Local Probabilistic Models: Tabular CPDs

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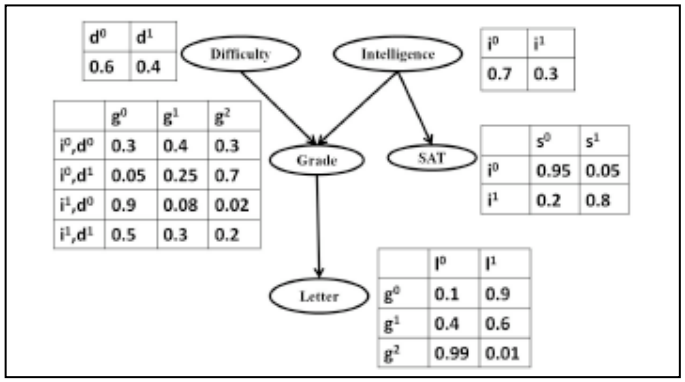
Topics

- Local Probabilistic Models
  1. Tabular CPDs
  2. Deterministic CPDs
  3. Context-Specific CPDs
     - (1)Tree CPD (Printer Diagnosis), (2) Rule CPD
  4. Independence of Causal Influence
     - (1) Noisy-OR, (2) Generalized Linear Models
  5. Continuous Variables: Robotics
     - Hybrid Models: Thermostat
  6. Conditional BNs: Computer Network

Local Probabilistic Models

- Bayesian Networks capture *global properties* of independence of variables



➔

$$I(G) = \{(\mathrm{D} \perp I \mid \phi),\ (G \perp S \mid D, I),\ (S \perp D, G, L \mid I),\ (D \perp I, S \mid \phi)\ (L \perp I, D, S \mid G)\}$$

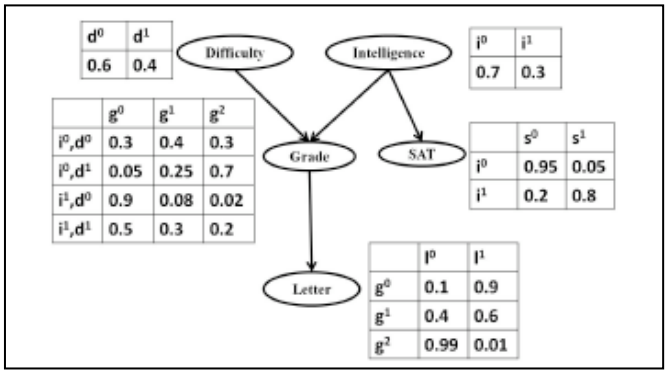
- Properties of independence allow us to:
  - factorize high-dimensional joint distribution into product of lower-dimensional CPDs (or factors)

$$P(D,I,G,S,L) = P(D)P(I)P(G \mid D,I)P(S \mid I)P(L \mid G)$$

- Next: exploit additional regularities in CPDs

Tabular CPDs

- When we have only Discrete Valued Random Variables



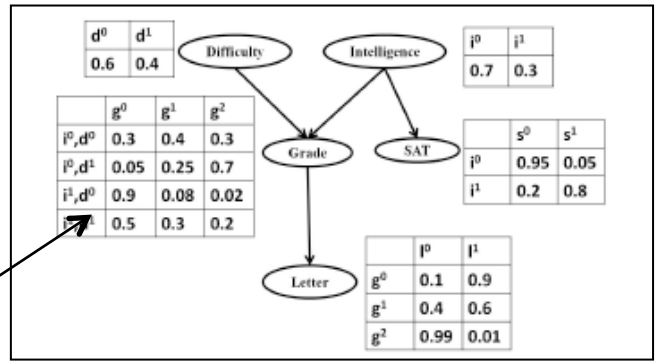
- Encode $P(X|\mathrm{pa}_X)$ as a table
  - Contains an entry for each assignment to X and pa_X
  - Proper CPD requires all non-negative values and

$$\sum_{x \in Val(x)} P\left(x \mid \mathrm{pa}_X\right) = 1$$

- Inference algorithms can use table CPDs in a natural way
  - Leads to perception that table CPDs are inherent to BNs, but…….

Disadvantages of Tabular CPDs

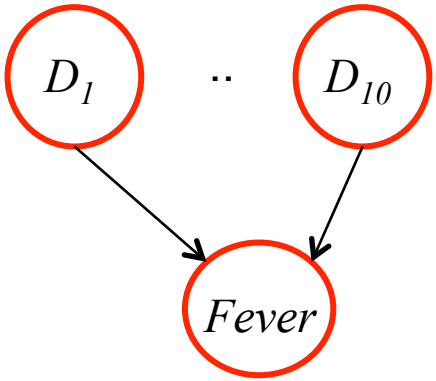
- R.v.s with inf. domains, e.g.,continuous values
  - cannot store each conditional probability in a table
- Even in discrete case there are difficulties
  - No. of parameters grows exponentially with no. of parents: for binary variable X with n binary *parents* we need 2^n values



Two binary
Parents
Four values

Unwieldiness of CPTs

- Tabular representation becomes rapidly large as the no. of parents grows
- It is a serious one in many settings
  - If Fever is caused by 10 diseases, we need to ask expert to answer $1{,}024$ questions– tiresome!



- Regularity among CPDs is not exploited
  - When D_1 is true, $Fever$ is certain irrespective of others

Solution: Different viewpoint

- A CPD needs to specify a conditional probability $P(x|\mathrm{pa}_X)$ for every assignment of values pa_X and x but does not have to do so by listing each such value explicitly
- View the CPDs not as tables listing all conditional probabilities but as functions that given pa_X and x return the conditional probability $P(x|\mathrm{pa}_X)$