

PROLOG

- PROLOG is a programming language
- PROLOG: PROgramming in LOGic
- Invented by Alain Colmerauer in 1972 in Marseille (France)
- Use of mathematical logic to represent knowledge
- Declarative language



Syntax

- **Constants** are character strings that begin with minuscule letters
 - e.g. `alice`, `edward`, `james_bond_007`
- **Variables** are character strings that begin with a capital or a `' '`
 - e.g. `X`, `_G341`, `Stranger`, `The_Older...`
- **Clauses** begin by the positive literals, then the sign `‘:-’` and the sequence of negative literals separated by comma (`“,”`); it always end with a dot (`“.”`)

```
soeur(X, Y) :- femme( X),  
              parents( X, Mere, Pere),  
              parents( Y, Mere, Pere).
```

```
soeur( X, Y)  $\vee$   $\neg$  femme( X)  $\vee$   
   $\neg$  parents( X, Mere, Pere)  $\vee$   $\neg$  parents( Y, Mere, Pere).
```

```
femme( X)  $\wedge$  parents( X, Mere, Pere)  $\wedge$   
  parents( Y, Mere, Pere)  $\Rightarrow$  soeur( X, Y)
```

- **Mode “consult”**

```
femme(alice) .  
femme(victoria) .  
homme(albert) .  
homme(edward) .  
parents(edward, victoria, albert) .  
parents(alice, victoria, albert) .
```

Set of clauses

- **Mode “query”**

```
?- femme(alice) .  
Yes  
?- homme(X) .  
X = albert  
Yes  
?- femme(X), parents(X, U, I) .  
X = alice  
U = victoria  
I = albert ;  
No
```

Data Bases in PROLOG

Atom Conjunction

- **Mode “consult”**

```
femme(alice).
```

```
femme(victoria).
```

```
homme(albert).
```

```
homme(edward).
```

```
parents(edward, victoria, albert).
```

```
parents(alice, victoria, albert).
```

```
soeur( X, Y) :- femme( X), parents( X, Mere, Pere),  
                parents( Y, Mere, Pere).
```

- **Mode “query”**

```
?- soeur(U, edward).
```

```
U = alice ;
```

```
No
```

```
?- soeur(U, V).
```

```
U = alice
```

```
V = edward ;
```

```
U = alice
```

```
V = alice ;
```

```
No
```

Set of Horn clauses

Deduction in PROLOG

Atom conjunction

Definition: a **positive Horn clause** is a clause (i.e. A disjunction of literals) that owns one and only one positive literal

Examples:

Positive Horn Clauses

femme(victoria) .

homme(edward) .

parents(alice, victoria, albert) .

soeur(X, Y) :- femme(X), parents(X, Mere, Pere),
parents(Y, Mere, Pere) .

Are Horn clauses

$\neg \text{barber}(X) \vee \text{shave}(X, Y) \vee \text{shave}(Y, Y)$ which means:

“the barbers shave those who don't shave by themselves” is not a Horn clause

$\neg \text{barber}(X) \vee \neg \text{shave}(X, Y) \vee \neg \text{shave}(Y, Y)$ which means *“there is no barber who shaves someone who shaves himself”* is not a positive Horn clause

Declarative programming

- Description of predicate properties:

```
nombre(0) .
```

```
nombre(s(X)) :- nombre(X) .
```

```
addition(0, X, X) .
```

```
addition(s(X), Y, s(Z)) :- addition(X, Y, Z) .
```

- Call in query mode:

```
?- nombre(s(s(0))) .
```

Yes

```
?- addition(s(s(0)), s(s(s(0))) (Z) .
```

Output

```
Z = s(s(s(s(s(0)))))
```

Yes

Result



- **Mode “consult”**

`nombre(0).` **`nombre(0) →`** Set of Horn clauses = **Rules**
`nombre(s(X)) :- nombre(X).` **`nombre(s(X)) → nombre(x)`**
`addition(0, X, X).` **`addition(0,X,X) →`**
`addition(s(X), Y, s(Z)) :- addition(X, Y, Z).`
 `addition(s(X), Y, s(Z)) → addition(X, Y, Z)`

- **Mode “query”**

Atom conjunction

`?- nombre(s(s(s(0))).`
`?- addition(s(s(0)), s(s(s(0))), X).`

- **Work of the Interpreter**

*The query is rewritten using rules
until it becomes empty.*

PROLOG for
“babies”

