Deep Belief Nets

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Topics

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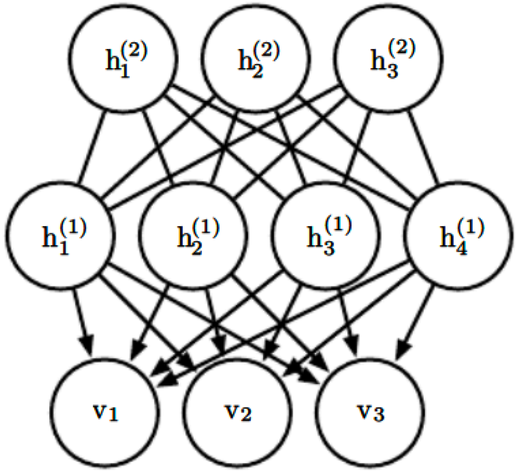
History of Deep Belief Networks

- One of the first non-convolutional models to admit training of deep architectures
  - Deep belief networks started the current deep learning renaissance
  - Prior to this deep modes were considered too difficult to optimize
    - Kernel machines with convex objective functions dominated the landscape
  - Demonstrated that deep architectures outperformed kernelized SVM on MNIST
- Today deep belief networks have fallen out of favor and rarely used

What are deep belief networks?

- They are generative models with several layers of latent variables
  - Latent variables are typically binary
  - Visible layers can be binary or real
  - There are no intra-layer connections
- Connections between top two layers are undirected
- Connections between all other layers is directed, pointing towards data

An example of a DBN



- It is a hybrid graphical model involving both directed and undirected connections
  - No intra-layer connections
  - Has multiple hidden layers

Distribution represented by a DBM

- A DBN with ℓ hidden layers has ℓ weight matrices $\mathrm{W}^{(1)},..,\mathrm{W}^{(\ell)}$.
- It contains $\ell+1$ bias vectors $\boldsymbol{b}^{(1)},..,\boldsymbol{b}^{(\ell)}$
- Bias $\boldsymbol{b}^{(0)}$ provides biases for the visible layer
- The probability distribution represented is:

$$\boxed{\begin{aligned} P(\boldsymbol{h}^{(l)}, \boldsymbol{h}^{(l-1)}) &\propto \exp\left(\boldsymbol{b}^{(l)\top}\boldsymbol{h}^{(l)} + \boldsymbol{b}^{(l-1)\top}\boldsymbol{h}^{(l-1)} + \boldsymbol{h}^{(l-1)\top}\boldsymbol{W}^{(l)}\boldsymbol{h}^{(l)}\right), \\ P(h_i^{(k)} = 1 \mid \boldsymbol{h}^{(k+1)}) &= \sigma\left(b_i^{(k)} + \boldsymbol{W}_{:,i}^{(k+1)\top}\boldsymbol{h}^{(k+1)}\right) \forall i, \forall k \in 1, \ldots, l-2, \\ P(v_i = 1 \mid \boldsymbol{h}^{(1)}) &= \sigma\left(b_i^{(0)} + \boldsymbol{W}_{:,i}^{(1)\top}\boldsymbol{h}^{(1)}\right) \forall i. \end{aligned}}$$

  - In the case of real-valued variables

$$\boxed{\mathbf{v} \sim \mathcal{N}\left(\boldsymbol{v}; \boldsymbol{b}^{(0)} + \boldsymbol{W}^{(1)\top}\boldsymbol{h}^{(1)}, \boldsymbol{\beta}^{-1}\right)}$$

Sampling from a DBN

- To sample from a DBN:
- First run several steps of Gibbs sampling from the top two hidden layers
  - This stage is drawing a sample from the RBM defined by the top two layers
- Then use a single pass of ancestral sampling through rest of the model
  - to draw a sample from the visible units

Inference in a DBN

- Intractability of inference is due to:
  - the explaining away effect within each directed layer
  - Interaction between two hidden layers that have undirected connections
- Evaluating or maximizing standard evidence bound on the log-likelihood is also intractable
  - Beacause evidence bound takes the expectation of cliques
    - whose size is equal to network width

Training a DBN

- Begin by training an RBM to maximize $\mathbb{E}_{\mathbf{v}\sim p_{\text{data}}} \log p(\boldsymbol{v})$
  - Using contrastive divergence or stochastic maximum likelihood
    - Parameters of RBM then define parameters of first layer of DBN
- Next, a second RBM is trained to maximize

$$\mathbb{E}_{\mathbf{v}\sim p_{\text{data}}} \mathbb{E}_{\mathbf{h}^{(1)}\sim p^{(1)}(\boldsymbol{h}^{(1)}|\boldsymbol{v})} \log p^{(2)}(\boldsymbol{h}^{(1)})$$

  - Where $p^{(1)}$ and $p^{(2)}$: probability distributions represented by the two RBMs
    - In effect second RBM is trained to model the distribution defined by sampling the hidden units of the first RBM

Using a DBN

- The trained DBN may be directly used as a generative model
- But most interest arose from classification problems
- We can use weights of DBN to define an MLP

$$\boldsymbol{h}^{(1)} = \sigma\left(b^{(1)} + \boldsymbol{v}^\top \boldsymbol{W}^{(1)}\right).$$

$$\boldsymbol{h}^{(l)} = \sigma\left(b_i^{(l)} + \boldsymbol{h}^{(l-1)\top} \boldsymbol{W}^{(l)}\right) \forall l \in 2, \ldots, m,$$