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Course Number:	532
Semester/Year (e.g.F2016)	F 2021

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<i>Assignment/Lab Number:</i>	3
<i>Assignment/Lab Title:</i>	System Properties and Convolution

<i>Submission Date :</i>	Nov 21 st /2021
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<http://www.ryerson.ca/senate/current/pol60.pdf>

$$\begin{aligned}
 A1) \quad x_1(t) &= \cos \frac{3\pi}{10} t + \frac{1}{2} \cos \frac{\pi}{10} t \\
 &= \frac{1}{2} e^{j\frac{3\pi}{10}t} + \frac{1}{2} e^{-j\frac{3\pi}{10}t} + \frac{1}{2} \left(\frac{1}{2} e^{j\frac{\pi}{10}t} + \frac{1}{2} e^{-j\frac{\pi}{10}t} \right) \\
 &= \frac{1}{2} e^{j\frac{3\pi}{10}t} + \frac{1}{2} e^{-j\frac{3\pi}{10}t} + \frac{1}{4} e^{j\frac{\pi}{10}t} + \frac{1}{4} e^{-j\frac{\pi}{10}t}
 \end{aligned}$$

Fundamental Frequency

$$\frac{3\pi}{10} \left(\frac{10}{\pi} \right) = 3 \quad \omega_{01} = \frac{3\pi}{10} \quad \omega_{02} = \frac{\pi}{10} \quad \frac{\text{GCF}}{\text{LCM}} = \frac{\pi}{10}$$

$$T = \frac{2\pi}{\frac{\pi}{10}} = 20$$

$$D_3 = \frac{1}{2} \quad D_{-3} = \frac{1}{2} \quad D_1 = \frac{1}{4} \quad D_{-1} = \frac{1}{4}$$

$$\Rightarrow D_n = \frac{1}{20} \int_{-10}^{10} \left[\frac{1}{2} e^{j\frac{3\pi}{10}t} + \frac{1}{2} e^{-j\frac{3\pi}{10}t} + \frac{1}{4} e^{j\frac{\pi}{10}t} + \frac{1}{4} e^{-j\frac{\pi}{10}t} \right] e^{-j\frac{\pi}{10}nt} dt$$

$$\begin{aligned}
 &= \frac{1}{20} \left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\frac{\pi}{10}} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\frac{\pi}{10}} \right. \\
 &\quad \left. + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\frac{\pi}{10}} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\frac{\pi}{10}} \right]
 \end{aligned}$$

$$= \frac{1}{2} [\sin[(3-n)\pi] + \sin[(3+n)\pi] + \frac{1}{2} \sin[(1+n)\pi] + \frac{1}{2} \sin[(1-n)\pi]]$$

A2) $x_2(t)$: $T_0 = 20$ $\omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$\begin{aligned} D_n &= \frac{1}{20} \left[\int_{-5}^5 (1) e^{-jn\frac{\pi}{10}t} dt \right] = \frac{1}{20} \left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\frac{\pi}{10}t} \right]_{-5}^5 \\ &= \frac{1}{20} \left[\frac{-10}{jn\pi} e^{-jn\frac{\pi}{2}} + \frac{10}{jn\pi} e^{jn\frac{\pi}{2}} \right] \\ &= \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right) \end{aligned}$$

$x_3(t)$: $T_0 = 40$ $\omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

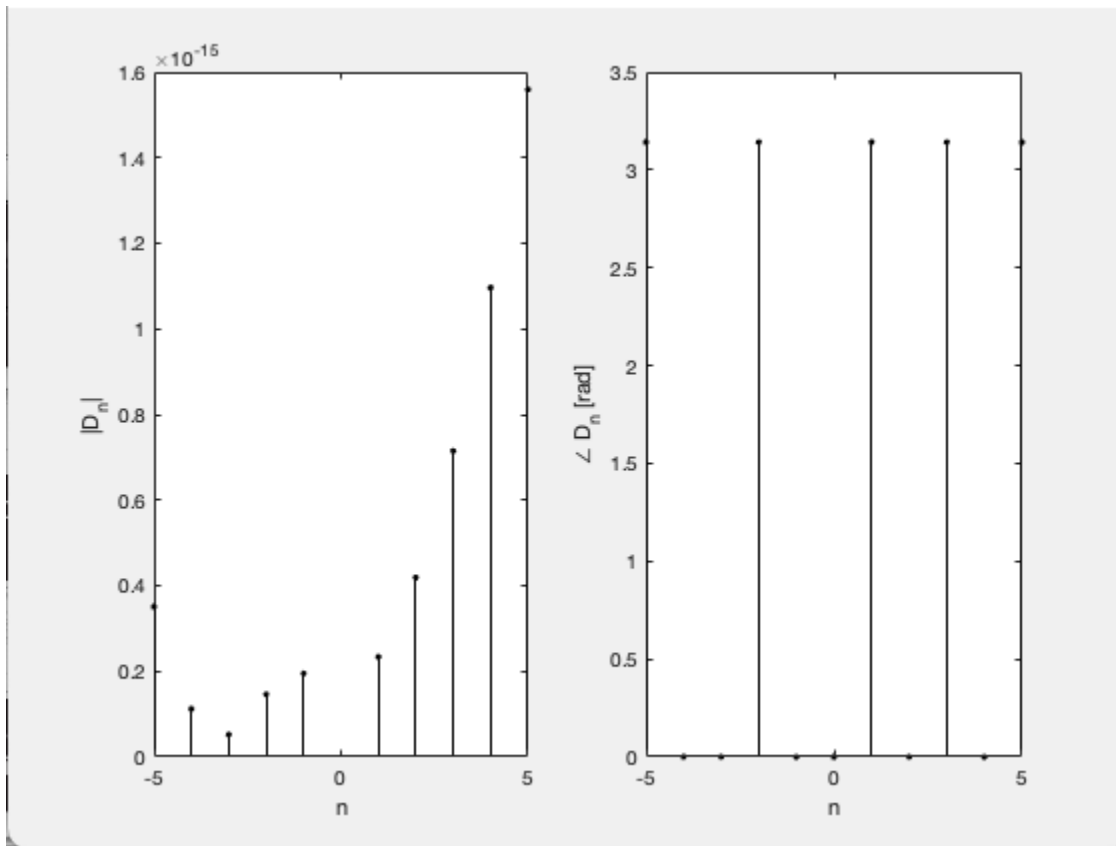
$$\begin{aligned} D_n &= \frac{1}{40} \int_{-5}^5 (1) e^{-jn\frac{\pi}{20}t} dt \\ &= \frac{1}{40} \left[\frac{1}{-jn\frac{\pi}{20}} e^{-jn\frac{\pi}{20}t} \right]_{-5}^5 \\ &= \frac{1}{40} \left[\frac{-20}{jn\pi} e^{-jn\frac{\pi}{4}} + \frac{20}{jn\pi} e^{jn\frac{\pi}{4}} \right] \\ &= \frac{1}{n\pi} \sin\left(\frac{n\pi}{4}\right) \end{aligned}$$