PROLOG strategy

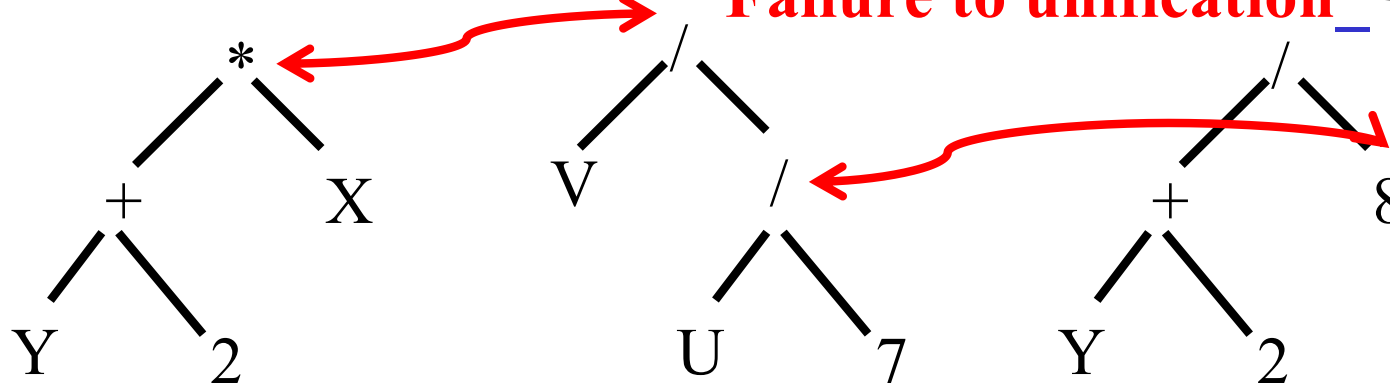
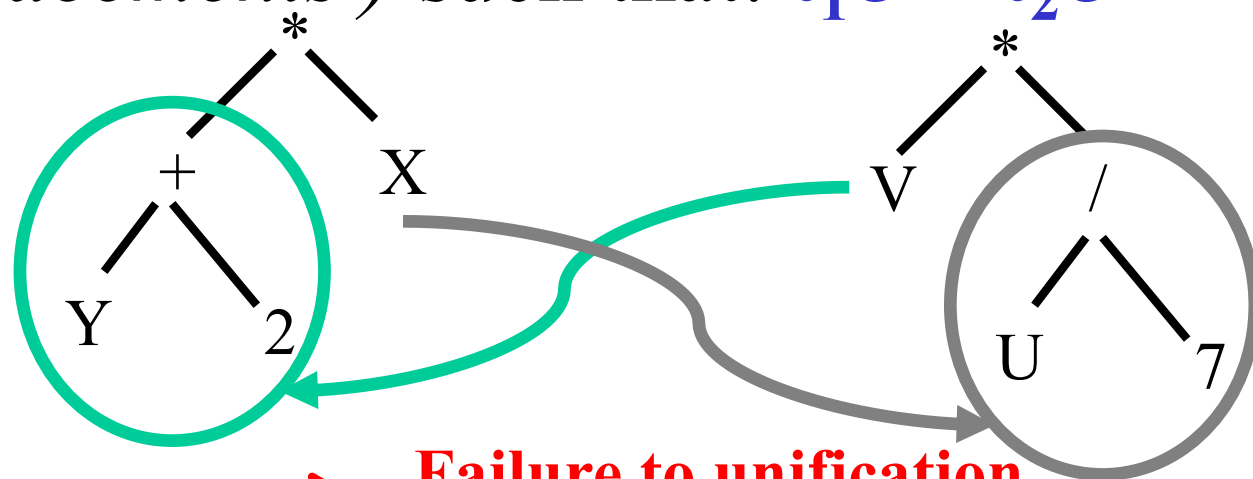
- PROLOG try to apply the “rules” (i.e. the clause) on the atoms of the “query”
- PROLOG makes use of the “unification” mechanism
- The atoms of the “query” are solved sequentially, from left to right.
- The clauses are considered one after one, from the first until the last.



