# Projet RGM

May 18, 2020

## Contents

# 1 Equation

Starting point

$$(\hat{H}_0 + \hat{U}) \mid \Psi >= E \mid \Psi > \qquad \hat{U} = \hat{V} + \hat{W}$$

where we split the local and non-local part of the potential $U = V + W$.

In configuration space, we aim to solve the scattering solutions of the Schrodinger equation having the form

$$(H_0 - E)\ \Psi(\vec{R}) + V(\vec{R})\ \Psi(\vec{R}) + \int d\vec{R}'\ W(\vec{R}, \vec{R}'; E)\ \Psi(\vec{R}') = 0 \tag{1}$$

where

$$H_0 = -\frac{\hbar^2}{2\mu}\Delta_{\vec{R}}$$

the local potential

$$V(\vec{R}) = \sum_{n=1}^{3} \eta_n e^{-\kappa_n R^2} \tag{2}$$

and a non-local and E-dependent term that we will write in the form

$$W(\vec{R}, \vec{R}'; E) = -\sum_{i=1}^{4} c_i\ W_i(R, R', \vec{R}\cdot\vec{R}'; E)\ e^{-\left(\alpha_i R^2 + \beta_i \vec{R}\cdot\vec{R}' + \gamma_i R'^2\right)} \tag{3}$$

In detail

$$\begin{aligned}
W(\vec{R}, \vec{R}'; E) \quad &= \quad c_1 \left[\frac{\hbar^2}{2\mu}\left(4\alpha_1^2 R^2 + \beta_1^2 R'^2 + 4\alpha_1\beta_1 \vec{R}\cdot\vec{R}' - 2\alpha_1\right) + E\right]\ e^{-\left(\alpha_1 R^2 + \beta_1 \vec{R}\cdot\vec{R}' + \gamma_i R'^2\right)} && (4)\\
&- \quad c_2 \qquad e^{-\left(\alpha_2 R^2 + \beta_2 R'^2 + \gamma_2 \vec{R}\cdot\vec{R}'\right)} && (5)\\
&- \quad c_3 \qquad e^{-\left(\alpha_3 R^2 + \beta_3 R'^2 + \gamma_3 \vec{R}\cdot\vec{R}'\right)} && (6)\\
&- \quad c_4 \qquad e^{-\left(\alpha_4 R^2 + \beta_4 R'^2 + \gamma_4 \vec{R}\cdot\vec{R}'\right)} && (7)
\end{aligned}$$

That is having a radial dependence $W(\vec{R}, \vec{R}'; E) \equiv f(R, R', \vec{R}\cdot\vec{R}'; E)$

1. It depends on $3 \times 2 + 4 \times 4 = 20$ constants and the effective mass $\mu$
2. Usually the RGM equation have the form

$$E \int dr'\ N(r, r')\chi(r') = \int dr' H(r, r')\chi(r')$$

   that is with a "norm term" $N(r, r')$. Is it absent in your case ?

## 2 Partial wave solution

After projecting, the **reduced radial equation** takes the form

$$-\frac{\hbar^2}{2\mu}u_L''(R) - Eu_L(R) + \left[V(R) + \frac{\hbar^2}{2\mu}\frac{L(L+1)}{R^2}\right]u_L(R) + \int dR' W_L(R,R';E)u_L(R') = 0 \tag{8}$$

with the local potential

$$V(R) = \sum_{n=1}^{3} \eta_n e^{-\kappa_n R^2} \tag{9}$$

and the non-local E-dependent one

$$W_L(R,R';E) = F_L(R,R') + \sum_{n=1}^{4} 4\pi\ i^L\ c_n\ \{E\delta_{1n} + \bar{\delta}_{1n}\}\ j_L(i\beta_n RR')e^{-(\alpha_n R^2 + \gamma_n R'^2)}\ RR' \tag{10}$$

