IV. Line Formation in Expanding Atmospheres

1. Doppler - Effect

Stellar winds → velocity fields $\vec{V}(\vec{r})$ → need to be accounted for in radiative transfer

$$\frac{dI_\nu}{ds} = -\kappa_\nu(s) I_\nu + \epsilon_\nu(s) \quad \longrightarrow \quad \frac{dI_\nu}{ds} = -\kappa_\nu(s,\vec{V}) I_\nu + \epsilon_\nu(s,\vec{V})$$

velocity fields affect $\kappa_\nu,\ \epsilon_\nu$ through Doppler - Effect

$$\nu^{'} = \nu\, \gamma\, (1 - \beta\ cos\theta)$$

$$\gamma = \ (1-\beta)^{1/2}$$

$$\beta = \ |\,\vec{V}\,|\,/c$$

$\nu^{'}$ = frequency of photon in moving system
ν = observer
θ = angle between photon direction and $\vec{V}(\vec{r})$

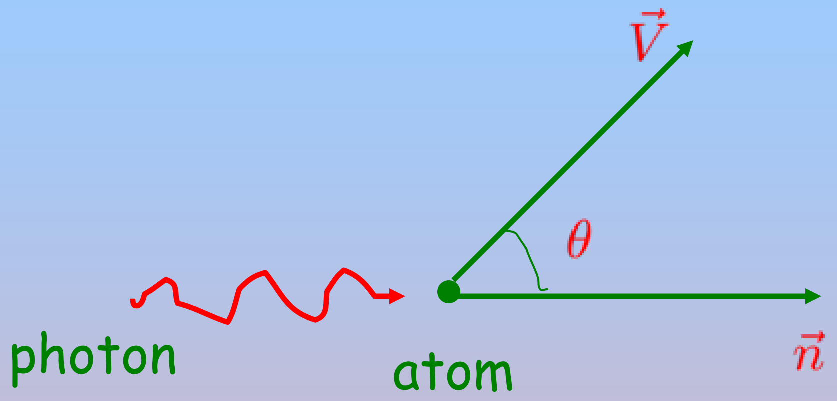
Relativistic formula including "transversal" effect at θ=90°

stellar winds are non- relativistiv with V/c << 1

$$\nu' = \nu\,(1 - \beta\; cos\theta)$$

$$\nu' = \nu\,(1 - \frac{|\vec{V}|}{c} cos\theta)$$

$$\nu' = \nu\,(1 - \frac{1}{c}\vec{V} \bullet \vec{n})$$



Atom moving with $\vec{V}$ "sees" photon with ν'

non-moving observer "sees" photon with ν

non-moving photosphere emits photon with ν

2. The line absorption coefficient

line absorption coefficient in static case with $\vec{V} = 0$

$$\kappa_\nu^{line} = \frac{\pi e^2}{mc} f_{lu} \{ n_l(s) - \frac{g_l}{g_u} n_u(s) \} \phi(\nu)$$

"classical" cross section

oscillator strength

occupation numbers l,u

line broadening

$\phi(\nu)$ contains thermal line broadening
natural
pressure

$$\phi(\nu) = \frac{1}{\Delta\nu_D} \frac{1}{\sqrt{\pi}} e^{-(\frac{\nu-\nu_0}{\Delta\nu_D})^2}$$

$$\Delta\nu_D = \nu_0 \frac{v_{th}}{c} = \frac{\nu_0}{c} \{ \frac{2kT}{m_{ion}} \}^{1/2}$$

in winds → thermal broadening

thermal Doppler width

Introducing the "dimensionless frequency" x

$$x = \frac{\nu - \nu_0}{\Delta\nu_D} \qquad \phi(x) = \frac{1}{\pi^{1/2}} e^{-x^2}$$

and the line profile φ(x)

$$\kappa_\nu^{line}(s) = k(s)\ \phi(x)$$

$$k(s) = \frac{\pi e^2}{mc} f_{lu} \{ n_l - \frac{g_l}{g_u} n_u \} \frac{1}{\Delta\nu_D}$$

we obtain the static line absorption coefficient

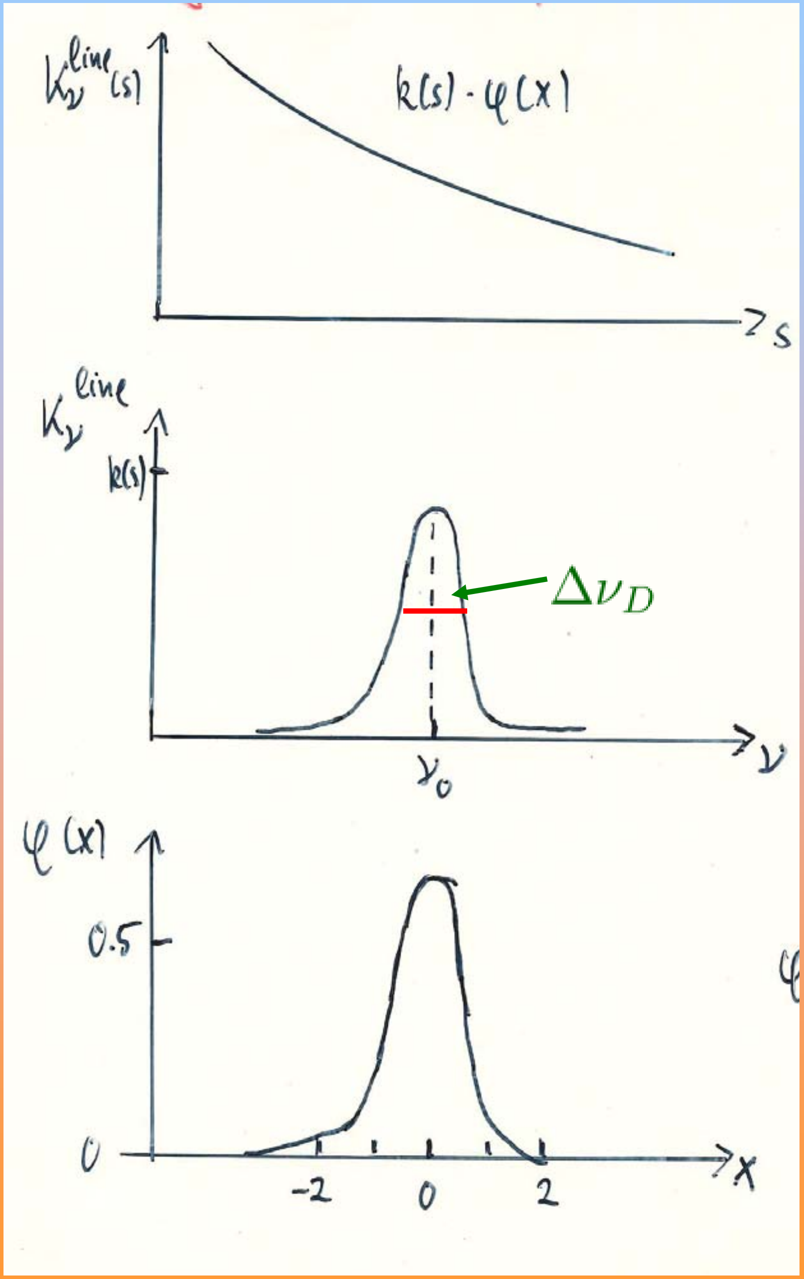
Note: the static line absorption coefficient is

(i) is isotropic (no dependence on angle θ)

(ii) varies smoothly as a function of s through k(s)

(iii) varies strongly as a function of x through φ(x)

hydrostatic atmosphere



smooth function of s

strong function of ν
width determined
by $\Delta\nu_D$

$$\phi(x)$$

Hydrodynamic atmosphere with $\vec{V} \neq 0$

Atom moving with $\vec{V}(s)$ sees shifted frequencies

$$\nu \longrightarrow \nu^{'}$$

$$\nu' - \nu = -\nu_0 \; \mu \; \frac{V(s)}{c}$$

$$\mu = cos\ \theta \qquad V(s) = \mid \vec{V}(s) \mid$$

profile function changes from

$$\phi\ (x) = \pi^{-1/2}\ e^{-x^2} = \pi^{-1/2}\ e^{-(\frac{\nu-\nu_0}{\Delta\nu_D})^2}$$

to

$$\phi\ (x - \frac{\nu_0}{c}\frac{\mu V(s)}{\Delta\nu_D}) = \pi^{-1/2}\ e^{-(\frac{\nu-\nu_0}{\Delta\nu_D} - \frac{\nu_0}{c}\frac{\mu V(s)}{\Delta\nu_D})^2}$$

Introducing

$$\vec{v}(r) = \frac{1}{v_{th}} \vec{V}(r)$$

velocity in units of thermal velocity

and using $v_{th} = \frac{c}{\nu_0} \Delta\nu_D$

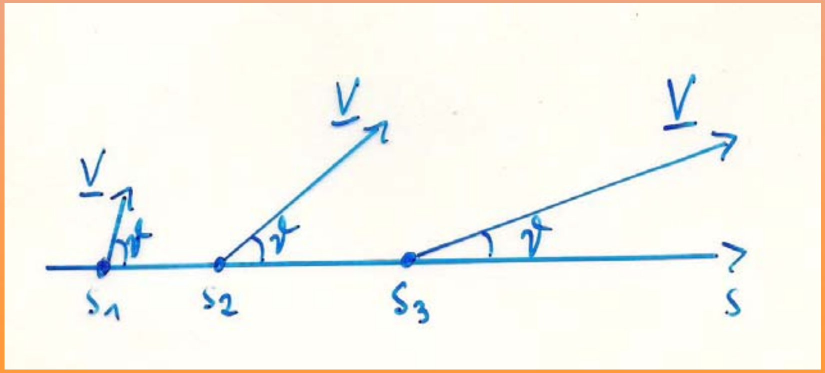
$$\phi\ (x) \longrightarrow \phi\ (x - \mu\ v(r)) = \pi^{-1/2}\ e^{-(x-\mu v)^2}$$

we obtain the absorption coeffient in the hydrodynamic case

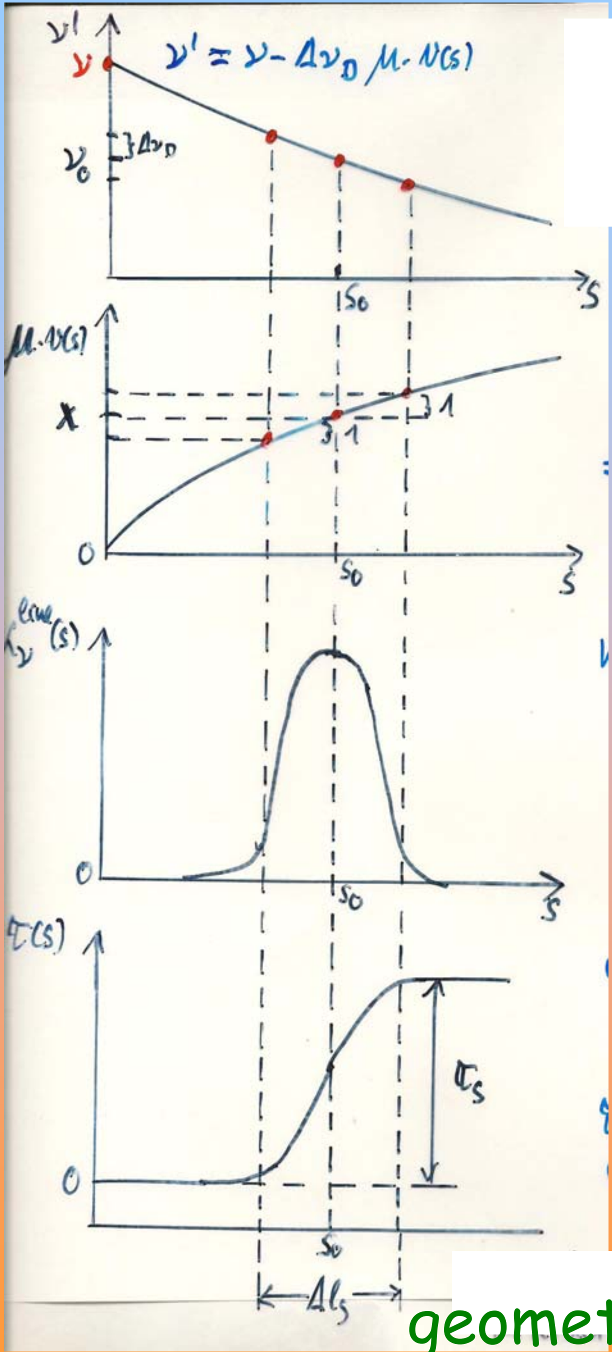
$$\kappa_\nu^{line} = k(s)\ \phi(x - \mu v(s))$$

$\phi\ (x - \mu\ v(s))$ ➡ anisotropic

$\phi\ (x - \mu\ v(s))$ ➡ extreme depth dependent



light ray passes through velocity field absorbed by atoms with different Doppler shifts



frequency of photon as seen by gas

at s = s_0: $\nu' = \nu_0$ resonance absorption

projected velocity

at s = s_0: $x = \mu\ v$ resonance absorption

κ_ν has strong maximum at s = s_0:

$\tau(s) = \int \kappa ds$ optical depth is step function

τ_S optical thickness of interaction region

geometrical width of interaction region

radiative line transfer with $V(r) \gg v_{th}$ →

photon – gas interaction restricted to thin interaction zone around the resonance condition defined by

$$\boxed{\begin{gathered} x = \mu \; v(r) \\ \text{or} \\ x = \vec{n} \bullet \vec{v}(r) \end{gathered}}$$

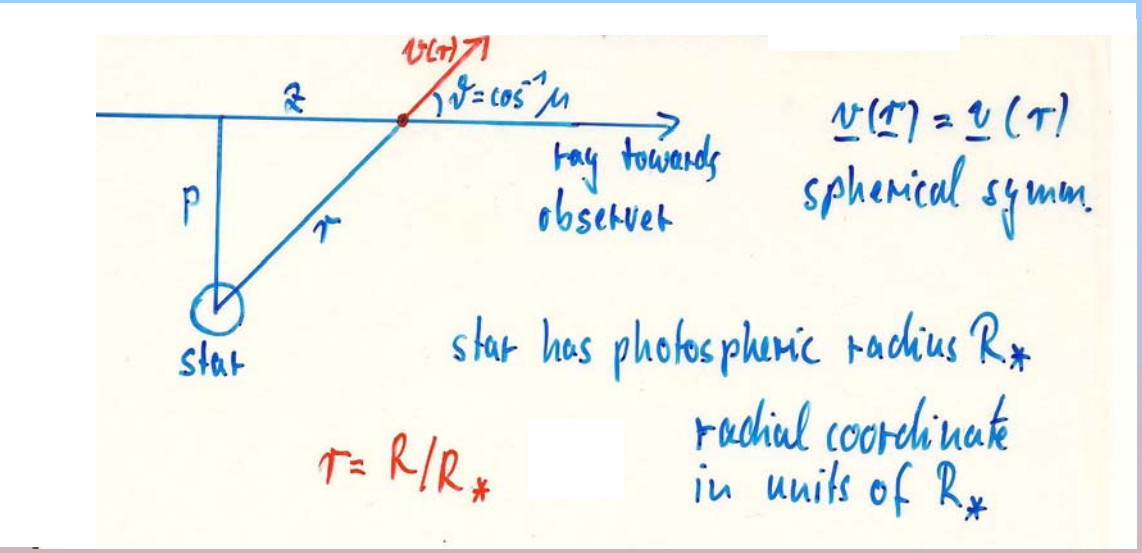
Note: hydrostatic radiative line transfer

photons reach observer from layers with $\tau \approx 1$

hydrodynamic radiative line transfer

photons come from interaction zone defined by resonance condition independent of value of τ

