Financial Economics

Lecture 05. Valuation of bonds

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Outline

- **Introduction to bonds**
- PV formulas and valuation of known cash flows
- Pure discount bonds (zero-coupon bonds)
- Coupon bonds
- What determines yield

Fixed income security

- **A fixed-income security** refers to an investment that provides **a fixed stream of income** to the investor over **a predetermined period of time**.
  - Bonds are a common type of fixed-income security.
  - Other than bonds:
    - Certificates of Deposit (CDs)
    - Convertible Bonds
    - Asset-based securities
    - and so on

Introduction to bonds

- The issuer of bonds agrees to
  - Pay a fixed amount of interest periodically to the holder.
    - **Coupons:** quarterly, semiannually, annually
  - Repay a fixed amount of principal at the date of maturity.
    - **Par value** or **face value**
- **Maturity**
  - Bond: $\geq$10 yrs; notes: 1~10 yrs; money market: <1 yr
- **Yield**
  - Annualized return
  - Coupon yield (or coupon rate)=annual coupon/par value
  - Current yield=annual coupon/current price
  - Yield to maturity: the annualized return if holding the bond to maturity

Types of bonds

- Secured vs unsecured bonds
  - Secured bonds: backed by specified assets. Bond holder can claim the ownership of the specified property in the case of issuer's default. (Senior vs. Junior)
  - Unsecured: backed by promise and based on the general credit of the issuer
- Issuer's type
  - Government treasury bonds: issued directly by the nation's government
  - Municipal bonds: issued by the local governments. Risk can be much higher than government treasury bonds
  - Corporate bonds: issued by the companies. They have a wide risk range. (Yield rate can rage from 7%-8% to 30+%)

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- About bonds
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Using PV formulas to value known flows

- You have been offered the opportunity to purchase a mortgage.
  - It is a mortgage-based security
- The remaining life of the mortgage is 60 months, with payment of $400. Your required rate of return is **1.5% / month**
- How much would you like to offer?

Calculation

- Using the present value of an annuity formula, you will pay no more than

$$
\begin{aligned}
PV &= \frac{pmt}{i}\left(1-\left(\frac{1}{1+i}\right)^{n}\right) \\
&= \frac{400}{0.015}\left(1-\left(\frac{1}{1.015}\right)^{60}\right) \\
&= \$15{,}752.11
\end{aligned}
$$

Change in required rate

- If your required rate of return increased to **1.6% / month**

N	I	PV	PMT	FV
60	1.6%	? 15,354.66	-400	0

Using PV formulas to value known flows

- Observe that the maximum you would pay for the mortgage has decreased
- An increase in the required rate of return always leads to a decrease in the value of a **fixed-income security**
- The proof is very easy
  - Write the PV of the fixed income security as the sum terms

$$PV = \sum_{j=1}^{n}\left(pmt_j * \left(\frac{1}{1+i}\right)^j \right)$$
$$= pmt_1 * \left(\frac{1}{1+i}\right)^1 + pmt_2 * \left(\frac{1}{1+i}\right)^2 + \ldots + pmt_{n-1} * \left(\frac{1}{1+i}\right)^{n-1} + pmt_n * \left(\frac{1}{1+i}\right)^n$$

  - If i goes up, $1 + i$ goes up, $1/(1 + i)$ goes down, $(1/(1 + i))^j$ goes down for $i > 0$. So if the payments are positive, then the sum must also go down
  - Similarly, i down -> PV up

Using PV formulas to value known flows

- Basic principle in evaluating known flows
  - A change in market interest rates causes a change in the *opposite* direction in the market values of all existing contracts promising fixed payments in the future
- Volatile market rates imply volatile market values

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- PV formulas and valuation of known cash flows
- Introduction to bonds
- **Pure discount bonds (zero-coupon bonds)**
- Coupon bonds
- Why yields differ

Pure discount bonds

- Also called “zero-coupon bonds”
- A pure discount bond is a security that promises to pay a specified single cash payment (*face value* or *par value*) at its maturity date
  - There is no cash flow associated with interest before maturity
  - Pure discount bonds are purchased at a discount from their face or par value
- Yield to maturity (YTM) on a pure discount bond is the annualized rate of return to investors who buy it and hold it until matures

