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THE PHOTON SCATTERING BY ORIENTED NUCLEI

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Abstract: The general formalism of photon scattering by oriented nuclei is developed for electric dipole and electric quadrupole radiation. The differential scattering cross section can be expanded in terms of the orientation parameters f_ν, the coefficients of which contain the interference of the different polarizabilities. Explicit formulae are given for dipole radiation. The photon scattering by aligned ^{165}Ho nuclei is discussed in detail in the framework of the dynamic collective model showing the importance of the scalar-tensor polarizability interference for direct observation of the optical anisotropy of nuclei.

1. Introduction

The scattering of photons by nuclei is, and will be in the near future, an interesting and promising research field in nuclear structure physics. Especially the development of monochromatic photon beams e.g. using the annihilation of positrons in flight will greatly improve the quality of data available from photon scattering and absorption experiments. The main physical information and insight into the structure of nuclei subject to investigation by photons can be divided into the following groups:

(i) detailed structure of giant multipole resonances and other nuclear levels having appreciable strength to the ground state and low excited states,

(ii) the coupling of giant multipole resonances to low energetic collective modes (collective correlations of nucleons). The detailed structure of the wave functions with collective correlations will be sensitive to the various nuclear polarizabilities.

(iii) test of nuclear models,

(iv) test of the interaction of nuclei with the radiation field especially for high energetic photons where the Siegert theorem fails ($kR \approx 1$).

In a recent paper [1]) we have treated the photon scattering by heavy deformed nuclei in the giant resonance region. These nuclei are anisotropic oscillators with a large tensor polarizability [2]). The tensor polarizability causes a large inelastic differential scattering cross section for photon scattering into the rotational levels of the ground state band and the γ-bands [1, 3]). It contributes also to the elastic scattering cross section for odd-mass nuclei. The first experimental hint of the existence of the tensor polarizability was the discrepancy between the measured total quasi-elastic scattering cross section and the cross section calculated from the absorption cross

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section via the optical theorem [4-7]). For unoriented nuclei only the scalar polarizability contributes to the absorption cross section while both the scalar and tensor polarizabilities contribute to the quasie elastic scattering cross section. Recently the optical anisotropy of deformed nuclei associated with the tensor polarizability has been observed [8]) by showing that the γ-absorption cross section for ^{165}Ho depends upon the alignment of the nuclei with respect to the γ-ray beam.

In this paper we will discuss another possible method of direct observation of the tensor polarizability of odd-mass nuclei, namely the elastic photon scattering by oriented nuclei where interference between scalar and tensor polarizability occurs. To this end, we develop in sect. 2 the general formalism for photon scattering by oriented nuclei for electric dipole and electric quadrupole radiation. Partial polarization of the radiation is included. In sect. 3 we specialize to unpolarized radiation and oriented nuclei and compute the differential cross section in terms of the orientation parameters f_ν. Explicit formulae for electric dipole radiation are given. An application to photon scattering by aligned ^{165}Ho nuclei is presented in sect. 4. For two cases the differential cross sections as a function of energy are treated in detail. (i) The alignment is perpendicular to the scattering plane and (ii) the alignment is in the scattering plane and perpendicular to the incident photon. The different behaviour of the scalar-tensor interference term in the two cases is shown. Finally, the angular distribution of the scattered radiation for different energies is discussed.

2. General Formulae

A second-order perturbation calculation leads to the following expression for the differential scattering cross section for the scattering of incoming photons with the wave vector $\boldsymbol{k}$ and circular polarization $p\ (=\pm 1)$ into outgoing photons $(\boldsymbol{k}', p')$ (refs. [9,10]))

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{k'}{k}\,|K_{\mathrm{if}}^{\mathrm{pp}'}|^2, \tag{1}$$

where the transition amplitude is given by

$$\begin{aligned} K_{\mathrm{if}}^{\mathrm{pp}'} = & -\frac{Z^2e^2}{AMc^2}\,\boldsymbol{e}^*_{\boldsymbol{k}'p'}\boldsymbol{e}_{\boldsymbol{k}p}\delta_{\mathrm{if}} \\ & +\frac{1}{c^2}\sum_n\Big[\langle \mathrm{f}|\int \boldsymbol{j}\cdot\boldsymbol{A}^*(\boldsymbol{k}')|n\rangle\,\frac{1}{E_{ni}-E-\frac{1}{2}i\Gamma_n}\,\langle n|\int \boldsymbol{j}\cdot\boldsymbol{A}(\boldsymbol{k})|\mathrm{i}\rangle \\ & +\langle \mathrm{f}|\int \boldsymbol{j}\cdot\boldsymbol{A}(\boldsymbol{k})|n\rangle\,\frac{1}{E_{ni}+E'+\frac{1}{2}i\Gamma_n}\,\langle n|\int \boldsymbol{j}\cdot\boldsymbol{A}^*(\boldsymbol{k}')|\mathrm{i}\rangle\Big], \end{aligned} \tag{2}$$

$\boldsymbol{e}_{\boldsymbol{k}p}$ and $\boldsymbol{e}_{\boldsymbol{k}'p'}$ are the polarization unit vectors for circular polarization, E and E' the energies of the incoming and outgoing photons, respectively, $\boldsymbol{j}$ the nuclear current density, M the nucleon mass, A the mass number and $\boldsymbol{A}(\boldsymbol{k})$ the Fourier component of the vector potential

$$A(k) = e_{kp} e^{ik \cdot r}. \tag{3}$$

We expand $A(k)$ and $A^*(k')$ into multipoles with respect to an arbitrary quantization axis

$$A(k) = -\sqrt{2\pi} \sum_{L,M} i^L \sqrt{2L+1} B_{LM}(k) D^L_{Mp}(\alpha, \beta, \gamma), \tag{4a}$$

$$A^*(k') = \sqrt{2\pi} \sum_{L,M} (-)^{L-p'} i^L \sqrt{2L+1} B_{LM}(k') D^L_{M,-p'}(\alpha', \beta', \gamma'), \tag{4b}$$

where

$$B_{LM}(k) = pA_{LM}(\mathrm{M}) + iA_{LM}(\mathrm{E}), \tag{5}$$

$A_{LM}(\mathrm{M})$ and $A_{LM}(\mathrm{E})$ are the magnetic and electric multipole fields of the order L. The Euler angles α, β, γ and α', β', γ' describe the rotations R and R' of the quantization axis n into the direction of k and k', respectively (see fig. 1). We use for the rotation matrices the convention of Rose [11]). Further we have

$$e_{kp} \cdot e^*_{k'p'} = (-)^{p-p'} D^1_{-p,p'}(\alpha_r, \beta_r, \gamma_r), \tag{6}$$

if $R_r(\alpha_r, \beta_r, \gamma_r)$ rotates the direction of the incident photons into the direction of the

