Formula Sheet

1 Definition 1.1

Mean of a sample with n values:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

2 Definition 1.2

Variance of a sample: overall distance of values from the mean:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

3 Definition 1.3

Standard Deviation (s): square root of variance (s^2):

$$s = \sqrt{s^2}$$

4 Theorem 2.5

Multiplicative Law of Probability: The probability of the intersection of two events:

1. If dependent, $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$.
2. If independent, $P(A \cap B) = P(A)P(B)$.

Additive Law of Probability: The probability of the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If $P(A \cap B) = 0$ (mutually exclusive events), then $P(A \cup B) = P(A) + P(B)$.

5 Definition 2.6

Probability of an event A within a sample space S, such that $A \subseteq S$. The following are true:

1. $P(A) \geq 0$
2. $P(S) = 1$
3. If $(A_1, A_2, \ldots, A_n)$ are mutually exclusive, then $P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i)$.

6 Definition 2.7

Permutation: ordered arrangement of r distinct objects, with n possible orders:

$$P_n^r = \frac{n!}{(n-r)!}$$

7 Theorem 2.7

If A is an event, then $P(A) = 1 - P(\bar{A})$.

8 Definition 2.8

Combination: number of subsets of size r that can be formed from n objects:

$$C_n^r = \frac{n!}{r!(n-r)!}$$

9 Theorem 2.8

Total probability: assume Definition 2.11. Then, for any event A:

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

10 Definition 2.9

Conditional probability: chance event A has occurred, given event B has occurred (where $P(B) > 0$):

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

11 Definition 2.10

Independence: The following must be true for events A and B to be independent. Otherwise, they are dependent:

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A \cap B) = P(A)P(B)$

12 Definition 2.11

Partition: for any positive integer k, $\{B_1, B_2, \ldots, B_k\}$ is a partition of sample space S if:

1. $S = B_1 \cup B_2 \cup \ldots \cup B_k$
2. $B_i \cap B_j = \emptyset$ for all $i \neq j$

13 Bayes' Theorem

For events A and B in space S when $P(A) > 0$ and $P(B) > 0$:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

14 Definition 3.3

Probability distribution for each value y of random variable Y, given $0 \leq P(y) \leq 1$:

$$p(y) = P(Y = y)$$

15 Definition 3.4

Expected value of discrete random variable Y:

$$\mu = E(Y) = \sum_y yp(y)$$

16 Definition 3.5

Variance of discrete random variable Y:

$$\sigma^2 = V(Y) = E(Y - \mu)^2$$

Standard deviation of discrete random variable Y:

$$\sigma = \sqrt{E(Y - \mu)^2}$$

17 Binomial Distribution

Probability Mass Function for binomial variable y with n trials, success probability p, and failure probability q:

$$P(Y = y) = \binom{n}{y} p^y q^{n-y}$$

18 Theorem 3.7

Expected value of binomial random variable Y:

$$\mu = E(Y) = np$$

Variance of Y:

$$\sigma^2 = V(Y) = npq$$

Standard deviation of Y:

$$\sigma = \sqrt{npq}$$

19 Definition 3.8

Geometric probability distribution mass function, with success probability p and failure probability q:

$$P(Y = y) = q^{y-1}p$$

20 Theorem 3.8

Expected value of geometric random variable Y:

$$\mu = E(Y) = \frac{1}{p}$$

Variance of Y:

$$\sigma^2 = V(Y) = \frac{1-p}{p^2}$$

Standard deviation of Y:

$$\sigma = \sqrt{\frac{1-p}{p^2}}$$

21 Definition 3.9

Negative binomial probability distribution, given $y = r, r+1, r+2, \ldots$ and $0 \leq P(y) \leq 1$. y represents the trial where the rth success occurs, with success probability p:

$$P(Y = y) = \binom{y-1}{r-1} p^r q^{y-r}$$

22 Theorem 3.9

Expected value of negative binomial random variable Y:

$$\mu = E(Y) = \frac{r}{p}$$

Variance of Y:

$$\sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

Standard deviation of Y:

$$\sigma = \sqrt{\frac{r(1-p)}{p^2}}$$

23 Definition 3.10

Hypergeometric probability distribution, given $y = 0, 1, 2, \ldots, n, y \leq r, n - y \leq N - r$. n items are selected from N, with r objects of desired type:

$$P(Y = y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

24 Theorem 3.10

Expected value of hypergeometric random variable Y:

$$\mu = E(Y) = \frac{nr}{N}$$

Variance of Y:

$$\sigma^2 = V(Y) = n\frac{r}{N}\frac{N-r}{N}\frac{N-n}{N-1}$$

Standard deviation of Y:

$$\sigma = \sqrt{n\frac{r}{N}\frac{N-r}{N}\frac{N-n}{N-1}}$$

25 Definition 3.11

Poisson probability distribution of random variable Y:

$$P(Y = y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

26 Theorem 3.11

Expected value of Poisson distribution:

$$E(Y) = \mu = \lambda$$

Variance of Poisson distribution:

$$V(Y) = \sigma^2 = \lambda$$

27 Theorem 3.14

Tchebysheff's Theorem for random variable Y, mean μ, and variance σ^2. For any constant $k > 0$:

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

or

$$P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

28 Definition 4.1

Distribution function of Y (any random variable):

$$F(y) = P(Y \leq y) \quad \text{for all } -\infty < y < \infty$$

29 Theorem 4.1

Properties of a distribution function:

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$
3. $F(y)$ is a nondecreasing function of y.

30 Definition 4.2

If $F(y)$ is continuous on $-\infty < y < \infty$, Y is a continuous random variable.

