Derivation of an inter-cluster potential from a two-fermion contact interaction

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The potential acting between a point particle and a bound state of A particles is related to the interaction between single particles. Specifically, the A-particle state is totally symmetric in coordinate space ("bosonic"), and the single particles should be amenable to a first-order description with two- and three-body contact/zero-range interactions. Furthermore, the internal space of the $A+1$ equal-mass particles is given as A-dimensional, which demands an $A+1$ body state of mixed symmetry.

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The inter-cluster potential In order to study the large-N limit, we develop a model inspired by the resonating-group formalism ([1],[2]). That entails the assumption of a frozen A-body core whose spatially symmetric wave function we parametrize with a single parameter a via

$$\phi_A := e^{-\frac{a}{2}\sum_{i=1}^{A}(\boldsymbol{r}_i-\boldsymbol{R}_A)^2} \quad ; \begin{array}{l}\boldsymbol{r}_i \ : \text{single-particle coordinates}\\ \boldsymbol{R}_A \ : \text{core centre of mass}\end{array} . \tag{1}$$

The system is thereby reduced to only three degrees of freedom, namely the relative distance between core and the odd particle. The respective equation of motion reads in terms of the effective mass μ, the relative kinetic energy E between core and odd particle:

$$\int \left\{ \phi_A^* \left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}_R^2 - E + \mathcal{V} \right) \mathcal{A}\left[\phi_A \psi(\boldsymbol{R})\right] \right\} d\boldsymbol{r}_{1\ldots A} = 0 \ . \tag{2}$$

Antisymmetrization is required between two particles only, $\mathcal{A} = \mathbb{1} - P_{A,A+1}$, and the interaction is effective, only if it involves the odd particle:

$$\mathcal{V} = C_0(\Lambda) \sum_{i}^{A-1} \delta_\Lambda^{(3)}\big(\boldsymbol{r}_i - \boldsymbol{r}_{A+1}\big) \tag{3}$$

$$+ D_1(\Lambda) \sum_{i<j}^{A-1} \Big[\delta_\Lambda^{(3)}\big(\boldsymbol{r}_i - \boldsymbol{r}_j\big)\, \delta_\Lambda^{(3)}\big(\boldsymbol{r}_i - \boldsymbol{r}_{A+1}\big) \tag{4}$$

$$+ \delta_\Lambda^{(3)}\big(\boldsymbol{r}_i - \boldsymbol{r}_{A+1}\big)\, \delta_\Lambda^{(3)}\big(\boldsymbol{r}_j - \boldsymbol{r}_{A+1}\big) \Big] \ . \tag{5}$$

The contribution from the identical copy of the $(A+1)^{\text{th}}$ particle – without loss of generality identified with label A – interacting is excluded, because we anticipate its vanishing because of antisymmetrization. As in our calculations, the zero-range contact forces are smeared out in order to obtain regular solutions – in contrast to, *e.g.* , Bethe-Peierls [3] boundary conditions – this *a priori* exclusion introduces artefacts. We assume that those are insignificant relative to the other terms from the inter-fragment interaction.

The integration in (2) yields a three-dimensional Schrödinger equation with a non-local potential

$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}_R^2 - E\right)\psi(\boldsymbol{R}) + \sum_{n=1}^{3}\eta_n\, e^{-\kappa_n \boldsymbol{R}^2}\psi(\boldsymbol{R}) - \sum_{n=1}^{4}\zeta_n \int \left\{ \widehat{\mathfrak{O}}_n e^{-\alpha_n \boldsymbol{R}^2 - \beta_n \boldsymbol{R}\cdot\boldsymbol{R}' - \gamma_n \boldsymbol{R}'^2} \right\} \psi(\boldsymbol{R}')d\boldsymbol{R}' = 0 \ . \tag{6}$$

The coefficients $\alpha, \ldots, \kappa$ are functions of the number of core particles A, the core size a, the interaction regulator Λ, and the single-particle mass m, and

$$\widehat{\mathfrak{O}}_1 = -\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}_R^2 - E \tag{7}$$

$$\to -\frac{\hbar^2}{2\mu}\left(4\alpha_1^2\boldsymbol{R}^2 + \beta_1^2\boldsymbol{R}'^2 + 4\alpha_1\beta_1\boldsymbol{R}\cdot\boldsymbol{R}' - 2\alpha_1\right) - E \ ,$$

while $\widehat{\mathfrak{O}}_{2,3,4} = \mathbb{1}$. The sign (+) of the first sum – the direct interaction – is that of the microscopic LEC. In contrast, the second, non-local part reverts the sign. For example, the effect of an attractive 2-body potential becomes a repulsive contribution due to antisymmetrization in this term.