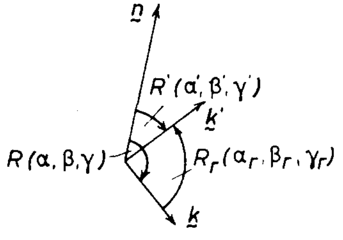

Fig. 1. The three directions (incident photon k, scattered photon k', quantization axis n) and the three rotations R, R', R_r involved in the problem.

outgoing photons (see fig. 1). Inserting (4a), (4b) and (6) into eq. (2), we get for the transition amplitude the general formula

$$\begin{aligned} K^{pp'}_{\mathrm{if}} = & -\frac{Z^2e^2}{AMc^2}(-)^{p-p'} D^1_{-p,p'}(R_r)\delta_{\mathrm{if}} \\ & -\sum_{n,L,L',M,M'} (-)^{L'-p'} \left[C^{LL'}_{1n} \langle \mathrm{f} | \int j \cdot A^*(k') | n \rangle \langle n | \int j \cdot A(k) | \mathrm{i} \rangle \right. \\ & \left. + C^{LL'}_{2n} \langle \mathrm{f} | \int j \cdot A(k) | n \rangle \langle n | \int j \cdot A^*(k') | \mathrm{i} \rangle \right] D^L_{M,p}(R) D^{L'}_{M',-p'}(R'), \end{aligned} \tag{7}$$

where

$$C^{LL'}_{1n} = \frac{2\pi}{c^2} i^{L+L'} \sqrt{(2L+1)(2L'+1)} \frac{1}{E_{ni} - E - \frac{1}{2}i\Gamma_n}, \tag{8a}$$

$$C^{LL'}_{2n} = \frac{2\pi}{c^2} i^{L+L'} \sqrt{(2L+1)(2L'+1)} \frac{1}{E_{ni} + E' + \frac{1}{2}i\Gamma_n}. \tag{8b}$$

Now, we restrict ourselves to the most important case of electric dipole and electric quadrupole radiation. Introducing the nuclear polarizabilities [12,1]) $P^L_{L'}$ defined by

$$P^L_{L'} = (-)^L \sum_n \begin{Bmatrix} L' & I_f & I_i \\ I_n & L & L \end{Bmatrix} (\hat{C}^L_{2n} + (-)^{L'} \hat{C}^L_{1n}) + \delta_{if}\delta_{L1}\delta_{L'0}(-)^{2I_i}\sqrt{3(2I_i+1)}\frac{Z^2e^2}{AMc^2}, \quad (9)$$

where

$$\hat{C}^L_{1n} = C^{LL}_{1n}\langle I_f\alpha_f|| \int \boldsymbol{j}\cdot \boldsymbol{A}_L||I_n\alpha_n\rangle\langle I_n\alpha_n|| \int \boldsymbol{j}\cdot \boldsymbol{A}_L||I_i\alpha_i\rangle, \quad (9a)$$

$$\hat{C}^L_{2n} = C^{LL}_{2n}\langle I_f\alpha_f|| \int \boldsymbol{j}\cdot \boldsymbol{A}_L||I_n\alpha_n\rangle\langle I_n\alpha_n|| \int \boldsymbol{j}\cdot \boldsymbol{A}_L||I_i\alpha_i\rangle, \quad (9b)$$

we get

$$K^{pp'}_{if} = (-)^{p'} \sum_{\substack{L=1 \\ M,M'}}^{2} \sum_{L'=0}^{2L} (-)^{I_f-M_i}(2L'+1) \times \begin{pmatrix} L & L & L' \\ M & M' & -N \end{pmatrix}\begin{pmatrix} L' & I_f & I_i \\ N & -M_f & M_i \end{pmatrix} P^L_{L'} D^L_{M,p}(R) D^L_{M',-p'}(R'). \quad (10)$$

Usually the incoming photon and the initial nuclear states have no fixed polarization p and magnetic quantum number M_i.

In general we have incoherent mixtures of different p and M_i states. In order to take into account these various possibilities we use the density matrices [13,14]) σ and ρ of the initial photon and nuclear states, respectively. Furthermore, one has to sum over the different possible final states and include a polarization analysis of the outgoing photons. Taking all these into account, we obtain

$$\frac{d\sigma}{d\Omega} = \frac{k'}{k} \sum_{p,q,p',q'} \sum_{M_i,M'_i,M_f} \sigma_{pq}\rho_{M_iM'_i} K^{pp'}_{M_iM_f} K^{qq'*}_{M'_iM_f} \sigma'_{q'p'}, \quad (11)$$

σ' is the density matrix of the detector analyser. Note that Tr σ' is normalized to maximum response efficiency and not to 1. We have from eq. (10)

$$\begin{aligned} K^{pp'}_{M_iM_f} K^{qq'*}_{M'_iM_f} &= \sum_{K,L=1}^{2} \sum_{K',L',M,N} (-)^{M_i-M'_i}(2L'+1)(2K'+1) \\ &\times \begin{pmatrix} L & L & L' \\ M & M'-M & -M' \end{pmatrix}\begin{pmatrix} K & K & K' \\ N & N'-N & -N' \end{pmatrix}\begin{pmatrix} L' & I_f & I_i \\ M' & -M_f & M_i \end{pmatrix} \\ &\times \begin{pmatrix} K' & I_f & I_i \\ N' & -M_f & M'_i \end{pmatrix} P^L_{L'} P^{K*}_{K'} D^L_{M,p}(R) D^{K*}_{N,q}(R) D^L_{M'-M,-p'}(R') D^{K*}_{N'-N,-q'}(R) \\ &= \sum_{K,L,I,J,K',L',M,N} (-)^{M_i-M'_i+M'-p+p'}(2L'+1)(2K'+1)(2I+1)(2J+1) \\ &\times \begin{pmatrix} L & L & L' \\ M & M'-M & -M' \end{pmatrix}\begin{pmatrix} K & K & K' \\ N & N'-N & -N' \end{pmatrix}\begin{pmatrix} L' & I_f & I_i \\ M' & -M_f & M_i \end{pmatrix} \end{aligned}$$

