Ryerson University

Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title:	ELE
Course Number:	532
Semester/Year (e.g.F2016)	F 2021

Instructor:	Javad Alirezaie

Assignment/Lab Number:	3
Assignment/Lab Title:	System Properties and Convolution

Submission Date :	Nov 21st /2021
Due Date:	Nov 21st /2021

Student LAST Name	**Student FIRST Name**	**Student Number**	**Section**	**Signature***
Youssef	Kirolos	500968175	01	K. Y
Miceli	Julian	500984182	01	J. M

*By signing above you attest that you have contributed to this written lab report and confirm that all work you have contributed to this lab report is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a "0" on the work, an "F" in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: http://www.ryerson.ca/senate/current/pol60.pdf

A1) $x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$

$$= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{2}\left(\frac{1}{2}e^{j\pi/10\,t} + \frac{1}{2}e^{-j\pi/10\,t}\right)$$

$$= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\pi/10\,t}$$

Fundamental Frequency

$\frac{3\cancel{\pi}}{\cancel{10}}\left(\frac{\cancel{10}}{\cancel{\pi}}\right) = 3 \quad \omega_{0_1} = \frac{3\pi}{10} \quad \omega_{02} = \frac{\pi}{10} \quad \frac{GCF}{LCM} = \frac{\pi}{10}$

$T = \frac{2\pi}{\frac{\pi}{10}} = 20$

$D_3 = \frac{1}{2} \quad D_{-3} = \frac{1}{2} \qquad D_1 = \frac{1}{4} \quad D_{-1} = \frac{1}{4}$

$$\Rightarrow D_n = \frac{1}{20}\int_{-10}^{10}\left[\frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}\right]e^{-j\frac{\pi}{10}nt}\,dt$$

$$= \frac{1}{20}\left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\pi/10} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\pi/10} + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\pi/10} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\pi/10}\right.$$

$$= \frac{1}{2}\Big[\sin[(3-n)\pi] + \sin[(3+n)\pi] + \frac{1}{2}\sin[(1+n)\pi]$$

$$+ \frac{1}{2}\sin[(1-n)\pi]\Big]$$

A2) $x_2(t)$: $T_0 = 20$ $\quad \omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20}\left[\int_{-5}^{5}(1)\, e^{-jn\frac{\pi}{10}t}\, dt\right] = \frac{1}{20}\left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5}$$

$$= \frac{1}{20}\left[\frac{-10}{jn\pi} e^{-jn\frac{\pi}{2}} + \frac{10}{jn\pi} e^{jn\frac{\pi}{2}}\right]$$

$$= \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$x_3(t)$: $T_0 = 40$ $\quad \omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

$$D_n = \frac{1}{40}\int_{-5}^{5}(1)\, e^{-jn\frac{x}{20}t}\, dt$$

$$= \frac{1}{40}\left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5}$$

$$= \frac{1}{40}\left[\frac{-20}{jn\pi} e^{-jn\frac{\pi}{4}} + \frac{20}{jn\pi} e^{jn\frac{\pi}{4}}\right]$$

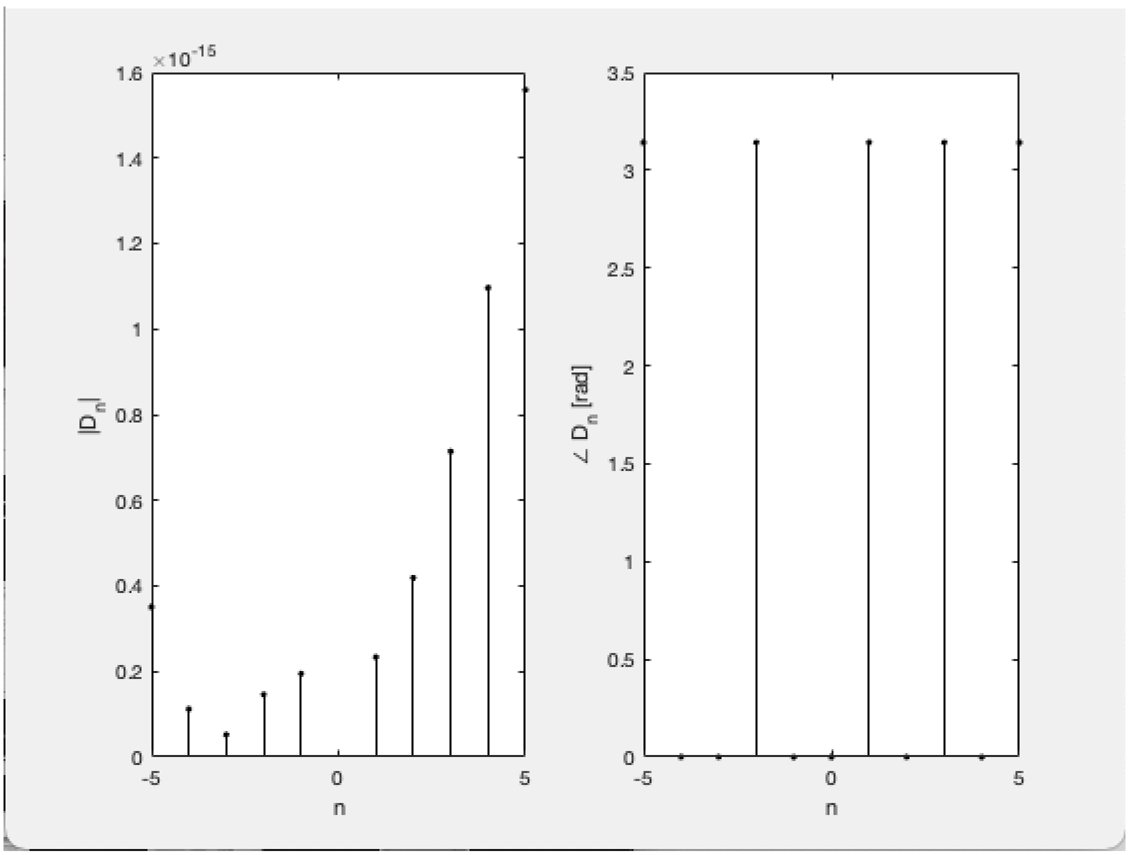
$$= \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