Pure discount bonds

- Solving this, the yThe pure discount bond is an example of the present value of a lump sum equation we analyzed in lecture 2
- Yield-to-maturity on a pure discount bond is given by the relationship:

$$F = P(1+i)^n \quad \Rightarrow \quad i = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1$$

- In this equation,
  - P is the present value or price of the bond
  - F is the face or future value
  - N is the investment period
  - i is the yield-to-maturity

- Example:
- A pure discount bond with a face value of $1,000 maturing in one year is sold at a price of $950
- What is the YTM?

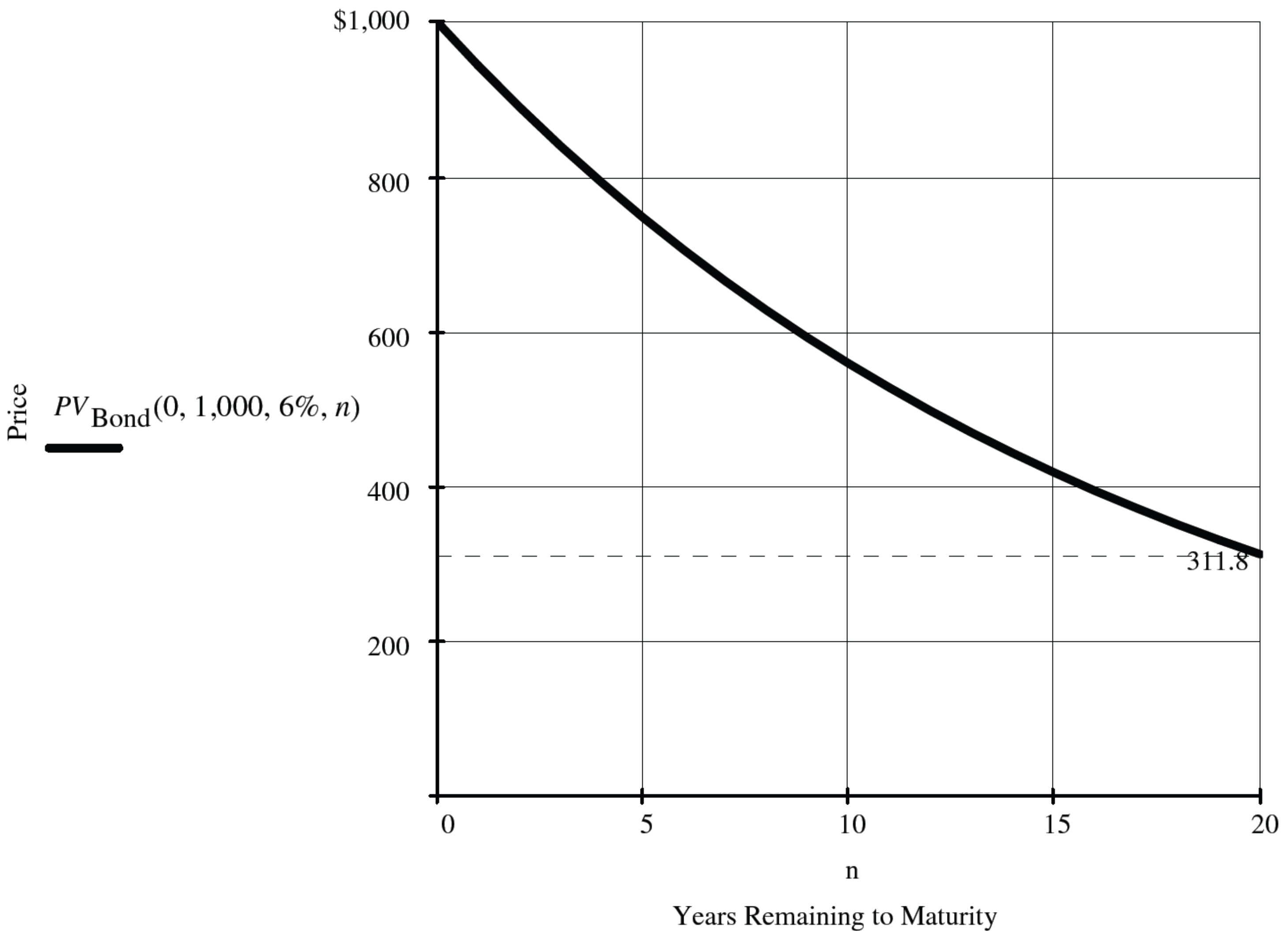
$$i = \left(\frac{F}{P}\right) - 1 = \frac{F - P}{P} = \frac{\$1{,}000 - \$950}{\$1{,}000} = 5.26\%$$

