

a. The “LIT equation”

$$\left( \hat{H}_{\text{nuclear}} - \underbrace{E_0 - \mathcal{R}[\sigma] - i \mathcal{I}[\sigma]}_{:= -\mathcal{E}} \right) \Psi_{\text{LIT}}^{J^\pi m_j} = \left[ \hat{\mathcal{O}}_{Lm_L} \{ |\mathbf{k}|, \mathbf{j}_v \} \otimes \Psi_0^{J_0^{\pi_0}} \right]^{J^\pi m_j} . \quad (1)$$

with

$$v(\text{ertex}) \in \{ \mathbf{j}_o(\mathbf{x}) = \dots, \mathbf{j}_s(\mathbf{x}) = \dots, \mathbf{j}_{mec}(\mathbf{x}) = \dots, \dots \} ; \quad (2)$$

b. The variational basis

$$\Psi_{\text{LIT}}^{J^\pi m_j} = \sum_n u_n \phi_n^{J^\pi m_j} . \quad (3)$$

with

$$\phi_n^{J^\pi m_j} \in \left\{ [\xi_{S_n} \otimes \mathcal{Y}_{l_n}(\boldsymbol{\rho})]^{Jm_j} e^{-\gamma_n \boldsymbol{\rho}^2} \mid \gamma \in \mathbb{R}_+, s \in \mathbb{N}^{A-1} + \left(\frac{\mathbb{N}}{2}\right)^{A-2}, l \in \mathbb{N}^{A-1} + \mathbb{N}^{A-2} \right\} \quad (4)$$

c. The matrix form of the “LIT equation”

$$\sum_{s=1}^{N_{\text{LIT}}} \phi_r^{J^\pi m_j} \left( \hat{H}_{\text{nuclear}} - \mathcal{E} \right) \phi_s^{J^\pi m_j} u_s = \sum_{n=1}^{N_0} \sum_{m_L} c_n \underbrace{\langle LJ_0; m_L m_j - m_L \mid Jm_j \rangle}_{\text{Eq.(17)}} \phi_r^{J^\pi m_j} \hat{\mathcal{O}}_{Lm_L} \phi_n^{J_0^{\pi_0}(m_j - m_L)} . \quad (5)$$

with

$$N_{\text{LIT}} : \text{number of basis states used to expand the LIT state, e.g. } \Psi_{\text{LIT}}^{2-} ; \quad (6)$$

$$N_0 \leq N_{\text{LIT}} : \text{number of basis states used to expand the target, e.g. , the deuteron;} \quad (7)$$

$$(8)$$

d. The matrix element

$$\phi_m^{J^\pi m_j} \hat{\mathcal{O}}_{Lm_L} \phi_n^{J_0^{\pi_0} m_{j_0}} := \langle m; l_l S_l J_l m_{j_l} \mid \mathcal{A} \mathcal{O}_{Lm_L} \mid l_r S_r J_r m_{j_r}; n \rangle \quad (9)$$

$$= (-1)^{L-J_r+J_l} \underbrace{\langle LJ_r; m_L m_{j_r} \mid J_l m_{j_l} \rangle}_{\text{enemb:600ff}} \underbrace{\langle m; l_l S_l J_l \parallel \mathcal{A} \mathcal{O}_L \parallel l_r S_r J_r; n \rangle}_{\downarrow} \quad (10)$$

$$\underbrace{\hat{J}_r \hat{J}_l \hat{L} \left\{ \begin{matrix} l_l^m & l_r^n & p \\ S_l^m & S_r^n & q \\ J_l & J_r & L \end{matrix} \right\}}_{\text{enemb:ecce}} \sum_{\text{dc}} \sum_{\mathbf{p} \in \text{dc}} \underbrace{\langle m; l_l^m \parallel \mathcal{O}_p^o \parallel \mathcal{A}_{\text{dc}} l_r^n; n \rangle}_{\text{luise}} \cdot \underbrace{\langle m; S_l^m \parallel \mathcal{O}_q^s \parallel \mathcal{A}_{\mathbf{p}} S_r^n; n \rangle}_{\text{obem}} \quad (11)$$

$$= \hat{J}_r \hat{L} \langle \mathbf{J}_r \mathbf{L}; \mathbf{m}_{j_r} \mathbf{m}_L \mid J_l m_{j_l} \rangle \cdot \left\{ \begin{matrix} l_l^m & l_r^n & p \\ S_l^m & S_r^n & q \\ J_l & J_r & L \end{matrix} \right\} \quad (12)$$

