

Multifermionic universal systems in contact effective field theory

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Abstract

We present a study of the behavior of renormalizable momentumless contact effective field theories in isomassive many-fermionic systems with more particles than fermionic flavors. The theory is structured to include two- and three-body spin-independent contact operators and relative low energy constant fitted on two- and three-body shallow bound states. The fitting inputs are varied to generalize our study to the largest possible ensemble of universal systems. We conclude that in the contact limit none of the studied P-wave systems are stable and that the theory predicts their decay in products with spatially symmetric wave functions. This result has several implications, not only in atomic physics but also in nuclear structure, where it proves that a momentumless contact interaction can hardly effectively describe P-wave stable systems (e.g. ${}^6\text{He}$).

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1. Introduction

Many nonrelativistic quantum systems are observed to follow similar behavior regardless of the energy scale of the problem or the detail of the inter-particle interaction. Systems with a scattering length much larger than the typical size of the particles involved fall into this category and are called to be universal. This leads to the situation in which all the observed few-body features are fully determined only by the energy of the two- and three-particles systems. During the years many studies have been performed in those systems, however, how universality is translated in the many-body sector is a problem that has still to be completely understood. Nonetheless, it is of fundamental importance since many physical universal systems, from nuclei to atomic condensate, show non-trivial many-body features. Since in universal systems the two-body scattering length is much larger than the effective range ($a_0 \gg r_0$) and any other low energy scattering parameter, contact effective field theory (EFT) is the most natural theoretical framework to study their properties. The renormalizable nature of the theory allows also to study the behavior of such systems independently from the detail of the interaction, and the powercounting expansion permits to quantitatively study real systems that are close but not precisely, to the unitary limit.

In this work, we study universal systems many-body behavior in the simplest non-trivial class of contact EFTs in which only momentum independent central two- and three-body contact interactions are taken into account in the theory leading order (LO). This theory and truncation were proven to be properly renormalized and to give good results both in bosonic systems up to 60 particles [1] and four nucleons [2] predicting the presence of spatially symmetric boundstates which became more bound with the increase of considered particles. However, the picture changes when more fermions of the same species are present in the same system. O. I. Kartavtsev et al. [3] showed that in the case of A^2B fermionic system (two of one flavor and the third of a second kind) contact EFT predicts a stable bound state only if the mass ratio between the two kinds of fermions is larger than $m_B/m_A \gtrsim 13$. However, this is not the case when the particles show a (quasi-)symmetry in the fermionic flavors as in the nuclear case or for the same fermionic atom specie. D. S. Petrov et al. [4, 5, 6] extended this result showing that the scattering length of two dibaryons composed of two isomassive fermionic species ($AB + AB$) has the same sign as the $A + AB$ one. These two general statements show that, in the unitary limit and with a contact interaction,

three and four two-flavor fermions are not stable against breaking in spatially symmetric pairs. In nuclear physics, the situation is more complex since the number of fermionic flavors is four and more particles can be arranged in a symmetric spatial wave function. However, ^{16}O nucleus appears to be unstable with respect four- α breaking within contact EFT framework [7]. Recently, Gattobigio et al. [8] showed that also ^6He is not stabilized by a momentumless short-range interaction with respect deuterium and α decay, but a stable bound state can be found if P-wave interactions are included in the potential. It is still not clear if the impossibility to stabilize P-wave¹ states is a general feature of contact EFTs, however, there is no doubt that this is a fundamental requirement for a many-body theory, since many systems, as nuclear ones, are close to universality and present many-body bound states.

In this paper, we explore the possibility to bind P-wave systems with a contact EFT studying systems with A fermionic flavors and $A + 1$ and $A + 2$ particles. This study is performed varying the input of the theory in the proximity of the unitary limit, and specifically to the nuclear case, to ensure the most general possible conclusions. For simplicity all the calculations have been made with fermions of mass 938.858 MeV, corresponding to the averaged nucleonic mass. However, the results remain general since the energy scales of the systems can be rescaled in the concept of system universality.

The paper is so structured: first, the theory and the fitting procedures used in this work are described. Follows, in the result section, the calculation outcomes for various systems with a particular focus on the nuclear cases experimentally known to be bound. In the conclusive section, an analysis of the work done and the consequence to renormalizable EFTs is performed. Finally, in the appendices, the numerical and analytically methods used in this work are briefly described.

¹Systems with more fermions than fermionic flavors are referred as P-wave systems as, in a shell model framework, they would require at least a particle in $L > 0$ single-particle state to ensure correct total antisymmetric wavefunction. It can also be noticed that these states are naturally found as fermionic ground states, but many-bosons excitation share the same spacial proprieties.

