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Meson Exchange and Isobar Current Effects in Nuclear Photon Scattering

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Explicit expressions of meson exchange and isobar current contributions to the nuclear two-photon amplitude are derived in the non-relativistic limit. In connection with this amplitude the requirement of gauge invariance on the photon scattering amplitude and on the nuclear electromagnetic interaction operator is discussed in detail.

1. Introduction

Already shortly after Yukawa had proposed the meson theory of nuclear forces, it had been noted that the mediation of the nuclear forces by the exchange of mesons would lead to the existence of additional currents, the meson exchange currents (MEC) [1]. Despite this early recognition, however, a long time passed until the lowest order MEC as required by gauge invariance had been calculated reliably.

Beginning with the success of Brown and Riska [2] in explaining a large fraction of the long standing discrepancy in thermal n-p capture by π-MEC, a whole series of investigations on contributions of MEC started [3]. At present the lowest order isovector MEC is on safe grounds and the best evidence for it has been found in electrodisintegration of deuterium near threshold $d(e,e')$ [4] and the magnetic moments of ^{3}H and ^{3}He besides the thermal n-p capture.

Recently Christillin and Rosa-Clot [5, 6] have claimed that nuclear Compton scattering is another process strongly influenced by MEC. They have estimated a sizable contribution to the Compton scattering cross section at an energy of 100 MeV and above. However, their estimate is based on low energy theorems the extrapolation of which to such high energies is questionable. In view of the success of present models for MEC it seems worthwhile to use these models also for an explicit construction of MEC contribution to the photon scattering amplitude in order to put the low energy estimates on quantitative grounds.

We would like to point out that these models of MEC are usually static potential models and do not describe properly retardation effects of the meson exchange. Therefore, they cannot describe the real pion production channel for photon energies above the pion production threshold. This means furthermore, that in the two-photon exchange amplitude the pion never can be on mass shell and, therefore, the amplitude will be real in the forward direction even above pion-production threshold where the imaginary part is given by the absorptive process of pion production according to the optical theorem. This will also have some bearing on the use of dispersion relations as will be discussed in some detail. Consequently, the present treatment is limited to the non-relativistic domain below pion-production threshold.

The purpose of this paper is to derive explicit expressions of MEC contributions to the photon scattering amplitude which can be used in an explicit numerical evaluation. In Sect. 2 we will shortly review the general features of the photon scattering amplitude and discuss the consequences of gauge invariance. We then specialize to a non-relativistic system of nucleons interacting via two-body forces in Sect. 3. In particular, the consequences of gauge invariance on the Hamiltonian up to second order with respect to an external electromagnetic field will be discussed. The non-relativistic photon scattering amplitude is given in Sect. 4 and in Sect. 5 the explicit

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MEC contributions to the two-photon amplitude are derived by minimal substitution. Dispersion relations are considered in Sect. 6 and finally isobar current effects are discussed in Sect. 7.

2. The Photon Scattering Amplitude

In this section we will shortly review the general features of the photon scattering amplitude. For the process, where a photon with 4-momentum $k=(k_0=\omega,\mathbf{k})$ and polarization λ is scattered off a particle with 4-momentum $P_i=(E_i,\mathbf{P}_i)$ leading to a photon $\{k',\lambda'\}$ and a final particle momentum P_f, the amplitude up to order e^2 is given by

$$\begin{aligned}&\langle P_f k'\lambda'|S|P_i k\lambda\rangle=\delta_{k'k}\delta_{\lambda'\lambda}\delta_{fi}\\&-2\pi i(4\omega'\omega)^{-\frac{1}{2}}\delta^{(4)}(P_f+k'-P_i-k)\varepsilon^*_\mu(k'\lambda')\\&\cdot T^{\mu\nu}_{fi}(k',k)\varepsilon_\nu(k\lambda),\end{aligned}\tag{1}$$

where $\varepsilon_\mu(k\lambda)$ and $\varepsilon_\nu(k'\lambda')$ denote the initial and final photon polarization vectors, respectively. The T-matrix has the form

$$\begin{aligned}T^{\mu\nu}_{fi}(k',k)&=-i\int d^4x\,e^{ik'x}\langle P_f(T(\hat{j}^\mu(x),\hat{j}^\nu(0))|P_i\rangle\\&+\int d^4x\,e^{ik'x}\langle P_f|\hat{B}^{\mu\nu}(x,0)|P_i\rangle\end{aligned}\tag{2}$$

if the electromagnetic interaction of the system is given up to second order in the electromagnetic potential $A_\mu(x)$ by

$$\begin{aligned}\hat{H}_{\text{e.m.}}(A_\mu)&=\int d^4x\,\hat{j}^\mu(x)A_\mu(x)\\&+\tfrac{1}{2}\int d^4x\,d^4y\,A_\mu(x)\hat{B}^{\mu\nu}(x,y)A_\nu(y).\end{aligned}\tag{3}$$

The two-photon operator $\hat{B}^{\mu\nu}(x,y)$ describes first order contributions to two-photon processes, e.g., seagull terms (see Fig. 1).
The current operator $\hat{j}_\mu$ and the two-photon operator $\hat{B}^{\mu\nu}$ are not independent, because gauge invariance requires

$$k'_\mu T^{\mu\nu}_{fi}(k',k)=T^{\nu\mu}_{fi}(k',k)k_\mu=0.\tag{4}$$

Using for $T^{\mu\nu}_{fi}$ the explicit form of (2), one has

$$\begin{aligned}k'_\mu T^{\mu\nu}_{fi}(k',k)&=\langle P_f|\int d^4x\,e^{ik'x}\{i\partial_\mu\hat{B}^{\mu\nu}(x,0)\\&+\frac{1}{2\pi}\int\frac{dz}{z-i\varepsilon}\frac{1}{i}\partial_\mu(e^{izx_0}\hat{j}^\mu(x)\hat{j}^\nu(0)\\&+e^{-izx_0}\hat{j}^\nu(0)\hat{j}^\mu(x))\}|P_i\rangle\\&=\langle P_f|\int d^4x\,e^{ik'x}\{i\partial_\mu\hat{B}^{\mu\nu}(x,0)\\&+\frac{1}{2\pi}\int dz\,e^{izx_0}[\hat{j}^0(x),\hat{j}^\nu(0)]\}|P_i\rangle,\end{aligned}\tag{5}$$

where we have made use of current conservation

$$\partial_\mu\hat{j}^\mu=0.\tag{6}$$

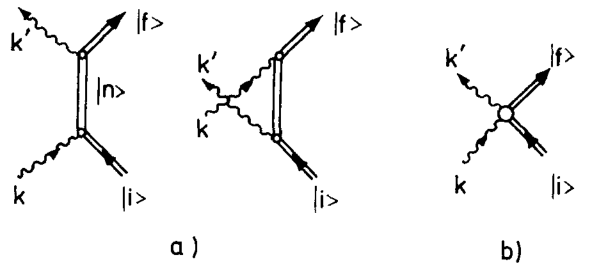

Fig. 1. Photon scattering diagrams: **a** second order resonance and crossed process, **b** first order two-photon process (seagull term)

Thus, one has the following gauge condition for the two-photon operator [5]

$$\begin{aligned}&\langle P_f|\int d^3x\,e^{-i\mathbf{k}'\cdot\mathbf{x}}[\hat{j}^0(0,\mathbf{x}),\hat{j}^\nu(0)]|P_i\rangle\\&=-i\langle P_f|\int d^4x\,e^{ik'x}\partial_\mu\hat{B}^{\mu\nu}(x,0)|P_i\rangle\end{aligned}\tag{7}$$

for $k+P_i=k'+P_f$, which connects the two-photon operator with the charge and current density operators.

3. Gauge Invariance of a Non-Relativistic Hamiltonian and the Two-Photon Operator

We will now consider a non-relativistic Hamiltonian for nucleons interacting via two-body forces

$$\begin{aligned}\hat{H}_0&=\sum_i p_i^2/2M+\tfrac{1}{2}\sum_{i\neq j}V_{ij}\\&=T+V.\end{aligned}\tag{8}$$

In the presence of an external electromagnetic field A_μ the Hamiltonian is a functional of A_μ and, similar to (3), one has up to second order in A_μ

$$\begin{aligned}\hat{H}(A_\mu)&=\hat{H}_0+\int d^3x\,\hat{j}^\mu(x)A_\mu(x)\\&+\tfrac{1}{2}\int d^3x\,d^3y\,A_\mu(x)\hat{B}^{\mu\nu}(x,y)A_\nu(y),\end{aligned}\tag{9}$$

where

$$\hat{j}^\mu=\left.\frac{\delta\hat{H}(A_\mu)}{\delta A_\mu}\right|_{A_\mu=0}\tag{10}$$

is the total nuclear current density operator and

$$\hat{B}^{\mu\nu}(x,y)=\left.\frac{\delta^2\hat{H}(A_\mu)}{\delta A_\mu(x)\,\delta A_\nu(y)}\right|_{A_\mu=0}\tag{11}$$

the two-photon operator. We will now discuss the consequences of gauge invariance on the current and two-photon operators from the requirement that the system is invariant under gauge transformations. Gauge invariance means that for a gauge transformation $A_\mu\to A_\mu-\partial_\mu\lambda$ one has the following relation for the Hamilton operator

$$e^{-i\hat{\chi}}\hat{H}(A_\mu-\partial_\mu\lambda)e^{i\hat{\chi}}+\frac{\partial\hat{\chi}}{\partial t}=\hat{H}(A_\mu), \tag{12}$$

where

$$\hat{\chi}=\int d^3x\,\hat{j}^0(\mathbf{x})\,\lambda(\mathbf{x},t), \qquad \frac{\partial\hat{\chi}}{\partial t}=\int d^3x\,\hat{j}^0(\mathbf{x})\frac{\partial\lambda}{\partial t}. \tag{13}$$

