Compton scattering off the deuteron at low and intermediate energies

M. Weyrauch*
National Institute for Nuclear Physics and High Energy Physics, Section K (NIKHEF-K), NL-1009 DB Amsterdam, The Netherlands
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Compton scattering off the deuteron is studied for photon energies up to about 100 MeV. This energy limit reflects the fact that only intermediate nucleon-nucleon (*NN*) states are considered. The *NN* propagator is constructed in a separable potential model, the parameters of which are fitted to describe the experimental *NN* scattering phase shifts. The problem of gauge invariance of the Compton amplitude is analyzed and the role of nonlocal currents is discussed. The low-energy theorem is satisfied. Our approach enables a direct calculation of both the real and imaginary part of the Compton scattering amplitude. It turns out that the strongest multipoles are dominated by the Born terms. Numerical results are compared to a previous dispersion theoretical calculation, and we find a strong disagreement between both calculations. We are led to conclude that certain assumptions made in the dispersion theoretical calculation are not justified.

I. INTRODUCTION

Investigations of the two-nucleon system play an important role in nuclear physics. Besides providing the simplest system to study the fundamental nucleon-nucleon (*NN*) interaction, the two-nucleon system is well suited for investigations of non-nucleonic degrees of freedom in nuclei. One reason for this is that within the different models of nuclear dynamics either exact calculations can be performed, or the necessary approximations are well under control.

In particular, studies of the *NN* system with electromagnetic probes helped to achieve a good understanding of nonnucleonic dynamics: A clear signature of meson exchange currents has first been found in radiative *n-p* capture.[1] More detailed information on meson and Δ dynamics could later on be obtained by investigating photodisintegration and electrodisintegration of the deuteron.[2] More recently some discussion started about possible signatures of quantum chromodynamics in electron deuteron elastic scattering.[3]

All the above-mentioned studies of the two-nucleon system with electromagnetic probes concern single-photon processes. Not much work up to now has been spent on deuteron Compton scattering and other two-photon reactions with the *NN* system. This partly can be explained by considerable difficulties which up to now have hampered two-photon experiments. But if these difficulties can be overcome, two-photon reactions and, in particular, Compton scattering may prove to be very valuable tools. A reason for this is that with Compton scattering one probes both the imaginary and real parts of the amplitude, while photoabsorption and electron scattering (to first order) only involve the imaginary part. To say it in more technical terms: With Compton scattering the full propagator is investigated.

In this respect Compton scattering is closely related to elastic pion scattering and pion photoproduction, i.e., all these processes in principle probe the very same nuclear dynamics. But in comparison to pionic reactions Compton scattering has the advantage, that the electromagnetic vertex of the photon is well understood. Furthermore, since only real photons are involved, no problems arise with initial and/or final state interactions. Therefore, Compton scattering is a uniquely "clean" probe of the complete dynamics involved in the two-nucleon system. Eventually it would be desirable to study all three processes $^2\mathrm{H}(\pi,\pi)^2\mathrm{H}$, $^2\mathrm{H}(\gamma,\pi)^2\mathrm{H}$, and $^2\mathrm{H}(\gamma,\gamma)^2\mathrm{H}$ simultaneously, as has been recently done[4] for the elementary processes $N(\pi,\pi)N$, $N(\gamma,\pi)N$, and $N(\gamma,\gamma)N$.

It is the goal of this paper to provide for the first time an explicit calculation of the deuteron Compton amplitude below about 100-MeV photon energy and to compare the results with a previous dispersion theoretical analysis. The dynamics involved in the present calculation is graphically represented in Fig. 1. The Born terms of the direct and crossed processes are shown in Figs. 1(a) and 1(b), and the rescattering processes are depicted in Figs. 1(c) and 1(d). Of course, the crossed diagrams contribute to the real part of the amplitude only, and the imaginary part of the amplitude is related to the photoabsorption cross section via the optical theorem. The diagram shown in Fig. 1(e) represents the contribution of the two-photon amplitude, which is purely real. Consideration of this term is important in order to have the Compton amplitude gauge invariant and consequently the low-energy theorem satisfied.