2. Theoretical framework

The goal of this work is to study the behavior of many-body systems of non-relativistic fermions with unnatural large scattering length and negligible effective range parameter. The most natural theoretical framework to describe those systems is contact EFT. For a deeper insight about contact EFT, and its renormalization we suggest the reading of [9, 10, 11, 12, 13, 14]. The most general Lagrangian that satisfies the wanted symmetries (Particle number conservation, spin and isospin symmetry, parity, \dots) can be written as

$$L = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - \frac{c_0^{(0)}}{2} (\psi^\dagger \psi)^2 - \frac{d_0^{(0)}}{6} (\psi^\dagger \psi)^3 + \dots \quad (1)$$

where C and D are low energy constants (LEC) and ellipsis represent terms with more derivatives. The Lagrangian terms can be sorted with respect to powers of the expansion parameter Q/M , with Q the exchanged momentum between particles and M the smallest mass scale present in the system. This arranges the interaction in operators gradually less important increasing the theory order. For simplicity, no spin or isospin symmetry breaking is included in our description. This is correct in atomic systems (e.g. ^3He [] or ^{40}K atoms [to Johannes: there is a paper about potassium? It looks to be quite unstable (but bananas are good nevertheless)]) which do not break spin symmetry and we do not expect to change qualitatively results in nuclei since nuclear interaction is close to being SU4 symmetric. Notice that the goal of this work is not to study the consequences of long-range forces, like meson exchanges, in the theory. Therefore, the smallest breaking scale present is the particle mass and, in the case of finite two-body binding energy, the scattering length of the system. Therefore, we expect subleading orders of the theory to be negligible.

In the presence of two-body shallow poles, two-body contact interaction associated with the LEC $c_0^{(0)}$ needs to be promoted at LO and treated non-perturbatively. A three-body contact term, associated with $d_0^{(0)}$, is also needed to be promoted at LO to avoid Thomas collapse in the 3+ body

Name	B_2 [MeV]	B_3 [MeV]
A3	1	3
A4	1	4
Nuc	2.22	8.48
Uni	0	1

Table 1: Different three to two-body ratios used in this work.

systems [11]. To regularize the interaction we choose a Gaussian regulator $\delta_\Lambda(x) = \left(\frac{\Lambda}{2\sqrt{\pi}}\right) e^{-\frac{x^2\Lambda^2}{4}}$ introducing an ultraviolet cut-off Λ . The resulting LO Hamiltonian reads:

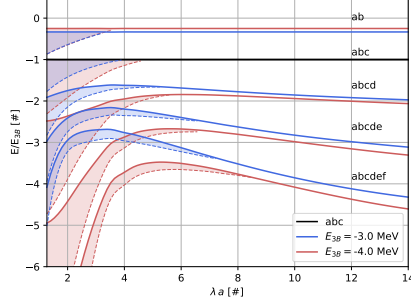
$$H = - \sum_i \frac{\hbar^2}{2m} \nabla^2 + C_0^\Lambda \sum_{i < j} \delta_\Lambda(r_{ij}) + D_0^\Lambda \sum_{i < j \neq k} \sum_{cyc} \delta_\Lambda(r_{ij}) \delta_\Lambda(r_{ik}). \quad (2)$$

In the spirit of contact EFT, Λ has to be taken larger than the breaking scale of the theory. If the theory is properly renormalized, any observables calculated will depend only on inverse powers of the cut-off. On one hand, the cut-off defines the maximum momentum resolved by the interaction, but, in the other, it also determines the effective range of the two-body system. In fact, in the contact limit, the effective range is always $r_0 \leq 0$ [15, 16, 17], and approaches zero in case of momentumless interaction. In analogy to the square well interaction, that can be computed analytically (see specifically [17] for the derivation), the effective range dependence with a Gaussian regulator can be written as

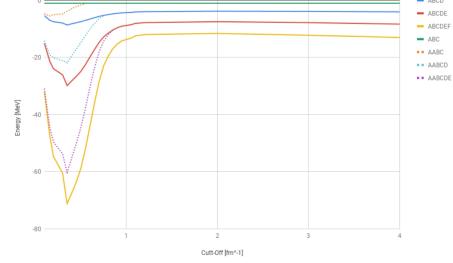
$$\frac{r_0}{a} = \frac{\alpha}{\Lambda a} + \frac{\beta}{(\Lambda a)^2} + \frac{\gamma}{(\Lambda a)^3} \xrightarrow{a \rightarrow \infty} \frac{\alpha}{\Lambda a}. \quad (3)$$

The contact EFT employed in this work does not include momentum dependent operators as P-wave interaction, tensor force, spin-orbit, or relativistic contributions; or coulomb forces at LO. The treatment of subleading operators in perturbation theory is, in this framework, a safe way of preserving renormalizability beyond LO (see [18, 19] for an alternative way of including subleading operators in the theory). However, new poles of the system T-matrix can not be created adding operators in perturbation theory. Therefore, we require the theory to give a good qualitative description of the quantum states of the systems of interest already at LO.