Expanding the *lhs* of (12) in powers of the gauge function λ up to second order gives as conditions

$$\begin{aligned}&\frac{\partial\hat{\chi}}{\partial t}+i[\hat{H},\hat{\chi}]-\int d^3x\,\hat{j}^\mu\partial_\mu\lambda\\&=\int d^3x\,d^3y\,\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})\,A_\mu(\mathbf{x})\,\partial_\nu A(y)\end{aligned} \tag{14}$$

and

$$\begin{aligned}&[\hat{\chi},[\hat{H},\hat{\chi}]]+\int d^3x\,d^3y\,\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})\,\partial_\mu\lambda(\mathbf{x})\,\partial_\nu\lambda(\mathbf{y})\\&=2i[\int d^3x\,d^3y\,\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})\,A_\mu(\mathbf{x})\,\partial_\nu\lambda(\mathbf{y})\\&\quad+\int d^3x\,\hat{j}^\mu\partial_\mu\lambda,\hat{\chi}].\end{aligned} \tag{15}$$

Here we have made use of the symmetry property of the two-photon operator

$$\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})=\hat{B}^{\nu\mu}(\mathbf{y},\mathbf{x}). \tag{16}$$

With the help of the first condition we can reduce the second (15) to

$$\int d^3x\,d^3y\,\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})\,\partial_\mu\lambda(\mathbf{x})\,\partial_\nu\lambda(y)=[\hat{\chi},[\hat{H},\hat{\chi}]], \tag{17}$$

since

$$\left[\frac{\partial\hat{\chi}}{\partial t},\hat{\chi}\right]=0. \tag{18}$$

The first condition, (14), can be simplified using

$$\int d^3x\,\hat{j}^\mu\partial_\mu\lambda=\frac{\partial\hat{\chi}}{\partial t}-\int d^3x\,\lambda\,\nabla\cdot\hat{\mathbf{j}} \tag{19}$$

and put into the form

$$\begin{aligned}&i[\hat{H},\hat{\chi}]+\int d^3x\,\lambda\,\nabla\cdot\hat{\mathbf{j}}\\&=\int d^3x\,d^3y\,\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})\,A_\mu(\mathbf{x})\,\partial_\nu\lambda(\mathbf{y}).\end{aligned} \tag{20}$$

Since this should hold for any gauge function λ one obtains

$$i[\hat{H},\hat{\rho}(\mathbf{x})]+\nabla\cdot\hat{\mathbf{j}}(\mathbf{x})=-\int d^3y\,\partial_\mu\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})\,A_\nu(\mathbf{y}). \tag{21}$$

Using the explicit form of $\hat{H}(A_\mu)$ from (8) and comparing the terms up to second order in A_μ one finally obtains as gauge conditions

$$\nabla\cdot\hat{\mathbf{j}}(\mathbf{x})=i[\hat{\rho}(\mathbf{x}),\hat{H}_0], \tag{22}$$

i.e., the continuity equation, and furthermore, for the two-photon operator

$$\frac{\partial}{\partial x^\mu}\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y})=i[\hat{\rho}(\mathbf{x}),\hat{j}^\nu(\mathbf{y})] \tag{23}$$

from which $\hat{B}^{\nu 0}=\hat{B}^{0\nu}=0$ follows, since $[\hat{\rho}(\mathbf{x}),\hat{\rho}(\mathbf{y})]=0$ has been assumed, and

$$[\hat{B}^{\mu\nu}(\mathbf{x},\mathbf{y}),\hat{\rho}(\mathbf{z})]=0. \tag{24}$$

It is not surprising that we arrive at the same condition as from the gauge invariance requirement on the scattering amplitude. Analogously, (17) leads to the condition

$$\partial_k\partial_l\hat{B}^{kl}(\mathbf{x},\mathbf{y})=[\hat{\rho}(\mathbf{x}),[\hat{H}_0,\hat{\rho}(\mathbf{y})]], \tag{25}$$

which, however, is not an additional condition, but follows directly from (23).

We will now exploit the fact that we deal with a nonrelativistic system. First of all, one can separate center-of-mass and internal motion, because the two-body interaction does not depend on center-of-mass coordinates

$$\hat{H}_0=\hat{H}_0^{\mathrm{cm}}+\hat{H}_0^{\mathrm{in}}, \tag{26}$$

where

$$\hat{H}_0^{\mathrm{cm}}=\hat{P}^2/2AM, \qquad \hat{\mathbf{P}}=\sum_j\hat{\mathbf{p}}_j, \tag{27}$$

$$\hat{H}_0^{\mathrm{in}}=\sum_j\hat{T}'_j+\hat{V}=\hat{T}^{\mathrm{in}}+\hat{V}, \tag{28}$$

$$\hat{T}'_j=\hat{p}'^2_j/2M, \qquad \hat{\mathbf{p}}'_j=\hat{\mathbf{p}}_j-\hat{\mathbf{P}}/A. \tag{29}$$

Consequently, the eigenfunctions factorize in cm- and internal motion, i.e., one can characterize each eigenstate of $\hat{H}_0$ by the total cm-momentum $\mathbf{P}$ and an internal quantum number n: $|\mathbf{P},n\rangle$. The one-body convection current can be obtained from the minimal substitution

$$\hat{\mathbf{p}}_j\to\hat{\mathbf{p}}_j-e_j\hat{\mathbf{A}}(\mathbf{r}_j), \tag{30}$$

or

$$\hat{\mathbf{P}}\to\hat{\mathbf{P}}-\sum_j e_j\hat{\mathbf{A}}(\mathbf{r}_j), \tag{31}$$

and

$$\hat{\mathbf{p}}'_j\to\hat{\mathbf{p}}'_j-e_j\hat{\mathbf{A}}(\mathbf{r}_j)+\frac{1}{A}\sum_l e_l\hat{\mathbf{A}}(\mathbf{r}_l), \tag{32}$$

yielding the well-known expressions

$$\begin{aligned}\hat{\mathbf{j}}^c_{[1]}(\mathbf{x})&=-\left.\frac{\delta\hat{T}(\mathbf{A})}{\delta\mathbf{A}(\mathbf{x})}\right|_{\mathbf{A}\equiv 0}\\&=\frac{1}{2M}\sum_j e_j\{\hat{\mathbf{p}}_j,\delta(\mathbf{x}-\mathbf{r}_j)\}=\hat{\mathbf{j}}^{\mathrm{cm},c}+\hat{\mathbf{j}}^{\mathrm{in},c}\end{aligned} \tag{33}$$

$$\hat{\mathbf{j}}^{\mathrm{cm},c}(\mathbf{x})=\frac{1}{2AM}\{\hat{\mathbf{P}},\hat{\rho}_{[1]}(\mathbf{x})\},$$

$$\hat{\rho}_{[1]}(\mathbf{x})=\sum_j e_j\,\delta(\mathbf{x}-\mathbf{r}_j) \tag{34}$$

$$\hat{\mathbf{j}}^{\mathrm{in},c}(\mathbf{x})=\frac{1}{2M}\sum_j e_j\{\hat{\mathbf{p}}'_j,\,\delta(\mathbf{x}-\mathbf{r}_j)\} \tag{35}$$

to which one has to add the spin current

$$\hat{\mathbf{j}}^{s}_{[1]}(\mathbf{x})=\frac{i}{2M}\sum_j \mu_j\boldsymbol{\sigma}_j\times[\hat{\mathbf{p}}_j,\delta(\mathbf{x}-\mathbf{r}_j)]. \tag{36}$$

For a nucleus with non-vanishing ground state spin one could separate also a spin current

$$\hat{\mathbf{j}}^{\mathrm{cm},s}(\mathbf{x})=i\boldsymbol{\nabla}\times\hat{\mathbf{M}}(\mathbf{x}), \tag{37}$$

where $\hat{\mathbf{M}}(\mathbf{x})$ is the ground state magnetization density. Furthermore, it is well known that a two-body current is required from gauge invariance (see (22)), if the interaction is momentum dependent and/or contains an exchange part. In the case that the interaction is constructed from an underlying meson field theory, this two-body meson exchange current can be obtained from the reduction of the relevant diagrams or also by minimal substitution [7, 8] if one writes the interaction in such a form that the degrees of freedom of the exchanged mesons are explicitly exhibited. Then

$$\hat{\mathbf{j}}^{\mathrm{MEC}}_{[2]}(\mathbf{x})=-\left.\frac{\delta\hat{V}(\mathbf{A})}{\delta\mathbf{A}(\mathbf{x})}\right|_{\mathbf{A}\equiv 0} \tag{38}$$

and this current satisfies the relation

$$\boldsymbol{\nabla}\cdot\hat{\mathbf{j}}^{\mathrm{MEC}}_{[2]}(\mathbf{x})=i[\hat{\rho}_{[1]}(\mathbf{x}),\hat{V}]. \tag{39}$$

