Multi-fermion systems with contact theories

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Abstract

The particle-stability of isomassive fermionic systems is analysed for constituent numbers which exceed the number of accessible flavour states. The interaction is taken as an effective field theory of momentum- and flavour-independent two- and three-body regulated contact terms which are renormalized to shallowly bound dimer and trimer states. The regulator dependence of the stability of these systems is assessed as a function of particle number and the proximity of the interaction to two-body unitarity.

We find no system of mixed-spatial symmetry stable with respect to a decay into fragments with spatially symmetric wave functions if the range of the interaction is shorter than a number- and renormalization-condition-dependent critical range. We elaborate on the consequences of our results for the systematic description of atomic and nuclear systems. For the latter, in particular, the study provides strong evidence for an inclusion of momentum-dependent interaction terms in order to describe P-wave stable systems such as ^{6}Li and ^{8}Be effectively.

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1. Introduction

The entirety of observables on an A-body system can be split into two classes. In one, there are those which can be parametrized with certain properties of subsystems ($A_{\rm sub} < A$) by solving the same dynamical equation for the respective number of particles. In the other class, one finds all observables characteristic to the interaction of A particles which cannot be related to a "few" parameters of the subsystems. A systematic way to assign a given observable to either of these classes is to our best knowledge unknown.

However, progress has been made to categorize particular nuclear and atomic quantities. Employing the effective-field-theory (EFT) formalism, it has been shown [1], for instance, that the binding energy of three bosons (trimer) is unrelated to two-body data if the latter sustains a bound state with spatial correlations significantly larger than the two-particle interaction which generates it (unitary system). The (linear-in-a-finite-interval) dependence of the ground-state energy of the four-boson system on the three-boson energy (Tjon line), in contrast, was identified as a universal consequence of this two-body unitarity [2]. For bosonic atoms, a generalization of this correlation has been established numerically (see Ref.[3] for the first step) by identifying a pair of one deep and one shallow $A+1$ boson state attached to a universal A-boson state starting with a fixed trimer. Thereby, a non-trivial property of A particles can be predicted solely with two- and three-body data.

It is the aim of this work, to assess the dependence of such a correlation between macroscopic and microscopic dynamics on any deviation from the purely bosonic character of the particles. The latter allows all particles to occupy the same spatial state which they cannot if the system comprises more elements than internal flavours[1]. To study the essence of the problem, we commence by considering the stability of systems with only one particle more than flavour states accessible as they depend on the short-distance structure of the same two- and three-body interaction which is known to universally create such stable states below a threshold corresponding to the

[1] We generalize the notation (see *e.g.* Ref. [4]) of a $p+q$ fermionic problem in which p identical fermions interact with another set of q identical fermions such that $\underbrace{1+\ldots+1}_{A \text{ terms}=:1^A}$ is the A-boson problem, $2+1^3$ being two spin-up and one spin-down neutron(s), and two protons, one spin-up and one spin-down, *i.e.* , 5-helium, *etc.*

separation of one particle. This model system comprises $A+1$ equal-mass particles, two of which are forced into the same flavour, *i.e.* , spin-isospin, state whose dynamics are dictated by an $A+1$-body Schrödinger equation which represents the leading order of an effective field theory as pertinent to a bosonic system. Thus, the problem is specified through five parameters, the particle's mass (m), the number A, a dimer and trimer binding energy which calibrate the two interactions strengths, and a regulator scale which represents a particular short-distance behaviour *wrt.* which the dimer and trimer are invariant by definition.

We focus the study on systems which are close to the so-called unitary limit where the dimer resides on or close to the two-body threshold under conditions which a three-body Efimov spectrum accumulates at the same threshold. Besides numerous atomic systems which exhibit such spectra in appropriately tuned traps, we address nuclei, in particular. The latter are amenable to a description with neutrons and protons and therefore do allow for a totally spatial symmetric state only up to $A = 4$ (ground state of the α particle). The generic stability problem states above translates into that of the short-distance/renormalization-group invariance of the resonant 5-helium, and the 6-lithium and 8-beryllium stable ground states.

We begin with a description of the employed interaction theory in terms of its Hamiltonian, regularization, and renormalization. Then, results on the regulator dependence of the stability of P-wave systems with up to $A > 120$ as they follow from this theory are presented. The nuclear systems 6-lithium and 8-beryllium are treated separately. Based on the results, we comment on potential consequences for the nuclear contact EFT. In an appendix, we present the structure and a succinct guide to an effective two-fragment interaction which allows for the description of larger systems.

2. Theoretical framework

The object of this work is a many-body system of non-relativistic fermions whose two-body scattering length (a_0) is large relative to the support of the interaction potential, *i.e.* , negligible compared with the effective range (r_0). The established theoretical framework pertinent to such systems is contact EFT (see *e.g.* Refs.[5, 6, 1, 7, 8, 9]). Its Lagrangian satisfies the symmetries of choice, namely, particle-number conservation, spin and isospin symmetry

[1], parity, and Galilei invariance, and reads

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m} \right) \psi + C_0 \left(\psi^\dagger \psi \right)^2 + D_0 \left(\psi^\dagger \psi \right)^3 \ + \ \ldots \tag{1}$$

with two low-energy constants (LEC) C_0 and D_0 representing the leading order (LO), while ellipsis represent any other symmetry-allowed structure which would be needed to, either, attain more accurate results at a higher order, or, to remove unphysical regulator dependences which might arise in certain amplitudes. The significance of these operators, which refine the leading order, is estimated via their mass dimension set by a product of ratios between external momenta Q and mass scales M. Intuitively, an upper bound for the Q's for the observable/amplitude under investigation can be estimated, while the emergence of scales which are small compared with the already accounted-for ones but arise only in an amplitude between states with more than some critical number of particles cannot be foreseen trivially. This latter possibility is of particular importance for this work, in which such operators are present as regulator artefacts and could produce a pole in one of the mixed-symmetry amplitudes which vanishes [2] with the strength of this spurious operator being reduced to zero in the course of the zero-range limit (see below).

