*a. The "LIT equation"*

$$\left(\hat{H}_{\mathrm{nuclear}}\underbrace{-E_0-\mathcal{R}\left[\sigma\right]-i\;\mathcal{I}\left[\sigma\right]}_{:=-\mathcal{E}}\right)\Psi_{\mathrm{LIT}}^{J^\pi m_j}=\left[\hat{\mathcal{O}}_{Lm_L}\left\{\left|\boldsymbol{k}\right|,\boldsymbol{j}_v\right\}\otimes\Psi_0^{J_0^{\pi 0}}\right]^{J^\pi m_j}\quad . \tag{1}$$

with

$$v(\mathrm{ertex})\in\{\;\boldsymbol{j}_o(\boldsymbol{x})=\ldots\;,\;\boldsymbol{j}_s(\boldsymbol{x})=\ldots\;,\;\boldsymbol{j}_{mec}(\boldsymbol{x})=\ldots\;,\;\ldots\}\quad ; \tag{2}$$

*b. The variational basis*

$$\Psi_{\mathrm{LIT}}^{J^\pi m_j}=\sum_n u_n\;\phi_n^{J^\pi m_j}\quad . \tag{3}$$

with

$$\phi_n^{J^\pi m_j}=\left[\xi_{S_n}\otimes\mathcal{Y}_{l_n}(\boldsymbol{\rho})\right]^{Jm_j}\;e^{-\gamma_n\boldsymbol{\rho}^2}\qquad(i.e.\;,\;\text{LS coupling})\quad ; \tag{4}$$

*c. The matrix form of the "LIT equation"*

$$\sum_{s=1}^{N_{\mathrm{LIT}}}\phi_r^{J^\pi m_j}\left(\hat{H}_{\mathrm{nuclear}}-\mathcal{E}\right)\phi_s^{J^\pi m_j}\;u_s=\sum_{n=1}^{N_0}\sum_{m_L}c_n\;\underbrace{\left(\,L\;\;m_L\;\;J_0\;\;m_j-m_L\,\middle|\,J\;\;m_j\,\right)}_{\texttt{enemb:ecce}}\;\phi_r^{J^\pi m_j}\hat{\mathcal{O}}_{Lm_L}\,\phi_n^{J_0^{\pi 0}(m_j-m_L)}\quad . \tag{5}$$

with

$$N_{\mathrm{LIT}}\;:\text{ number of basis states used to expand the LIT state, }e.g.\;,\Psi_{\mathrm{LIT}}^{2^-}\quad ; \tag{6}$$

$$N_0\leq N_{\mathrm{LIT}}\;:\text{ number of basis states used to expand the target, }e.g.\;,\text{ the deuteron;} \tag{7}$$

$$\tag{8}$$

*d. The matrix element*

$$\phi_m^{J^\pi m_j}\;\hat{\mathcal{O}}_{Lm_L}\;\phi_n^{J_0^{\pi 0}m_{j0}}:=\left\langle\;m;l_lS_lJ_lm_{j_l}\;\;\middle|\;\mathcal{A}\;\mathcal{O}_{Lm_L}\;\;\middle|\;l_rS_rJ_rm_{j_r};n\;\right\rangle \tag{9}$$

$$=\underbrace{(-1)^{L-J_r+J_l}\frac{\left(\,L\;\;m_L\;\;J_r\;\;m_{j_r}\,\middle|\,J_l\;\;m_{j_r}+m_L\,\right)}{\hat{J}_l}}_{\texttt{enemb:600ff}}\underbrace{\left\langle\;m;l_lS_lJ_l\;\;\middle\|\;\mathcal{A}\;\mathcal{O}_L\;\;\middle\|\;l_rS_rJ_r;n\;\;\right\rangle}\tag{10}$$

$$\downarrow$$

$$\underbrace{\hat{J}_r\hat{J}_l\hat{L}\begin{Bmatrix}l_l^m & l_r^n & p\\ S_l^m & S_r^n & q\\ J_l & J_r & L\end{Bmatrix}}_{\texttt{enemb:ecce}}\sum_{\mathrm{dc}}\sum_{\mathcal{P}\in\mathrm{dc}}\underbrace{\left\langle\;m;l_l^m\;\;\middle\|\;\mathcal{O}_p^o\;\;\middle\|\;\mathcal{A}_{\mathrm{dc}}l_r^n;n\;\;\right\rangle}_{\texttt{luise}}\cdot\underbrace{\left\langle\;m;S_l^m\;\;\middle\|\;\mathcal{O}_q^s\;\;\middle\|\;\mathcal{A}_{\mathcal{P}}S_r^n;n\;\;\right\rangle}_{\texttt{obem}} \tag{11}$$

