

I. APPENDIX: RESONATING GROUP METHOD AND INTERACTIONS

The non-relativistic dynamics of the four particles is given by the solutions to

$$\left(-\frac{\hbar^2}{2\mu}\Delta_R - E + \mathcal{V}\right)|\Psi\rangle \Leftrightarrow \langle\delta\Psi|\hat{H} - E|\Psi\rangle = 0 \quad . \quad (1)$$

The second, viz. the projection/variational equation is equivalent to the Schrödinger equation if $\delta\Psi$ denotes an arbitrary variation of the state. The resonating-group methods expresses the total wave function of this state in the form

$$\begin{aligned} \Psi = \mathcal{A} \Big\{ & \sum_i \phi(A_i) \phi(B_i) \chi_i(\mathbf{R}_i) \\ & + \sum_j \phi(A_j) \phi(B_j) \phi(C_j) \chi_j(\mathbf{R}_{j1}, \mathbf{R}_{j2}) + \sum_k \phi(A_k) \phi(B_k) \phi(C_k) \phi(D_k) \chi_k(\mathbf{R}_{k1}, \mathbf{R}_{k2}, \mathbf{R}_{k3}) \\ & + \dots + \sum_m c_m \eta_m \Big\} \zeta(\mathbf{R}_{\text{c.m.}}) \quad , \end{aligned} \quad (2)$$

where each term corresponds to a particular fragmentation of the particles into two, three, four, ... cluster. The internal motion of such a cluster of A particles is expanded with a complete set of functions $\phi(A_i)$, and the relative motion between the clusters is encoded in the χ_i 's. The last term is included to improve the expansion if the fragment-internal basis is not complete and/or if the cluster expansion is truncated such that a more complicated shape of the state for small separations between the clusters cannot be accounted for.

For the elastic scattering of a neutron off a three-nucleon targets whose ground state is $\Delta\epsilon_{\text{nuclear}} \approx 6.2$ MeV and $\Delta\epsilon_{\text{unitary}} \approx 8.4$ MeV, respectively, below the lowest dissociation thresholds, and the relative energies between the fragments $E_r \ll \Delta\epsilon$, we chose to express the wave function of the system with a two-cluster, single-channel ansatz without specific distortion (*cf.* eq. (5), first term):

$$\Psi = \mathcal{A}_{31} [\phi(3)\phi(1) \chi(\mathbf{R}) \zeta(\mathbf{R}_{\text{c.m.}})] = \int \mathcal{A}_{31} [\phi(3)\phi(1) \delta(\mathbf{R} - \mathbf{R}') \zeta(\mathbf{R}_{\text{c.m.}})] \chi(\mathbf{R}') d\mathbf{R}' \quad . \quad (3)$$

The parameter representation employed with the second equality in eq. (3) will yield a defining equation for the relative motion independent of the antisymmetrizer \mathcal{A} . Assuming an inert triton core amounts to a variation of the relative wave function, only:

$$\delta\Psi = \int \delta\chi(\mathbf{R}') \mathcal{A}_{AB} [\phi_A \phi_B \delta(\mathbf{R} - \mathbf{R}') \zeta(\mathbf{R}_{\text{c.m.}})] d\mathbf{R}' \equiv \oint \delta a_n \Phi_n \quad . \quad (4)$$

The latter notation allows for the following concise form of the projection equation

$$\left\langle \oint \delta a_m \Phi_m \mid \hat{H} - E \mid \oint a_n \Phi_n \right\rangle = 0 \rightarrow \langle \phi(3) \mid \hat{H} - E \mid \mathcal{A}_{31} [\chi(\mathbf{R}') \phi(3)] \rangle_{\mathbf{R}} = 0 \quad (5)$$

where the *ket* subscript average indicates that the average over the relative radius was already taken, and the remaining integrals are over the fragment-internal coordinates. If the cluster-internal states are properly anti-symmetrized, it remains to consider the inter-fragment anti-symmetrization via $\mathcal{A}_{31} = \mathbb{1} - \hat{P}_{14}$. We chose nucleons 1 and 4 to be the neutrons with identical spin orientation, and hence need to permute only those particles. All other permutations \hat{P}_{ij} will yield zero matrix elements because of the orthogonality of the internal states.

