Multi-fermion systems with contact theories

M. Schäfer

Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering, Břehová 7, 11519 Prague 1, Czech Republic

L. Contessi*

Racah Institute of Physics, The Hebrew university, 91904 Jerusalem, Israel

ESNT, IRFU, CEA, Universite Paris Saclay, F-91191 Gif-sur-Yvette, France

J. Kirscher*

Theoretical Physics Division, School of Physics and Astronomy, The University of Manchester, Manchester, M13 9PL, United Kingdom

Abstract

We study ground-state properties of isomassive systems of fermions whose number of constituents exceeds the number of fermionic flavours momentum-independent contact effective field theories. The regularized two- and three-body spin-independent contact operators are renormalized to yield relatively shallowly bound dimer and trimer states, respectively. The effect on the many-fermion systems of scale set by the ratio between the dimer and trimer binding energies is analysed to generalize our study to the largest possible ensemble of universal systems. In the zero-range limit none of the studied P-wave systems are stable with respect to a decay in fragments with spatially symmetric wave functions. We elaborate on the consequences of our results for the systematic description of atomic and nuclear systems. For the latter, in particular, the study provides strong evidence for an inclusion of momentum-dependent interaction terms in order to describe P-wave stable systems (^{6}He, *e.g.*), effectively.

*lorenzo.contessi@cea.fr

Preprint submitted to Elsevier *February 8, 2020*

1. Introduction

Many nonrelativistic quantum systems are observed to follow similar behavior regardless of the energy scale of the problem or the detail of the interparticle interaction. Systems with a scattering length much larger than the typical size of the particles involved fall into this category and are called to be universal. This leads to the situation in which all the observed few-body features are fully determined only by the energy of the two- and three-particles systems. During the years many studies have been performed in those systems[], however, how universality is translated in the many-body sector is a problem that has still to be completely understood. Nonetheless, it is of fundamental importance since many physical universal systems, from nuclei to atomic condensate, show non-trivial many-body features. Since in universal systems the two-body scattering length is much larger than the effective range ($a_0 \gg r_0$) and any other low energy scattering parameter, contact effective field theory (EFT) is the most natural theoretical framework to study their proprieties. The renormalizable nature of the theory allows also to study the behavior of such systems independently from the detail of the interaction, and the powercounting expansion permits to quantitatively study real systems that are close but not precisely, to the unitary limit.

In this work, we study universal systems many-body behavior in the simplest non-trivial class of contact EFTs in which only momentum independent central two- and three-body contact interactions are taken into account in the theory leading order (LO). This theory and truncation were proven to be properly renormalized and to give good results both in bosonic systems up to 60 particles [1] and four nucleons [2] predicting the presence of spatially symmetric boundstates which became more bound with the increase of considered particles. However, the picture changes when more fermions of the same species are present in the same system. O. I. Kartavtsev et al. [3] showed that in the case of A^2B fermionic system (two of one flavor and the third of a second kind) contact EFT predicts a stable bound state only if the mass ratio between the two kinds of fermions is larger than $m_B/m_A \gtrapprox 13$. However, this is not the case when the particles show a (quasi-)symmetry in the fermionic flavors as in the nuclear case or for the same fermionic atom specie. D. S. Petrov et al. [4, 5, 6] extended this result showing that the scattering length of two dibaryons composed of two isomassive fermionic species ($AB + AB$) has the same sign as the $A + AB$ one. These two general statements show that, in the unitary limit and with a contact interaction, three and four two-flavor fermions are not stable against breaking in spatially symmetric pairs. In nuclear physics, the situation is more complex since the number of fermionic flavors is four and more particles can be arranged in a symmetric spatial wave function. However, ^{16}O nucleus appears to be unstable with respect four-α breaking within contact EFT framework [7]. Recently, Gattobigio et al. [8] showed that also ^{6}He is not stabilized by a momentumless short-range interaction with respect deuterium and α decay, but a stable bound state can be found if P-wave interactions are included in the potential. It is still not clear if the impossibility to stabilize P-wave[1] states is a general feature of contact EFTs, however, there is no doubt that this is a fundamental requirement for a many-body theory, since many systems, as nuclear ones, are close to universality and present many-body bound states.

In this paper, we explore the possibility to bind P-wave systems with a contact EFT studying systems with A fermionic flavors and $A+1$ and $A+2$ particles. This study is performed varying the input of the theory in the proximity of the unitary limit, and specifically to the nuclear case, to ensure the most general possible conclusions. For simplicity all the calculations have been made with fermions of mass 938.858 MeV, corresponding to the averaged nucleonic mass. However, the results remain general since the energy scales of the systems can be rescaled in the concept of system universality.

The paper is so structured: first, the theory and the fitting procedures used in this work are described. Follows, in the result section, the calculation outcomes for various systems with a particular focus on the nuclear cases experimentally known to be bound. In the conclusive section, an analysis of the work done and the consequence to renormalizable EFTs is performed. Finally, in the appendices, the numerical and analytically methods used in this work are briefly described.

[1]Systems with more fermions than fermionic flavors are referred as P-wave systems as, in a shell model framework, they would require at least a particle in $L > 0$ single-particle state to ensure correct total antisymmetric wavefunction. It can also be noticed that these states are naturally found as fermionic ground states, but many-bosons excitation share the same spacial proprieties.