A few comments are in order. Firstly, $\zeta_{1\ldots4} = 0$ if $\mathcal{A} = \mathbb{1}$, *i.e.* , the inter-cluster potential is local. The first non-local term encodes the so-called exchange interaction. It is non-zero even in the absence of inter-particle forces. The two- and three-body contact forces affect the inter-cluster potential structurally in the same way. However, the respective coefficients differ significantly in their dependence on A, a and Λ. It is the combination of both, the two- and three-body terms, which results in the changing character of the interaction, the formation of attractive and repulsive regions, and thereby the possibility to bind the odd particle to the core. We postpone a detailed analysis of the sensitivity of the inter-cluster potential, for now, and continue with the discussion of the emerging spectrum of (6).

Partial-wave projection We solve Eq.(6) by expanding the total wave function[1]

$$\psi(\boldsymbol{R}) = R^{-1} \sum_{lm} \phi_{lm}(R) Y_{lm}(\hat{\boldsymbol{R}}) \tag{8}$$

and a projection with the integral operator

$$\int d^2\hat{\boldsymbol{R}}\; Y^*_{lm}(\hat{\boldsymbol{R}}) \tag{9}$$

from the left. The uncoupling of different partial waves becomes explicit when

$$e^{-\beta \boldsymbol{R}\cdot\boldsymbol{R}'} = 4\pi \sum_{LM} i^L j_L(i\beta R R') Y^*_{LM}(\hat{\boldsymbol{R}}) Y_{LM}(\hat{\boldsymbol{R}}') \tag{10}$$

is substituted. In the $n = 1$ part, which encodes the exchange effect on the free Hamiltonian, we apply the Laplacian before making the above and following substitutions:

$$\boldsymbol{R}\cdot\boldsymbol{R}' = -\sqrt{3}\left[\,\boldsymbol{R}_p \otimes \boldsymbol{R}'_{-p}\,\right]^{00} \tag{11}$$

with $\boldsymbol{r}_m = \sqrt{\frac{4\pi}{3}} r Y_{1,m}(\hat{\boldsymbol{r}})$. Using Eq.(4.6.3) and Eq.(3.7.8) of [4], we obtain the equation of motion for a single partial wave:

$$\left(\frac{\hbar^2}{2\mu}\left[-\partial_R^2 + \frac{l(l+1)}{R^2}\right] - E\right)\phi_{lm}(R) \tag{12a}$$

$$-(-E)\int \left(4\pi\, i^l \cdot \zeta_1 \cdot j_l(i\beta_1 R R')\right) \cdot e^{-\alpha_1 \boldsymbol{R}^2 - \gamma_1 \boldsymbol{R}'^2} \phi_{lm}(R')\; RR' dR' \tag{12b}$$

$$-\left(\frac{\hbar^2}{2\mu}\right)\int (4\pi\cdot\zeta_1)\cdot e^{-\alpha_n \boldsymbol{R}^2 - \gamma_n \boldsymbol{R}'^2} \cdot \left\{ \left[-(4\alpha_1^2 R^2 + \beta_1^2 R'^2 - 2\alpha_1) + \frac{l(l+1)}{R^2}\right] i^l j_l(i\beta_1 R R') \right.$$
$$\left. +(4\alpha_1\beta_1)\cdot RR' \cdot \left(i^{l-1} j_{l-1}(i\beta_1 RR')(2l-3) \begin{pmatrix} 1 & l-1 & l \\ 0 & 0 & 0 \end{pmatrix}^2 + i^{l+1} j_{l+1}(i\beta_1 RR')(2l-1) \begin{pmatrix} 1 & l+1 & l \\ 0 & 0 & 0 \end{pmatrix}^2 \right) \right\} \phi_{lm}(R')\; RR' dR' \tag{12c}$$

$$-\sum_{n=2}^{4} \zeta_n \int (4\pi\, i^l)\cdot j_l(i\beta_n RR')\cdot e^{-\alpha_n \boldsymbol{R}^2 - \gamma_n \boldsymbol{R}'^2}\; \phi_{lm}(R')\; RR'\, dR' \tag{12d}$$

$$+\sum_{n=1}^{3} \eta_n\; e^{-\kappa_n \boldsymbol{R}^2} \phi_{lm}(R) \tag{12e}$$

$$= 0 \quad , \text{with } [\eta_i] = \text{MeV} \quad [\zeta_i] = \text{fm}^{-3}$$