The theory has two degrees of freedom, the two- and the three-body LECs, that are fitted on few-body observables. The two-body LECs are fitted on small but finite two-body binding $B_2 = 1$ MeV equivalent to $a_0 \approx 6.7$ fm in the limit of large cut-off, to the nuclear binding $B_2 = 2.22$ MeV and to the unitary limit $B_2 \rightarrow 0$. The fit has been performed using a precise variational diagonalization method in case of finite binding and using the Numerov method in the unitary limit. The three-body LEC is fitted to reproduce one shallow three-body bound state with binding $B_3 = 3, 4, 8.48$,



(a) Colors online. Ground-state energy behaviour of A (solid line) and $A + 1$ (dashed line) A -flavours fermionic system with respect to the cut-off used. The blue line represent the theory fitted on the configuration A3 and the red represent the A4 one (see table 1 and text for the configuration specifics).



(b) Ground-state energy behaviour of A (solid line) and $A + 1$ (dashed line) A -flavours fermionic system with respect to the cut-off used in the unitary limit. The two-body LEC is fitted to zero energy threshold, the three body LEC is fitted to reproduce a unique $B_3 = 4$ MeV three body bound state. See text for the figure description.

Figure 1

1 MeV as on view in table 1. The three-body binding breaks the universality of the theory, therefore, we expect the theory behavior to be more dependent on the choice of the three-body binding than to the choice of the two-body scattering length or to the particle mass. All the three-body calculations have been performed using SVM.

3. Results

Defining an effective theory as in equation 2 with C_0 and D_0 fitted to reproduce the two- and three-body energies defined in table 1, the energy of various P-wave systems are calculated. Using SVM the ground state energies of up to $A + 1 = 7$ isomassive fermions with $A \leq 6$ flavors are calculated. The same is done up to $A + 1 \sim 60$ A -flavour fermionic systems using resonating group method (RGM). In the case of A *different* fermions the ground state is spatially symmetric. No spin dependence is included in our model, therefore, those systems behaves as if composed of bosons (it is boson-like) and are

equivalent to the case discussed in [1]. Convergence of the ground state energy of bosonic-like system with respect to the cut-off is observed for all the theory input used in this work. We observe an increasing of the ground state binding with the number of particles. The point radius $\langle r^2 \rangle$ has been calculated for $A \leq 6$, for a number of systems too small to extrapolate the behavior for large A , however, we expect it to behave as $A^{1/3}$ as predicted by the neutron drop model and observed in [1].

Adding an extra fermion with the same flavor as one in core to the system, we observe the groundstate wavefunction to be spatially asymmetric, and with angular momentum $L=1$. However, this state remains stable only for small cut-off after which the system breaks in the symmetric core and the extra fermion. This is a common behaviour of all the systems and input configurations considered. In figure 1a, are compared the ground-state energy of the $A \leq 6$ bosonic-like fermionic system and the $L=1$ $A+1$ bindings for $B_3 = 3$ and 4 MeV. For each number of particles a critical cut-off Λ_c^A , corresponding to the cut-off in which the $A+1$ state becomes unstable, exists. For larger cut-offs, the ground-state energy coincide with the one of the A -fermion systems and the system breaks in a A -particles symmetric subsystem and a free fermion. The result for A^2B fermions is in agreement with the one found by Kartavtsev et al. [3] in the particular case in which the fermions are isomassive.