In addition to the ordering scheme of the symmetry-allowed vertices, their contribution to the iterative solution of the equation of motion has to be assessed. By themselves, loop diagrams with C_0 vertices, do not generate the analytic structure required by the existence of shallow two-body poles. Consequently, the solution must not be truncated at any order in these LECs. The momentum-independent three-body contact term $\propto D_0$ must be treated non-perturbatively[1], too. However, for a different reason, which is the divergence of the pole location in the three-body amplitude, in other words, the Thomas collapse. This catastrophic pole trajectory results from the iteration of the regularized two-body vertex in the zero-range limit. To extract predictions with the theory – defined thus far via a vertex structure and

[1] Besides its pertinence to bosonic systems, the ensuing theory is useful to some extend (see Ref. [10]) for four-flavour fermionic systems in which a relatively shallow state exists in all four two-fermion S-wave channels, *i.e.* , nuclei, too.

[2] It vanishes precisely because the operator is assumed to scale naturally and thus its regulator dependence is implicitly set by C_0 and D_0 which are renormalized to consider the unnatural two- and three-body scales, only.

an ordering scheme – a regularization scheme has to be defined which assumes "something" about the structure of the amplitude in domains which are of no interest, *i.e.* , momenta exceeding a presumably known breakdown scale. Finally, a prescription to assess the independence of the amplitude within the domains of interest on this "something" has to be specified. Here, we regularize the interaction with a Gaussian regulator $\delta_\Lambda(\boldsymbol{x}) \propto C_0 \Lambda^3 e^{-\frac{\Lambda^2}{4}\boldsymbol{x}^2}$ and thereby introduce the regulator scale Λ. The resulting LO Hamiltonian reads:

$$H = -\sum_i \frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + C_0^\Lambda \sum_{i<j} \delta_\Lambda(\boldsymbol{r}_i - \boldsymbol{r}_j) + D_0^\Lambda \sum_{\substack{i<j<k \\ \text{cyc}}} \delta_\Lambda(\boldsymbol{r}_i - \boldsymbol{r}_j)\delta_\Lambda(\boldsymbol{r}_i - \boldsymbol{r}_k). \quad (2)$$

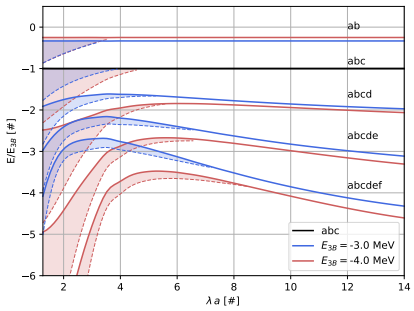
Regarding the assessment of the independence of observables of interest on this particular "something" Λ, we demand of a useful theory that its predictions are independent only if $\Lambda^{-1} \ll$ the smallest characteristic correlation of particles within the system under consideration. Hence, for nuclei in particular, we demand any Λ dependence to vanish for $\Lambda \gg 1\,\mathrm{fm}^{-1} = \mathcal{O}(r_{\mathrm{rms}}(\alpha))$.

While Λ act thus as a cutoff for momenta with which structure is resolved, it also introduces a non-zero effective range for the two-body system. For a square well interaction, the relation between cutoff and effective range can be computed analytically (see *e.g.* Ref. [11]),

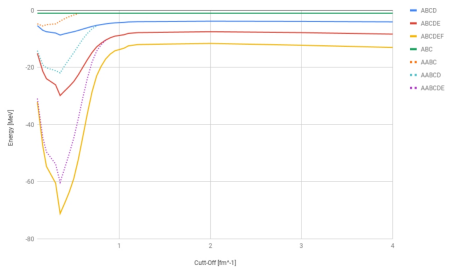
$$\frac{r_0}{a_0} = \frac{\text{const.}}{\Lambda a_0} + \frac{\text{const.}}{(\Lambda a_0)^2} + \frac{\text{const.}}{(\Lambda a_0)^3} \xrightarrow{a\to\infty} \frac{\text{const.}}{\Lambda a_0} \quad . \quad (3)$$

We use this relation as an approximation for the effective range of the Gaussian regulator when presenting the results.

With the formal structure of the theory thus summarized, it is applied to a particular system of equal mass particles by setting m and the two low-energy constants to yield Λ-independent predictions for two uncorrelated observables. Throughout this work, he mass is set by an average nucleon mass of $m = 938$ MeV. For the LECs, we consider multiple scenarios. First, to study the generic $2{+}1^{(A-1)}$ system, C_0^Λ is calibrated with a small but finite two-body binding $B(2) = 1$ MeV which corresponds to $a_0 \approx 6.7$ fm for large cutoffs and $B(2) = \epsilon^-$ MeV, *i.e.* , $a_0 \approx -\infty$. For $B(2) = 1$ MeV, we fit three three-body binding energies via D_0^Λ. These three choices, $B(3) \in \{1.5,\, 3,\, 4\}$ MeV approach the unitary limit differently by increasing the numerator instead of the denominator of the ratio $B(3)/B(2) := \Lambda^*$.