A3:

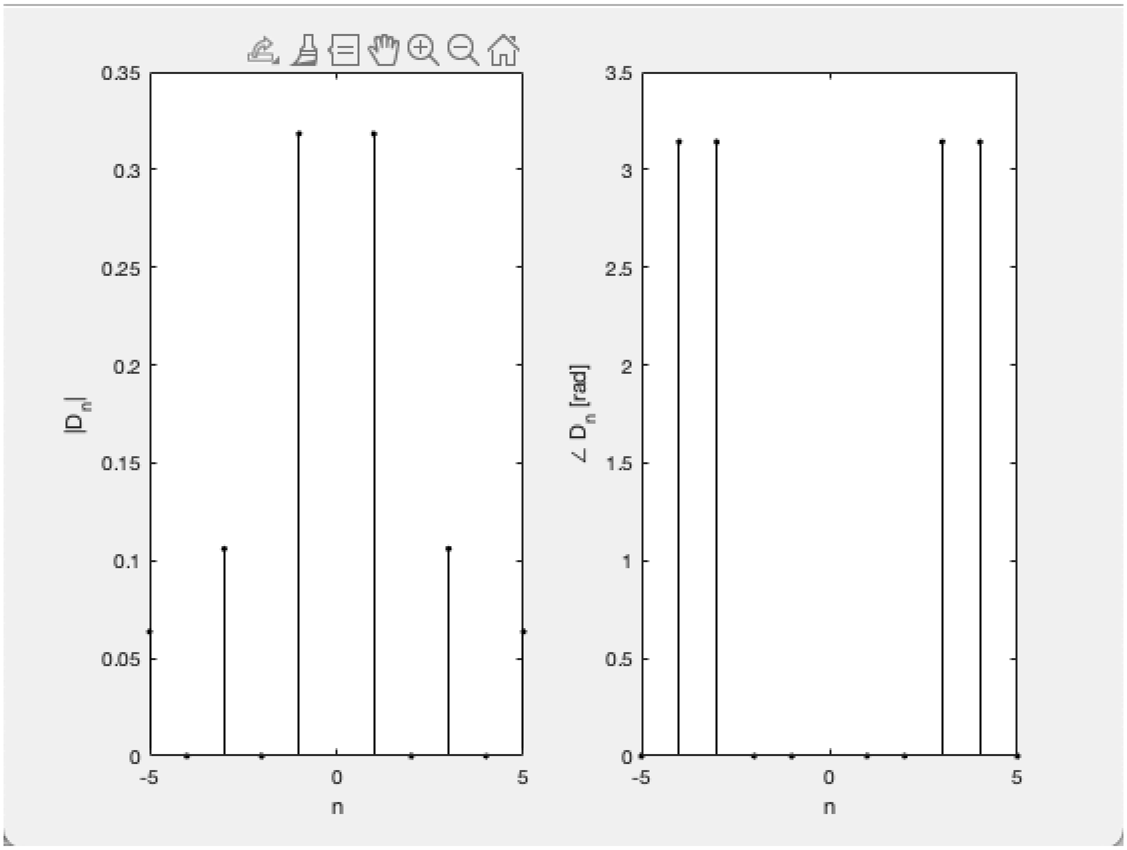
The Code for A3

```
d=1;
n=3;
function [D]=Dn(d,n)
D1 = (1/2)*(sin((3-n)*pi))+(sin((3+n)*pi))+((1/2)*(sin((1+n)*pi)))+((1/2)*(sin((1-n)*pi)));
D2 = (1/(n.*pi)*sin((n*pi)/2));
D3 = (1/(n.*pi)*sin((n*pi)/4));
if (d==1)
D=D1;
end
if (d==2)
D=D2;
end
if (d==3)
D=D3;
end
end
```

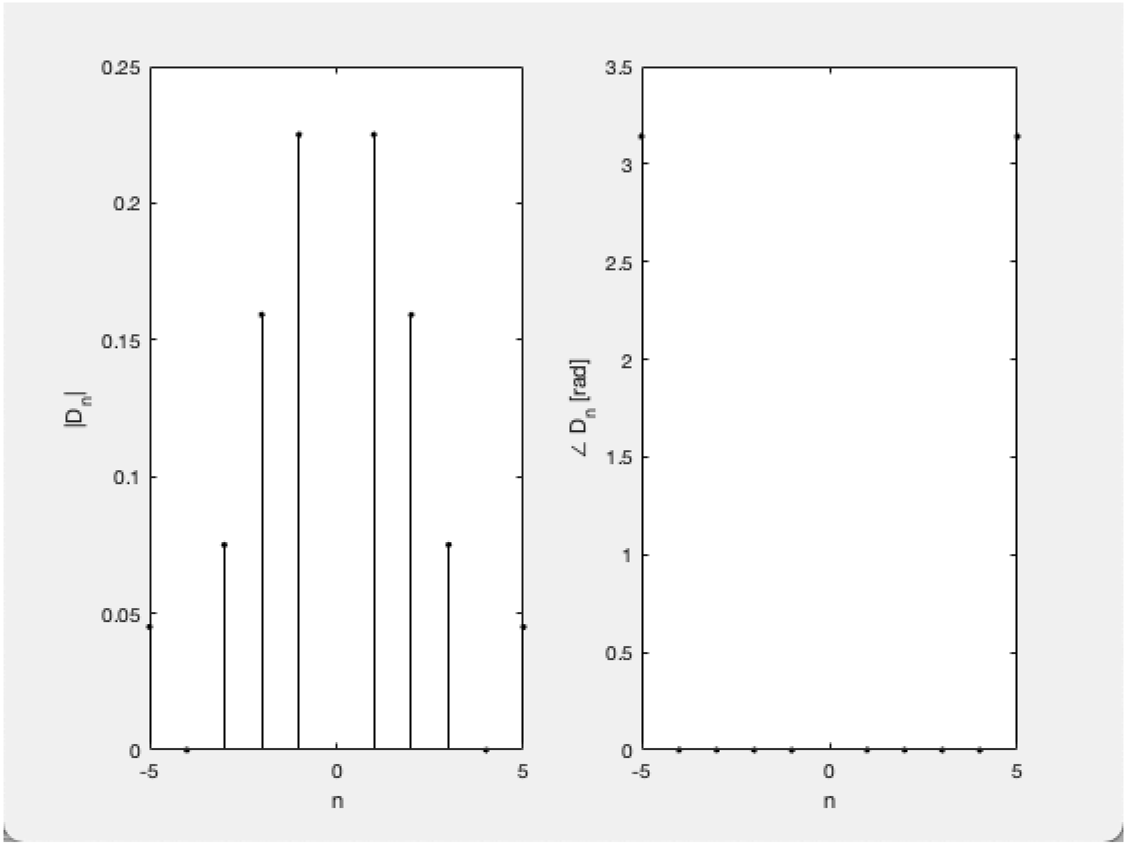
A4:



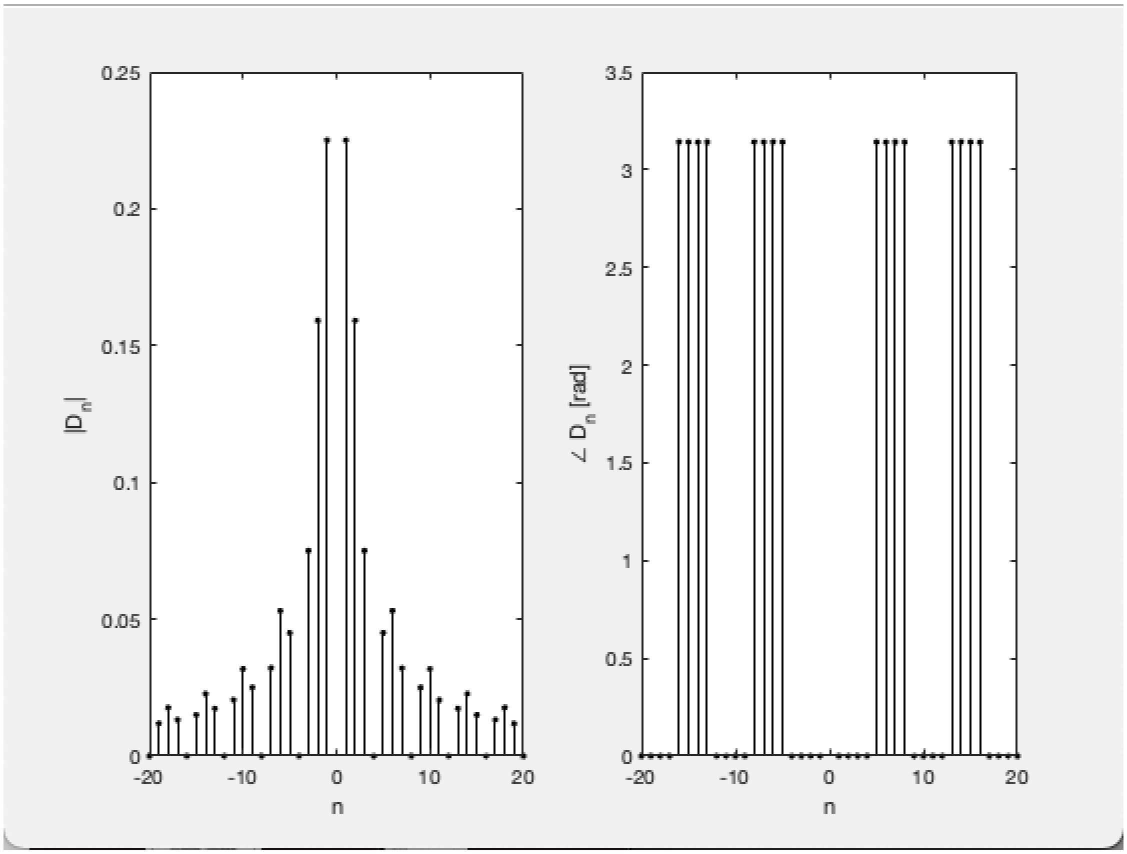
```
figure
clf;
n = (-5:5);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');

figure
```



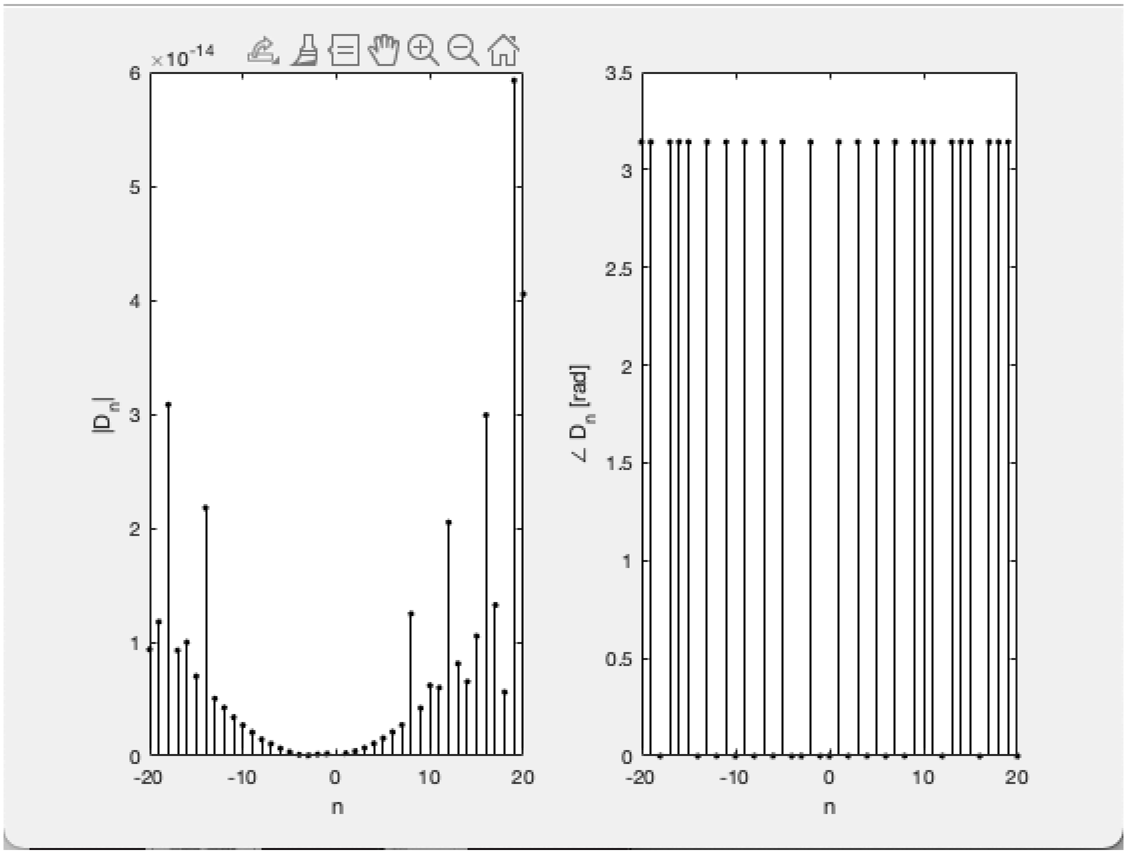

```
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');

figure
```