A3:

The Code for A3

```
1 - d=1;
2 - n=3;
3 - function [D]=Dn(d,n)
4 - D1 = (1/2)*(sin((3-n)*pi))+(sin((3+n)*pi))+((1/2)*(sin((1+n)*pi)))+(1/2)*(sin((1-n)*pi));
5 - D2 = (1/(n.*pi)*sin((n*pi)/2));
6 - D3 = (1/(n.*pi)*sin((n*pi)/4));
7 - if (d==1)
8 - D=D1;
9 - end
10 - if (d==2)
11 - D=D2;
12 - end
13 - if (d==3)
14 - D=D3;
15 - end
16 - end
```

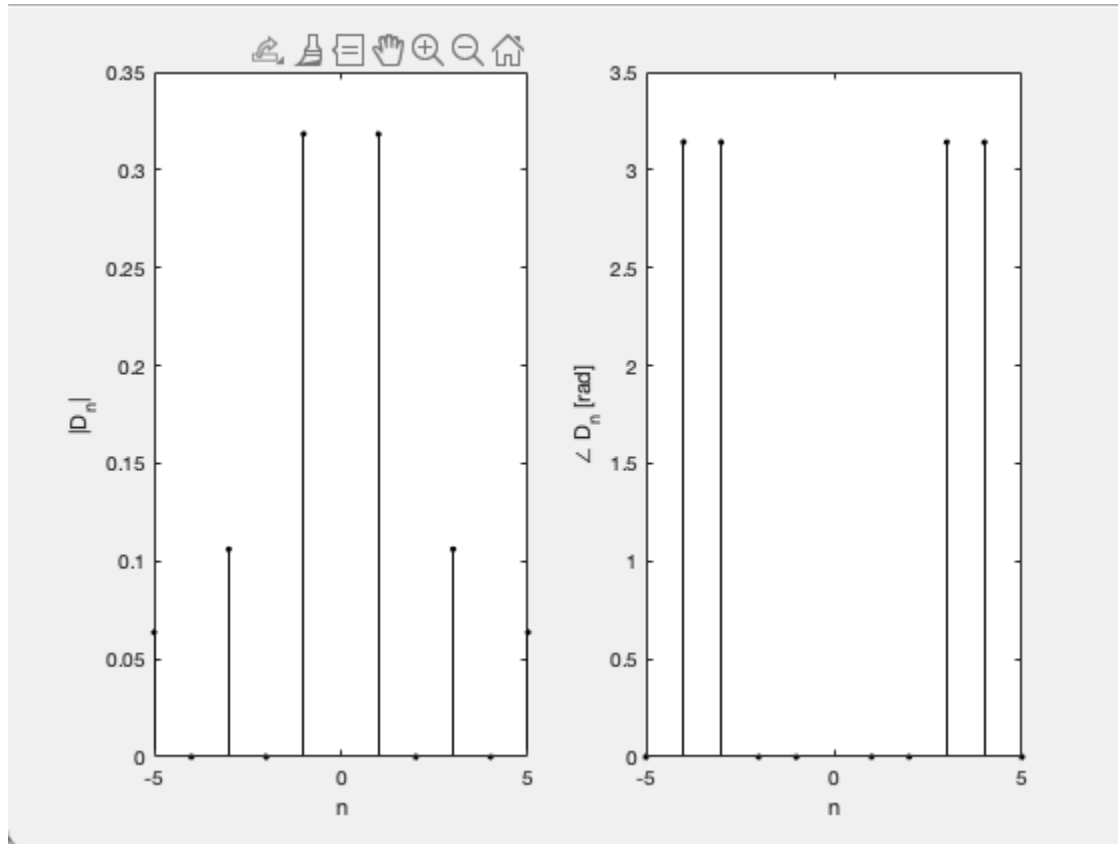
A4:



```

1 - figure
2 - clf;
3 - n = (-5:5);
4 - D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi)).*sin((1+n).*pi) + (1./(2.*n.*pi)).*sin((1-n).*pi) ;
5 - subplot(1,2,1); stem(n,abs(D_n),'.k');
6 - xlabel('n'); ylabel('|D_n|');
7 - subplot(1,2,2); stem(n,angle(D_n),'.k');