$$\cdot \sum_{\text{dc}} \sum_{\mathbf{p} \in \text{dc}} \langle m; l_l^m \parallel \mathcal{O}_p^o \parallel \mathcal{A}_{\text{dc}} l_r^n; n \rangle \cdot \langle m; S_l^m \parallel \mathcal{O}_q^s \parallel \mathcal{A}_{\mathbf{p}} S_r^n; n \rangle \quad (13)$$

with

$$\hat{a} := \sqrt{2a+1} ; \quad (14)$$

$$\mathcal{A} = \sum_{\mathbf{p} \in \mathcal{S}_{A-1}} (-1)^{\text{sgn}(\mathbf{p})} \hat{\mathbf{p}} = \oplus_{\text{dc}} \quad (15)$$

$$\text{dc} : \text{double co-set} \quad (16)$$

1 *e. The calculation*

(i) Solve

$$\hat{H}_{\text{nuclear}} \Psi^{J_0^{\pi_0}} = E_0 \Psi^{J_0^{\pi_0}}$$

with ansatz

$$\Psi^{J_0^{\pi_0}} = \sum_n c_n \phi_n^{J_0^{\pi_0}} .$$

2 If  $\hat{H}_{\text{nuclear}}$  is a spherical rank-0 operator – a condition which most practical nuclear potentials satisfy –  
3  $\Psi^{J_0^{\pi_0}} \neq f(m_{j_0})$ . We obtain  $\Psi^{J_0 J_0}$ , in practice.

(ii) Calculate

$$\mathbb{H}_{rs} := \left\langle \phi_r^{J^\pi} \left| \hat{H}_{\text{nuclear}} \right| \phi_s^{J^\pi} \right\rangle \quad \text{and} \quad \mathbb{N}_{rs} := \left\langle \phi_r^{J^\pi} \left| \phi_s^{J^\pi} \right\rangle \quad \forall |L - J_0| \leq J \leq |L + J_0|$$

(iii)  $\forall m_j \& m_L$ , calculate

$$S_{rs, m_L}^{J m_j J_0} := \left\langle \phi_r^{J^\pi m_j} \left| \hat{O}_{L m_L} \right| \phi_s^{J_0^{\pi_0} J_0} \right\rangle ,$$

4 and superimpose these matrix elements according to Eq.(5)

$$S_r^{J m_j} := \sum_{m_L} \langle L J_0 ; m_L m_j - m_L | J m_j \rangle \underbrace{\sum_n^{N_0} c_n S_{rn, m_L}^{J m_j J_0}}_{\text{enemb.f OUT}} . \quad (17)$$

5 Ecce,  $\Psi^{J_0^{\pi_0}} \neq f(m_{j_0})$  does not allow for an elimination of  $m_j$  from this equation!

6 (iv) Solve the (complex) linear matrix equation

$$(\mathbb{H}_{rs} - \mathcal{E} \mathbb{N}_{rs}) u_s^{J m_j} = S_r^{J m_j} \quad (18)$$

7 to obtain the LIT state

$$\psi_{J_i(\text{initial})/f(\text{inal}); J(i)n(\text{termediate})m_n}^{v(\text{ertex}), (\mu)L(\text{tipolarity})}(k, \sigma) = \psi_{J_0; J m_j}^{v, L}(k, \sigma) := \Psi_{\text{LIT}}^{J^\pi m_j} \left( \underbrace{|\mathbf{k}|, v, L}_{\text{vertex quantum numbers}} ; \underbrace{E_0, J_0}_{\text{initial/final-state quantum numbers}} ; \mathcal{R}[\sigma], \mathcal{I}[\sigma] \right) . \quad (19)$$

(v) The inner product

$$\mathcal{L}_{v' L', v L}^{J_f, J_i; J}(k', k, \sigma) = (-1)^{J - J_i + L - L' + v'} N_{J, \sigma} \sum_{m_j} \underbrace{\left\langle \psi_{J_f; J m_j}^{v', L'}(k', \sigma) \left| \psi_{J_i; J m_j}^{v, L}(k, \sigma) \right\rangle}_{= \sum_{r, s} (u_r^{J m_j})^* u_s^{J m_j} \mathbb{N}_{rs}} \quad (20)$$

with  $N_{J, \sigma}$  being the multiplicity of Lorentz states for given  $J$  and  $\sigma$ .