2. Theoretical framework

The goal of this work is to study the behavior of many-body systems of non-relativistic fermions with unnatural large scattering length and negligible effective range parameter. The most natural theoretical framework to describe those systems is contact EFT. For a deeper insight about contact EFT, and its renormalization we suggest the reading of [9, 10, 11, 12, 13, 14]. The most general Lagrangian that satisfies the wanted symmetries (Particle number conservation, spin and isospin symmetry, parity, $\cdots$) can be written as

Name	B_2 [MeV]	B_3 [MeV]
A3	1	3
A4	1	4
Nuc	2.22	8.48
Uni	0	1

Table 1: Different three to two-body ratios used in this work.

$$L = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - \frac{c_0^{(0)}}{2} \left(\psi^\dagger \psi\right)^2 - \frac{d_0^{(0)}}{6} \left(\psi^\dagger \psi\right)^3 + \cdots \tag{1}$$

where C and D are low energy constants (LEC) and ellipsis represent terms with more derivatives. The Lagrangian terms can be sorted with respect to powers of the expansion parameter Q/M, with Q the exchanged momentum between particles and M the smallest mass scale present in the system. This arranges the interaction in operators gradually less important increasing the theory order. For simplicity, no spin or isospin symmetry breaking is included in our description. This is correct in atomic systems (e.g. ^{3}He [] or ^{40}K atoms [to Johannes: there is a paper about potassium? It looks to be quite unstable (but bananas are good nevertheless)]) which do not break spin symmetry and we do not expect to change qualitatively results in nuclei since nuclear interaction is close to being SU4 symmetric. Notice that the goal of this work is not to study the consequences of long-range forces, like meson exchanges, in the theory. Therefore, the smallest breaking scale present is the particle mass and, in the case of finite two-body binding energy, the scattering length of the system. Therefore, we expect subleading orders of the theory to be negligible.

In the presence of two-body shallow poles, two-body contact interaction associated with the LEC $c_0^{(0)}$ needs to be promoted at LO and treated non-perturbatively. A three-body contact term, associated with $d_0^{(0)}$, is also needed to be promoted at LO to avoid Thomas collapse in the 3+ body systems [11]. To regularize the interaction we choose a Gaussian regulator $\delta_\Lambda(x) = \left(\frac{\Lambda}{2\sqrt{\pi}}\right) e^{-\frac{x^2\Lambda^2}{4}}$ introducing an ultraviolet cut-off Λ. The resulting LO Hamiltonian reads:

$$H = -\sum_i \frac{\hbar^2}{2m}\nabla^2 + C_0^\Lambda \sum_{i<j} \delta_\Lambda(r_{ij}) + D_0^\Lambda \sum_{i<j\neq<k} \sum_{cyc} \delta_\Lambda(r_{ij})\delta_\Lambda(r_{ik}). \tag{2}$$

In the spirit of contact EFT, Λ has to be taken larger than the breaking scale of the theory. If the theory is properly renormalized, any observables calculated will depend only on inverse powers of the cut-off. On one hand, the cut-off defines the maximum momentum resolved by the interaction, but, in the other, it also determines the effective range of the two-body system. In fact, in the contact limit, the effective range is always $r_0 \leq 0$ [15, 16, 17], and approaches zero in case of momentumless interaction. In analogy to the square well interaction, that can be computed analytically (see specifically [17] for the derivation), the effective range dependence with a Gaussian regulator can be written as

$$\frac{r_0}{a} = \frac{\alpha}{\Lambda a} + \frac{\beta}{(\Lambda a)^2} + \frac{\gamma}{(\Lambda a)^3} \xrightarrow{a\to\infty} \frac{\alpha}{\Lambda a}. \tag{3}$$

The contact EFT employed in this work does not include momentum dependent operators as P-wave interaction, tensor force, spin-orbit, or relativistic contributions; or coulomb forces at LO. The treatment of subleading operators in perturbation theory is, in this framework, a safe way of preserving renormalizability beyond LO (see [18, 19] for an alternative way of including subleading operators in the theory). However, new poles of the system T-matrix can not be created adding operators in perturbation theory. Therefore, we require the theory to give a good qualitative description of the quantum states of the systems of interest already at LO.

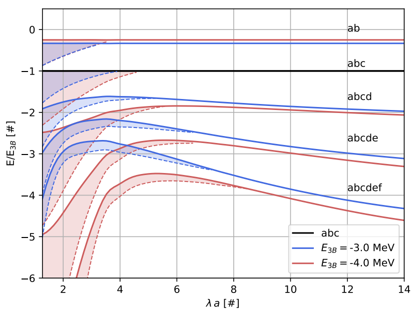
The theory has two degrees of freedom, the two- and the three-body LECs, that are fitted on few-body observables. The two-body LECs are fitted on small but finite two-body binding $B_2 = 1$ MeV equivalent to $a_0 \approx 6.7$ fm in the limit of large cut-off, to the nuclear binding $B_2 = 2.22$ MeV and to the unitary limit $B_2 \to 0$. The fit has been performed using a precise variational diagonalization method in case of finite binding and using the Numerov method in the unitary limit. The three-body LEC is fitted to reproduce one shallow three-body bound state with binding $B_3 = 3$, 4, 8.48, 1 MeV as on view in table 1. The three-body binding breaks the universality of the theory, therefore, we expect the theory behavior to be more dependent on the choice of the three-body binding than to the choice of the two-body scattering length or to the particle mass. All the three-body calculations have been performed using SVM.

3. Results

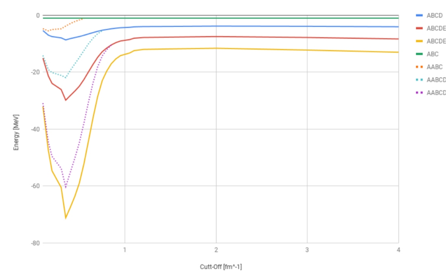
We discuss results of the theory for A-body systems with at least A fermionic flavours, *i.e.* , those with a bosonic ground state, before considering systems which contain one and two interacting particles more than accessible flavours. For each system, we analyse its dependence on the interaction-range-characterizing regulator parameter (Λ), the proximity of the theory to two-body unitarity, which we parametrize with the ratio between dimer and trimer binding energies ($B(3)/B(2) := \Lambda^*$), and we study the spatial symmetry of the ground states. Finally, the many-body limit $A \gg 1$ is investigated.

