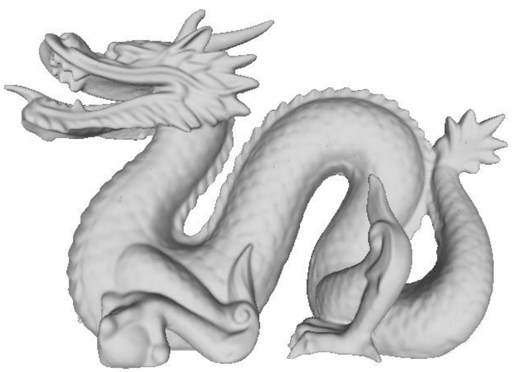
n is the number of faces containing d1

网格细分

- **Catmull-Clark Subdivision**

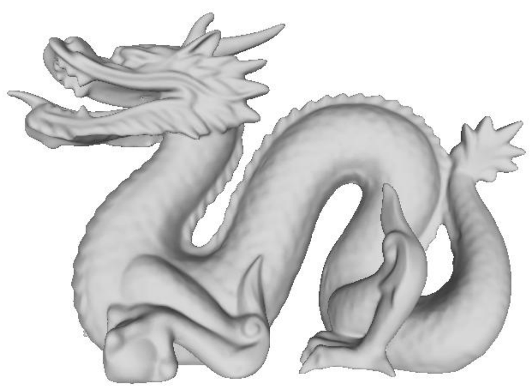


网格简化

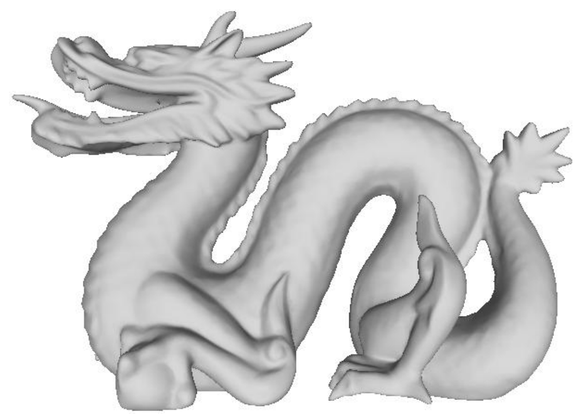


Vertex : 437645