$$\times \begin{pmatrix} K' & I_f & I_i \\ N' & -M_f & M_i' \end{pmatrix} \begin{pmatrix} L & K & J \\ M & -N & N-M \end{pmatrix}$$

$$\times \begin{pmatrix} L & K & J \\ p & -q & q-p \end{pmatrix} \begin{pmatrix} L & K & I \\ M'-M & N-N' & M-N+N'-M' \end{pmatrix} \begin{pmatrix} L & K & I \\ -p' & q' & p'-q' \end{pmatrix}$$

$$\times P^L_{L'} P^{K*}_{K'} D^J_{M-N,\, p-q}(R) D^I_{N-M+M'-N',\, q'-p'}(R'). \tag{12}$$

The sum over M, N can be performed using the relation of de-Shalit [15])

$$\sum_{M,N} \begin{pmatrix} L & L & L' \\ M & M'-M & -M' \end{pmatrix} \begin{pmatrix} K & K & K' \\ N & N'-N & -N' \end{pmatrix}$$

$$\times \begin{pmatrix} L & K & J \\ M & -N & N-M \end{pmatrix} \begin{pmatrix} L & K & I \\ M'-M & N-N' & M \end{pmatrix}$$

$$= (-)^{K'} \sum_{X,M} (2X+1) \begin{pmatrix} L' & K' & X \\ -M' & N' & M'-N' \end{pmatrix} \begin{pmatrix} X & J & I \\ M'-N' & M & N'-M'-M \end{pmatrix}$$

$$\times \begin{Bmatrix} L' & K' & X \\ L & K & J \\ L & K & I \end{Bmatrix}. \tag{13}$$

We obtain

$$K^{pp'}_{M_iM_f} K^{qq'*}_{M'_iM_f} = \sum_{K,L,I,J,K',L',M,X} (-)^{M_i - M'_i + M' - p + p' + K'} (1L'+1)(2K'+1)(2I+1)$$

$$\times (2J+1)(2X+1)$$

$$\times \begin{pmatrix} L' & I_f & I_i \\ M' & -M_f & M_i \end{pmatrix} \begin{pmatrix} K' & I_f & I_i \\ N' & -M_f & M_i' \end{pmatrix} \begin{pmatrix} L & K & J \\ p & -q & q-p \end{pmatrix} \begin{pmatrix} L & K & I \\ -p' & q' & p'-q' \end{pmatrix}$$

$$\times \begin{pmatrix} L' & K' & X \\ -M' & N' & M'-N' \end{pmatrix} \begin{pmatrix} X & J & I \\ M'-N' & M & N'-M'-M \end{pmatrix} \begin{Bmatrix} L' & K' & X \\ L & K & J \\ L & K & I \end{Bmatrix}$$

$$\times P^L_{L'} P^{K*}_{K'} D^J_{-M,\, p-q}(R) D^I_{M-N'+M',\, q'-p'}(R'). \tag{14}$$

The summation over M_f gives

$$\sum_{M_f} K^{pp'}_{M_iM_f} K^{qq'*}_{M'_iM_f} = \sum_{K,L,I,J,K',L',X,M} (-)^{-p+p'-I_f-M_i+X+K'} (2L'+1)(2K'+1)(2I+1)$$

$$\times (2J+1)(2X+1)$$

$$\times \begin{pmatrix} L & K & J \\ p & -q & q-p \end{pmatrix} \begin{pmatrix} L & K & I \\ -p' & q' & p'-q' \end{pmatrix} \begin{pmatrix} I_i & I_i & X \\ M_i & -M_i' & M_i'-M_i \end{pmatrix}$$

$$\times \begin{pmatrix} X & J & I \\ M_i'-M_i & M & M_i-M_i'-M \end{pmatrix} \begin{Bmatrix} I_i & I_i & X \\ K' & L' & I_f \end{Bmatrix}$$

$$\times \begin{Bmatrix} L' & K' & X \\ L & K & J \\ L & K & I \end{Bmatrix} P^L_{L'} P^{K*}_{K'} D^J_{-M,\, p-q}(R) D^I_{M+M'_i-M_i,\, q'-p'}(R'). \tag{15}$$

The scattering cross section now takes the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{k'}{k} \sum_{L, K, L', K'} P^L_{L'} P^{K*}_{K'} G^{LK}_{L'K'}(\rho, \sigma, \sigma', R, R'), \tag{16}$$

where

$$\begin{aligned} G^{LK}_{L'K'}&(\rho, \sigma, \sigma', R, R') \\ &= \sum_{p,q,p',q'} \sum_{M_i, M'_i} \sigma_{pq}\sigma'_{q'p'}\rho_{M_i M'_i} \sum_{I, J, X, M} (-)^{X+K'+I_f+M_i}(2L'+1)(2K'+1) \\ &\times (2I+1)(2J+1)(2X+1) \begin{pmatrix} L & K & J \\ p & -q & q-p \end{pmatrix} \begin{pmatrix} L & K & I \\ -p' & q' & p'-q' \end{pmatrix} \\ &\times \begin{pmatrix} I_i & I_i & X \\ M_i & -M'_i & M'_i - M_i \end{pmatrix} \begin{pmatrix} X & J & I \\ M'_i - M_i & M & M_i - M'_i - M \end{pmatrix} \begin{Bmatrix} I_i & I_i & X \\ K' & L' & I_f \end{Bmatrix} \\ &\times \begin{Bmatrix} L' & K' & X \\ L & K & J \\ L & K & I \end{Bmatrix} D^J_{-M, p-q}(R) D^I_{M'_i - M_i + M, q'-p'}(R'). \end{aligned} \tag{17}$$

Eq. (16) shows clearly the separation between the dynamical properties of the nucleus contained in the polarizabilities $P^L_{L'}$ and the geometrical properties of the scattering problem described by $G^{LK}_{L'K'}$. The quantities $G^{LK}_{L'K'}$ are completely determined by specification of the three directions (of the incident photon, the scattered photon and the axis of quantization) and by the density matrices involved in the experimental arrangement. The polarizabilities $P^L_{L'}$ are the interesting quantities, which give information about nuclear structure. One has to determine these from the differential cross sections and compare with the predictions given by some specific nuclear model.