3.1. Bosonic 1^A systems

The A-boson ground states of a theory characterized by a Hamiltonian of type (2) has been numerically analysed in detail (see *e.g.* [20, 21, 22, 23, 24]). In contrast to these studies, the bosonic interaction of this work supports exactly one bound dimer and one trimer. The universal accumulation of trimers at the $1+1$ body threshold is not considered. Furthermore, for all Λ^* considered, only one $(A+1)$-boson state is found below the A-boson threshold and no second as established numerically, *e.g.* , in Refs. [25, 26, 23]. Comparing the $A < 7$ body spectra of $\Lambda^* = 4$ with $\Lambda^* = 3$ in fig. 1a, $B(A)/B(3)$ reduces, too. This indicates, that these states resemble the shallow components of the conjectured universal pair. This hypothesis is based on the argument that $\Lambda^* \to 0$ is realized with an increasingly repulsive three-body interaction which precludes the emergence of further A-boson bound states but has an enhanced effect in a larger system as the number of triplets grows with A. Therefore, one naïvely expects a more rapid decrease of $B(A)$ *wrt.* $B(A-1)$ which compensates the initially found wider gap. To parametrize the number A^* at which $B(A^*+1) < B(A^*)$, *i.e.* , stable (A^*+1)-body cluster cease to exist, remains an open question. For this work, we assume that $A^* = 2$, which means that for moving the trimer bound state closer to the dimer-boson threshold the larger systems remain stable but



(a) Colors online. Ground-state energy behaviour of A (solid line) and $A+1$ (dashed line) $A-$flavours fermionic system with respect to the cut-off used. The blue line represent the theory fitted on the configuration A3 and the red represent the A4 one (see table 1 and text for the configuration specifics).



(b) Ground-state energy behaviour of A (solid line) and $A+1$ (dashed line) $A-$flavours fermionic system with respect to the cut-off used in the unitary limit. The two-body LEC is fitted to zero energy threshold, the three body LEC is fitted to reproduce a unique $B_3 = 4$ MeV three body bound state. See text for the figure description.

Figure 1

they accumulate at the trimer-$(A-3)$-boson threshold. This accumulation resembles the set of shallow states of the alleged universal pair.

These considerations which identify the observed A-body states as ground states are useful to characterize their spatial structure. The variational bases expand only states with $L_{\text{total}} = 0$. Yet, spatial configurations with mixed symmetry are, in principle, allowed. However, interaction matrix elements of such mixed states, *e.g.* , particles occupying different oscillator S-shells $\begin{array}{|c|}\hline \mathrm{1s} \\ \hline \mathrm{2s} \\ \hline \end{array}$, should be of higher energy than a totally symmetric state $\begin{array}{|c|c|}\hline \mathrm{1s} & \mathrm{1s} \\ \hline \end{array}$. For the three and four-body states we verified this conjecture numerically for three- and four-body systems. To that end, a resonating-group calculation which, in addition to $L_{\text{total}} = 0$, projected all relative motions between the particles (in Jacobi coordinates) onto $L = 0$ was employed. The contribution of configurations to $L_{\text{total}} = 0$ from Jacobi motion with $L > 0$ was found increasingly insignificant with $\Lambda \to \infty$.

To summarize, we found the ground state of up to 7 bosons bound with respect to the $B(A-1)$ threshold and spatially totally symmetric. The convergence rate of $B(A)$ and $\langle r(A) \rangle$ (within the considered cutoff range $\Lambda \in [0.1, 14]\,\text{fm}^{-1}$) increases with Λ^*, *i.e.* , closer to the two-body unitarity limit, the bosonic ground states become less sensitive with respect to details of the two- and three-body interactions. The finite dimer binding energy is identified as a parameter which controls the existence of the universal pair of A-boson states – shallow and deep *wrt.* a universal $(A-1)$-boson state. For the considered values of $B(2)$ and Λ^*, we identify only one element of the universal pair.

3.2. Fermionic $2{+}1^{(A-1)}$ *systems*

Now add one particle with an identical mass and flavour-equal[1] to one of the constituents of the above A-boson systems.

In a naïve approach, we commence with spatial variational-basis states constrained to a total orbital angular momentum $L_{\text{total}} = 0$, as above. Consequently, our SVM implementation with the antisymmetric constraint on a pair of particles yields $B(A+1) < 0$, *i.e.* , no bound states. Although, this

[1] The identity of two of the $A+1 \in [3,7]$ particles was enforced on the SVM basis states which where totally antisymmetric with an A-dimensional internal space. For $2+1$ particles, *e.g.* , the spin-up/down states of a neutron suffice, while for $4+1$, the spin and isospin formalism can be invoked.

can be proven analytically[1] for contact interactions, finite interaction ranges, *i.e.* , $\forall\Lambda < \infty$, especially for $\Lambda \ll 1\,\mathrm{fm}^{-1}$, such bound states cannot be ruled out, *a priori* because: one, a state with $L_{\mathrm{total}} = 0$ has non-zero overlap with mixed-symmetry states (as they are enforced by the internal wave-function component), and two, the finite range allows for non-zero matrix elements of the interaction even if two particles reside in different orbitals. However, the results demonstrate the smallness of such transitions which are necessary for the binding for reasonable ranges.