8 - xlabel('n'); ylabel('\angle D_n [rad]');
9 -
10 - figure

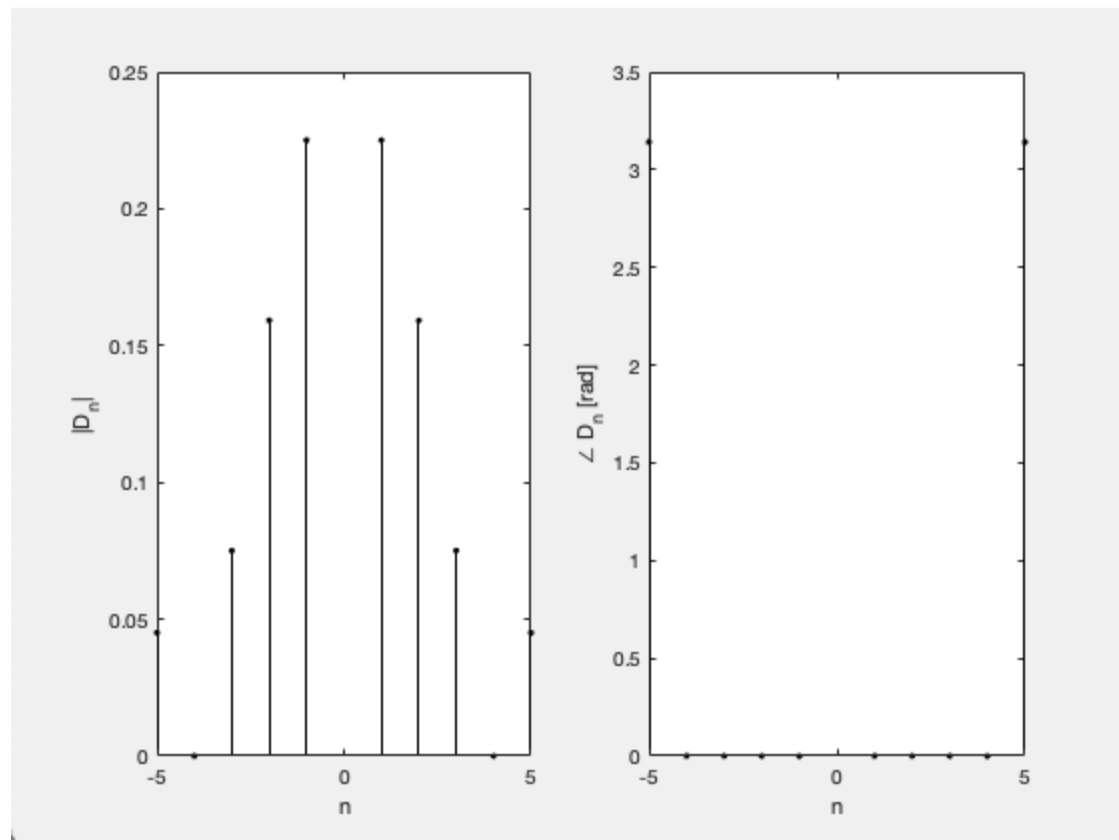
```



```

12 - clf;
13 - n = (-5:5);
14 - D_n = (1./(n.*pi)).*sin((n.*pi)./2));
15 - subplot(1,2,1); stem(n,abs(D_n),'.k');
16 - xlabel('n'); ylabel('|D_n|');
17 - subplot(1,2,2); stem(n,angle(D_n),'.k');
18 - xlabel('n'); ylabel('\angle D_n [rad]');
19 -
20 - figure

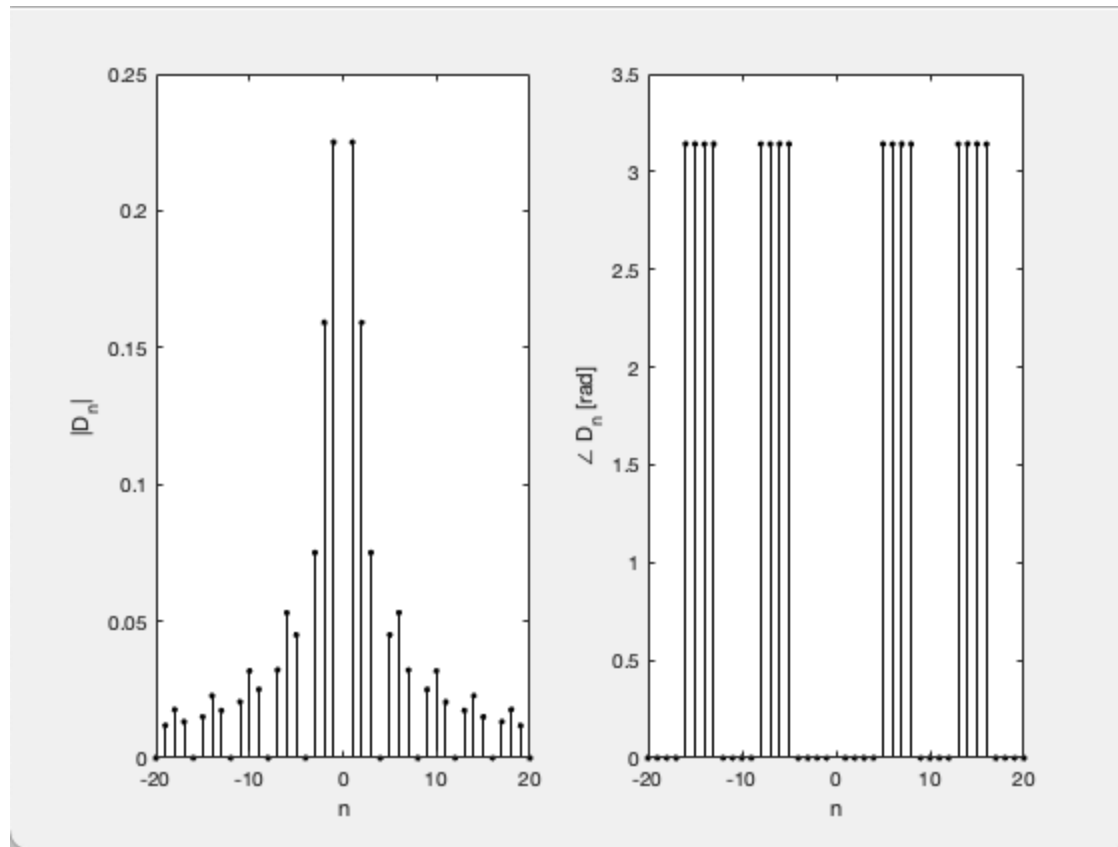
```



```

22 - clf;
23 - n = (-5:5);
24 - D_n = (1./(n.*pi).*sin((n.*pi)./4));
25 - subplot(1,2,1); stem(n,abs(D_n),'.k');
26 - xlabel('n'); ylabel('|D_n|');
27 - subplot(1,2,2); stem(n,angle(D_n),'.k');
28 - xlabel('n'); ylabel('\angle D_n [rad]');