Pure discount bonds

- Example
  - You can purchase a pure discount bond for $9,000, and it matures in two years with a face value of $10,000
  - What is the YTM?

$$i = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1 = \left(\frac{10000}{9000}\right)^{\frac{1}{2}} - 1 = 5.41\%$$

A pure discount bond's price over time



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Coupon bonds

- A coupon bond obligates the issuer to
  - Make periodic payments of interest (called ***coupon payments***) to the bond holder until the bond matures
  - **Pay the face value** of the bond to the bond holder at the time of mature

Coupon rate

- The *coupon rate* is the interest rate applied to the face value to compute the coupon payment
  - A bond with a face value of $1,000 and a coupon rate of 10% pays an annual coupon of $100
  - At maturity, the payment is $1,000+$100
- Cash flows for 10% $1,000 coupon bond

Year	0	1	2	3	4	5	6
Coupon Face Value		100	100	100	100	100	100
							1,000

Current yield and yield-to-maturity

- The current yield is the coupon over price

$$Current\ Yield = \frac{Coupon}{Current\ Price}$$

- The *yield-to-maturity* is the discount rate that makes the present value of the cash flows from the bond equal to the current price of the bond

$$P = \sum_{t=1}^{T} \frac{Coupon}{(1+i)^t} + \frac{F}{(1+i)^n}$$

- In this equation,
  - P is the present value or price of the bond
  - F is the face or future value
  - N is the investment period
  - i is the yield-to-maturity
- We have the price of the coupon bond, and the timing and magnitude of its future cash flows, so we can determine its YTM

Par, premium, and discount bonds

- A coupon bond with its current price equal to its par value is a *par bond*
- If it is trading below par it is a *discount bond*
- If it is trading above par it is a *premium bond*

Bond pricing principle #1: (par bonds)

- If a bond's price equals its face value, then its yield-to-maturity = current yield = coupon rate.
- Proof:

$$P = \frac{pmt}{i}\left(1-\left(\frac{1}{1+i}\right)^n\right) + F\left(\frac{1}{1+i}\right)^n \quad \& \quad P = F \quad \Rightarrow$$