Being more explicit, the motion of the two fragments relative to each other is approximated by

$$\int \left\{ \phi(3) \left(-\frac{\hbar^2}{2\mu} \Delta_R - E + B(3) + \mathcal{V}_{31} \right) \mathcal{A}_{31} [\phi(3) \chi(\mathbf{R})] \right\} d\mathbf{r}_{1,2,3} = 0 \quad . \quad (6)$$

This expression is further evaluated by: (i) employing the parameter representation, $\chi(\mathbf{R}) = \int \delta(\mathbf{R} - \mathbf{R}') \chi(\mathbf{R}') d\mathbf{R}'$; (ii) expanding the triton ground state in Gaussians, $\phi(3) = \sum_n^{N_f} c_n \cdot e^{-\alpha_n \sum_{i=1}^3 (\mathbf{r}_i - \mathbf{R}_3)^2}$ with norm $\mathbb{N}_3 = \sum_{i,j=1}^{N_f} c_i c_j \left(\frac{\pi}{\alpha_i + \alpha_j} \right)^{3/2}$; (iii) substituting the leading-order EFT(\not{p}) potential as inter-cluster interaction,

n	$\eta^{(ij)}$	$w^{(ij)}$		
1	$\frac{2C(\lambda)}{[2(\alpha_i+\alpha_j)(3\alpha_i+3\alpha_j+2\lambda)]^{3/2}}$	$\frac{3(\alpha_i+\alpha_j)\lambda}{3(\alpha_i+\alpha_j)+2\lambda}$		
2	$\frac{D(\lambda)}{[6(\alpha_i+\alpha_j)^2+16\lambda(\alpha_i+\alpha_j)+2\lambda^2]^{3/2}}$	$\frac{3(\alpha_i+\alpha_j)\lambda((\alpha_i+\alpha_j)+2\lambda)}{3(\alpha_i+\alpha_j)^2+8(\alpha_i+\alpha_j)\lambda+\lambda^2}$		
	$\zeta^{(ij)} \otimes \hat{\zeta}$	$a^{(ij)}$	$b^{(ij)}$	$c^{(ij)}$
1	$\frac{\hbar^2}{2\mu} \frac{3^3}{2^6} \frac{1}{[2(\alpha_i+\alpha_j)]^{3/2}} \otimes (-\mathbf{\Delta}_R - \mathbf{p}^2)$	$\frac{3}{32}(\alpha_i + 9\alpha_j)$	$\frac{9}{16}(\alpha_i + \alpha_j)$	$\frac{3}{32}(9\alpha_i + \alpha_j)$
2	$-\frac{3^3}{2^5} \frac{C(\lambda)}{[2(\alpha_i+\alpha_j)+\lambda]^{3/2}} \otimes \mathbb{1}$	$\frac{3(\alpha_i^2+9\alpha_j^2+10\alpha_i\alpha_j+\lambda(14\alpha_i+18\alpha_j))}{16(2(\alpha_i+\alpha_j)+\lambda)}$	$\frac{18(\alpha_i+\alpha_j)(\alpha_i+\alpha_j)+2\lambda}{16(2(\alpha_i+\alpha_j)+\lambda)}$	$\frac{3(9\alpha_i^2+\alpha_j^2+10\alpha_i\alpha_j+\lambda(6\alpha_i+2\alpha_j))}{16(2(\alpha_i+\alpha_j)+\lambda)}$
3	$-\frac{3^3}{2^6} \frac{D(\lambda)}{[2(\alpha_i+\alpha_j)+5\lambda]^{3/2}} \otimes \mathbb{1}$	$\frac{3(\alpha_i^2+9\alpha_j^2+10\alpha_i\alpha_j+\lambda(16\alpha_i+36\alpha_j)+27\lambda^2)}{16(2(\alpha_i+\alpha_j)+5\lambda)}$	$\frac{18((\alpha_i+\alpha_j)^2+4\lambda(\alpha_i+\alpha_j)+3\lambda^2)}{16(2(\alpha_i+\alpha_j)+5\lambda)}$	$\frac{3(9\alpha_i^2+\alpha_j^2+10\alpha_i\alpha_j+\lambda(24\alpha_i+4\alpha_j)+3\lambda^2)}{16(2(\alpha_i+\alpha_j)+5\lambda)}$