If the spatial component of the variational basis is projected onto $L_{\mathrm{total}} = 1$, the eigenvalue spectra of the $A + 1 \in [3, 7]$ particle systems do contain a negative value ($B(A + 1) > B(A)$ and $L_{\mathrm{total}} = 1$ is understood from here on unless noted otherwise) for $\Lambda \approx 0.1\,\mathrm{fm}^{-1}$. In prose, if the regularized interaction provides enough attraction beyond a repulsive region in which an effective angular momentum barrier drives the particles apart, the system's ground state is bound. In order to assess the universal character of such a stable bound state with mixed symmetry – in other words, is $B(A + 1)$ correlated with $B(2, 3)$ (neither of which is $f(\Lambda)$ by construction)? – the cutoff is varied: $\Lambda \in [0.1, 14]\,\mathrm{fm}^{-1}$.

With increasing cutoff, *i.e.* , decreasing interaction range, *i.e.* , approaching the contact limit, $B(A + 1)$ is found to decrease and vanish for some critical value λ_c. This critical range increases linearly with A (see fig. 3a). The more particles in the bosonic ground-state core, the shorter-ranged the microscopic two- and three-body interaction has to be in order not to stabilize the $A + 1$ mixed-symmetry state. As $\lambda_c(A = 2 - 6)$ are arguably small relative to a scale above which the nuclear contact theory (EFT($\not\pi$)) would be useful, the result implies:

The absence of $L = 1$, $S = 1/2$, $A + 1$-body bound states in an A-flavour theory is a consequence of flavour-independent contact interactions which are renormalized to one bound dimer and one bound trimer.

Strictly speaking, the results presented thus far support this conclusion about the universal instability of the $A + 1$ body state only for $A = 2 - 6$. In order to gain insight whether the linear dependence of the critical range holds for $A > 6$, we employ a single-channel, effective two-fragments resonating-group approximation (see *e.g.* Refs. [27, 28]). This approximation turns the $A + 1$ body problem into a two-body problem between a "frozen" core

---

[1]

and one of the original particles of the few-body problem. What makes this seemingly drastic simplification appropriate is the halo character of the problem, namely, the increasingly large gap $\lim_{\Lambda\to\infty}[B(A)-B(A-1)]=\infty$, which does not allow for excitations of the bosonic core induced by the $A+1$-th particle if the energy of the latter does not exceed the scale set by this gap. It is helpful to consider this treatment as a generalization of the time-honoured description of the 5-nucleon system around the Nucleon-α threshold (see *e.g.* [29, 30] for the EFT formulation).

However, the RGM method does not model the core as point-like but retains its finite size. It assumes an independent motion of the core particles but takes into account (anti) symmetrization. The independent motion allows for a representation of the core by its A-body ground state as it exists in the absence of the $A+1$-th particle. As this state was shown to be spherically symmetric with $L_{\text{total}}=0$, we choose to retain only one component of an harmonic-oscillator (HO) Slater-determinant expansion, *i.e.* , a product of four single-particle HO ground-state orbitals. The $A+1$-body Schrödinger equation reduces to its two-body form by disregarding variations of this wave function while minimizing the corresponding functional only *wrt.* the component which describes the relative motion. In this course, the effect of the antisymmetrization between two of the particles has to be considered and is reflected in isolated components of the effective two-body potential.

We detail the parametrization of these various components of the core-particle interaction with the microscopic coupling strengths C_0, D_0, the single-particle mass m, and the regulator parameter Λ in an appendix 5. There, we split the effective potential in the customary three terms: The direct potential, which averages the interaction of the orbiter with the core particles. This interaction is local and resembles the character of the two-particle interaction for the fragment-relative coordinate. It does not consider the statistical properties of the particles. These properties lead to the non-local exchange interaction which contains an energy-dependent part which is conventionally called the exchange kernel.

First, we shall discuss the case which disregards the exchange of the orbiter with one of the core constituents (local approximation). The increase of the critical Gaussian cutoff λ_c with the number of particles which we found by explicit A-body SVM calculations for $A<8$ continues in the RGM extrapolation up to a maximum number $\lambda_c(A^*)$. Both, A^* and the associated $\lambda_c(A^*)$ increase with Λ^* (compare maxima of the three curves in fig. 3b). The significance of this finding depends on the magnitude of λ_c

relative the the breakdown scale ($\Lambda_{\rm Hi}$) – any $\Lambda \lesssim \Lambda_{\rm Hi}$ affects the interaction at a range where universal states are known to reside for $\Lambda \to \infty$ but would surely "feel" the small-Λ changes – of the employed theory. For point particles, we estimate $\Lambda_{\rm Hi} \approx m^{-1} = \mathcal{O}(10^{-1}\,{\rm fm}^{-1})$. We find $\Lambda_{\rm Hi} < \lambda_c$ for $A > 3$ and conclude that the stability of a fermionic system of point particles with mass m and finite Λ^* depends on characteristics of the interaction which cannot be observed in the bosonic ground states. For particles whose substructure is known, *e.g.* , nucleons as composites whose excitations become relevant at energies of about the pion mass (m_π), the above conclusion, namely $\Lambda_{\rm Hi} \approx m_\pi^{-1} = \mathcal{O}(1\,{\rm fm}^{-1}) < \lambda_c$ is satisfied for $A \gtrsim 7$. The unbound character of five- and six-nucleons are thus universal consequences of their particular two- and three-body subsystems.

To substantiate this conjecture, we fit the experimental deuteron and triton binding energies, $B(2) = 2.22$ MeV and $B(3) = 8.48$ MeV, respectively. The spin singlet dibaryon (*e.g.* , the dineutron) also bound with $B(2)$ as a consequence of the assumed spin-independence of the leading-order theory. The effect of a spin-dependent LO interaction which discriminates between a real deuteron, and virtual singlets, has been found small (see *e.g.* []), and in particular, as the interaction in the other two-body channels is still attractive – the observed phenomena rest on the combinatorial enhancement of attractive two- and three-body matrix elements (see appendix 5) and depend only in magnitude on those – our qualitative results shall not change.