Analogously, the two-photon operator is obtained in the form $(k,l=1,2,3)$

$$\hat{B}_{kl}(\mathbf{x},\mathbf{y})=\hat{B}^{\mathrm{cm}}_{kl}(\mathbf{x},\mathbf{y})+\hat{B}^{\mathrm{in,kin}}_{kl}(\mathbf{x},\mathbf{y})
+\hat{B}^{\mathrm{MEC}}_{kl}(\mathbf{x},\mathbf{y}), \tag{40}$$

where

$$\hat{B}^{\mathrm{cm}}_{kl}(\mathbf{x},\mathbf{y})=\left.\frac{\delta^2\hat{T}^{\mathrm{cm}}(\mathbf{A})}{\delta A_k(\mathbf{x})\,\delta A_l(\mathbf{y})}\right|_{\mathbf{A}\equiv 0}
=\frac{\delta_{kl}}{AM}\sum_{jj'} e_j e_{j'}\,\delta(\mathbf{x}-\mathbf{r}_j)\,\delta(\mathbf{y}-\mathbf{r}_{j'}) \tag{41}$$

$$\hat{B}^{\mathrm{in,kin}}_{kl}(\mathbf{x},\mathbf{y})=\frac{\delta_{kl}}{M}\sum_i\left(e_i\,\delta(\mathbf{x}-\mathbf{r}_i)-\frac{1}{A}\sum_j e_j\,\delta(\mathbf{x}-\mathbf{r}_j)\right)
\times\left(e_i\,\delta(\mathbf{y}-\mathbf{r}_i)-\frac{1}{A}\sum_{j'} e_{j'}\,\delta(\mathbf{y}-\mathbf{r}_{j'})\right) \tag{42}$$

$$\hat{B}^{\mathrm{MEC}}_{kl}(\mathbf{x},\mathbf{y})=\left.\frac{\delta^2\hat{V}(\mathbf{A})}{\delta A_k(\mathbf{x})\,\delta A_l(\mathbf{y})}\right|_{\mathbf{A}\equiv 0}. \tag{43}$$

If one does not want to separate the cm-two-photon operator one gets the somewhat simpler expression for the kinetic two-photon operator

$$\hat{B}^{\mathrm{kin}}_{kl}(\mathbf{x},\mathbf{y})=\hat{B}^{\mathrm{cm}}_{kl}(\mathbf{x},\mathbf{y})+\hat{B}^{\mathrm{in,kin}}_{kl}(\mathbf{x},\mathbf{y})
=\frac{\delta_{kl}}{M}\sum_j e_j^2\,\delta(\mathbf{x}-\mathbf{r}_j)\,\delta(\mathbf{y}-\mathbf{r}_j). \tag{44}$$

The evaluation of the gauge condition of (23) gives for the kinetic part

$$\begin{aligned}
\frac{\partial}{\partial x_k}\hat{B}^{\mathrm{kin}}_{kl}(\mathbf{x},\mathbf{y})&=\frac{1}{M}\sum_j e_j^2\frac{\partial}{\partial x_l}\delta(\mathbf{x}-\mathbf{r}_j)\,\delta(\mathbf{y}-\mathbf{r}_j)\\
&=-\frac{i}{2M}\sum_{jj'} e_j e_{j'}\{[\hat{p}_{j,l},\delta(\mathbf{x}-\mathbf{r}_{j'})],\delta(\mathbf{y}-\mathbf{r}_j)\}\\
&=i\left[\sum_{j'} e_{j'}\,\delta(\mathbf{x}-\mathbf{r}_{j'}),\frac{1}{2M}\sum_j e_j\{\hat{p}_{j,l},\delta(\mathbf{y}-\mathbf{r}_j)\}\right]\\
&=i[\hat{\rho}_{[1]}(\mathbf{x}),\hat{j}^{c}_{[1]l}(\mathbf{y})]\\
&=i[\hat{\rho}_{[1]}(\mathbf{x}),\hat{j}_{[1]l}(\mathbf{y})].
\end{aligned} \tag{45}$$

Therefore, the MEC two-photon operator is related to the commutator of the charge density with the MEC current density operator

$$\frac{\partial}{\partial x_k}\hat{B}^{\mathrm{MEC}}_{kl}(\mathbf{x},\mathbf{y})=i[\hat{\rho}_{[1]}(\mathbf{x}),\hat{j}^{\mathrm{MEC}}_{[2]l}(\mathbf{y})]. \tag{46}$$

With respect to cm and internal motion we note furthermore

$$\frac{\partial}{\partial x_k}\hat{B}^{\mathrm{cm}}_{kl}(\mathbf{x},\mathbf{y})=i[\hat{\rho}_{[1]}(\mathbf{x}),\hat{j}^{\mathrm{cm}}_{[1]l}(\mathbf{y})] \tag{47}$$

$$\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_l}\hat{B}^{\mathrm{cm}}_{kl}(\mathbf{x},\mathbf{y})=[\hat{\rho}_{[1]}(\mathbf{x}),[\hat{T}^{\mathrm{cm}},\hat{\rho}_{[1]}(\mathbf{y})]], \tag{48}$$

and thus

$$\frac{\partial}{\partial x_k}\hat{B}^{\mathrm{in,kin}}_{kl}(\mathbf{x},\mathbf{y})=i[\hat{\rho}_{[1]}(\mathbf{x}),\hat{j}^{\mathrm{in}}_{[1]l}(\mathbf{y})]. \tag{49}$$

These separate relations for the cm and internal two-photon operators are important for the low energy limit of the scattering amplitude, which is solely described by the two-photon cm-amplitude giving the well-known Thomson limit at zero photon energy.

4. The Non-Relativistic Photon Scattering Amplitude

From the general expression for the photon scattering amplitude of (2) one obtains for the non-relativistic system under consideration

$$\begin{aligned}
&T_{fi,\lambda'\lambda}(\mathbf{k}',\mathbf{k})=B^{\mathrm{kin}}_{fi,\lambda'\lambda}(\mathbf{k}',\mathbf{k})+B^{\mathrm{MEC}}_{fi,\lambda'\lambda}(\mathbf{k}',\mathbf{k})\\
&-(2\pi)^3\sum_n\int d^3P_n\Big\{\delta(\mathbf{P}_n-\mathbf{P}_i-\mathbf{k})\\
&\cdot\frac{\langle P_f|\boldsymbol{\varepsilon}'^*_{\lambda'}\cdot\hat{\mathbf{j}}(0)|P_n\rangle\langle P_n|\boldsymbol{\varepsilon}_\lambda\cdot\hat{\mathbf{j}}(0)|P_i\rangle}{E_n-E_i-k-i\varepsilon}+\delta(\mathbf{P}_n-\mathbf{P}_i+\mathbf{k}')\\
&\cdot\frac{\langle P_f|\boldsymbol{\varepsilon}_\lambda\cdot\hat{\mathbf{j}}(0)|P_n\rangle\langle P_n|\boldsymbol{\varepsilon}'^*_{\lambda'}\cdot\hat{\mathbf{j}}(0)|P_i\rangle}{E_n-E_i+k'-i\varepsilon}\Big\}\\
&\mathbf{P}_f+\mathbf{k}'=\mathbf{P}_i+\mathbf{k},\ E_f+k'=E_i+k,
\end{aligned}\tag{50}$$

where

$$B^{\mathrm{kin}}_{fi,\lambda'\lambda}=\frac{\boldsymbol{\varepsilon}'^*_{\lambda'}\cdot\boldsymbol{\varepsilon}_\lambda}{M}\langle f|\sum_j e_j^2\,e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_j}|i\rangle \tag{51}$$

$$B^{\mathrm{MEC}}_{fi,\lambda'\lambda}=\int d^3x\,e^{-i\mathbf{k}'\cdot\mathbf{x}}\langle P_f|\varepsilon'^*_{\lambda' l'}\hat{B}^{\mathrm{MEC}}_{l'l}(\mathbf{x},0)\,\varepsilon_{\lambda l}|P_i\rangle. \tag{52}$$

The current matrix elements have the general structure

$$\begin{aligned}
\langle P_a|\hat{\mathbf{j}}(0)|P_b\rangle&=\mathbf{j}_{ab}(\mathbf{q},\mathbf{P})\\
&=\mathbf{j}^{\mathrm{cm}}_{ab}+\mathbf{j}^{\mathrm{in}}_{ab}
\end{aligned}\tag{53}$$

where

$$\mathbf{j}^{\mathrm{cm}}_{ab}(\mathbf{q},\mathbf{P})=\frac{\hat{\rho}(q)}{2AM}\mathbf{P}, \tag{54}$$

$$\hat{\mathbf{j}}^{\mathrm{in}}_{ab}(\mathbf{q},\mathbf{P})=\hat{\mathbf{j}}^{\mathrm{in}}_{ab}(\mathbf{q}), \tag{55}$$

and

$$\begin{aligned}
\mathbf{q}&=\mathbf{P}_a-\mathbf{P}_b,\\
\mathbf{P}&=\mathbf{P}_a+\mathbf{P}_b.
\end{aligned}\tag{56}$$

It is not difficult to show explicitly that the scattering amplitude of (50) fulfills the gauge invariance condition, provided that the MEC two-photon amplitude satisfies (46).