TABLE I: Local (η, w) and non-local (ζ, a, b, c) components of the resonating-group eq. (9).

$$\begin{aligned}
\mathcal{V}_{31} &= C(\lambda) \sum_{\substack{i \in A \\ j \in B}} \delta_\lambda^{(3)}(\mathbf{r}_i - \mathbf{r}_j) + D(\lambda) \sum_{\substack{i, j, k \\ i \in A \Rightarrow j \vee k \in B}} \delta_\lambda^{(3)}(\mathbf{r}_i - \mathbf{r}_j) \delta_\lambda^{(3)}(\mathbf{r}_i - \mathbf{r}_k) \\
&= C(\lambda) \cdot \left(\delta_\lambda^{(3)}(\mathbf{r}_2 - \mathbf{r}_4) + \delta_\lambda^{(3)}(\mathbf{r}_3 - \mathbf{r}_4) \right) + D(\lambda) \cdot \left(\delta_\lambda^{(3)}(\mathbf{r}_2 - \mathbf{r}_3) \delta_\lambda^{(3)}(\mathbf{r}_2 - \mathbf{r}_4) \right) \quad ; \quad (7)
\end{aligned}$$

(iv) acting with \mathcal{A}_{31} on ϕ_3 and $\delta(\mathbf{R} - \mathbf{R}')$; and finally, (v) doing the Gaussian integrals for the triton-internal single-particle coordinates $\mathbf{r}_{1,2,3}$. Thereby, one obtains the form

$$(\hat{T} - E) \chi(\mathbf{R}) + \mathcal{V}^{(1)}(\mathbf{R}) \chi(\mathbf{R}) + \int d^{(3)}\mathbf{R}' \mathcal{V}^{(2)}(\mathbf{R}, \mathbf{R}', E) \chi(\mathbf{R}') = 0 \quad . \quad (8)$$

More conveniently, we do express this equation as

$$\mathbb{N}_3 \cdot (-\mathbf{\Delta}_R - \mathbf{p}^2) \chi(\mathbf{R}) + \sum_{i,j=1}^{N_f} c_i c_j \left[\sum_{n=1}^2 \eta_n^{(ij)} e^{-w_n^{(ij)} \mathbf{R}^2} \chi(\mathbf{R}) - \sum_{n=1}^3 \int \left\{ \zeta_n^{(ij)} \hat{\zeta}_n e^{-a_n^{(ij)} \mathbf{R}^2 - b_n^{(ij)} \mathbf{R} \cdot \mathbf{R}' - c_n^{(ij)} \mathbf{R}'^2} \right\} \chi(\mathbf{R}') d\mathbf{R}' \right] = 0 \quad (9)$$

with $\hat{\eta}_n, \hat{\zeta}_n, w_n, a_n, b_n, c_n$ dependent upon, (i) the low-energy constants: $C(\lambda), D(\lambda)$, (ii) the triton-expansion parameters: α_n, c_n , (iii) the regulator parameter: λ , and (iv) the relative momentum: $\mathbf{p}^2 = 2\mu E$, as given in table (I).

The character of the interaction is expected to depend strongly on the spatial symmetry of the wave function, and as the parities of the triton core and the point neutron are positive, the overall parity is set by the relative motion χ . In order to investigate the parity dependence, we project into partial waves: Expanding $\chi(\mathbf{R}) = R^{-1} \sum_{lm} \phi_{lm}(R) Y_{lm}(\hat{\mathbf{R}})$ and projecting from the left with $R \int d^2 \hat{\mathbf{R}} Y_{lm}^*(\hat{\mathbf{R}})$ before substituting

$$e^{-b\mathbf{R} \cdot \mathbf{R}'} = 4\pi \sum_{LM} i^L j_L(ibRR') Y_{LM}^*(\hat{\mathbf{R}}) Y_{LM}(\hat{\mathbf{R}}') \quad , \quad (10)$$