For $A \leq 4$, we observe instability pattern qualitatively identical to those shown in fig. 1a for the corresponding $A + 1$ systems. Hence, the three-parameter theory predicts correctly the experimentally established instability of nuclei in the ${}^3p$, ${}^3n$, ^{4}H, ^{4}Li, and ^{5}He channels.

In contrast, ^{6}Li is known to sustain a $J^\pi = 1^+$ bound state approximately 1.5 MeV below the α-deuteron threshold. In order to make predictions with contact theories for this three-proton and three-neutron (2+2+1+1) nucleus, the (iso)spin component of the SVM wave function was chosen appropriately. We find a particle-stable ^{6}Li below a critical cutoff $\lambda_c \approx 1.5\,{\rm fm}^{-1}$, while for larger cutoffs, *i.e.* , a smaller interaction range, the ground state energy is smaller than the sum of the consistently assessed deuteron and alpha-particle binding energies (see fig. 4). Analogously, we find ^{8}Be (2+2+2+2) to become unstable *wrt.* an alpha decay at a $\lambda_c \approx 1\,{\rm fm}^{-1}$, *i.e.* , the same order as a nuclear interaction range assumed to be mediated by pion exchange (see discussion above). In contrast to the $2+1^{(A-1)}$ systems, λ_c decreases with the number of particles.

To understand this, we return to the one-particle-fragment motion relative to an A-particle core. For this motion being in an odd partial wave, repulsive components in the effective – direct and non-local exchange terms act analogous in the sense that the attractive or repulsive character of one of their components is set solely by the respective two- or three-body low-energy constant indicating its origin – interaction resulting from an assumed[1] repulsive three-body contact are small compared with the attractive two-body terms for small A and Λ. Increasing any of the two parameters increases the repulsive three-body-related term, either because the running of D_0^Λ exceeds Λ^6, eventually, or via a dominance of the triplets over pairs in the many-body system. While this dependence on A and Λ suggests that for larger A the weakening of attractive along an enhancement of the repulsive components of the potential also reduces λ_c the exponentially increase in range of both through the implicit dependence of a on A is larger in case of the attractive two-body remnant. The slower extension of its range, however, eventually is compensated by the quadratic growth of the strength of the repulsive three-body parts (given reasonable assumptions, *e.g.* , liquid drop, about the $a(A)$ relation). This explains qualitatively the initial increase of λ_c with A up to an A^* (range dominance) beyond which number, λ_c should decrease (as discussed in fig. 3b) in consequence of the quadratic strength increase of the repulsive interaction component (combinatorial strength dominance).

In an even partial wave, the effective interaction strengths between the core and a single particle are reduced by considering the exchange of the particle with one core element. In addition, the independent motion of the particles during the exchange introduces and interaction-independent repulsion in an even partial wave (exchange kernel). These two effect combined disfavour a stable state.

If, however, there is an even number of particles exchanged between the fragments (two in the case of ^{6}Li, and four for ^{8}Be), the effective potential changes by additional terms pertaining to these elements of the antisymmetrizer. Those new components introduce the opposite behaviour, namely, an enhancement of a coupling strength in an even partial wave and a mitigation in an odd one.

Even relative angular momentum, at low Λ, weakens the attractive C_0^Λ-proportional and the repulsive D_0^Λ-proportional term. The component of the

---
[1]

potential which supports binding is thereby reduced and the combinatorially-enhanced balance relative to the repulsive three-body term is dominated by the latter already for smaller A.

The meaning of λ_c, however, is unchanged. Once the cutoff exceeds these critical values in the course of taking the zero-range limit, neither ^{6}Li nor ^{8}Be are predicted particle stable by the leading-order theory.

The implication of this result for the usefulness of the pionless leading-order effective field theory for the description of these nuclei, the trajectory of the bound-state poles through the respective thresholds at λ_c is crucial. To this end, we apply the method of analytical continuation in the coupling constant (ACCC, see, *e.g.* Ref. [31]) to the $2+1$ case which represents, *e.g.* , a three-neutron system. Thereby, an attractive three-body contact term is introduces with strength d_0^Λ of the same structure as the one which renormalizes the bosonic three-body system in (2). The delta functions of this term use a cutoff parameter Λ which is identical to the two-body cutoff. For a range of cutoffs (see fig. 2), the initial d_0^Λ was chosen to bind the $2+1$ system before taking the limit $d_0^\Lambda \to 0$ while following the bound state pole on its way on the physical (energy) sheet through the branch cut starting at $E=0$ from above onto the fourth quadrant of the unphysical sheet. On The latter (hatched area in fig. 2), it represents a resonance. The pole remains on this sheet for $\Lambda \lesssim 4\,\mathrm{fm}^{-1}$. For larger cutoffs, the pole passes through another branch cut and leaves the first unphysical sheet. Thus, it no longer represents a dimer-particle resonance. Furthermore, its trajectory does not indicate to converge to a point on this second unphysical sheet. This behaviour suggests that an analogue of the dynamical pole generated (not generated by a single diagram but rather an infinite geometric series) by the contact theory (2) in the two- and three-boson systems, does not exist in $2+1^{(A-1)}$, neither as a bound nor resonant state.

To relate this observation to EFT($\not\pi$), in particular, the magnitude of the critical cutoff relative the scales in the theory is of importance. λ_c for ^{6}Li is arguably of the same order as the breakdown scale $\lambda_c \sim 1.5$ fm^{-1} $\approx$ 300 MeV set by the pion mass. The EFT framework demands to study cutoff/renormalization-group independence of the theory at values significantly larger than any presumed breakdown scale. Thereby, the interpretation of the stable six-body bound states as a cutoff artefact is justified.

