A nucleon contact theory for P-wave dominated nuclei

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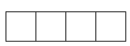
We analyse EFT($\not\pi$) at leading order in few-nucleon systems which are characterized by an amplitude pole whose imaginary part is small relative

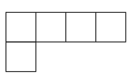
I. WHAT IS A P-WAVE STATE?

First, the imaginary part of the pole location which characterises the state must be small compared with the energy difference to the lowest threshold at which the creation of particles which are not element of the theory becomes energetically possible. Second, the state cannot reside in a totally symmetric spatial configuration.

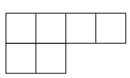
Ecce I would like to include an explicit calculation which relates an >2-body wave function, in some relative-coordinate basis, to a single-particle basis which can be identified with a young tableaux. I think the technical term for that is an inverse *Talmi transformation*.



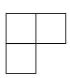
^{4}He — no P-wave state — $\exists \lim_{\Lambda\to\infty} \mathrm{EFT}(\not\pi)$



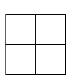
^{5}He — P-wave state — $\nexists \lim_{\Lambda\to\infty} \mathrm{EFT}(\not\pi)$



^{6}Li — P-wave state — $\nexists \vee \exists \lim_{\Lambda\to\infty} \mathrm{EFT}(\not\pi)$



^3n — P-wave tri-neutron state — $\nexists \lim_{\Lambda\to\infty} \mathrm{EFT}(\not\pi)^{[\text{we show}]}$



^4n — P-wave tetra-neutron state — $\nexists \lim_{\Lambda\to\infty} \mathrm{EFT}(\not\pi)^{[\text{ref needed}]}$

For ^{5}He, EFT($\not\pi$) cannot sustain anything but a free ^{4}He-neutron continuum state because any contribution from the fifth particle vanishes in the zero-range limit. From the tableaux, that particle is in an odd partial wave, and matrix elements of those are zero for contact interaction which do not depend on the momentum. ^{6}Li, on the other hand, might be bound because of the additional non-zero contribution from the symmetric pair.

In order for an unpaired nucleon to yield a pole structure which differs from that which is given by the "paired" part, one has to augment the interaction such that its zero-range limit is non-zero for the matrix element

$$\left\langle \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \middle| \hat{H}_{\mathrm{LO}} \middle| \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \right\rangle .$$

The additional term, $\hat{H}_{\mathrm{LO}} = \hat{H}^{\not\pi}_{\mathrm{LO}} + \delta\hat{H}$, must not obstruct, through its iteration, S-wave observables which EFT($\not\pi$) is devised to describe at leading order.

The following structure satisfies these constraints if its coupling constants are renormalized properly (and I have, as of now, only a faint idea what that means and how to achieve it) with a set of operators of identical, minimal mass dimension:

II. ^2n-n AND ^{5}He WITH EFT($\not\pi$) AND EFT($\not\pi$)*

First, one observes that the EFT($\not\pi$) induces an effective attraction between the neutron projectile and the dineutron and ^{4}He target, respectively, which is non-zero even if $\Lambda \to \infty$. The pertinent phase shifts do clearly rule out the possibility of a resonance at low energy.

If we invoke EFT($\not\pi$)* and calibrate its LEC's to ???, a steep rise in the phase shifts below an energy at which the α's structure is resolved is stable in the limit $\Lambda \to \infty$.

III. ON THE EFFECT OF AN ENHANCED 2-BODY INTERACTION IN AN A-BODY SYSTEM

Above, we showed results of the dependence of the dineutron-neutron and 4-helium-neutron spectra on the zero-range limit of a momentum-independent 2-body interaction. It remains to be shown that the found absence of any kind of pole of the amplitude is robust *w.r.t.* to the renormalization of the three interaction parameters. But, assuming that, *e.g.* , a replacement of the deuteron binding energy with the triplet-S-wave *np* scattering length as input to one of the LECs does not produce a pole in the P-wave structures considered above, the interaction *must* be augmented in order to represent a useful theory for the description of systems which fall into the ^{5}He-class.

Definition We assign all systems which exhibit correlated behaviour which can be observed at scales which are too small to resolve substructure of its constituting elements **and** can only be adequately described by a spatial configuration which is asymmetric under the exchange of all particles which occupy *different* single-particle states ("shell shuffling") to the ^{5}He-class.