$$\mathbf{R} \cdot \mathbf{R}' = -\sqrt{3} [\mathbf{R}_p \otimes \mathbf{R}'_{-p}]^{00} = \frac{4\pi}{3} RR' \sum_p (-)^p Y_{1p} Y'_{1-p} \quad , \quad \text{and} \quad \mathbf{r}_m = \sqrt{\frac{4\pi}{3}} r Y_{1,m}(\hat{\mathbf{r}}) \quad , \quad (11)$$

$$(12)$$

yields

$$\begin{aligned}
0 = & \mathbb{N}_3 \cdot \left(\partial_R^2 - \frac{l(l+1)}{R^2} + \mathbf{p}^2 \right) \phi_{lm}(R) - \frac{2\mu}{\hbar^2} \sum_{i,j=1}^{N_f} c_i c_j \left(\sum_{n=1}^2 \eta_n^{(ij)} e^{-w_n^{(ij)} R^2} \phi_{lm}(R) \right. \\
& - \int dR' \phi_{lm}(R') (4\pi R R') \left[\zeta_1^{(ij)} \cdot e^{-a_1^{(ij)} R^2 - c_1^{(ij)} R'^2} \cdot \left\{ \left[-(4(a_1^{(ij)})^2 R^2 + (b_1^{(ij)})^2 R'^2 - 2a_1^{(ij)}) + \frac{l(l+1)}{R^2} - \frac{2\mu}{\hbar^2} E \right] i^l j_l(i b_1^{(ij)} R R') \right. \right. \\
& \left. \left. - (4a_1^{(ij)} b_1^{(ij)}) \cdot R R' \cdot \sum_L i^L j_L(i b_1^{(ij)} R R') \hat{L} \hat{l} \begin{pmatrix} L & 1 & l \\ 0 & 0 & 0 \end{pmatrix}^2 \Delta_{L1l} \right\} + \sum_{n=2}^3 \zeta_n^{(ij)} i^l j_l(i b_n R R') \cdot e^{-a_n^{(ij)} R^2 - c_n^{(ij)} R'^2} \right] \Bigg) \quad (13)
\end{aligned}$$

with a 3- j symbol $\neq 0 \Leftrightarrow L+1+l = \text{even}$, and $\Delta_{j_1 j_2 j_3} \neq 0 \Leftrightarrow j_3 = |j_1 - j_2|, \dots, j_1 + j_2$. From this equation, we obtain solutions for S - and P -waves on a discrete, equidistant coordinate grid: $R, R' \in \{r_0, r_0 + \epsilon, r_0 + 2\epsilon, \dots, \underbrace{r_0 + N_{\text{grid}} \epsilon}_{=: r_{N_{\text{grid}}+1}}\}$. We

chose a maximal grid dimension such that for $R' = r_{N_{\text{grid}}+1}$ both local and non-local interactions vanish. Thence, we solve eq. (13) for the *principal-value* wave function ($\phi^{(P)}$) with the boundary condition

$$\phi_{l,p}^{(P)}(R) \stackrel{R \rightarrow \infty}{\simeq} \sin\left(pR - \frac{\pi l}{2}\right) + K_l \cdot \cos\left(pR - \frac{\pi l}{2}\right) \quad (14)$$

$$= \lim_{pR \rightarrow \infty} [u_l(pR)] + \tan \delta_l \cdot \lim_{pR \rightarrow \infty} [v_l(pR)] = \begin{cases} l=0: \sin(pR) - \tan \delta_l \cos(pR) \\ l=1: \frac{\sin(pR)}{pR} - \cos(pR) - \tan \delta_l \left[\frac{\cos(pR)}{pR} + \sin(pR) \right] \end{cases} \quad (15)$$

For the Riccati-Bessel, $u_l(z) = z j_l(z)$, and the Riccati-Neumann Functions, $v_l(z) = z n_l(z)$, we use [] as reference, where $j_l(n_l)$ is the proper spherical Bessel(Neumann) function. Note, the relation between the Neumann and the spherical Bessel function of the second kind: $n_l(z) = (-)^l y_l(z)$. Now, we obtain the phase shifts from

$$\tan \delta_l = \frac{\phi_{l,p}^{(P)}(r_{N_{\text{grid}}+1})}{v_l(pr_{N_{\text{grid}}+1})} - \frac{u_l(pr_{N_{\text{grid}}+1})}{v_l(pr_{N_{\text{grid}}+1})} \quad (16)$$