For the conception of an extension of the EFT($\not\pi$)which predicts also the particle-stable character of ^{6}Li and ^{8}Be in the zero-range limit, we analyse the mechanism behind the stability of these systems for $\Lambda < \lambda_c$. Of all artefacts

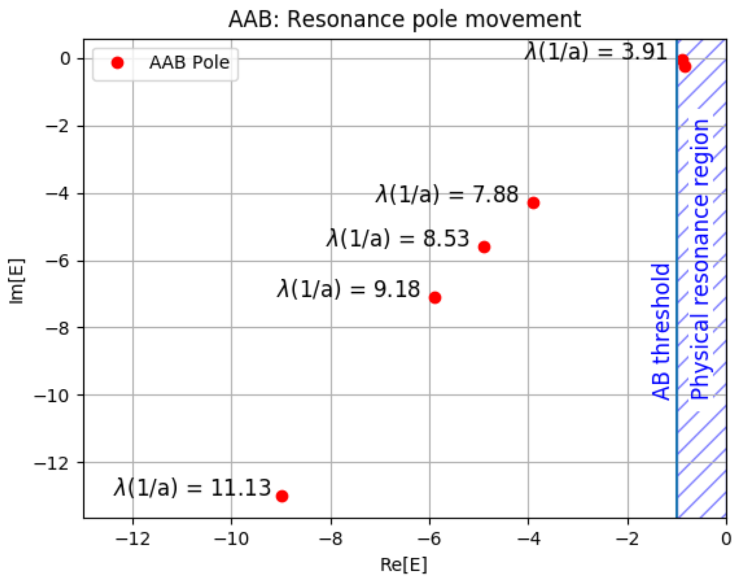
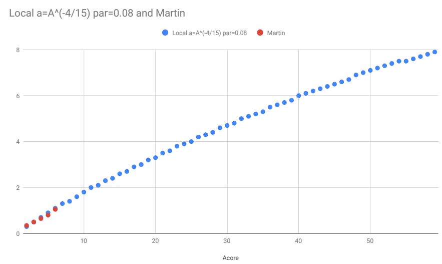


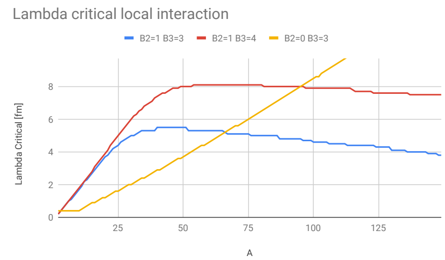
Figure 2: Lowest Hamiltonian eigenvalue analytically continued below the lowest two-/three-fragment breakup (ordinate/vertical at $\mathrm{Re}[E] = -1$ MeV) threshold for a $2+1$ system. Eigenvalues in the hatched region correspond to resonances which disappear with increasing regulator cutoff (λ label) through the three-fragment breakup branch cut.

introduced by the finite range of the regulated contact interaction, we single out: First, a finite effective range in the two-body S-wave channel, and second, a non-zero, attractive two-body P-wave interaction. Both contribute to the attraction in the $2{+}1^{(A-1)}$ system but their relative significance in this role is obscure. In other words, the finite-range interaction does not only describe a finite but large S-wave scattering length but also other finite parameters of the effective-range expansion of the S-wave amplitude, the S-wave effective range r_0 being one of these. Furthermore, the scattering volume a_p of the two-nucleon P-wave amplitude is non-zero. By studying the sensitivity of the bound $2{+}1^{(A-1)}$system below λ_c*wrt.* a variation of either of those parameters instead of the combined change as induced by varying Λ as in the analysis presented thus far, insight about the order of a potential enhancement of the corresponding higher-order-in-LO-EFT($\not\pi$)vertices is gained.

First, we project the two-body interaction in a spin 0, *i.e.* , an asymmetric internal state. This forces two interacting particles into an even spatial state. Non-zero matrix elements between states in odd partial waves cancel. The



(a)



(b)

Figure 3: Dependence of the critical cutoff (λ_c) on the number of core particles (A). Red dots in (a) mark SVM few-body calculations. All other curves represent results obtained within a single-channel resonating-group method. In (b), the effect of a change in the ratio between two- and three-boson binding energies on λ_c is shown. For $B(2) = 0$, the respective scattering length $\to \infty$.

effect of removing this contribution is shown in fig. 5. Namely, a significant reduction of λ_c^A. Without the residual attraction in odd partial waves, the binding is attributed mainly to the finite r_0 and a coupling to even larger relative two-body angular momenta.

What is our proposal for a contact theory which describes all what EFT($\not\pi$) does and 6Li et al.?

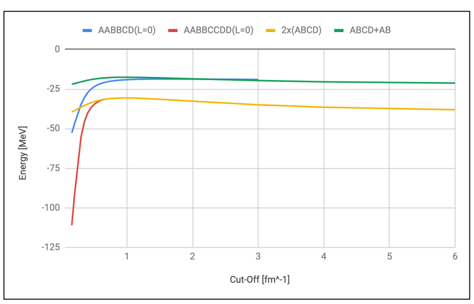


Figure 4: Energy of ^{8}Be, ^{6}Li and the relative thresholds with respect the cut-off. The instability of P-wave system is observed also if the theory is fitted to reproduce nuclear two- and three-body bindings.