29 -
30 - figure

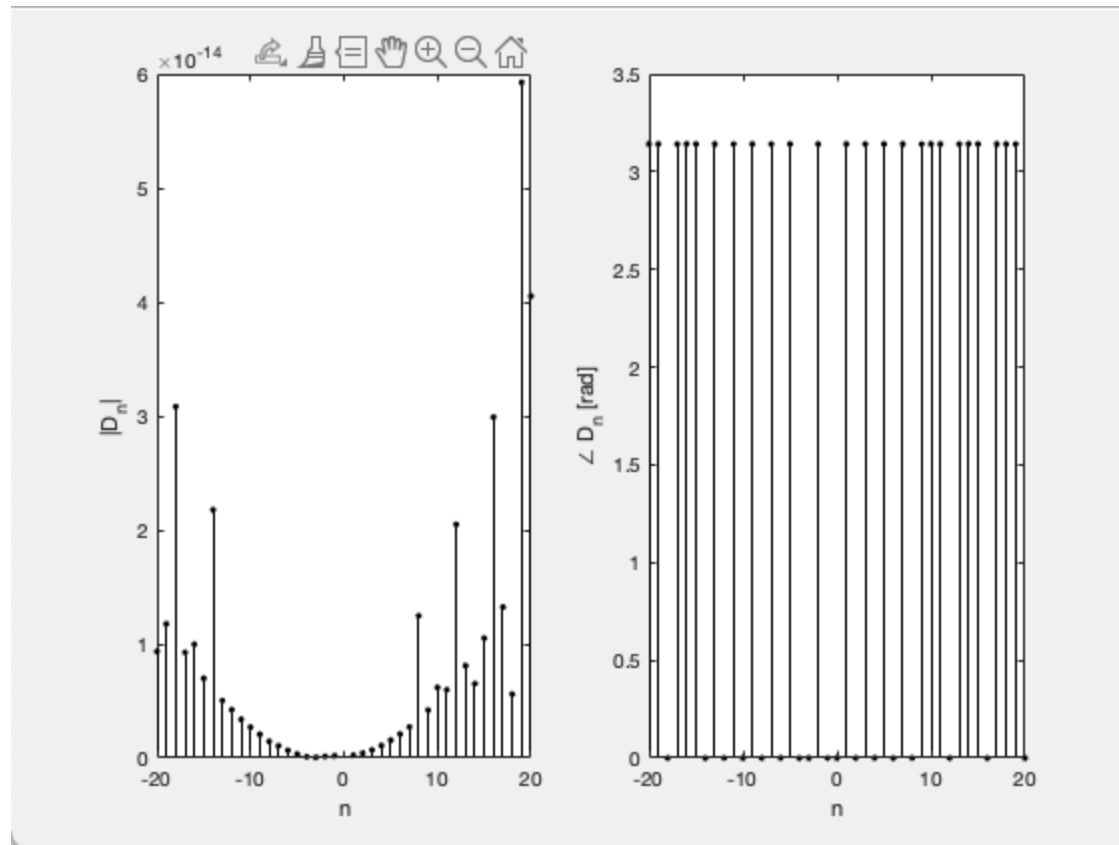
```



```

32 - clf;
33 - n = (-20:20);
34 - D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
35 - subplot(1,2,1); stem(n,abs(D_n),'k');
36 - xlabel('n'); ylabel('|D_n|');
37 - subplot(1,2,2); stem(n,angle(D_n),'k');
38 - xlabel('n'); ylabel('\angle D_n [rad]');
39 -
40 - figure

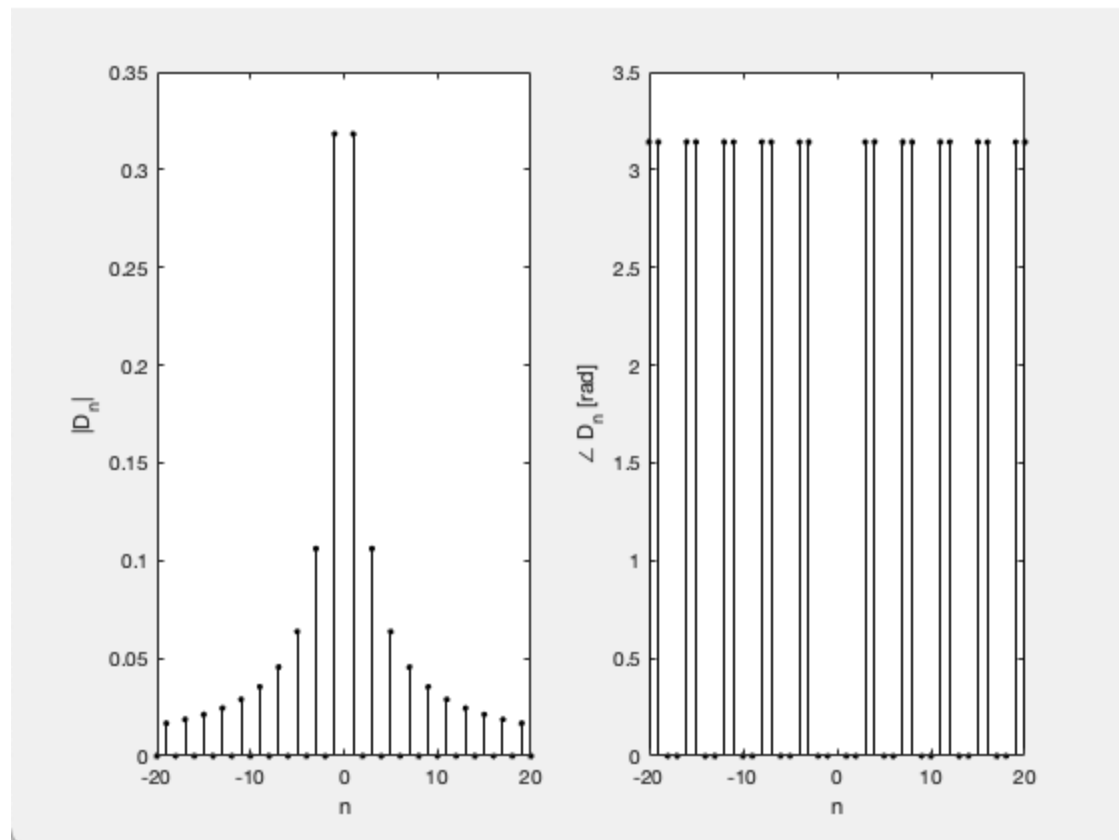
```



```

42 - clf;
43 - n = (-20:20);
44 - D_n = (1./(n.*pi)).*sin((n.*pi)./2));
45 - subplot(1,2,1); stem(n,abs(D_n),'.k');
46 - xlabel('n'); ylabel('|D_n|');
47 - subplot(1,2,2); stem(n,angle(D_n),'.k');
48 - xlabel('n'); ylabel('\angle D_n [rad]');
49 -
50 - figure