As we are interested primarily in the description of ^{5}He itself, in- and outgoing states contain P-waves. Thus, we assume that the same interaction which is constrained in the 2-nucleon sector by P-wave data, namely EFT($\not\pi$) at next^2-to-leading order – due to the large S-wave scales one set of momentum dependent operators is promoted over the other, $\boldsymbol{q}^2$ is next-to-leading order while $\boldsymbol{q}\cdot\boldsymbol{q}'$ is next^2-to-leading order if understood as part of an amplitude – is minimally needed.

Is 2-body P-wave data correlated with ^{5}He? We want to test this hypothesis by including the N^{2}LO operator structure in the LO EFT($\not\pi$) potential. Thereby, the two 2-body and the one 3-body contact interactions are iterated along a tensor, a spin-orbit, and a $\boldsymbol{r}^2$ operator.

In a first empirical study, we were tempted to assess the dependence of the α-neutron system on the iteration of solely the spin-orbit term. The pertinent phase shifts were found remarkably robust, and a significant change was only observed at coupling strengths were the 2-body sector developed a bound state in some P-wave channel (for EFT($\not\pi$)-LO with $c_{LS}<0$, in the $^2P_0$ channel). To understand this, consider the triton, a $J^\pi=\frac{1}{2}^+$ state, which has small, but non-zero overlap with a state in which both relative motions reside in P orbitals. Hence, the spin-orbit force affects the triton. This effect, however, was found also of significance for much stronger spin-orbit strengths, than those which bind the 2-body system. It is counter intuitive that a system with more pairs in an attractive configuration is more robust.

To be concrete, consider the spin-orbit force in the 2-body system:

$$c_{ls}\langle{}^3P_0|\boldsymbol{L}\cdot\boldsymbol{S}|{}^3P_0\rangle=-2\;c_{ls}\quad.$$

Once this attraction becomes large enough to form a pocket in the repulsive angular momentum well, the system develops, first a virtual or resonant, and from it eventually a bound-state pole. For another pole to emerge in the 3-nucleon amplitude, the pertinent matrix element is

$$\left\langle\left[0^-\otimes\frac{1}{2}^+\right]^{\frac{1}{2}^+}\middle|\boldsymbol{L}\cdot\boldsymbol{S}\middle|\left[0^-\otimes\frac{1}{2}^+\right]^{\frac{1}{2}^+}\right\rangle=-1\;c_{ls}\quad.$$

In words, when increasing the spin-orbit force, the formation of a bound P-wave dimer does not stabilize the triton because the interaction is weaker between the dimer and the third nucleon and, from this rough estimate, must be twice as large to form a bound dimer between the 0^- neutron-proton and a neutron. At this point, it is also understandable how increasing the strength of a 2-body interaction does not necessarily stabilize any larger system. The interaction between a specific $(SL)J$-dimer and the other parts of the system, here the third neutron might become repulsive. Here, this precludes the formation of a $J^\pi=\frac{3}{2}^+$ nnp state via this mechanism.

So how can the combination of spin-orbit, tensor, and $\boldsymbol{r}^2$ interaction yield an effect at some strength in systems of different size? If we would have singled out the tensor operator for the above exercise – this is complicated because of the constraint of the invariant deuteron state but assuming that this is then taken into consideration with $\boldsymbol{r}^2$, we can consider

$$c_T\langle{}^3P_0|Y_2|{}^3P_0\rangle=2\;c_T\quad,$$

while it is zero for this specific dimer-neutron matrix element. The tensor can therefore be used to balance the effect of the spin-orbit term in the 2-body sector, while it is ineffectual in the 3-body case.

IV. SEARCH FOR THE MINIMAL NUCLEON-NUCLEON THEORY

We demand that the theory shall be constrained with $A\leq 4$-body data, and thereby is constructive in its prediction of a pole in the 5-body system which is within the convergence radius of the newly construed EFT.