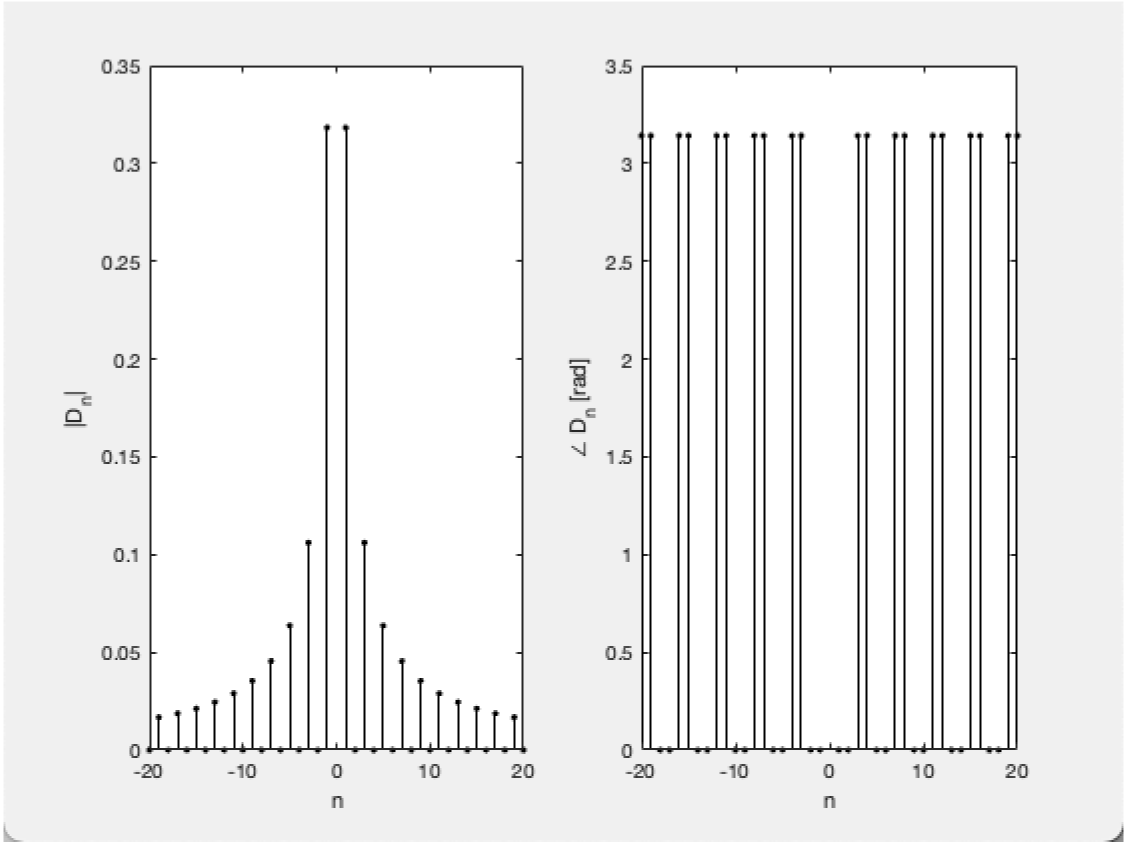

```
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');

figure
```



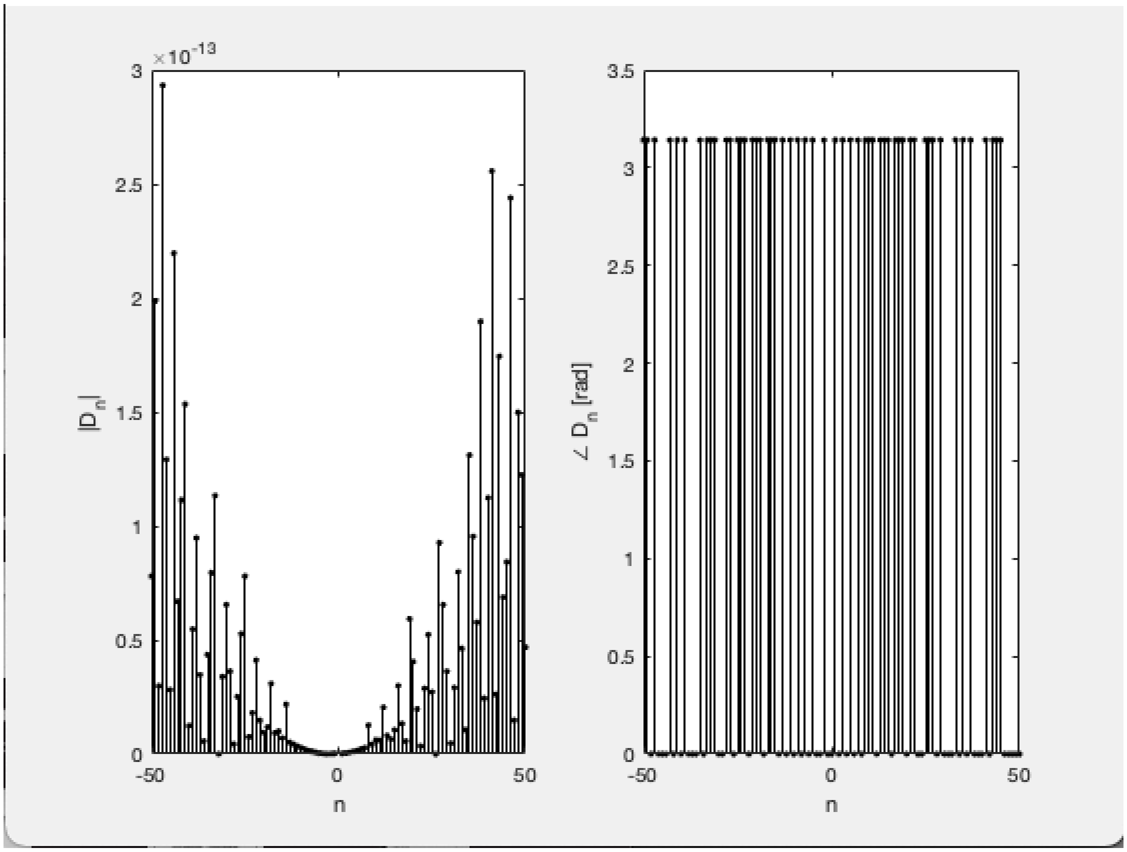

```matlab
clf;
n = (-20:20);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');

figure
```