For the low energy limit of the elastic scattering amplitude one neglects the momenta of initial and final photon (long wave length limit) and, thus, can use the Siegert-theorem for the current matrix elements

$$\begin{aligned}
\langle P_n|\hat{\mathbf{j}}(0)|P_i\rangle&=i\,\langle n|[\hat{H}^{\mathrm{in}}_0,\hat{\mathbf{D}}]\,|i\rangle\\
&=i\,\varepsilon_{ni}\,\langle n|\hat{\mathbf{D}}|i\rangle\\
&=i\,\varepsilon_{ni}\,\mathbf{D}_{ni},
\end{aligned}\tag{57}$$

where $\hat{\mathbf{D}}$ is the intrinsic dipole operator and ε_n denotes the intrinsic energy. Then one obtains the well-known expression for the elastic scattering amplitude in the long wave length approximation

$$\begin{aligned}
&T_{ii,\lambda'\lambda}(\mathbf{k}',\mathbf{k})=B_{ii,\lambda'\lambda}(0,0)\\
&-\sum_n\varepsilon^2_{ni}\left(\frac{\boldsymbol{\varepsilon}'^*_{\lambda'}\cdot\mathbf{D}_{in}\,\mathbf{D}_{ni}\cdot\boldsymbol{\varepsilon}_\lambda}{\varepsilon_{ni}-\omega}+\frac{\boldsymbol{\varepsilon}_\lambda\cdot\mathbf{D}_{in}\,\mathbf{D}_{ni}\cdot\boldsymbol{\varepsilon}'^*_{\lambda'}}{\varepsilon_{ni}+\omega}\right)\\
&=B_{ii,\lambda'\lambda}-\langle i|[\boldsymbol{\varepsilon}'^*_{\lambda'}\cdot\hat{\mathbf{D}},[\hat{H}^{\mathrm{in}}_0,\hat{\mathbf{D}}\cdot\boldsymbol{\varepsilon}_\lambda]]\,|i\rangle\\
&-\omega^2\sum_n\left(\frac{\boldsymbol{\varepsilon}'^*_{\lambda'}\cdot\mathbf{D}_{in}\,\mathbf{D}_{ni}\cdot\boldsymbol{\varepsilon}_\lambda}{\varepsilon_{ni}-\omega}+\frac{\boldsymbol{\varepsilon}_\lambda\cdot\mathbf{D}_{in}\,\mathbf{D}_{ni}\cdot\boldsymbol{\varepsilon}'^*_{\lambda'}}{\varepsilon_{ni}+\omega}\right).
\end{aligned}\tag{58}$$

Now the gauge condition of (25) gives

$$B^{\mathrm{in}}_{ii,\lambda'\lambda}=\langle i|[\boldsymbol{\varepsilon}^*_{\lambda'}\cdot\hat{\mathbf{D}},[\hat{H}^{\mathrm{in}}_0,\hat{\mathbf{D}}\cdot\boldsymbol{\varepsilon}_\lambda]]\,|i\rangle \tag{59}$$

It means that the two-photon amplitude of the internal motion is cancelled by the zero-frequency limit of the resonance amplitude. Thus, one arrives at the well-known expression for the photon scattering amplitude for the low energy region in the dipole approximation

$$\begin{aligned}
&T_{fi,\lambda'\lambda}=\delta_{if}\,B^{\mathrm{cm}}_{ii,\lambda'\lambda}\\
&-\omega^2\sum_n\left(\frac{\boldsymbol{\varepsilon}'^*_{\lambda'}\cdot\mathbf{D}_{in}\,\mathbf{D}_{ni}\cdot\boldsymbol{\varepsilon}_\lambda}{\varepsilon_{ni}-\omega}+\frac{\boldsymbol{\varepsilon}_\lambda\cdot\mathbf{D}_{in}\,\mathbf{D}_{ni}\cdot\boldsymbol{\varepsilon}'^*_{\lambda'}}{\varepsilon_{ni}+\omega}\right),
\end{aligned}\tag{60}$$

where the limit $\omega\to 0$ is determined by the two-photon amplitude of the cm-motion alone, giving the classical Thomson limit. Therefore, as long as the long wave length approximation is reliable, there are no explicit meson exchange corrections even though in replacing the current matrix elements by the corresponding dipole transitions (see (57)) one has implicitly taken into account the MEC contributions.

On the other hand, if one goes to higher photon energies where the long wave limit is no longer applicable, one has to consider explicitly the MEC-two-photon amplitude as defined in (43). Since the resonance amplitude is expected to go to zero with increasing energy it has been suggested by Christillin and Rosa-Clot [5, 6] that at higher energies, say around 100 MeV, the MEC-two-photon amplitude is important and may give a relatively strong contribution to the total scattering amplitude. Extrapolations, however, from low energy expressions do not appear reliable to estimate the relative importance of $B^{\mathrm{MEC}}_{fi\lambda'\lambda}(\mathbf{k}',\mathbf{k})$ and it is safer to use an explicit model of MEC effects.

5. Construction of the MEC Two-Photon Amplitude

In the case that the NN-force is obtained from an explicit meson exchange model, the corresponding meson exchange current $\hat{\mathbf{j}}^{\mathrm{MEC}}_{[2]}$ and two-photon operator $\hat{B}^{\mathrm{MEC}}_{kl}(\mathbf{x},\mathbf{y})$ can be calculated reliably by either explicit evaluation of the proper diagrams or by the principle of minimal substitution [7]. The latter method will be pursued in this paper and exemplified for the pion exchange contribution. To this end, we write the OPE-potential in a somewhat unconventional form so that all implicit nucleon and meson momentum and isospin dependence is exhibited explicitly [8].

$$V^{\mathrm{OPE}}(\mathbf{r}_{12};(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_\pi)) = \frac{f^2}{m^2}[\boldsymbol{\sigma}_1\cdot\mathbf{p}_1,[\boldsymbol{\sigma}_2\cdot\mathbf{p}_2,\sum_\mu(-)^\mu\tau_{1,\mu} \cdot\langle\mathbf{r}_1,\mu|g(\mathbf{p}_\pi)\,G_0(\mathbf{p}_\pi)\,g(\mathbf{p}_\pi)|\mathbf{r}_2,\mu\rangle\,\tau_{2,-\mu}]], \tag{61}$$

where

$$G_0(\mathbf{p}_\pi)=(\mathbf{p}_\pi^2+m^2)^{-1} \tag{62}$$

is the propagator of the exchanged pion and $g(\mathbf{p}_\pi)$ a phenomenological vertex form factor. For simplicity we will put $g(\mathbf{p}_\pi)=1$. Furthermore, we have introduced in (61) a pion wave function $|\mathbf{r},\mu\rangle$ denoting the isospin projection by μ. Then the method of minimal substitution gives the meson exchange current from the functional derivative of the OPE-potential with respect to $\mathbf{A}$

$$\hat{\mathbf{j}}^{\mathrm{MEC}}_{[2]}(\mathbf{x})=-\frac{\delta}{\delta\mathbf{A}(\mathbf{x})}V^{\mathrm{OPE}}(\mathbf{r}_{12};(\mathbf{p}_1-e_1\mathbf{A}_1,\mathbf{p}_2 -e_2\mathbf{A}_2,\mathbf{p}_\pi-e_\pi\mathbf{A}_\pi))|_{\mathbf{A}\equiv 0}, \tag{63}$$

where

$$\mathbf{A}_j=\mathbf{A}(\mathbf{r}_j),\quad e_j=\frac{e}{2}(1+\tau_{j3}),$$
$$\mathbf{A}_\pi=\mathbf{A}(\mathbf{r}_\pi),\quad e_\pi=e\,t_{\pi 3}, \tag{64}$$

and the MEC two-photon amplitude from the second functional derivative with respect to $\mathbf{A}$ (see (43)). Using the explicit form of (61), one obtains the following expression as second order term of V^{OPE} with respect to the vector potential

$$\begin{aligned}\frac{f^2}{m^2}\{&[e_1\boldsymbol{\sigma}_1\cdot\mathbf{A}_1,[e_2\boldsymbol{\sigma}_2\cdot\mathbf{A}_2,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\langle\mathbf{r}_1|G_0|\mathbf{r}_2\rangle]]\\ &+[e_1\boldsymbol{\sigma}_1\cdot\mathbf{A}_1,[\boldsymbol{\sigma}_2\cdot\mathbf{p}_2,e\,i\,T^1_{12}\\ &\cdot\langle\mathbf{r}_1|G_0\{\mathbf{p}_\pi;\mathbf{A}_\pi\}\,G_0|r_2\rangle]]\\ &+[\boldsymbol{\sigma}_1\cdot\mathbf{p}_1,[e_2\boldsymbol{\sigma}_2\cdot\mathbf{A}_2,e\,i\,T^1_{12}\\ &\cdot\langle\mathbf{r}_1|G_0\{\mathbf{p}_\pi;\mathbf{A}_\pi\}\,G_0|\mathbf{r}_2\rangle]]\\ &+[\boldsymbol{\sigma}_1\cdot\mathbf{p}_1,[\boldsymbol{\sigma}_2\cdot\mathbf{p}_2,e^2\,T^0_{12}\\ &\cdot\langle\mathbf{r}_1|G_0\{\mathbf{p}_\pi;\mathbf{A}_\pi\}\,G_0\{\mathbf{p}_\pi;\mathbf{A}_\pi\}\,G_0|\mathbf{r}_2\rangle]]\\ &+[\boldsymbol{\sigma}_1\cdot\mathbf{p}_1,[\boldsymbol{\sigma}_2\cdot\mathbf{p}_2,e^2\,T^0_{12}\\ &\cdot\langle\mathbf{r}_1|G_0\,\mathbf{A}_\pi^2\,G_0|\mathbf{r}_2\rangle]]\}.\end{aligned} \tag{65}$$