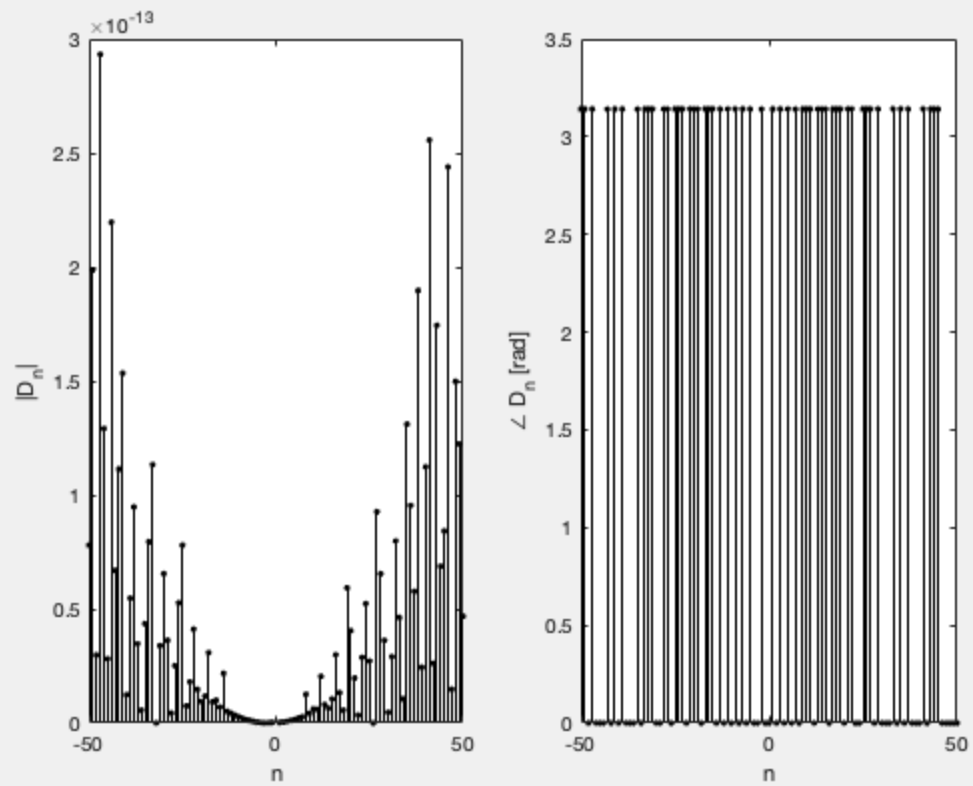
```

```

52 -   clf;
53 -   n = (-20:20);
54 -   D_n = (1./(n.*pi).*(sin((n.*pi)./4)));
55 -   subplot(1,2,1); stem(n,abs(D_n),'.k');
56 -   xlabel('n'); ylabel('|D_n|');
57 -   subplot(1,2,2); stem(n,angle(D_n),'.k');
58 -   xlabel('n'); ylabel('\angle D_n [rad]');

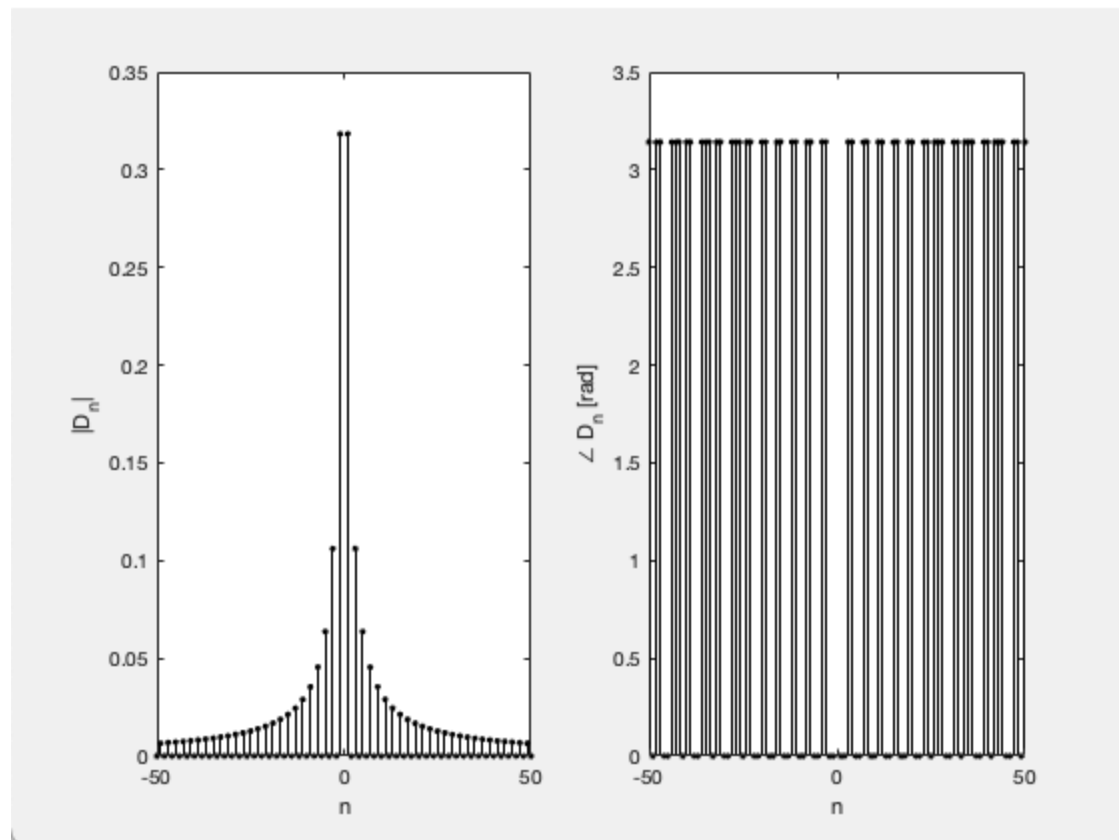
```



```

62 - figure
63 - clf;
64 - n = (-50:50);
65 - D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi)) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi)).*sin((1+n).*pi) + (1./(2.*n.*pi)).*sin((1-n).*pi);
66 - subplot(1,2,1); stem(n,abs(D_n),'.k');
67 - xlabel('n'); ylabel('|D_n|');
