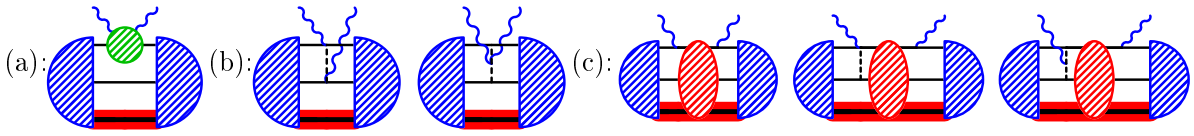


Figure 2: Representatives of the 3 contributing classes in $X\ {}^A Z \to Y\ {}^A Z$, illustrated by Compton scattering off nuclei (crossed graphs not shown). (a): onebody; (b): twobody; (c): rescattering; thick (red/black) line: spectator nucleons; see **PO20**.

### 1.2.4 Methodology/Motivation: Few-Nucleon Systems with External Probes

Dynamical Response Functions (DRFs) are time-tested parametrisations [**?** ] of the collective response of a nucleus ${}^A Z$ to an external probe which may change between initial and final state while the nucleus is unchanged, $X\ {}^A Z \to Y\ {}^A Z$. Assuming that the probes $X$ and $Y$ interact with the nucleus perturbatively via $H_{\rm int}^{X,Y}$, the system propagates coherently in the intermediate state; in symbolic notation suppressing quantum numbers (*cf.* fig. 2 (c)):

$$P_{YX}^{\rm out,in}(\omega) = \sum_\chi \langle {}^A Z_{\rm out}|\ H_{\rm int}^Y\ \underbrace{|\chi\rangle\ \langle\chi| \frac{1}{H - E(\omega,\vec{q})} |\psi\rangle\ \langle\psi|}_{\text{kernel}}\ H_{\rm int}^Y\ |A Z_{\rm in}\rangle + (X \leftrightarrow Y) \qquad (1.5)$$

The intermediate-state energy $E(\omega, \vec{q})$ is a function of the energy $\omega$ of the initial probe $X$ and the momentum transfer $\vec{q}$; $H$ is the Hamiltonian of the ${}^A Z$ system, and $|\chi\rangle, |\psi\rangle$ are its eigenstates for energy $E$. DRFs depend also on the quantum numbers of both nucleus and probes $X, Y$. In Compton scattering, these are the dynamical polarisabilities of the nucleus in external electromagnetic fields of definite multipolarity, with $\vec{q}$ found from the energy and scattering angle of the photons in the centre-of-mass system [**?** ], but the concept applies of course more generally. DRFs are quite economical for (semi-)inclusive processes like rescattering but not for one- and two-nucleon contributions. They therefore complement our transition-density approach which is uneconomical for rescattering. Their combination describes highly efficiently any perturbative probe of nuclei over a wide energy range.

We developed [10] a novel approach inspired by the Lorentz Integral Transformation (LIT) method [204–207]. While **PO3** below discusses the publication, the following outlines the idea. The eigenstates $|\chi\rangle, |\psi\rangle$ to $H$ are usually not square-integrable, but we only need their overlap with the nuclear states $|{}^A Z\rangle$ and these are. ~~We therefore construct~~ a Gaußian (and hence square-integrable) basis for the kernel via the Stochastic Variational Method (SVM) [69, 70]. The number of basis states and their widths are optimised by a genetic algorithm [**?** ] with penalty/fitness functions as priors which emphasise certain aspects of the intermediate basis, like the expectation that clusters should be important. Some of these are "learned" by the algorithm from analysing the bases of previous choices. For example, in a response of $^3$He, it is reasonable to assume that a continuum deuteron-proton intermediate state should be prominent, and some bases will mimic that better than others. One must of course avoid bias in the basis choice ("incestuous basis generation").

We see as an important development that we use not one optimised intermediate-state basis, but many. Often, one tries to increase numerical accuracy by enlarging the basis space, at exponentially greater computational complexity. We have seen that an alternative way provides very good results at smaller cost. We keep the number of basis states relatively small but test many bases and interpret the result statistically, assuming that the "most likely" response function is the one which is most consistent with the set of responses from all bases that were tested. There is no guarantee that this is true, but we see again that it compares

very well with both analytical models and other methods while costing less computer power and converging much more quickly. This approach would clearly fail for a Rayleigh-Ritz-like variational principle in which the "true" answer is an absolute lower (or upper) bound to the stochastic set, so that all different bases would likely scatter around a suboptimal answer off the correct one. Fortunately, this is not such a case. While the previous application proposed to directly employ the LIT, we grew sceptical of numerical convergence already for the deuteron but definitely for the 3-nucleon system. Our new way is numerically more stable and needs less creativity since the LIT inversion is avoided altogether.

In the end, this method has, like so many inverse problems, no well-defined solution. Therefore, we perform careful checks of convergence, independence from the priors/basis choices, and comparison to both quasi-analytic models as well as other methods and data as available. For Compton scattering, for example, restoration of the Thomson limit and photodissociation provide stringent tests; see **??**. In addition, we compute the real and imaginary parts of a DRF independently and check one from the other by the optical theorem.