Particular emphasis will be put on studying the real part of the deuteron Compton amplitude, since the imaginary part is already well understood from previous calculations of deuteron photodisintegration. The real part is an interesting and important quantity since it is directly related to the off-shell structure of the *NN* system. Furthermore, dynamical effects, which contribute to the imaginary part at higher energies only, show up in the real part already at low and intermediate energies (e.g., π production, $\bar{N}$ production), since here the particles have to be produced only virtually. We have to include these effects, e.g., in terms of the two-photon amplitude mentioned above. A direct calculation of the real part of the

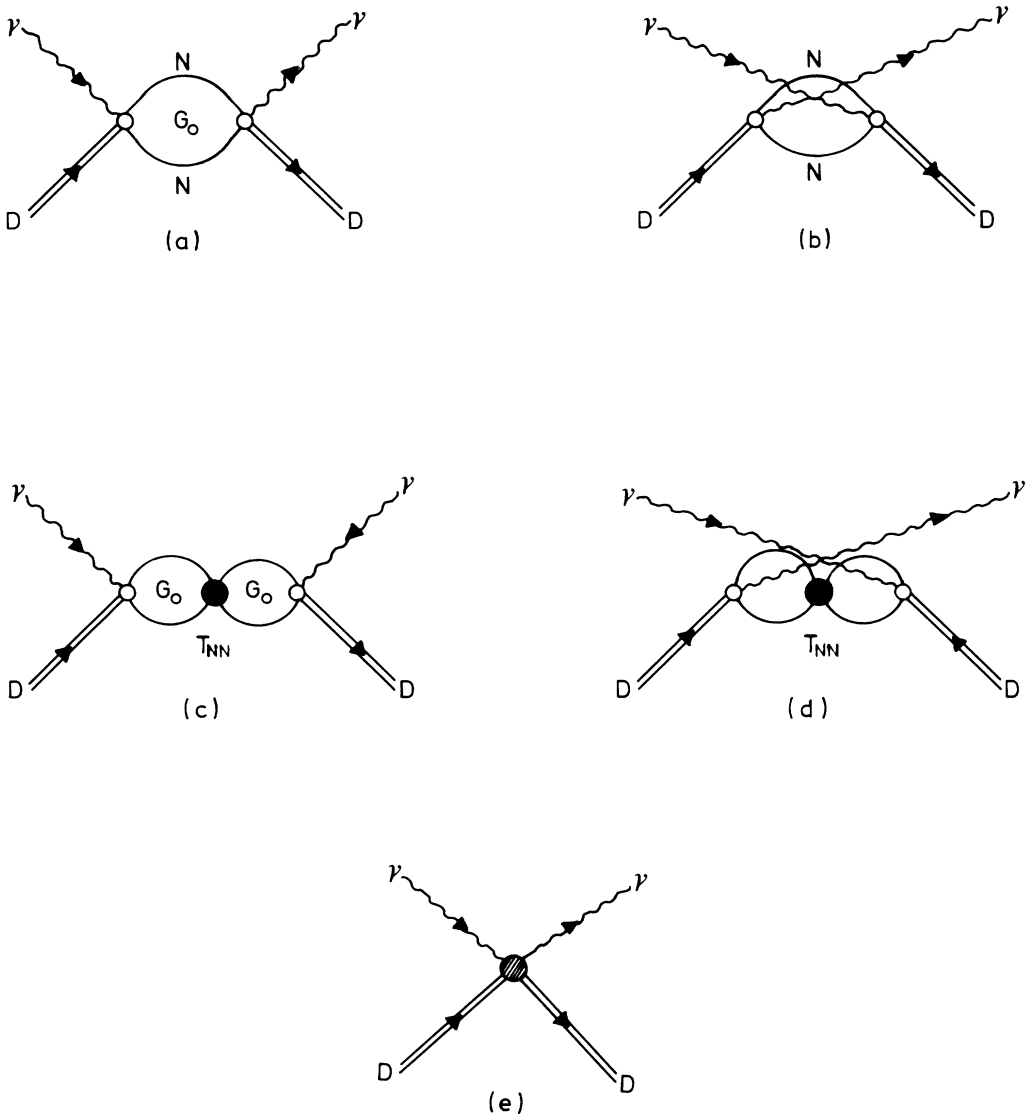

FIG. 1. The deuteron Compton scattering amplitude: Born terms [direct (a) and crossed (b)], rescattering terms [direct (c) and crossed (d)], two-photon amplitude (e), D and γ stand for deuteron and photon, respectively, N for nucleon, G_0 denotes the free NN propagator and T_{NN} the full off-shell NN scattering amplitude.