Unification

Unify: to make unique

Unification: two terms t_1 and t_2 unify if and only if there exists a substitution σ (*i.e. a list of variable replacements*) such that: $t_1\sigma = t_2\sigma$



Example: addition

`:- addition(s(s(0)), s(s(s(0))), z)`

C1

C2

`:- addition(s(0), s(s(s(0))), z1)`

C1

C2

`:- addition(0, s(s(s(0))), z2)`

C1

C2



C1: `addition(0, X, X).`

C2: `addition(s(X), Y, s(Z)) :- addition(X, Y, Z).`

Function Inversion

- Description of predicate properties:

```
nombre(0) .
```

```
nombre(s(X)) :- nombre(X) .
```

```
addition(0, X, X) .
```

```
addition(s(X), Y, s(Z)) :- addition(X, Y, Z) .
```

- Call in query mode:

```
?- addition(s(s(0)), U, s(s(s(s(s(0)))))) .
```

```
U = s(s(s(0)))
```

```
Yes
```

```
?- nombre(X) .
```

```
X = 0 ;
```

```
X = s(0) ;
```

```
X = s(s(0)) ;
```

```
X = s(s(s(0))) ;
```

```
X = s(s(s(s(0))))
```



Program

- A program is a set of clauses of which head literal has the same predicate name and the same arity

- Examples

- Program addition: *[addition(X, Y, Z) means $X+Y=Z$]*

`addition(0, X, X).`

`addition(s(X), Y, s(Z)) :- addition(X, Y, Z).`

- Program multiplication: *[multiply(X, Y, Z) means $Z=X.Y$]*

`multiply(0, X, 0).`

`multiply(s(N), P, R) :- multiply(N, P, Q),
addition(P, Q, R).`

- Programme factorial: *[fact_s(N, P) means $P = N!$]*

`fact_s(0, s(0)).`

`fact_s(s(0), s(0)).`

`fact_s(s(N), R) :- fact_s(N, Q),
multiply(s(N), Q, R).`



Procedure call

- A call to a procedure is an atom in the “query” mode:

Examples :

?- fact_s(s(s(s(0))), U).

U = s(s(s(s(s(s(0))))))

Yes

?- multiply(s(s(0)), s(s(s(0))), R).

R = s(s(s(s(s(s(0))))))

Yes

?- multiply(s(s(0)), H s(s(s(s(s(s(0)))))).

H = s(s(s(0)))

Yes

Output

Input



Example addition and/or tree

`:- addition(s(s(0)), s(s(s(0))), Z)`

|
C2 $\{X1 = s(0),$
 $Z = s(Z1)\}$
|

`:- addition(s(0), s(s(s(0))), Z1)`

|
C2 $\{X2 = 0,$
 $Z = s(Z1) = s(s(Z2))\}$
|

`:- addition(0, s(s(s(0))), Z2)`

|
C1 $\{Z2 = s(s(s(0)))\}$
|

□ **1st solution:**
 $Z = s(s(Z2)) = s(s(s(s(0))))$



C1: addition(0, X, X) .

C2: addition(s(X), Y, s(Z)) :- addition(X, Y, Z) .