This equation defines a generalized Eigenvalue problem

$$\int dR'\; \hat{\mathcal{D}}_{RR'}\; \phi_{lm}(R') = E \int dR' \left(\delta_{RR'} + \hat{\mathcal{K}}_{RR'}\right) \phi_{lm}(R') \tag{13}$$

whose solution we expand in a finite set of harmonic oscillator functions[2].

[2] $\phi_{nl\nu}(R) = N_{nl\nu} R^{l+1} e^{-\nu R^2} L_n^{l+1/2}(2\nu R^2)$ in terms of a normalizing constant $N_{nl\nu} = \left(\left(\frac{2\nu^3}{\pi}\right)^{1/2} \cdot \frac{2^{2l+n+3} n! \nu^l}{(2n+2l+1)!!}\right)^{1/2}$ and gener-

n	η_n	κ_n		
1				
2				
3				
n	ζ_n	α_n	β_n	γ_n
1				
2				
3				
4				

(14)

TABLE I: Numerical values which specify the core-particle motion via Eq. (12) for $\Lambda = 4\,\mathrm{fm}^{-1}$, $A = 4$, $m_N = 938$ MeV, $a = 0.56$ fm, $\hbar c = 197$ MeV·fm.

Example: 5-body system In the following, we analyse features of the effective interaction for the specific case of four internal degrees of freedom and a system of five particles with identical masses. With the numerical values as given in table 14, this example pertains to nuclear physics as described in leading order with the pionless effective field theory.

APPENDIX:

The normalized radial wave function corresponding to the state (nl) in a *three-dimensional isotropic oscillator potential* has the form

$$\begin{aligned}
\phi_{nl}(r) &= C_{nl}\; e^{-\frac{\alpha^2}{2}r^2}\; r^l\; F\big[1-n, l+\frac{3}{2}; \alpha^2 r^2\big] \\
&= C_{nl}\; e^{-\frac{\alpha^2}{2}r^2}\; r^l\; \left(1 + \frac{(1-n)}{(l+\frac{3}{2})}\cdot\frac{\alpha^2 r^2}{1!} + \frac{(1-n)(2-n)}{(l+\frac{3}{2})(l+\frac{5}{2})}\cdot\frac{(\alpha^2 r^2)^2}{2!} + \frac{(1-n)(2-n)(3-n)}{(l+\frac{3}{2})(l+\frac{5}{2})(l+\frac{7}{2})}\cdot\frac{(\alpha^2 r^2)^3}{3!} + \dots\right) \\
&\overset{l=0}{=} C_n\; e^{-\frac{\alpha^2}{2}r^2}\; \left(1 + \frac{(1-n)}{(\frac{3}{2})}\cdot\frac{\alpha^2 r^2}{1!} + \frac{(1-n)(2-n)}{(\frac{3}{2})(\frac{5}{2})}\cdot\frac{(\alpha^2 r^2)^2}{2!} + \frac{(1-n)(2-n)(3-n)}{(\frac{3}{2})(\frac{5}{2})(\frac{7}{2})}\cdot\frac{(\alpha^2 r^2)^3}{3!} + \dots\right) \\
&= e^{-\frac{\alpha^2}{2}r^2}\sum_{i=1}^{n}(-)^{i+1}\; r^{2(i-1)}
\end{aligned}$$

with $n = 1, 2, 3, \dots$, $l \geq 0$, and normalization $C_{nl} = \frac{1}{\Gamma(l+3/2)}\sqrt{\frac{2\Gamma(1+n+1/2)}{\Gamma(n)}}\sqrt{(M\omega/\hbar)^{l+3/2}}$. The part of the function in brackets is > 0 at $r = 0$, and intersects the x axis $n-1$ times before diverging with $(-)^{n+1}\; r^{2(n-1)}$.