Two cases are of special interest,

(i) unoriented nuclei and at least partially polarized incident radiation with a polarization analysis of the scattered radiation,

(ii) oriented nuclei and unpolarized incident radiation and no polarization analysis of the scattered radiation.

The first case has been discussed in detail by Fano [12]). It turns out that in this case one can only determine the absolute values of the polarizabilities and not the phases. This is easily seen from (17) because we have $\rho_{M_i M'_i} = (2I_i+1)^{-1}\delta_{M_i M'_i}$ for unoriented nuclei and therefore

$$\sum_{M_i} (-)^{M_i} \begin{pmatrix} I_i & I_i & X \\ M_i & -M_i & 0 \end{pmatrix} = (-)^{I_i}\sqrt{2I_i+1}\,\delta_{X0}. \tag{18}$$

This leads to $G^{LK}_{L'K'} \propto \delta_{L'K'}$ because of the relation $\Delta(L'K'X)$ and, therefore, the interference terms between different polarizabilities vanish. Necessarily, one needs oriented nuclei in order to get information about the phases of the polarizabilities through the interference terms. Therefore we will proceed in discussing the second case.

3. Oriented Nuclei and Unpolarized Radiation

In general the density matrix of the initial state with respect to an arbitrary quantization axis will be non-diagonal. But we can choose a special direction so that the matrix becomes diagonal, e.g. the direction of an external magnetic field or the symmetry axis of the electric field of a crystal. By the quantization axis we mean henceforth this direction. Then we have

$$\rho_{M_i M'_i} = a_{M_i}\delta_{M_i M'_i}, \qquad \sum_{M_i} a_{M_i} = 1. \tag{19}$$

Here a_{M_i} are the probabilities for finding the state $|I_i M_i\rangle$ as an initial nuclear state.

The density matrices σ_0, σ'_0 of the photon states are multiples of the unit matrix

$$\sigma_0 = \tfrac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \qquad \sigma'_0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{20}$$

These matrices correspond to the experimental situation where we are averaging over both initial polarizations (equal probability for both) and are summing over the polarizations of the final photon (detection of the final photons independent of their polarization).

With (19) and (20) we get from (17)

$$\begin{aligned}
&G^{LK}_{L'K'}(\rho, \sigma_0, \sigma'_0, R, R')\\
&= \tfrac{1}{2}\sum_{M_i} a_{M_i} \sum_{I,J,X,M} (-)^{X+K'+I_f+M_i}(2L'+1)(2K'+1)(2I+1)(2J+1)\\
&\times(2X+1)\begin{pmatrix}L & K & J\\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix}L & K & I\\ 1 & -1 & 0\end{pmatrix}(1+(-)^{L+K+J})(1+(-)^{L+K+I})\\
&\times\begin{pmatrix}I_i & I_i & X\\ M_i & -M_i & 0\end{pmatrix}\begin{pmatrix}X & J & I\\ 0 & M & -M\end{pmatrix}\begin{Bmatrix}I_i & I_i & X\\ K' & L' & I_f\end{Bmatrix}\begin{Bmatrix}L' & K' & X\\ L & K & J\\ L & K & I\end{Bmatrix} D^J_{-M,0}(R)D^I_{M,0}(R').
\end{aligned} \tag{21}$$

It is convenient to introduce the $2I_i$ orientation parameters f_ν (ref. [13]) instead of the probabilities a_{M_i} of (19). Both sets of parameters are related to each other:

$$f_\nu = \binom{2\nu}{\nu}^{-1} I_i^{-\nu}\left(\frac{(2I_i+\nu+1)!}{(2I_i-\nu)!}\right)^{\frac{1}{2}}\sum_{M_i}(-)^{I_i-M_i}\begin{pmatrix}I_i & I_i & \nu\\ M_i & -M_i & 0\end{pmatrix} a_{M_i}. \tag{22}$$

Note that for aligned nuclei, where $a_{-M_i} = a_{M_i}$, the f_ν with odd ν vanish.

Then we have from (21)

$$G^{LK}_{L'K'}(\rho, \sigma_0, \sigma'_0, R, R') = \sum_\nu f_\nu g^{LK}_{L'K'\nu}(R, R'), \tag{23}$$

with

$$g^{LK}_{L'K'\nu}(R,R') = \tfrac{1}{2}\binom{2\nu}{\nu} I_{\rm i}^{\nu}\left(\frac{(2I_{\rm i}-\nu)!}{(2I_{\rm i}+\nu+1)!}\right)^{\frac{1}{2}}(2\nu+1)\begin{Bmatrix} I_{\rm i} & I_{\rm i} & \nu \\ K' & L' & I_{\rm f}\end{Bmatrix}$$
$$\times \sum_{I,J,M}(-)^{\nu+K'+I_{\rm f}+I_{\rm i}}(2L'+1)(2K'+1)(2I+1)(2J+1)(1+(-)^{L+K+I})(1+(-)^{L+K+J})$$
$$\times \begin{pmatrix} L & K & I \\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix} L & K & J \\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix} \nu & J & I \\ 0 & M & -M\end{pmatrix}\begin{Bmatrix} L' & K' & \nu \\ L & K & J \\ L & K & I\end{Bmatrix} D^{J}_{-M,0}(R)D^{I}_{M,0}(R'). \quad (24)$$

Some simple properties of the $g^{LK}_{L'K'\nu}$ are immediately seen. We have

$$g^{LK}_{L'K'\nu} = 0, \quad \text{if not} \quad \Delta(L'K'\nu), \tag{24a}$$
$$g^{LK*}_{L'K'\nu} = (-)^{\nu} g^{LK}_{L'K'\nu} = g^{KL}_{K'L'\nu}, \tag{24b}$$
$$g^{KK}_{K'K'\nu} = 0, \quad \text{if} \quad \nu \text{ is odd.} \tag{24c}$$

Expression (24c) follows from (24b). Relation (24b) means that $g^{LK}_{L'K'\nu}$ is real for even ν and pure imaginary for odd ν. Therefore the even ν terms (see (26)) determine the real parts of $P^{L}_{L'}P^{K*}_{K'}$ and the odd-ν terms the imaginary parts. In the special case $L' = 0$, $K' = 2$, i.e. scalar-tensor interference, one can determine only Re $P^{L}_{0}P^{K*}_{2}$, because of property (24a).

