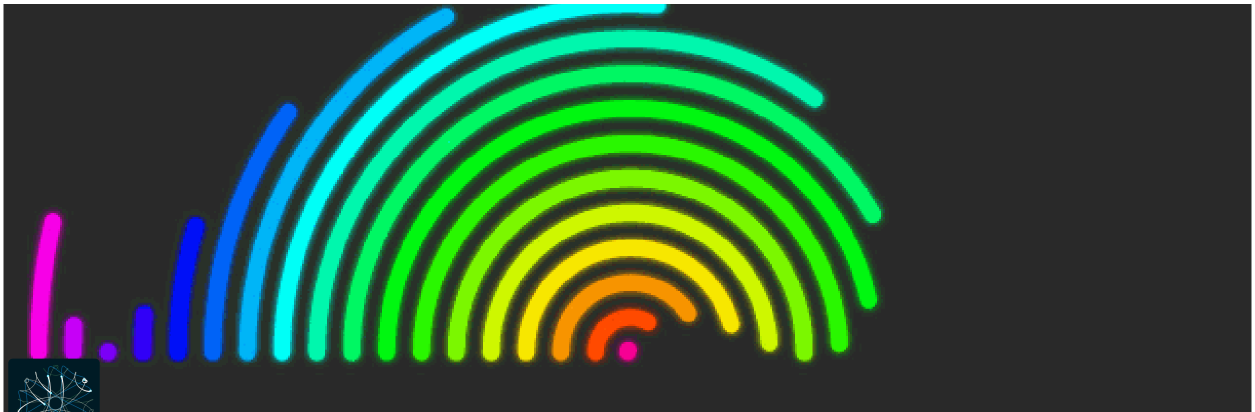


Discreet Mathematics

refer these books:

https://www.math.uh.edu/~dlabate/settheory_Ashlock.pdf

https://web.stanford.edu/class/cs103x/cs103x-notes.pdf

deep set theory (not recommended, but quite interesting 😅):

Set Theory

Set theory, as a separate mathematical discipline, begins in the work of Georg Cantor. One might say that set theory was born in late 1873, when he made the amazing discovery that the linear continuum, that is, the real line, is not countable, meaning that its points cannot be

https://plato.stanford.edu/entries/set-theory/#AxiSetThe

> "sets are the best shit there is "-me

Set theory :

everything is a set and can be represented by sets , but what is a set , I'm glad you asked

Set :

A set is an unordered collection of distinct objects. The objects in a
set are called the elements, or members, of the set. A set is said to contain its elements.

there are many types of sets :

Empty set / null set{ Φ }:

an empty set Φ is just a set with nothing in it , just like your life .

Singleton set :

a set with a single elements in it . just one .

{1} , {2}, {3}

Universal set {∞} :

a set with everything in it , all the possible outcomes . everything .

{∞}

Subset / Superset :

a sub set is just a set with elements similar to a bigger sets . all the elements of subsets are present in the bigger set called the super set

for eg

Arr A[ ] = {1,2,3,4,45};

Arr B[ ] = {1,2,3};

soo we can say that **B is a subset of A .**

or B $\subseteq$ A.

or we could also say that **A is a super set of B .**

Power set {2^A}:

power sets are sets of all subsets of a set .

let arr A[ ] = {2, 4, 17, 23};

then power set of a is

2^A **= {∅, {2}, {4}, {17}, {23}, {2, 4}, {2, 17}, {2, 23}, {4, 17}, {4, 23}, {17, 23}, {2, 4, 17}, {2, 4, 23}, {2, 17, 23} {4, 17, 23}, {2, 4, 17, 23}}**

then notice power set has length = 2^ length of A.

Compliment of a set {Ac (or A′)}:

basically everything else except the set

A′ = U - A

U = universal set

different set algebra :

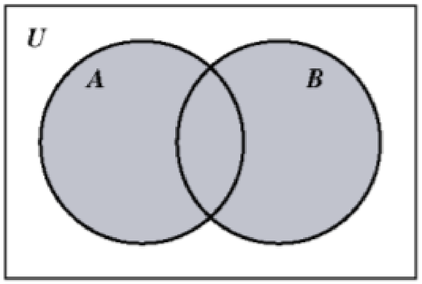
union :

a ⊔ b is nothing but , all elements of both sets together

a = {1,3,4,5}

b = {1,2,3,4}

a ⊔ b = {1,2,3,4,5}



$A \cup B$

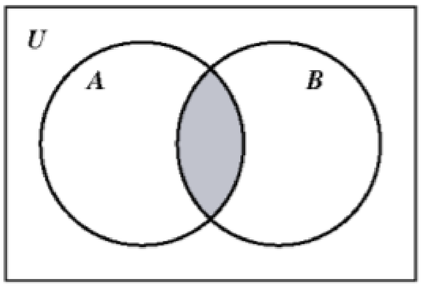
intersection :

a ∩ b is nothing but , the elements common in both sets .

a = { 1,2,3,4}

b = {2,4,6,8}

a ∩ b = {2,4}



$A \cap B$

Difference:

The difference of A and B, written A – B, is the set whose elements are the elements of A which are not in B.

A – B ={x | x in A and x not in B}.

