Definitions and Theorems

Mean of a sample with n values:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Variance of a sample (overall distance of values from the mean):

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

Standard Deviation (s): square root of variance s^2:

$$s = \sqrt{s^2}$$

Probability of an event A within a sample space S, such that $A \subseteq S$:

- $P(A) \geq 0$
- $P(S) = 1$
- If $A_1, A_2, \ldots, A_n$ are mutually exclusive:

$$P(A_1, A_2, \ldots, A_n) = \sum_{i=1}^{n} P(A_i)$$

Permutation (ordered arrangement of r distinct objects, with n possible orders):

$$P_n^r = \frac{n!}{(n-r)!}$$

Combination (number of subsets of size r that can be formed from n objects):

$$C_n^r = \frac{n!}{r!(n-r)!}$$

Conditional probability (chance that event A has occurred, given event B has occurred):

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Independence: All 3 of these must be true: Dependence: Any one of these 3 must be false:

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A \cap B) = P(A)P(B)$

Multiplicative Law of Probability (The probability of the intersection of two events):

- If dependent, $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$
- If independent, $P(A \cap B) = P(A)P(B)$

Additive Law of Probability (The probability of the union of two events):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If $P(A \cap B) = 0$: - The event is mutually exclusive - $P(A \cup B) = P(A) + P(B)$

If A is an event, then $P(A) = 1 - P(\bar{A})$.

For any positive integer k, $\{B_1, B_2, \ldots, B_k\}$ is a partition of sample space S if:

- $S = B_1 \cup B_2 \cup \cdots \cup B_k$
- $B_i \cap B_j = \emptyset$ for all $i \neq j$

Total probability (assuming the above is true):

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Bayes' Theorem

For events A and B in the space S where $P(A) > 0$ and $P(B) > 0$:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Probability distribution for each value y of random variable Y, given $0 \leq P(y) \leq 1$:

$$p(y) = P(Y = y)$$

Expected value of discrete random variable Y:

$$\mu = E(Y) = \sum_{Y=y} yp(y)$$

Variance of discrete random variable Y:

$$\sigma^2 = V(Y) = E(Y - \mu)^2$$

Standard deviation of discrete random variable Y:

$$\sigma = \sqrt{E(Y - \mu)^2}$$

Binomial Distribution

Probability Mass Function for binomial variable y with n trials, success probability p, and failure probability q:

$$p(y) = P(Y = y) = \binom{n}{y} p^y q^{n-y}$$

Expected value of binomial random variable Y:

$$\mu = E(Y) = np$$

Binomial variance of Y:

$$\sigma^2 = V(Y) = npq$$

Binomial standard deviation of Y:

$$\sigma = \sqrt{npq}$$

0.1 Geometric probability distribution mass function

- with success probability p and failure probability q:

$$p(y) = q^{y-1} p$$

- Expected value of geometric random variable Y:

$$\mu = E(Y) = \frac{1}{p}$$

- Geometric variance of Y:

$$\sigma^2 = V(Y) = \frac{1-p}{p^2}$$

- Geometric standard deviation of Y:

$$\sigma = \sqrt{\frac{1-p}{p^2}}$$