Larger systems are not accessible with ab-initio few-body methods like SVM. For this purpose an extension of resonating group method (RGM) [20, 21] has been developed. From the fundamental two- and three-body interaction, a potential between the spatially symmetric core and the remaining fermion is extracted. In principle this permits to reduce a complex many-body calculation in a simpler two-body problem at the price of having non-local and energy dependent interaction. The derivation is done assuming that the wave function of the core is frozen to be the harmonic oscillator (HO) Hartree Fock (HF) ground state with respect to the system center of mass. The oscillator parameter is fixed to reproduce the HO ground state point radius $\sqrt{\langle r^2 \rangle} \propto A^{1/3}$. α is fitted to reproduce the $\Lambda_c^{3 \leq A \leq 6}$ from SVM calculations. In figure 2 is shown the behavior for $A \leq 60$ using RGM with local approximation for different configuration of the theory. We observe an increasing of Λ_c^A with A , remaining finite for all the systems considered. In one hand, this means that adding more particles in the spatially symmetric core of the system, the P-wave states becomes more stable and for any r_0 there is a minimal A after which it stabilize (see equation 3 for $\Lambda \Leftrightarrow r_0$ cor-

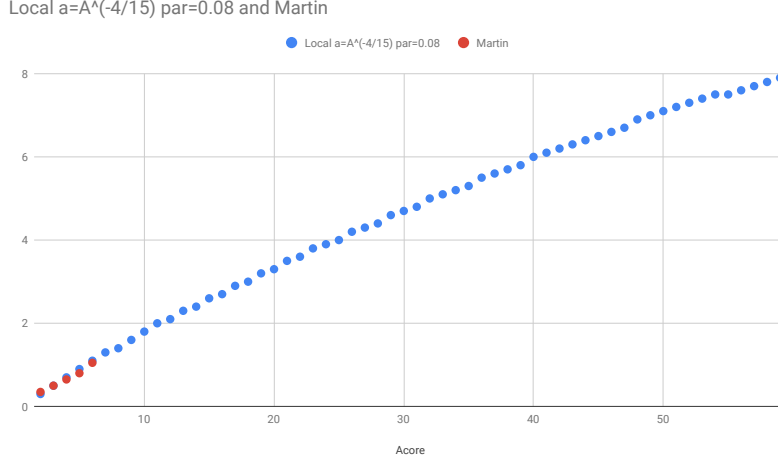


Figure 2: Critical cut-off behaviour with the number of particle in the spatially symmetric core when one extra fermion with the same flavour as one in the core is added. Red dots refer to SVM few-body calculation, the blue ones to the RGM result.

response). On the other one, there is always a Λ_c^A for which the P-wave state becomes unstable and in the contact limit all P-wave systems decay in the symmetric core and one particle. Comparing Λ_c^A with the scale of the system (the mass of the particles), it results that the instability always happens far from the expected renormalization invariant region. This remains true also when we consider nuclear mesonic scales. In facts, introducing a new breaking scale $M_{hi} \sim M_\pi \approx 140$ MeV we expect to reach renormalization invariance for $\Lambda \ll M_\pi a \sim 10 - 40$ which is larger or comparable with the critical cut-off we observe.

— This behavior can be seen for different configuration of two and three body bindings close to universality. Studying Λ_c^A behavior with the non-universal parameter $B_3/B_2 \dots$ need more data here to do the comparison.

The nuclear case is studied fitting the two-body energy to the deuterium binding $B_2 = 2.22$ MeV and B_3 to the $B(^3He) = 8.48$ MeV ground state. No spin dependence is included in the interaction, therefore, spin singlet dibaryon (e.g. the dineutron) results as bound as the deuterium. However, nuclear physics do not hardly break spin symmetry and we expect our result to be qualitatively accurate. We observe the same instability pattern shown in figure 1a for all the $A+1$ systems with $A \leq 4$. However, in the

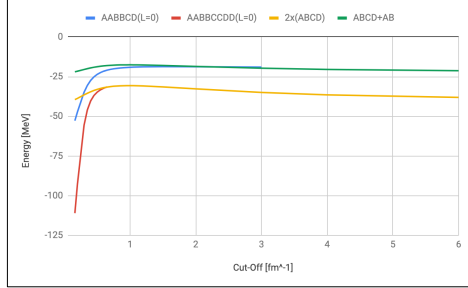


Figure 3: Energy of ${}^8\text{Be}$, ${}^6\text{Li}$ and the relative thresholds with respect the cut-off. The instability of P-wave system is observed also if the theory is fitted to reproduce nuclear two- and three-body bindings.