(a) (Colors online) Cutoff dependence of ground-state energies of A-boson (solid line) and $2+1^{(A-1)}$(dashed line) systems for two trimer binding energies, $B(3) = 3$ MeV (blue) and $B(3) = 4$ MeV (red), and a single dimer constraint to $B(2) = 1$ MeV.



(b) Cutoff dependence of ground-state energies of A-boson (solid line) and $2+1^{(A-1)}$(dashed line) systems close the the unitary limit at which the dimer binding energy is at threshold $B(2) \to 0$, and the trimer is kept at $B(3) = 4$ MeV.

Figure 1

The pionless, nuclear EFT is renormalized separately to yield the deuteron and triton binding energies of $B(2) = 2.22$ MeV and $B(3) = 8.48$ MeV, respectively. The fits employ precise SVM and RGM variational diagonalization methods for D_0^Λ, while C_0^Λ was determined via a Numerov-type integration of the appropriate one-dimensional radial Schrödinger equation.

3. Results

We discuss results of the theory for A-body systems with at least A fermionic flavours, *i.e.* , those with a bosonic ground state, before considering systems which contain one and two interacting particles more than accessible flavours. For each system, we analyse its dependence on the interaction-range-characterizing regulator parameter (Λ), the proximity of the theory to two-body unitarity, which we parametrize with the ratio between dimer and trimer binding energies ($B(3)/B(2) := \Lambda^*$), and we study the spatial symmetry of the ground states. Finally, the many-body limit $A \gg 1$ is investigated.

3.1. Bosonic 1^A systems

The A-boson ground states of a theory characterized by a Hamiltonian of type (2) has been numerically analysed in detail (see *e.g.* Refs.[12, 13, 14, 15, 16]). In contrast to these studies, the bosonic interaction of this work supports exactly one bound dimer and one trimer. The universal accumulation of trimers at the $1+1$ body threshold is not considered. Furthermore, for all Λ^* considered, only one $(A+1)$-boson state is found below the A-boson threshold and no second as established numerically, *e.g.* , in Refs. [17, 18, 15]. Comparing the $A < 7$ body spectra of $\Lambda^* = 4$ with $\Lambda^* = 3$ in fig. 1a, $B(A)/B(3)$ reduces, too. This indicates, that these states resemble the shallow components of the conjectured universal pair. This hypothesis is based on the argument that $\Lambda^* \to 0$ is realized with an increasingly repulsive three-body interaction which precludes the emergence of further A-boson bound states but has an enhanced effect in a larger system as the number of triplets grows with A. Therefore, one naïvely expects a more rapid decrease of $B(A)$ *wrt.* $B(A-1)$ which compensates the initially found wider gap. To parametrize the number A^* at which $B(A^*+1) < B(A^*)$, *i.e.* , stable (A^*+1)-body cluster cease to exist, remains an open question. For this work, we assume that $A^* = 2$, which means that for moving the trimer bound state closer to the dimer-boson threshold the larger systems remain stable but they accumulate at the trimer-$(A-3)$-boson threshold. This accumulation resembles the set of shallow states of the alleged universal pair.

These considerations which identify the observed A-body states as ground states are useful to characterize their spatial structure. The variational bases expand only states with $L_{\text{total}} = 0$. Yet, spatial configurations with mixed symmetry are, in principle, allowed. However, interaction matrix elements of such mixed states, *e.g.* , particles occupying different oscillator S-shells $\boxed{\begin{smallmatrix}1_s\\2_s\end{smallmatrix}}$, should be of higher energy than a totally symmetric state $\boxed{1_s}\boxed{1_s}$. For the three and four-body states we verified this conjecture numerically for three- and four-body systems. To that end, a resonating-group calculation which, in addition to $L_{\text{total}} = 0$, projected all relative motions between the particles (in Jacobi coordinates) onto $L = 0$ was employed. The contribution of configurations to $L_{\text{total}} = 0$ from Jacobi motion with $L > 0$ was found increasingly insignificant with $\Lambda \to \infty$.

To summarize, we found the ground state of up to 7 bosons bound with respect to the $B(A-1)$ threshold and spatially totally symmetric. The

convergence rate of $B(A)$ and $\langle r(A)\rangle$ (within the considered cutoff range $\Lambda \in [0.1, 14]\,\mathrm{fm}^{-1}$) increases with Λ^*, *i.e.* , closer to the two-body unitarity limit, the bosonic ground states become less sensitive with respect to details of the two- and three-body interactions. The finite dimer binding energy is identified as a parameter which controls the existence of the universal pair of A-boson states – shallow and deep *wrt.* a universal $(A-1)$-boson state. For the considered values of $B(2)$ and Λ^*, we identify only one element of the universal pair.

3.2. Fermionic $2{+}1^{(A-1)}$ *systems*

Now add one particle with an identical mass and flavour-equal[1] to one of the constituents of the above A-boson systems.