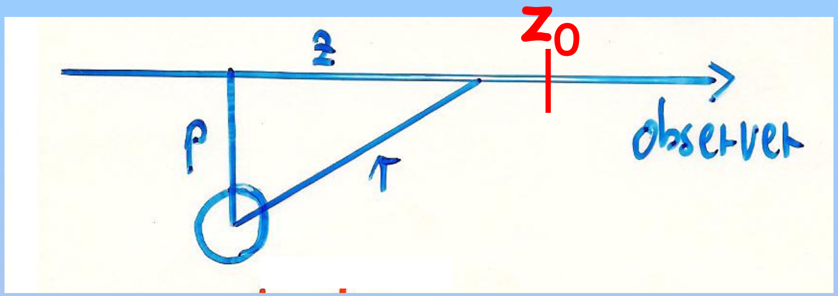
3. Interaction surfaces in spherical symmetry



"(p,z) – geometry"

$$r^2 = p^2 + z^2$$
$$z = \mu\, r$$
$$p = r\,(1-\mu^2)^{1/2}$$

- all rays characterized by impact parameter p
- z coordinate along ray
- p, z, r all in units of stellar photospheric radius $R_\star$



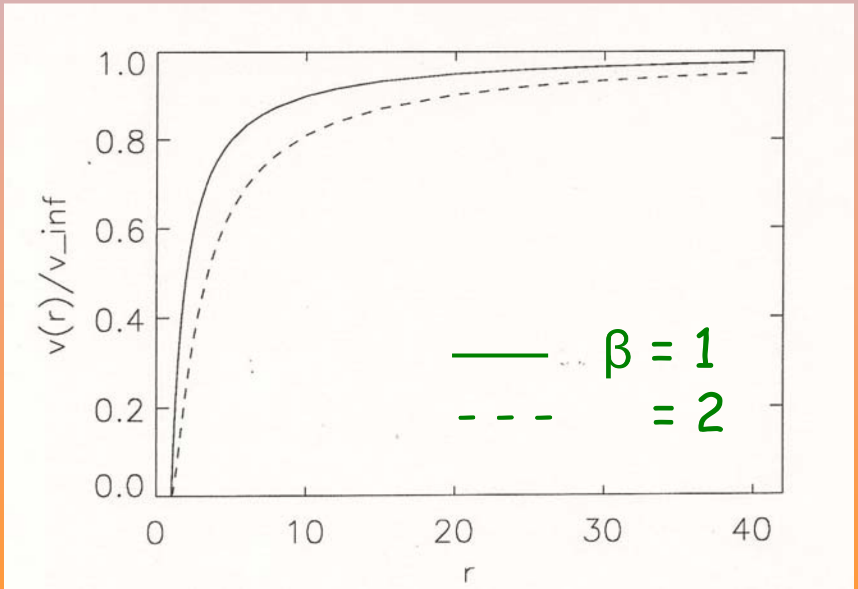
For each ray with frequency x and impact parameter p exists interaction point z_0 where

$$x = \mu \, v$$

the set of all interaction points is $z_0(x,p)$ "interaction surface"

the shape of $z_0(x,p)$ depends on the velocity field v(r)

observed v(r) for hot stars

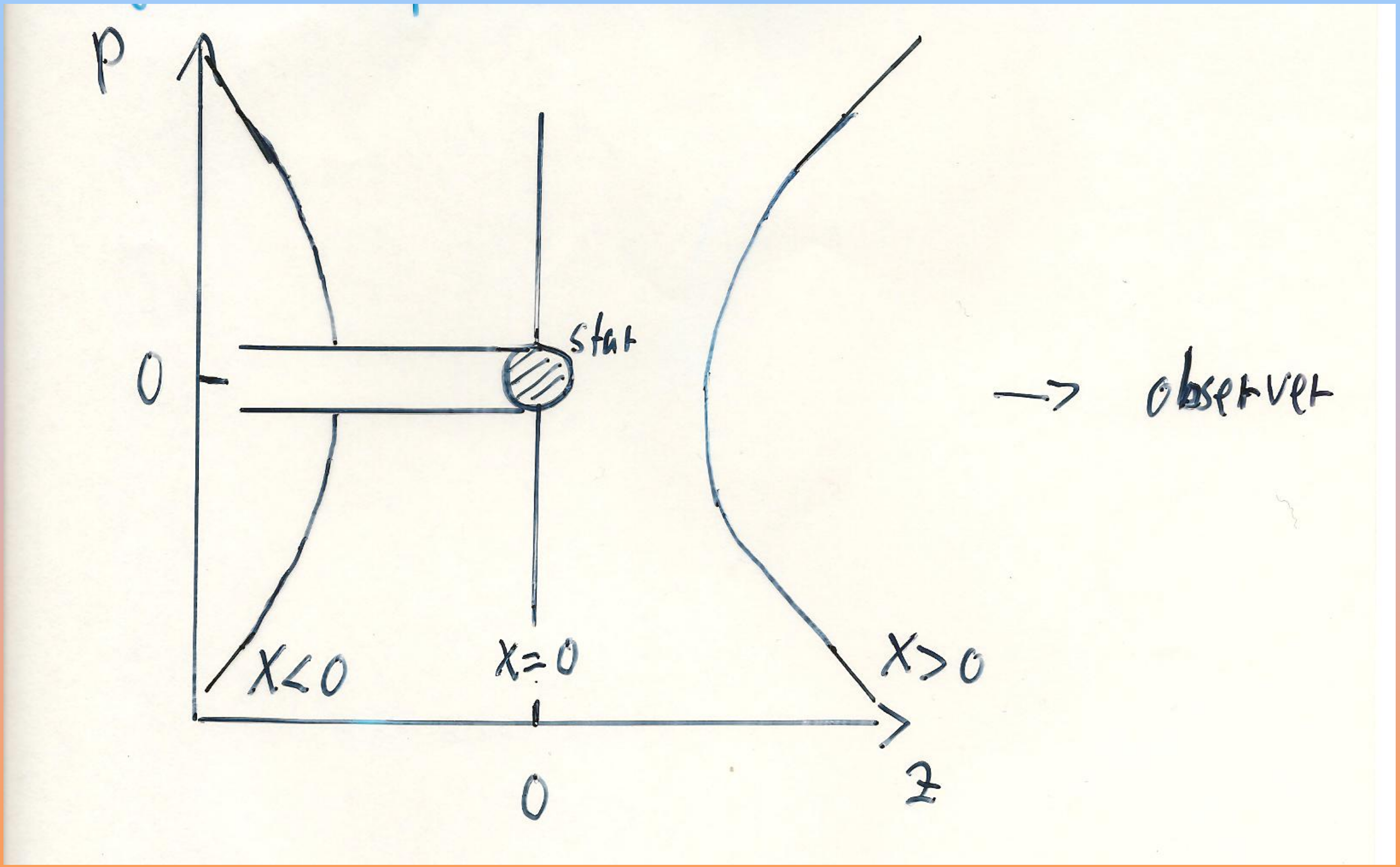


$$v(r) = v_\infty (1 - \frac{1}{r})^\beta$$

$$\beta = 0.5..4.0$$

$$\frac{dv}{dr} \geq 0 \quad v \to v_\infty, \; r \gg 1$$

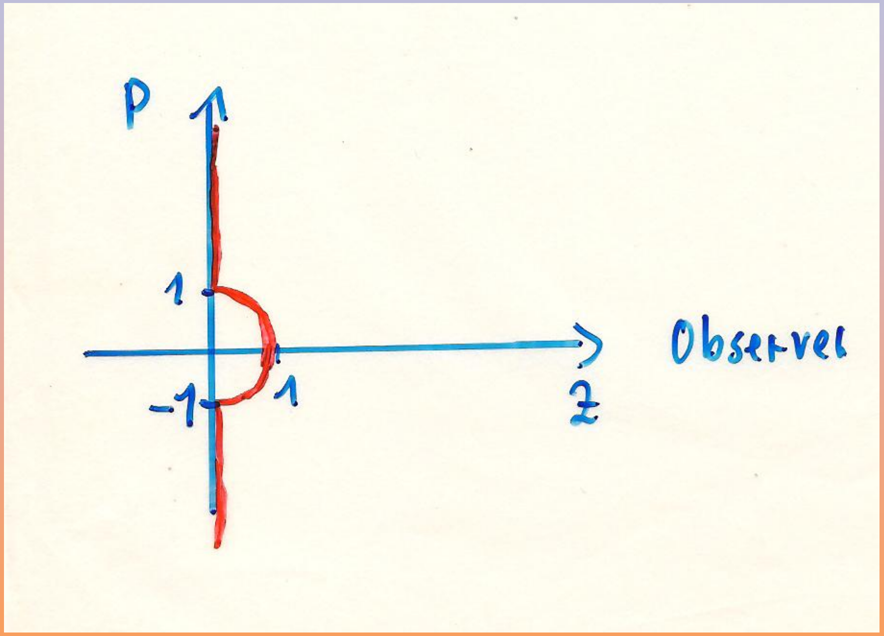
general shape of interaction surfaces



discussion of $z_0(x, p)$

a) line center frequency x=0

resonance condition $$x = \mu\ v(r) = \frac{z}{r} v(r) \qquad \mu = \frac{z}{r}$$



interaction surface

for p ≤ 1 : r =1, since v(r) = 0
p > 1: z = 0, since r > 1
and v(r) > 0

b) radial axis towards observer with p = 0, x ≠ 0

$x = \mu v(r)$ but $p = 0$ ➡ $\mu = 1$

$$p = r\,(1-\mu^2)^{1/2}$$

➡ $$x = v(r)$$

$$r^2 = p^2 + z^2$$

➡ $$x = v_\infty\Big(1 - \frac{1}{(p^2 + z_0^2)^{1/2}}\Big)^\beta = v_\infty\Big(1 - \frac{1}{z_0}\Big)^\beta$$

➡ $$z_{0,p=0} = \frac{1}{1 - \big(\frac{x}{v_\infty}\big)^{\frac{1}{\beta}}}$$

for $x < x_{max} = v_\infty$

max. Doppler-shift

c) p $\neq$ 0, but $x/v_\infty \longrightarrow 1$

x/v_∞ approaching 1 ➡ $z_0(x,p) \gg 1$

$r^2 = p^2 + z^2$

$\mu = z/r$

$$x = \frac{z_0}{(p^2+z_0^2)^{1/2}}\, v_\infty (1 - \frac{1}{(p^2+z_0^2)^{1/2}})^\beta$$

resonance condition
x = μv(r)

➡

$$(\frac{x}{v_\infty})^2\ (p^2+z_0^2) = z_0^2\ (1 - \frac{1}{(p^2+z_0^2)^{1/2}})^{2\beta}$$

$$\approx z_0^2\ (1 - \frac{2\beta}{(p^2+z_0^2)^{1/2}})$$

$(1+\epsilon)^n \approx 1 + n\epsilon$

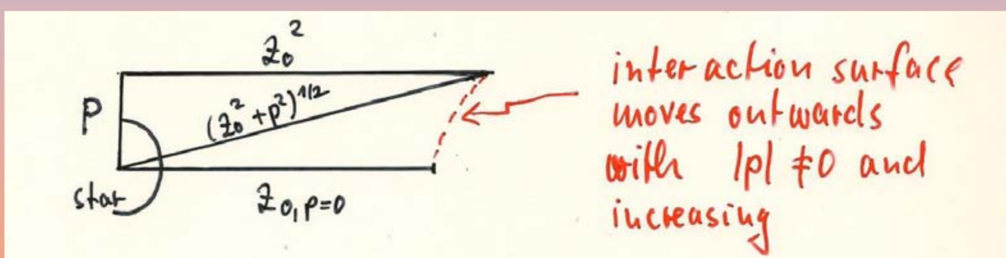
$|\,\epsilon\,| \ll 1$

solve for z_0^2 ➡

$$\rightarrow \quad z_0^2(x,p) = \frac{(x/v_\infty)^2}{\{1-\frac{x^2}{v_\infty^2}-\frac{2\beta}{(p^2+z_0^2)^{1/2}}\}}\; p^2$$

for p =0 → { } = 0 → $z_{0,p=0} = \dfrac{2\beta}{1-\frac{x^2}{v_\infty^2}} = \dfrac{2}{1+\frac{x}{v_\infty}}\;\dfrac{\beta}{1-\frac{x}{v_\infty}}$

|p| > 0 → { } > 0 → $\{\,\} = 2\beta\,\left(\dfrac{1}{z_{0,p=0}} - \dfrac{1}{(z_0^2+p^2)^{1/2}}\right)$