Systems with more particles than accessible internal states require a spatial state with mixed symmetry, *i.e.* , at least one particle has to reside in a state which is different from the one the others are in, *e.g.* , A core particles in a $n = l = 0$ state, and one particle in an $n \neq 0$ but still $l = 0$ state. Matrix elements of contact interaction between such configurations are zero, and we shall need to know whether the same is true for a one-parameter, spherically symmetric, finite-range representation of the contact, $\lim_{\Lambda\to\infty} g_\Lambda(r) = \delta(r)$.

APPENDIX: THE CORE WAVE FUNCTION

Our ansatz for an A-boson system in its ground state (symmetric in space *wrt.* particle exchange) is

$$\phi_A := e^{-\frac{a}{2}\sum_{i=1}^{A}\bar{\boldsymbol{r}}_{\mathrm{i}}^2} = e^{-a\sum^{A-1}\bar{\boldsymbol{r}}_{\mathrm{i}}^2 - a\sum_{i<j}^{A-1}\bar{\boldsymbol{r}}_{\mathrm{i}}\cdot\bar{\boldsymbol{r}}_{\mathrm{j}}} \quad . \tag{15}$$

Instead of single-particle coordinates, the usage of cluster coordinates identifies the centre of mass as the origin of a "harmonic", effective potential, in which the particles reside in their independent-particle/mean-field ground state. All observables of the system in this state depend thus on a single parameter, the (oscillator) width a. The root-mean-square radius, in particular, relates to a via

$$\begin{aligned}
\mathfrak{r}^2 =& \frac{\int_{\mathbb{R}^{3(A-1)}} d(\bar{\boldsymbol{r}}_1, \dots, \bar{\boldsymbol{r}}_{\text{A-1}})\; \sum_{i=1}^{A-1}\bar{\boldsymbol{r}}_{\mathrm{i}}^2\phi_A^2}{\int_{\mathbb{R}^{3(A-1)}} d(\bar{\boldsymbol{r}}_1, \dots, \bar{\boldsymbol{r}}_{\text{A-1}})\; \phi_A^2} && (16) \\
=& \frac{3}{2}\cdot\frac{(A-1)^2}{A}\cdot a^{-1} \overset{A\gg 1}{=} \frac{3}{2}\cdot A\cdot a^{-1} \quad . && (17)
\end{aligned}$$

We assume the correlation between the radius $\mathfrak{r}$ and the ground-state wave function of the 3-nucleon system, to hold for any spatially symmetric A-boson system. However, the functional relation, $\mathfrak{r} = f\,[A, B(3), B(2)]$, is unknown (to the best of our knowledge).

We employ the assumption of a radius which grows almost proportional to the volume of A spheres, *i.e.* , $\propto A^{\frac{1}{3}+\delta_V}$. With the parameter δ_V, deviations from the

liquid-drop approximation are allowed,

$$\delta_V \begin{cases} > 0 & \text{faster than voluminous growth} \\ < 0 & \text{less than volume growth} \end{cases}$$

By "SVM measuring" r_0 for some A, we implicitly relate the parameters of the theory, namely, the cutoff λ, the mass m_N, and the LECs/few-body renormalization conditions to a.

APPENDIX: INTER-CLUSTER POTENTIAL

Consider two compound systems, whose relative motion is much slower compared with the motion of the particles within each of these fragments. Within an arbitrarily small time interval, the probability of a given particle to interact/hit/overlap with another particle is then enhanced for the partner belonging to the same compound.

$$P_{dt}(\text{intra}) \gg P_{dt}(\text{inter})$$

or

$$\#(\text{internal collisions}) \gg \#(\text{inter-cluster collisions}) \ .$$

The relative motion of particles within each cluster is then decoupled from, *one*, the relative motion *wrt.* the other cluster(s), *and two*, the internal motion in those. In turn, the internal structure affects cluster-relative motion. This reasoning underlies the single-channel resonating-group approximation and also foreshadows the non-hermitian character of the effective interaction.

$$\widehat{M}_{\text{D}} = \begin{pmatrix} 4a & & & \\ & 4a & (2a)_{\triangledown} & \\ & (2a)_{\triangle} & \ddots & \\ & & & 4a \end{pmatrix} \ ; \ \boldsymbol{\mathcal{S}}_{\text{D}} = \mathbf{0} \ ; \ B_{\text{D}} = (R - R') \tag{18}$$

Properties of the effective $2{+}1^{(A-1)}$ *potential* Strength factors and exponents, *i.e.* , ranges, of the effective two-body potential between an A-boson bound state and a particle which is flavour- and mass-equal to one of the bosons are listed in table **??**.