Now the scattering cross section (16) can be expressed in terms of the orientation parameters, which shows the advantage of introducing these parameters

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_{\nu} f_{\nu}\frac{\mathrm{d}\sigma_{\nu}}{\mathrm{d}\Omega}, \tag{25}$$

$$\frac{\mathrm{d}\sigma_{\nu}}{\mathrm{d}\Omega} = \frac{k'}{k}\sum_{L,K,L',K'} P^{L}_{L'}P^{K*}_{K'}g^{LK}_{L'K'\nu}. \tag{26}$$

For dipole radiation we will give explicit formulae by separating the spin-dependent terms

$$g^{11}_{L'K'\nu}(R,R') = (-)^{I_{\rm i}+I_{\rm f}}I_{\rm i}^{\nu}\left(\frac{(2I_{\rm i}-\nu)!}{(2I_{\rm i}+\nu+1)!}\right)^{\frac{1}{2}}\begin{Bmatrix} I_{\rm i} & I_{\rm i} & \nu \\ K' & L' & I_{\rm f}\end{Bmatrix}\hat{g}^{1}_{L'K'\nu}(R,R'), \tag{27}$$

with

$$\hat{g}^{1}_{L'K'\nu}(R,R') = \tfrac{1}{2}\binom{2\nu}{\nu}(2L'+1)(2K'+1)(2\nu+1)(-)^{\nu+K'}\sum_{I,J,M}(2I+1)(2J+1)$$
$$\times(1+(-)^{I})(1+(-)^{J})\begin{pmatrix} 1 & 1 & I \\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix} 1 & 1 & J \\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix} \nu & J & I \\ 0 & M & -M\end{pmatrix}\begin{Bmatrix} L' & K' & \nu \\ 1 & 1 & J \\ 1 & 1 & I\end{Bmatrix}$$
$$\times D^{J}_{-M,0}(R)D^{J}_{M,0}(R'). \tag{28}$$

Straightforward calculation leads to

$$\nu = 0 \qquad \hat{g}^{1}_{000} = \tfrac{1}{6}(1+\cos^2\theta),$$
$$\hat{g}^{1}_{110} = -\tfrac{1}{4}\sqrt{3}(2+\sin^2\theta),$$
$$\hat{g}^{1}_{220} = \tfrac{1}{12}\sqrt{5}(13+\cos^2\theta); \tag{29}$$

$$\nu = 1 \qquad \hat{g}^1_{011} = -i\sqrt{\tfrac{3}{2}}\sin\beta\sin\beta'\sin\varphi\cos\theta,$$
$$\hat{g}^1_{121} = -i\sqrt{\tfrac{3}{2}}\sin\beta\sin\beta'\sin\varphi\cos\theta; \tag{30}$$

$$\nu = 2 \qquad \hat{g}^1_{022} = \tfrac{1}{\sqrt{2}}5(\cos^2\theta+3\cos^2\beta+3\cos^2\beta'-3\cos\theta\cos\beta\cos\beta'-2),$$
$$\hat{g}^1_{112} = -3\sqrt{\tfrac{15}{2}}(\cos^2\theta-3\cos\theta\cos\beta\cos\beta'),$$
$$\hat{g}^1_{122} = -15\sqrt{\tfrac{3}{2}}(\cos^2\beta-\cos^2\beta'),$$
$$\hat{g}^1_{222} = 5\sqrt{\tfrac{5}{14}}(6\cos^2\beta+6\cos^2\beta'-\cos^2\theta+3\cos\theta\cos\beta\cos\beta'-4); \tag{31}$$

$$\nu = 3 \qquad \hat{g}^1_{123} = -i\,5\sqrt{21}(5\cos\beta\cos\beta'-\cos\theta)\sin\beta\sin\beta'\sin\varphi; \tag{32}$$

$$\nu = 4 \qquad \hat{g}^1_{224} = \tfrac{15}{2}\sqrt{\tfrac{35}{2}}(2\cos^2\theta-5\cos^2\beta-5\cos^2\beta'+35\cos^2\beta\cos^2\beta' -20\cos\theta\cos\beta\cos\beta'+1). \tag{33}$$

All the other terms vanish because of (24a). The geometrical meaning of the different angles occurring in (29)-(33) is shown in fig. 2.

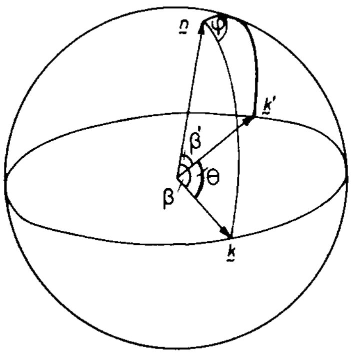

Fig. 2. The geometrical meaning of the angles occurring in eqs. (29)-(33).

Then the different scattering cross sections (26) take for dipole radiation the form

$$\frac{d\sigma_0}{d\Omega} = \sum_{L'=0}^{2} |P^1_{L'}|^2 g^{11}_{L'L'0}, \tag{34}$$

$$\frac{d\sigma_1}{d\Omega} = (P^1_0 P^{1*}_1 - P^{1*}_0 P^1_1) g^{11}_{011} + (P^1_1 P^{1*}_2 - P^{1*}_1 P^1_2) g^{11}_{121}, \tag{35}$$

$$\frac{d\sigma_2}{d\Omega} = (P^1_0 P^{1*}_2 + P^{1*}_0 P^1_2) g^{11}_{022} + |P^1_1|^2 g^{11}_{112} + (P^1_1 P^{1*}_2 + P^{1*}_1 P^1_2) g^{11}_{122} + |P^1_2|^2 g^{11}_{222}, \tag{36}$$

$$\frac{d\sigma_3}{d\Omega} = (P^1_1 P^{1*}_2 - P^{1*}_1 P^1_2) g^{11}_{123}, \tag{37}$$

$$\frac{d\sigma_4}{d\Omega} = |P^1_2|^2 g^{11}_{224}. \tag{38}$$

4. Elastic Photon Scattering by Aligned Ho Nuclei

As an application of the developed formalism we treat the elastic photon scattering by aligned ^{165}Ho nuclei in the giant resonance region. The nucleus ^{165}Ho has a large tensor polarizability [1, 5–7]), which is expected because of the strongly deformed shape. The tensor polarizability is therefore expected to contribute remarkably via the scalar - tensor interference term, which occurs in $d\sigma_2/d\Omega$ only (see (34)-(38)).