Example addition: derivation tree

`:- addition(s(s(0)), s(s(s(0))), z)`

C1

C2

`:- addition(s(0), s(s(s(0))), z1)`

C1

C2

`:- addition(0, s(s(s(0))), z2)`

C1

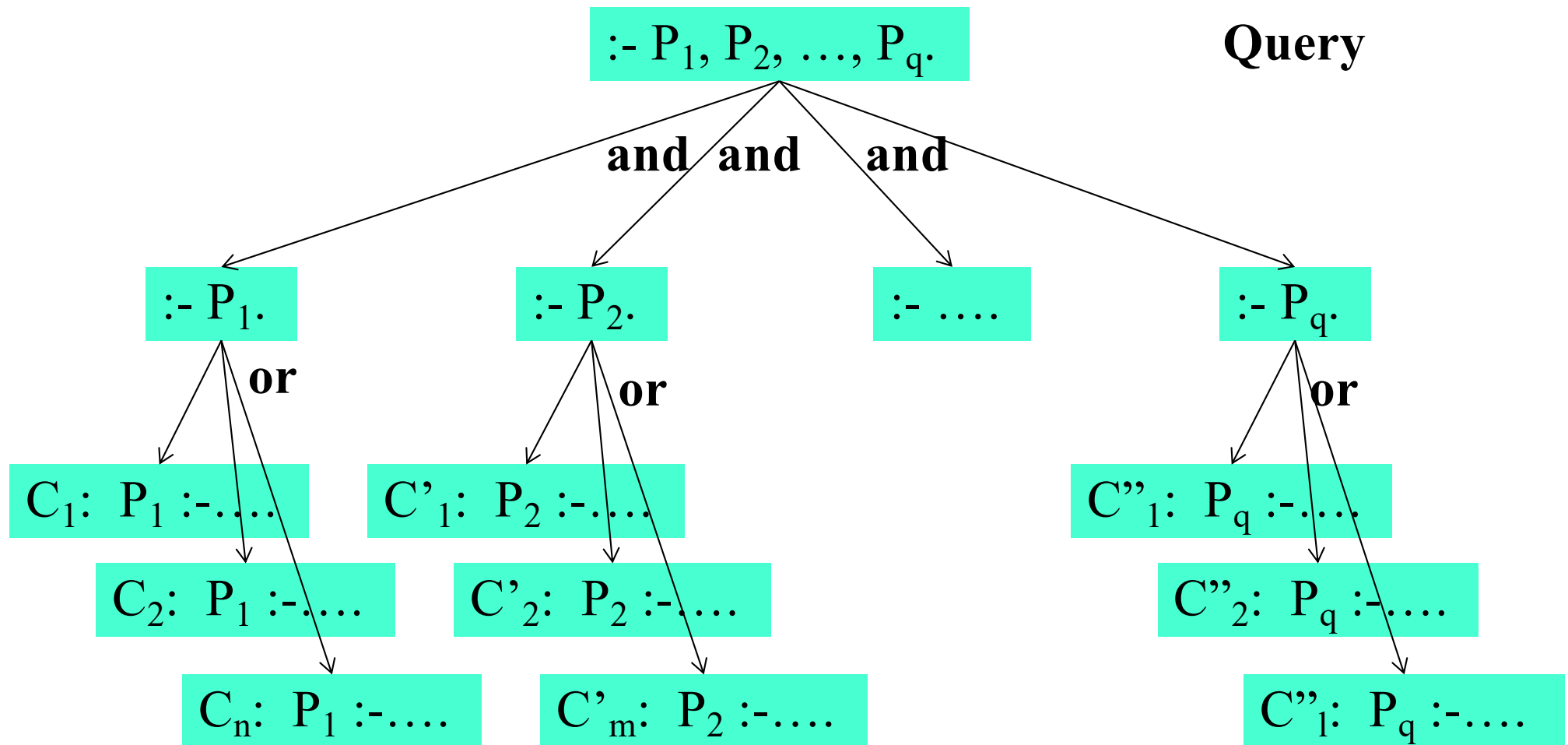
C2



C1: `addition(0, X, X).`

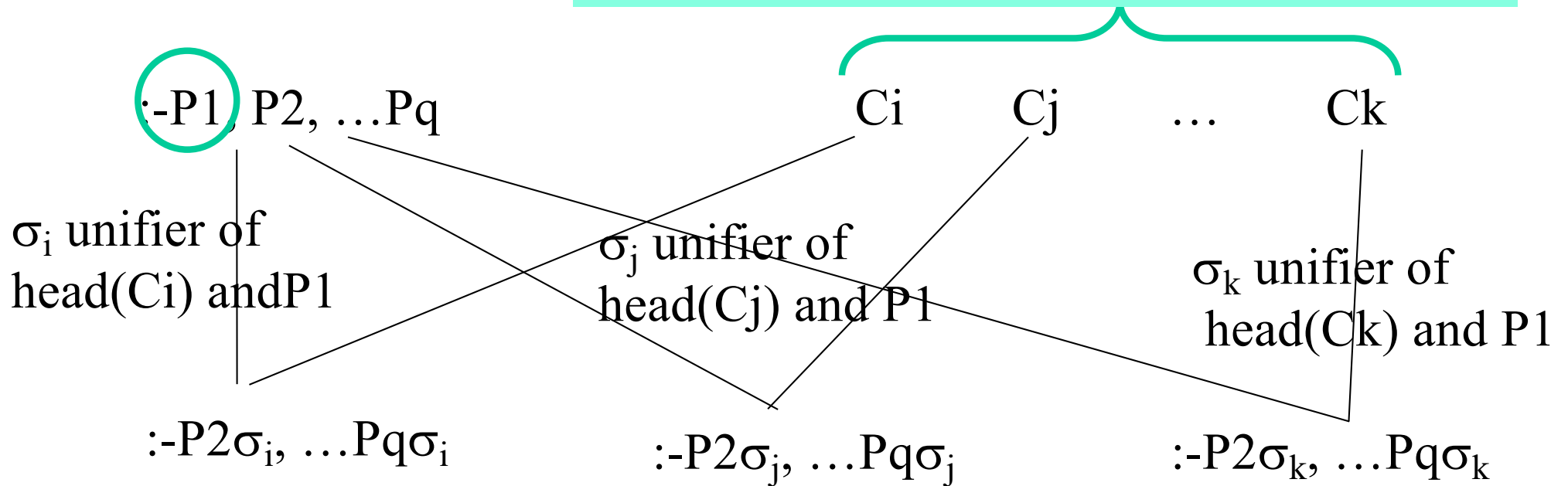
C2: `addition(s(X), Y, s(Z)) :- addition(X, Y, Z).`