where

$$\bar{\delta}_{1n} \equiv 1 - \delta_{1n}$$

$$\begin{aligned}
F_L(R,R') &= A(R,R')\,[B_L(R,R') + C_L(R,R') + D_L(R,R')] \\
A(R,R') &= -\frac{\hbar^2}{2\mu}4\pi c_1 e^{-(\alpha_1 R^2 + \gamma_1 R'^2)}RR' \\
B_L(R,R') &= \left[-4\alpha_1^2 R^2 - \beta_1^2 R'^2 + 2\alpha_1 + \frac{L(L+1)}{R^2}\right] i^L j_L(i\beta_1 RR') \\
C_L(R,R') &= \bar{\delta}_{L0}\ 4\alpha_1\beta_1\ i^{L-1} j_{L-1}(i\beta_1 RR')(2L-3)\begin{pmatrix} 1 & L-1 & L \\ 0 & 0 & 0 \end{pmatrix}^2 RR' \\
D_L(R,R') &= 4\alpha_1\beta_1\ i^{L+1} j_{L+1}(i\beta_1 RR')(2L-1)\begin{pmatrix} 1 & L+1 & L \\ 0 & 0 & 0 \end{pmatrix}^2 RR'
\end{aligned}$$

1. Although not explicit, i think that $W_L$ must be real
2. Since $j_L(z) \approx z^L$, $W_L$ vanishes when $R, R \to 0$ and when $R, R' \to \infty$
3. Same remark concerning the absence of "norm term"
4. In practical solutions I prefer multiply equation (8) by $(2\mu/\hbar^2)$, introduce the wave number $q$ driving the assymptotics, and write it in the form

$$\boxed{u_L''(R) + \left[q^2 - v(R) - \frac{L(L+1)}{R^2}\right]u_L(R) - \int dR'\ w_L(R,R';E)\ u_L(R') = 0} \tag{11}$$

where

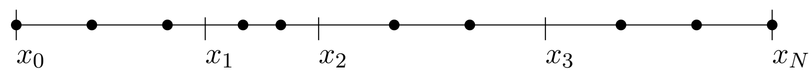
$$v = \frac{2\mu}{\hbar^2}V \qquad w = \frac{2\mu}{\hbar^2}W \qquad q^2 = \frac{2\mu}{\hbar^2}E$$

### 2.1 Solution using splines

We aim to solve

$$\varphi''(x) + [q^2 - v(x)]\varphi(x) - \int_0^\infty w(x,x')\varphi(x')dx' = 0 \tag{12}$$

on a given grid with of N+1 points $G = \{x_0, x_1, \dots, x_N\}$ not necessarily equidistants.




We search the solution of (12) in the form

$$\varphi(x) = \sum_{j=0}^{2N+1} c_j S_j(x) \tag{13}$$

where $S_j$ is a set of 2(N+1) given functions, depending on G, and $c_j$ 2(N+1) coefficients to determine.

By inserting this expression in (12) we obtain

$$\sum_j [\hat{L}S_j](x)\; c_j \equiv 0 \tag{14}$$

where $\hat{L}$ denote some integro-differential operator acting on $S_i$

In order to transform (14) in a linear system for $c_i$, we "validate" this expression – that is we assume its validity – in a set of 2(N+1) "well chosen" points $\bar{x}_{i=0,1,\dots,2N+1}$. One usually chose two inside each of the $N$ intervals, eventually supplemented with the two extremes of the grid G.
Each collocation point $\bar{x}_i$ gives rise to a linear equation and this gives the $(2N+2)\times(2N+2)$ square linear system:

$$\sum_{i,j=0}^{2N+2} A_{ij} c_j = 0 \tag{15}$$

where

$$\begin{aligned} A_{ij} &= S_j''(\bar{x}_i) + [q^2 - v(\bar{x}_i)]S_j(\bar{x}_i) - w_{ij} \\ w_{ij} &= \int_0^\infty dx'\; w(\bar{x}_i, x')\; S_j(x') \end{aligned}$$

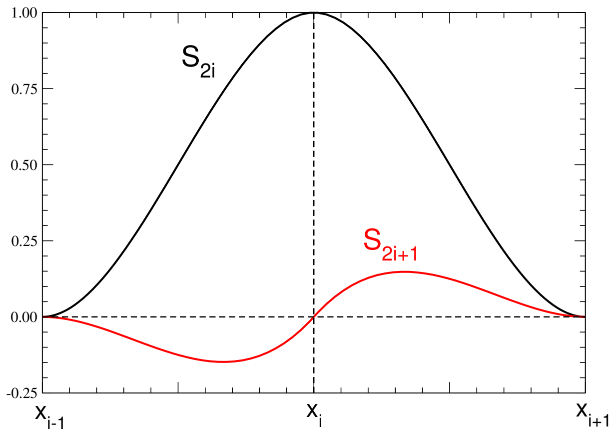
The linear system (15) will be still modified when introducing the appropriate boundary conditions.