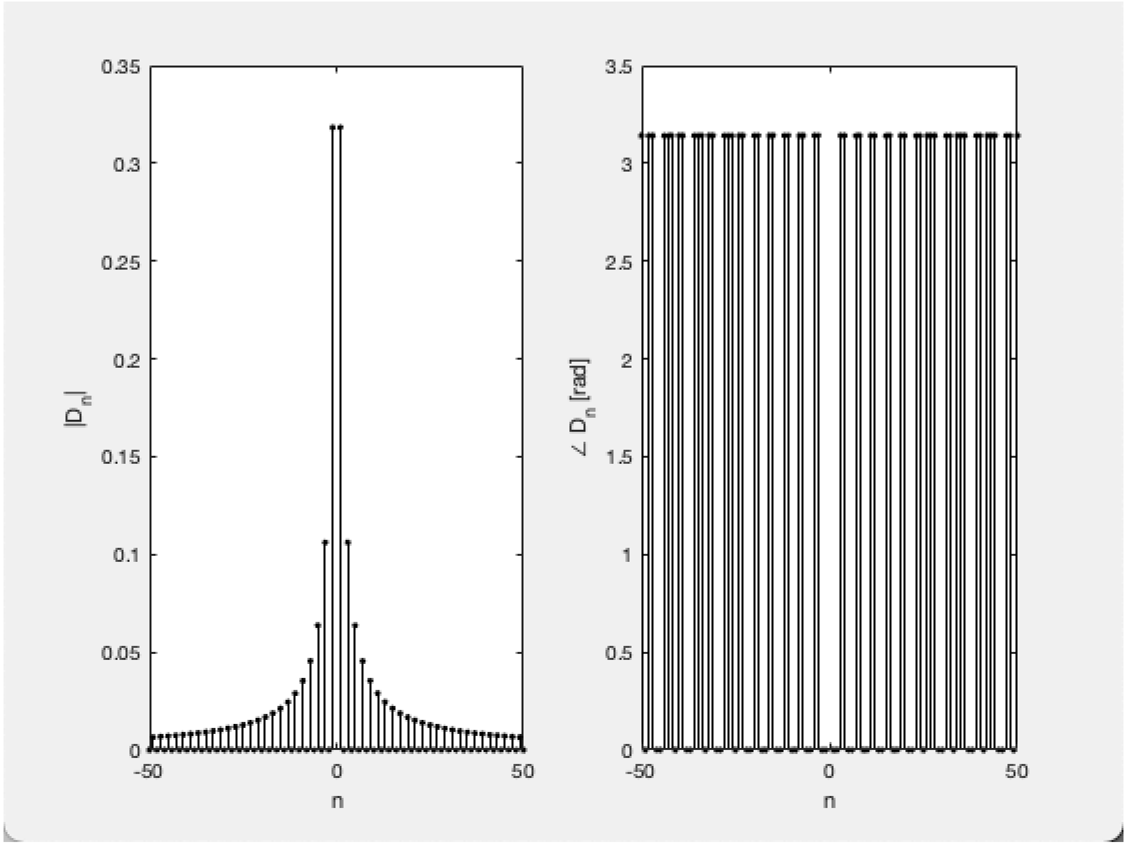

```
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');

figure
```



