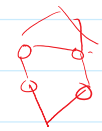
Lattice & Boolean Algebra

23 September 2020 17:53

Unit-3. Lattice and Boolean Algebra

Unit-3:
- Lattice theory → POSET. ($(S, \leq)$) — ↑ → Diagraph
  - Product partial order.
  - Hasse diagram →
  - Sup(s) ✓ Max. ✓ Greatest
  - Inf(s) ✓ Min. ✓ Least
  - → Lattice → Defn →
- Boolean Algebra →
- Appl^n of Lattice theory.



Boolean Algebra → Digital Logic →
- → K-map →
- → SOP
- → POS

Unit-4 → ($L, *$) →

Lattice → Boolean Algebra →

Lattice ⇄ Boolean Algebra

24 September 2020 16:11

Lattice → $Sup(a,b) = a \vee b$ = Read ('a join b') = $LUB(a,b)$ → LCM

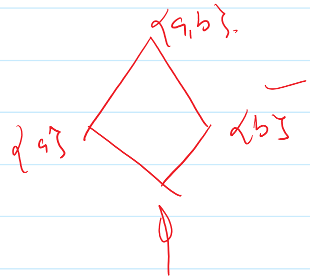
$Inf(a,b) = a \wedge b$ = Read it as ('a meet b') = $GLB(a,b)$ → GCD

→ $a \to A$, $b \to B$

$Sub(A,B) = LUB(A,B) = A \cup B$ → $L = \{P(A), \subseteq)$

$Inf(A,B) = GLB(A,B) = A \cap B$

→ Simple set operations.



→ $A = \{a,b\}$, $(P(A), \subseteq)$

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Lattice :- $\vee, \wedge$ P.L. < B.A. >

Important Results.

L, $a, b, c \in L$

1) $a \wedge b \leq a$, $b \leq a \vee b$

2) $a \leq b \Leftrightarrow a \wedge b = a$ Consistency

$a \leq b \Leftrightarrow a \vee b = b$

3) Idempotency. $a \wedge a = a$

$a \vee a = a$

Absorption law: $\rightarrow$ $a \wedge (a \vee b) = a$

$a \vee (a \wedge b) = a$

4) $a \wedge b = b \wedge a$ Commutative

$a \vee b = b \vee a$

5) Associative

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

$a \vee (b \vee c) = (a \vee b) \vee c$

6). Domination laws. $\rightarrow$ greatest $\rightarrow$ 1

If $0, 1 \in L$. least $\rightarrow$ 0

$0 \wedge a = 0$, $0 \vee a = a$

$1 \wedge a = a$, $1 \vee a = 1$

Distributive inequalities law:

a, b, c.

$a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

$a \wedge (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$.

Another def^n of a lattice :→

→ A non-empty set L with two binary operation V and $\wedge$ is said to form a lattice iff $\forall a, b, c \in L$,

Lattice

1) Idempotency → $a \wedge a = a$, $a \vee a = a$
2) Commutative → $a \wedge b = b \wedge a$, $a \vee b = b \vee a$
3) Associative → $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ — $\vee$: join.
4) Absorption →

$$a \wedge (a \vee b) = a$$

$$a \vee (a \wedge b) = a$$

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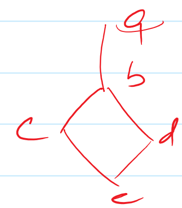
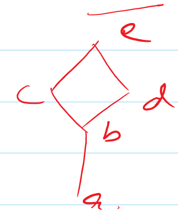
1. Dual of a lattice is also a lattice.





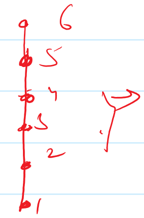
2. Product → → lattice.

(set, Relation)

3. Every chain is a lattice

$(Z^+, \le)$

1, 2, 3, 4, 5, 6

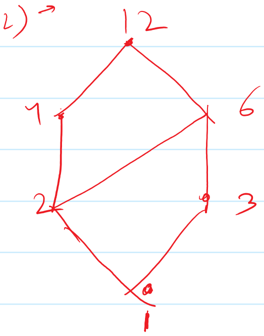


4. $(D_n, |)$ → Division

Set of divisors of n,

$(D_{12}, |)$ → Hasse diagram

(1, 2, 3, 4, 6, 12) →



$(D_{25}, |)$

{always a lattice}

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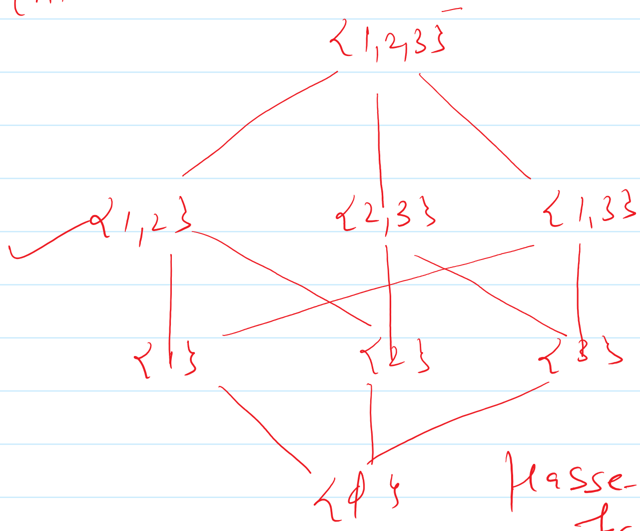
Sup (A,B)