Procedural interpretation: and/or tree



Procedural interpretation: derivation tree

Clauses of which head literal unify with P1



Data structures

- Terms:
 - Variables
 - Function with arguments: $f(a, g(h, I), K)$
- Atoms:
 - Character strings (beginning with a minuscule)
 - Numbers
- Lists:
 - Empty lists, list sequences
- Strings
- Numbers



List structure

- Defined by the *constructor*, **cons** and by the *empty list*, **nil**
 - Form: **cons**(<head>, <queue>)
 - <head> is any literal
 - <queue> is a list

Examples:

- The list (a b c) can be represented by:
cons(a, cons(b, cons(c, nil)))
- The tree (+ (- b c) (* d e)) can be represented by:

```
cons(+, cons( cons(-, cons(b, cons(c, nil))),  
      cons ( cons(*, cons(d, cons(e, nil))),  
          nil),  
      nil),  
      nil)
```



List structure (2)

- Simplification:
 - The list $(a\ b\ c)$ is written $[a, b, c]$, which means $\text{cons}(a, \text{cons}(b, \text{cons}(c, \text{nil})))$
 - In the same way, $(+ (- b\ c) (* d\ e))$ is written $[+, [-, b, c], [*, d, e]]$
- List constructor: ‘|’

Example: the list (a, b, c) is written:

- $[a, b, c]$
- $[a\ |\ [b, c]]$
- $[a\ |\ [b\ |\ [c\ |\ []]]]$
- $[a, b\ |\ [c]]$



Program example

test if an element belong to a list

- Program « **appartient** » with terms:

```
appartient_t(A, cons(A, B)) .  
appartient_t(A, cons(B, C)) :-  
    appartient_t(A, C) .
```

- Program « **appartient** » with PROLOG representation :

```
appartient0(A, [A|B]) .  
appartient0(A, [B|C]) :- appartient0(A, C) .
```



Execution

C1: appartient (X, [X|L]).

```
C2: appartient(X, [T|L]) :- appartient(X, L).
```

```
:- appartient(3, [1, 2, 3, 4])
```

c2

$$\sigma_1 = \{x_1/3, y_1/1, l_1/[2, 3, 4]\}$$

```
:- appartient(3, [2, 3, 4])
```

c2

$$\sigma_2 = \{x_2/3, y_2/2, l_2/[3, 4]\}$$

```
:- appartient(3, [3, 4])
```

C1

c2

$$\sigma_3 = \{x_3/3, \mathbb{L}_3/[4]\}$$


Other examples with lists

Concatenation of two lists:

```
concat([], M, M) .
```

```
concat([A|L], M, [A|N]) :- concat(L, M, N) .
```

List inversion:

```
miroirnaif([], []).
```

```
miroirnaif([A|Q], R) :- miroirnaif(Q, P),  
                        concat(P, [A], R) .
```



Execution

C1: `concat([], M, M).`

C2: `concat([A|L], M, [A|N]) :- concat(L, M, N).`

`:- concat([a, b], [c, d, e], R)`

C2

$\sigma_1 = \{A1/a, L1/[b], M1/[c, d, e], R/[A1|N1]\}$

`:- concat([b], [c, d, e], N1)`

C2

$\sigma_2 = \{A2/b, L2/[], M2/[c, d, e], N1/[A2|N2]\}$

`:- concat([], [c, d, e], N2)`

C1

$\sigma_3 = \{N2=M3=[c, d, e]\}$



$R = [A1|N1] = [a|[A2|N2]] = [a | [b | [c, d, e]]] = [a, b, c, d, e]$

Univ (“=..”): from list to terms

- If L is a list $X =.. L$ unifies X with a term of which functor is the L head and arguments the L tail

?- $X = [f, a, b, c].$

$X = f(a, b, c)$

- If T is a term $T =.. L$ unifies L with a list of which head is the T functor and tail is the list of T arguments

?- $g(a, b, c, d) = L.$

$L = g(a, b, c, d)$

