

I. DIMER-DIMER SCATTERING

In the single-channel approximation, the resonating-group equation assumes the non-local form

$$(\hat{T} - E) \chi(\mathbf{r}) + \mathcal{V}^{(1)}(\mathbf{r}) \chi(\mathbf{r}) + \int d^{(3)}\mathbf{r}' \mathcal{V}^{(2)}(\mathbf{r}, \mathbf{r}', E) \chi(\mathbf{r}') = 0 \quad (1)$$

with the radial coordinates denoting the spatial separation between the two fragments. If these fragments are two-body S -wave bound states comprised of equal-mass fermions, the effective potentials which derive from a zero-range fermion-fermion interaction are given for a two- and three-species system. We denote the former as $(ab):(ab)$ (scale invariant), and the latter $(ab):(ca)$ (discretely scale invariant, Thomas collapse of (abc)). The characteristic three-body scale in an $(ab):(ca)$ system flows into the effective dimer-dimer potentials, while in the absence of such a scale in the zero-range two-body limit, the effective potentials are parametrized by the dimer, *i.e.*, a two-body observable, only.

In detail,

$(ab):(ab):$

$$\mathcal{V}_{(ab):(ab)}^{(1)}(\mathbf{r}) = 2 C_0(\lambda) \cdot \left(\frac{2\alpha}{2\alpha + \lambda} \right)^{3/2} \cdot e^{-\frac{2\alpha\lambda}{2\alpha + \lambda} \mathbf{r}^2}, \quad (2)$$

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$$\mathcal{V}_{(ab):(ab)}^{(2)}(\mathbf{r}, \mathbf{r}', E) = 8 \alpha^{3/2} \cdot e^{-\alpha \mathbf{r}'^2} \cdot \left[\frac{\hbar^2}{2\mu} (4\alpha^2 \mathbf{r}^2 - 2\alpha) \cdot e^{-\alpha \mathbf{r}^2} + E \cdot e^{-\alpha \mathbf{r}^2} - 2 C_0(\lambda) \cdot \left(\frac{2\alpha}{2\alpha + \lambda} \right)^{3/2} \cdot e^{-\alpha \cdot \frac{2\alpha + 3\lambda}{2\alpha + \lambda} \mathbf{r}^2} \right]. \quad (3)$$

$(ab):(ca):$

$$\mathcal{V}_{(ab):(ca)}^{(1)}(\mathbf{r}) = 3 \cdot C_0(\lambda) \cdot \left(\frac{2\alpha}{2\alpha + \lambda} \right)^{3/2} \cdot e^{-\frac{2\alpha\lambda}{2\alpha + \lambda} \mathbf{r}^2} \quad (4)$$

$$+ D_0(\lambda) \cdot \left(\left(\frac{2\alpha}{2\alpha + \lambda} \right)^3 \cdot e^{-\frac{4\alpha\lambda}{2\alpha + \lambda} \mathbf{r}^2} + \left(\frac{2\alpha}{\sqrt{(2\alpha + \lambda)^2 + 2\alpha\lambda}} \right)^3 \cdot e^{-\frac{4\alpha\lambda(\alpha + \lambda)}{4\alpha^2 + 6\alpha\lambda + \lambda^2} \mathbf{r}^2} \right) \quad (5)$$

$$\mathcal{V}_{(ab):(ca)}^{(2)}(\mathbf{r}, \mathbf{r}', E) = 8 \alpha^{3/2} \cdot \left(e^{-\alpha \mathbf{r}'^2} \cdot \left[\frac{\hbar^2}{2\mu} (4\alpha^2 \mathbf{r}^2 - 2\alpha) \cdot e^{-\alpha \mathbf{r}^2} + E \cdot e^{-\alpha \mathbf{r}^2} \right] \right. \quad (6)$$

$$\left. - C_0(\lambda) \cdot e^{-(\alpha + \lambda)(\mathbf{r}^2 + \mathbf{r}'^2) - 2\lambda \mathbf{r}' \cdot \mathbf{r}} - 2 C_0(\lambda) \cdot \left(\frac{2\alpha}{2\alpha + \lambda} \right)^{3/2} \cdot e^{-\alpha \cdot (\mathbf{r}'^2 + \frac{2\alpha + 3\lambda}{2\alpha + \lambda} \mathbf{r}^2)} \right) \quad (7)$$