Here we have used the following relations for the isospin matrix elements

$$\sum_\mu(-)^\mu\tau_{1,\mu}\langle\mu|\,t_{\pi 3}\,|\mu\rangle\,\tau_{2;\mu}=i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_3 =i\,T^1_{12}, \tag{66}$$

$$\sum_\mu(-)^\mu\tau_{1,\mu}\langle\mu|\,t^2_{\pi 3}\,|\mu\rangle\,\tau_{2;\mu}=\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2-\tau_{1,3}\,\tau_{2,3} =T^0_{12}. \tag{67}$$

Furthermore, one needs

$$[\tau_{1,3},\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]=-2i\,T^1_{12},$$
$$[\tau_{1,3},T^1_{12}]=-2i\,T^0_{12}. \tag{68}$$

Then one obtains from (65) essentially four different contributions to the two-photon operator

$$\hat{B}^{\pi\mathrm{MEC}}_{kl}=\sum_{\alpha=1}^{4}\hat{B}^{\pi\mathrm{MEC},\alpha}_{kl}, \tag{69}$$

where the four amplitudes are:

1) pair-pair current

$$\hat{B}^{\pi\mathrm{MEC},1}_{kl}(\mathbf{x},\mathbf{y})=e^2\frac{f^2}{m^2}\sum_{jj'}J^0_{jj'}\,\sigma_{j,k}\,\delta_j(\mathbf{x}) \cdot\sigma_{j',l}\,\delta_{j'}(\mathbf{y})\,T^1(\mathbf{r}_{jj'}) \tag{70}$$

$$\delta_j(\mathbf{x})=\delta(\mathbf{x}-\mathbf{r}_j) \tag{71}$$

$$J^1(\mathbf{r})=\frac{e^{-mr}}{4\pi r}. \tag{72}$$

2) pair-(π-current)

$$\hat{B}^{\pi\mathrm{MEC},2}_{kl}(\mathbf{x},\mathbf{y})=-e^2\frac{f^2}{m^2}\sum_{jj'}T^0_{jj'}\Big(\sigma_{j,k}\,\delta_j(\mathbf{x}) \cdot\boldsymbol{\sigma}_{j'}\cdot\boldsymbol{\nabla}_{j'}\,J^2_l(\mathbf{r}_j,\mathbf{y},\mathbf{r}_{j'}) +\begin{pmatrix}l\leftrightarrow k\\ \mathbf{y}\leftrightarrow\mathbf{x}\end{pmatrix}\Big) \tag{73}$$

$$\mathbf{J}^2(\mathbf{r}_j,\mathbf{y},\mathbf{r}_{j'})=J^1(\mathbf{r}_j-\mathbf{y})\,\overleftrightarrow{\boldsymbol{\nabla}}_y\,J^1(\mathbf{y}-\mathbf{r}_{j'}). \tag{74}$$

3) (π-current)-(π-current)

$$\hat{B}^{\pi\mathrm{MEC},3}_{kl}(\mathbf{x},\mathbf{y})=-\frac{e^2}{2}\frac{f^2}{m^2}\sum_{jj'}T^0_{jj'}\,\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla}_j\,\boldsymbol{\sigma}_{j'}\cdot\boldsymbol{\nabla}_{j'} \cdot\Big(J^3_{kl}(\mathbf{r}_j,\mathbf{x},\mathbf{y},\mathbf{r}_{j'})+\begin{pmatrix}l\leftrightarrow k\\ \mathbf{y}\leftrightarrow\mathbf{x}\end{pmatrix}\Big) \tag{75}$$

$$\begin{aligned}J^3_{kl}(\mathbf{r},\mathbf{x},\mathbf{y},\mathbf{r}')&=J^1(\mathbf{r}-\mathbf{x})\,\overleftrightarrow{\nabla}_{x,k}\,J^1(\mathbf{x}-\mathbf{y})\,\overleftrightarrow{\nabla}_{y,l}\,J^1(\mathbf{y}-\mathbf{r}')\\ &=J^2_k(\mathbf{r},\mathbf{x},\mathbf{y})\,\overleftrightarrow{\nabla}_{y,l}\,J^1(\mathbf{y}-\mathbf{r}')\\ &=J^1(\mathbf{r}-\mathbf{x})\,\overleftrightarrow{\nabla}_{x,k}\,J^2_l(\mathbf{x},\mathbf{y},\mathbf{r}').\end{aligned} \tag{76}$$

4) π-two-photon amplitude

$$B^{\pi\mathrm{MEC},4}_{kl}(\mathbf{x},\mathbf{y})=-e^2\frac{f^2}{m^2}\delta_{kl}\,\delta(\mathbf{x}-\mathbf{y})\,T^0_{jj'} \cdot\sum_{jj'}\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla}_j\,\boldsymbol{\sigma}_{j'}\cdot\boldsymbol{\nabla}_{j'}\,J^1(\mathbf{x}-\mathbf{r}_j)\,J^1(\mathbf{x}-\mathbf{r}_{j'}) \tag{77}$$

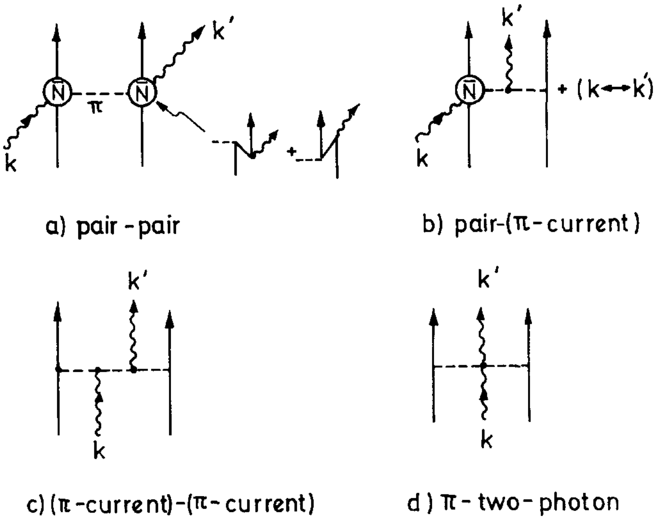

Fig. 2. The various contributions to the π-MEC two-photon amplitude

The corresponding diagrams are shown in Fig. 2. The same expression for $B^{\pi\text{MEC}}_{kl}(\mathbf{x}, \mathbf{y})$ has been obtained by Friar [9] by explicit evaluation of these diagrams. Using the relations

$$(\Delta_x - m^2)\, J^1(\mathbf{x}-\mathbf{r}) = -\delta(\mathbf{x}-\mathbf{r}) \tag{78}$$

$$\nabla_x \cdot \mathbf{J}^2(\mathbf{r}, \mathbf{x}, \mathbf{r}') = (\delta(\mathbf{x}-\mathbf{r}) - \delta(\mathbf{x}-\mathbf{r}'))\, J^1(\mathbf{r}-\mathbf{r}') \tag{79}$$

$$\begin{aligned}&\nabla_{x,k}\, J^3_{kl}(\mathbf{r}, \mathbf{x}, \mathbf{y}, \mathbf{r}')\\ &= (\delta(\mathbf{x}-\mathbf{r}) - \delta(\mathbf{x}-\mathbf{y}))\, J^2_l(\mathbf{r}, \mathbf{y}, \mathbf{r}')\\ &\quad - \nabla_{x,l}\, \delta(\mathbf{x}-\mathbf{y})\, J^1(\mathbf{r}-\mathbf{y})\, J^1(\mathbf{y}-\mathbf{r}'),\end{aligned} \tag{80}$$

one easily can verify that the gauge condition

$$\frac{\partial}{\partial x_k} \hat{B}^{\pi\text{MEC}}_{kl}(\mathbf{x}, \mathbf{y}) = i\, [\hat{\rho}_{[1]}(\mathbf{x}), \hat{j}^{\pi\text{MEC}}_l(\mathbf{y})] \tag{81}$$

is satisfied with the π-exchange current operator

$$\begin{aligned}\hat{j}^{\pi\text{MEC}}_l(\mathbf{y}) &= e\, \frac{f^2}{m^2} \sum_{jj'} T^1_{jj'} (-\tfrac{1}{2} \boldsymbol{\sigma}_j \cdot \nabla_j\, \boldsymbol{\sigma}_{j'} \cdot \nabla_{j'}\\ &\cdot J^2_l(\mathbf{r}_j, \mathbf{y}, \mathbf{r}_{j'}) + \boldsymbol{\sigma}_j \cdot \nabla_j\, \sigma_{j',l}\, \delta(\mathbf{r}_{j'}-\mathbf{y})\, J^1(\mathbf{r}_j - \mathbf{r}_{j'})).\end{aligned} \tag{82}$$