TABLE 1

Calculated elastic dipole polarizabilities of ^{165}Ho from 8 to 23 MeV incident photon energy

E(MeV)	Re P_0^1	Im P_0^1(10^{-2} fm)	Re P_1^1	Im P_1^1(10^{-2} fm)	ReP_2^1	Im P_2^1(10^{-2} fm)
8.0	− 5.79	2.58	−0.02	−0.01	−1.96	− 0.51
8.5	− 2.73	3.63	−0.03	−0.01	−2.51	− 0.78
9.0	1.03	5.19	−0.04	−0.03	−3.23	− 1.21
9.5	5.67	7.64	−0.06	−0.05	−4.19	− 1.95
10.0	11.39	11.65	−0.08	−0.09	−5.44	− 3.27
10.5	18.12	18.62	−0.10	−0.17	−6.90	− 5.74
11.0	24.37	31.07	−0.05	−0.33	−7.81	−10.41
11.5	23.76	50.83	0.25	−0.48	−5.18	−17.86
12.0	7.10	67.62	0.67	−0.14	4.89	−22.92
12.5	−11.79	65.35	0.43	0.36	15.60	−18.63
13.0	−16.43	57.60	0.07	0.33	20.14	−11.00
13.5	−14.21	56.92	−0.04	0.16	21.26	− 4.36
14.0	−13.59	63.12	−0.01	0.05	20.61	1.55
14.5	−18.66	71.51	0.06	0.03	18.23	6.45
15.0	−27.65	77.22	0.08	0.05	14.97	9.71
15.5	−37.47	80.76	0.09	0.05	11.69	11.88
16.0	−49.27	83.09	0.12	0.06	8.12	13.34
16.5	−63.69	81.28	0.15	0.13	4.15	13.60
17.0	−76.73	73.11	0.10	0.20	0.68	12.29
17.5	−84.49	61.45	0.02	0.21	−1.50	10.17
18.0	−87.23	50.19	−0.03	0.19	−2.54	8.09
18.5	−86.98	40.93	−0.06	0.16	−2.88	6.39
19.0	−85.31	33.76	−0.07	0.13	−2.88	5.09
19.5	−83.11	28.31	−0.08	0.10	−2.73	4.11
20.0	−80.83	24.12	−0.08	0.08	−2.52	3.38
20.5	−78.65	20.86	−0.07	0.07	−2.31	2.82
21.0	−76.63	18.26	−0.07	0.06	−2.10	2.38
21.5	−74.97	16.16	−0.06	0.05	−1.91	2.03
22.0	−73.12	14.44	−0.06	0.04	−1.74	1.75
22.5	−71.60	13.01	−0.06	0.04	−1.58	1.53
23.0	−70.22	11.81	−0.05	0.03	−1.43	1.34

This term can be seen in aligned nuclei, where the odd-ν orientation parameters vanish (22). One method to get large values of nuclear alignment for ^{165}Ho is described in ref. [8]). With the values given there one obtains $f_2 = 0.30$, $f_4 = 0.01$ at a temperature $T = 0.28°$K. With these values we have made the calculations.

The differential scattering cross section is then

$$\frac{d\sigma}{d\Omega} = \sum_{L'} |P^1_{L'}|^2 g^{11}_{L'L'0} + f_2[(P^1_0 P^{1*}_2 + P^{1*}_0 P^1_2) g^{11}_{022} + |P^1_1|^2 g^{11}_{112}$$
$$+ (P^1_1 P^{1*}_2 + P^{1*}_1 P^1_2) g^{11}_{122} + |P^1_2|^2 g^{11}_{222}] + f_4 |P^1_2|^2 g^{11}_{224}. \quad (39)$$

We have computed the nuclear polarizabilities in the framework of the dynamic collective theory [1, 16, 17]) using the parameters of ref. [1]). The polarizabilities are given

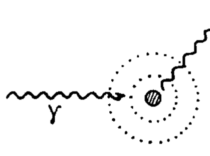

Fig. 3. Geometrical arrangement in case (i). The scattering plane is the paper plane. The magnetic field or the symmetry axis of the crystal field (quantization axis) is perpendicular indicated by the dots.

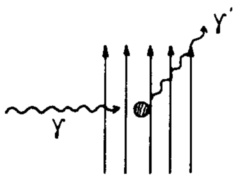

Fig. 4. Geometrical arrangement in case (ii). The magnetic field or the symmetry axis of the crystal field is in the scattering plane (paper plane) indicated by the arrows.

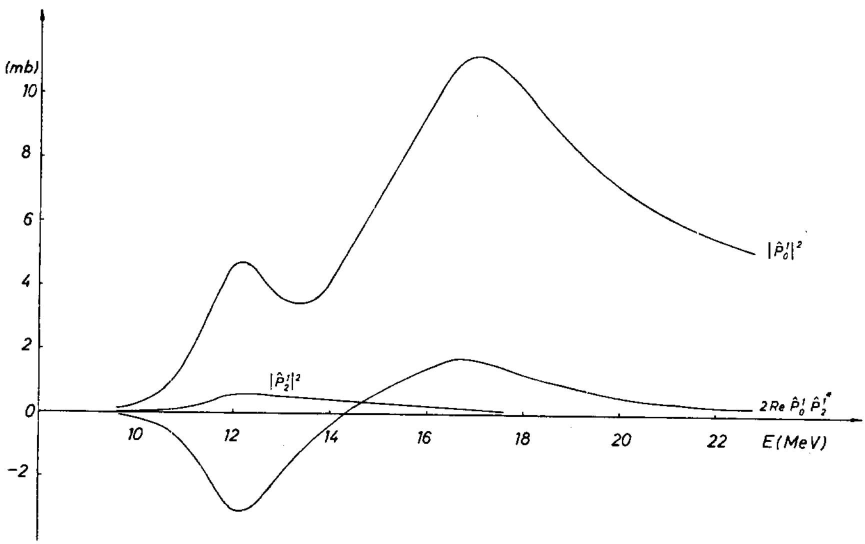

Fig. 5. $|P_0{}^1|^2$, $|P_2{}^1|^2$ and $2\text{Re}\, P_0{}^1 P_2{}^{1*}$ as functions of incident photon energy E.

in table 1 as function of the incident photon energy. In fig. 5 are plotted the different coefficients of the angular functions entering in (39). It is remarkable that the scalar - tensor interference term is so large and that it changes sign by variation of the photon energy. We notice that the vector polarizability given by this model is nearly zero.