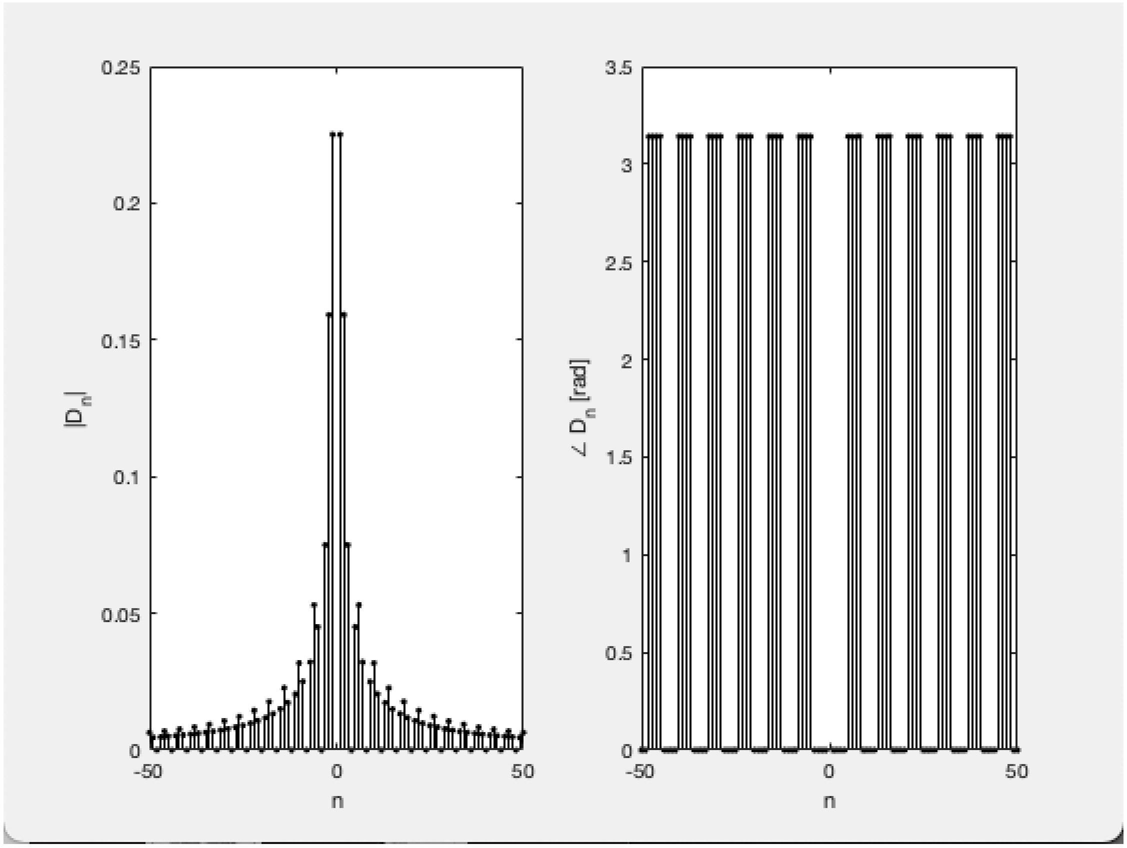

```
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



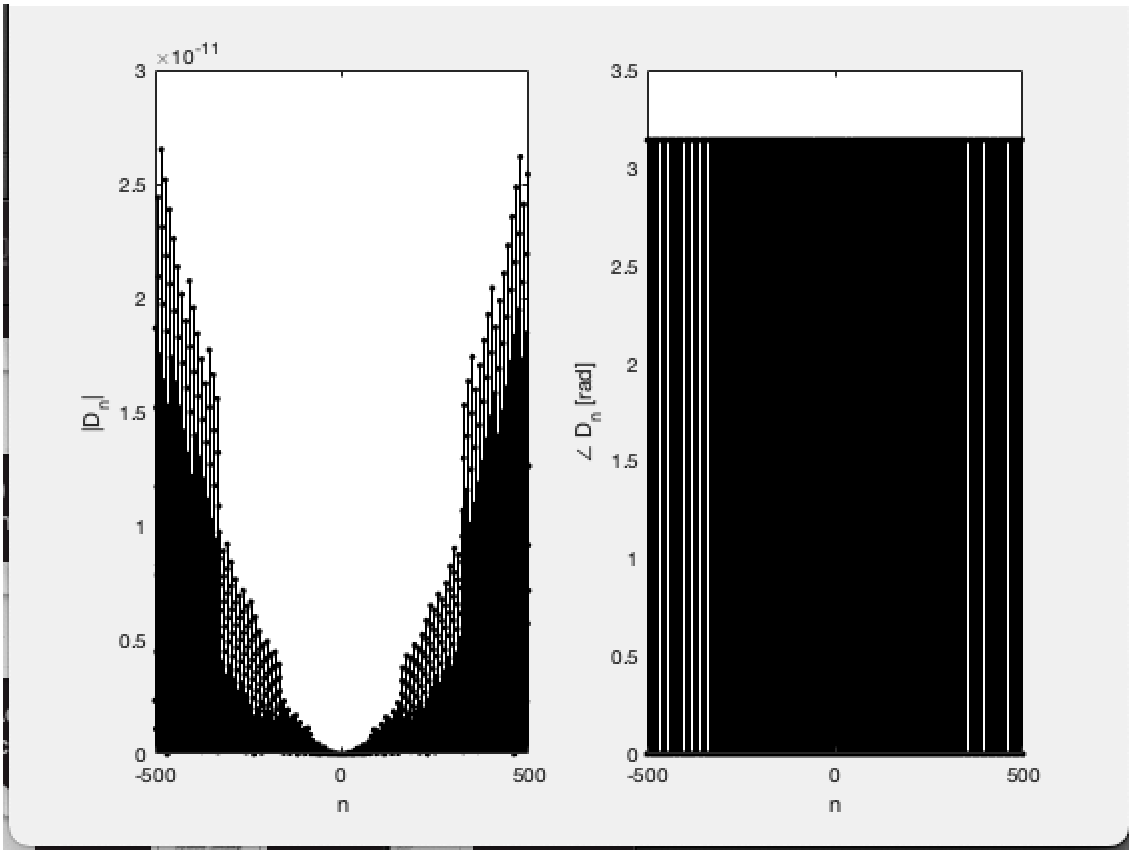

```
figure
clf;
n = (-50:50);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



