

$$V_{\text{local}}(R; \Lambda, A, a) = \frac{Z_{\text{loc}}}{k} N^{-1} \left[C(\Lambda-1) \left(\frac{(2\pi)^{\Lambda-1}}{|M_1|} \right)^{\frac{1}{2}} e^{-\alpha_1^+ R^2} + \frac{(\Lambda-1)(\Lambda-2)}{2} D_1 \left(\frac{(2\pi)^{\Lambda-1}}{|M_1|} \right)^{\frac{1}{2}} e^{-\alpha_1^+ R^2} \right]$$

$$\left(\frac{(2\pi)^{\frac{\Lambda}{2}} \cdot \frac{1}{2\alpha} \cdot (2\pi)^{\Lambda-1} \cdot \frac{1}{\sqrt{|M_1|}} \right)^{\frac{1}{2}} \Rightarrow \left(\frac{(2\pi)^{\Lambda-2}}{|M_1|} \right)^{\frac{1}{2}}$$

$$V_{\text{local}}(E, R, R'; \Lambda, A, a) = i 4\pi N^{-1} \left[f_1 R e^{-\alpha_1^+ R^2} \left(-R^2 \partial_R R^2 \partial_{R'} E + \frac{R(R-1)}{R^2} \right) R' e^{-\alpha_1^+ R'^2} j_x(i\beta_1 R R') \right. \\ \left. + \frac{Z_{\text{loc}}}{k} \left(f_2 R e^{-\alpha_2^+ R^2} j_x(i\beta_2 R R') R' e^{-\alpha_2^+ R'^2} + f_3 R e^{-\alpha_3^+ R^2} j_x(i\beta_3 R R') R' e^{-\alpha_3^+ R'^2} \right) \right]$$

$\uparrow \qquad \qquad \qquad \uparrow \qquad \qquad \qquad \uparrow$
 $\langle \phi_A | (\hat{V}_3^{\text{atom}} + \hat{V}_3^{\text{slow}}) \hat{P}[\phi_A S(R-R')] \rangle$

to be used in:

$$\left(\partial_R^2 - E_{\text{rel}} + \frac{\ell(\ell+1)}{R^2} \right) \psi_{\text{em}}(R) = \hat{V}_{\text{local}}(R) \psi_{\text{em}}(R) + \int dR' \left[\hat{V}_{\text{loc}}^{(0)}(R, R') + E \hat{V}_{\text{loc}}^{(1)}(R, R') \right] \psi_{\text{em}}(R')$$

$$\Leftrightarrow \left[\left(\partial_R^2 + \frac{\ell(\ell+1)}{R^2} - \hat{V}_{\text{local}} \right) S(R-R') - \int dR' \hat{V}_{\text{loc}}^{(0)}(R, R') \right] \psi_{\text{em}}(R') = E_{\text{rel}} \left[S(R-R') + \int dR' \hat{V}_{\text{loc}}^{(1)} \right] \psi_{\text{em}}(R')$$

$$\equiv H_{RR'} \underbrace{\psi_{\text{em}}^{R'}}_{= \sum_i c_i f_i} = E K_{RR'} \psi_{\text{em}}^{R'} \\ = \sum_i c_i f_i \quad \text{and} \quad \int dR f_j \Rightarrow c_i f_i^R H_{RR'} f_i^{R'} = E f_i^R K_{RR'} f_i^{R'}$$