(Z^+, |) → lattice

lattice

($P(A)$, $\subseteq$) → lattice

$A = \{1,2,3\}$ Hasse-diagram



Sup(_ _) = $A \cup B$

Inf (_ _) = $A \cap B$

$\{1,2\}, \{2,3\} \Rightarrow = \{1,2,3\}$

Inf

$A \cap B$

($P(A)$, $\subseteq$) ⇒ lattice

Hasse-diagram for it

Types of lattice :→

Bounded lattice :> $(L, \leq)$ → greatest - 1 $\quad \forall a \in L, \quad 0 \leq a \leq 1$

least - 0 $\quad 0 \wedge a = 0 \quad 0 \vee a = a$

$(P(A), \subseteq)$ → $A = \{a, b\}$ $\quad \phi, \{a, b\}$ $\quad 1 \wedge a = a \quad 1 \vee a = 1$

Complemented lattice $a \in L; \; a' \in L$ for each element $a \in L$, atleast one Complement exist, then → Complemented lattice

Ex → $a \vee a' = 1$ → greatest

→ $a \wedge a' = 0$ → least

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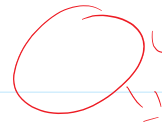


lattice

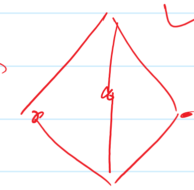
Distributive Lattice :-

Distributive laws.

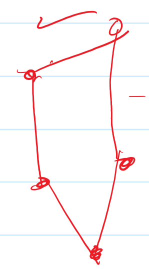
$\forall\, a, b, c \in L.$

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$



Kite



pentagon

To check:

→ A lattice is non-distributive iff it contains a sublattice, isomorphic to one of the non-distributive lattices.

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Bounded,

Distributive

Complemented lattice → ϕ →

Complete → $(L, \leq)$ → 1, lattice

every subset of L → Sup → linear.

Inf →

Sub-lattice :→

Lattice L.

↳ S, → $a, b \in S \Rightarrow a \wedge b, a \vee b \in S$

$(S, \vee, \wedge) \rightarrow$ Sub-lattice to lattice L.

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Boolean Algebra → ~~Small break~~ till 05:05 pm }.

Boolean Algebra

Bounded. A Lattice is a boolean Algebra if it is Bounded

Complemented

Distributive

BCD Lattice is a B.A.

Another Def^n of B.A. →

	Inf	Sup
set,	$\wedge$,	$\vee$
	$\cdot$,	$+$

6 - Axioms:

1) Closure $\forall a, b \in S$, $a + b \in S$, $a \cdot b \in S$

2) Commutativity: $\forall a, b \in S$, $a + b = b + a$, $a \cdot b = b \cdot a$

3) Associativity - {CCADEE}

$S, +, \cdot$

4) Distributivity → $\forall a, b, c \in S$ $a + (b \cdot c) = (a + b) \cdot (a + c)$

$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

B.A

5) Existence of Identity. $\forall a \in S$, $\exists e$ (unique) such that $a + e = e + a = a$.

6) Existence of Complement → $\forall a \in S$, $a + a' = a' + a = 1$ →

$a \cdot a' = a' \cdot a = 0$

Other derived laws for B.A.

$\vee \to +$

$\wedge \to \cdot$

BCD lattice is

Another diff

difficulty

1. Idempotent $a+a=a$
$a \cdot a = a$

2. $(a')' = a$

3. $a + a \cdot b = a$
$a \cdot (a+b) = a$

4. $(a+b)' = a' \cdot b'$
$(a \cdot b)' = a' + b'$ } De-morgan's law

$\} \to$ B.A. $\}$

Lattice Axioms

5. $a+0=a$ $a \cdot 0 = 0$
$a+1=1$, $a \cdot 1 = a$ } Domination's laws

24 September 2020 17:15

Operator precedance in Boolean Expressions. } ⇒ Digital logic

1. ⟶

Logic - Circuits

2. ( ) highest precedance

Complement

·

+ lowest

Algebra ⟶ { K-map }

B.A ⇒ $(S, \wedge, \vee)$

$(S, +, \cdot)$

(a, b) ⟶ }

$A + B \cdot C \Rightarrow A + (B \cdot C)$

$\overline{(A+B)} \rightarrow (A+B) \rightarrow$ Complement ⇒

B. A

24 September 2020 17:19

→ Simplification of Boolean Expression

$(a \oplus a b)'(a \oplus b) = ??$ → $a'b$

Known

→ Lattice fn

Lattice

Unit 3. Lattice L — Ch 15 Schaum's Series DM → (General

— Boole A → Kenneth Rosen

→ Appⁿ of lattice → Kenneth Rosen Ch 9

→ 9.6 & 7