$(z_0^2+p^2)^{1/2} > z_{0,p=0}$

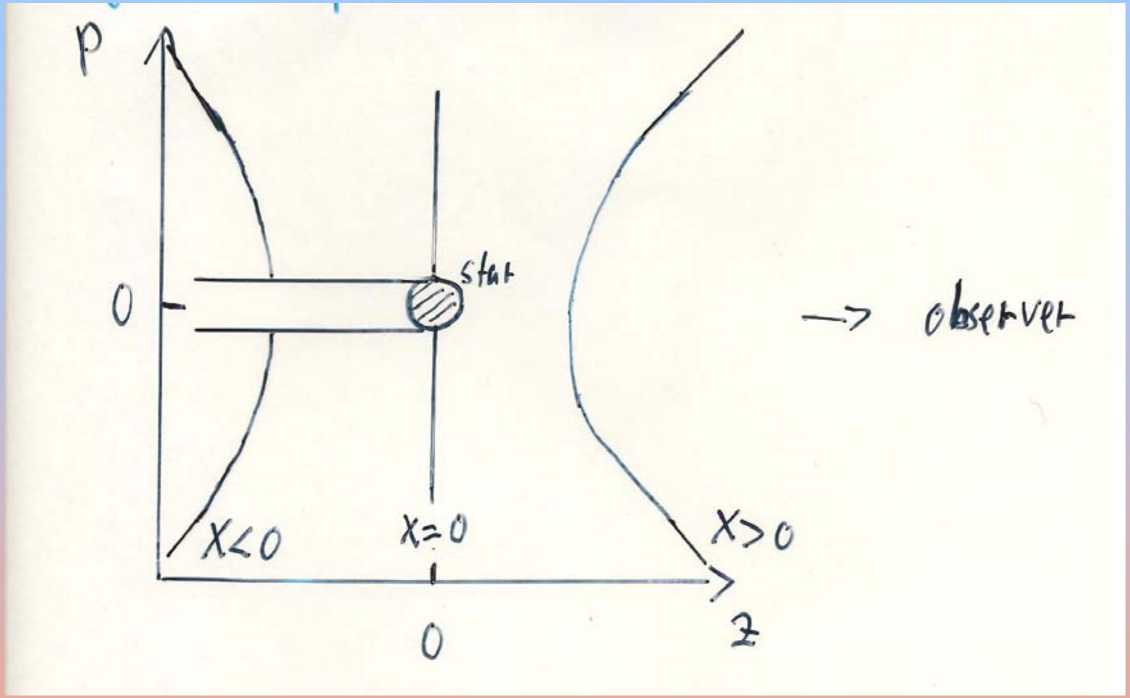
|p| >> 1 →

$(z_0^2+p^2)^{1/2} \gg z_{0,p=0}$

$$z_0(x,p) = \frac{x/v_\infty}{(1-\frac{x^2}{v_\infty^2})^{1/2}}\; p$$

interaction surface linear with p at large p

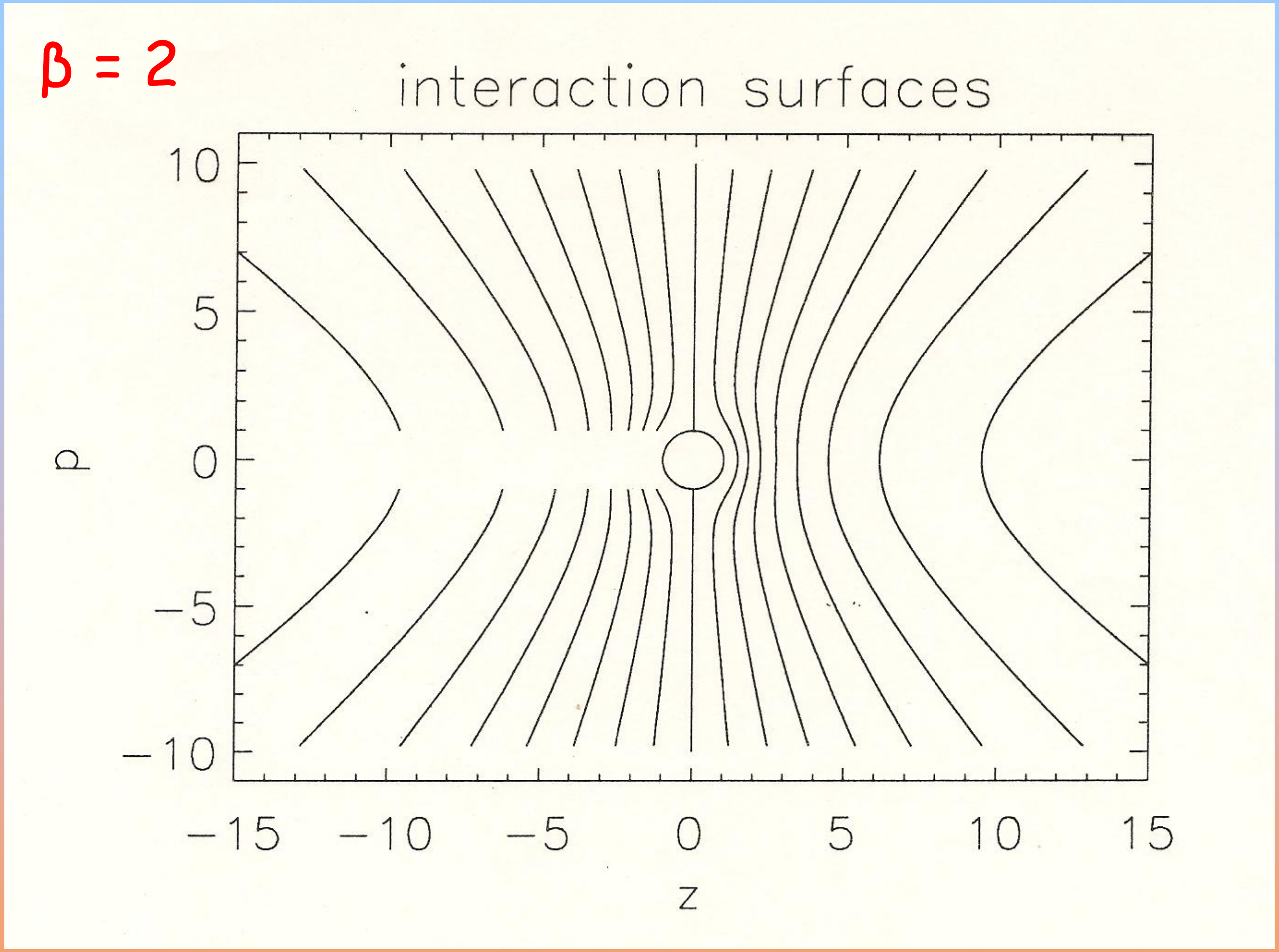
general shape of interaction surfaces



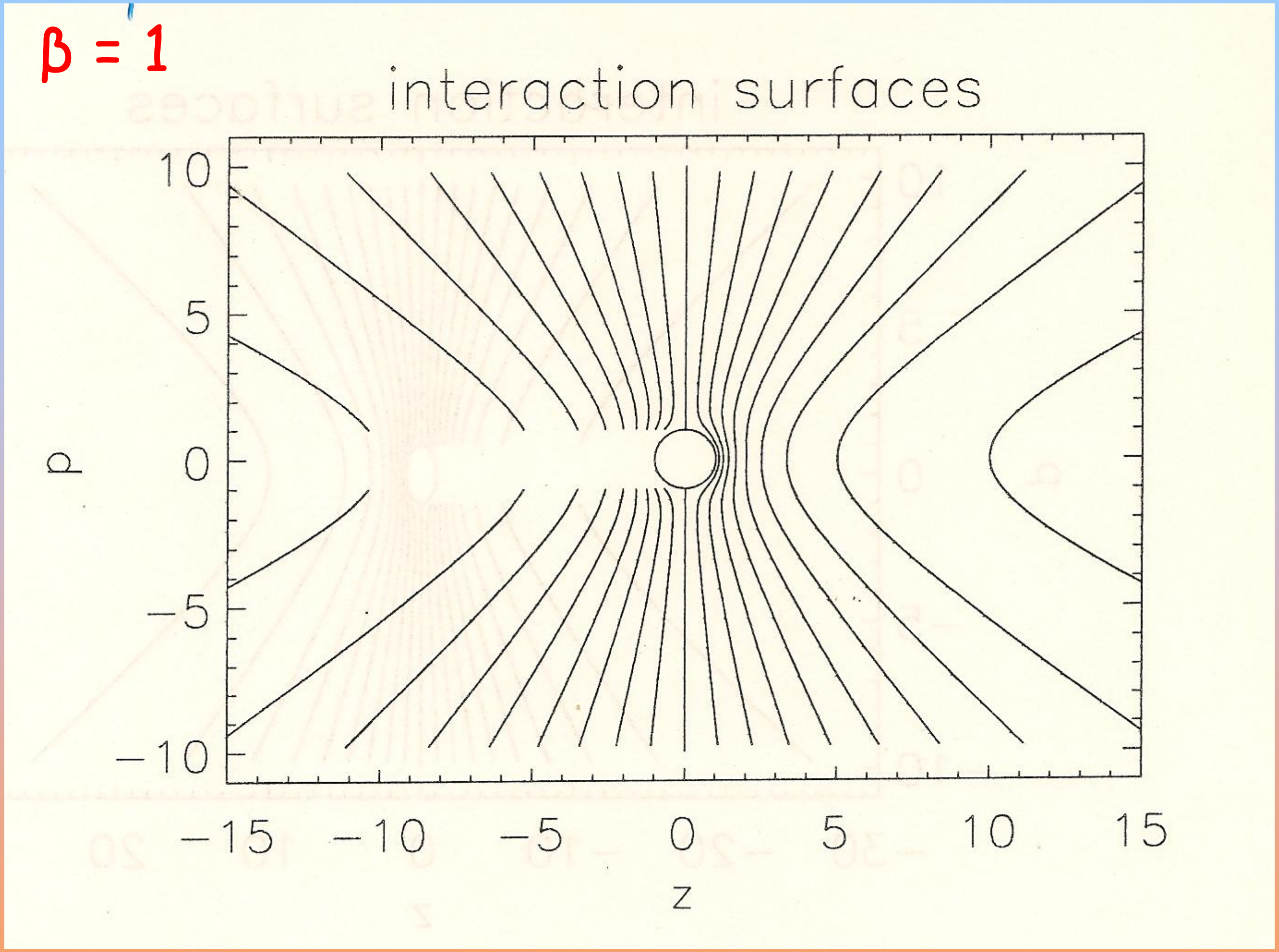
detailed calculation by numerical solution of

$$f(z_0, x, p, \beta) = 0$$

$$f = \frac{x}{v_\infty} - \frac{z_0}{r_0}\left(1 - \frac{1}{r_0}\right)^\beta \qquad r_0 = (z_0^2 + p^2)^{1/2}$$



x/v_{∞} = -0.9, -0.8, -0.7 … +0.7, +0.8, +0.9



x/v_∞ = -0.9, -0.8, -0.7 … +0.7, +0.8, +0.9

a very simple case: linear velocity fields

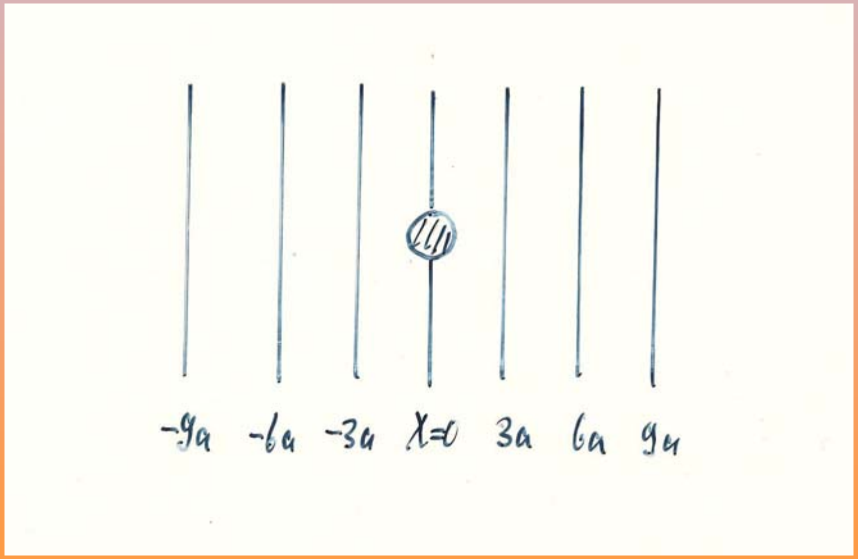
$$v(r) = a \bullet r$$

resonance condition ➡

$$x = \mu\, v(r) = \frac{z_0}{r}\, v(r)$$

$$x = \frac{z_0}{r}\, a\, r = z_0\, a$$

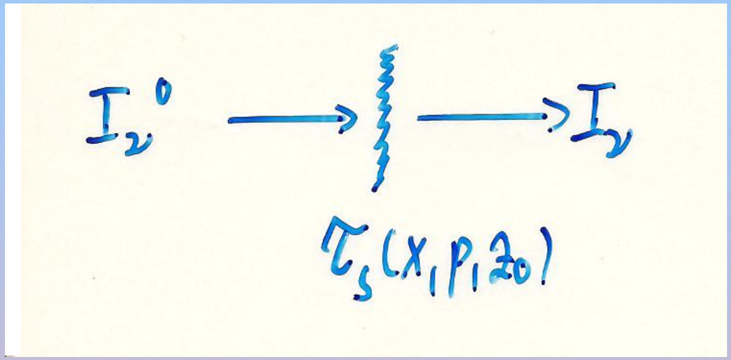
➡ $$z_0 = \frac{x}{a}$$ independent of p !!!



plane interaction surfaces

Supernovae have linear velocity fields

4. Optical thickness of interaction surface



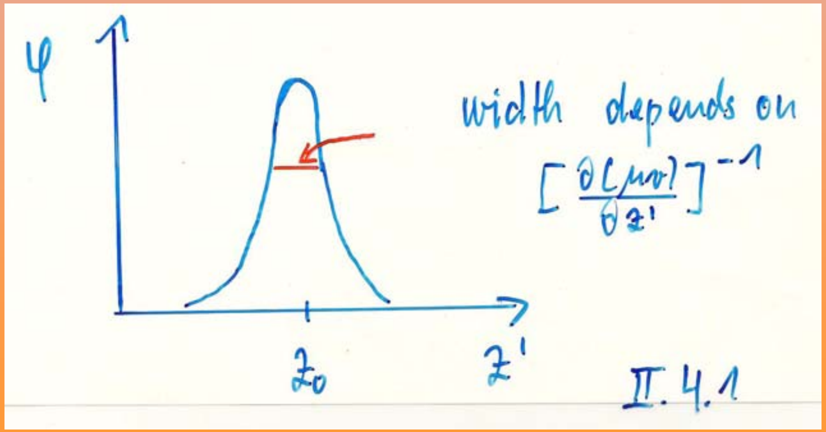
Optical thickness of interaction surface determines how much is absorbed and re-emitted

➡ calculate τ along ray with p = const.

$$\tau(x,p,z) = R_* \int_{z_{min}}^{z} k[r(\tilde z,p)]\ \phi[x - \mu(\tilde z,p) v(r(\tilde z,p))] d\tilde z$$

integrand has peak at interaction point z_0, where

$$x - \mu(z_0,p)\ v(r(z_0,p)) = 0$$



$\frac{\partial(\mu v)}{\partial z}$ large ➡ peak narrow

➡ $k(r)=k(r_0)$=const over peak

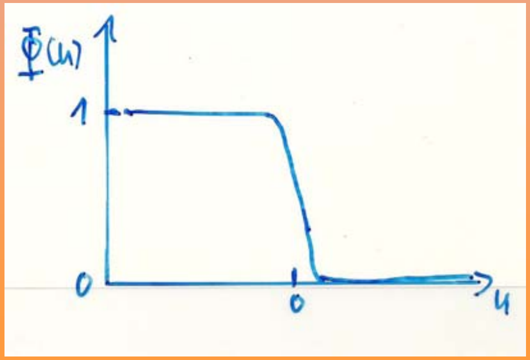
substitute $u = x - \mu\, v$ → $d\tilde{z} = -\{\frac{\partial(\mu v)}{\partial \tilde{z}}\}^{-1} du$