The two-photon scattering amplitude is obtained by a Fourier transformation

$$\begin{aligned}&B^{\pi\text{MEC}}_{fi,\lambda'\lambda}(\mathbf{k}', \mathbf{k}) = \langle P_f | \int d^3x\, e^{-i\mathbf{k}'\cdot\mathbf{x}}\\ &\cdot \varepsilon'^{*}_{\lambda',l'}\, \hat{B}^{\pi\text{MEC}}_{l'l}(\mathbf{x}, 0)\, \varepsilon_{\lambda,l} | P_i \rangle\\ &= e^2 \frac{f^2}{m^2}\, \varepsilon'^{*}_{\lambda',l'}\, \varepsilon_{\lambda,l} \langle f | \sum_{jj'} T^0_{jj'} (\sigma_{j'l'}\, \sigma_{jl}\, e^{i(\mathbf{k}\cdot\mathbf{r}_j - \mathbf{k}'\cdot\mathbf{r}_{j'})}\, J^1(\mathbf{r}_{jj'})\\ &- \left(\sigma_{j'l'}\, e^{-i\mathbf{k}'\cdot\mathbf{r}_{j'}}\, \boldsymbol{\sigma}_j \cdot \nabla_j\, G^2_l(\mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j) + \begin{Bmatrix} l' \leftrightarrow l \\ -\mathbf{k}' \leftrightarrow \mathbf{k} \end{Bmatrix}\right)\\ &- \tfrac{1}{2} \boldsymbol{\sigma}_{j'} \cdot \nabla_{j'}\, \boldsymbol{\sigma}_j \cdot \nabla_j \left(G^3_{l'l}(-\mathbf{k}', \mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j) + \begin{Bmatrix} l' \leftrightarrow l \\ -\mathbf{k}' \leftrightarrow \mathbf{k} \end{Bmatrix}\right)\\ &- \delta_{l'l}\, \boldsymbol{\sigma}_{j'} \cdot \nabla_{j'}\, \boldsymbol{\sigma}_j \cdot \nabla_j\, G^2(\mathbf{k}-\mathbf{k}', \mathbf{r}_{j'}, \mathbf{r}_j)) | i \rangle,\end{aligned} \tag{83}$$

where

$$\begin{aligned}G^2_l(\mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j) &= \int d^3x\, J^2_l(\mathbf{r}_{j'}, \mathbf{x}, \mathbf{r}_j)\, e^{i\mathbf{k}\cdot\mathbf{x}}\\ &= (\nabla_{j',l} - \nabla_{j,l})\, G^2(\mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j)\end{aligned} \tag{84}$$

$$\begin{aligned}G^2(\mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j) &= \int d^3x\, J^1(\mathbf{r}_{j'} - \mathbf{x})\, J^1(\mathbf{x}-\mathbf{r}_j)\, e^{i\mathbf{k}\cdot\mathbf{x}}\\ &= \frac{1}{(2\pi)^3}\, e^{i\mathbf{k}\cdot(\mathbf{r}_{j'}+\mathbf{r}_j)/2} \int d^3q\\ &\cdot \frac{e^{i\mathbf{q}\cdot(\mathbf{r}_{j'}-\mathbf{r}_j)}}{((\mathbf{q}-\mathbf{k}/2)^2 + m^2)((\mathbf{q}+\mathbf{k}/2)^2 + m^2)}\end{aligned} \tag{85}$$

$$\begin{aligned}&G^3_{l'l}(\mathbf{k}', \mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j)\\ &= (2\nabla_{j'} - i\mathbf{k}')_{l'} (2\nabla_j - i\mathbf{k})_l\, G^3(\mathbf{k}', \mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j)\end{aligned} \tag{86}$$

$$\begin{aligned}G^3(\mathbf{k}', \mathbf{k}, \mathbf{r}_{j'}, \mathbf{r}_j) &= \int d^3x\, d^3y\, J^1(\mathbf{r}_{j'} - \mathbf{x})\, J^1(\mathbf{x}-\mathbf{y})\\ &\cdot J^1(\mathbf{y}-\mathbf{r}_j)\, e^{i(\mathbf{k}'\cdot\mathbf{x} + \mathbf{k}\cdot\mathbf{y})}\\ &= \frac{1}{(2\pi)^3}\, e^{i(\mathbf{k}'+\mathbf{k})\cdot(\mathbf{r}_{j'}+\mathbf{r}_j)/2} \int d^3q\\ &\cdot \frac{e^{i\mathbf{q}\cdot(\mathbf{r}_{j'}-\mathbf{r}_j)}}{((\mathbf{q}+\frac{1}{2}(\mathbf{k}+\mathbf{k}'))^2 + m^2)((\mathbf{q}-\frac{1}{2}(\mathbf{k}+\mathbf{k}'))^2 + m^2)}\\ &\cdot \frac{1}{((\mathbf{q}+\frac{1}{2}(\mathbf{k}-\mathbf{k}'))^2 + m^2)}.\end{aligned} \tag{87}$$

In the same manner the two-photon amplitudes for ρ and ω exchange can be obtained. It is not difficult to show explicitly that the two-photon amplitude $B^{\pi\text{MEC}}_{ii\lambda'\lambda}$ of (83) satisfies the low-energy-limit

$$B^{\pi\text{MEC}}_{ii\lambda'\lambda}(0, 0) = \langle i | [\boldsymbol{\varepsilon}'^{*}_{\lambda'} \cdot \hat{\mathbf{D}}, [\hat{V}^{\text{OPE}}, \hat{\mathbf{D}} \cdot \boldsymbol{\varepsilon}_\lambda]] | i \rangle. \tag{88}$$

On the other hand, in the limit $k \to \infty$ only the last term of the forward two-photon exchange amplitude (83) survives corresponding to the two-photon pion amplitude (Fig. 1d), the Thomson scattering off a charged pion:

$$B^{\pi\text{MEC}}_{ii\lambda\lambda}(\mathbf{k}, \mathbf{k}) \xrightarrow[k \to \infty]{} \frac{e^2}{m}\, n_\pi, \tag{89}$$

where

$$\begin{aligned}n_\pi &= -\frac{f^2}{m} \langle i | \sum_{jj'} T^0_{jj'}\, \boldsymbol{\sigma}_j \cdot \nabla_j\, \boldsymbol{\sigma}_{j'} \cdot \nabla_{j'}\, \frac{1}{(2\pi)^3}\\ &\cdot \int d^3q\, \frac{e^{i\mathbf{q}\cdot(\mathbf{r}_j - \mathbf{r}_{j'})}}{(q^2+m^2)^2} | i \rangle.\end{aligned} \tag{90}$$

This result has a simple meaning, because we can interpret n_π as the average number of the exchanged charged pions and, thus, the limit of $B^{\pi\text{MEC}}_{ii\lambda\lambda}$ for $k \to \infty$ is just the free scattering amplitude of an ensemble of n_π free pions.

Therefore, in the high energy limit the forward scattering amplitude of the non-relativistic system approaches the sum of the scattering amplitudes of Z free protons and n_π free pions, if the resonant amplitude tends to zero in this limit as it usually does.

$$T_{ii\lambda\lambda}(\omega) \xrightarrow[\omega\to\infty]{} Z\frac{e^2}{M}+n_\pi \frac{e^2}{m}. \tag{91}$$

This is a nice result, however, rather academic since we have neglected up to now the internal nucleon degrees of freedom, pion production and relativistic effects as well, which will destroy this simple result. Before discussing contributions from intrinsic nucleon degrees of freedom, we will shortly consider dispersion relations.

6. Dispersion Relations for the Scattering Amplitude

Gell-Mann, Goldberger and Thirring [10] have derived the following subtracted dispersion relation for the elastic forward scattering amplitude from causality conditions

$$\begin{aligned}\mathcal{Re}(T_{ii}(\omega)-T_{ii}(0)) &= \frac{2\omega^2}{\pi} P\int_0^\infty d\omega' \frac{\mathcal{Im}\, T_{ii}(\omega')}{\omega'(\omega'^2-\omega^2)}\\ &= -\frac{\omega^2}{2\pi^2} P\int_0^\infty d\omega' \frac{\sigma_{\text{tot}}(\omega')}{\omega'^2-\omega^2}\end{aligned} \tag{92}$$

using crossing symmetry and the optical theorem connecting the total absorption cross section $\sigma_{\text{tot}}(\omega)$ with the imaginary part of the elastic forward scattering amplitude. Even though it has been shown that for a nonrelativistic system causality is violated [11–14] and, thus, this simple relation does not hold exactly, the violation is not so strong as to prevent its use for estimates. For the present formal discussion we will neglect this violation and, furthermore, the internal nucleon structure, which would make it necessary to modify (92) [10].