In a naïve approach, we commence with spatial variational-basis states constrained to a total orbital angular momentum $L_{\mathrm{total}} = 0$, as above. Consequently, our SVM implementation with the antisymmetric constraint on a pair of particles yields $B(A+1) < 0$, *i.e.* , no bound states. Although, this can be proven analytically[2] for contact interactions, finite interaction ranges, *i.e.* , $\forall \Lambda < \infty$, especially for $\Lambda \ll 1\,\mathrm{fm}^{-1}$, such bound states cannot be ruled out, *a priori* because: one, a state with $L_{\mathrm{total}} = 0$ has non-zero overlap with mixed-symmetry states (as they are enforced by the internal wave-function component), and two, the finite range allows for non-zero matrix elements of the interaction even if two particles reside in different orbitals. However, the results demonstrate the smallness of such transitions which are necessary for the binding for reasonable ranges.

If the spatial component of the variational basis is projected onto $L_{\mathrm{total}} = 1$, the eigenvalue spectra of the $A+1 \in [3, 7]$ particle systems do contain a negative value ($B(A+1) > B(A)$ and $L_{\mathrm{total}} = 1$ is understood from here on unless noted otherwise) for $\Lambda \approx 0.1\,\mathrm{fm}^{-1}$. In prose, if the regularized interaction provides enough attraction beyond a repulsive region in which an effective angular momentum barrier drives the particles apart, the system's ground state is bound. In order to assess the universal character of such a stable bound state with mixed symmetry – in other words, is $B(A+1)$

[1] The identity of two out of 3 and 7 particles was enforced on the SVM basis states, while these internal states were made totally antisymmetric for any A-boson system. For $2+1^1$ particles, *e.g.* , the spin-up/down states of a neutron suffice, while for $2+1^3$, the spin and isospin formalism can be invoked.

[2]

correlated with $B(2,3)$ (neither of which is $f(\Lambda)$ by construction)? – the cutoff is varied: $\Lambda \in [0.1, 14]\,\mathrm{fm}^{-1}$.

With increasing cutoff, *i.e.* , decreasing interaction range, *i.e.* , approaching the contact limit, $B(A+1)$ is found to decrease and vanish for some critical value λ_c. This critical range increases linearly with A (see fig. 3a). The more particles in the bosonic ground-state core, the shorter-ranged the microscopic two- and three-body interaction has to be in order not to stabilize the $A+1$ mixed-symmetry state. As $\lambda_c(A=2-6)$ are arguably small relative to a scale above which the nuclear contact theory (EFT($\not\pi$)) would be useful, the result implies:

The absence of $L=1$, $S=1/2$, $A+1$-body bound states in an A-flavour theory is a consequence of flavour-independent contact interactions which are renormalized to one bound dimer and one bound trimer.

Strictly speaking, the results presented thus far support this conclusion about the universal instability of the $A+1$ body state only for $A=2-6$. In order to gain insight whether the linear dependence of the critical range holds for $A>6$, we employ a single-channel, effective two-fragments resonating-group approximation (see *e.g.* Refs. [19, 20]). This approximation turns the $A+1$ body problem into a two-body problem between a "frozen" core and one of the original particles of the few-body problem. What makes this seemingly drastic simplification appropriate is the halo character of the problem, namely, the increasingly large gap $\lim_{\Lambda\to\infty}[B(A)-B(A-1)]=\infty$, which does not allow for excitations of the bosonic core induced by the $A+1$-th particle if the energy of the latter does not exceed the scale set by this gap. It is helpful to consider this treatment as a generalization of the time-honoured description of the 5-nucleon system around the Nucleon-α threshold (see *e.g.* Refs.[21, 22] for the EFT formulation).

However, the RGM method does not model the core as point-like but retains its finite size. It assumes an independent motion of the core particles but takes into account (anti) symmetrization. The independent motion allows for a representation of the core by its A-body ground state as it exists in the absence of the $A+1$-th particle. As this state was shown to be spherically symmetric with $L_{\mathrm{total}}=0$, we choose to retain only one component of an harmonic-oscillator (HO) Slater-determinant expansion, *i.e.* , a product of four single-particle HO ground-state orbitals. The $A+1$-body Schrödinger equation reduces to its two-body form by disregarding variations of this wave function while minimizing the corresponding functional only *wrt.* the component which describes the relative motion. In this course, the effect of the

antisymmetrization between two of the particles has to be considered and is reflected in isolated components of the effective two-body potential.

We detail the parametrization of these various components of the core-particle interaction with the microscopic coupling strengths C_0, D_0, the single-particle mass m, and the regulator parameter Λ in an appendix 5. There, we split the effective potential in the customary three terms: The direct potential, which averages the interaction of the orbiter with the core particles. This interaction is local and resembles the character of the two-particle interaction for the fragment-relative coordinate. It does not consider the statistical properties of the particles. These properties lead to the non-local exchange interaction which contains an energy-dependent part which is conventionally called the exchange kernel.

First, we shall discuss the case which disregards the exchange of the orbiter with one of the core constituents (local approximation). The increase of the critical Gaussian cutoff λ_c with the number of particles which we found by explicit A-body SVM calculations for $A < 8$ continues in the RGM extrapolation up to a maximum number $\lambda_c(A^*)$. Both, A^* and the associated $\lambda_c(A^*)$ increase with Λ^* (compare maxima of the three curves in fig. 3b). The significance of this finding depends on the magnitude of λ_c relative the the breakdown scale ($\Lambda_{\rm Hi}$) – any $\Lambda \lesssim \Lambda_{\rm Hi}$ affects the interaction at a range where universal states are known to reside for $\Lambda \to \infty$ but would surely "feel" the small-Λ changes – of the employed theory. For point particles, we estimate $\Lambda_{\rm Hi} \approx m^{-1} = \mathcal{O}(10^{-1}\,{\rm fm}^{-1})$. We find $\Lambda_{\rm Hi} < \lambda_c$ for $A > 3$ and conclude that the stability of a fermionic system of point particles with mass m and finite Λ^* depends on characteristics of the interaction which cannot be observed in the bosonic ground states. For particles whose substructure is known, *e.g.* , nucleons as composites whose excitations become relevant at energies of about the pion mass (m_π), the above conclusion, namely $\Lambda_{\rm Hi} \approx m_\pi^{-1} = \mathcal{O}(1\,{\rm fm}^{-1}) < \lambda_c$ is satisfied for $A \gtrsim 7$. The unbound character of five- and six-nucleons are thus universal consequences of their particular two- and three-body subsystems.

