Local Probabilistic Models: Deterministic CPDs

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Topics

- Local Probabilistic Models
  1. Tabular CPDs
  2. Deterministic CPDs
  3. Context-Specific CPDs
     - (1)Tree CPD (Printer Diagnosis), (2) Rule CPD
  4. Independence of Causal Influence
     - (1) Noisy-OR, (2) Generalized Linear Models
  5. Continuous Variables: Robotics
     - Hybrid Models: Thermostat
  6. Conditional BNs: Computer Network

Deterministic CPDs

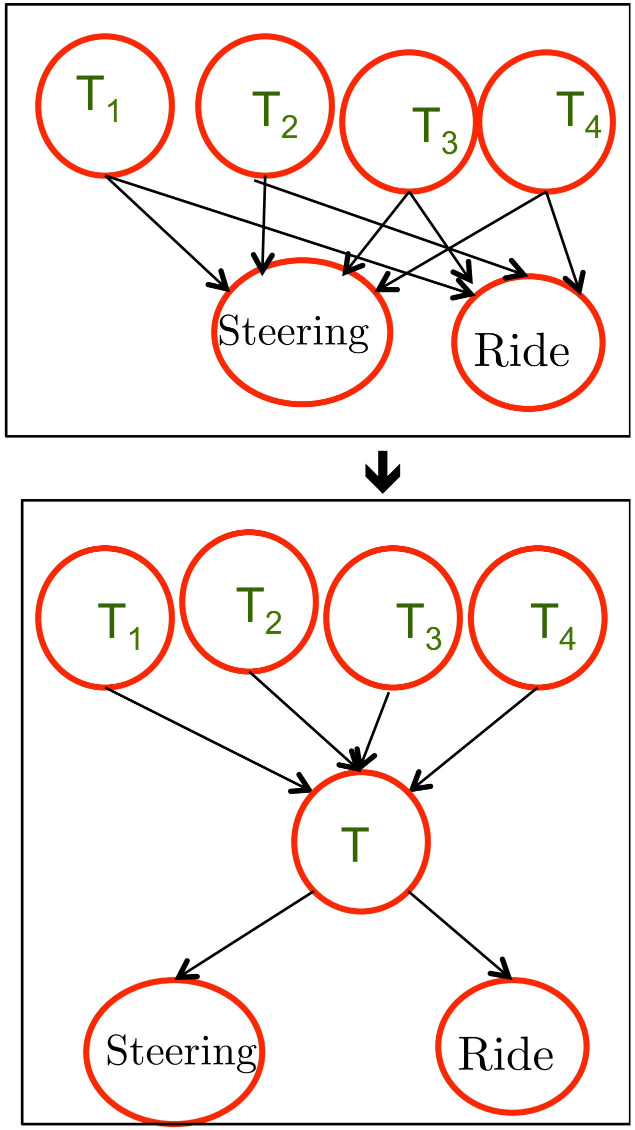
- Simplest non-tabular CPD
- A variable X is a deterministic function of its parents pa_X,
  - i.e., there is a function f such that
  - f: $Val(\mathrm{pa}_X) \rightarrow Val(X)$

$$P\left(x \mid \mathrm{pa}_X\right) = \begin{cases} 1 & x = f(\mathrm{pa}_X) \\ 0 & \text{otherwise} \end{cases}$$

- Example of binary-valued variables: X is "or" of parents: f : $P(X|Y,Z) = Y \vee Z$
- Example of Continuous domain: We want to assert in $P(X|Y,Z)$
  - that X is sum of parent values

Ex of Deterministic CPD: Modeling a car

- Tire variables T_1, T_2, T_3, T_4
  - Effects of flat: Steering, Ride,…
- Instead of effects having as parents all T_is, have them depend on single variable T
  - Which is a deterministic Or of its parents $\boxed{T=T_1 \vee T_2 \vee T_3 \vee T_4}$
- Advantages
  - Reduced indegree (8 vs 2)
    - Each effect has 1 instead 4
  - If there are more dependencies, considerable savings

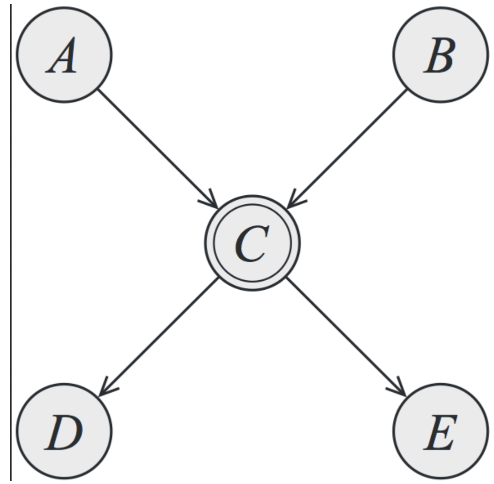


Deterministic CPDs & Independencies

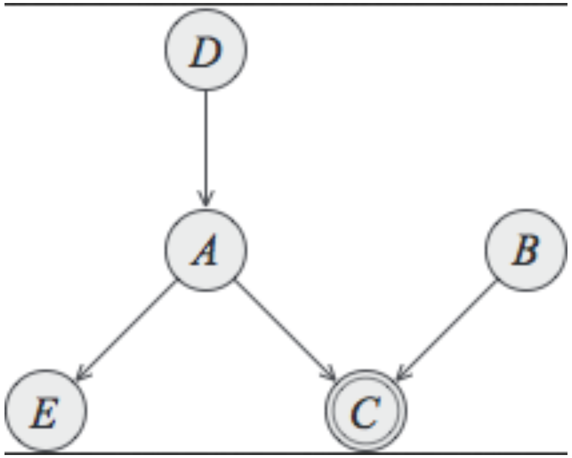
- Determining Independencies in a BN are slightly different with deterministic CPDs
- Recall that conditional independence is a numeric property
- Although defined using equality of probabilities
  - graph structure allows us to deduce some independencies without looking at the numbers
- Need to modify D-separation for determinism

Ex: Modifying D-separation

- If C is a deterministic function of A and B, what new independencies exist?
- If A and B are known, C is known, so D and E are independent: $(D \perp E \mid A, B)$
- Not necessarily true if C were not deterministic



More Complex Example with Deterministic CPDs



- C is exclusive or of A and B
- If B and C are known, A is known.
  - Therefore D and E are independent: $(D \perp E | B, C)$