System	$2+2$	$2+2+1+1$
L_{total}	r_c [fm]	r_c [fm]
0	4.16	1.23
1	5.21	2.96
2	4.81	2.30

Table 2: Minimal effective range required to stabilize the $2+2$ (dimer-dimer) and the $2+2+1+1$ (^{6}Li nucleus) systems in different partial waves.

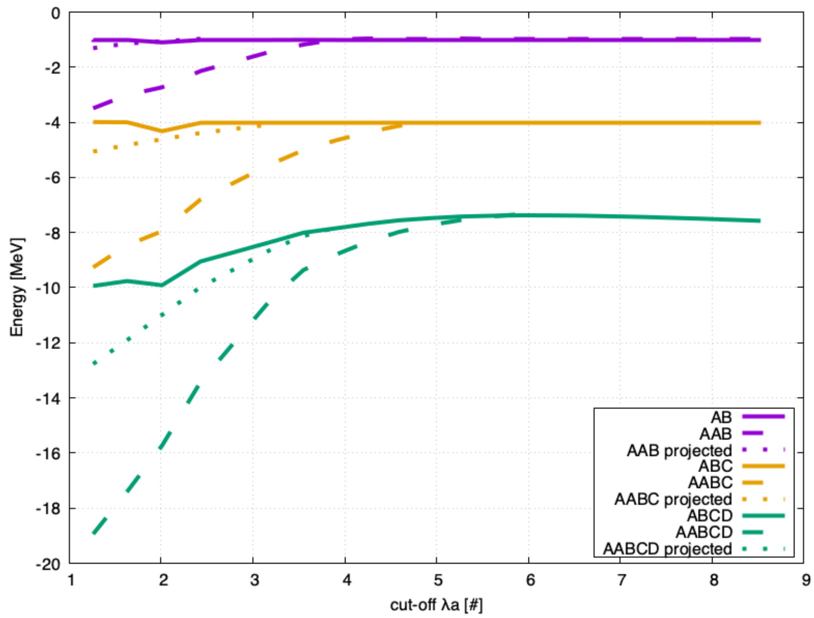


Figure 5: Energy of the A+1 fermionic systems with projected (dotted line) and not projected (dashed lines) compared witht the energy of the bosonic like core with A spatially symmetric fermions.

4. Conclusion

A non-relativistic system of $A+1$ particles with identical masses and an A-dimensional internal flavour space cannot sustain a bound state if its dynamics is constrained by representations of two- and three-particle momentum-independent contact interactions which are renormalized to yield a bound/unitary two-body state and a bound three-body state and whose residual finite-range is below a critical value (λ_c^1) which decreases with increasing particle content A of the system. If the range of the regulated contact interactions, however, surpasses the critical range, the ground state of the $A+1$ particles is bound with respect to breakup in the bosonic, spatially symmetric A-body ground state and a single free particle. The total orbital angular momentum of this $A+1$ bound state is $L_{\text{total}} = 1$. For any finite ratio $\Lambda^* < \infty$, the critical range reaches a minimum at a certain number of particles A.

5. Appendix: Inter-cluster Potential

$$\int \Bigg\{ \frac{\hbar^2}{2\mu} \left[-\partial_R^2 \left(\mathbb{1} + (\mathfrak{o}_E \leftrightarrow \mathfrak{o}_\mu)\right) + \frac{l(l+1)}{R^2} \left(\mathbb{1} + (\mathfrak{o}_E \leftrightarrow \mathfrak{o}_L)\right) \right] \phi_{lm}(R') \tag{4a}$$

$$-E \left(\delta(R-R') + (-)^{l+1} \mathfrak{o}_E(1) \left(\frac{a}{\pi}\right)^{3/2} e^{+\frac{1}{2}\mathfrak{o}_E(aA^{-1})RR' - \frac{1}{2}\mathfrak{o}_E(a)\boldsymbol{R}^2 - \frac{1}{2}\mathfrak{o}_E(a)\boldsymbol{R}'^2} \right) \tag{4b}$$

$$+\mathfrak{o}_2(A) \cdot \frac{C_0^\Lambda}{\Lambda^3} \cdot a^{\frac{3}{2}} \cdot e^{-\mathfrak{o}_2(a)\boldsymbol{R}^2} \left(\delta(R-R') + (-)^{l+1} \mathfrak{o}_2(1) \left(\frac{a}{\pi}\right)^{3/2} e^{+\mathfrak{o}_2(aA^{-1})RR' - \mathfrak{o}_2(a)\boldsymbol{R}'^2} \right) \tag{4c}$$

$$+2 \cdot \mathfrak{o}_3(A^2) \cdot \frac{D_0^\Lambda}{\Lambda^6} \cdot a^3 \cdot e^{-2\mathfrak{o}_3(a)\boldsymbol{R}^2} \left(\delta(R-R') + (-)^{l+1} \mathfrak{o}_3(1) \left(\frac{a}{\pi}\right)^{3/2} e^{+2\mathfrak{o}_3(aA^{-1})RR' - 2\mathfrak{o}_3(a)\boldsymbol{R}'^2} \right) \Bigg\} \phi_{lm}(R') \tag{4d}$$

$$= 0 \quad .$$

References

[1] J. Carlson, S. Gandolfi, U. van Kolck, and S. A. Vitiello, "Ground-state properties of unitary bosons: From clusters to matter," *Phys. Rev. Lett.*, vol. 119, p. 223002, Nov 2017.

[2] N. Barnea, L. Contessi, D. Gazit, F. Pederiva, and U. van Kolck, "Effective Field Theory for Lattice Nuclei," *Phys. Rev. Lett.*, vol. 114, no. 5, p. 052501, 2015.

[3] O. I. Kartavtsev and A. V. Malykh, "Low-energy three-body dynamics in binary quantum gases," *Journal of Physics B: Atomic, Molecular and Optical Physics*, vol. 40, pp. 1429–1441, mar 2007.