However, one caveat should be mentioned in applying (92) to the scattering amplitude of (50). Gell-Mann, Goldberger and Thirring have assumed that only the resonance amplitude contributes to the scattering process and did not consider a non-resonant real two-photon amplitude as we are discussing here. If one naively applies the dispersion relation of (92) to the total scattering amplitude, i.e., resonant plus non-resonant part, one arrives at an inconsistency for the high energy limit which is according to (92)

$$\mathcal{Re}\, T_{ii}(\omega) \xrightarrow[\omega\to\infty]{} \mathcal{Re}\, T_{ii}(0)+\frac{1}{2\pi^2}\int_0^\infty d\omega'\, \sigma_{\text{tot}}(\omega'), \tag{93}$$

provided, the integrated cross section converges as we will assume. Now one has

$$T_{ii}(0)=\frac{(Ze)^2}{AM} \tag{94}$$

from the low energy theorem and

$$\begin{aligned}\frac{1}{2\pi^2}\int_0^\infty d\omega'\, \sigma_{\text{tot}}(\omega') &= \langle i|[\hat{D}_z, [\hat{H}_0^{\text{in}}, \hat{D}_z]]|i\rangle\\ &= \frac{NZ}{AM}e^2+B_{ii}^{\text{MEC}}(0,0)\end{aligned} \tag{95}$$

according to Gerasimov [15]. This then gives

$$\mathcal{Re}\, T_{ii}(\infty)=Z\frac{e^2}{M}+B_{ii}^{\text{MEC}}(0,0) \tag{96}$$

in contrast to (91). The error lies in assuming a dispersion relation for the total amplitude. In fact, a consistent picture emerges if one uses the dispersion relation for the resonant part alone [15]

$$\mathcal{Re}(T_{ii}^{\text{res}}(\omega)-T_{ii}^{\text{res}}(0)) = -\frac{\omega^2}{2\pi^2} P\int_0^\infty d\omega' \frac{\sigma_{\text{tot}}(\omega')}{\omega'^2-\omega^2}. \tag{97}$$

Using from (58) [15]

$$T_{ii}^{\text{res}}(0)=-\langle i|[\hat{D}_z, [\hat{H}_0^{\text{in}}, \hat{D}_z]]|i\rangle, \tag{98}$$

one obtains

$$\begin{aligned}\mathcal{Re}\, T_{ii}(\omega) &= \mathcal{Re}(T_{ii}^{\text{res}}(\omega)+B_{ii}(\omega))\\ &= -\langle i|[\hat{D}_z, [\hat{H}_0^{\text{in}}, \hat{D}_z]]|i\rangle+B_{ii}(\omega)\\ &\quad -\frac{\omega^2}{2\pi^2} P\int_0^\infty d\omega' \frac{\sigma_{\text{tot}}(\omega')}{\omega'^2-\omega^2}.\end{aligned} \tag{99}$$

From which follows

$$\mathcal{Re}\, T_{ii}(\infty)=B_{ii}(\infty)+T_{ii}^{\text{res}}(0)+\frac{1}{2\pi^2}\int_0^\infty d\omega'\, \sigma_{\text{tot}}(\omega') \tag{100}$$

and if $T_{ii}^{\text{res}}(\infty)=0$, we recover the result of Gerasimov [15]

$$\begin{aligned}\int_0^\infty d\omega'\, \sigma_{\text{tot}}(\omega') &= 2\pi^2\, T_{ii}^{\text{res}}(0)\\ &= 2\pi^2\langle i|[\hat{D}_z, [\hat{H}_0^{\text{in}}, \hat{D}_z]]|i\rangle\end{aligned} \tag{101}$$

and $T_{ii}(\infty)$ is given by (91).

This problem is connected with the static limit of the non-relativistic treatment neglecting retardation effects in the meson exchange processes as mentioned in the introduction. The absence of real pion production as absorptive process prohibits the use of dispersion relations for this part of the amplitude. Therefore, in estimating the relative importance of the MEC-photon amplitude by dispersion relations in the non-relativistic model one should use (99).

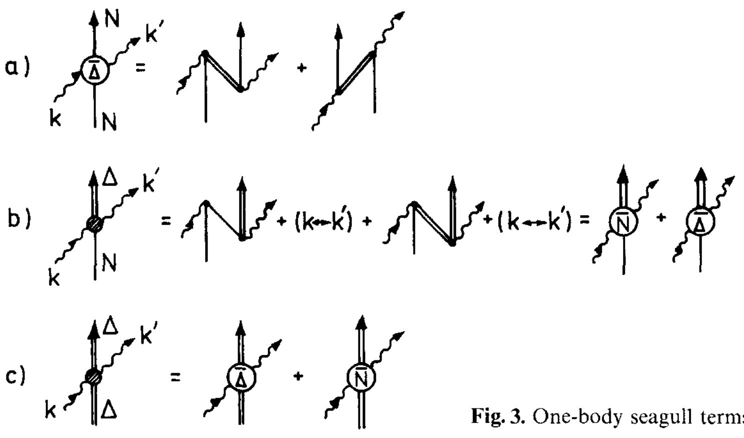

Fig. 3. One-body seagull terms involving Δ and $\bar{\Delta}$ isobars

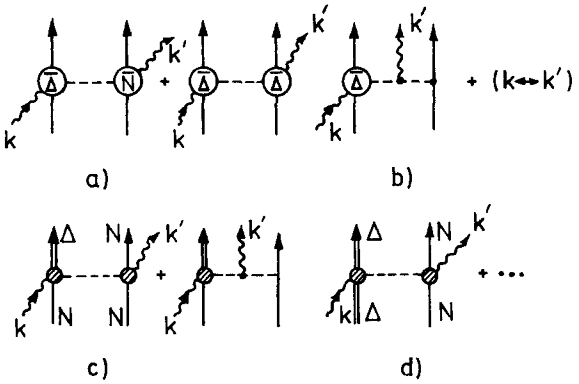

Fig. 4. Two-body pion exchange seagull terms involving Δ and $\bar{\Delta}$ isobars

7. Isobar Current Contributions

In this last section we will discuss the contributions of isobar currents to the two-photon amplitude. For simplicity we will restrict our considerations to the $\Delta(1232)$ isobar. The results can easily be generalized to other isobar states. Again one has two kinds of two-photon amplitudes:

(i) One-body contributions from intermediate antiparticles, e.g., anti-Δ, giving a two-photon operator in the non-relativistic reduction similar to the non-relativistic Thomson amplitude. They are displayed in Fig. 3. The diagrams with incoming and/or outgoing Δ (Fig. 3b, c) contribute only, if nuclear isobar configurations [16, 17] are included in the nuclear wave functions.

(ii) Two-body contributions from meson exchange processes as shown in Fig. 4. First there is an additional nucleon two-photon amplitude corresponding to the diagrams a) and b) in Fig. 2, but with an intermediate anti-Δ instead of an antinucleon (see Fig. 4a, b). Furthermore, there are two-body meson exchange contributions involving incoming and/or outgoing isobars which again appear only if the wave functions contain isobar configurations. A few examples are shown in the diagrams c) and d) of Fig. 4.

We will now briefly derive the most important nucleon amplitudes corresponding to the diagrams of Fig. 3a and Fig. 4a in the non-relativistic limit. First we need the nucleon-anti-Δ transition current. One has, similar to the N-Δ current [17-19],

$$\langle \bar{\Delta}\, p' s' | \hat{j}_\nu(0) | N p s\rangle = \bar{u}^\mu_{\bar{\Delta}}(p', s')\, O_{\mu\nu}\, u_N(p, s), \tag{102}$$

where the gauge invariant operator $O_{\mu\nu}$ is given by

$$O_{\mu\nu} = (a_1(g_{\mu\nu} q^2 - q_\mu q_\nu) + a_2(g_{\mu\nu}\, q\cdot P - q_\mu P_\nu) + a_3(g_{\mu\nu}\, \not{q} - q_\mu \gamma_\nu))\gamma_5, \tag{103}$$

and $q = p' - p$, $P = p' + p$. The non-relativistic reduction analogous to the N-Δ current [19] gives then

$$\langle \bar{\Delta} | \hat{j}_0(0) | N\rangle = \sqrt{\tfrac{3}{2}}\, \bar{a}\, \tau_{\Delta N,3}\, \boldsymbol{\sigma}_{\Delta N}\cdot \mathbf{q},$$

$$\langle \bar{\Delta} | \hat{\mathbf{j}}(0) | N\rangle = \sqrt{\tfrac{3}{2}}\, \bar{a}\, q_0\, \tau_{\Delta N,3}\, \boldsymbol{\sigma}_{\Delta N}, \tag{104}$$

where

$$\bar{a} = -a_1 q_0 - a_2(q_0 + 2M) + a_3. \tag{105}$$

It may be expressed by the on-shell $M1$ and $E2$ transition moments [17]

$$G^{\Delta N}_{M1} = \frac{2}{e}(a - 2b) \qquad G^{\Delta N}_{E2} = \frac{4}{a}\, a$$

$$a = -\tfrac{1}{4}(a_1(M_\Delta - M) + a_2(M_\Delta + M) + a_3)(M_\Delta - M)^\star$$

$$b = \tfrac{1}{2} M a_3 \tag{106}$$

giving

$$\bar{a} \approx -\frac{1}{M} G^{\Delta N}_{M1} + \frac{M_\Delta + M}{2M(M_\Delta - M)} G^{\Delta N}_{E2}$$

$$\approx -\frac{1}{M} G^{\Delta N}_{M1} + \frac{1}{M_\Delta - M} G^{\Delta N}_{E2}. \tag{107}$$

⋆ In [17] the (6.6) for a contains an error.

