Local Probabilistic Models: Conditional Bayesian Networks

Sargur Srihari
srihari@cedar.buffalo.edu

Topics

- Local Probabilistic Models
  – Conditional BNs
  – Example: Computer Network

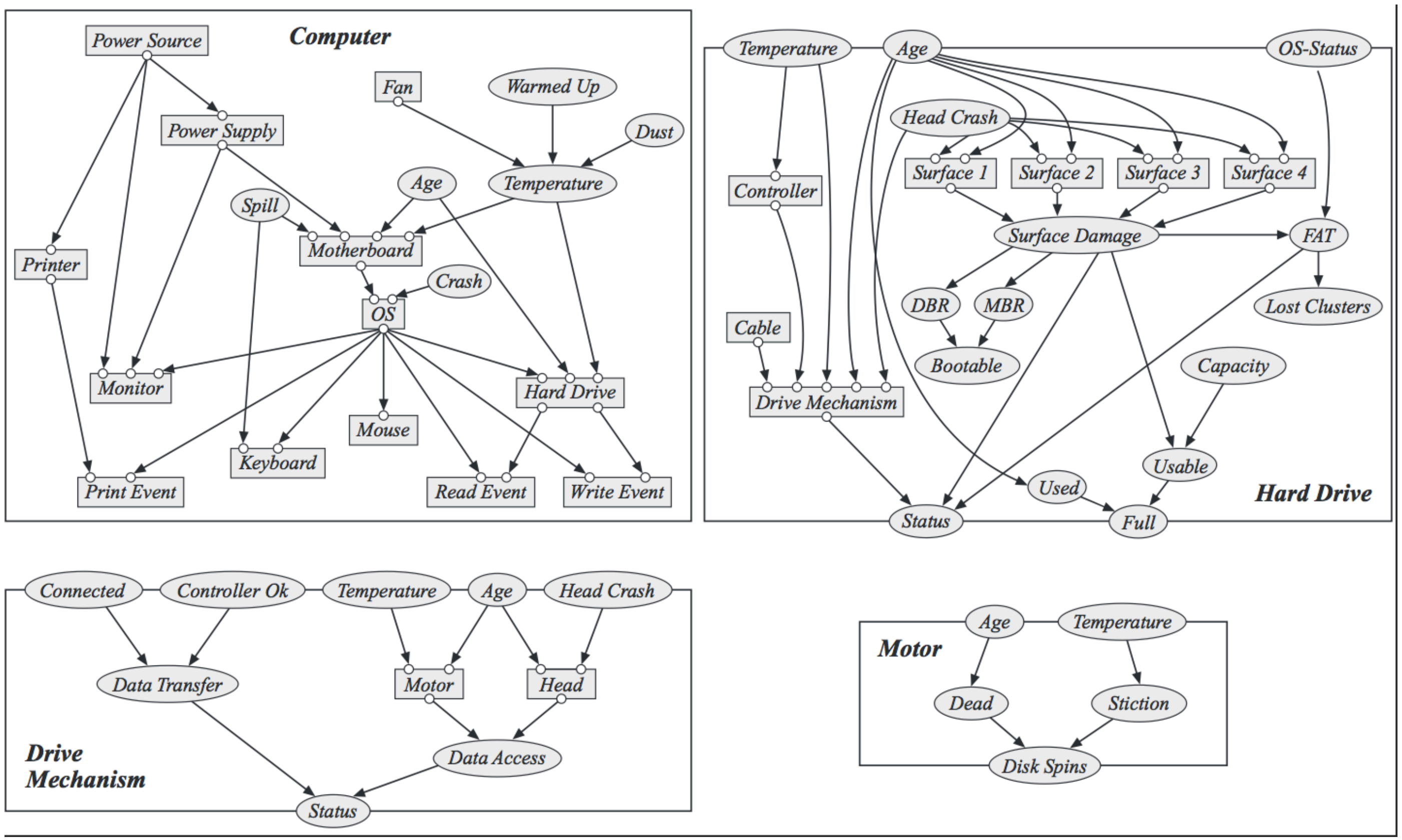
Conditional Bayesian Network

- Another compact representation of a CPD
- Bayesian Network Fragment
- Does not represent a full distribution but a conditional distribution
- The undirected analog is called a conditional random field

Encapsulated CPD

- To describe a complex system where components are composed of other lower-level subsystems
- We wish to model each subsystem separately
- Modeling a computer for fault diagnosis:
- Components are hard drive, disk surfaces within drive, etc.

