I. A TALE OF TWO METHODS TO SCATTER PHOTONS OF A DEUTERON

Ref. [1] does not explicitly calculate scattering amplitudes – transition probabilities between states with one deuteron and one photon in the initial and final states as parametrized in Eq. (27) and Fig. (2) of Ref. [1] – which are output of calculations by Hildebrandt [2] who (I am not sure this is true) was the first to resolve the problem to consider the interacting propagation of the two deuteron-comprising nucleons between the absorption and creation of the photons in the framework of chiral perturbation theory. The latter method employs Siegert's theorem – the approximation of the photon field $\boldsymbol{e}_\lambda\ e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \approx \boldsymbol{\nabla} S(\boldsymbol{r})$, *i.e.* , as a pure nabla field in order to utilize the conservation of the nuclear current – to facilitate the calculation of the amplitude which can alternatively be obtained with the *Lorentz Integral Transform Method* [3]. The latter's application to larger nuclei is, barring potentially prohibitive numerical complications, viable. To asses its practicality for *us*, we reconstruct the amplitude from polarizabilities[†] via

$$\begin{aligned} T^{fi}_{\lambda'\lambda}(\boldsymbol{k}',\boldsymbol{k}) =& (-)^{1+\lambda'+I_f-M_i} \sum_{\substack{L^{(\prime)},M^{(\prime)}\\ J}} (-)^{L+L'}(2J+1) \begin{pmatrix} I_f & J & I_i \\ -M_f & m & M_i \end{pmatrix} \begin{pmatrix} L & L' & J \\ M & M' & -m \end{pmatrix} \\ & \underbrace{P^{LL'\lambda\lambda'}_{if,J}(k',k)}_{\substack{= \\ \sum_{\nu,\nu'} \lambda'^{\nu'}\lambda^{\nu} P_{if,J}(M^{\nu'}L',M^{\nu}L,k',k)}} \qquad D^L_{M,\lambda}(\boldsymbol{k}\to\boldsymbol{e}_z) D^{L'}_{M',-\lambda'}(\boldsymbol{k}'\to\boldsymbol{e}_z) \end{aligned}$$

with the dipole approx. as used in Ref. [1], *i.e.* , electric ($\nu^{(\prime)}=0$) $\Rightarrow L^{(\prime)}=1$ transitions, $k=k'$ in c.m. system, and the total spin of the deuteron $I_{i/f}=1$ and associated z-projections $M_{i/f}$, this reads

$$=(-)^{\lambda'-M_i} \sum_{M^{(\prime)},J} (2J+1) \begin{pmatrix} 1 & J & 1 \\ -M_f & m & M_i \end{pmatrix} \begin{pmatrix} 1 & 1 & J \\ M & M' & -m \end{pmatrix} P_{if,J}(E1,k) D^1_{M,\lambda}(\boldsymbol{k}\to\boldsymbol{e}_z) D^1_{M',-\lambda'}(\boldsymbol{k}'\to\boldsymbol{e}_z) \quad .$$

The rotation matrices align the respective $\boldsymbol{k}$ vector with $t$he quantization axis, which is $\boldsymbol{e}_z$, here. It is common to represent the effect of the two photons on the nucleus in an angular-momentum basis. First, each photon field is multipole expanded seperately, before the various terms of the product of both expansions are combined to form spherical tensor operators of rank $|L-L'| \leq J \leq |L+L'|$. These may induce a change of angular momentum – In general, the photon can induce spin- and orbital-angular-momentum transitions of arbitrary magnitude regardless of its nature as a $S=1$ gauge boson. Only if the interaction between this vector particle and fermions is constrained to momentum- and spin-vector independent couplings, one can disregard ΔS, while ΔL might be limited through the multipole expansion of A_μ. – of the deuteron by $\leq J$. In the considered approximation, $|I_i - I_f| \in \{0,1,2\}$. The projection m is completely fixed by $M_{i/f}$, and $M^{(\prime)}$ are the z-projections of the multipole $E1$ components of the photons.

$$=(-)^{\lambda'-M_i} \sum_{M',J} (2J+1) \begin{pmatrix} 1 & J & 1 \\ -M_f & m & M_i \end{pmatrix} \begin{pmatrix} 1 & 1 & J \\ M & M' & -m \end{pmatrix} P_{if,J}(E1,k)\ d^{(1)}_{M',-\lambda'}(-\theta) = T^{fi}_{\lambda'}(k,\theta) \qquad (1)$$

We chose $\boldsymbol{k} \parallel \boldsymbol{e}_z$. Therefore, there is no need to rotate $\boldsymbol{k}$ and $D^1_{M,\lambda}(\alpha,\beta,\gamma)=1$. Note that in the considered dipole approximation, this implies the λ independence of the amplitude?! The scattering angle θ parameterizes the propagation direction of the outgoing photon, and the scattering plane is then defined such that a rotation about the y-axis of $\boldsymbol{k}'$ by $-\theta$ aligns $\boldsymbol{k}'$ with $\boldsymbol{e}_z$.

$$D^1_{M',-\lambda'}(\alpha_z=0,\beta_y=-\theta,\gamma_z=0) = d^{(1)}_{M',-\lambda'}(-\theta) \quad .$$

[†]If not stated differently, we use conventions and notation consistent with Ref. [1].