→
$$\tau(x,p,z) = R_* \int_{u(x,p,z)}^{u(x,p,z_{min})} k(r)\ \{\frac{\partial(\mu v)}{\partial \tilde{z}}\}^{-1} \phi(u) du$$

$$\approx \underbrace{R_*\ k(r(z_0,p))\ \{\frac{\partial(\mu v)}{\partial z}\}^{-1}}_{\tau_s(p,z_0)} \underbrace{\int_u^{u(z_{min})} \phi(u) du}_{\Phi(u)}$$

since $u(z_{min}) \gg 1$ → $\Phi(u) \approx \int_u^\infty \phi(\tilde{u}) d\tilde{u} = \pi^{-1/2} \int_u^\infty e^{-\tilde{u}^2} d\tilde{u} = Erfc(u)$

complementary error function



step function

$\Phi(u) = 0,\ u > 0$

$\Phi(u) = 1,\ u \le 0$

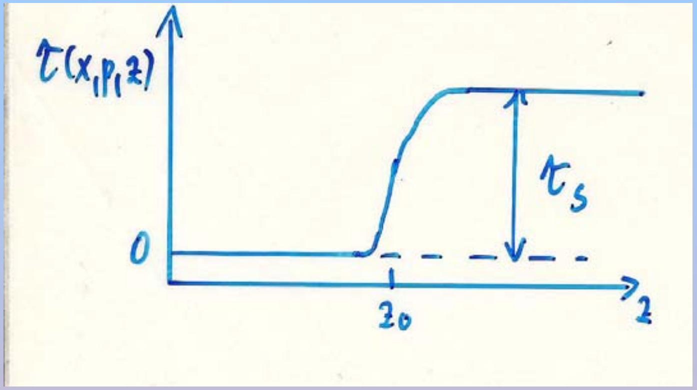
explanation for previous page

u as a function of z

$$u = x - \mu \; v = x - \frac{z}{(z^2 + p^2)^{1/2}} \; v_\infty (1 - \frac{1}{r})^\beta$$

For $z = z_{min} \rightarrow -\infty$ ➡ $u \rightarrow u = x + v_\infty$

$$\tau(x,p,z) \approx \tau_s(p,z_0)\ \Phi[u(x,p,z)]$$



τ_s optical thickness of interaction surface for ray with p, x

$$\tau_s = R_* k(r) \frac{\partial(\mu v)}{\partial z}^{-1}$$

p = const.

$$\frac{\partial}{\partial z}\{\mu(z,p)\ v(r(z,p))\} = \mu \frac{\partial v}{\partial r}\frac{\partial r}{\partial z} + v\frac{\partial \mu}{\partial z}$$

$$p^2 + z^2 = r^2 \qquad z = r\mu$$

$$\rightarrow\ r\partial r = z\partial z$$

$$\rightarrow\ \frac{\partial r}{\partial z} = \frac{z}{r} = \mu$$

$$\frac{\partial \mu}{\partial z} = \frac{1}{r} - \frac{z}{r^2}\mu$$

$$\rightarrow\ \frac{\partial}{\partial z}(\mu v) = \mu^2\frac{\partial v}{\partial r} + \frac{v}{r}(1-\mu^2)$$

$$\boxed{\tau_s = R_* \frac{k(r)}{Q(r,\mu)} = R_* \frac{k(r)}{\mu^2 \frac{\partial v}{\partial r} + (1-\mu^2)\frac{v}{r}}}$$

discussion:

$$\tau_s = R_* \frac{k(r)}{Q(r,\mu)} = R_* \frac{k(r)}{\mu^2 \frac{\partial v}{\partial r} + (1-\mu^2)\frac{v}{r}}$$

$$Q(r,\mu) = \mu^2 \frac{\partial v}{\partial r} + (1-\mu^2)\frac{v}{r}$$

symmetric in μ

radial tangential velocity gradient

$$Q(r,\mu) = \frac{v}{r}(1+\sigma\mu^2) \qquad \sigma = \frac{\partial\ ln\ v}{\partial\ ln\ r} - 1$$

departure from homologous expansion

$v \propto r$

$$\tau_S \approx R_*\ k\ \Delta l_s$$

$$\Delta l_s = \frac{1}{Q(r,\mu)}$$

geometrical width of interaction region (dimensionless)

5. The Sobolev Approximation

we made approximation

$$\tau(x,p,z) \approx R_* k(r(z_0,p)) \int \phi(x-\mu v) d\tilde{z}$$

only justified, if k(r) ~ const over peak of integrand
width of peak is

$$\Delta l_s = \frac{1}{Q(r,\mu)}$$

k(r) varies with density ρ or on comparable scale

$$\Delta l_{dyn} = \left| \frac{d\rho}{dr}\frac{1}{\rho} \right|^{-1}$$

approximation is ok, if

$$\Delta l_s \ll \Delta l_{dyn}$$

Sobolev approximation

in winds (as we will learn later)

$$\rho(r) \propto r^{-n}, n = 1...4$$

$$\Delta l_{dyn} = \frac{r}{n}$$

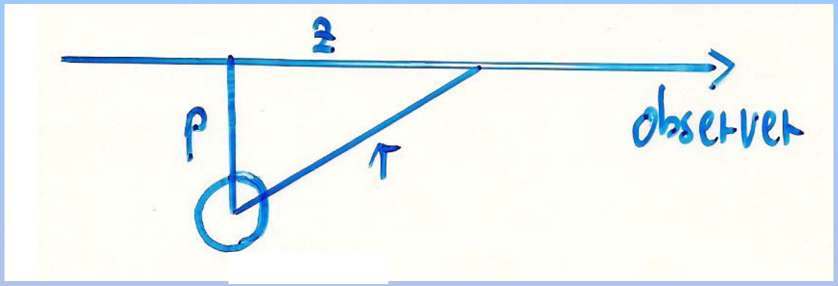
on the other hand

$$\Delta l_s \approx \frac{r}{v}$$

if $v \gg n$ which is the case for strongly supersonic winds with $v \gg 1$

Sobolev approximation is ok

6. Radiative transfer



solution of transfer eq. in (p,z) - geometry

$$\frac{1}{R_*}\frac{dI(x,p,z)}{dz} = \kappa_x(r(p,z))\{I(x,p,z) - S(r(p,z))\}$$

$$\kappa_x = \kappa_\nu^{bg} + n_E\,\sigma_E + k(r)\;\phi(x-\mu v)$$

total absorption coefficient

background opacity/ emissivity

Thomson scattering

line absorption emission

$$\epsilon_x = \epsilon_\nu^{bg} + n_E\,\sigma_E\;J_\nu + \epsilon(r)\;\phi(x-\mu v)$$

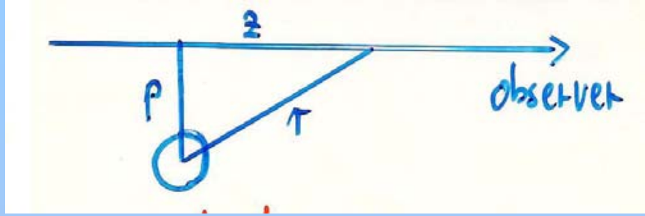
total emission coefficient

$$S(r) = \frac{\kappa_\nu^{bg}}{\kappa_x} S_{bg}(r) + \frac{n_E\,\sigma_E}{\kappa_x}\;J_\nu + \frac{k(r)}{\kappa_x}\;S_L(r)$$

total source function

$$S_{bg}(r) = \frac{\epsilon_\nu^{bg}}{\kappa_\nu^{bg}},\quad S_L = \frac{\epsilon(r)}{k(r)}$$

formal solution along ray with p = const.



$$I(x,p,z) = R_* \int_{z_{min}}^{z} S(r(p,\tilde z)) e^{-\Delta\tau(x,p,\tilde z)} \kappa_x(r(p,\tilde z)) d\tilde z + I_{z_{min}} e^{-\tau(x,p,z)}$$

$\tau(x,p,\tilde z)$ $\qquad\qquad$ $\Delta\tau(x,p,\tilde z)$

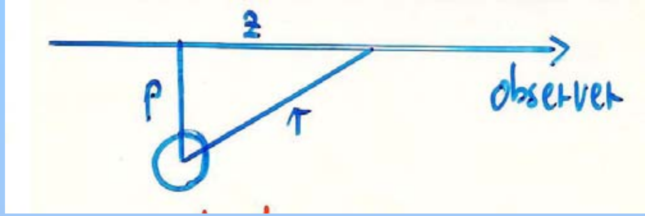
z_{min} $\qquad$ $\tilde z$ $\qquad$ z

$\tau(x,p,z)$

$$\tau(x,p,z) = R_* \int_{z_{min}}^{z} \kappa_x(r(p,\tilde z)) d\tilde z$$ optical path z_{min} to z

$$\Delta\tau(x,p,\tilde z) = \tau(x,p,z,) - \tau(x,p,\tilde z)$$ optical path $\tilde z$ to z

discussion of formal solution



$$I(x,p,z) = R_* \int_{z_{min}}^{z} S(r(p,\tilde{z}))e^{-\Delta\tau(x,p,\tilde{z})}\kappa_x(r(p,\tilde{z}))d\tilde{z} + I_{z_{min}}e^{-\tau(x,p,z)}$$

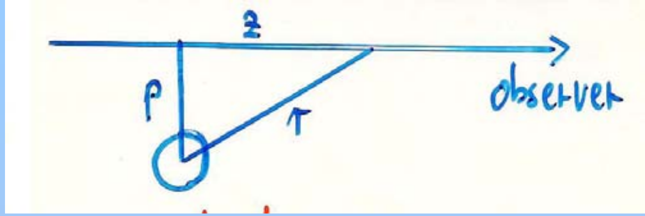
$I_{z_{min}}$ incident radiation at initial point

$I_{z_{min}}e^{-\tau(\tilde{z})}$ fraction of incident radiation arriving at z weakened by absorption

$S(\tilde{z})\kappa_x(\tilde{z}) = \epsilon_x(\tilde{z})$ re-emitted intensity at $\tilde{z}$

$S(\tilde{z})\kappa_x(\tilde{z})e^{-\Delta\tau(\tilde{z})}$ fraction of intensity emitted at $\tilde{z}$ arriving at z ; weakened by absorption between $\tilde{z}$ and z

$R_* \int_{z_{min}}^{z} ...d\tilde{z}$ Integration of contributions from all $\tilde{z}$



$$I(x,p,z) = R_* \int_{z_{min}}^{z} S(r(p,\tilde{z}))e^{-\Delta\tau(x,p,\tilde{z})}\kappa_x(r(p,\tilde{z}))d\tilde{z} + I_{z_{min}} e^{-\tau(x,p,z)}$$

$I_{z_{min}}(p)$ depends on impact parameter p

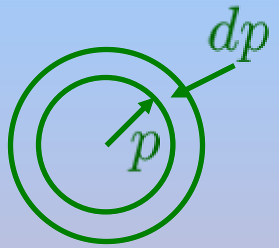
$0 \leq \ | p | \ \leq 1$ $\quad I_{z_{min}}(p) = I_c(p)$ continuum intensity from stellar core

$1 \leq \ | p |$ $\quad$ integration starts at $z = -\infty$

$$I_{z_{min}}(p) = 0$$

With $\kappa_x(r)$ and $S(r)$ known we can calculate

$I(x, p, z = z_{max})$ intensity arriving at observer



observer sees integral over all p weighted by area from ring with ~2pdp

observer sees flux $$F(x) \propto R_*^2 \int_0^\infty I(x, p, z_{max}) 2pdp$$

continuum flux from stellar surface $$F_{cont}(x) \propto R_*^2 \int_0^1 I_c(x, p) 2pdp$$

observed normalized line profile

$$\boxed{P(x) = \frac{\int_0^\infty I(x, p, z = z_{max}) pdp}{\int_0^1 I_c(p) pdp}}$$

standard stellar wind diagnostics

$\kappa_x(r)$

$S(r)$

from solution of NLTE rate equations

then numerical solution to calculate line profile P(x)

→ spectral diagnostics of line profiles

→ determination of $v(r),\ \rho(r),\ \dot{M}$

for better understanding of wind diagnostics

2 approximations in the following

1. Line opacity dominates

$$k(r) \gg \kappa_\nu^{bg}, n_E \sigma_E$$

2. assumption $\frac{dv}{dr}$ large

→ Sobolev approximation

radiative transfer only in interaction zones

Approximation 1

$$\kappa_x = k(r)\ \phi(x-\mu v) \qquad \epsilon_x = \epsilon(r)\ \phi(x-\mu v)$$

$$S(r) = S_L(r) = \frac{\epsilon(r)}{k(r)}$$

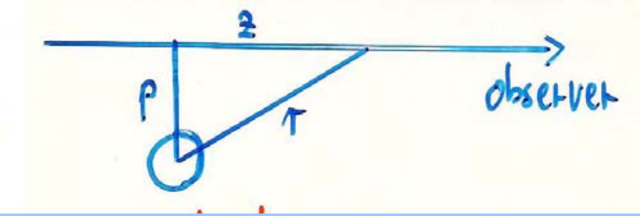
with exakt expression for $\epsilon(r),\ k(r)$

$$k(r) = \frac{\pi e^2}{mc} f_{lu} \{n_l - \frac{g_l}{g_u} n_u\} \frac{1}{\Delta\nu_D}$$

$$\epsilon(r) = \frac{\pi e^2}{mc} f_{lu} \frac{2h\nu^3}{c^2} \frac{g_l}{g_u} n_u \frac{1}{\Delta\nu_D}$$

$$\Rightarrow \qquad S_L(r) = \frac{2h\nu^3}{c^2} \frac{1}{\frac{n_l/g_l}{n_u/g_u} - 1}$$

formal solution in p-z geometry (p. 28)



$$I(x,p,z) = R_* \int_{z_{min}}^{z} S(r(p,\tilde{z}))e^{-\Delta\tau(x,p,\tilde{z})}\kappa_x(r(p,\tilde{z}))d\tilde{z} + I_{z_{min}}e^{-\tau(x,p,z)}$$

changes to

$$I(x,p,z) = R_* \int_{z_{min}}^{z} S_L(r)e^{-\Delta\tau(x,p,\tilde{z})}k(r)\phi(x-\mu v)d\tilde{z} + I_{z_{min}}e^{-\tau(x,p,z)}$$

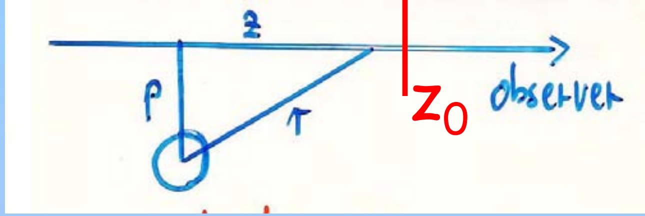
$$\tau(x,p,z) = R_* \int_{z_{min}}^{z} k(r)\phi(x-\mu v)d\tilde{z}$$