- All terms in the effective potential retain a finite range even for zero-range two- and three-body interactions ($\Lambda \to \infty$).
- The range of terms originating from the two-body interaction is $\approx 2\times$ as large as the one of the terms proportional to D_1^{Λ}.
- For $A \to \infty$, the interaction becomes local, *i.e.* , $\beta \to 0$.
- For $A \gg 1$, the terms η_1 and ζ_2 grow linearly with A consistent with the number of pairwise interactions between the orbiter and a core constituent. The terms $\eta_{2,3}$ and $\zeta_{3,4}$) increase in magnitude quadratically with A which reflects the $(A-1)(A-2)$ possible ways triplets can be selected including the orbiter and two core particles.
- The strength of the exchange term (ζ_1) is independent of A and Λ. It depends linearly on the energy and is repulsive in even, and attractive in odd partial waves ($i^l j_l(ir) \gtrless 0 \ \forall r$ for $\substack{\text{even}\\\text{odd}} \ l$).

$$
\begin{aligned}
\mathfrak{v}(\boldsymbol{R},\boldsymbol{R}') = \;& C_0^{\Lambda}\cdot\frac{8A'(aA)^{3/2}}{\left(4aA+A'\Lambda^2\right)^{3/2}} && \cdot e^{-\frac{aA\Lambda^2}{4aA+A'\Lambda^2}\boldsymbol{R}^2} && \left(V_{dir}^{(2)}\right)\\
&+D_1^{\Lambda}\cdot\frac{32a^3A''A'A^{3/2}}{\left(16a^2A+4a(3A-1)\Lambda^2+A''\Lambda^4\right)^{3/2}} && \cdot e^{-\frac{2aA\Lambda^2\left(2a+\Lambda^2\right)}{16a^2A+4a(3A-1)\Lambda^2+A''\Lambda^4}\boldsymbol{R}^2} && \left(V_{dir}^{(3_a)}\right)\\
&+D_1^{\Lambda}\cdot\frac{32a^3A''A'A^{3/2}}{\left(16a^2A+8a(A-1)\Lambda^2+A''\Lambda^4\right)^{3/2}} && \cdot e^{-\frac{2aA\Lambda^2}{4aA+A''\Lambda^2}\boldsymbol{R}^2} && \left(V_{dir}^{(3_*)}\right)\\
\hline
&+\frac{2\sqrt{2}(aA^3)^{3/2}}{\left(A'(A+1)^2\right)^{3/2}}\cdot\widehat{\mathfrak{O}}_1 && \cdot e^{-\frac{a\left(A^3+A\right)}{2A'(A+1)^2}\boldsymbol{R}^2-\frac{2aA^2}{A'(A+1)^2}\boldsymbol{R}\cdot\boldsymbol{R}'-\frac{a\left(A^3+A\right)}{2A'(A+1)^2}\boldsymbol{R}'^2} && \left(V_{Ex}^{(\mathbb{1})}\right)\\
\hline
&+C_0^{\Lambda}\cdot\frac{8\pi^{-3/2}(A+1)^{-3}A'(aA)^{9/2}}{\left(4a^2A'+aA''\Lambda^2\right)^{3/2}} && \cdot e^{-\frac{aA\left(4a\left(A^2+1\right)+\left(3A^2+A+2\right)\Lambda^2\right)}{2(A+1)^2\left(4aA'+A''\Lambda^2\right)}\boldsymbol{R}^2} && \\
& && \cdot e^{-\frac{4aA^2\left(2a+\Lambda^2\right)}{(A+1)^2\left(4aA'+A''\Lambda^2\right)}\boldsymbol{R}\cdot\boldsymbol{R}'} && \\
& && \cdot e^{-\frac{aA\left(4a\left(A^2+1\right)+\left(A^2-A+2\right)\Lambda^2\right)}{2(A+1)^2\left(4aA'+A''\Lambda^2\right)}\boldsymbol{R}'^2} && \left(V_{Ex}^{(2)}\right)\\
&+D_1^{\Lambda}\cdot\frac{32\pi^{-\frac{3}{2}}(A+1)^{-3}A''A'(aA)^{9/2}}{\left(16a^2A'+4a(3A-4)\Lambda^2+A'''\Lambda^4\right)^{3/2}} && \cdot e^{-\frac{aA\left(16a^2\left(A^2+1\right)+4a\left(5A^2+A+4\right)\Lambda^2+\left(5A^2+2A+3\right)\Lambda^4\right)}{2(A+1)^2\left(16a^2A'+4a(3A-4)\Lambda^2+A'''\Lambda^4\right)}\boldsymbol{R}^2} && \\
& && \cdot e^{-\frac{2aA^2\left(16a^2+16a\Lambda^2+3\Lambda^4\right)}{(A+1)^2\left(16a^2A'+4a(3A-4)\Lambda^2+A'''\Lambda^4\right)}\boldsymbol{R}\cdot\boldsymbol{R}'} && \\
& && \cdot e^{-\frac{aA\left(16a^2\left(A^2+1\right)+4a\left(3A^2-A+4\right)\Lambda^2+\left(A^2-2A+3\right)\Lambda^4\right)}{2(A+1)^2\left(16a^2A'+4a(3A-4)\Lambda^2+A'''\Lambda^4\right)}\boldsymbol{R}'^2} && \left(V_{Ex}^{(3_a)}\right)\\
&+D_1^{\Lambda}\cdot\frac{32\pi^{-\frac{3}{2}}(A+1)^{-3}A''A'(aA)^{9/2}}{\left(16a^2A'+4a(2A-4)\Lambda^2+A'''\Lambda^4\right)^{3/2}} && \cdot e^{-\frac{aA\left(4a\left(A^2+1\right)+\left(5A^2+2A+3\right)\Lambda^2\right)}{2(A+1)^2\left(4aA'+A'''\Lambda^2\right)}\boldsymbol{R}^2} && \\
& && \cdot e^{-\frac{2aA^2\left(4a+3\Lambda^2\right)}{(A+1)^2\left(4aA'+A'''\Lambda^2\right)}\boldsymbol{R}\cdot\boldsymbol{R}'} && \\
& && \cdot e^{-\frac{aA\left(4a\left(A^2+1\right)+\left(A^2-2A+3\right)\Lambda^2\right)}{2(A+1)^2\left(4aA'+A'''\Lambda^2\right)}\boldsymbol{R}'^2} && \left(V_{Ex}^{(3_*)}\right)
\end{aligned}
$$