68 - subplot(1,2,2); stem(n,angle(D_n),'.k');
69 - xlabel('n'); ylabel('\angle D_n [rad]');

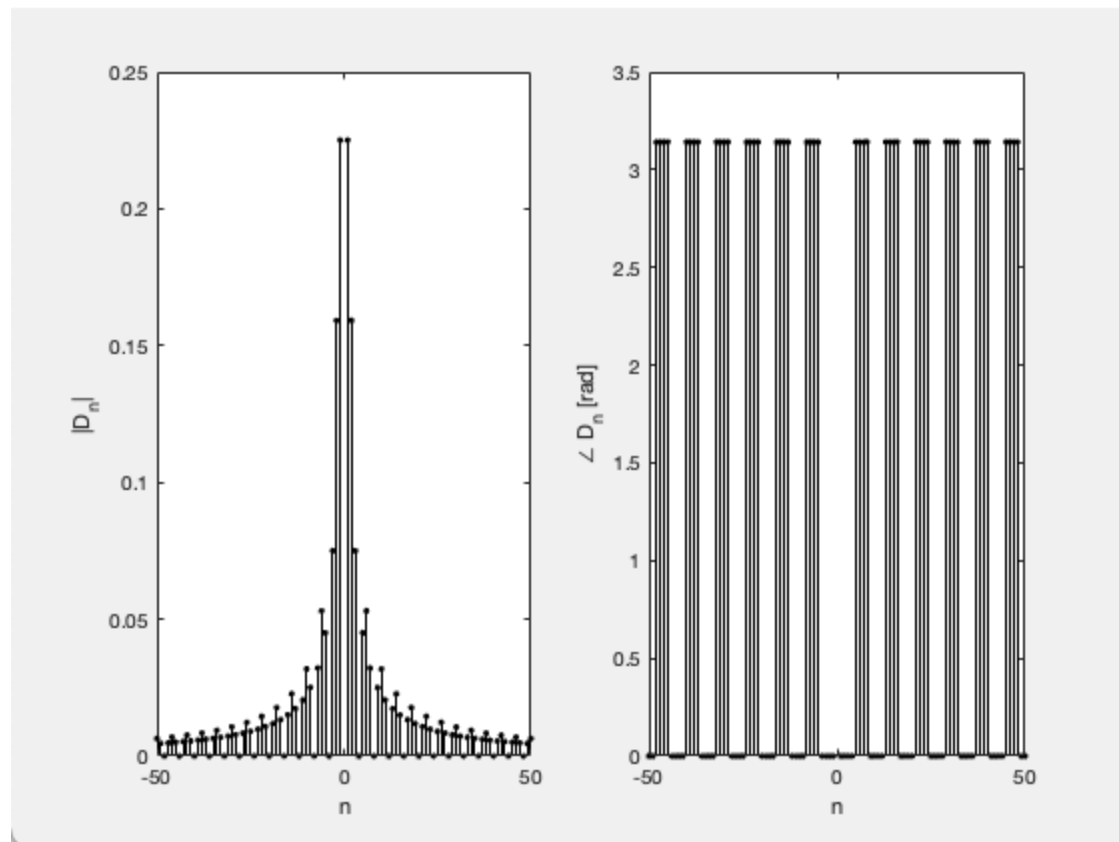
```



```

71 - figure
72 - clf;
73 - n = (-50:50);
74 - D_n = (1./(n.*pi).*(sin((n.*pi)./2)));
75 - subplot(1,2,1); stem(n,abs(D_n),'.k');
76 - xlabel('n'); ylabel('|D_n|');
77 - subplot(1,2,2); stem(n,angle(D_n),'.k');
78 - xlabel('n'); ylabel('\angle D_n [rad]');

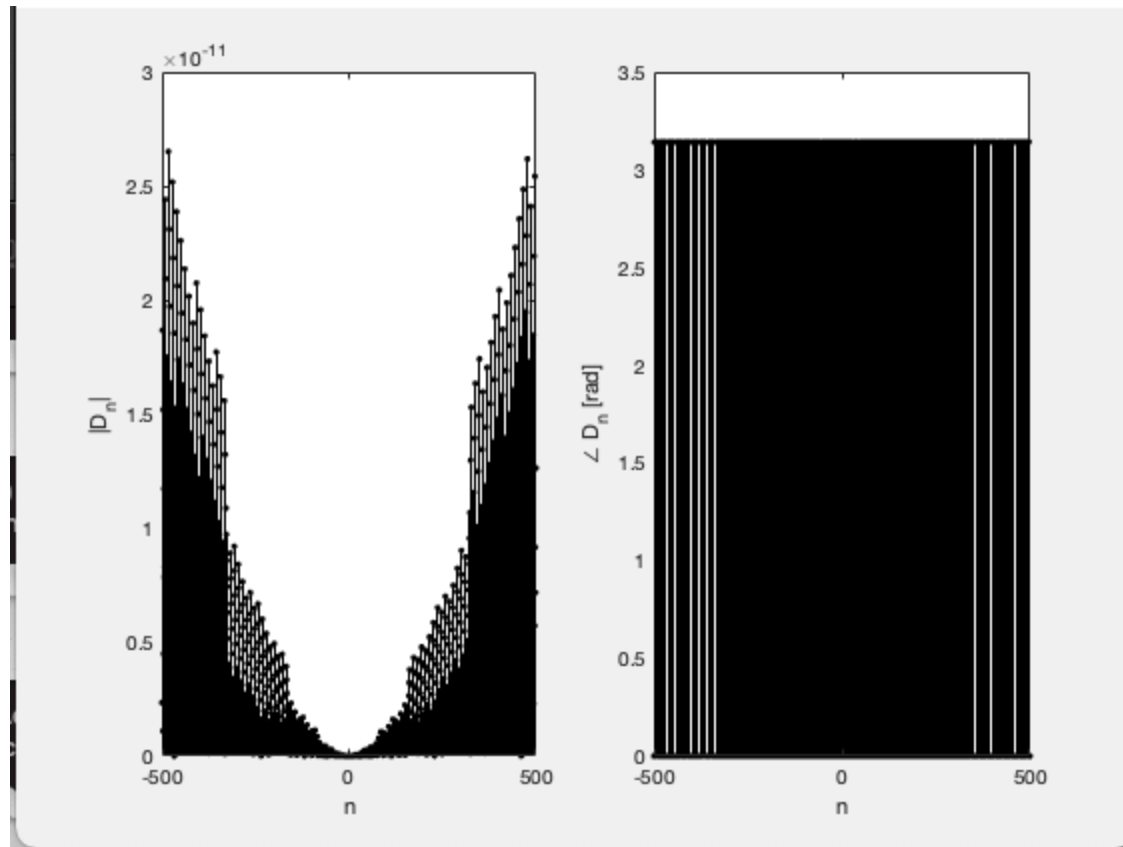
```



```

80 - figure
81 - clf;
82 - n = (-50:50);
83 - D_n = (1./(n.*pi)).*sin((n.*pi)./4);
84 - subplot(1,2,1); stem(n,abs(D_n),'.k');
85 - xlabel('n'); ylabel('|D_n|');
86 - subplot(1,2,2); stem(n,angle(D_n),'.k');
