I. DIMER-DIMER SCATTERING

In the single-channel approximation, the resonating-group equation assumes the non-local form

$$(\hat{T}-E)\;\chi(\boldsymbol{r})+\mathcal{V}^{(1)}(\boldsymbol{r})\;\chi(\boldsymbol{r})+\int d^{(3)}\boldsymbol{r}'\;\mathcal{V}^{(2)}(\boldsymbol{r},\boldsymbol{r}',E)\;\chi(\boldsymbol{r}')=0 \tag{1}$$

with the radial coordinates denoting the spatial separation between the two fragments. If these fragments are two-body S-wave bound states comprised of equal-mass fermions, the effective potentials which derive from a zero-range fermion-fermion interaction are given for a two- and three-species system. We denote the former as (ab):(ab) (scale invariant), and the latter (ab)∴(ca)(discretely scale invariant, Thomas collapse of (abc)). The characteristic three-body scale in an (ab)∴(ca) system flows into the effective dimer-dimer potentials, while in the absence of such a scale in the zero-range two-body limit, the effective potentials are parametrized by the dimer, *i.e.* , a two-body observable, only.

In detail,

(ab):(ab)**:**

$$\mathcal{V}^{(1)}_{(ab):(ab)}(\boldsymbol{r})=2\,C_0(\lambda)\cdot\left(\frac{2\alpha}{2\alpha+\lambda}\right)^{3/2}\cdot e^{-\frac{2\alpha\lambda}{2\alpha+\lambda}\boldsymbol{r}^2}\quad, \tag{2}$$

$$\mathcal{V}^{(2)}_{(ab):(ab)}(\boldsymbol{r},\boldsymbol{r}',E)=\;8\,\alpha^{3/2}\cdot e^{-\alpha\boldsymbol{r}'^2}\cdot\left[\frac{\hbar^2}{2\mu}\;(4\alpha^2\boldsymbol{r}^2-2\alpha)\cdot e^{-\alpha\boldsymbol{r}^2}+E\cdot e^{-\alpha\boldsymbol{r}^2}-2\;C_0(\lambda)\cdot\left(\frac{2\alpha}{2\alpha+\lambda}\right)^{3/2}\cdot e^{-\alpha\cdot\frac{2\alpha+3\lambda}{2\alpha+\lambda}\boldsymbol{r}^2}\right]\;. \tag{3}$$

(ab)**∴**(ca)**:**

$$\mathcal{V}^{(1)}_{(ab)\therefore(ca)}(\boldsymbol{r})=3\cdot C_0(\lambda)\cdot\left(\frac{2\alpha}{2\alpha+\lambda}\right)^{3/2}\cdot e^{-\frac{2\alpha\lambda}{2\alpha+\lambda}\boldsymbol{r}^2} \tag{4}$$

$$+D_0(\lambda)\cdot\left(\left(\frac{2\alpha}{2\alpha+\lambda}\right)^3\cdot e^{-\frac{4\alpha\lambda}{2\alpha+\lambda}\boldsymbol{r}^2}+\left(\frac{2\alpha}{\sqrt{(2\alpha+\lambda)^2+2\alpha\lambda}}\right)^3\cdot e^{-\frac{4\alpha\lambda(\alpha+\lambda)}{4\alpha^2+6\alpha\lambda+\lambda^2}\boldsymbol{r}^2}\right) \tag{5}$$

$$\mathcal{V}^{(2)}_{(ab)\therefore(ca)}(\boldsymbol{r},\boldsymbol{r}',E)=8\;\alpha^{3/2}\cdot\left(e^{-\alpha\boldsymbol{r}'^2}\cdot\left[\frac{\hbar^2}{2\mu}\;(4\alpha^2\boldsymbol{r}^2-2\alpha)\cdot e^{-\alpha\boldsymbol{r}^2}+E\cdot e^{-\alpha\boldsymbol{r}^2}\right]\right. \tag{6}$$

$$-C_0(\lambda)\cdot e^{-(\alpha+\lambda)(\boldsymbol{r}^2+\boldsymbol{r}'^2)-2\lambda\boldsymbol{r}'\cdot\boldsymbol{r}}-2\;C_0(\lambda)\cdot\left(\frac{2\alpha}{2\alpha+\lambda}\right)^{3/2}\cdot e^{-\alpha\cdot\left(\boldsymbol{r}'^2+\frac{2\alpha+3\lambda}{2\alpha+\lambda}\boldsymbol{r}^2\right)} \tag{7}$$

$$-\;D_0(\lambda)\cdot\left(\frac{\alpha}{\alpha+\lambda}\right)^{3/2}\cdot e^{-\frac{2\alpha^2+4\alpha\lambda+\lambda^2}{2(\alpha+\lambda)}(\boldsymbol{r}^2+\boldsymbol{r}'^2)-\frac{\lambda^2}{\alpha+\lambda}\boldsymbol{r}\cdot\boldsymbol{r}'} \tag{8}$$

$$\left.-D_0(\lambda)\cdot\left(\frac{2\alpha(\alpha+\lambda)}{2\alpha^2+3\alpha\lambda+\lambda^2}\right)^{3/2}\cdot e^{-\frac{2\alpha^2+5\alpha\lambda+\lambda^2}{2(\alpha+\lambda)}\boldsymbol{r}^2-(\alpha+\lambda)\boldsymbol{r}'^2-2\lambda\boldsymbol{r}\cdot\boldsymbol{r}'}\right) \tag{9}$$