We have, furthermore, calculated for two cases the elastic differential cross sections as functions of the photon energy for scattering angles of 90° and 135° and the angular distributions. The two cases are

(i) the quantization axis (symmetry axis of the crystal electric field) is perpendicular to the scattering plane (see fig. 3) and

(ii) the quantization axis lies in the scattering plane and is perpendicular to the incident photon direction (see fig. 4).

The angular functions are ($I_i = \frac{7}{2}$) in case (i)

$$
\begin{aligned}
g_{022}^{11} &= \tfrac{7}{192}\sqrt{\tfrac{7}{6}}(\cos^2\theta-2),\\
g_{112}^{11} &= -\tfrac{7}{96}\cos^2\theta,\\
g_{122}^{11} &= 0,\\
g_{222}^{11} &= \tfrac{1}{72}(\cos^2\theta+4),\\
g_{224}^{11} &= \frac{7^3}{2^{10}\cdot 3^2}(2\cos^2\theta+1),
\end{aligned}
\tag{40}
$$

and in case (ii)

$$
\begin{aligned}
g_{022}^{11} &= \tfrac{7}{192}\sqrt{\tfrac{7}{6}}(2\sin^2\theta-1),\\
g_{112}^{11} &= -\tfrac{7}{96}\cos^2\theta,\\
g_{122}^{11} &= -\frac{7}{64\sqrt{3}}\sin^2\theta,\\
g_{222}^{11} &= -\tfrac{1}{72}(7\sin^2\theta-5),\\
g_{224}^{11} &= \frac{7^3}{2^{10}\cdot 3^2}(7\cos^2\theta-4).
\end{aligned}
\tag{41}
$$

The differential cross sections are plotted in fig. 6 ($\theta = 90°$) and fig. 7 ($\theta = 135°$). For comparison we have also plotted the differential cross section for unoriented nuclei. At an angle $\theta = 135°$ the angular function g_{022}^{11} of the scalar-tensor interference term vanishes in case (ii) as is seen from (41) and the other f_2 and f_4 contributions are nearly negligible. Therefore the differential cross section is close to the differential cross section for unoriented nuclei. For $\theta = 90°$ the angular function g_{022}^{11} of the scalar-tensor interference term has different signs in the two cases and is twice as large in case (i) than in case (ii). This is shown in fig. 6. Therefore one can determine the scalar-tensor interference term by simple subtraction of one cross section from the other.

Finally we have computed the angular distribution of the elastic scattered radiation

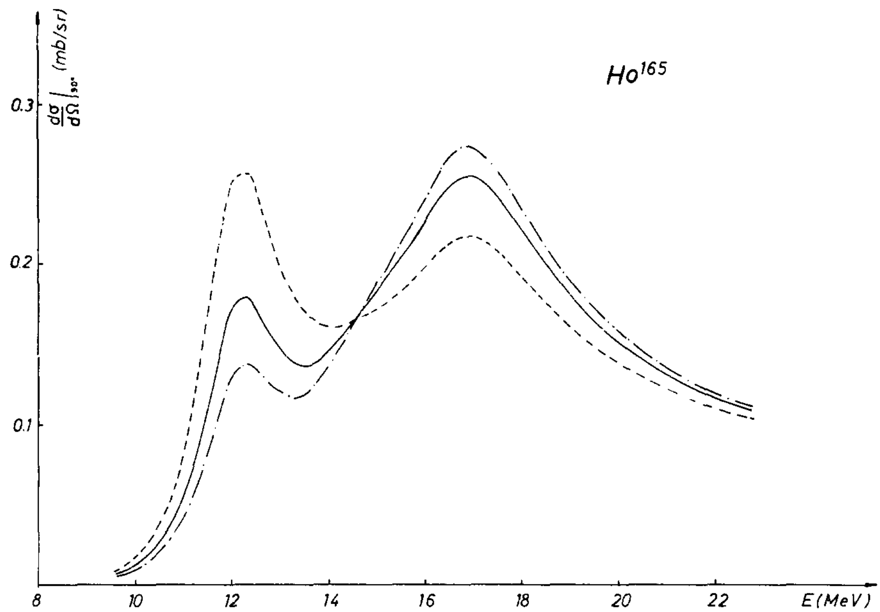

Fig. 6. The elastic differential cross section of ^{165}Ho for unoriented (full curve) and aligned target (case (i): dashed curve, case (ii): dot-and-dash curve) at a scattering angle $\theta = 90°, f_2 = 0.3$.

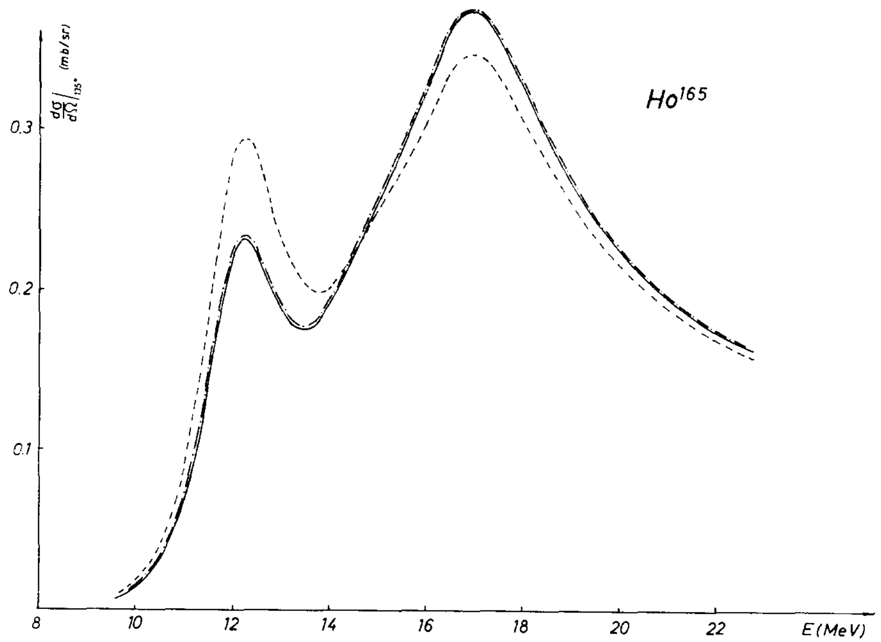

Fig. 7. The elastic differential cross section of ^{165}Ho for unoriented (full curve) and aligned target (case (i): dashed curve, case (ii): dot-and-dashed curve) at a scattering angle $\theta = 135°, f_2 = 0.3$.

for photon energy of 12, 14, 16 and 18 MeV. The results are shown in fig. 8. The different behaviour for the two cases, because of the different sign of the scalar-tensor interference term, is obvious.