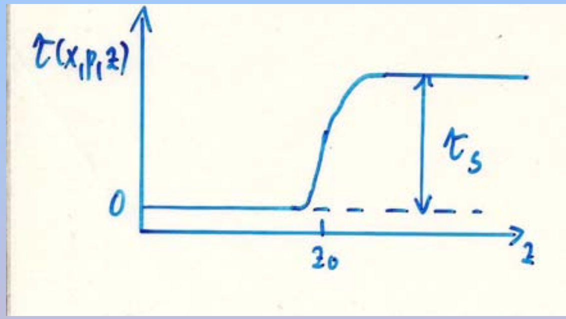
$$\Delta\tau(x,p,\tilde{z}) = \tau(x,p,z,) - \tau(x,p,\tilde{z})$$

$$r = r(p,\tilde{z})$$

$$\mu = \mu(p,\tilde{z})$$

Approximation 2 see pages 20 to 26





$$\tau(x,p,z) \approx \tau_s(p,z_0)\ \Phi[u(x,p,z)]$$

$$\Phi \approx 0,\ z < z_0 \qquad \Phi \approx 1,\ z \geq z_0$$

$$R_* \int_{z_{min}}^{z} S_L(r) e^{-\Delta\tau(x,p,\tilde z)} k(r)\phi(x-\mu v)d\tilde z$$

$$\approx R_*\ S_L(r(p,z_0)) \int_{z_{min}}^{z} e^{-\Delta\tau(x,p,\tilde z)} k(r)\phi(x-\mu v)d\tilde z$$

substituting $\tilde z \rightarrow \Delta\tau(\tilde z)$

$$\Delta\tau(\tilde z) = \tau(z) - \tau(\tilde z)$$

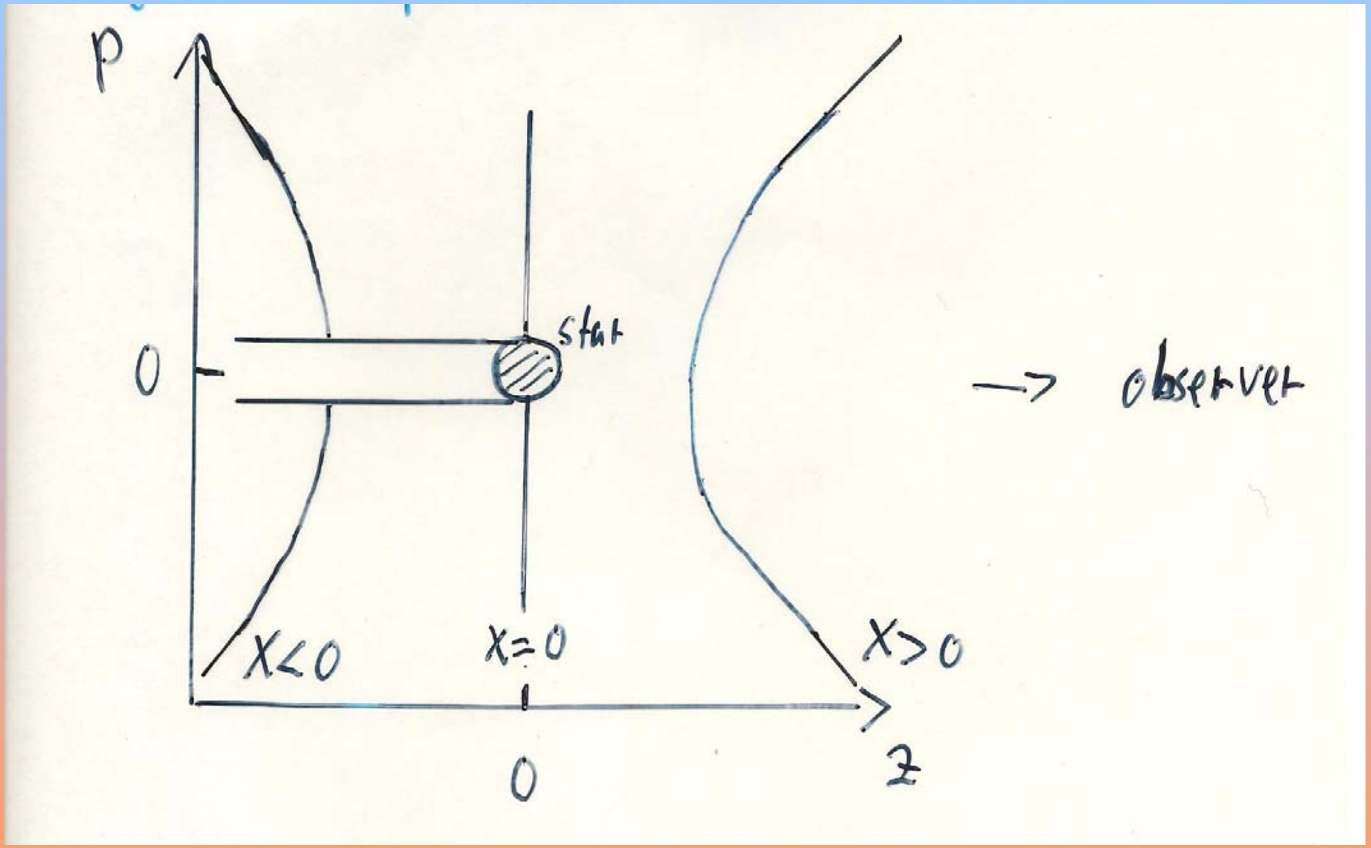
$\Rightarrow$ $d(\Delta\tau) = -R_*\ k(\tilde z)\phi(x-\mu v)d\tilde z$

$$\Delta\tau(z) = 0 \qquad \Delta\tau(z_{min}) = \tau(z) = \tau_S \Phi$$

$$= S_L(r_0(p,z_0)) \int_0^{\tau_S \phi} e^{-\Delta\tau} d\Delta\tau = S_L(r_0(p,z_0))(1 - e^{-\tau_S \Phi})$$

note: z_0 is z-value of interaction zone along p = const. for frequency x

Remember: general shape of interaction surfaces



now we can calculate line profile by integrating over pdp along interaction surface

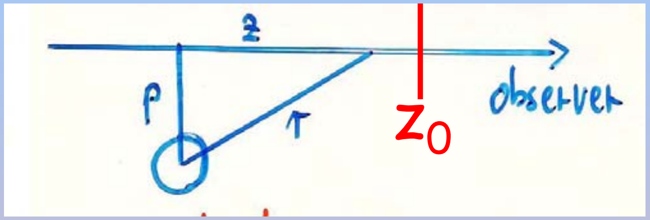
➡ Intensity determined by $S_L(r_0), \tau_S(r_0)$ at interaction zone

$$I(x,p,z) = S_L(r_0(p,z_0))(1 - e^{-\tau_S(p,z_0)\Phi(z)}) + I_{z_{min}} e^{-\tau_S(p,z_0)\Phi(z)}$$

with, see p. 30

$$I_{z_{min}}(p) = 0 \qquad 1 \le \ |p|$$

$$I_{z_{min}}(p) = I_c(p) \qquad 0 \le \ |p| \ \le 1$$



Intensity arriving at telescope has $\Phi(z \ge z_0) = 1$ ➡

$x \ge 0$ blue frequencies

$$I(x,p,z=\infty) = S_L(r_0(p,z_0))(1 - e^{-\tau_S(p,z_0)}) + I_c e^{-\tau_S(p,z_0)} \qquad 0 \le |p| \le 1$$

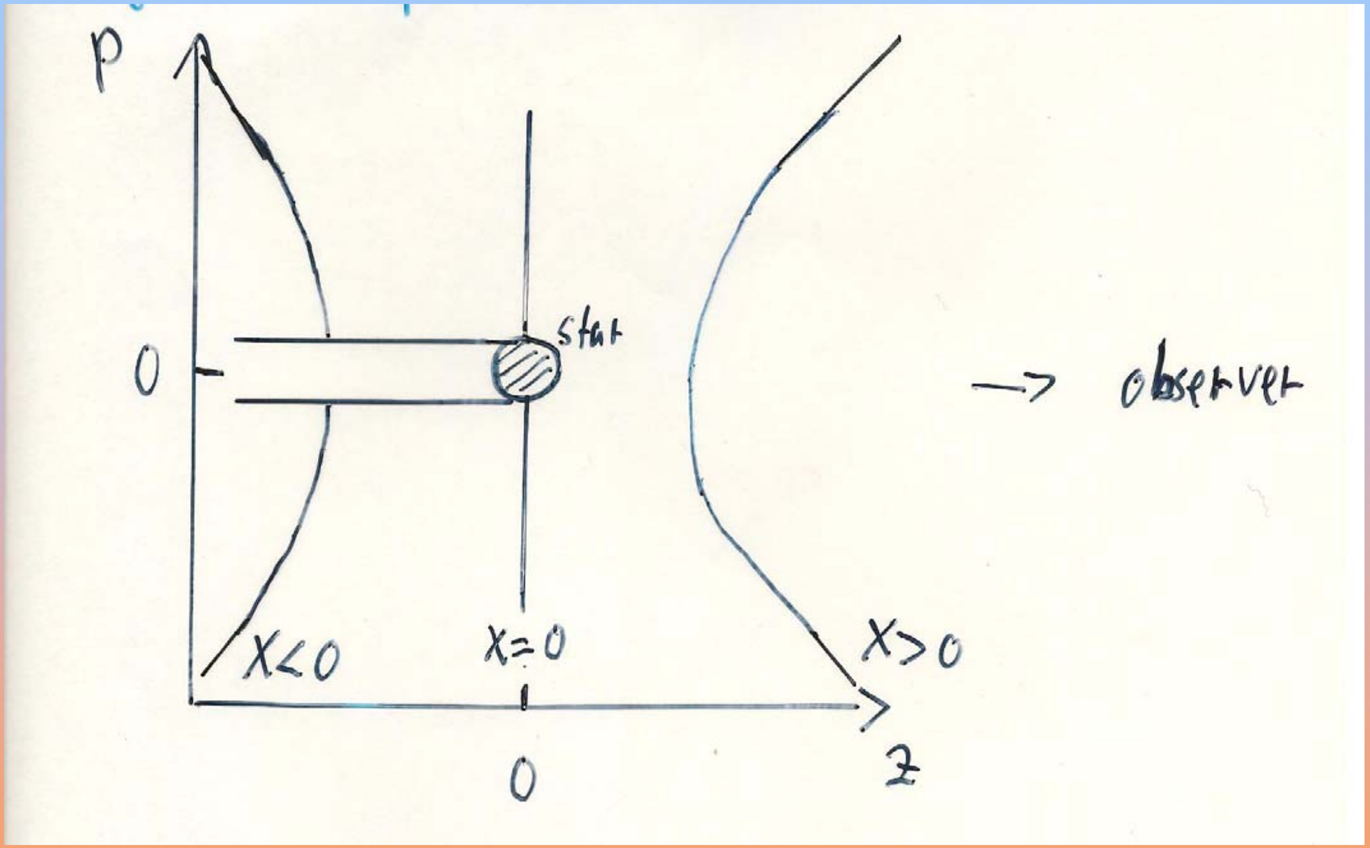
$$I(x,p,z=\infty) = S_L(r_0(p,z_0))(1 - e^{-\tau_S(p,z_0)}) + 0 \qquad |p| \ge 1$$

$x \le 0$ red frequencies

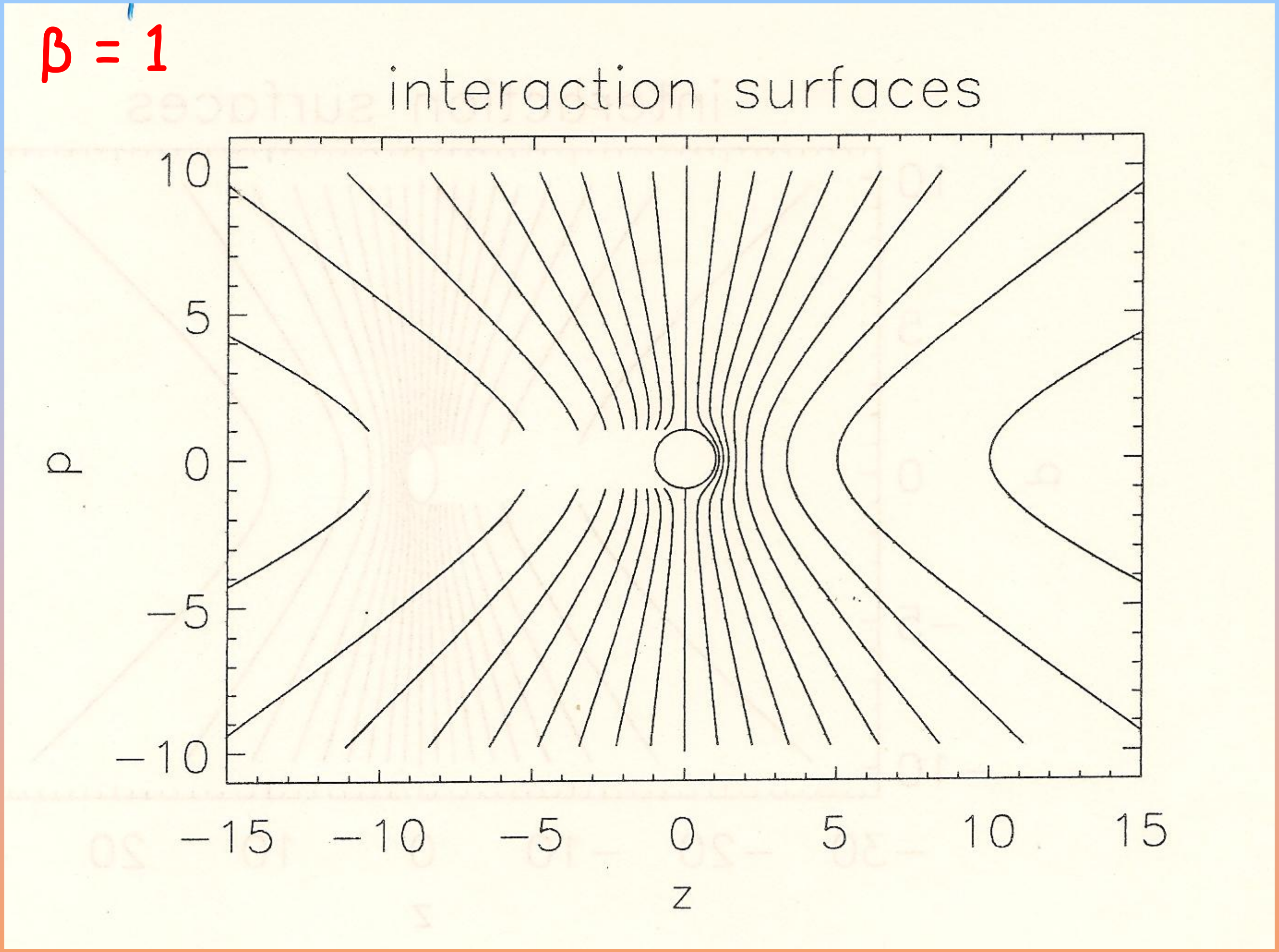
$$I(x,p,z=\infty) = I_c(p) \qquad 0 \le |p| \le 1$$

$$I(x,p,z=\infty) = S_L(r_0(p,z_0))(1 - e^{-\tau_S(p,z_0)}) \qquad |p| \ge 1$$

Remember: general shape of interaction surfaces



now we can calculate line profile by integrating over pdp along interaction surface



x/v_∞ = -0.9, -0.8, -0.7 … +0.7, +0.8, +0.9

now we can calculate the line profile

$$P(x) = \frac{\int_0^\infty I(x,p,z=\infty)pdp}{\int_0^1 I_c(p)pdp}$$

If we neglect limb-darkening of continuum radiation from stellar photosphere, then $I_c(p) = I_c$ = const. $\rightarrow$ $\int_0^1 I_c\, p\, dp = \frac{1}{2} I_c$