To substantiate this conjecture, we fit the experimental deuteron and triton binding energies, $B(2) = 2.22$ MeV and $B(3) = 8.48$ MeV, respectively. The spin singlet dibaryon (*e.g.* , the dineutron) also bound with $B(2)$ as a consequence of the assumed spin-independence of the leading-order theory. The effect of a spin-dependent LO interaction which discriminates between a real deuteron, and virtual singlets, has been found small (see *e.g.* Refs.[23, 10]), and in particular, as the interaction in the other two-body

channels is still attractive – the observed phenomena rest on the combinatorial enhancement of attractive two- and three-body matrix elements (see appendix 5) and depend only in magnitude on those – our qualitative results shall not change.

For $A \leq 4$, we observe instability pattern qualitatively identical to those shown in fig. 1a for the corresponding $A + 1$ systems. Hence, the three-parameter theory predicts correctly the experimentally established instability of nuclei in the ${}^3p$, ${}^3n$, ^{4}H, ^{4}Li, and ^{5}He channels.

In contrast, ^{6}Li is known to sustain a $J^\pi = 1^+$ bound state approximately 1.5 MeV below the α-deuteron threshold. In order to make predictions with contact theories for this three-proton and three-neutron (2+2+1+1) nucleus, the (iso)spin component of the SVM wave function was chosen appropriately. We find a particle-stable ^{6}Li below a critical cutoff $\lambda_c \approx 1.5\,\mathrm{fm}^{-1}$, while for larger cutoffs, *i.e.* , a smaller interaction range, the ground state energy is smaller than the sum of the consistently assessed deuteron and α-particle binding energies (see fig. 4). Analogously, we find ^{8}Be $(2 + 2 + 2 + 2)$ to become unstable *wrt.* an α-decay at a $\lambda_c \approx 1\,\mathrm{fm}^{-1}$, *i.e.* , the same order as a nuclear interaction range assumed to be mediated by pion exchange (see discussion above). In contrast to the $2+1^{(A-1)}$ systems, λ_c decreases with the number of particles.

To understand this, we return to the one-particle-fragment motion relative to an A-particle core. For this motion being in an odd partial wave, repulsive components in the effective – direct and non-local exchange terms act analogous in the sense that the attractive or repulsive character of one of their components is set solely by the respective two- or three-body low-energy constant indicating its origin – interaction resulting from an assumed[1] repulsive three-body contact are small compared with the attractive two-body terms for small A and Λ. Increasing any of the two parameters increases the repulsive three-body-related term, either because the running of D_0^Λ exceeds Λ^6, eventually, or via a dominance of the triplets over pairs in the many-body system. While this dependence on A and Λ suggests that for larger A the weakening of attractive along an enhancement of the repulsive components of the potential also reduces λ_c the exponentially increase in range of both through the implicit dependence of a on A is larger in case of the attractive two-body remnant. The slower extension of its range, however, eventually is

[1]

compensated by the quadratic growth of the strength of the repulsive three-body parts (given reasonable assumptions, *e.g.* , liquid drop, about the $a(A)$ relation). This explains qualitatively the initial increase of λ_c with A up to an A^* (range dominance) beyond which number, λ_c should decrease (as discussed in fig. 3b) in consequence of the quadratic strength increase of the repulsive interaction component (combinatorial strength dominance).

In an even partial wave, the effective interaction strengths between the core and a single particle are reduced by considering the exchange of the particle with one core element. In addition, the independent motion of the particles during the exchange introduces and interaction-independent repulsion in an even partial wave (exchange kernel). These two effect combined disfavour a stable state.

If, however, there is an even number of particles exchanged between the fragments (two in the case of ^{6}Li, and four for ^{8}Be), the effective potential changes by additional terms pertaining to these elements of the antisymmetrizer. Those new components introduce the opposite behaviour, namely, an enhancement of a coupling strength in an even partial wave and a mitigation in an odd one.

Even relative angular momentum, at low Λ, weakens the attractive C_0^Λ-proportional and the repulsive D_0^Λ-proportional term. The component of the potential which supports binding is thereby reduced and the combinatorially-enhanced balance relative to the repulsive three-body term is dominated by the latter already for smaller A.

The meaning of λ_c, however, is unchanged. Once the cutoff exceeds these critical values in the course of taking the zero-range limit, neither ^{6}Li nor ^{8}Be are predicted particle stable by the leading-order theory.