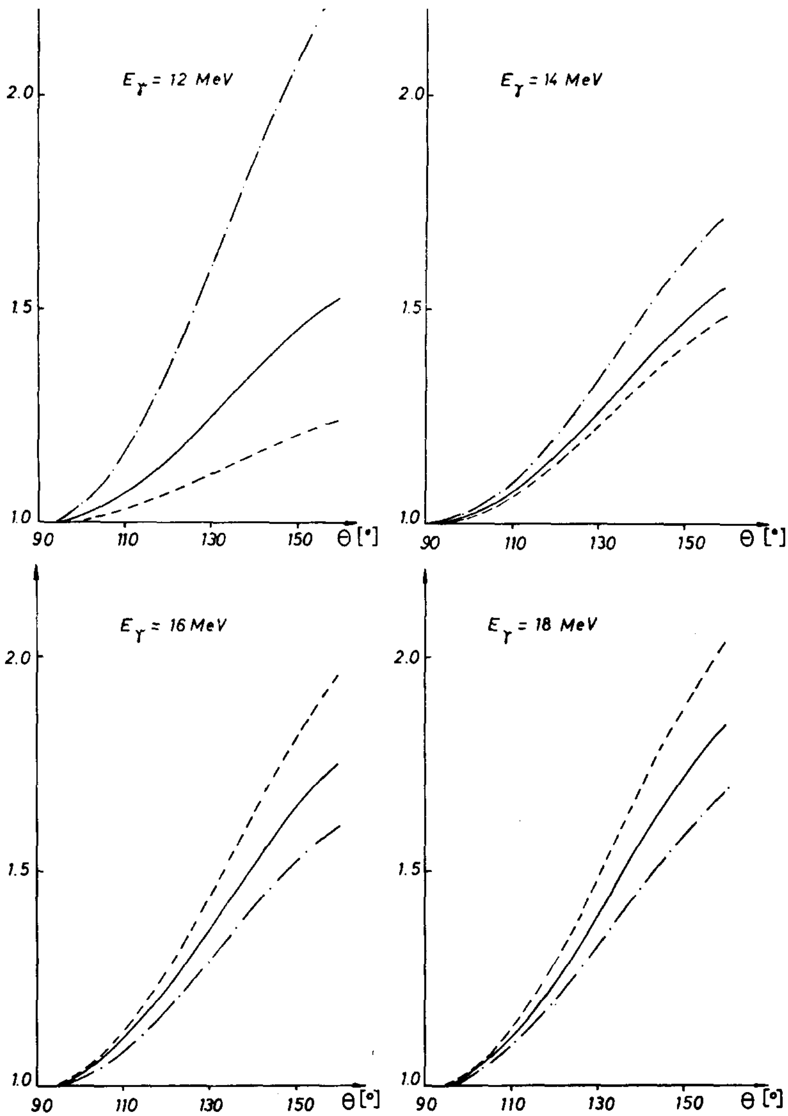

Fig. 8. Angular distributions of elastic scattered photons by unoriented (full curve) and aligned ^{165}Ho targets (case (i): dashed curves, case (ii): dot-and-dash curves) for incident photon energies $E = 12$, 14, 16 and 18 MeV, normalized by $d\sigma/d\Omega|90°$.

5. Summary

The results of the general formalism developed in sects. 2 and 3 are the following: The differential cross section for scattering of unpolarized photons by oriented nuclei can be represented in terms of the orientation parameters f_ν (see eq. (26)). The coefficients $d\sigma_\nu/d\Omega$ contain for $\nu > 0$ the interference terms $P^L_{L'} P^{K*}_{K'}$ if the triangular relation $\Delta(L'K'\nu)$ is fulfilled (see eq. (24)). Because the vector polarizability nearly vanishes the scalar-tensor interference term is the only one which contributes appreciably. This term occurs in $d\sigma_2/d\Omega$ only because of the selection rule (24) so that one needs alignment only in order to observe this interference experimentally. The other contributions are one order of magnitude or more lower.

The discussion of ^{165}Ho in sect. 4 shows that the scalar-tensor interference influences appreciably the elastic scattering cross section seen in the experiment. Therefore one can extract this term from experiments with different orientations of the alignment axis with respect to the photon at different angles. The photon scattering by oriented nuclei seems, therefore, to be a useful tool for direct observation of the tensor polarizability of nuclei. To decide experimentally the question of existence of a nuclear vector polarizability a scattering experiment with a polarized target ($f_1 \neq 0$) seems to be suitable [18]. In such an experiment the vector polarizability would manifest itself in an asymmetric angular distribution of the scattered radiation around 0° (see eqs. (35) and (30)).

Furthermore, we remark that the inelastic scattering cross sections will not be affected seriously by the orientation of the target. The reason for this is that the inelastic scattering with an appreciable cross section leads to states with spin different from the ground state spin [1]. Therefore, the scalar polarizability vanishes for these cases (see eq. (9)) and the most important scalar-tensor interference too. The elastic scalar-tensor interference can, however, be also measured by the quasi-elastic scattering cross section.

A final remark is made about vibrational nuclei. Because of the spherical shape one expects the tensor polarizability to be zero. But both anharmonicity of the surface vibrations and coupling of surface quadrupoles with giant resonance dipoles [19,20] will destroy the spherical symmetry and therefore cause non-vanishing tensor polarizability to be seen in γ-absorption and scattering experiments with aligned targets.

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