$x \geq 0$ blue frequencies

$$P(x) = \frac{2}{I_c}\{\int_0^\infty S_L(r_0)(1-e^{-\tau_S(r_0)})pdp + I_c\int_0^1 e^{-\tau_S(r_0)}pdp\}$$

$$= \int_0^\infty \frac{S_L(r_0)}{I_c}(1-e^{-\tau_S(r_0)})2pdp + \int_0^1 e^{-\tau_S(r_0)}2pdp$$

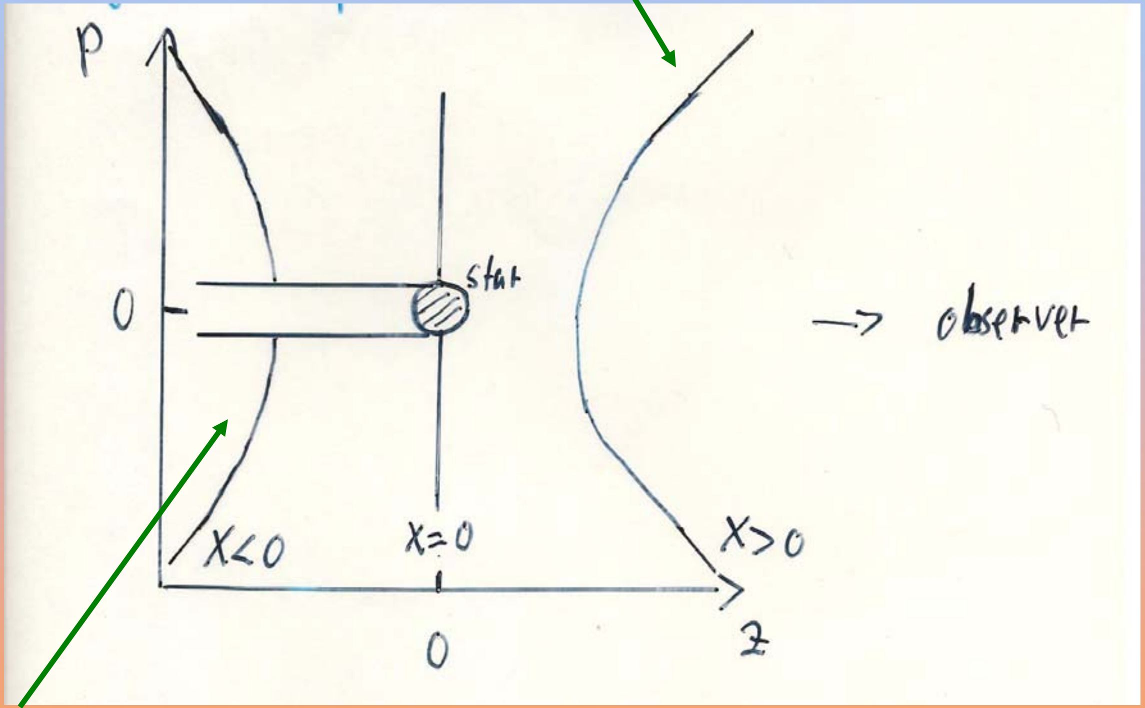
$x \leq 0$ red frequencies

$$P(x) = \frac{2}{I_c}\{\int_0^1 I_c pdp + \int_1^\infty S_L(r_0)(1-e^{-\tau_S(r_0)})pdp\}$$

$$= 1 + \int_1^\infty \frac{S_L(r_0)}{I_c}(1-e^{-\tau_S(r_0)})2pdp$$

$$P(x) = \int_0^\infty \frac{S_L(r_0)}{I_c}(1 - e^{-\tau_S(r_0)})2pdp + \int_0^1 e^{-\tau_S(r_0)}2pdp$$

blue emission integral from 0 to ∞



red emission integral from 1 to ∞

$$P(x) = 1 + \int_1^\infty \frac{S_L(r_0)}{I_c}(1 - e^{-\tau_S(r_0)})2pdp$$

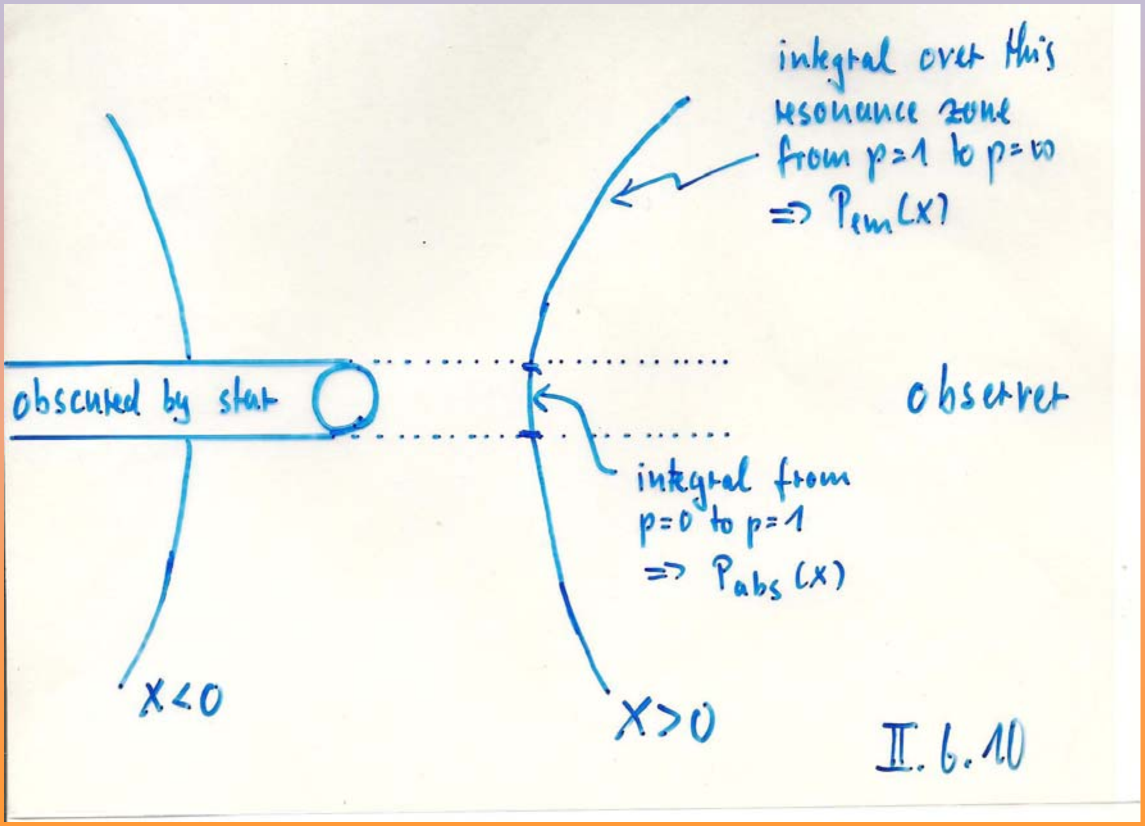
with definitions

$$P_{em}(x) = \int_1^\infty \frac{S_L}{I_c}(p, z_0)(1 - e^{-\tau_S(p,z_0)})2pdp$$

$$P_{abs}(x) = \int_0^1 \{\frac{S_L}{I_c}(p, z_0)(1 - e^{-\tau_S(p,z_0)}) + e^{-\tau(p,z_0)}\}2pdp$$

→ line profiles $x \leq 0$ red $x \geq 0$ blue

$P(x) = 1 + P_{em}(x)$ $P(x) = P_{abs}(x) + P_{em}(x)$



Discussion 1. Red part of line profile $x < 0$

since $P_{em}(x) > 0 \rightarrow P(x) > 1$

always in emission!!!!!

2. Blue part of line profile $x > 0$

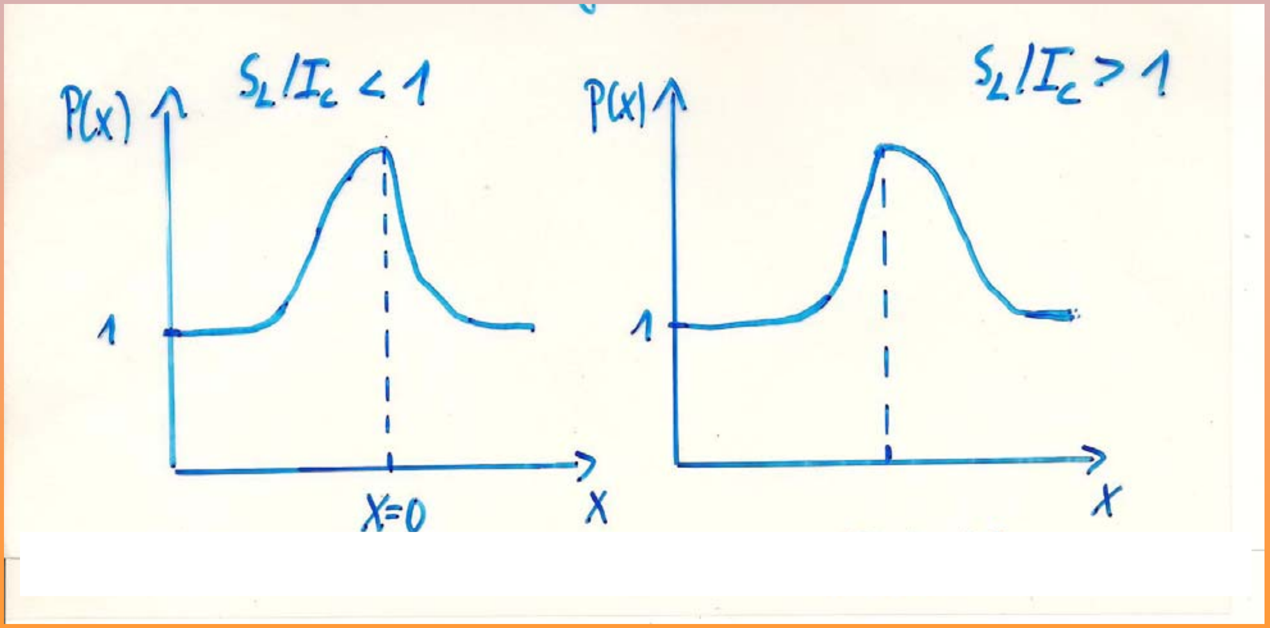
$S_L/I_c < 1 \rightarrow P_{abs}(x) < 1$

$= 1 \qquad = 1$

$> 1 \qquad > 1$

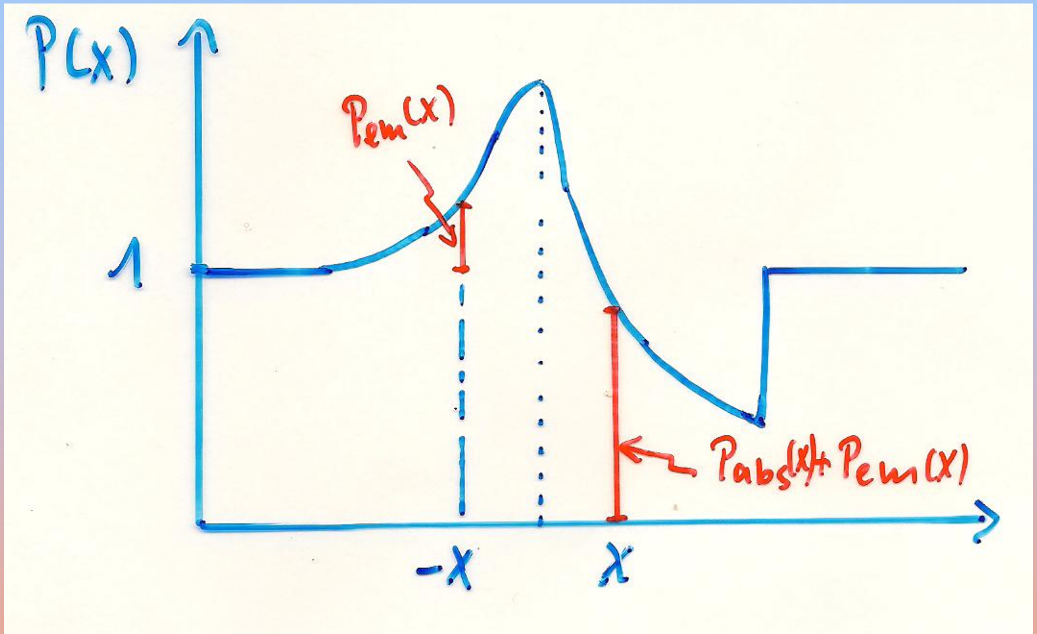
S_L/I_c determines symmetry of line profile

red/blue equal for $S_L/I_c=1$ (good test of programs)



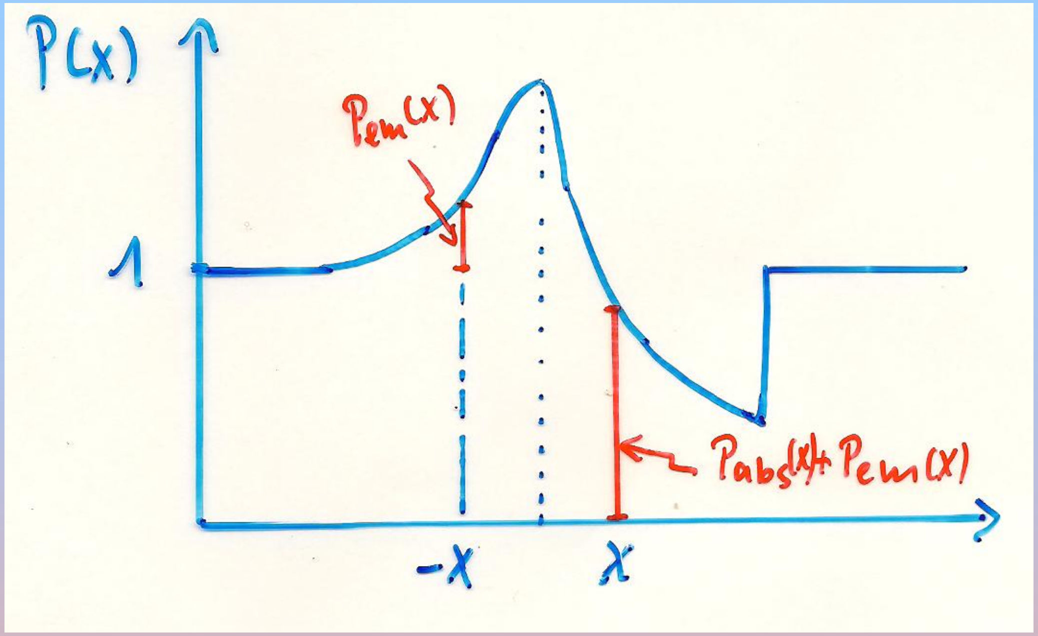
3. If $S_L/I_c < 1$ and if in addition $P_{abs}(x) + P_{em}(x) < 1$

→ blue absorption!!!!