The implication of this result for the usefulness of the pionless leading-order effective field theory for the description of these nuclei, the trajectory of the bound-state poles through the respective thresholds at λ_c is crucial. To this end, we apply the method of analytical continuation in the coupling constant (ACCC, see, *e.g.* Ref. [24]) to the $2+1$ case which represents, *e.g.* , a three-neutron system. Thereby, an attractive three-body contact term is introduces with strength d_0^Λ of the same structure as the one which renormalizes the bosonic three-body system in (2). The delta functions of this term use a cutoff parameter Λ which is identical to the two-body cutoff. For a range of cutoffs (see fig. 2), the initial d_0^Λ was chosen to bind the $2+1$ system before taking the limit $d_0^\Lambda \to 0$ while following the bound state pole on its way on the physical (energy) sheet through the branch cut starting at

$E = 0$ from above onto the fourth quadrant of the unphysical sheet. On The latter (hatched area in fig. 2), it represents a resonance. The pole remains on this sheet for $\Lambda \lesssim 4\,\mathrm{fm}^{-1}$. For larger cutoffs, the pole passes through another branch cut and leaves the first unphysical sheet. Thus, it no longer represents a dimer-particle resonance. Furthermore, its trajectory does not indicate to converge to a point on this second unphysical sheet. This behaviour suggests that an analogue of the dynamical pole generated (not generated by a single diagram but rather an infinite geometric series) by the contact theory (2) in the two- and three-boson systems, does not exist in $2{+}1^{(A-1)}$, neither as a bound nor resonant state.

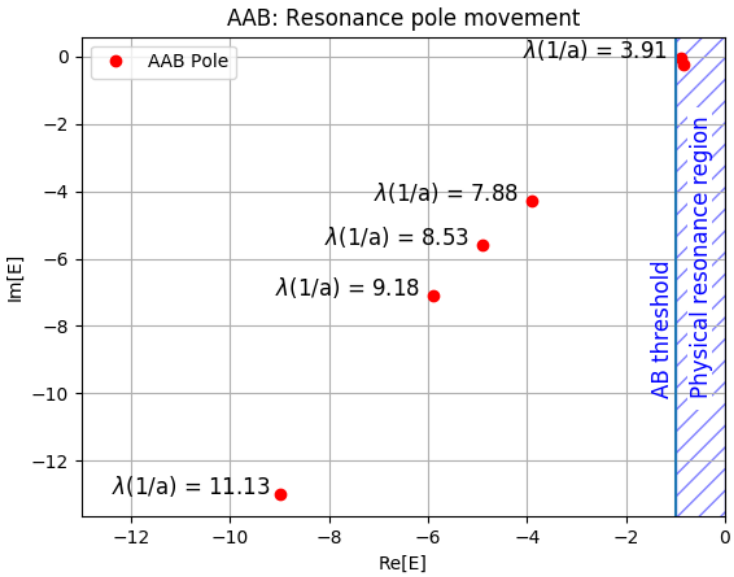


Figure 2: Lowest Hamiltonian eigenvalue analytically continued below the lowest two-/three-fragment breakup (ordinate/vertical at $\mathrm{Re}[E] = -1$ MeV) threshold for a $2+1$ system. Eigenvalues in the hatched region correspond to resonances which disappear with increasing regulator cutoff (λ label) through the three-fragment breakup branch cut.

To relate this observation to EFT($\not\pi$), in particular, the magnitude of the critical cutoff relative the scales in the theory is of importance. λ_c for ^{6}Li is arguably of the same order as the breakdown scale $\lambda_c \sim 1.5$ $\mathrm{fm}^{-1} \approx 300$ MeV set by the pion mass. The EFT framework demands to study cutoff/renormalization-group independence of the theory at values significantly larger than any presumed breakdown scale. Thereby, the inter-

pretation of the stable six-body bound states as a cutoff artefact is justified.

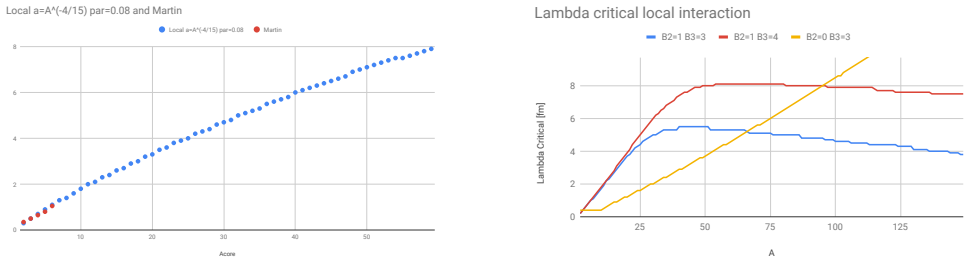
For the conception of an extension of the EFT($\not\pi$)which predicts also the particle-stable character of ^{6}Li and ^{8}Be in the zero-range limit, we analyse the mechanism behind the stability of these systems for $\Lambda < \lambda_c$. Of all artefacts introduced by the finite range of the regulated contact interaction, we single out: First, a finite effective range in the two-body S-wave channel, and second, a non-zero, attractive two-body P-wave interaction. Both contribute to the attraction in the $2{+}1^{(A-1)}$ system but their relative significance in this role is obscure. In other words, the finite-range interaction does not only describe a finite but large S-wave scattering length but also other finite parameters of the effective-range expansion of the S-wave amplitude, the S-wave effective range r_0 being one of these. Furthermore, the scattering volume a_1 of the two-nucleon P-wave amplitude is non-zero. By studying the sensitivity of the bound $2{+}1^{(A-1)}$system below λ_c *wrt.* a variation of either of those parameters instead of the combined change as induced by varying Λ as in the analysis presented thus far, insight about the order of a potential enhancement of the corresponding higher-order-in-LO-EFT($\not\pi$)vertices is gained.