a. Operator structure We postulate

$$\begin{aligned}\hat{V}=&\left[C_1^\Lambda\;\hat{P}({}^1S_0)+C_2^\Lambda\;\hat{P}({}^3S_1)\right]\;e^{-\frac{\Lambda^2}{4}r_{ij}^2}\\&+D_0^\Lambda\;\hat{P}(S={}^1\!/_2)\;e^{-\frac{\Lambda^2}{4}\left(r_{ij}^2+r_{ik}^2\right)}\\&+\left[C_3^\Lambda\;\hat{P}({}^1S_0)+C_4^\Lambda\;\hat{P}({}^3S_1)\right]\;\boldsymbol{r}^2\;e^{-\frac{\Lambda^2}{4}r_{ij}^2}\\&+-\frac{i}{2}C_5\;(\boldsymbol{r}\times\boldsymbol{\nabla})\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\;e^{-\frac{\Lambda^2}{4}r_{ij}^2}\\&+C_6\;\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{r}\;\boldsymbol{\sigma}_2\cdot\boldsymbol{r}-\frac{1}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\;\boldsymbol{r}^2\right]\;e^{-\frac{\Lambda^2}{4}r_{ij}^2}\end{aligned}$$

as the minimal theory which predicts a P-wave state in the 5-body system, yields RG-invariant results for $A\leq 4$ observables which are represented by amplitude poles whose location is close enough to the respective threshold, that only particles which are either explicit DoF or resemble renormalization conditions, c.f., the deuteron, the triton, or the virtual nn singlet, can be produced.

b. Renormalization In addition to the three EFT($\not\pi$)-LO LECs come 4 constants which correlate to 4 independent nn P-wave channels, and the phenomena which must be RG invariant are represented by the following list of observables:

structure-full channels:

$$\left\{{}^1S_0,{}^3S_1,\epsilon\right\}_{nn}\text{ and }\left\{J^\pi=\tfrac{1}{2}^+\right\}_{nnn}$$

structure-empty channels:

$$\left\{{}^{1(3)}P_{1(0,1,2)}\right\}_{nn}\text{ and }\left\{J^\pi=\tfrac{1(3)}{2}^{-(\pm)}\right\}_{nnn}$$

Comments are in order. The interaction enables transition between S- and D-waves, and thus the deuteron is already at LO a superposition of to partial waves. The number of constraints exceeds the number of LEC's and hence the absence of a pole must be a consequence of the location of the included poles, and the structure of the interaction. A trivial example is the vanishing nn P-wave phase shift at $\mathrm{EFT}(\not\pi)$-LO. Before considering, *e.g.* , why the augmented theory bears the chance to be "well behaved" in channels where there must be no shallow poles, we discuss the mechanism which protects the nnn $^3\!/_2{}^+$ channel from sustaining a shallow pole in the zero-range limit of $\mathrm{EFT}(\not\pi)$.

In the nnp system, total spin $^3\!/_2{}^+ \;\widehat{=}\; \ydiagram{3}_s$ which cannot be occupied in a totally symmetric spatial state $\ydiagram{3}_r$ because the corresponding iso-spin state would have to be anti-symmetric in all three pairs, which is impossible for $T=\frac{1}{2}$. So, our argument above applies here, too. In an arbitrary single-particle basis, the state can be expressed in terms of vectors with the permutation-group structure

$$\Big|\,(nnp)^3\!/_2{}^+\,\Big\rangle = \ydiagram{3}_s \otimes \ydiagram{2,1}_r \otimes \ydiagram{2,1}_I \; + \; \ydiagram{3}_s \otimes \ydiagram{1,1,1}_r \otimes \ydiagram{3}_I \quad . \tag{1}$$

In the zero-range limit, matrix elements in this basis vanish for the interacting pair residing in an asymmetric configuration. What remains is an exchange interaction. Note that the positive parity does not preclude $\ydiagram{1,1,1}_r$ or $\ydiagram{2,1}_r$, because the single-particle shells which relate to the rows of the tableaux might be of either parity.

To illustrate the potential failure of the augmented theory, we assume its LEC's to be calibrated to all the structure-full observables and three nn P-waves. The latter we realize by demanding that a phase shift at some $E_{\mathrm{cm}} < 1$ MeV reproduces data. Thereby, a weak interaction is enforced. Whereas the momentum-independent contact terms vanish, matrix elements for an interacting asymmetric pair with the additional terms contribute

V. CONCLUSION

Now we conjecture that this interaction produces non-trivial, *i.e.* , not just projectile-target continuum amplitudes for:

Ecce list of observables we deem amenable to this theory, *e.g.* , ^{6}Li and ^{16}O