(21)

## I. FORMULAS AND CONSTANTS

(Wigner) 3- $j$  symbol: 
$$\begin{pmatrix} L & S & J \\ m_l & m_s & -m_j \end{pmatrix} = (-1)^{L-S+m_j} (2J+1)^{-\frac{1}{2}} \langle LS; m_l m_s | J m_j \rangle \quad (22)$$

Matrix for single-axis rotation: 
$$\begin{aligned} \mathcal{D}_{m',m}^{(j)}(0 \ \beta \ 0) &\equiv d_{m',m}^{(j)}(\beta) \\ &= \left[ \frac{(j+m')!(j-m')!}{(j+m)!(j-m)!} \right]^{\frac{1}{2}} \\ &\quad \cdot \sum_{\sigma} \begin{pmatrix} j+m \\ j-m'-\sigma \end{pmatrix} \begin{pmatrix} j-m \\ \sigma \end{pmatrix} (-1)^{j-m'-\sigma} \\ &\quad \cdot \left( \cos \frac{\beta}{2} \right)^{2\sigma+m+m'} \left( \sin \frac{\beta}{2} \right)^{2j-2\sigma-m-m'} \end{aligned} \quad (23)$$

(24)

### a. The Lorentz transformation of a Lorentzian

The strength functions which constitute the Compton amplitudes are themselves composed of scalar functions of an energy parameter. I assume that these functions can be expanded to any desired accuracy in a Lorentzian/-Cauchy basis as follows:

$$r(e) = \sum_n c_n \frac{\theta(e - e_{\text{th}})}{(a_n - e)^2 + b_n^2} := \sum_n c_n \theta(e - e_{\text{th}}) f_n. \quad (25)$$

As the process is non-trivial only if the photon's energy exceeds a minimum threshold energy  $e_{\text{th}}$ , it is in order to include a system-characteristic step function.

In practice, we obtain an integral transformation of this quantity with a Lorentzian kernel,

$$L(\sigma) := \int_{-\infty}^{\infty} \frac{r(e)}{(\sigma_r - e)^2 + \sigma_i^2} de = \sum_n c_n \int_{e_{\text{th}}}^{\infty} \frac{f_n(e)}{(\sigma_r - e)^2 + \sigma_i^2} de. \quad (26)$$

If basis basis functions<sup>①</sup> which we define through the integral

$$\begin{aligned} \int_{e_{\text{th}}}^{\infty} \frac{f_n(e)}{(\sigma_r - e)^2 + \sigma_i^2} de &= \int_{e_{\text{th}}}^{\infty} \frac{1}{(\sigma_r - e)^2 + \sigma_i^2} \cdot \frac{1}{(a_n - e)^2 + b_n^2} de \\ &\quad \vdots \\ &= [(\sigma_r - a_n)^2 + (\sigma_i - b_n)^2]^{-1} \cdot [(\sigma_r - a_n)^2 + (\sigma_i + b_n)^2]^{-1} \\ &\quad \cdot \left\{ \sigma_i^{-1} ((\sigma_r - a_n)^2 + b_n^2 - \sigma_i^2) \left( \frac{\pi}{2} + \tan^{-1} \left( \frac{\sigma'_r}{\sigma_i} \right) \right) \right. \\ &\quad \left. + b_n^{-1} ((\sigma_r - a_n)^2 - b_n^2 + \sigma_i^2) \left( \frac{\pi}{2} + \tan^{-1} \left( \frac{a'_n}{b_n} \right) \right) \right. \\ &\quad \left. + (\sigma_r - a_n) \ln \left( \frac{\sigma_r'^2 + \sigma_i^2}{a_n'^2 + b_n^2} \right) \right\} \\ &:= L_n(\sigma, e_{\text{th}}) \end{aligned}$$

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<sup>①</sup>  $x' := x - e_{\text{th}}$

$\alpha = \frac{e^2}{4\pi}$	dimensionless	$\frac{1}{137.03604}$
$\hbar c$		197.32858 MeV · fm <sup>2</sup>

(28)

TABLE I. Implemented numerical values.

allow for an accurate expansion of a given, physical function  $L(\sigma)$ , namely for a set  $\mathfrak{B} = \{(a_n, b_n) \in \mathbb{R}_{0+}\}_{n=1, \dots, d}$ , we seek

$$\min_{\mathbf{c}} \left| L(\sigma) - \sum_n c_n L_n(\sigma, e_{\text{th}}) \right| := \mathbf{c}^*. \quad (27)$$

The set of optimal parameters  $\mathbf{c}^*$  represents via Eq.(25) an expansion of the response function in Lorentzians. If  $\mathfrak{B}$  is “numerically” complete, the dependence of  $r(e)$  on changes in this basis should be negligible, *i.e.*, the problem is **not** ill-posed. This shall now be demonstrated.