The strengths of the exchange-interaction terms for $A \gg 1$ and/or $\Lambda \gg 1\,\mathrm{fm}^{-1}$is then related to the corresponding direct-interaction strength via

$$\zeta_{n+1} = \eta_n \cdot \left(\frac{A}{A+1}\right)^3 \left(\frac{a}{\pi}\right)^{3/2} \quad . \tag{19}$$

The fraction $\left(\frac{a}{\pi}\right)^{3/2}$ is the inverse of an integration of the non-local terms assuming a constant wave function (exponents scale with a which produces an $\sqrt{\pi/a}$ upon integration, and the spherical Bessel functions provide the remaining a^{-1}).

In order to study the zero-range limit and the related renormalization-group invariance, the large Λ behaviour is of interest. In this limit, the potential scales as

$$\lim_{\Lambda\to\infty}\mathfrak{v}(\boldsymbol{R},\boldsymbol{R}') = \begin{aligned} &\mathcal{O}(A)\cdot\frac{C_0^\Lambda}{\Lambda^3}\cdot a^{\frac{3}{2}}\cdot e^{-a\boldsymbol{R}^2} && +2\cdot\mathcal{O}(A^2)\cdot\frac{D_1^\Lambda}{\Lambda^6}\cdot a^3\cdot e^{-2a\boldsymbol{R}^2}\\ &+\mathcal{O}(A)\cdot\frac{C_0^\Lambda}{\Lambda^3}\cdot a^3\cdot e^{-\frac{3}{2}a\boldsymbol{R}^2-\frac{4a}{\mathcal{O}(A)}\boldsymbol{R}\cdot\boldsymbol{R}'-\frac{3}{2}a\boldsymbol{R}'^2} && +2\cdot\mathcal{O}(A^2)\cdot\frac{D_1^\Lambda}{\Lambda^6}\cdot a^{\frac{9}{2}}\cdot e^{-\frac{5}{2}a\boldsymbol{R}^2-\frac{6a}{\mathcal{O}(A)}\boldsymbol{R}\cdot\boldsymbol{R}'-\frac{5}{2}a\boldsymbol{R}'^2}\\ &+\mathcal{O}(1)\cdot a^{\frac{3}{2}}\cdot e^{-\frac{a}{2}\boldsymbol{R}^2-2a\boldsymbol{R}\cdot\boldsymbol{R}'-\frac{a}{2}\boldsymbol{R}'^2} && \end{aligned},$$