#### 2.1.1 Cubic "splines"

The essentials about "splines"[1]

1. There are two "spline" functions associated to each grid point $x_i$, $S_{2i}$ and $S_{2i+1}$.
2. Their "support" consists in two consecutive intervals surrounding $x_i$ : $D_i = [x_{i-1}, x_{i+1}]$ (vanish outside)
3. They are piece-wise cubic polynomials on each interval $[x_i, x_{i-1}]$ with $C^1$ matching conditions among them

[1] Although everybody call them "splines" they are in fact Cubic Hermite Interpolation Polynomials and hence "spline". (See for instance Few-Body Syst. (2011) 49: 205-222 )




4. Interesting properties are

$$
\begin{aligned}
S_j(x_i) &= \delta_{j,2i} \\
S'_j(x_i) &= \delta_{j,2i+1}
\end{aligned}
$$

Because of that, the coeficients of an spline expansion of a function $f$ on a grid $G = \{x_0, x_1, ..., x_N\}$

$$f(x) = \sum_{i=0}^{2N+1} c_i S_i(x)$$

have a simple interpretation in terms of the grid values:

$$
\begin{aligned}
f(x_i) &= c_{2i} \\
f'(x_i) &= c_{2i+1}
\end{aligned}
$$

5. Their analytic expresions are:

$$
\begin{aligned}
S_{2i}(x) &= \begin{cases} 3\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)^2 - 2\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)^3 & \text{if } x \in [x_{i-1}, x_i] \\ 3\left(\frac{x_{i+1}-x}{x_{i+1}-x_i}\right)^2 - 2\left(\frac{x_{i+1}-x}{x_{i+1}-x_i}\right)^3 & \text{if } x \in [x_i, x_{i+1}] \end{cases} \\
S_{2i+1}(x) &= \begin{cases} \left[-\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)^2 + \left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)^3\right](x_i - x_{i-1}) & \text{if } x \in [x_{i-1}, x_i] \\ \left[+\left(\frac{x_{i+1}-x}{x_{i+1}-x_i}\right)^2 - \left(\frac{x_{i+1}-x}{x_{i+1}-x_i}\right)^3\right](x_{i+1} - x_i) & \text{if } x \in [x_i, x_{i+1}] \end{cases}
\end{aligned}
$$

and simalr expressiuon exist for their first and second derivatives.

They are represented in the figure above

### 2.1.2 Boundary conditions

We will see that the boundary conditions fixe two coefficients in expansion (13) and the number of unknown coefficients is in fact 2N

1. At the origin: since we are using the reduced radial equation one must fulfil

$$\phi(0) = 0 \quad \Longleftrightarrow \quad c_0 = 0$$

This means that $c_0$ is absent in the expansion (13) as well as the equation corresponding to $\bar{x}_0$

2. Asymptotic ($r \to \infty$) for scattering problem: **we impose at** $x = x_N$

$$
\begin{aligned}
\varphi(x) &= \varepsilon F_1(z) + C F_2(z) \qquad z = qx &(16)\\
\varphi'(x) &= \varepsilon q F'_1(z) + C q F'_2(z) &(17)
\end{aligned}
$$

with

- $F_i$ two **known solutions** of the free equation (e.g. $\hat{j}_L, \hat{n}_L, \hat{h}^+_L, ...$) linearly independent
- $\varepsilon$=0,1 to cover all possibilities, including purely outgoing waves (e.g. resonances)