## II. “HOW TO DO AN INTEGRAL”

### a. The multipole operators

- convection current<sup>①</sup>

$$\mathbf{j}_o(\mathbf{x}) = \frac{e}{2m} \sum_i \frac{1}{2} (1 + \tau_z(i)) \left\{ \mathbf{p}(i), \delta^{(3)}(\mathbf{x} - \mathbf{r}(i)) \right\} \quad (29a)$$

$$\text{(electric)} \quad \mathcal{O}_{Lm_L}^{\text{el}} = \frac{e\hbar}{mc} \hat{L}^{-1} \sum_i g_l(i) \left[ \sqrt{L} \Delta_{LM}^{L+1}(\mathbf{r}(i)) - \sqrt{L+1} \Delta_{LM}^{L-1}(\mathbf{r}(i)) \right] \quad (29b)$$

$$\text{(magnetic)} \quad \mathcal{O}_{Lm_L}^{\text{mag}} = i \frac{e\hbar}{mc} \sum_i g_l(i) \Delta_{LM}^L(\mathbf{r}(i)) \quad (29c)$$

- spin current

$$\mathbf{j}_s(\mathbf{x}) = \frac{e\hbar}{2m} \sum_i \frac{1}{2} (g_{s_p} (1 + \tau_z(i)) + g_{s_n} (1 - \tau_z(i))) \boldsymbol{\sigma}(i) \times \boldsymbol{\nabla}(i) \delta^{(3)}(\mathbf{x} - \mathbf{r}(i)) \quad (30a)$$

$$\text{(electric)} \quad \mathcal{O}_{Lm_L}^{s\text{el}} = - \frac{e\hbar |\mathbf{k}|}{2mc} \sum_i g_s(i) \sum_{M,\nu} \langle L1; M\nu | Lm_L \rangle \cdot \boldsymbol{\sigma}_\nu(i) \Phi_{LM}(\mathbf{r}(i)) \quad (30b)$$

$$\text{(magnetic)} \quad \mathcal{O}_{Lm_L}^{s\text{mag}} = i \frac{e\hbar |\mathbf{k}|}{2mc} \hat{L}^{-1} \sum_i g_s(i) \sum_{M,\nu} \left[ \sqrt{L} \langle L+11; M\nu | Lm_L \rangle \boldsymbol{\sigma}_\nu(i) \Phi_{L+1M}(\mathbf{r}(i)) \right. \quad (30c)$$

$$\left. - \sqrt{L+1} \langle L-11; M\nu | Lm_L \rangle \boldsymbol{\sigma}_\nu(i) \Phi_{L-1M}(\mathbf{r}(i)) \right] \quad (30d)$$

<sup>①</sup>Indices referring to particles are put in brackets.

with

$$\Phi_{Lm_L}(\mathbf{r}) = j_L(kr) Y_{Lm_L}(\Omega_r) \quad \textcircled{1} \quad (31)$$

$$\Delta_{Lm_L}^J(\mathbf{r}) = \sum_{M,\nu} \langle L1; M\nu | Jm_L \rangle \Phi_{Lm_L}(\mathbf{r}) \mathbf{p}_\nu \quad \textcircled{2} \quad (32)$$

24

b. *Siegert form*

$$\left\langle f \left| \left( \frac{1}{ck} \int d\mathbf{x} \mathbf{j}(\mathbf{x}) \cdot \nabla_x \times \mathbf{L} [j_L(kx) Y_{LM}(\Omega_x)] \right) \right| i \right\rangle \quad (33)$$

$$\stackrel{k \rightarrow 0}{=} \frac{i}{k} \frac{L+1}{L} \left\langle f \left| \left( \int d\mathbf{x} \mathbf{j}(\mathbf{x}) \cdot \nabla_x [j_L(kx) Y_{LM}(\Omega_x)] \right) \right| i \right\rangle \quad (34)$$