87 - xlabel('n'); ylabel('\angle D_n [rad]');

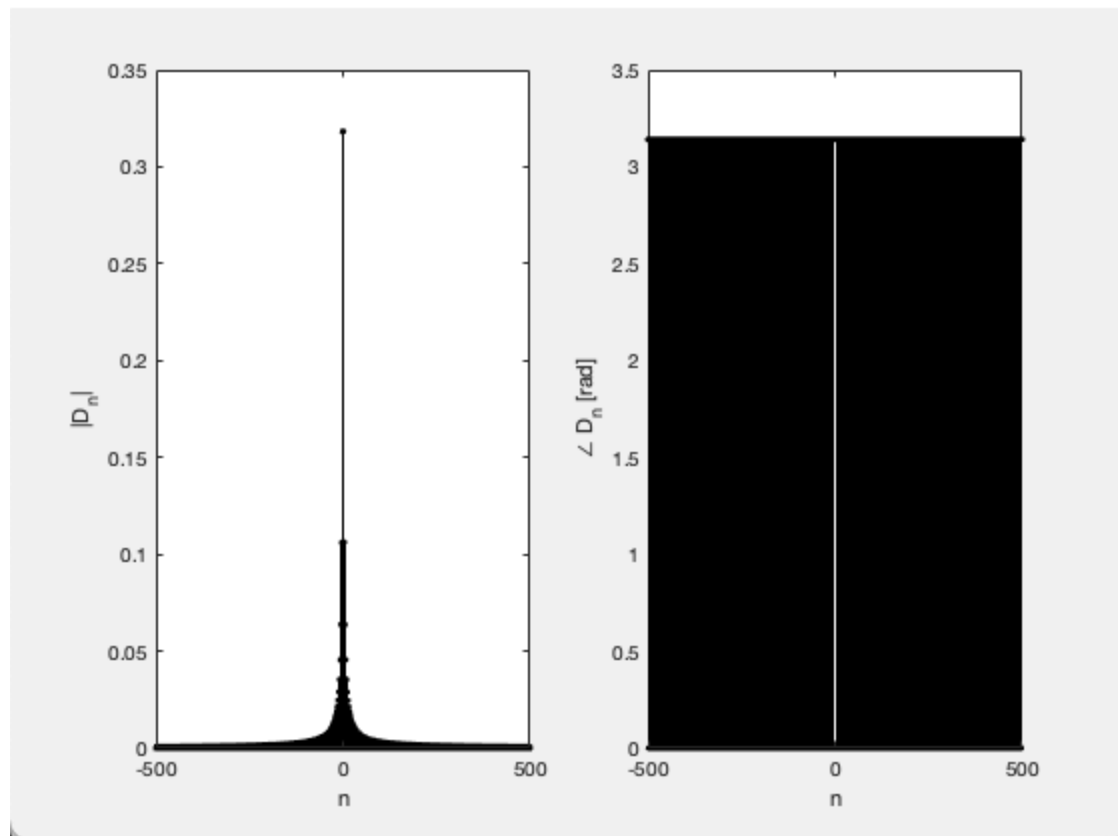
```



```

91 - figure
92 - clf;
93 - n = (-500:500);
94 - D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
95 - subplot(1,2,1); stem(n,abs(D_n),'k');
96 - xlabel('n'); ylabel('|D_n|');
97 - subplot(1,2,2); stem(n,angle(D_n),'k');
98 - xlabel('n'); ylabel('\angle D_n [rad]');

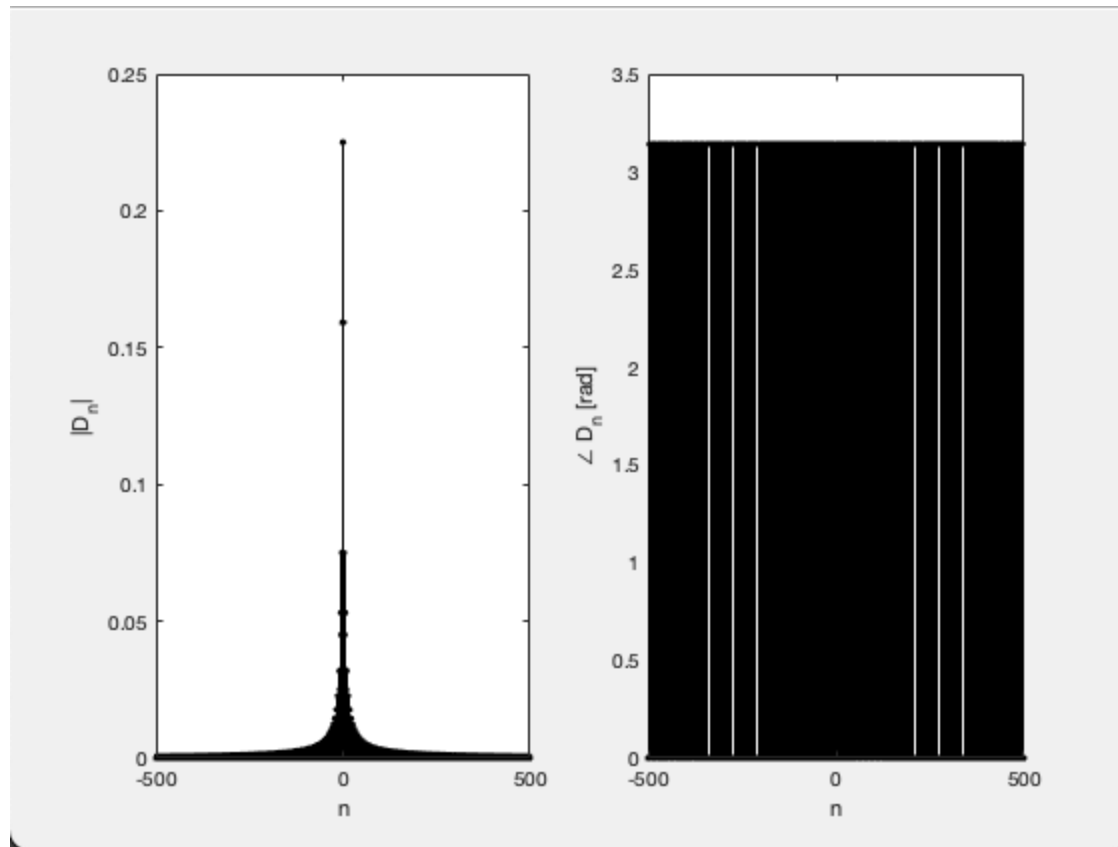
```



```

100 - figure
101 - clf;
102 - n = (-500:500);
103 - D_n = (1./(n.*pi).*sin((n.*pi)./2));
104 - subplot(1,2,1); stem(n,abs(D_n),'.k');
105 - xlabel('n'); ylabel('|D_n|');
106 - subplot(1,2,2); stem(n,angle(D_n),'.k');
107 - xlabel('n'); ylabel('\angle D_n [rad]');