By eliminating $C$ from (16) et (17) one gets a relation between the solution and its derivative at $x = x_N$

$$\varphi' = q\frac{F_2'}{F_2}\,\varphi - \varepsilon q F_1 \left[\frac{F_2'}{F_2} - \frac{F_1'}{F_1}\right]$$

We will write this in the generic form

$$\varphi'(x_N) = \Delta_N\ \varphi(x_N) - \Delta_N' \tag{18}$$

$$\Delta_N = q\frac{F_2'}{F_2} \tag{19}$$

$$\Delta_N' = \varepsilon q F_1 \left[\frac{F_2'}{F_2} - \frac{F_1'}{F_1}\right] \tag{20}$$

In terms of spline coefficients (18) reads:

$$\boxed{c_{2N+1} = \Delta_N\ c_{2N} - \Delta_N'} \tag{21}$$

This relation allows us to eliminate $c_{2N+1}$ in the expansion (13), eliminates the equation corresponding to $\bar{x}_{2N+1}$ in (15) and introduces two wain differences in the remaining $2N \times 2N$ linear system:

$$\sum_{i,j=0}^{2N} A_{ij}c_j = 0 \tag{22}$$

- Change the matrix elements involving $A_{*,2N+1}$

$$A_{2N-1,2N} \;\rightarrow\; \hat{A}_{2N-1,2N} = A_{2N-1,2N} + A_{2N-1,2N+1}\Delta_N \tag{23}$$

$$A_{2N,2N} \;\rightarrow\; \hat{A}_{2N,2N} = A_{2N,2N} + A_{2N,2N+1}\ \Delta_N \tag{24}$$

- **Introduces an inhomogeneus term** in , which becomes

$$Ac = b \qquad b = \begin{pmatrix} 0 \\ 0 \\ .. \\ y_{2N} \\ y_{2N+1} \end{pmatrix}$$

with

$$y_{2N-1} = \Delta_N'\ A_{2N-1,2N+1} \tag{25}$$

$$y_{2N} = \Delta_N'\ A_{2N,2N+1} \tag{26}$$

## 2.2 Solution using Finite Differences

We aim to solve

$$u''(x) + \frac{2\mu}{\hbar^2}\left[E - V(x)\right] u(x) = 0$$

that will be written in the form

$$u''(x) + \left[q^2 - v(x)\right] u(x) = 0 \tag{27}$$

with the usual notations

$$q^2 = \frac{2\mu}{\hbar^2}E \qquad v(x) = \frac{2\mu}{\hbar^2}V$$

In case of a non local terms it becomms

$$u''(x) + \left[q^2 - v(x)\right] u(x) + \int dx' w(x,x') u(x') = 0 \qquad\qquad w = \frac{2\mu}{\hbar^2}W$$

We search the unknown fonction $u(x)$ on an equidistant grid G=$\{x_0, x_1, \ldots, x_N\}$ with spacing $h$, i.e. $x_i = i.h$. This means that we want to determine u=$\{u_0, u_1, ..u_N\}$ where $u_i \equiv u(i.h)$

## 2.3 Recurrence Algorithm

Using the **symmetric discretisation** of the second derivative

$$h^2 u''(x) = u(x-h) - 2u(x) + u(x+h) + h^2 o(h^2)$$

equation (27) results into the **recurrence relation**

$$\boxed{F_{i-1}u_{i-1} + D_i u_i + F_{i+1}u_{i+1} = 0 \qquad i = 1, \ldots N}$$

with[2]

$$\begin{aligned} F_i &= 1 & (28)\\ D_i &= -2 + h^2(q^2 - v_i) & (29) \end{aligned}$$

Since $u_0 = 0$ there are N unknowns $u_1, ....u_N$. By developing the recurrence relation one obtains:

$$\begin{array}{ll} i=1 & D_1u_1 + F_2u_2 \qquad\quad = 0 \\ i=2 & F_1u_1 + D_2u_2 + F_1u_3 = 0 \\ i=3 & F_2u_2 + D_3u_3 + F_4u_4 = 0 \\ \ldots & \ldots \\ i=N & F_{N-1}u_{N-1} + D_Nu_N = -F_{N+1}u_{N+1} \end{array}$$