$$- D_0(\lambda) \cdot \left(\frac{\alpha}{\alpha + \lambda} \right)^{3/2} \cdot e^{-\frac{2\alpha^2 + 4\alpha\lambda + \lambda^2}{2(\alpha + \lambda)} (\mathbf{r}^2 + \mathbf{r}'^2) - \frac{\lambda^2}{\alpha + \lambda} \mathbf{r} \cdot \mathbf{r}'} \quad (8)$$

$$- D_0(\lambda) \cdot \left(\frac{2\alpha(\alpha + \lambda)}{2\alpha^2 + 3\alpha\lambda + \lambda^2} \right)^{3/2} \cdot e^{-\frac{2\alpha^2 + 5\alpha\lambda + \lambda^2}{2(\alpha + \lambda)} \mathbf{r}^2 - (\alpha + \lambda) \mathbf{r}'^2 - 2\lambda \mathbf{r} \cdot \mathbf{r}'} \quad (9)$$

It is in order to consider the following limits:

zero-range or contact limit: $\lambda \gg \alpha$

local approximation: $\int d^{(3)}\mathbf{r}' \mathcal{V}^{(2)}(\mathbf{r}, \mathbf{r}', E) \chi(\mathbf{r}') \xrightarrow{E \rightarrow 0} \chi(\mathbf{r}) \cdot v^{(2)}(\mathbf{r}) \cdot \int d^{(3)}\mathbf{r}' v^{(2)}(\mathbf{r}')$

Assuming an unnaturally large dimer scale emergent from a relatively short-ranged fermion-fermion interaction, the zero-range approximation is justified and the ensuing dimer-dimer potentials read:

(zero-range) (ab):(ab):

$$\mathcal{V}_{(ab):(ab)}^{(1)}(\mathbf{r}) = 2 (2\alpha)^{3/2} \frac{C_0(\lambda)}{\lambda^{3/2}} \cdot e^{-2\alpha\mathbf{r}^2} , \quad (10)$$

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$$\mathcal{V}_{(ab):(ab)}^{(2)}(\mathbf{r}, \mathbf{r}', E) = 8 \alpha^{3/2} \cdot e^{-\alpha\mathbf{r}'^2} \cdot \left[\frac{\hbar^2}{2\mu} (4\alpha^2\mathbf{r}^2 - 2\alpha) \cdot e^{-\alpha\mathbf{r}^2} + E \cdot e^{-\alpha\mathbf{r}^2} - 2 (2\alpha)^{3/2} \frac{C_0(\lambda)}{\lambda^{3/2}} \cdot e^{-3\alpha\mathbf{r}^2} \right] . \quad (11)$$

(zero-range) (ab):.(ca):

$$\mathcal{V}_{(ab):.(ca)}^{(1)}(\mathbf{r}) = 3 (2\alpha)^{3/2} \frac{C_0(\lambda)}{\lambda^{3/2}} \cdot e^{-2\alpha\mathbf{r}^2} + 2 (2\alpha)^3 \frac{D_0(\lambda)}{\lambda^3} \cdot e^{-4\alpha\mathbf{r}^2} \quad (12)$$

$$\mathcal{V}_{(ab):.(ca)}^{(2)}(\mathbf{r}, \mathbf{r}', E) = 8 \alpha^{3/2} \cdot \left(e^{-\alpha\mathbf{r}'^2} \cdot \left[\frac{\hbar^2}{2\mu} (4\alpha^2\mathbf{r}^2 - 2\alpha) \cdot e^{-\alpha\mathbf{r}^2} + E \cdot e^{-\alpha\mathbf{r}^2} \right] \right. \quad (13)$$

$$\left. - C_0(\lambda) \cdot e^{-\lambda(\mathbf{r}+\mathbf{r}')^2} - 2 (2\alpha)^{3/2} \frac{C_0(\lambda)}{\lambda^{3/2}} \cdot e^{-\alpha\mathbf{r}'^2 - 3\alpha\mathbf{r}^2} \right. \quad (14)$$

$$\left. - \alpha^{3/2}(1 + 2^{3/2}) \frac{D_0(\lambda)}{\lambda^{3/2}} \cdot e^{-\frac{\lambda}{2}(\mathbf{r}+\mathbf{r}')^2} \right) \quad (15)$$