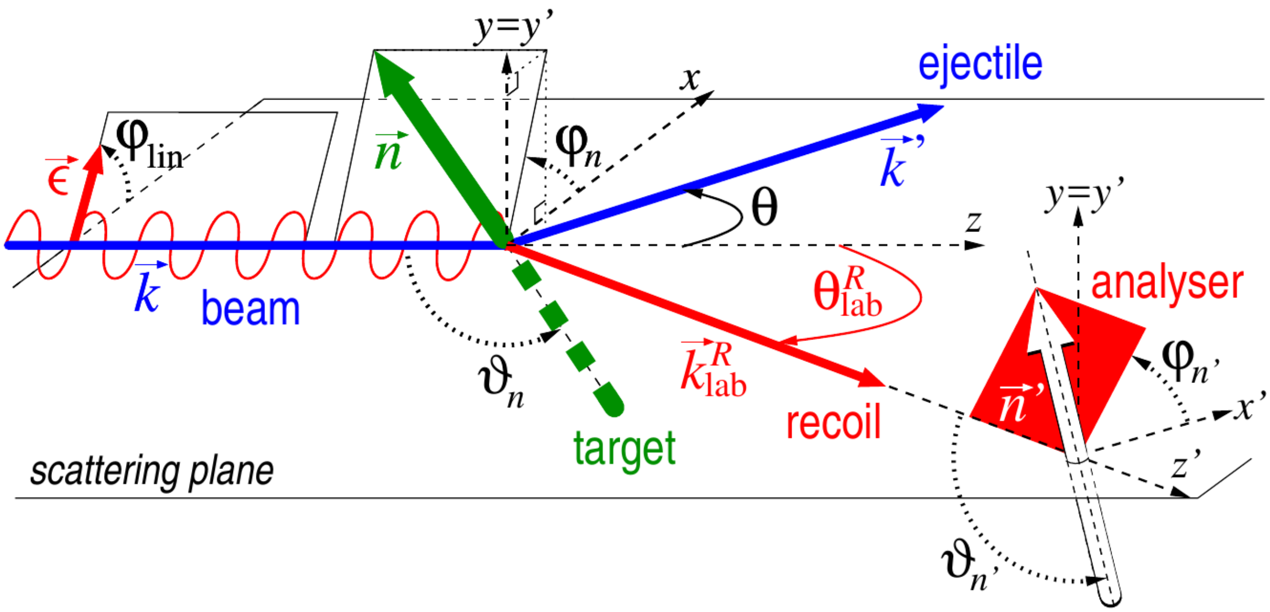

FIG. 1. Scattering geometry as taken from Ref. [4].

II. FORMULAS AND CONSTANTS

$$\text{(Wigner) 3-}j\text{ symbol:}\qquad \begin{pmatrix} L & S & J \\ m_l & m_s & -m_j \end{pmatrix} = (-1)^{L-S+m_j}\,(2J+1)^{-\frac{1}{2}}\,(Lm_l\;Sm_s \mid LS\;Jm_j) \tag{2}$$

$$\begin{aligned}\text{Matrix for single-axis rotation:}\qquad \mathcal{D}^{(j)}_{m',m}(0\;\beta\;0) &\equiv d^{(j)}_{m',m}(\beta) \\ &= \left[\frac{(j+m')!(j-m')!}{(j+m)!(j-m)!}\right]^{\frac{1}{2}} \\ &\quad\cdot \sum_{\sigma} \begin{pmatrix} j+m \\ j-m'-\sigma \end{pmatrix} \begin{pmatrix} j-m \\ \sigma \end{pmatrix} (-1)^{j-m'-\sigma} \\ &\quad\cdot \left(\cos\frac{\beta}{2}\right)^{2\sigma+m+m'} \left(\sin\frac{\beta}{2}\right)^{2j-2\sigma-m-m'}\end{aligned} \tag{3}$$

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1 G. Bampa, W. Leidemann, and H. Arenhovel, Phys. Rev. **C84**, 034005 (2011), arXiv:1107.2320 [nucl-th].

2 R. P. Hildebrandt, *Elastic Compton Scattering from the Nucleon and Deuteron*, Ph.D. thesis, Munich, Tech. U. (2005), arXiv:nucl-th/0512064 [nucl-th].

3 V. D. Efros, W. Leidemann, G. Orlandini, and N. Barnea, Journal of Physics G: Nuclear and Particle Physics **34**, R459 (2007).

4 H. W. Griesshammer, J. A. McGovern, and D. R. Phillips, Eur. Phys. J. **A54**, 37 (2018), arXiv:1711.11546 [nucl-th].