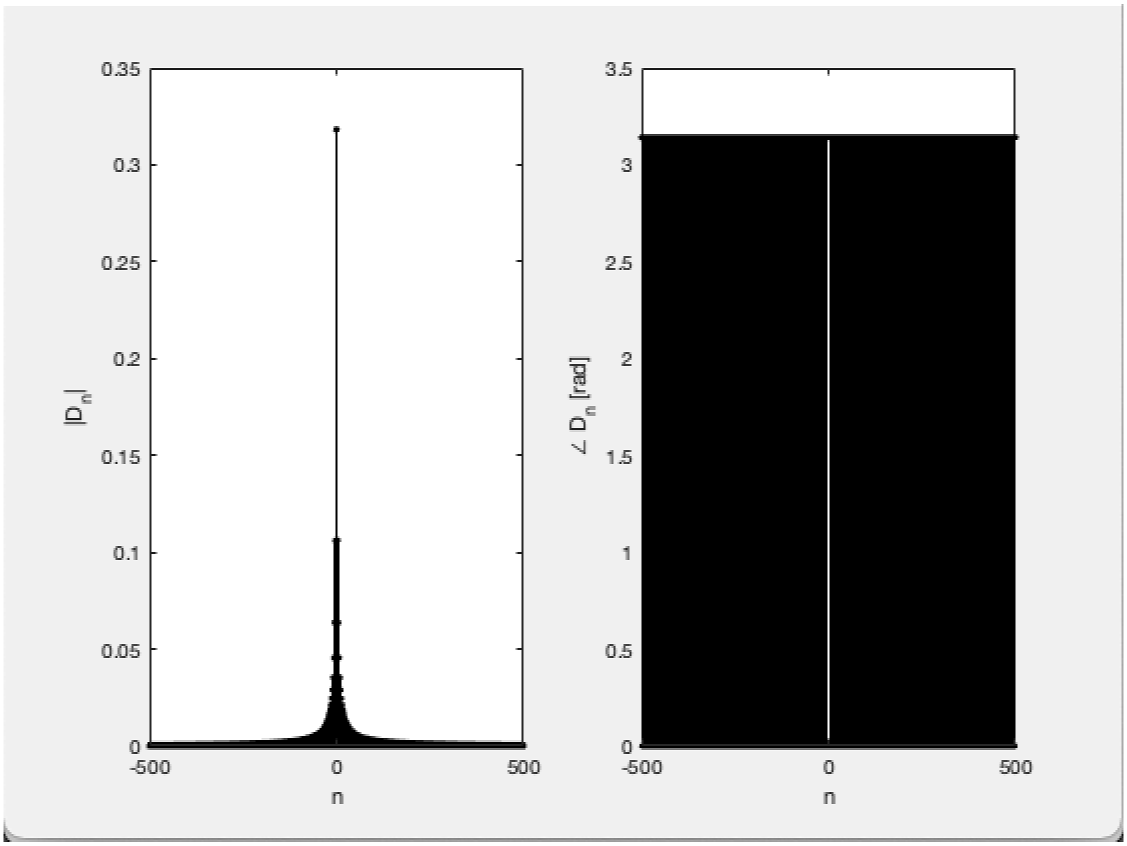

```
figure
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



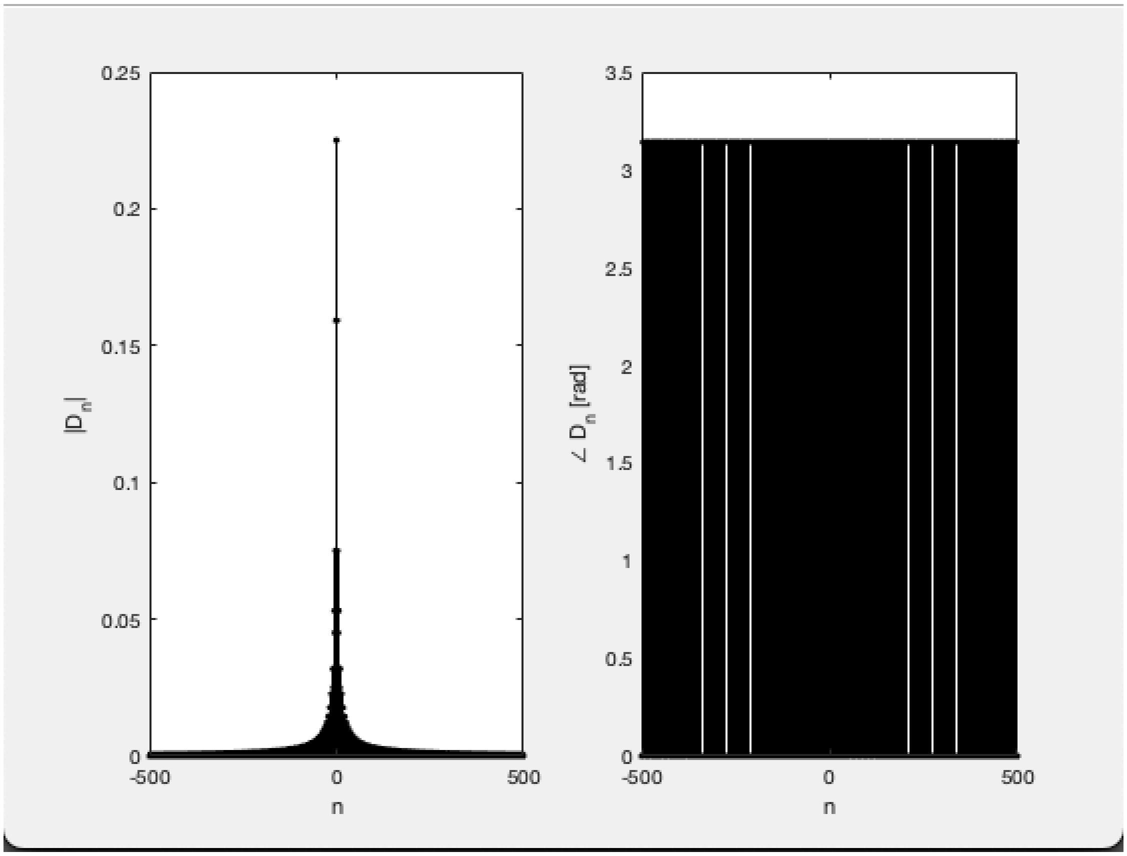

```
figure
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```




```
figure
clf;
n = (-500:500);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```




