Formula Sheet

1 Definition 3.11

Poisson probability distribution of random variable Y:

$$P(Y = y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

2 Theorem 3.11

Expected value of Poisson distribution:

$$E(Y) = \mu = \lambda$$

Variance of Poisson distribution:

$$V(Y) = \sigma^2 = \lambda$$

3 Theorem 3.14

Tchebysheff's Theorem for random variable Y, mean μ, and variance σ^2. For any constant $k > 0$:

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

or

$$P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

4 Definition 4.1

Distribution function of Y (any random variable):

$$F(y) = P(Y \leq y) \quad \text{for all } -\infty < y < \infty$$

5 Theorem 4.1

Properties of a distribution function:

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$
3. $F(y)$ is a nondecreasing function of y.

6 Definition 4.2

If $F(y)$ is continuous on $-\infty < y < \infty$, Y is a continuous random variable.

7 Theorem 4.2

Properties of a density function $f(y)$:

1. $f(y) \geq 0 \quad \text{for all} \ -\infty < y < \infty$
2. $\int_{-\infty}^{\infty} f(y)\, dy = 1$

8 Definition 4.3

Probability density function of Y:

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

9 Theorem 4.3

If Y has a density function $f(y)$ and bounds $a < b$, the probability that Y is on interval $[a, b]$:

$$P(a \leq Y \leq b) = \int_a^b f(y)\, dy$$

10 Theorem 4.4

Expected value of a function $g(Y)$ of a continuous random variable Y:

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y) f(y)\, dy$$

11 Definition 4.5

Expected value of continuous random variable Y:

$$E(Y) = \int_{-\infty}^{\infty} y f(y)\, dy$$

12 Theorem 4.5

Let c be a constant and $g(Y), g_1(Y), \ldots, g_k(Y)$ be functions of Y:

1. $E(c) = c$
2. $E(cg(Y)) = cE[g(Y)]$
3. $E(g_1(Y) + g_2(Y) + \cdots + g_k(Y)) = E[g_1(Y)] + E[g_2(Y)] + \cdots + E[g_k(Y)]$

13 Definition 4.6

Y has a continuous uniform probability distribution if its density function is:

$$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 \le y \le \theta_2 \\ 0, & \text{elsewhere} \end{cases}$$

14 Theorem 4.6

If $\theta_1 < \theta_2$ and Y is uniformly distributed on (θ_1, θ_2), then:

1. $E(Y) = \mu = \frac{\theta_1 + \theta_2}{2}$
2. $V(Y) = \sigma^2 = \frac{(\theta_2 - \theta_1)^2}{12}$

15 Theorem 4.7

Expected value of normally distributed random variable Y:

$$E(Y) = \mu$$

Variance of normally distributed random variable Y:

$$V(Y) = \sigma^2$$

16 Definition 4.8

Y has a normal probability distribution if its density function is:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)} \quad \text{when } \sigma > 0, -\infty < \mu < \infty, \text{ and } -\infty < y < \infty$$

17 Theorem 4.8

Expected value of gamma distributed random variable Y:

$$E(Y) = \mu = \alpha\beta$$

Variance of gamma distributed random variable Y:

$$V(Y) = \sigma^2 = \alpha\beta^2$$

18 Definition 4.9

Y has a gamma distribution with parameters $\alpha > 0$ and $\beta > 0$ if the density function is:

$$f(y) = \begin{cases} \frac{y^{\alpha-1} e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, & 0 \le y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt$.

19 Theorem 4.10

Expected value of exponentially distributed random variable Y:

$$E(Y) = \mu = \beta$$

Variance of exponentially distributed random variable Y:

$$V(Y) = \sigma^2 = \beta^2$$

20 Definition 4.11

Y has an exponential distribution with parameter $\beta > 0$ if the density function is:

$$f(y) = \begin{cases} \frac{1}{\beta}e^{-y/\beta}, & 0 \leq y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

21 Definition 5.1

Joint probability function of Y_1 and Y_2:

$$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2) \quad \text{when } -\infty < y_1, y_2 < \infty$$

22 Theorem 5.1

If Y_1 and Y_2 have a joint probability function, then:

1. $p(y_1, y_2) \geq 0$
2. $\sum_{y_1, y_2} p(y_1, y_2) = 1$

23 Definition 5.2

Joint distribution function of Y_1 and Y_2:

$$F(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2) \quad \text{when } -\infty < y_1, y_2 < \infty$$

24 Theorem 5.2

If Y_1 and Y_2 have a joint distribution function, then:

1. $F(-\infty, -\infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$
2. $F(\infty, \infty) = 1$
3. $f(y_1, y_2) \geq 0$
4. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1\, dy_2 = 1$

25 Definition 5.3

Jointly continuous random variables Y_1 and Y_2:

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2)\, dt_2\, dt_1$$

26 Definition 5.4

Marginal probability function of Y_1 and Y_2:

$$p_1(y_1) = \sum_{\text{all } y_2} p(y_1, y_2) \quad \text{and} \quad p_2(y_2) = \sum_{\text{all } y_1} p(y_1, y_2)$$

Marginal density function of Y_1 and Y_2:

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_2 \quad \text{and} \quad f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1$$

27 Theorem 5.4

Marginal probability variables Y_1 and Y_2 are independent if and only if:

$$p(y_1, y_2) = p_1(y_1)p_2(y_2) \quad \text{for every } (y_1, y_2)$$

Otherwise, Y_1 and Y_2 are dependent.

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28 Definition 5.5

Conditional discrete probability function of Y_1 and Y_2:

$$p(y_1|y_2) = \frac{p(y_1, y_2)}{p_2(y_2)} \quad \text{when } p_2(y_2) > 0$$

29 Definition 5.6

Conditional distribution function of Y_1 and Y_2:

$$F(y_1|y_2) = P(Y_1 \leq y_1 | Y_2 = y_2)$$

30 Definition 5.7

Conditional density of Y_1 given $Y_2 = y_2$:

$$f(y_1|y_2) = \frac{f(y_1, y_2)}{f_2(y_2)}$$

Conditional density of Y_2 given $Y_1 = y_1$:

$$f(y_2|y_1) = \frac{f(y_1, y_2)}{f_1(y_1)}$$

31 Definition 5.8

Joint distribution variables Y_1 and Y_2 are independent if and only if:

$$F(y_1, y_2) = F_1(y_1)F_2(y_2) \quad \text{for every } (y_1, y_2)$$

Otherwise, Y_1 and Y_2 are dependent.