$$= \frac{1}{\hbar k} \frac{L+1}{L} \left\langle f \left| \int d\mathbf{x} [\rho(\mathbf{x}), \hat{H}_{\text{nuclear}}] j_L(kx) Y_{LM}(\Omega_x) \right| i \right\rangle \quad (35)$$

$$\stackrel{L=1}{=} \stackrel{\& \rho = \rho^{(1)}}{=} \frac{2}{\hbar k} \left\langle f \left| \sum_i^A q(i) [j_1(kr(i)) Y_{1M}(\Omega_{\mathbf{r}(i)}), \hat{H}_{\text{nuclear}}] \right| i \right\rangle \quad (36)$$

with

$$\mathbf{L} = -i\hbar (\mathbf{x} \times \nabla_x) \quad (37)$$

$$\rho^{(1)}(\mathbf{x}) = \sum_i^A \underbrace{\frac{e}{2} (1 + \tau_z(i))}_{:=q(i)} \delta^{(3)}(\mathbf{x} - \mathbf{r}_i) \quad (38)$$

$$\lim_{x \rightarrow 0} j_l(x) = \frac{x^l}{(2l+1)!!} \quad (39)$$

c. *The non-trivial matrix element (which serves **2-body** currents, too)*

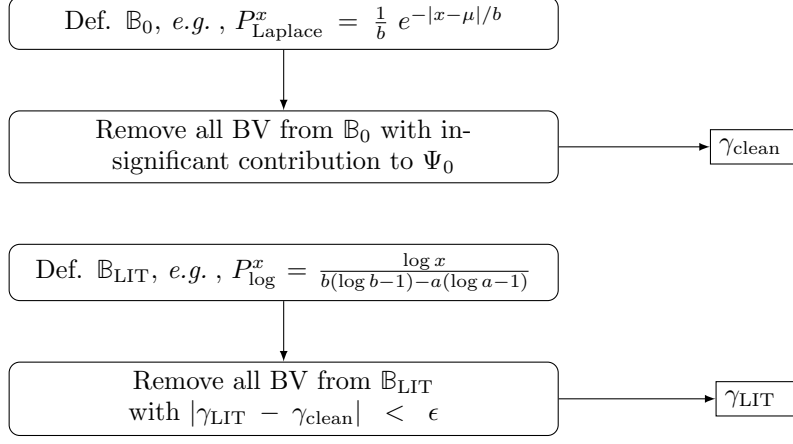
$$\left\langle m; l_l m_{l_l} \left| \Phi_{Lm_L}(\boldsymbol{\rho}_\nu) e^{-\beta \mathbf{r}_{ij}} \prod_N^{N_{\text{op}}} \mathcal{Y}_{L_N M_N}(\mathbf{r}_{ij}) \right| l_r m_{l_r}; n \right\rangle \quad (40)$$

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<sup>②</sup>rank-L spherical  $\mathbf{L}^2$  tensor

<sup>④</sup>linear combination of spherical  $\mathbf{L}^2$  rank-L tensors, hence, itself a rank-L spherical  $\mathbf{L}^2$  tensor and **not** rank-J spherical  $\mathbf{L}^2$ .

### 25 III. NOTES ON THE RRGIM IMPLEMENTATION



27 **luise.f** → **qual.f**

28  $P_{dc}$  and width-independent quantities of  $\Gamma_{l_1 m_1, \dots, l_z m_z}$  with  $z = n_{cl} - 1 + n_{cr} - 1 + n_{ww}$ .

29 **obem.f** → **qual.f**

30 **qual.f** → **enemb.f**

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31 WRITE(NBAND1) NZF,MUL,(LREG(K),K=1,NZOPER),I,(NZRHO(K),K=1,NZF)
32
33 WRITE(NBAND1) N3,MMASSE(1,N1,K),MMASSE(2,N1,K),MLAD(1,N1,K),
34 1 MLAD(2,N1,K),MSS(1,N1,K),MSS(2,N1,K),MS(N1,K),
35 2 (LZWERT(L,N2,K),L=1,5),(RPAR(L,K),L=1,N3),KP(MC1,N4,K)
36
37 WRITE(NBAND1) NTE,NC,ND,ITV2
38
39 WRITE(NBAND1) ((IND(MM,NN),NN=1,JRHO),MM=1,IRHO)
40
41 WRITE(NBAND1) NUML,NUMR,IK1H,JK1H,LL1,
42 * ((F(K,L),(J-1,DM(K,L,J),J=1,LL1),L=1,JK1),
43 * K=1,IK1)
  
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