d. Example: dimer-dimer scattering – 2-component Fermions The scattering of two identical dimers, each comprised of equal-mass particles which interact resonantly (scattering length significantly larger than the effective range $a^{-1}r \to 0$), at an energy sufficiently low such that the effects of branch cuts due to dimer disintegration and excitation can be neglected, shall serve as a benchmark. For this experiment, the remarkable result

$$\frac{a_{dd}}{a} \approx 0.6 \tag{15}$$

for the ratio between S-wave scattering lengths of the dimer-dimer amplitude a_{dd} and the resonant two-fermion system a has been found[5] in a four-body calculation. We use this result as a standard in order to demonstrate the accuracy of the RGM approximation, *i.e.* , the reformulation of the dimer-dimer problem as a two-body system, as well as that of the numerical solution of the ensuing non-local equation.

Pertinent to this system are the following quantities:

$$A = 4 \quad , \quad \mu = m \text{ (single-particle mass)} \quad , \frac{\hbar^2}{2\mu} \overset{\text{nuclear}}{=} 20.7 \text{ MeV} \cdot \text{fm}^2 \tag{16}$$

$$\phi_{A(B)} = e^{-\alpha \bar{\boldsymbol{r}}^2_{1(3)}} \quad , \tag{17}$$

$$\mathcal{A} = \mathbb{1} - \hat{P}_{13} - \hat{P}_{24} + \hat{P}_{13}\hat{P}_{24} \quad , \tag{18}$$

$$\mathcal{V} = C_0(\Lambda) \sum_{(i,j)\in X} e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i - \boldsymbol{r}_j)^2} \quad , \quad X = \{(13),(24)\} \quad ; \tag{19}$$

The three-dimensional incarnation (2) which follows reads

$$(\hat{T} - E)\ \chi(\boldsymbol{r}) + \mathcal{V}^{(1)}(\boldsymbol{r})\ \chi(\boldsymbol{r}) + \int d^{(3)}\boldsymbol{r}'\ \mathcal{V}^{(2)}(\boldsymbol{r},\boldsymbol{r}',E)\ \chi(\boldsymbol{r}') = 0 \tag{20}$$

with a *local* potential which receives contributions from the identity and the complete particle exchange

$$\mathcal{V}^{(1)}(\boldsymbol{r}) = C_0(\lambda) \cdot \left(\frac{2\alpha}{2\alpha+\lambda}\right)^{3/2} \cdot e^{-\frac{2\alpha\lambda}{2\alpha+\lambda}\boldsymbol{r}^2} \tag{21}$$

and a *non-local*, energy-dependent potential feeding from the kinetic and potential acting on single, odd exchanges of a single atom between the clusters

$$\mathcal{V}^{(2)}(\boldsymbol{r},\boldsymbol{r}',E) = 8\ \alpha^{3/2} \cdot \left[\frac{\hbar^2}{2\mu}\ (4\alpha^2\boldsymbol{r}^2 - 2\alpha) + E\right] \cdot e^{-\alpha(\boldsymbol{r}'^2 + \boldsymbol{r}^2)} \tag{22}$$

$$- C_0(\lambda) \cdot \frac{32\sqrt{2}\alpha^3}{(2\alpha+\lambda)^{3/2}} \cdot e^{-\alpha\left(\boldsymbol{r}'^2 + \frac{2\alpha+3\lambda}{2\alpha+\lambda}\boldsymbol{r}^2\right)} \quad . \tag{23}$$

It is in order to consider the following limits:

- $\lambda \gg \alpha$
- $\int d^{(3)}\boldsymbol{r}'\ \mathcal{V}^{(2)}(\boldsymbol{r},\boldsymbol{r}',E)\ \chi(\boldsymbol{r}') \overset{E\to 0}{\approx} \chi(\boldsymbol{r}) \cdot v^{(2)}(\boldsymbol{r}) \cdot \int d^{(3)}\boldsymbol{r}'\ v^{(2)}(\boldsymbol{r}')$.

A comparison between the ensuing local, zero-range approximation

$$\mathcal{V}^{(0)}(\boldsymbol{r}) = \frac{\hbar^2}{2\mu}\ (4\alpha^2\boldsymbol{r}^2 - 2\alpha) \cdot 8\ \pi^{3/2} \cdot e^{-\alpha\boldsymbol{r}^2} + C_0(\lambda) \cdot \left(\frac{2\alpha}{2\alpha+\lambda}\right)^{3/2} \cdot \left(e^{-2\alpha\boldsymbol{r}^2} - 16\pi^{3/2}e^{-3\alpha\boldsymbol{r}^2}\right) \tag{24}$$

and the full RGM potential will enable us to quantify and trace the effect of the particle statistics.

ECCE the following:

- As the non-local potential factorizes, there is no mixing of partial waves and (20) applies to each partial wave

$$\left(-\frac{\hbar^2}{2\mu}\left(\partial_r^2 - \frac{l(l+1)}{r^2}\right) - E\right)\ \chi_l(r) + \mathcal{V}^{(1)}(r)\ \chi_l(r) + 4\pi \int dr'\ (rr')\ \mathcal{V}^{(2)}(r,r',E)\ \chi_l(r') = 0 \tag{25}$$

  with a strictly local angular momentum barrier and a factor of 4π which stems from the two independent angular integration averages of $\chi_l(r)$ and $\chi_l(r')$. We used $\chi(\boldsymbol{r}) = r^{-1}\sum_{lm}\chi_l(r)Y_{lm}(\hat{\boldsymbol{r}})$, and projected "from the left" with $r\int d^{(2)}\hat{\boldsymbol{r}}\ Y^*_{l'm'}(\hat{\boldsymbol{r}})$.

- Lorenzo, $\lambda = \frac{\Lambda^2}{4}$ and hence the pre-factor of the second term runs $\propto \Lambda^{-1}$ in EFT($\not{\pi}$) . Therefore, For $\Lambda \to \infty$, a model-independent interaction remains which encodes microscopic dimer characteristics via α.