First, we project the two-body interaction in a spin 0, *i.e.* , an asymmetric internal state. This forces two interacting particles into an even spatial state. Non-zero matrix elements between states in odd partial waves cancel. The effect of removing this contribution is shown in fig. 5. Namely, a significant reduction of λ_c^A. Without the residual attraction in odd partial waves, the binding is attributed mainly to the finite r_0 and a coupling to even larger relative two-body angular momenta.

What is our proposal for a contact theory which describes all what EFT($\not\pi$) does and 6Li et al.?

4. Conclusion

A non-relativistic system of $A+1$ particles with identical masses and an A-dimensional internal flavour space cannot sustain a bound state if its dynamics is constrained by representations of two- and three-particle momentum-independent contact interactions which are renormalized to yield a bound/unitary two-body state and a bound three-body state and whose residual finite-range is below a critical value (λ_c^1) which decreases with increasing particle content A of the system. If the range of the regulated contact interactions, however, surpasses the critical range, the ground state of the



(a) (b)

Figure 3: Dependence of the critical cutoff (λ_c) on the number of core particles (A). Red dots in (a) mark SVM few-body calculations. All other curves represent results obtained within a single-channel resonating-group method. In (b), the effect of a change in the ratio between two- and three-boson binding energies on λ_c is shown. For $B(2) = 0$, the respective scattering length $\to \infty$.

$A + 1$ particles is bound with respect to breakup in the bosonic, spatially symmetric A-body ground state and a single free particle. The total orbital angular momentum of this $A + 1$ bound state is $L_{\text{total}} = 1$. For any finite ratio $\Lambda^* < \infty$, the critical range reaches a minimum at a certain number of particles A.

5. Appendix: Inter-cluster Potential

We shall sketch the derivation of the potential between a core system of A equal-mass bosons (spatially symmetric in a ground state with energy $B(A)$) and a single particle (mass-equal to all but flavour identical to only one of the core constituents) as it follows from a two- and three-boson contact interaction as discussed above following (2). Such an effective 2-body potential is appropriate for the description of the $A + 1$ body amplitude close[1] to the $A + 1$ normal threshold. We assume that the wave functions of stable $2+1^{(A-1)}$ structures which were found in the explicit $A + 1$ body calculations either below λ_c or via the ACCC procedure at this Am branch point are dominated by a two-fragment configuration of an inert core and a single particle. This assumption is equivalent to the single-channel version of

[1]The energy of the relative motion is small relative to the splitting between $B(A)$ and the next-lowest A-body threshold

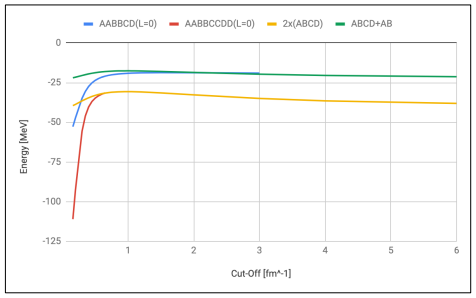


Figure 4: Cutoff dependence of the lowest Hamiltonian eigenvalue in the ^{8}Be (red) and ^{6}Li (blue) channels along with the lowest threshold, α-α (yellow) and α-deuteron (green), respectively.

System	$2+2$	$2+2+1+1$
L_{total}	r_c [fm]	r_c [fm]
0	4.16	1.23
1	5.21	2.96
2	4.81	2.30

Table 1: Minimal effective range to stabilize the $2+2$ (dimer-dimer) and the $2+2+1+1$ (^{6}Li nucleus) systems in different partial waves.

a resonating-group formulation of the many-body problem (see *e.g.* Ref.[25]) and reduces the $A+1$ body Schrödinger equation to

$$\left\langle \phi_A \middle| \left(\hat{T}_{\boldsymbol{R}} + \hat{V}_{\mathrm{A,A+1}} - E\right) \hat{A} \left[\phi_A \psi(\boldsymbol{R})\right] \right\rangle = 0 \quad , \tag{4}$$

which is an equation for the relative motion ψ (a function of the relative distance between the centre of mass between the core and the particle's location) between the core in state ϕ_A and a single particle. The average is taken *wrt.* the internal coordinates[1] of the fragment $\left\langle \dots \right\rangle = \prod_{i=1}^{A-1} \int d^3\bar{\boldsymbol{r}}_{\mathrm{i}}$, $E = E_{\text{total}} - B(A)$, $\hat{A} = \mathbb{1} - \hat{P}(\boldsymbol{r}_{\mathrm{A}} \leftrightarrow \boldsymbol{r}_{\mathrm{A+1}})$, and

$$\begin{aligned} \hat{V}_{\mathrm{A,A+1}} = \; & C_0^\Lambda \sum_{i \le A} \delta_\Lambda(\boldsymbol{r}_{\mathrm{i}} - \boldsymbol{r}_{\mathrm{A+1}}) \\ + D_1^\Lambda \sum_{\substack{i \neq j \\ \le A}} & \left\{ \delta_\Lambda(\boldsymbol{r}_{\mathrm{i}} - \boldsymbol{r}_{\mathrm{A+1}}) \delta_\Lambda(\boldsymbol{r}_{\mathrm{j}} - \boldsymbol{r}_{\mathrm{A+1}}) + \delta_\Lambda(\boldsymbol{r}_{\mathrm{i}} - \boldsymbol{r}_{\mathrm{A+1}}) \delta_\Lambda(\boldsymbol{r}_{\mathrm{j}} - \boldsymbol{r}_{\mathrm{j}}) \right\} \quad . \end{aligned} \tag{5}$$