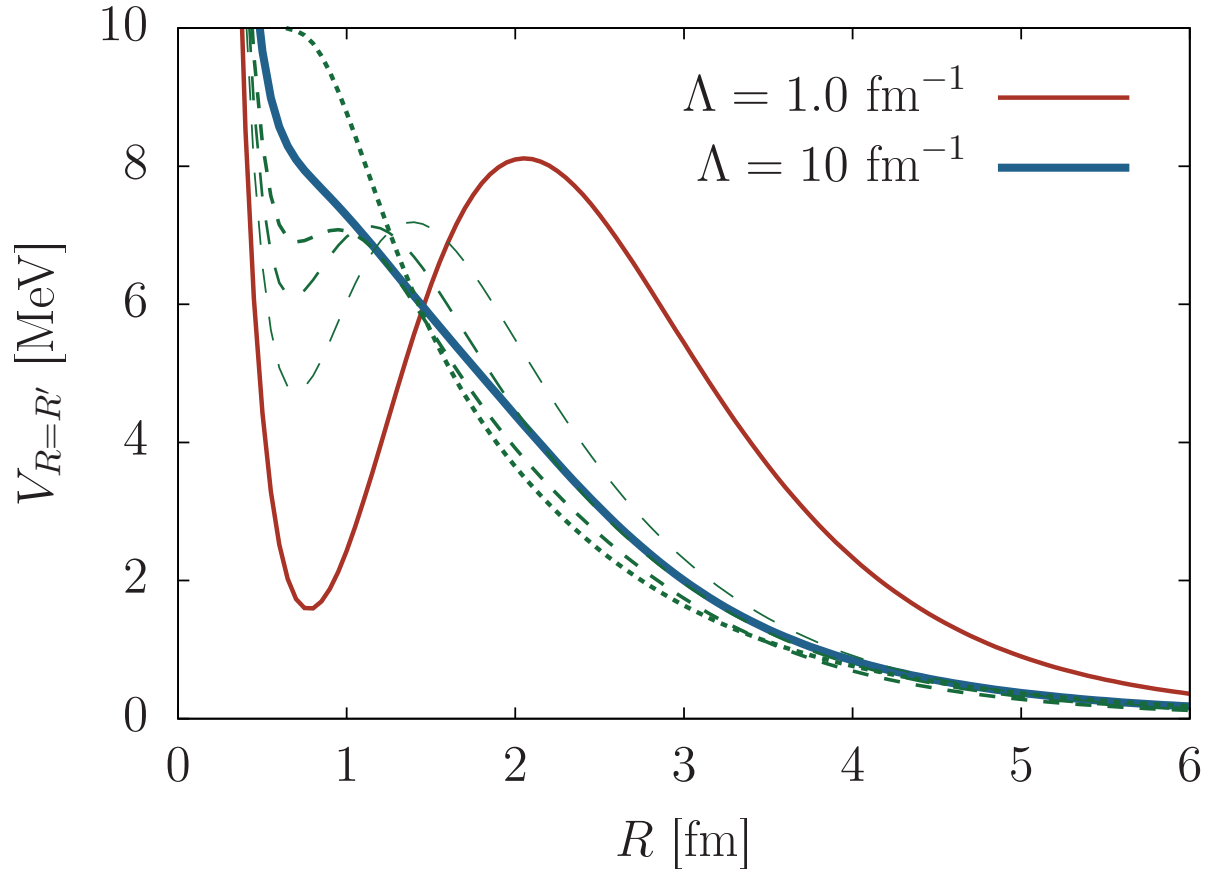


FIG. 1: Two-fragment potential as a function of the distance between a neutron and the center of mass of a triton. The plot shows the P -wave component of the interaction including local and non-local part of the kinetic-, pair-, and three-body interactions. The non-local parts are evaluated at $R = R'$. The height of the potential's barrier shrinks continuously from the smallest cutoff (red) and eventually vanishes at the largest (fat blue), with intermediate values represented by green, dashed curves.