This can be written in the matrix form

$$Au = b \qquad \Longleftrightarrow \qquad \begin{pmatrix} D_1 & F_2 & & 0 & 0 & 0 \\ F_1 & D_2 & F_3 & 0 & 0 & 0 \\ 0 & F_2 & D_3 & F_4 & 0 & 0 \\ 0 & 0 & F_3 & D_4 & F_5 & 0 \\ 0 & 0 & 0 & F_4 & D_5 & F_6 \\ 0 & 0 & 0 & 0 & F_5 & D_6 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \ldots \\ u_N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \ldots \\ -F_{N+1}u_{N+1} \end{pmatrix} \tag{30}$$

The term involving $u_{N+1}$, that we have placed in the right hand side, is mandatory to "close the system" and will incorporate the boundary conditions

[2]Althoug $F_i$ is here trivial, we prefer to keep this form, which extend to more elaborate discretisation schemes

In case of a non local interaction the matrix $A$ is changed into

$$A \to A = \begin{pmatrix} D_1 & F_2 & & 0 & 0 & 0 \\ F_1 & D_2 & F_3 & 0 & 0 & 0 \\ 0 & F_2 & D_3 & F_4 & 0 & 0 \\ 0 & 0 & F_3 & D_4 & F_5 & 0 \\ 0 & 0 & 0 & F_4 & D_5 & F_6 \\ 0 & 0 & 0 & 0 & F_{N-1} & D_N \end{pmatrix} + \begin{pmatrix} \bar{w}_{11} & \bar{w}_{12} & \bar{w}_{13} & \bar{w}_{14} & \bar{w}_{15} & \bar{w}_{1N} \\ \bar{w}_{21} & \bar{w}_{12} & \bar{w}_{13} & \bar{w}_{14} & \bar{w}_{15} & \bar{w}_{2N} \\ \bar{w}_{31} & \bar{w}_{12} & \bar{w}_{13} & \bar{w}_{14} & \bar{w}_{15} & \bar{w}_{3N} \\ \bar{w}_{41} & \bar{w}_{12} & \bar{w}_{13} & \bar{w}_{14} & \bar{w}_{15} & \bar{w}_{4N} \\ \bar{w}_{N-1,1} & \bar{w}_{12} & \bar{w}_{13} & \bar{w}_{14} & \bar{w}_{15} & \bar{w}_{N-1N} \\ \bar{w}_{N1} & \bar{w}_{12} & \bar{w}_{13} & \bar{w}_{14} & \bar{w}_{15} & \bar{w}_{NN} \end{pmatrix}$$

Where we used the discretisation

$$\int dx' w(x,x')u(x') \approx \sum_{i=1}^{N} h w_{ij} u_j \qquad \bar{w}_{ij} = h^3 w_{ij}$$

## 2.4 Scattering problem

1. **Zero energy** (compute the S-wave scattering length $a_0$)

   We search a solution which behaves asymptotically as

   $$u = r - a \tag{31}$$

   This means fulfilling the condition $u'_N = 1$

   Using the discretized derivative, this is equivalent to impose

   $$u_{N+1} = u_N + h$$

   This modifies slightly the las equation, by removing $u_{N+1}$ since expressed in terms of $u_N$

   $$D_N + \hat{w}_{NN} \to D_N + \hat{w}_{NN} + F_{N+1}$$

   and the inhomogeneous terms

   $$-F_{N+1}u_{N+1} \to -hF_{N+1}$$

   Once $u_i$ is determined, the scattering length is given by $a_0 = x_N - u_N$

2. **Non zero energy**

## 2.5 Dimer-Dimer Model (JK)

Two parameters $\alpha, \lambda$ (in $\text{fm}^{-2}$) and the reduced mass $\mu = 938.92$ MeV, which determine a numerical function $C_0(\lambda)$ (in MeV) given below,

- A local term

$$V(R) = C_0(\lambda)\, \frac{1}{(b\sqrt{\lambda})^3}\ e^{-\left(\frac{R}{b}\right)^2} \tag{32}$$

driven by a range parameter

$$\frac{1}{b^2} = \frac{2\alpha\lambda}{2\alpha+\lambda} \quad \Leftrightarrow \quad b^2 = \frac{1}{\lambda} + \frac{1}{2\alpha} \tag{33}$$

and having MeV dimension given by $C_0$ (notice that $b\sqrt{\lambda}$ dimensionless )