Examples: Let

K = {a, b}, then

L = {c, d} and K – L = {a, b}

M = {b, d} K – M = {a}

L – M = {c}

symmetric difference:

The symmetric difference of sets A and B, denoted by A□B, consists of those elements which belong to 'A or B' but not 'A and B'.

$(A \oplus B) = (A \sqcup B) - (A \cap B)$. We can also write

$(A \oplus B) = (A - B) \sqcup (B - A)$.

things that are new :

let A1,A2,A3 be three sets , then ,

$$\bigcap_{i=1}^{3} Ai \quad = A1 \cap A2 \cap A3$$

same with super union . nothing more .

cardinality :

cardinality of a set is basically the no of elements in the set , for eg:

A = {1,3,5,6}

cardinality of a = 4.

The cardinality of A × B

is N*M, where N is the Cardinality of A and M is the cardinality of B.

Disjoint

Two sets are said to be disjoint if their intersection is the empty set .i.e sets have no common elements.

Relations :

a relation is a similarity between two sets like a chain in between two seemengly unrelated objects for eg .

2 and 6 . have a relation that 2 X 3 = 6.

```
the relation between sets
a = {1,2,3,4,5}
b = {2,4,6,8,10}
is the relation that for all values in a , we have relation 2X for b
```

there are mainly 12 types of relations that we need to focus on (sab hard nahi h) :

- **null relation**

  there is no relation between the two sets , none at all .

- **one to one**

  A function is called one to one if for all elements a and b in A, **if f(a) = f(b)**,then it must be the case that **a = b**. It never maps distinct elements of its **domain**
  to the same element of its **co-domain**. mtlb agar a main h and b main bhi same number h toh one to one function nahi banate ok .

  this relation is also called as **injective function , this is the easiest explanation of this function**

  for eg →

```
let a = { 1,2,3,4,5};
let b = { -1,-2,-3,-4,-5};

then f(x) = x^2  is  a one-to-one function ;
f(a[0]) == f(b[0]);
same for all;
```

  another thing to note here is →

  if |A| < |B| then every function is one to one , because har ek element in A ke liye koi naa koi element h , bacha hua such that

  > f(a[0]) == f(b[0]); is satisfied

  similarly , if |A| >|B| then there is no onto function , like koi function har ek element in A ke liye har ek element in B bana skta

- **onto**

  Surjective function or onto is a function in which every element In the domain if B has atleast one element in the domain of A such that f(A)=B.

  for eg→

```
let a = {1,2,3,4,5};
let b = {25,50,75,100,125};

then f(x) = 25*x is an onto function ;
becoz f(a[0]) = b[0];
same for all ;
```

- **Bijective relation (both one to one and onto)**

- **Reflexive Relation**

  for math savvy people→ { a function is reflexive iff for a in A (a,a) in R }

  mtlb , for every element in the set A the relation is true for (a,a) for eg →

```
R = { a,b in whole| a=b} ; is a reflexive function becoz every element is = to itself
R = {a,b in whole| a^2 = a*b} ; is reflexive in nature . becoz every element squared is equal to a*a;
R = {a,b in natural | a = 2b}; is not reflexive because there is no  natural no whose twice is equal to itself .
```

Symmetric Relation

for math savvy people → {a function is reflexive when if (a, b) belongs to R then (b,a) also belongs to R}

arthat , for every element in set A , if (a,b) is true for relation R then , (b,a) is also true . for eg→

```
R = { a,b in natural| a+b !> a*b}; because in natural numbers there are no numbers such that
 their sum is greater than their product and becoz (a+b) = (b+a) , (a*b) = (b*a)
this relation is symmetric .

 R ={a,b in natural| a=b} ; is also symmetric . because a = b is same as b=a .
```

Anti-Symmetric Relation

Transitive relation

for phd in maths ppl → { for all in A if (a,b) in R and (b,c) in R then (a,c) in R }

ie , if (a,b) satisfies R , and (b,c) satisfies R then , (a,c) also satisfies R for eg→

```
R = {a , b in A | a= b } , here if a = b  , b = c , then a = c .  simple stuff
```

|A| = m and |B| = n, then

1. No. of functions from A to B = n^m (proof →

```
let A = {a,b,c} ;
let b = {1,2,3};
so for every element in A ;
it can have 3 relations -> for eg -> a = (a,1) , (a,2) ,(a,3) so it can have only 3 relations .
same for every other element
so , in total ,
no of relations = 3 * 3 * 3 ;
                = 3^3;
which would be the correct answer . hence by generalizing this eg in our formula
every element in A would have |B| relations
hence the answer would be |B| * |B| * ... till all elements in A .
hence |B|^|A|.
where n = |B| {no of elements in B }
m = |A| {every element in A}.
hence no of functions = n^m.
```

2. No. of one to one function = (n, P, m)

3. No. of onto function =n^m– (n, C, 1)*(n-1)^m+ (n, C, 2)*(n-2)^m…. +(-1)^m-(n, C, n-1), if m >= n; 0 otherwise

4. Necessary condition for bijective function |A| = |B|
5. The no. of bijection function =n!
6. No. of relations =2^mn
7. No. of reflexive relations =2^n(n-1)
8. No. of symmetric relations = 2^n(n+1)/2
9. No. of Anti Symmetric Relations = 2^n * 3^(n(n-1)/2)
10. No. of asymmetric relations = 3^(n(n-1)/2)
11. No. of irreflexive relations = 2^(n(n-1))