```



```

109 - figure
110 - clf;
111 - n = (-500:500);
112 - D_n = (1./(n.*pi).*sin((n.*pi)./4));
113 - subplot(1,2,1); stem(n,abs(D_n),'.k');
114 - xlabel('n'); ylabel('|D_n|');
115 - subplot(1,2,2); stem(n,angle(D_n),'.k');
116 - xlabel('n'); ylabel('\angle D_n [rad]');

```

A5:

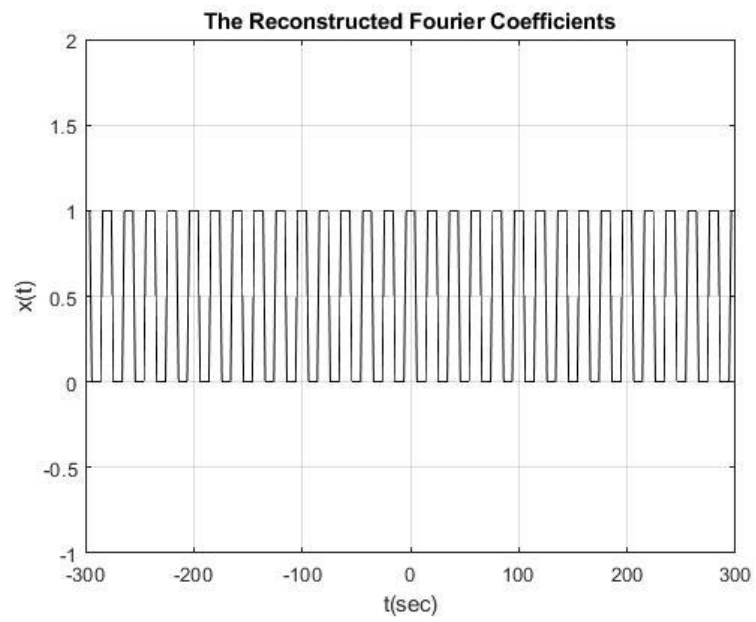
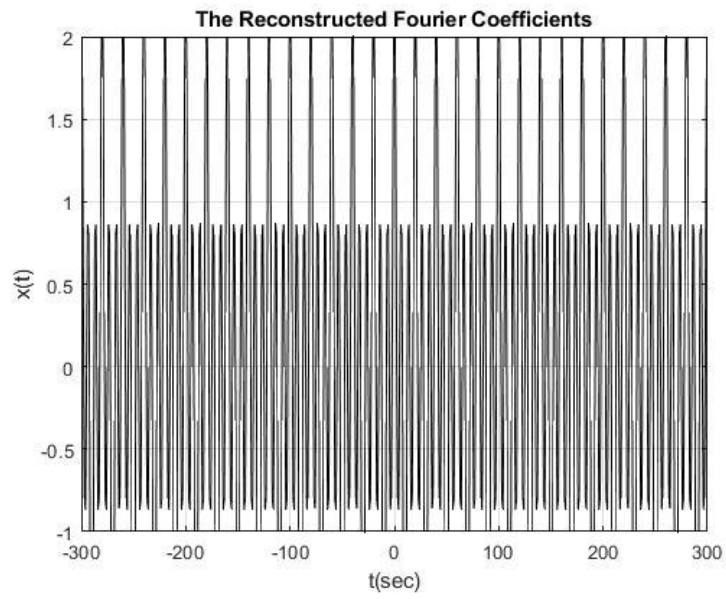
The Code for A5

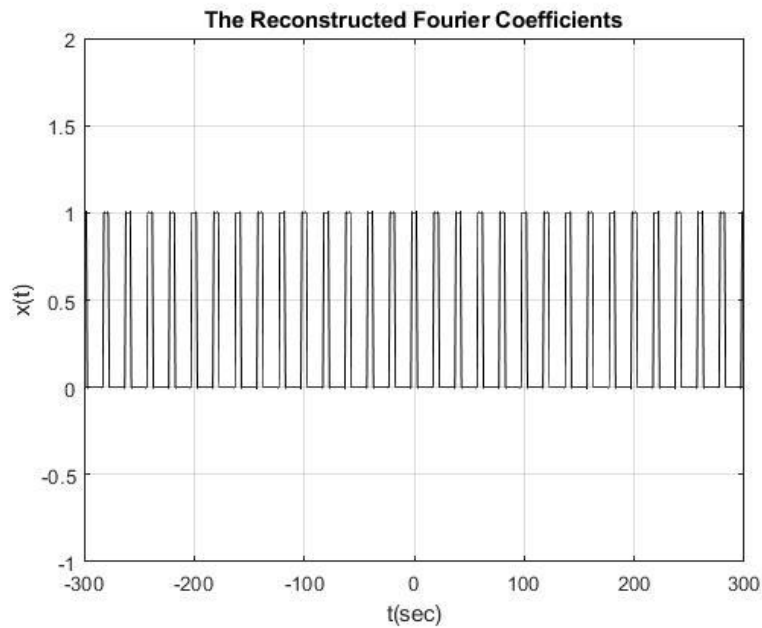
```

1  function [D] = A5(Dn)
2  -   n=-500:500;
3  -   D=Dn;
4  -   t=[-300:1:300];
5  -   w=pi*0.1;
6  -   x=zeros(size(t));
7  -   for i = 1:length(n)
8  -       for t=-300:1:300
9  -           x=x+D(i)*exp(1j*n(i)*w*t);
10 -       end
11 -   end
12 -   figure;
13 -   plot(t,x,'k')
14 -   xlabel('t(sec)');
15 -   ylabel('x(t)');
16 -   axis([-300 300 -1 2]);
17 -   title('The Reconstructed Fourier Coefficients');
18 -   grid;

```

A6:





The Code for A6

```

1 - D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi)) ;
2 - D_n2 = (1./(n.*pi).*sin((n.*pi)./2));
3 - D_n3 = (1./(n.*pi).*sin((n.*pi)./4));
4
5 - A5(D_n)
6 - A5(D_n2)
7 - A5(D_n3)

```

B1:

The following fundamental frequencies calculations are shown in part A1:

$x_1(t)$:

$$\omega_0 = \frac{\pi}{10}$$

$x_2(t)$:

$$\omega_0 = \frac{\pi}{10}$$

$x_3(t)$:

$$\omega_0 = \frac{\pi}{20}$$

B2:

$x_1(t)$ represents a sinc function as the denominator matches the expression within the sin portion of D_n . Also $x_1(t)$ has a distinct amount of coefficients whereas $x_2(t)$ and $x_3(t)$ has an infinite amount.

B3:

The fundamental frequencies between $x_2(t)$ and $x_3(t)$ differentiate in such a way that $x_3(t)$ has a smaller fundamental frequency in comparison to $x_2(t)$.

B4:

Based on the calculations done in **A2**, $x_2(t)$ has a D_o of 1. From this it can be observed that for these specific signals, the fourier coefficient, D_o , are dependent on the area of the functions. Since the area of $x_4(t)$ is 0.5, $x_4(t)$ has a D_o of 0.5.

B5:

Since $x_1(t)$ has a finite amount of fourier coefficients, nothing would change, however, for $x_2(t)$ increasing the fourier coefficients would increase the accuracy of the overall D_n .

B6:

For the graph in $x_1(t)$, it contains 4 fourier coefficients being ± 3 and ± 1 . For the graphs $x_2(t)$ and $x_3(t)$ they consist of an infinite amount of fourier coefficients.

B7:

It would be a viable option for $x_1(t)$ since it has a finite number of fourier coefficients, however, for $x_2(t)$ and $x_3(t)$ it would not be a viable option as they consist of an infinite amount of fourier coefficients. However, also considering the fundamental frequencies of the signals, $x_1(t)$, having a large fundamental frequency would take up alot of space in the hard drive. Therefore, it would only be viable if it has a smaller amount of fourier coefficients and a small fundamental frequency in order for space to not be wasted.