[4] D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, "Weakly bound dimers of fermionic atoms," *Phys. Rev. Lett.*, vol. 93, p. 090404, Aug 2004.

[5] D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, "Scattering properties of weakly bound dimers of fermionic atoms," *Phys. Rev.*, vol. A71, p. 012708, 2005.

[6] S. Endo and Y. Castin, "Absence of a four-body efimov effect in the $2 + 2$ fermionic problem," *Phys. Rev. A*, vol. 92, p. 053624, Nov 2015.

[7] L. Contessi, A. Lovato, F. Pederiva, A. Roggero, J. Kirscher, and U. van Kolck, "Ground-state properties of ^{4}He and ^{16}O extrapolated from lattice QCD with pionless EFT," *Phys. Lett.*, vol. B772, pp. 839–848, 2017.

[8] M. Gattobigio, A. Kievsky, and M. Viviani, "Embedding nuclear physics inside the unitary-limit window," *Phys. Rev.*, vol. C100, no. 3, p. 034004, 2019.

[9] G. P. Lepage, "How to renormalize the Schrodinger equation," in *Nuclear physics. Proceedings, 8th Jorge Andre Swieca Summer School, Sao Jose dos Campos, Campos do Jordao, Brazil, January 26-February 7, 1997*, pp. 135–180, 1997.

[10] U. van Kolck, "Effective field theory of nuclear forces," *Prog. Part. Nucl. Phys.*, vol. 43, pp. 337–418, 1999.

[11] P. F. Bedaque, H. W. Hammer, and U. van Kolck, "Renormalization of the three-body system with short range interactions," *Phys. Rev. Lett.*, vol. 82, pp. 463–467, 1999.

[12] E. Braaten and H. W. Hammer, "Universality in few-body systems with large scattering length," *Phys. Rept.*, vol. 428, pp. 259–390, 2006.

[13] H. W. Hammer, C. Ji, and D. R. Phillips, "Effective field theory description of halo nuclei," *J. Phys.*, vol. G44, no. 10, p. 103002, 2017.

[14] H. W. Hammer, S. Knig, and U. van Kolck, "Nuclear effective field theory: status and perspectives," 2019.

[15] E. P. Wigner, "Lower Limit for the Energy Derivative of the Scattering Phase Shift," *Phys. Rev.*, vol. 98, pp. 145–147, 1955.

[16] H. W. Hammer and D. Lee, "Causality and the effective range expansion," *Annals Phys.*, vol. 325, pp. 2212–2233, 2010.

[17] D. R. Phillips and T. D. Cohen, "How short is too short? Constraining contact interactions in nucleon-nucleon scattering," *Phys. Lett.*, vol. B390, pp. 7–12, 1997.

[18] S. Beck, B. Bazak, and N. Barnea, "Removing the Wigner bound in non-perturbative effective field theory," 2019.

[19] E. Epelbaum, A. M. Gasparyan, J. Gegelia, and U.-G. Meiner, "How (not) to renormalize integral equations with singular potentials in effective field theory," *Eur. Phys. J.*, vol. A54, no. 11, p. 186, 2018.

[20] B. Bazak, M. Eliyahu, and U. van Kolck, "Effective Field Theory for Few-Boson Systems," *Phys. Rev.*, vol. A94, no. 5, p. 052502, 2016.

[21] Y. Yan and D. Blume, "Energy and structural properties of N -boson clusters attached to three-body Efimov states: Two-body zero-range interactions and the role of the three-body regulator," *Phys. Rev.*, vol. A92, p. 033626, Sep 2015.

[22] M. Gattobigio, A. Kievsky, and M. Viviani, "Energy spectra of small bosonic clusters having a large two-body scattering length," *Phys. Rev.*, vol. A86, p. 042513, 2012.

[23] J. von Stecher, "Universal Five- and Six-Body Droplets Tied to an Efimov Trimer," *Phys. Rev. Lett.*, vol. 107, p. 200402, 2011.

[24] M. Gattobigio, A. Kievsky, and M. Viviani, "Spectra of helium clusters with up to six atoms using soft core potentials," *Phys. Rev.*, vol. A84, p. 052503, 2011.

[25] H. W. Hammer and L. Platter, "Universal Properties of the Four-Body System with Large Scattering Length," *Eur. Phys. J.*, vol. A32, pp. 113–120, 2007.

[26] J. von Stecher, J. P. D'Incao, and C. H. Greene, "Signatures of universal four-body phenomena and their relation to the Efimov effect," *Nature Physics*, vol. 5, pp. 417–421, Jun 2009.

[27] J. A. Wheeler, "Molecular viewpoints in nuclear structure," *Phys. Rev.*, vol. 52, pp. 1083–1106, Dec 1937.

[28] P. Naidon, S. Endo, and A. M. García-García, "Scattering of universal fermionic clusters in the resonating group method," *Journal of Physics B: Atomic, Molecular and Optical Physics*, vol. 49, p. 034002, jan 2016.

[29] C. A. Bertulani, H. W. Hammer, and U. Van Kolck, "Effective field theory for halo nuclei," *Nucl. Phys.*, vol. A712, pp. 37–58, 2002.

[30] L. S. Brown and G. M. Hale, "Field Theory of the $d+t \to n+\alpha$ Reaction Dominated by a $^5\mathrm{He}^*$ Unstable Particle," *Phys. Rev.*, vol. C89, no. 1, p. 014622, 2014.

[31] V. I. Kukulin and V. M. Krasnopol, "Description of few-body systems via analytical continuation in coupling constant," *Journal of Physics A: Mathematical and General*, vol. 10, pp. L33–L37, feb 1977.