and terms were organized their origin from pair or triplet interactions. The third-row term stems from the norm-exchange kernel. This skeleton of the potential is now fed into the partial-wave projected RGM equation (12) to yield

$$\int\left\{\frac{\hbar^2}{2\mu}\left[-\partial_R^2\left(\mathbb{1}+(\mathfrak{o}_E\leftrightarrow\mathfrak{o}_\mu)\right)+\frac{l(l+1)}{R^2}\left(\mathbb{1}+(\mathfrak{o}_E\leftrightarrow\mathfrak{o}_L)\right)\right]\phi_{lm}(R')\right. \tag{20a}$$

$$-E\left(\delta(R-R')+(-)^{l+1}\mathfrak{o}_E(1)\left(\frac{a}{\pi}\right)^{3/2}e^{+\frac{1}{2}\mathfrak{o}_E(aA^{-1})RR'-\frac{1}{2}\mathfrak{o}_E(a)\boldsymbol{R}^2-\frac{1}{2}\mathfrak{o}_E(a)\boldsymbol{R}'^2}\right) \tag{20b}$$

$$+\mathfrak{o}_2(A)\cdot\frac{C_0^\Lambda}{\Lambda^3}\cdot a^{\frac{3}{2}}\cdot e^{-\mathfrak{o}_2(a)\boldsymbol{R}^2}\left(\delta(R-R')+(-)^{l+1}\mathfrak{o}_2(1)\left(\frac{a}{\pi}\right)^{3/2}e^{+\mathfrak{o}_2(aA^{-1})RR'-\mathfrak{o}_2(a)\boldsymbol{R}'^2}\right) \tag{20c}$$

$$\left.+2\cdot\mathfrak{o}_3(A^2)\cdot\frac{D_1^\Lambda}{\Lambda^6}\cdot a^3\cdot e^{-2\mathfrak{o}_3(a)\boldsymbol{R}^2}\left(\delta(R-R')+(-)^{l+1}\mathfrak{o}_3(1)\left(\frac{a}{\pi}\right)^{3/2}e^{+2\mathfrak{o}_3(aA^{-1})RR'-2\mathfrak{o}_3(a)\boldsymbol{R}'^2}\right)\right\}\phi_{lm}(R')dR' \tag{20d}$$

$$=0\quad.$$

The first and second row of this equation demonstrate the effect of the antisymmetrization on the single-particle characteristics of the system which is solely the reduced mass. For even partial waves, the exchange of the particle amounts to an increased mass if the particle is close to the core, while for a large separation the original mass dictates the free propagation. The distance over which the mass is significantly enlarged is set by the core size a in the exponent of (20b). The mass/kinetic-energy correction due to exchange is thus characterized repulsive or attractive solely by the partial wave (only one single permutation comprises the antisymmetrization) with strength and range increasing with the size of the core.

The exchange has qualitatively the same effect on the terms associated with the two- and three-body contact interactions. Corrections to both (20c) and (20d) increase (odd) or decrease (even) the interaction strength without changing its attractive or repulsive character. Exchange effects do not turn an attractive interaction repulsive or vice versa. A potential destabilization is thus driven by a weakening of an otherwise sufficiently attractive direct interaction in combination with a genuine but interaction-independent repulsion as emergent from the kinetic part of the system. This insight in combination with the strengths of the interaction being much larger in magnitude for $\lim_{\Lambda\to\infty}$ compared with the kinetic-energy factors justify our qualitative conjectures in the discussion of fig. **??** which were based on the local approximation.

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