$$\langle \phi_A | (\hat{T}_{\underline{R}} - E_R + \hat{V}) | \hat{A}[\phi_A \chi_{\underline{R}}] \rangle = 0$$

$$\Leftrightarrow \quad \langle \phi_A | (\hat{T}_{\underline{R}} - E_R + \hat{V}) | (\mathbb{1} - \hat{P})[\phi_A \chi_{\underline{R}}] \rangle = 0$$

$$(\hat{T}_{\underline{R}} - E_R)\, \mathbb{N}\, \chi_{\underline{R}} + \langle \phi_A | \hat{V} (\mathbb{1} - \hat{P}) [\phi_A \chi_{\underline{R}}] \rangle - (\hat{T}_{\underline{R}} - E_R) \langle \phi_A | \hat{P} [\phi_A \chi_{\underline{R}}] \rangle = 0$$

$$\frac{\hbar^2}{2m}\left(-R^{-2} \partial_R R^2 \partial_R + \frac{L^2}{R^2}\right)$$

$$\mathbb{N} = \langle \phi_A | \phi_A \rangle \; ; \quad \chi_{\underline{R}} \equiv R^{-1} \sum_{\ell m} \psi_{\ell m}(R)\, Y_{\ell m}(\hat{\underline{R}}) \; ; \quad R \int d^2\hat{\underline{R}}\; Y_{\ell' m'}(\hat{\underline{R}}) \;\text{(projection)}, \quad m = \frac{A m \cdot m}{A m + m} = \frac{A}{A+1} m$$

$$\Rightarrow \quad \frac{\hbar^2}{2m}\left(\partial_R^2 - E_r + \frac{\ell(\ell+1)}{R^2}\right) \psi_{\ell m}(R) + \mathbb{N}^{-1} \underbrace{\langle \phi_A | \hat{V} | \phi_A \rangle}_{f^+_{2/3}\, e^{-\alpha^+_{2/3} R^2}} \psi_{\ell m}(R)$$

$$-\int d^2\hat{R}\, R\, Y^*_{\ell' m'}(\hat{R}) \left(+\frac{\hbar^2}{2m}\left(-R^{-2}\partial_R R^2 \partial_R - E + \frac{\hat{L}^2}{R^2}\right)\right) \underbrace{\int dR'\, R'^2 \int d^2\hat{\underline{R}}'}_{\int d^3\underline{R}} \underbrace{\langle \phi_A | \hat{P}[\delta(\underline{R} - \underline{R}') \phi_A] \rangle}\, R'^{-1} \sum_{\ell m} Y_{\ell m}(\hat{\underline{R}}') \psi_{\ell m}(R') \cdot \mathbb{N}^{-1}$$

$$= f_{1} \cdot e^{-\alpha_1 \underline{R}^2 - \beta_1 \underline{R}\cdot\underline{R}' - \gamma_1 \underline{R}'^2}$$

$$= f_1\, e^{-\alpha_1 R^2 - \gamma_1 R'^2} \cdot 4\pi \sum_{LM} i^L j_L(i\beta_1 R R')\, Y^*_{LM}(\hat{\underline{R}})\, Y_{LM}(\hat{\underline{R}}')$$

$$-\int d^2\hat{\underline{R}}\, R\, Y_{\ell' m'}(\hat{\underline{R}}) \int dR'\, R'^2 \int d^2\hat{\underline{R}}' \underbrace{\langle \phi_A | \hat{V}\, \hat{P}[\delta(\underline{R} - \underline{R}') \phi_A] \rangle}\, R'^{-1} \sum_{\ell m} Y_{\ell m}(\hat{\underline{R}}') \psi_{\ell m}(R') \cdot \mathbb{N}^{-1} = 0$$

$$= f^-_{2/3}\, e^{-\alpha^-_{2/3} R^2 - \beta^-_{2/3} \underline{R}\cdot\underline{R}' - \gamma^-_{2/3} R'^2}$$

$$\frac{2m}{\hbar^2} \mathbb{N}^{-1} \equiv n$$

$$\Rightarrow \quad \left(\partial_R^2 - E_{rel} + \frac{\ell(\ell+1)}{R^2}\right) \psi_{\ell m}(R) = n\left(f_2^+ e^{-\alpha_2^+ R^2} + f_3^+ e^{-\alpha_3^+ R^2}\right) \psi_{\ell m}(R)$$

$$- 4\pi f_1 \mathbb{N}^{-1} \int dR'\, R e^{-\gamma_1 R'^2} \left(-R^{-2}\partial_R R^2 \partial_R - E + \frac{\ell(\ell+1)}{R^2}\right) R' e^{-\alpha_1 R^2}\, i^\ell j_\ell(i\beta_1 R R')\; \psi_{\ell m}(R')$$

$$- n\, 4\pi \int dR'\, R R'\, i^\ell \left(j_\ell(i\beta_2 R R')\, f_2^- e^{-\alpha_2^- R^2 - \gamma_2^- R'^2} + j_\ell(i\beta_3 R R')\, f_3^- e^{-\alpha_3^- R^2 - \gamma_3^- R'^2}\right) \psi_{\ell m}(R')$$