It is in order to consider the following limits:

zero-range or contact limit: $\lambda\gg\alpha$

local approximation: $\int d^{(3)}\boldsymbol{r}'\;\mathcal{V}^{(2)}(\boldsymbol{r},\boldsymbol{r}',E)\;\chi(\boldsymbol{r}')\overset{E\to 0}{\approx}\chi(\boldsymbol{r})\cdot v^{(2)}(\boldsymbol{r})\cdot\int d^{(3)}\boldsymbol{r}'\;v^{(2)}(\boldsymbol{r}')$.

Assuming an unnaturally large dimer scale emergent from a relatively short-ranged fermion-fermion interaction, the zero-range approximation is justified and the ensuing dimer-dimer potentials read:

(zero-range) (ab):(ab)**:**

$$\mathcal{V}^{(1)}_{(ab):(ab)}(\boldsymbol{r}) = 2\,(2\alpha)^{3/2}\;\frac{C_0(\lambda)}{\lambda^{3/2}}\cdot e^{-2\alpha\boldsymbol{r}^2}\quad , \tag{10}$$

$$\mathcal{V}^{(2)}_{(ab):(ab)}(\boldsymbol{r},\boldsymbol{r}',E) = \;8\;\alpha^{3/2}\cdot e^{-\alpha\boldsymbol{r}'^2}\cdot\left[\frac{\hbar^2}{2\mu}\;(4\alpha^2\boldsymbol{r}^2-2\alpha)\cdot e^{-\alpha\boldsymbol{r}^2}+E\cdot e^{-\alpha\boldsymbol{r}^2}-2\,(2\alpha)^{3/2}\;\frac{C_0(\lambda)}{\lambda^{3/2}}\cdot e^{-3\alpha\boldsymbol{r}^2}\right]\quad . \tag{11}$$

(zero-range) (ab)**∴**(ca)**:**

$$\mathcal{V}^{(1)}_{(ab)\therefore(ca)}(\boldsymbol{r}) = 3\,(2\alpha)^{3/2}\;\frac{C_0(\lambda)}{\lambda^{3/2}}\cdot e^{-2\alpha\boldsymbol{r}^2}+2\,(2\alpha)^3\;\frac{D_0(\lambda)}{\lambda^3}\cdot e^{-4\alpha\boldsymbol{r}^2} \tag{12}$$

$$\mathcal{V}^{(2)}_{(ab)\therefore(ca)}(\boldsymbol{r},\boldsymbol{r}',E) = 8\;\alpha^{3/2}\cdot\Bigg(e^{-\alpha\boldsymbol{r}'^2}\cdot\left[\frac{\hbar^2}{2\mu}\;(4\alpha^2\boldsymbol{r}^2-2\alpha)\cdot e^{-\alpha\boldsymbol{r}^2}+E\cdot e^{-\alpha\boldsymbol{r}^2}\right] \tag{13}$$

$$-C_0(\lambda)\cdot e^{-\lambda(\boldsymbol{r}+\boldsymbol{r}')^2}-2\;(2\alpha)^{3/2}\,\frac{C_0(\lambda)}{\lambda^{3/2}}\cdot e^{-\alpha\boldsymbol{r}'^2-3\alpha\boldsymbol{r}^2} \tag{14}$$

$$-\alpha^{3/2}(1+2^{3/2})\;\frac{D_0(\lambda)}{\lambda^{3/2}}\cdot e^{-\frac{\lambda}{2}(\boldsymbol{r}+\boldsymbol{r}')^2}\Bigg) \tag{15}$$

We do now interpret these potentials as vertices of interacting dimer fields – the physical nature of the fields is inessential for the following; quite generally, we applied a transformation on a renormalized contact interaction, and we are now interested in whether or not this transformation, *i.e.* , the RGM averaging over fragment-internal, “frozen” degrees of freedom, preserves the renormalized character of amplitudes of the image theory – whose regularization is inherited from the renormalized fermion-fermion interaction.

We commence the analysis of the renormalizability of the transformed dimer-dimer theory under the assumption that the transformation does not affect the power-counting rules. That means, solutions of a Schrödinger equation with and interaction as given by the non-local potentials shall be well-behaved for $\lambda \to \infty$.