Encapsulated CPDs for Computer System Model



Input variables intersect top edge of box,
i.e., inputs are received outside the box
Output variables intersect the bottom

Encapsulated CPD

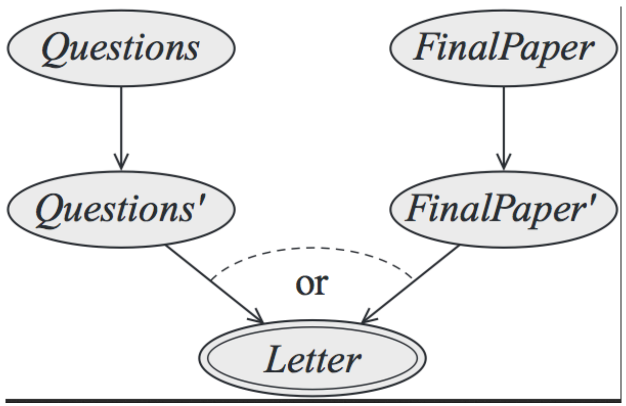
- Let Y be a variable with k parents $X_1,..X_k$.
- The CPD $P(Y|X_1,..X_k)$ is an encapsulated CPD if it is represented using a conditional Bayesian network over Y given $X_1,..X_k$
- Simplifies the model from a cognitive perspective
- Components are composed of other lower-level subsystems

Conditional Bayesian Networks

- Example of a conditional distribution
- It shows auxiliary variables not in the original network
- No CPD given for the parent variables *Questions* and *FinalPaper*
- Specifies a conditional distribution of *Letter* given *Questions* and *Final Paper*

CPD for Letter

Q	F	l^0	l^1
q^0	f^0	1	0
q^0	f^1	0.1	0.9
q^1	f^0	0.2	0.8
q^1	f^1	0.02	0.98



Definition of Conditional Bayesian Network

- A CBN $\mathfrak{B}$ over $\boldsymbol{Y}$ given $\boldsymbol{X}$ is an acyclic graph $\mathfrak{G}$ whose nodes are $\boldsymbol{X} \cup \boldsymbol{Y} \cup \boldsymbol{Z}$ where $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$ are disjoint
- $\boldsymbol{X}$ are inputs, $\boldsymbol{Y}$ outputs and $\boldsymbol{Z}$ are encapsulated
- Variables $\boldsymbol{X}$ have no parents in $\mathfrak{G}$
- Variables $\boldsymbol{Y} \cup \boldsymbol{Z}$ have a conditional distribution

$$P_{\mathfrak{B}}(\boldsymbol{Y},\boldsymbol{Z} \mid \boldsymbol{X}) = \prod_{X \in \boldsymbol{Y} \cup \boldsymbol{Z}} P\left(X \mid Pa_X^{\mathfrak{G}}\right)$$

Note:
X is dummy

  - The distribution $P_{\mathfrak{B}}(\boldsymbol{Y}|\boldsymbol{X})$ is defined as the marginal of $P_{\mathfrak{B}}(\boldsymbol{Y},\boldsymbol{Z}|\boldsymbol{X})$

$$P_{\mathfrak{B}}(\boldsymbol{Y} \mid \boldsymbol{X}) = \sum_{\boldsymbol{Z}} P_{\mathfrak{B}}\left(\boldsymbol{Y},\boldsymbol{Z} \mid \boldsymbol{X}\right)$$