TEST VALUES V(R)

| i | lambda | alpha | C0 | V(2) | V(10) | V(20) |
|---|---|---|---|---|---|---|
| 010 | 1.48320000 | 0.00119019 | -1.828446 | -0.116166D-03 | -0.924686D-04 | -0.453269D-04 |
| 100 | 4.49440000 | 0.00130720 | -357.319483 | -0.495666D-02 | -0.385702D-02 | -0.176126D-02 |
| 198 | 6.30870000 | 0.00136145 | -1382.322273 | -0.122529D-01 | -0.943546D-02 | -0.417017D-02 |

- A non local term[3]

$$W(R,R') = 32\pi\alpha^{3/2} RR'\, e^{-\left(\frac{R'}{b_1}\right)^2} \left\{ \left[\frac{\hbar^2}{2\mu}(4\alpha^2R^2 - 2\alpha) + E\right] e^{-\left(\frac{R}{b_1}\right)^2} - \frac{2C_0(\lambda)}{(b\sqrt{\lambda})^3} e^{-\left(\frac{R}{b_2}\right)^2} \right\} \tag{34}$$

with the two range parameters [4]

$$\frac{1}{b_1^2} = \alpha \quad \Leftrightarrow \quad b_1^2 = \frac{1}{\alpha} \qquad\qquad \frac{1}{b_2^2} = \frac{\alpha(2\alpha+3\lambda)}{2\alpha+\lambda} \quad \Leftrightarrow \quad b_2^2 = \frac{1}{\alpha}\frac{2\alpha+\lambda}{2\alpha+3\lambda} = b_1^2\frac{2\alpha+\lambda}{2\alpha+3\lambda}$$

TEST VALUES W(R,R')

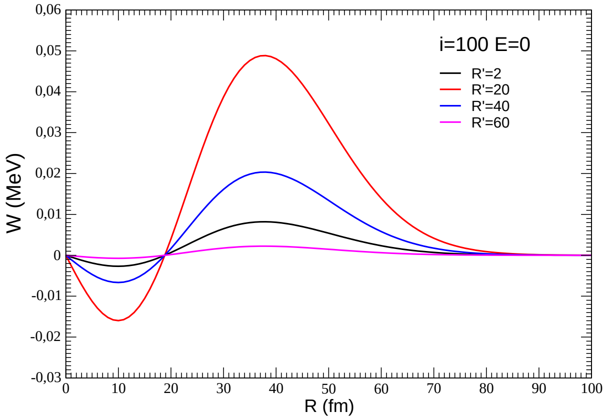
| i | lambda | alpha | C0 | W(2,30) | W(20,30) | W(40,20) |
|---|---|---|---|---|---|---|
| 010 | 1.48320000 | 0.00119019 | -1.828446 | -0.410905D-02 | -0.119713D-02 | 0.423542D-01 |
| 100 | 4.49440000 | 0.00130720 | -357.319483 | -0.382420D-02 | 0.312843D-02 | 0.480615D-01 |
| 198 | 6.30870000 | 0.00136145 | -1382.322273 | -0.277361D-02 | 0.690361D-02 | 0.503769D-01 |

  - Notice that $V(R,R')$ is not symmetric
  - The ranges $b, b_1, b_2$ are unusually large for a Nuclear Physics problem
  - The variation on R' is trivial $\sim x\exp(-x^2)$
  - The variation on R can have a structure (zero) depending on $\alpha$ and $E$. For $E = 0$ it is at $R \approx \sqrt{\frac{1}{2\alpha}}$

[3] A factor $4\pi RR'$ is introduced here in $W$ to be compatible with equation (8) and (11)

[4] We used

$$\frac{32\sqrt{2}\alpha^3}{(2\alpha+\lambda)^{3/2}} = \frac{16\alpha^{3/2}}{b^3\lambda^{3/2}}$$