System	$(AB)^2$	$(AB)^2CD$
Total L	r_c [fm]	r_c [fm]
0	4.16	1.23
1	5.21	2.96
2	4.81	2.30

Table 2: Minimum effective range that stabilize the 2+2 (two dimers) and the 4+2 (analogous to ${}^6\text{Li}$ in nuclear physics) for different partial waves. Despite the simple interaction the level order well represent the one observed for ${}^6\text{Li}$ [22]. Nonetheless, for small effective range (large cut-off) all these states become unstable.

nuclear case, none of these systems are expected to be stable. More interesting is ${}^6\text{Li}$, which is expected to have a stable bound-state with $L=1$. Our calculations show (see figure 3) that this state stability is supported only for a limited range of cut-off and the system breaks in a deuterium and an alpha particle. The calculation for ${}^8\text{Be}$ leads to similar results and an instability with respect to alpha decay. Studying the spectrum of the A^2B^2CD system (${}^6\text{Li}$ if fitted on nuclear observable), more than one state appear to be stable for very small Λ . All those states becomes unstable for large Λ . Studying the order of the states and the order of disappearance (see table 2), it well represent the expected one for the physical ${}^6\text{Li}$ despite the oversimplified interaction used.

Inspecting the scale of the critical cut-off with respect to the other present in the theory it can be argued that Λ_c for ${}^6\text{Li}$ is of the same order as the pionmass breaking scale $\Lambda_c \sim 1.5 \text{ fm}^{-1} \approx 300 \text{ MeV}$. In EFT the cut-off of the theory should be pushed to be much larger than the expected breaking scale in order to be able to define the powercounting. Therefore, we conclude that the presence of a stable six-body bound state is a cut-off artifact. Nonetheless, a less puristic approach of the theory with a finite range interaction might be still able to bind the six-body system. This methodology might be successful to describe small and even medium-large nuclei, but it

mixes different orders of the powercounting making hard to have a consistent estimation of theoretical uncertainties.

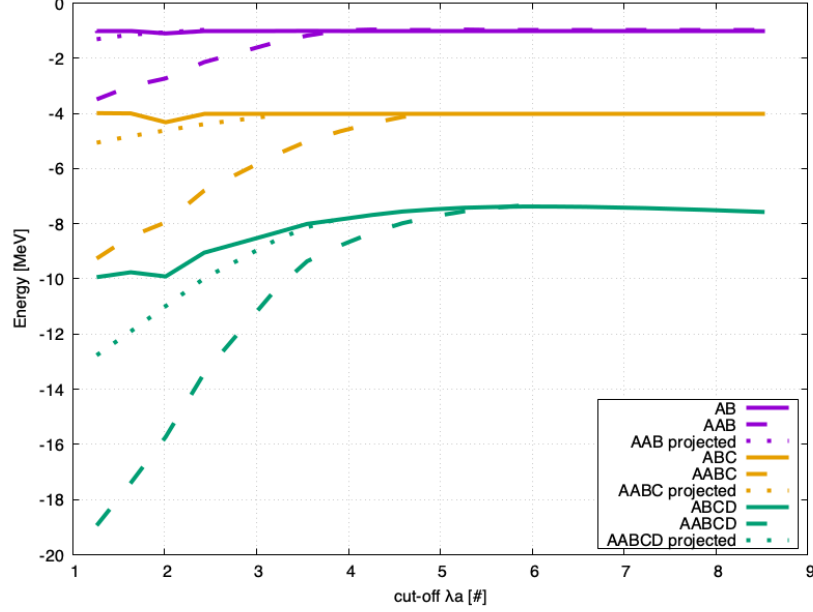


Figure 4: Energy of the A+1 fermionic systems with projected (dotted line) and not projected (dashed lines) compared with the energy of the bosonic like core with A spatially symmetric fermions.

Projecting the interaction discriminate between the contribution of different partial waves (and different orders of the theory) included into the finite cut-off interaction. In facts, the Gaussian regulator expanded in partial waves contains any-order partial waves. Larger angular momentum contributions disappear only when the cut-off is increased, but are still relevant in the region in which P-wave states are stable. In other words, finite range interaction does not only describe a finite and large S-wave scattering length but is also contaminated by finite larger-order ERE parameters in S-wave, as the S-wave effective range and the scattering volume for $L=1$. Both this contributions are expected to enter only subleading in the system description but contribute to the stabilization of P-wave systems at low cut-off. To study which partial waves mostly contribute to the system binding we project the four-flavor system in asymmetric spinorial state. This forces the spatial part to be symmetrical, (even total angular momentum). We observe a great reduction of Λ_c^A (shown in figure 4) which indicates that residual odd angular

momentums play a fundamental role in the system stability. The residual binding is due to S-wave effective range and large even angular momentum ($L=2$) contributions. This observation does not prove that a P-wave interaction will stabilize P-wave states above Λ_c^A , but suggests that it might help to extend contact EFT to multi-fermionic systems.

The pole movement with respect to Λ when the system becomes unstable is another important aspect of this study.

4. Discussion and conclusions

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