31 Theorem 4.2

Properties of a density function $f(y)$:

1. $f(y) \geq 0 \quad \text{for all } -\infty < y < \infty$
2. $\int_{-\infty}^{\infty} f(y)\, dy = 1$

32 Definition 4.3

Probability density function of Y:

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

33 Theorem 4.3

If Y has a density function $f(y)$ and bounds $a < b$, the probability that Y is on interval $[a, b]$:

$$P(a \leq Y \leq b) = \int_a^b f(y)\, dy$$

34 Theorem 4.4

Expected value of a function $g(Y)$ of a continuous random variable Y:

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y) f(y)\, dy$$

35 Definition 4.5

Expected value of continuous random variable Y:

$$E(Y) = \int_{-\infty}^{\infty} y f(y)\, dy$$

36 Theorem 4.5

Let c be a constant and $g(Y), g_1(Y), \ldots, g_k(Y)$ be functions of Y:

1. $E(c) = c$
2. $E(cg(Y)) = cE[g(Y)]$
3. $E(g_1(Y) + g_2(Y) + \cdots + g_k(Y)) = E[g_1(Y)] + E[g_2(Y)] + \cdots + E[g_k(Y)]$

37 Definition 4.6

Y has a continuous uniform probability distribution if its density function is:

$$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 \leq y \leq \theta_2 \\ 0, & \text{elsewhere} \end{cases}$$

38 Theorem 4.6

If $\theta_1 < \theta_2$ and Y is uniformly distributed on (θ_1, θ_2), then:

1. $E(Y) = \mu = \frac{\theta_1 + \theta_2}{2}$
2. $V(Y) = \sigma^2 = \frac{(\theta_2 - \theta_1)^2}{12}$

39 Theorem 4.7

Expected value of normally distributed random variable Y:

$$E(Y) = \mu$$

Variance of normally distributed random variable Y:

$$V(Y) = \sigma^2$$

40 Definition 4.8

Y has a normal probability distribution if its density function is:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)} \quad \text{when } \sigma > 0, -\infty < \mu < \infty, \text{ and } -\infty < y < \infty$$

41 Theorem 4.8

Expected value of gamma distributed random variable Y:

$$E(Y) = \mu = \alpha\beta$$

Variance of gamma distributed random variable Y:

$$V(Y) = \sigma^2 = \alpha\beta^2$$

42 Definition 4.9

Y has a gamma distribution with parameters $\alpha > 0$ and $\beta > 0$ if the density function is:

$$f(y) = \begin{cases} \frac{y^{\alpha-1}e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, & 0 \le y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}\,dt$.

43 Theorem 4.10

Expected value of exponentially distributed random variable Y:

$$E(Y) = \mu = \beta$$

Variance of exponentially distributed random variable Y:

$$V(Y) = \sigma^2 = \beta^2$$

44 Definition 4.11

Y has an exponential distribution with parameter $\beta > 0$ if the density function is:

$$f(y) = \begin{cases} \frac{1}{\beta}e^{-y/\beta}, & 0 \le y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

45 Definition 5.1

Joint probability function of Y_1 and Y_2:

$$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2) \quad \text{when} \quad -\infty < y_1, y_2 < \infty$$

46 Theorem 5.1

If Y_1 and Y_2 have a joint probability function, then:

1. $p(y_1, y_2) \ge 0$
2. $\sum_{y_1, y_2} p(y_1, y_2) = 1$

47 Definition 5.2

Joint distribution function of Y_1 and Y_2:

$$F(y_1, y_2) = P(Y_1 \le y_1, Y_2 \le y_2) \quad \text{when} \quad -\infty < y_1, y_2 < \infty$$

48 Theorem 5.2

If Y_1 and Y_2 have a joint distribution function, then:

1. $F(-\infty, -\infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$
2. $F(\infty, \infty) = 1$
3. $f(y_1, y_2) \geq 0$
4. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1\, dy_2 = 1$

49 Definition 5.3

Jointly continuous random variables Y_1 and Y_2:

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2)\, dt_2\, dt_1$$

50 Definition 5.4

Marginal probability function of Y_1 and Y_2:

$$p_1(y_1) = \sum_{\text{all } y_2} p(y_1, y_2) \quad \text{and} \quad p_2(y_2) = \sum_{\text{all } y_1} p(y_1, y_2)$$

Marginal density function of Y_1 and Y_2:

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_2 \quad \text{and} \quad f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1$$

51 Theorem 5.4

Marginal probability variables Y_1 and Y_2 are independent if and only if:

$$p(y_1, y_2) = p_1(y_1)p_2(y_2) \quad \text{for every } (y_1, y_2)$$

Otherwise, Y_1 and Y_2 are dependent.

Marginal density variables Y_1 and Y_2 are independent if and only if:

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52 Definition 5.5

Conditional discrete probability function of Y_1 and Y_2:

$$p(y_1|y_2) = \frac{p(y_1, y_2)}{p_2(y_2)} \quad \text{when } p_2(y_2) > 0$$

53 Definition 5.6

Conditional distribution function of Y_1 and Y_2:

$$F(y_1|y_2) = P(Y_1 \leq y_1 | Y_2 = y_2)$$

54 Definition 5.7

Conditional density of Y_1 given $Y_2 = y_2$:

$$f(y_1|y_2) = \frac{f(y_1, y_2)}{f_2(y_2)}$$

Conditional density of Y_2 given $Y_1 = y_1$:

$$f(y_2|y_1) = \frac{f(y_1, y_2)}{f_1(y_1)}$$

55 Definition 5.8

Joint distribution variables Y_1 and Y_2 are independent if and only if:

$$F(y_1, y_2) = F_1(y_1)F_2(y_2) \quad \text{for every } (y_1, y_2)$$

Otherwise, Y_1 and Y_2 are dependent.