Face : 871414

Vertex : 218121
Face : 435671

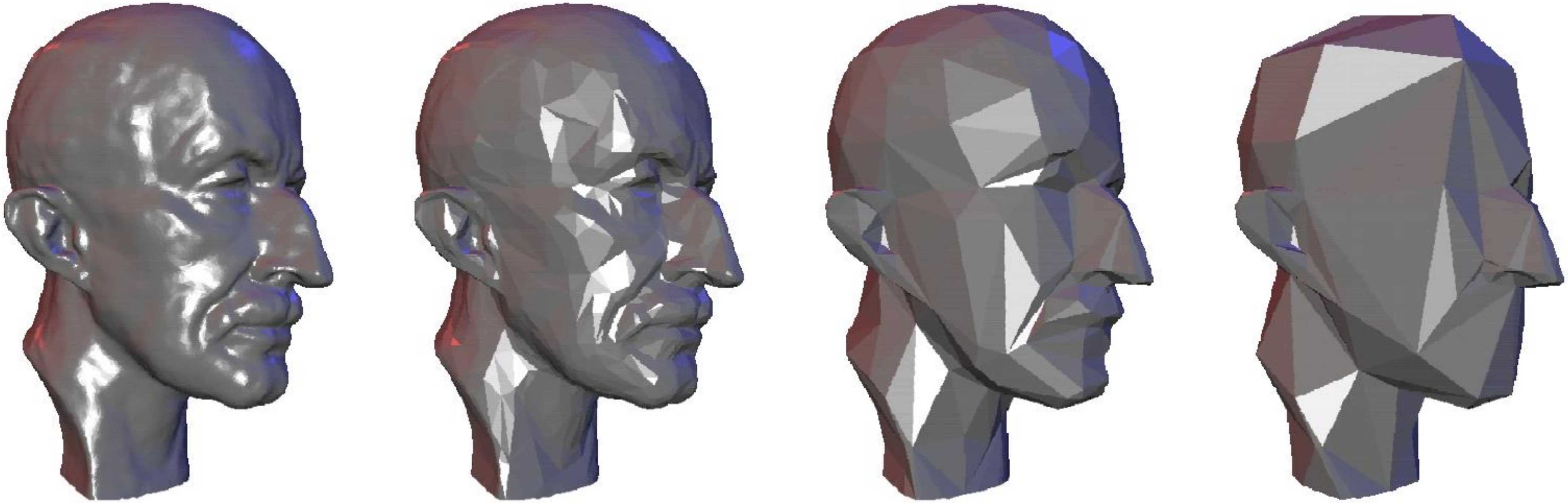


Vertex : 50073
Face : 99999



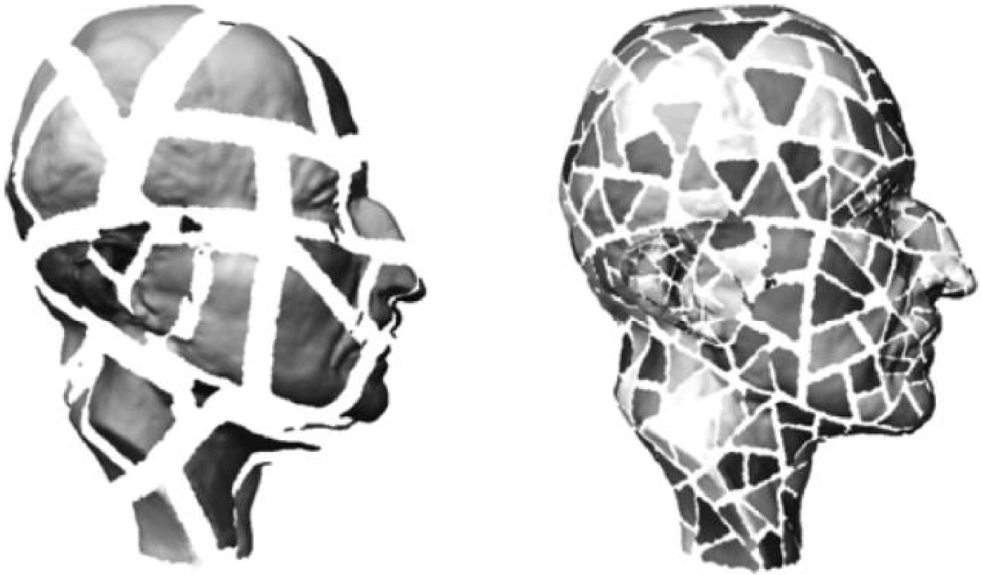
Vertex : 25000
Face : 49999

网格简化



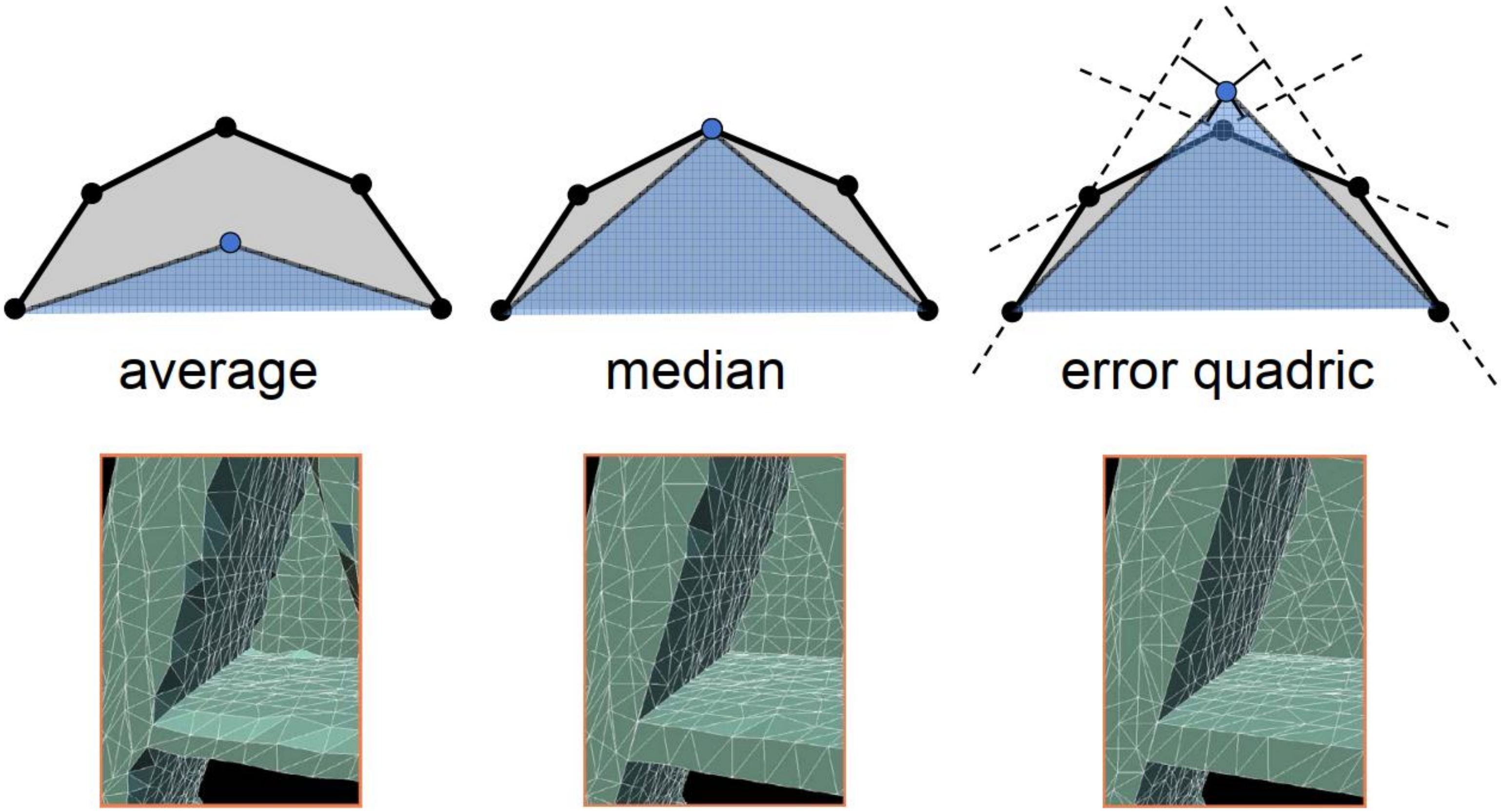