```
figure
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



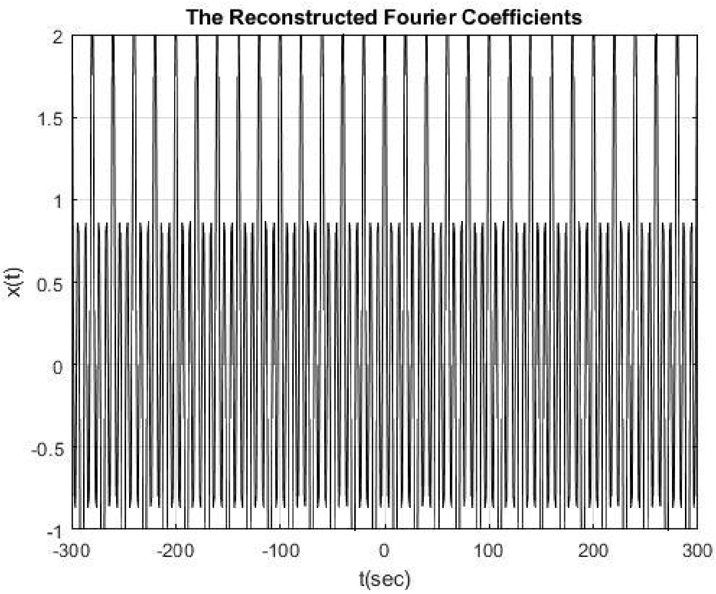

```
figure
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

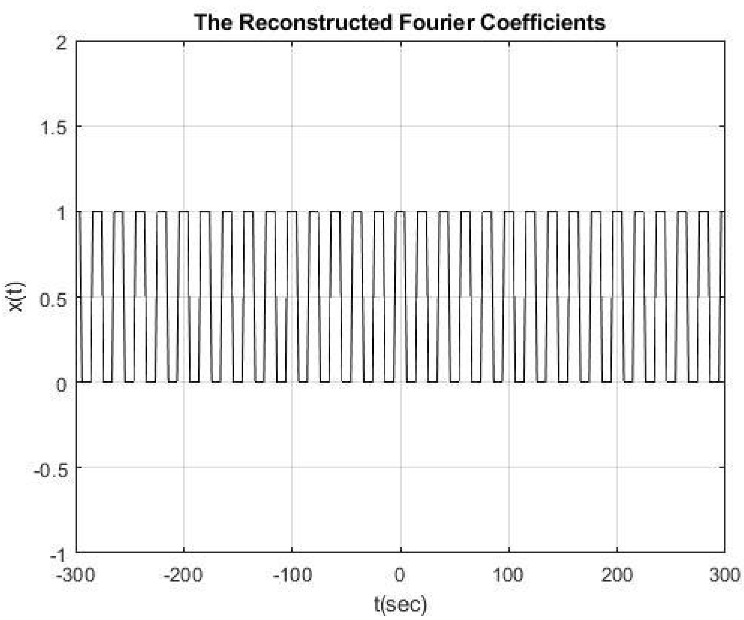
A5:

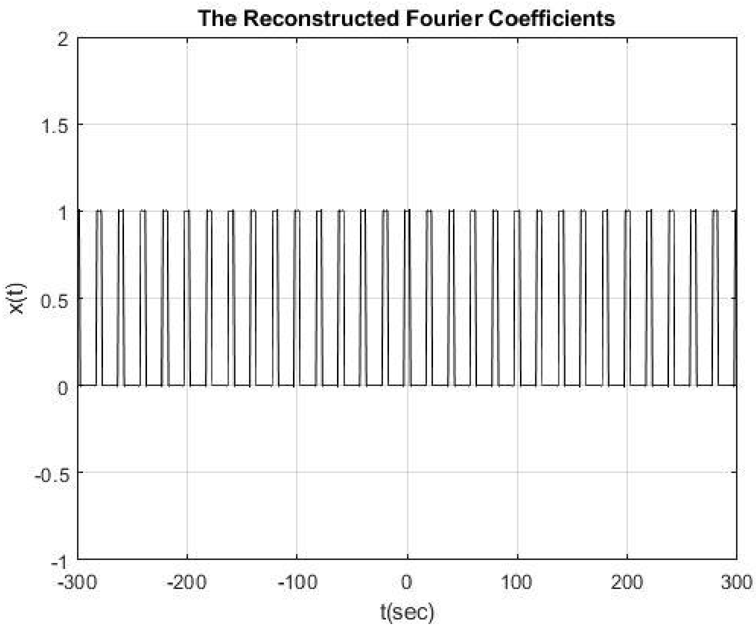
The Code for A5

```
function [D] = A5(Dn)
n=-500:500;
D=Dn;
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
    for t=-300:1:300
    x=x+D(i)*exp(1j*n(i)*w*t);
    end
end
figure;
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('The Reconstructed Fourier Coefficients');
grid;
```

A6:


The Reconstructed Fourier Coefficients
x(t)
t(sec)
2
1.5
1
0.5
0
-0.5
-1
-300
-200
-100
0
100
200
300


The Reconstructed Fourier Coefficients
x(t)
t(sec)
2
1.5
1
0.5
0
-0.5
-1
-300
-200
-100
0
100
200
300



The Code for A6

```
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
D_n2 = (1./(n.*pi).*sin((n.*pi)./2));
D_n3 = (1./(n.*pi).*sin((n.*pi)./4));

A5(D_n)
A5(D_n2)
A5(D_n3)
```

B1:

The following fundamental frequencies calculations are shown in part A1:

$x_1(t)$:

$$wo = \frac{\pi}{10}$$

$x_2(t)$:

$$wo = \frac{\pi}{10}$$

$x_3(t)$:

$$wo = \frac{\pi}{20}$$

B2:

$x_1(t)$ represents a sinc function as the denominator matches the expression within the sin portion of D_n. Also $x_1(t)$ has a distinct amount of coefficients whereas $x_2(t)$ and $x_3(t)$ has an infinite amount.

B3:

The fundamental frequencies between $x_2(t)$ and $x_3(t)$ differentiate in such a way that $x_3(t)$ has a smaller fundamental frequency in comparison to $x_2(t)$.

B4:

Based on the calculations done in **A2**, $x_2(t)$ has a D_o of 1. From this it can be observed that for these specific signals, the fourier coefficient, D_o, are dependent on the area of the functions. Since the area of $x_4(t)$ is 0.5, $x_4(t)$ has a D_o of 0.5.

B5:

Since $x_1(t)$ has a finite amount of fourier coefficients, nothing would change, however, for $x_2(t)$ increasing the fourier coefficients would increase the accuracy of the overall D_n.

B6:

For the graph in $x_1(t)$, it contains 4 fourier coefficients being $\pm\ 3$ and $\pm\ 1$. For the graphs $x_2(t)$ and $x_3(t)$ they consist of an infinite amount of fourier coefficients.

B7:

It would be a viable option for $x_1(t)$ since it has a finite number of fourier coefficients, however, for $x_2(t)$ and $x_3(t)$ it would not be a viable option as they consist of an infinite amount of fourier coefficients. However, also considering the fundamental frequencies of the signals, $x_1(t)$, having a large fundamental frequency would take up alot of space in the hard drive. Therefore, it would only be viable if it has a smaller amount of fourier coefficients and a small fundamental frequency in order for space to not be wasted.