Formula Sheet

1 Definition 1.1

Mean of a sample with n values:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

2 Definition 1.2

Variance of a sample: overall distance of values from the mean:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

3 Definition 1.3

Standard Deviation (s): square root of variance (s^2):

$$s = \sqrt{s^2}$$

4 Theorem 2.5

Multiplicative Law of Probability: The probability of the intersection of two events:

1. If dependent, $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$.
2. If independent, $P(A \cap B) = P(A)P(B)$.

Additive Law of Probability: The probability of the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If $P(A \cap B) = 0$ (mutually exclusive events), then $P(A \cup B) = P(A) + P(B)$.

5 Definition 2.6

Probability of an event A within a sample space S, such that $A \subseteq S$. The following are true:

1. $P(A) \geq 0$
2. $P(S) = 1$
3. If $(A_1, A_2, \ldots, A_n)$ are mutually exclusive, then $P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i)$.

6 Definition 2.7

Permutation: ordered arrangement of r distinct objects, with n possible orders:

$$P_n^r = \frac{n!}{(n-r)!}$$

7 Theorem 2.7

If A is an event, then $P(A) = 1 - P(\bar{A})$.

8 Definition 2.8

Combination: number of subsets of size r that can be formed from n objects:

$$C_n^r = \frac{n!}{r!(n-r)!}$$

9 Theorem 2.8

Total probability: assume Definition 2.11. Then, for any event A:

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

10 Definition 2.9

Conditional probability: chance event A has occurred, given event B has occurred (where $P(B) > 0$):

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

11 Definition 2.10

Independence: The following must be true for events A and B to be independent. Otherwise, they are dependent:

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A \cap B) = P(A)P(B)$

12 Definition 2.11

Partition: for any positive integer k, $\{B_1, B_2, \ldots, B_k\}$ is a partition of sample space S if:

1. $S = B_1 \cup B_2 \cup \ldots \cup B_k$
2. $B_i \cap B_j = \emptyset$ for all $i \neq j$

13 Bayes' Theorem

For events A and B in space S when $P(A) > 0$ and $P(B) > 0$:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

14 Definition 3.3

Probability distribution for each value y of random variable Y, given $0 \leq P(y) \leq 1$:

$$p(y) = P(Y = y)$$

15 Definition 3.4

Expected value of discrete random variable Y:

$$\mu = E(Y) = \sum_{y} yp(y)$$

16 Definition 3.5

Variance of discrete random variable Y:

$$\sigma^2 = V(Y) = E(Y - \mu)^2$$

Standard deviation of discrete random variable Y:

$$\sigma = \sqrt{E(Y - \mu)^2}$$

17 Binomial Distribution

Probability Mass Function for binomial variable y with n trials, success probability p, and failure probability q:

$$P(Y = y) = \binom{n}{y} p^y q^{n-y}$$

18 Theorem 3.7

Expected value of binomial random variable Y:

$$\mu = E(Y) = np$$

Variance of Y:

$$\sigma^2 = V(Y) = npq$$

Standard deviation of Y:

$$\sigma = \sqrt{npq}$$

19 Definition 3.8

Geometric probability distribution mass function, with success probability p and failure probability q:

$$P(Y = y) = q^{y-1}p$$

20 Theorem 3.8

Expected value of geometric random variable Y:

$$\mu = E(Y) = \frac{1}{p}$$

Variance of Y:

$$\sigma^2 = V(Y) = \frac{1-p}{p^2}$$

Standard deviation of Y:

$$\sigma = \sqrt{\frac{1-p}{p^2}}$$

21 Definition 3.9

Negative binomial probability distribution, given $y = r, r+1, r+2, \ldots$ and $0 \leq P(y) \leq 1$. y represents the trial where the rth success occurs, with success probability p:

$$P(Y = y) = \binom{y-1}{r-1} p^r q^{y-r}$$

22 Theorem 3.9

Expected value of negative binomial random variable Y:

$$\mu = E(Y) = \frac{r}{p}$$

Variance of Y:

$$\sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

Standard deviation of Y:

$$\sigma = \sqrt{\frac{r(1-p)}{p^2}}$$

23 Definition 3.10

Hypergeometric probability distribution, given $y = 0, 1, 2, \ldots, n, y \leq r, n - y \leq N - r$. n items are selected from N, with r objects of desired type:

$$P(Y = y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

24 Theorem 3.10

Expected value of hypergeometric random variable Y:

$$\mu = E(Y) = \frac{nr}{N}$$

Variance of Y:

$$\sigma^2 = V(Y) = n\frac{r}{N}\frac{N-r}{N}\frac{N-n}{N-1}$$

Standard deviation of Y:

$$\sigma = \sqrt{n\frac{r}{N}\frac{N-r}{N}\frac{N-n}{N-1}}$$