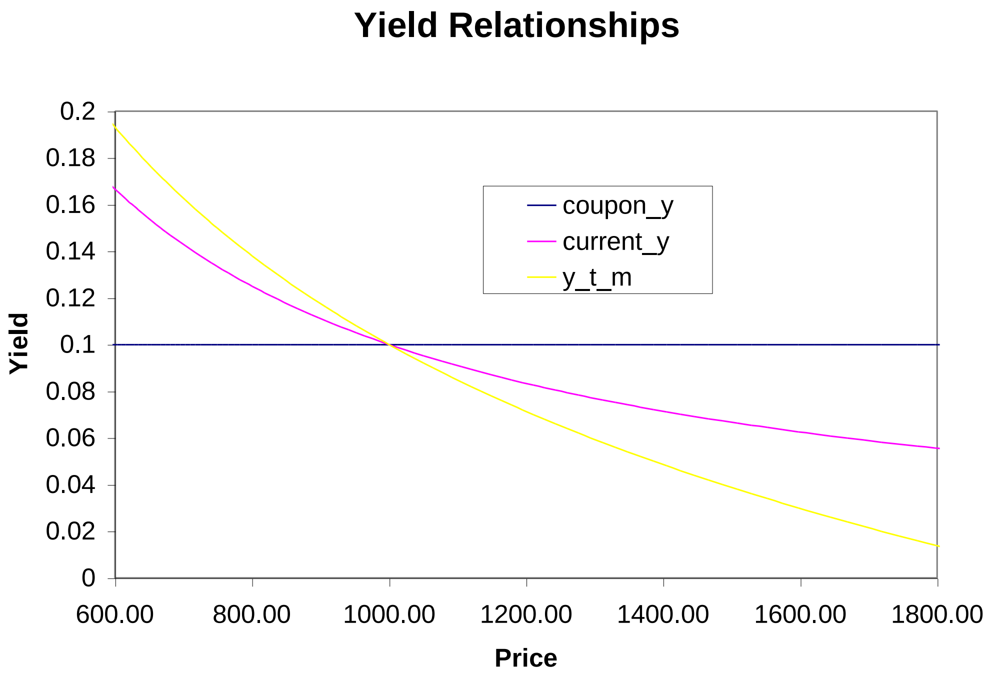
$$P\left(1-\left(\frac{1}{1+i}\right)^n\right) = \frac{pmt}{i}\left(1-\left(\frac{1}{1+i}\right)^n\right) \quad \Rightarrow \quad P = \frac{pmt}{i} = F$$

Bond pricing principle #2 & 3

- Bond pricing principle # 2: premium bonds
  - Bond price > face value $\Rightarrow$ YTM < current yield < coupon rate
- Bond pricing principle # 3: discount bonds
  - Bond price < face value $\Rightarrow$ YTM > current yield > coupon rate

How to remember principles

- Imagine that the bond was issued at par
  - The YTM moves from the coupon yield in the opposite direction to price
  - The coupon rate is unchanging
- This diagram may help:



Using pure discount bonds to value bonds

- Value a bond that pays its $100 coupon at the end of each year for 3-years, and its par value of $1,000 in 3-years
  - You have discovered three pure discount bonds (each with a $1,000 par value) that mature in 1, 2, and 3 years, and that are trading at $960, $890, and $810 respectively

Solution 1

- Use the fact that a coupon bond is the sum of pure discount bonds
- The coupon is equivalent to
  - a one-year pure discount bond with a face value of $100,
  - a two-year pure discount bond with a face value of $100,
  - a three-year pure discount bond with a face value of ($100+$1,000)

$$P = \frac{960}{1000}100 + \frac{890}{1000}100 + \frac{810}{1000}\left(1000 + 100\right)$$
$$P = \$1076.00$$

Solution 2

- First determine the yields-to-maturity of each discount bond
- Cash flows are then evaluated using them

$$i_{0,1} = \left(\frac{1{,}000}{960}\right)^{\frac{1}{1}} - 1 = 4.17\%$$

$$i_{0,2} = \left(\frac{1{,}000}{890}\right)^{\frac{1}{2}} - 1 = 6.00\%$$

$$i_{0,3} = \left(\frac{1{,}000}{810}\right)^{\frac{1}{3}} - 1 = 7.28\%$$

$$P = \frac{100}{1.0417} + \frac{100}{1.0600^2} + \frac{1000+100}{1.0728^3}$$

$$P = \$1{,}075.91$$

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Why yields for the same maturity differ

- The fundamental building block of bonds is the pure discount bond:
  - Coupon bonds may be viewed as a portfolio of discount bonds
- The rule of one price applies to bonds through pure discount bonds
- It is a mistake to assume that coupon bonds with the same life have the same yield
  - Their coupon rates differ, leading to a different % mix of discount bonds
  - Taxability
  - Callability
  - Convertibility
  - Risk

Are bonds risk free?

- Default risk (credit risk)
  - Moody's/S&P/Fitch ratings [link]
  - Highest AAA to Lowest C in Moody's/D in S&P/Fitch
- Interest rate risk/inflation risk
- Call risk/reinvestment risk
- Exchange rate risk
- Liquidity risk