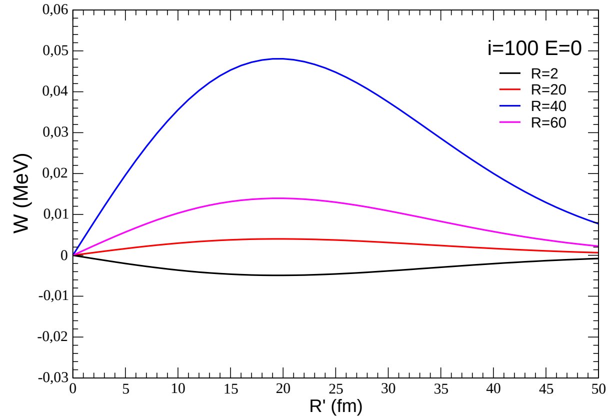




TABLE FOR C_0 (corresponding to mu=938 MeV)

| i | lambda [fm^-1] | alpha [fm^-2] | C0[MeV] | b | b1 | b2 |
|---|---|---|---|---|---|---|
| 1 | 0.63250000 | 0.00077171 | 0.83382111 | 25.485149 | 35.997549 | 20.800079 |
| 2 | 0.77460000 | 0.00105652 | 0.90776882 | 21.784003 | 30.765299 | 17.778483 |
| 3 | 0.89440000 | 0.00110491 | 0.91369538 | 21.298911 | 30.084067 | 17.383331 |
| 4 | 1.00000000 | 0.00109934 | 0.83396922 | 21.349903 | 30.160184 | 17.425738 |
| 5 | 1.09540000 | 0.00122357 | 1.37935968 | 20.237421 | 28.588120 | 16.517636 |
| 6 | 1.18320000 | 0.00116872 | 0.37470813 | 20.704194 | 29.251272 | 16.899340 |
| 7 | 1.26490000 | 0.00115710 | 0.00273375 | 20.806378 | 29.397781 | 16.983158 |
| 8 | 1.34160000 | 0.00120176 | -0.50265132 | 20.415725 | 28.846367 | 16.664395 |
| 9 | 1.41420000 | 0.00125485 | -1.12940812 | 19.979017 | 28.229559 | 16.307976 |
| 10 | 1.48320000 | 0.00119019 | -1.82844597 | 20.512806 | 28.986238 | 16.744158 |
| 11 | 1.54920000 | 0.00126792 | -2.72133438 | 19.874409 | 28.083684 | 16.222962 |
| 12 | 1.61250000 | 0.00121848 | -3.68073733 | 20.272333 | 28.647768 | 16.548123 |
| 13 | 1.67330000 | 0.00126717 | -4.83339523 | 19.879076 | 28.091993 | 16.227102 |
| 14 | 1.73210000 | 0.00130049 | -6.12301834 | 19.622635 | 27.729784 | 16.017806 |
| 15 | 1.78890000 | 0.00123366 | -7.48372394 | 20.145894 | 28.470970 | 16.445274 |
| 16 | 1.84390000 | 0.00127897 | -9.08293304 | 19.785902 | 27.962102 | 16.151387 |
| 17 | 1.89740000 | 0.00127640 | -10.79866171 | 19.805397 | 27.990239 | 16.167414 |
| 18 | 1.94940000 | 0.00132586 | -12.72678747 | 19.432620 | 27.463202 | 15.863071 |
| 19 | 2.00000000 | 0.00123879 | -14.67527162 | 20.102728 | 28.411957 | 16.410421 |
| 20 | 2.04940000 | 0.00122735 | -16.87450660 | 20.195784 | 28.544063 | 16.486498 |
| 21 | 2.09760000 | 0.00131155 | -14.92686728 | 19.537273 | 27.612617 | 15.948793 |
| 22 | 2.14480000 | 0.00124551 | -21.84398418 | 20.047649 | 28.335207 | 16.365669 |
| 23 | 2.19090000 | 0.00122351 | -24.55846572 | 20.226635 | 28.588821 | 16.511905 |
| 24 | 2.23610000 | 0.00134849 | -27.69202731 | 19.267390 | 27.231787 | 15.728596 |
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