网格简化

- 基本思路：
  - 顶点聚类，把相近的顶点聚成一个代表点
  - 计算代表点的位置
  - 重新连接边



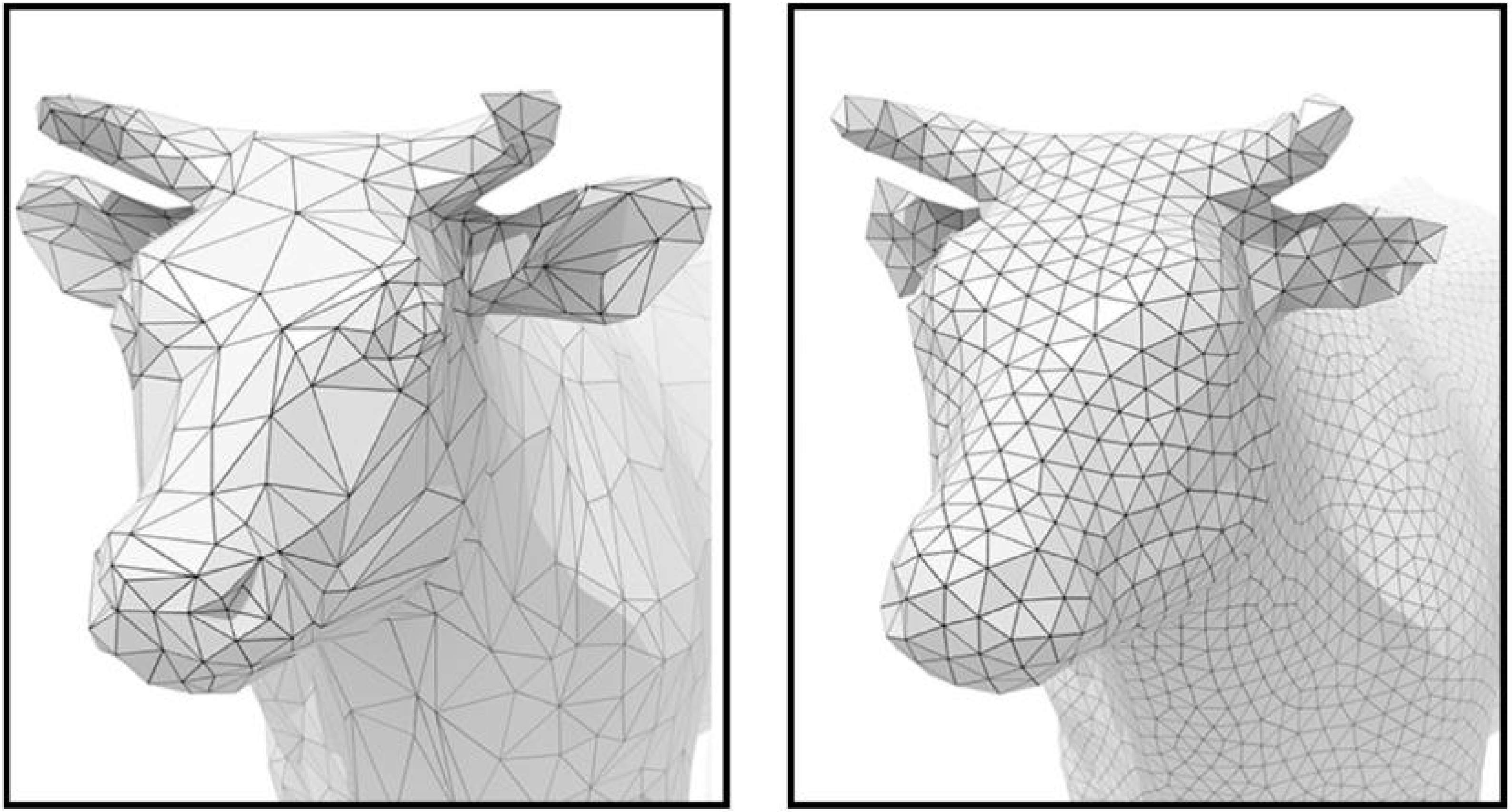
网格简化

- 计算代表点的位置



网格网格正则化（规范化）

- Mesh Regularization，改变网格的分布



Meshlab

- http://www.meshlab.net/