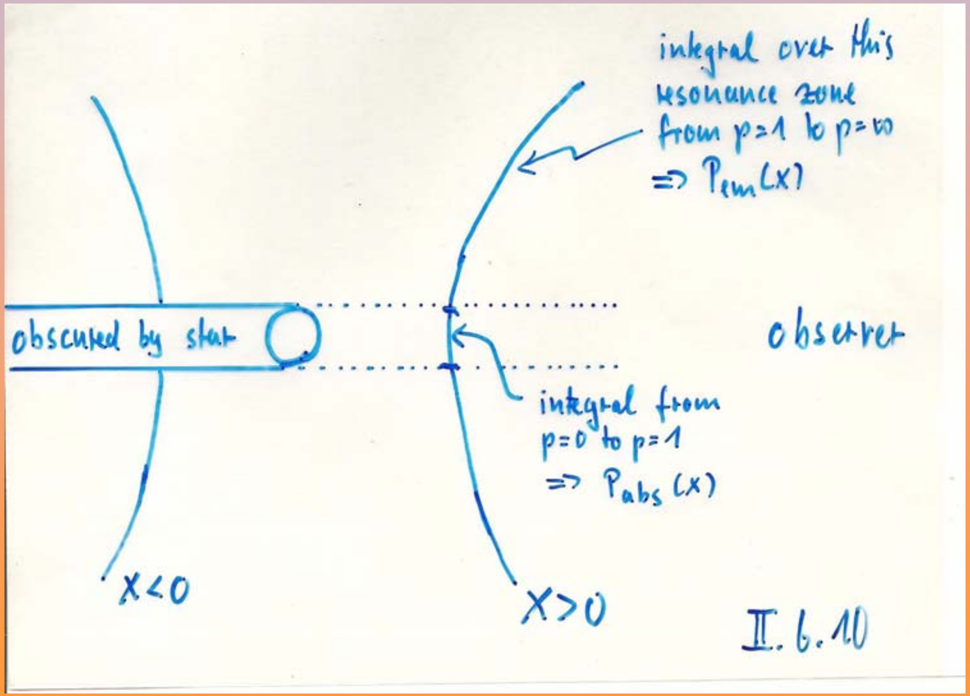


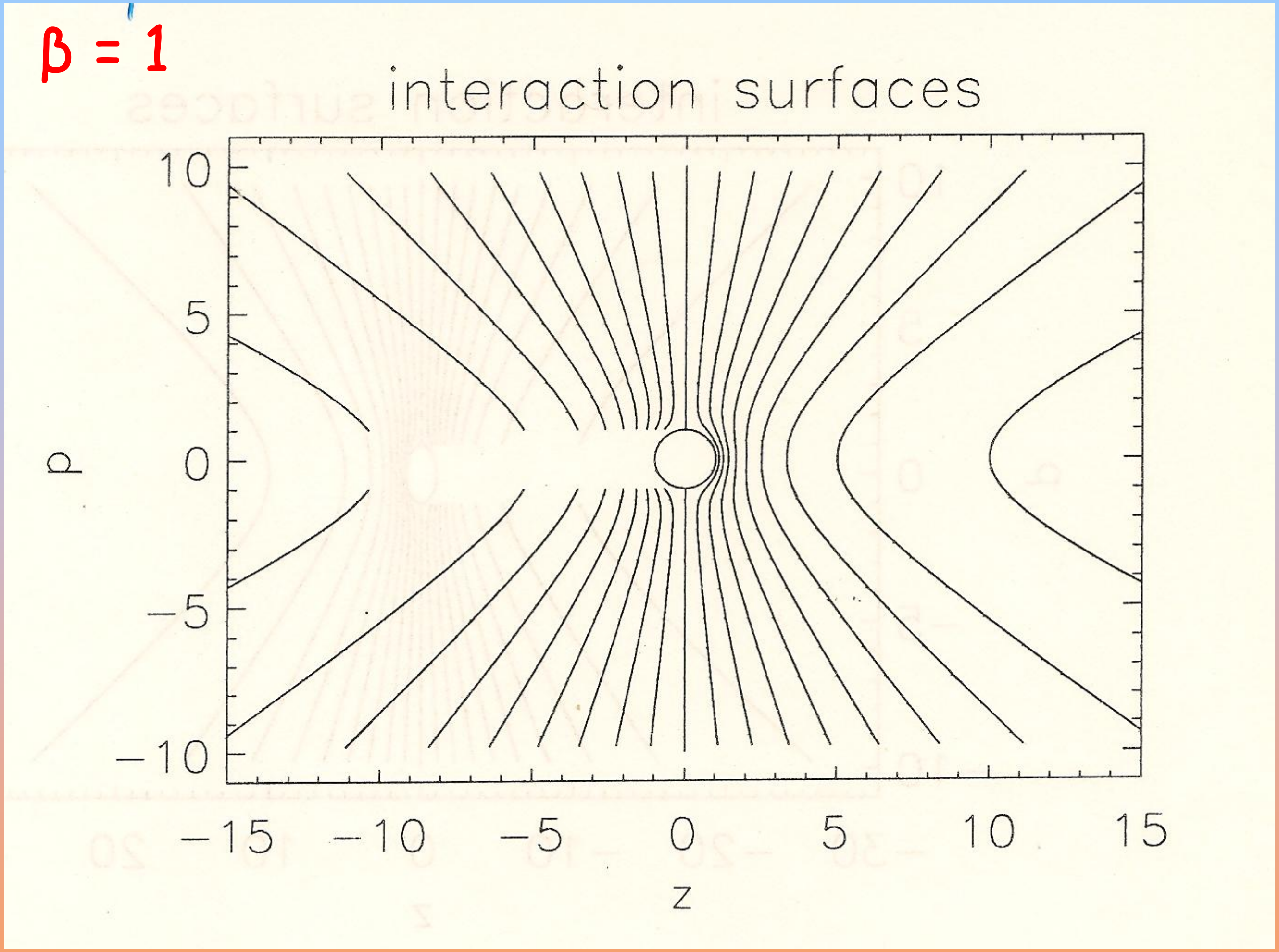
$$P_{abs}(x) = (P_{abs}(x) + P_{em}(x)) - P_{em}(x)$$

→ disentangling of emission and absorption integral possible!!!
wind tomography possible!!!!



wind tomography

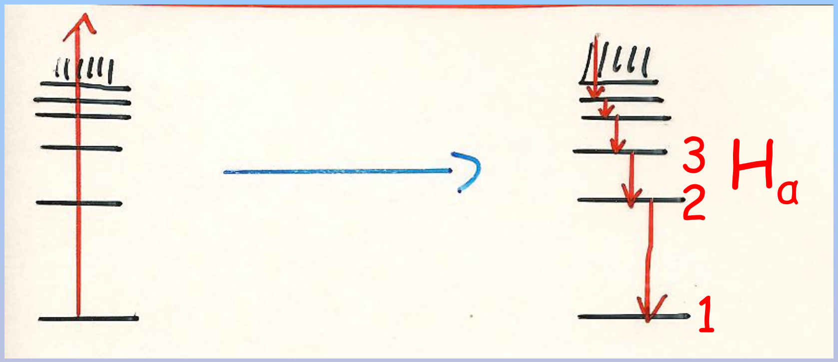




x/v_∞ = -0.9, -0.8, -0.7 … +0.7, +0.8, +0.9

Examples of stellar wind diagnostics

Example 1: H_α in O-stars



ionization from ground or excited level

cascade of subsequent spontaneous emissions

detailed NLTE calculations show n_3/n_2 ~ const. through winds

➡ $$S_L(r) = \frac{2h\nu^3}{c^2} \frac{1}{\frac{n_2}{n_3}\frac{g_3}{g_2} - 1} \approx const.$$

➡ S_L/I_c ~ const. We adopt S_L/I_c ~ 1 for simplicity

➡ $P_{abs}(x) = 1$

➡ emission line with similar red and blue part

P(x) = 1 + P_{em}(x)   strength of emission determined by P_{em}(x)

$$P_{em}(x) = \int_1^\infty \frac{S_L}{I_c}(p, z_0)(1 - e^{-\tau_S(p,z_0)})2pdp$$

$$\approx \int_1^\infty (1 - e^{-\tau_S(p,z_0)})2pdp$$

For O-stars $\tau_S^{H_\alpha} \ll 1$ optically thin interaction region

$$P_{em}(x) \approx \int_1^\infty \tau_S(p, z_0)2pdp$$

$$\tau_s = R_* \, \frac{k(r)}{\mu^2 \frac{\partial v}{\partial r} + (1-\mu^2)\frac{v}{r}} \propto R_* \, k(r)$$ p. 24

$$k(r) = \frac{\pi e^2}{mc} f_{lu} \, n_2(r)\{1 - \frac{n_3}{n_2}\frac{g_2}{g_3}\}\frac{1}{\Delta\nu_D} \propto n_2(r)$$ p. 34

detailed NLTE calculations show $n_2 \sim n_E n_p \sim \rho^2(r)$

→ $k(r) \sim \rho^2(r)$ with equation of continuity

$$\dot{M} = R_*^2 4\pi r^2 \rho v \quad \rightarrow \quad \rho^2(r) = (\frac{\dot{M}}{R_*^2})^2 \frac{1}{r^4 v^2}$$

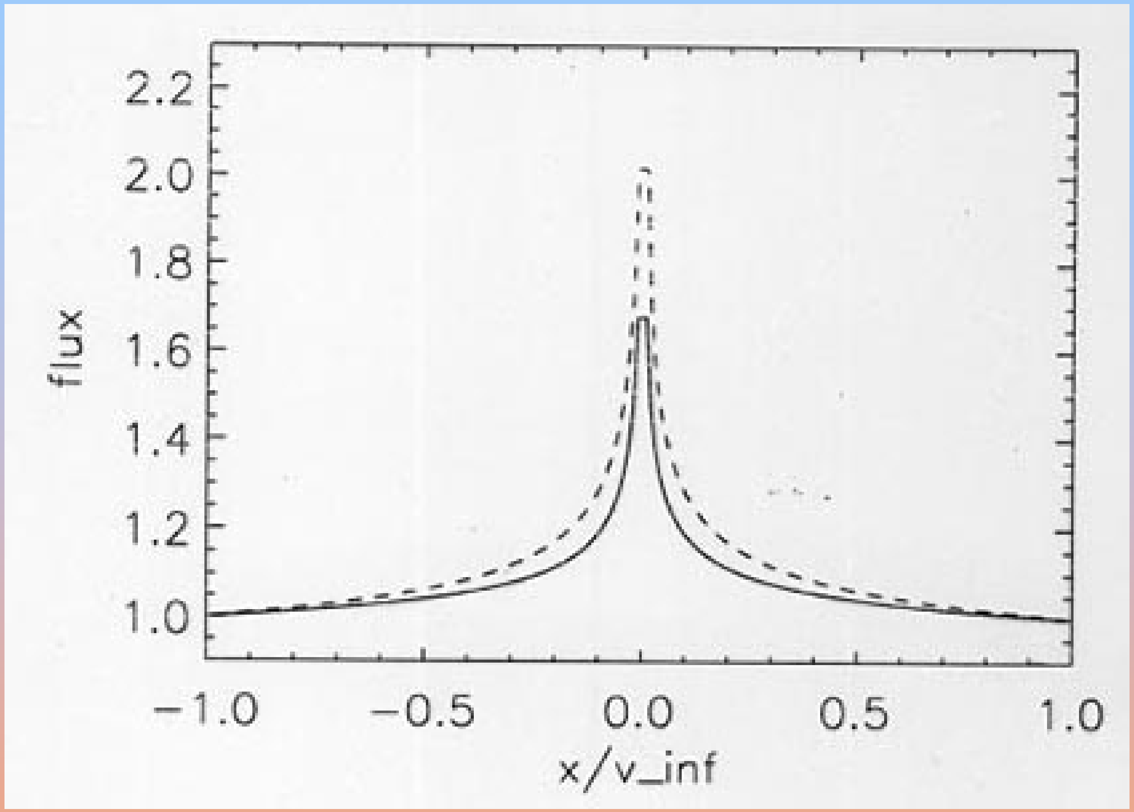
$$P_{em}(x) \approx \frac{S_L}{I_c} \int_1^\infty \tau_S(p, z_0) 2pdp \qquad \tau_s = R_* \frac{k(r)}{\mu^2 \frac{\partial v}{\partial r} + (1-\mu^2)\frac{v}{r}}$$

$$P_{em}(x) \propto (\frac{\dot{M}}{R_*^{3/2}})^2 \int_1^\infty \frac{1}{r^4 v^2(r)} \frac{1}{\mu^2 \frac{\partial v}{\partial r} + (1-\mu^2)\frac{v}{r}} 2pdp$$