with

$$\hat{a}:=\sqrt{2a+1}\quad ; \tag{12}$$

$$\mathcal{A}=\sum_{\mathcal{P}\in\mathcal{S}_{A-1}}(-1)^{\mathrm{sgn}(\mathcal{P})}\hat{\mathcal{P}}=\oplus_{\mathrm{dc}} \tag{13}$$

$$\mathrm{dc}\;\;:\;\;\text{double co-set} \tag{14}$$

*e. The calculation*

(i) Solve

$$\hat{H}_{\text{nuclear}}\ \Psi^{J_0^{\pi 0}} = E_0\ \Psi^{J_0^{\pi 0}}$$

with ansatz

$$\Psi^{J_0^{\pi 0}} = \sum_n c_n\ \phi_n^{J_0^{\pi 0}} \quad .$$

If $\hat{H}_{\text{nuclear}}$ is a spherical rank-0 operator – a condition which most practical nuclear potentials satisfy – $\Psi^{J_0^{\pi 0}} \neq f(m_{j_0})$. We obtain $\Psi^{J_0 J_0}$, in practice.

(ii) Calculate

$$H_{rs} := \left\langle\ \phi_r^{J^\pi}\ \middle|\ \hat{H}_{\text{nuclear}}\ \middle|\ \phi_s^{J^\pi}\ \right\rangle\ \text{ and }\ N_{rs} := \left\langle\ \phi_r^{J^\pi}\ \middle|\ \phi_s^{J^\pi}\ \right\rangle\ \ \forall\ |L-J_0| \leq J \leq |L+J_0|$$

(iii) Calculate

$$S_{rs}^{Jm_j} := \left\langle\ \phi_r^{J^\pi m_j}\ \middle|\ \hat{\mathcal{O}}_{Lm_L}\ \middle|\ \phi_s^{J_0^{\pi 0} m_{j_0}}\ \right\rangle\ \ ,$$

and superimpose these matrix elements according to Eq.(5)

$$S_r^{Jm_j} := \sum_{m_L} c_n\ \left(\ L\ \ m_L\ \ J_0\ \ m_{j_0} - m_L \,\middle|\, J\ \ m_j\ \right)\ S_{rn}^{Jm_j} \quad . \tag{15}$$

(iv) Solve the (complex) linear matrix equation

$$\left(H_{rs} - \mathcal{E} N_{rs}\right) u_s^{Jm_j} = S_r \tag{16}$$

to obtain the LIT state

$$\psi^{v(\text{ertex}),(\text{mu})L(\text{tipolarity})}_{J_{i(\text{nitial})/f(\text{inal})};J_{(\text{i})n(\text{termediate})}m_n}(k,\sigma) = \psi^{v,L}_{J_0;Jm_j}(k,\sigma) := \Psi_{\text{LIT}}^{J^\pi m_j}\Big(\ \underbrace{|\boldsymbol{k}|\,,v,L}_{\substack{\text{vertex}\\ \text{quantum numbers}}}\ \ ;\ \ \underbrace{E_0,J_0}_{\substack{\text{initial/final-state}\\ \text{quantum numbers}}}\ \ ;\mathcal{R}\left[\sigma\right],\mathcal{I}\left[\sigma\right]\Big) \quad . \tag{17}$$

(v) The inner product

$$\mathcal{L}^{J_f,J_i;J}_{v'L',vL}(k',k,\sigma) = (-1)^{J-J_i+L-L'+v'} N_{J,\sigma} \sum_{m_j} \underbrace{\left\langle\ \psi^{v',L'}_{J_f;Jm_j}(k',\sigma)\ \middle|\ \psi^{v,L}_{J_i;Jm_j}(k,\sigma)\ \right\rangle}_{=\sum_{r,s}(u_r^{Jm_j})^* u_s^{Jm_j}\ \ N_{rs}} \tag{18}$$

# I. FORMULAS AND CONSTANTS

(Wigner) 3-$j$ symbol:

$$\begin{pmatrix} L & S & J \\ m_l & m_s & -m_j \end{pmatrix} = (-1)^{L-S+m_j}\ (2J+1)^{-\frac{1}{2}}\ (Lm_l\ Sm_s \mid LS\ Jm_j) \tag{19}$$

Matrix for single-axis rotation:

$$\begin{aligned} \mathcal{D}^{(j)}_{m',m}(0\ \beta\ 0) &\equiv d^{(j)}_{m',m}(\beta) \\ &= \left[\frac{(j+m')!(j-m')!}{(j+m)!(j-m)!}\right]^{\frac{1}{2}} \\ &\quad\cdot \sum_{\sigma} \binom{j+m}{j-m'-\sigma} \binom{j-m}{\sigma} (-1)^{j-m'-\sigma} \\ &\quad\cdot \left(\cos\frac{\beta}{2}\right)^{2\sigma+m+m'} \left(\sin\frac{\beta}{2}\right)^{2j-2\sigma-m-m'} \end{aligned} \tag{20}$$