We approximate the ground-state core wave function with

$$\phi_A := e^{-\frac{a}{2} \sum_{i=1}^{A} \bar{\boldsymbol{r}}_{\mathrm{i}}^2} \quad , \tag{6}$$

[1]We chose coordinates relative to the centre of mass of the fragment, $\bar{\boldsymbol{r}}_{\mathrm{i}} := \boldsymbol{r}_{\mathrm{i}} - \frac{\sum_{i=1}^{A} \boldsymbol{r}_{\mathrm{i}}}{A} \quad i \in \{1, \dots, A\}$, and assume without loss of generality that particles A and $A+1$ are identical.

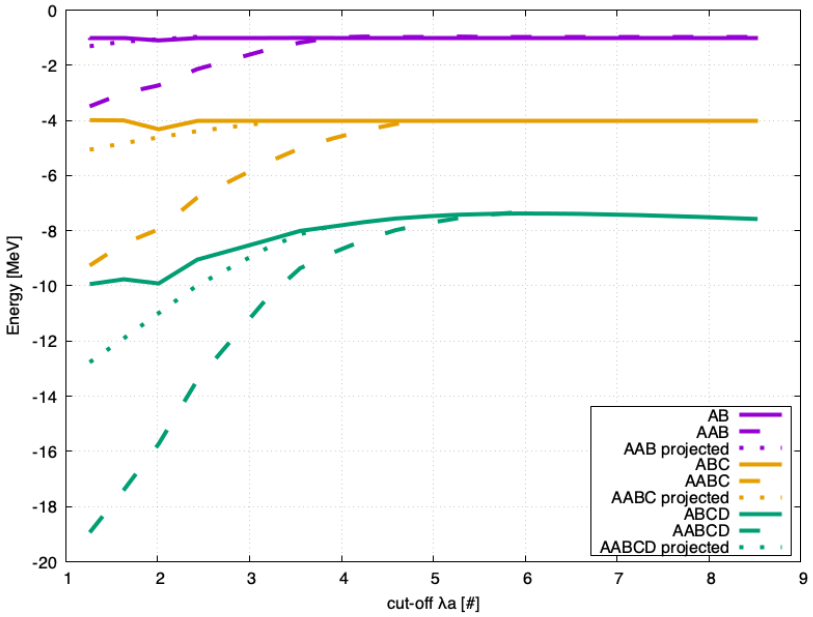


Figure 5: Cutoff dependence of the lowest energy eigenvalue of an $2+1^{(A-1)}$ fermionic systems ($A = 2$: purple, $A = 3$: beige, $A = 4$: green) as predicted with a projected (dotted line) and non-projected (dashed lines) interaction along with the lowest threshold set by the ground-state energy of the spatially symmetric A-boson core.

which is a localized, totally symmetric state parametrized by an oscillator strength of unknown dependence on A and Λ. The average over internal coordinates can then be evaluated analytically, and the resultant equation projected into partial waves assumes the final, non-local form

$$\int \left\{ \frac{\hbar^2}{2\mu} \left[-\partial_R^2 \left(\mathbb{1} + (\mathfrak{o}_E \leftrightarrow \mathfrak{o}_\mu) \right) + \frac{l(l+1)}{R^2} \left(\mathbb{1} + (\mathfrak{o}_E \leftrightarrow \mathfrak{o}_L) \right) \right] \phi_{lm}(R') \right. \tag{7a}$$

$$- E \left(\delta(R - R') + (-)^{l+1} \mathfrak{o}_E(1) \left(\frac{a}{\pi} \right)^{3/2} e^{+\frac{1}{2}\mathfrak{o}_E(aA^{-1})RR' - \frac{1}{2}\mathfrak{o}_E(a)\boldsymbol{R}^2 - \frac{1}{2}\mathfrak{o}_E(a)\boldsymbol{R}'^2} \right) \tag{7b}$$

$$+ \mathfrak{o}_2(A) \cdot \frac{C_0^\Lambda}{\Lambda^3} \cdot a^{\frac{3}{2}} \cdot e^{-\mathfrak{o}_2(a)\boldsymbol{R}^2} \left(\delta(R - R') + (-)^{l+1} \mathfrak{o}_2(1) \left(\frac{a}{\pi} \right)^{3/2} e^{+\mathfrak{o}_2(aA^{-1})RR' - \mathfrak{o}_2(a)\boldsymbol{R}'^2} \right) \tag{7c}$$

$$\left. + 2 \cdot \mathfrak{o}_3(A^2) \cdot \frac{D_0^\Lambda}{\Lambda^6} \cdot a^3 \cdot e^{-2\mathfrak{o}_3(a)\boldsymbol{R}^2} \left(\delta(R - R') + (-)^{l+1} \mathfrak{o}_3(1) \left(\frac{a}{\pi} \right)^{3/2} e^{+2\mathfrak{o}_3(aA^{-1})RR' - 2\mathfrak{o}_3(a)\boldsymbol{R}'^2} \right) \right\} \phi_{lm}(R') \tag{7d}$$

$$= 0 \quad .$$

The quantities $\mathfrak{o}_x$ represent numbers of order one times the arguments in braces.

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