Strength of stellar wind emission depends on $\frac{\dot{M}}{R_*^{3/2}}$

H_α excellent mass-loss indicator

H_α as mass-loss indicator

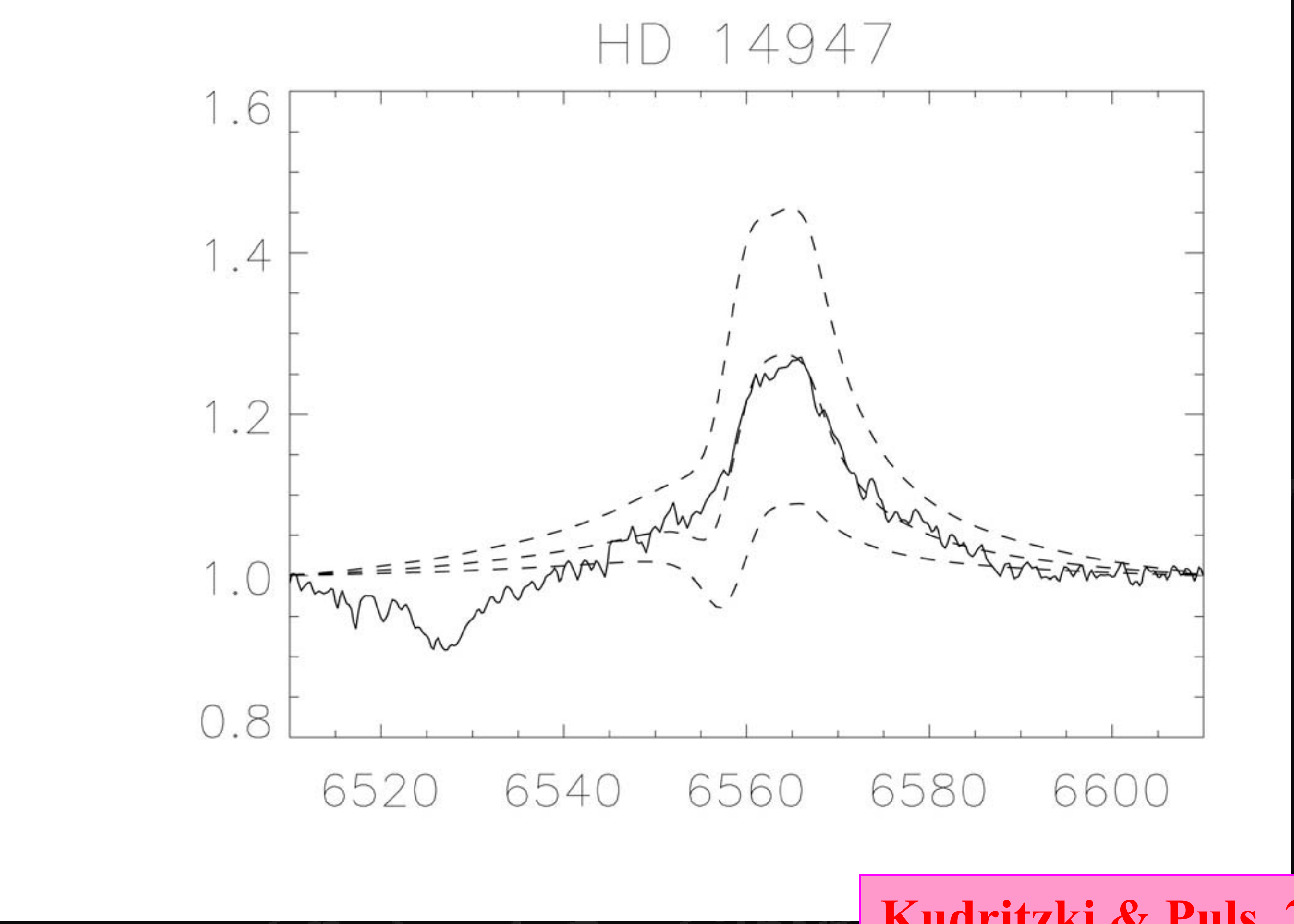


dashed profile calculated with25% higher mass-loss rate

exact integration of $P_{em}(x)$ along interaction surfaces for each frequency x yields line profile

velocity field adopted was $v(r) = v_\infty(1 - \frac{1}{r})^\beta, \ \beta = 2/3$

H_α emission O-star

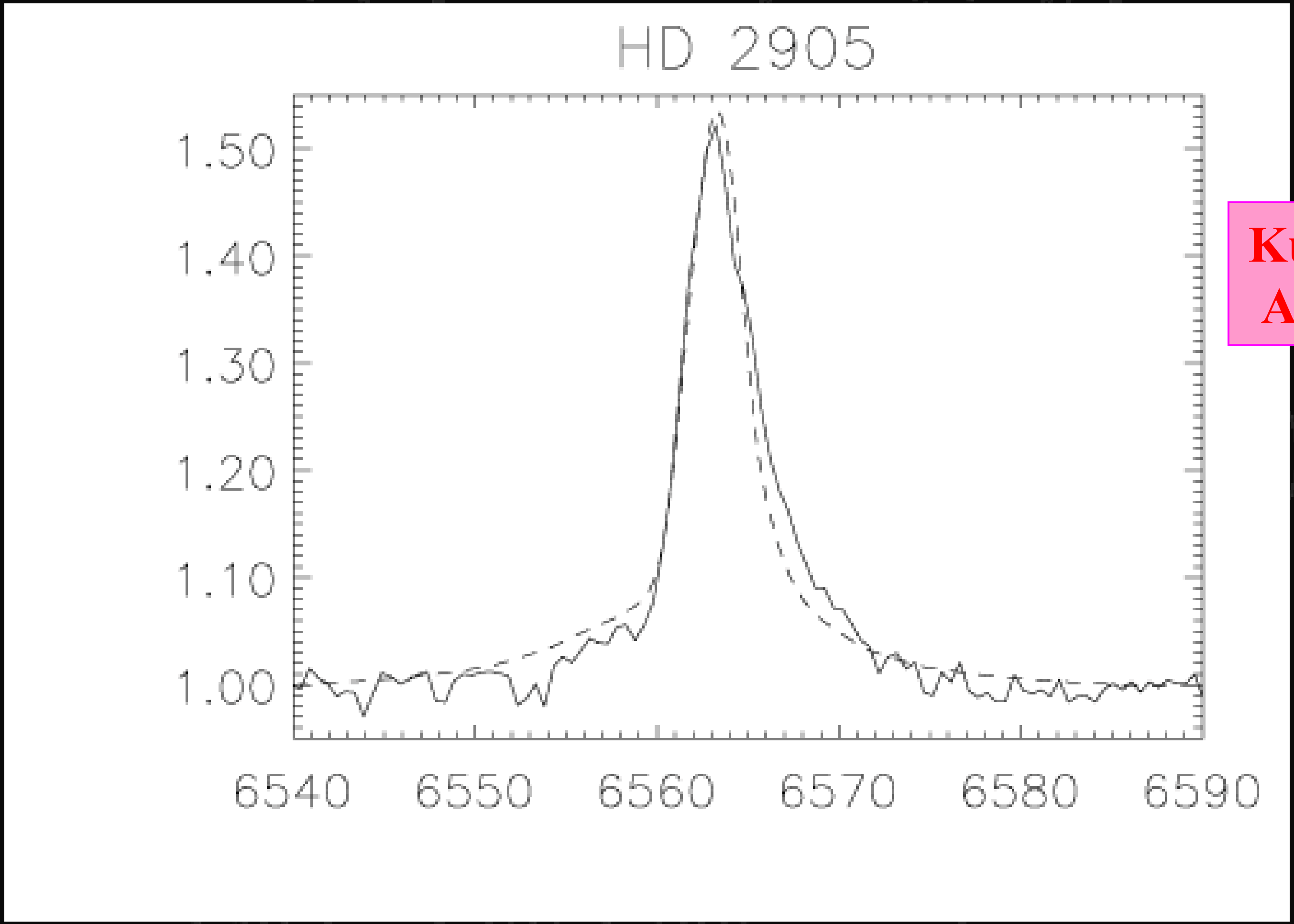


Exact NLTE model calculation

Variation of $\dot{M}$ by ± 20%

Kudritzki & Puls, 2000, AARA 38, 613

H_α emission B supergiant – stellar wind



Model calculation

Kudritzki et al. 1999, A&A 350, 970

Example 2: strong UV resonance line $\tau_S \gg 1$

resonance transition

absorption is followed by spontaneous emission
line scattering
detailed NLTE calculations show $S_L(r) \approx \frac{1}{2} I_c \frac{1}{r^3}$
is reasonable approximation

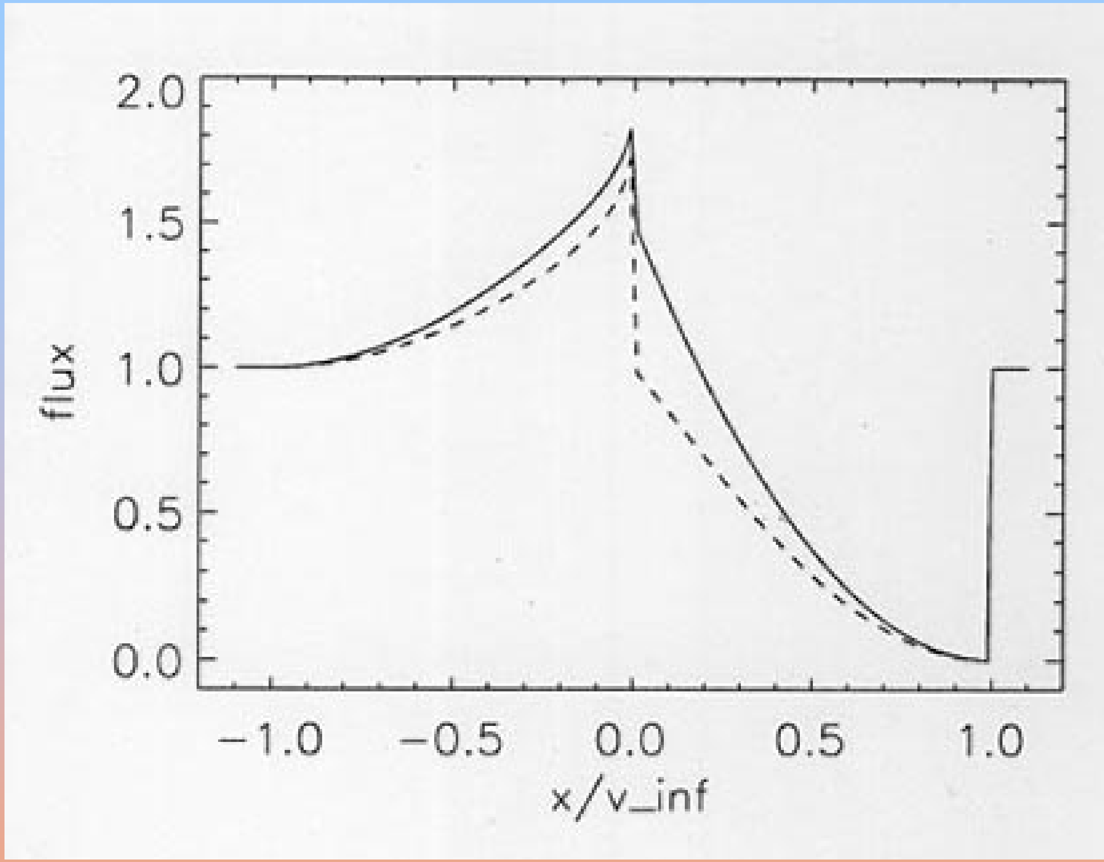
$$P_{em}(x) = \int_1^\infty \frac{S_L}{I_c}(p, z_0)(1 - e^{-\tau_S(p,z_0)}) 2pdp$$

$$\approx \int_1^\infty \frac{1}{r^3} pdp$$

in the same way

$$P_{abs} \approx \int_0^1 \frac{1}{r^3} pdp$$

P Cygni profile, because $P_{abs}(x) + P_{em}(x) < 1$
but no information about mass-loss rates, since $\tau_S \gg 1$
However the shape of velocity field v(r)
can be determined



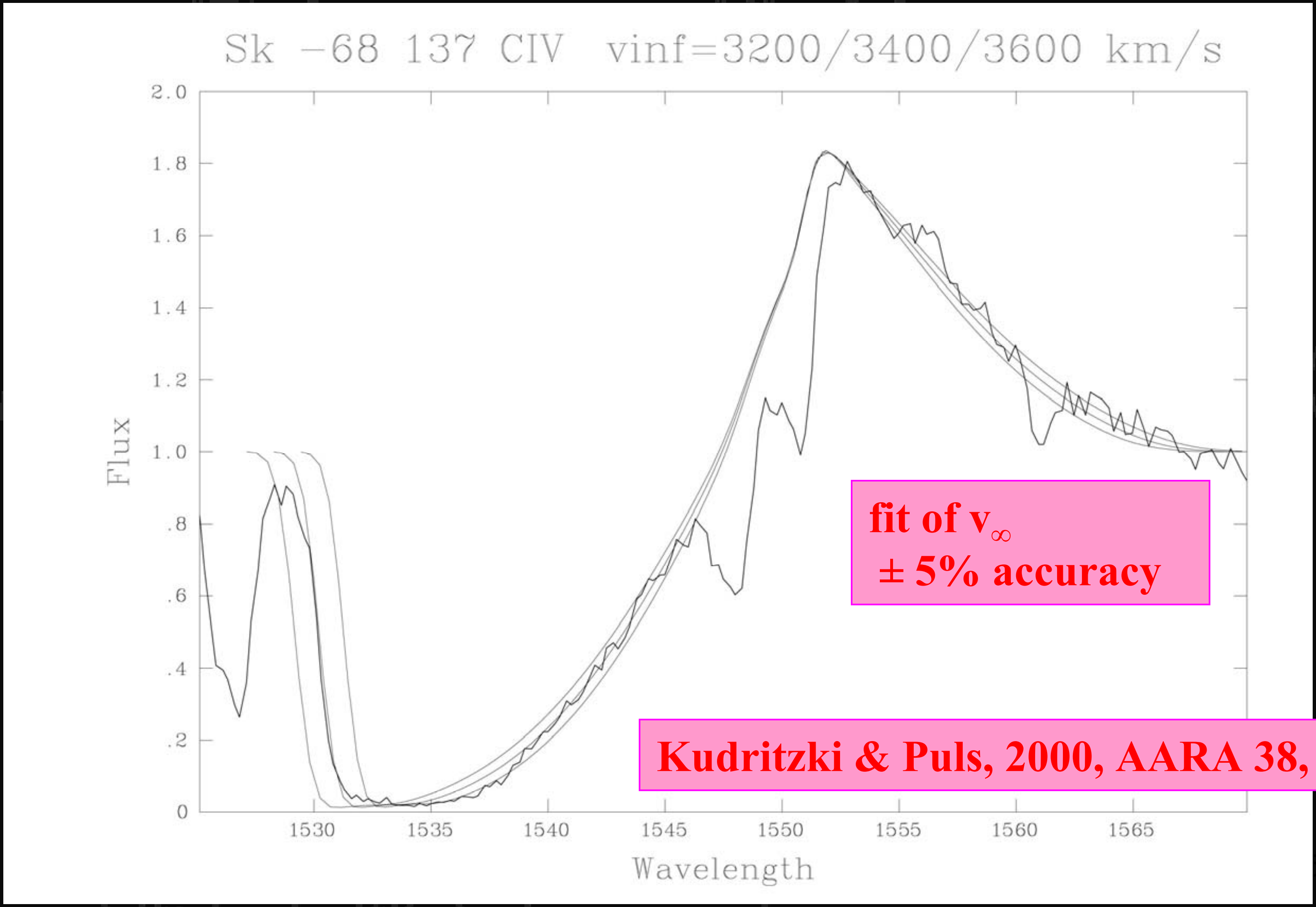
$$v(r) = v_\infty (1 - \frac{1}{r})^\beta$$

solid $\beta = 1$

dashed $\beta = 3/2$

exact integration of $P_{em}(x)$ and $P_{abs}(x)$ along interaction surfaces for each frequency x yields line profile

P Cygni profiles and $v_{infinity}$



fit of v_∞
$\pm$ 5% accuracy

Kudritzki & Puls, 2000, AARA 38, 613