Furthermore, $\boldsymbol{\sigma}_{\Delta N}$ is the usual transition spin operator with $\langle\frac{3}{2}\|\sigma_{\Delta N}^{[1]}\|\frac{1}{2}\rangle=2$. Summing the two diagrams of Fig. 3a one obtains as the corresponding two-photon amplitude

$$B_{fi,\lambda'\lambda}^{(\bar{\Delta})}(\mathbf{k}',\mathbf{k})=\langle f|\frac{k'k}{M_\Delta+M}\sum_j \bar{a}_j^2(\boldsymbol{\varepsilon}_{\lambda'}'^*\cdot\boldsymbol{\sigma}_{N\Delta}\boldsymbol{\sigma}_{\Delta N}\cdot\boldsymbol{\varepsilon}_\lambda$$
$$+\boldsymbol{\varepsilon}_\lambda\cdot\boldsymbol{\sigma}_{N\Delta}\boldsymbol{\sigma}_{\Delta N}\cdot\boldsymbol{\varepsilon}_{\lambda'}'^*)\,e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_j}|i\rangle$$
$$\approx\boldsymbol{\varepsilon}_{\lambda'}'^*\cdot\boldsymbol{\varepsilon}_\lambda\frac{4}{3}\frac{k'k\bar{a}^2}{2M}\langle f|\sum_j e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_j}|i\rangle. \tag{108}$$

The mass difference $M_\Delta-M$ has been neglected. Here we have used the identity

$$\mathbf{a}\cdot\boldsymbol{\sigma}_{N\Delta}\boldsymbol{\sigma}_{\Delta N}\cdot\mathbf{b}=\frac{2}{3}\mathbf{a}\cdot\mathbf{b}-\frac{i}{3}\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}). \tag{109}$$

This amplitude tends to zero for $k\to 0$ in accordance with the low energy theorem. It should only be used in the non-relativistic domain, i.e., for $k\ll M_N$. The high energy limit, where the amplitude of (108) diverges is not meaningful.

For the evaluation of the two-body MEC contribution of Fig. 4a one needs the $(\bar{\Delta}N\pi)$-vertex. With the usual $\Delta-N$ interaction Lagrangian [20] one obtains in the non-relativistic limit neglecting the $N-\Delta$ mass difference

$$\Gamma_\alpha^{\bar{\Delta}N\pi}(\mathbf{q})=-i\frac{f_{\Delta N\pi}}{2Mm}\tau_{\Delta N,\alpha}\boldsymbol{\sigma}_{\Delta N}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{q}g^{\bar{\Delta}N\pi}(\mathbf{q}). \tag{110}$$

Here we have introduced a phenomenological vertex form factor $g^{\bar{\Delta}N\pi}(q)$, which should be at least of dipole form in order to avoid divergences. Then the $(\gamma NN\pi)$-vertex function with intermediate anti-Δ is given by

$$\Gamma_{\gamma NN\pi,\alpha}^{(\bar{\Delta})}=-\sum_{\bar{\Delta}}\Big(\langle N|\hat{\mathbf{j}}(0)|\bar{\Delta}\rangle\frac{1}{M_\Delta+M}\Gamma^{\bar{\Delta}N\pi}$$
$$+\Gamma^{\bar{\Delta}N\pi*}\frac{1}{M_\Delta+M}\langle\bar{\Delta}|\hat{\mathbf{j}}(0)|N\rangle\Big)$$
$$=-i\frac{f_{\Delta N\pi}}{m(2M)^2}\sqrt{\frac{3}{2}}\,\bar{a}k(\boldsymbol{\sigma}_{N\Delta}\boldsymbol{\sigma}_{\Delta N}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{q}\tau_{N\Delta,3}\tau_{\Delta N,\alpha}$$
$$-\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}_{N\Delta}\cdot\mathbf{q}\boldsymbol{\sigma}_{\Delta N}\tau_{N\Delta,\alpha}\tau_{\Delta N,3}). \tag{111}$$

From (109) one obtains the following relation

$$\boldsymbol{\sigma}_{N\Delta}\boldsymbol{\sigma}_{\Delta N}\cdot\mathbf{q}=\frac{2}{3}\mathbf{q}+\frac{i}{3}\boldsymbol{\sigma}\times\mathbf{q} \tag{112}$$

and thus

$$\mathbf{S}(\boldsymbol{\sigma},\mathbf{q})=3\boldsymbol{\sigma}_{N\Delta}\boldsymbol{\sigma}_{\Delta N}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{q}$$
$$=3\boldsymbol{\sigma}\cdot\mathbf{q}\mathbf{q}-q^2\boldsymbol{\sigma}. \tag{113}$$

Furthermore with

$$\tau_{N\Delta,3}\tau_{\Delta N,\alpha}-\tau_{N\Delta,\alpha}\tau_{\Delta N,3}=-\tfrac{2}{3}i\tau_\gamma\varepsilon_{\gamma 3\alpha}$$
$$=-\tfrac{1}{3}[\tau_3,\tau_\alpha] \tag{114}$$

one obtains

$$\Gamma_{\gamma NN\pi}^{(\bar{\Delta})}=i\sqrt{\frac{3}{2}}\frac{f_{\Delta N\pi}}{9m}\frac{\bar{a}k}{(2M)^2}[\tau_3,\tau_\alpha]\mathbf{S}(\boldsymbol{\sigma},\mathbf{q})g^{\bar{\Delta}N\pi}(\mathbf{q}), \tag{115}$$

and similarly for an intermediate $\bar{N}$

$$\Gamma_{\gamma NN\pi}^{(\bar{N})}=-i\frac{f_{NN\pi}}{2m}[\tau_3,\tau_\alpha]\boldsymbol{\sigma}g^{\bar{N}N\pi}(\mathbf{q}). \tag{116}$$

With these expressions we finally get for the corresponding two-photon amplitudes of Fig. 4a

(i) $\bar{\Delta},\bar{N}$ intermediate

$$B_{fi,\lambda'\lambda}^{(\bar{\Delta}\bar{N})\mathrm{MEC}}(\mathbf{k}',\mathbf{k})=e^2\frac{\sqrt{6}}{9}\frac{f_{\Delta N\pi}f_{NN\pi}}{m^2}\frac{\bar{a}}{(2M)^2}$$
$$\cdot\langle f|\sum_{j\neq j'}T_{j'j}^0\Big(k'e^{i(\mathbf{k}\cdot\mathbf{r}_j-\mathbf{k}'\cdot\mathbf{r}_{j'})}\boldsymbol{\varepsilon}_{\lambda'}'^*\cdot\mathbf{S}(\boldsymbol{\sigma}_{j'},\boldsymbol{V}_{j'})$$
$$\cdot J^{1(\bar{\Delta}\bar{N})}(\mathbf{r}_{j'}-\mathbf{r}_j)\boldsymbol{\sigma}_j\cdot\boldsymbol{\varepsilon}_\lambda+\begin{Bmatrix}\lambda\leftrightarrow\lambda'\\ \mathbf{k}\leftrightarrow-\mathbf{k}'\end{Bmatrix}\Big)|i\rangle, \tag{117}$$

where

$$J^{1(\bar{\Delta}\bar{N})}(\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3q\frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2+m^2}g^{\bar{\Delta}N\pi}(\mathbf{q})g^{\bar{N}N\pi}(\mathbf{q}). \tag{118}$$

(ii) $\bar{\Delta},\bar{\Delta}$ intermediate

$$B_{fi,\lambda'\lambda}^{(\bar{\Delta}\bar{\Delta})\mathrm{MEC}}(\mathbf{k}',\mathbf{k})=e^2\frac{2}{27}\frac{f_{\Delta N\pi}^2}{m^2}\frac{\bar{a}^2k'k}{(2M)^4}$$
$$\cdot\langle f|\sum_{j\neq j'}T_{jj'}^0\boldsymbol{\varepsilon}_{\lambda'}'^*\cdot\mathbf{S}(\boldsymbol{\sigma}_{j'},\boldsymbol{V}_{j'})\mathbf{S}(\boldsymbol{\sigma}_j,\boldsymbol{V}_j)$$
$$\cdot e^{i(\mathbf{k}\cdot\mathbf{r}_j-\mathbf{k}'\cdot\mathbf{r}_{j'})}J^{1(\bar{\Delta}\bar{\Delta})}(\mathbf{r}_{j'j})|i\rangle \tag{119}$$

and $J^{1(\bar{\Delta}\bar{\Delta})}$ analogous to (118). These amplitudes are small in comparison to the corresponding amplitudes with intermediate $\bar{N}$ since they carry an extra factor of $\frac{k}{2M}$ or $\frac{kk'}{(2M)^2}$.

As mentioned before, these amplitudes with intermediate $\bar{\Delta}$ should only be used in the non-relativistic region. At present we are evaluating the various contributions to the two-photon amplitude explicitly. The results will be reported elsewhere.

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