deuteron Compton amplitude is not a simple task, since we have to sum over a continuum of intermediate excitations of the deuteron (as, e.g., NN, $NN\pi$, $N\Delta$, $\Delta\Delta$, and $N\Delta\pi$ states). In order to simplify our task, we have confined ourselves here to intermediate NN excitations only. This limits the applicability of our results to energies well below the pion production threshold. Furthermore, to make the calculation more easily tractable, the NN propagator is obtained in a separable potential model, the parameters of which have been fitted to reproduce the experimental NN phase shifts. Such a model provides a convenient off-shell extension of the NN scattering amplitude. Analogous methods have been used previously in calculations of NN bremsstrahlung,[5] pion deuteron scattering,[6] and nucleon deuteron scattering.[7]

An issue we are particularly concerned with here is the problem of gauge invariance of the Compton amplitude. We already mentioned, that, e.g., the low-energy theorem for the Compton amplitude is not satisfied, if the gauge symmetry is broken.[8,9] It is also known from previous studies of photo absorption on the deuteron,[10] that by using Siegert's theorem in calculating the electric multipoles of the current we implicitly take into account the dominant part of nonnucleonic currents. We discuss this issue with respect to Compton scattering and analyze in detail, how nonnucleonic contributions eventually modify our results.

In the next section we first present the general structure of the deuteron Compton amplitude. In connection with this we briefly review "off-shell" NN scattering. A partial wave decomposition of the Compton amplitude is derived in Sec. III. Equations (27), (33), and (34) of this section provide the basis of our numerical analysis. Gauge invariance and a rearrangement of the Compton

amplitude using Siegert's theorem will be discussed in Sec. IV. We present our numerical results in Sec. V and compare them with a previous dispersion theoretical calculation.[11] The validity of this dispersion theoretical method will be briefly investigated. As a test, and to give an idea of the accuracy of our technique, we also calculate the deuteron photoabsorption cross section. Finally, in Sec. VI, we put the present work into perspective and discuss further developments.

II. DEUTERON COMPTON SCATTERING

In this section we develop the necessary formalism in a form particularly useful for deuteron Compton scattering. We confine ourselves to intermediate nucleon-nucleon excitations, which limits our results to energies well below the pion production threshold.

A. The Compton scattering amplitude

Our first goal is to write down the Compton scattering amplitude in terms of the off-shell NN scattering T matrix by considering the graphs of Fig. 1. To this end we first define the (nonrelativistic) free NN propagator

$$G_0(E,q)=(E^+-E_q)^{-1}\ , \tag{1}$$

with E the total c.m. energy of the NN system ($E^+=E+i\epsilon$), $E_q=q^2/2\mu$, $\mathbf{q}$ the relative momentum, and μ the reduced mass of the two nucleons. The NN interaction is described by a potential V, so that the off-shell NN scattering T matrix can be obtained by solving a Lippmann-Schwinger equation,

$$T_{NN}=V+VG_0T_{NN}\ . \tag{2}$$

Since we will later on assume V to be a separable potential, the solution of (2) can be easily obtained analytically. Moreover, this procedure will provide us with a separable structure for T_{NN} itself, and in turn produces practical formulas for the Compton amplitude. For the more general discussion here the assumption of separability of the potential is not necessary.

The electromagnetic interaction of the real photons with the deuteron is described by the conserved current $\mathbf{j}(\mathbf{k})$ and the two-photon operator $B_{l'l}(\mathbf{k}',\mathbf{k})$. Consideration of the latter is required in order to have the amplitude for the two-photon process gauge invariant. (For example, for a nonrelativistic NN system without exchange of charged mesons the only two-photon contribution would be the well-known Thomson or "seagull" term.) At this point, however, we do not want to make any specific assumptions about the current and two-photon operators. We will discuss the structure of these operators and, in particular, the question of gauge invariance later in Sec. IV. Furthermore, we note that throughout this paper we use the transverse gauge of the electromagnetic field.

Here we only want to subdivide the two-photon amplitude further into a part related to the intrinsic motion and a part related to the center-of-mass (c.m.) motion of the system. The Compton amplitude therefore has three terms, the resonance amplitude R, which is related to intermediate excitations of the system and involves the current operator, the intrinsic two-photon amplitude $B^{\rm in}$, and the c.m. two-photon amplitude $B^{\rm c.m.}$,

$$T_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=R_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k}) +B^{\rm in}_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})+B^{\rm c.m.}_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})\ . \tag{3}$$

We have denoted the momenta and polarizations of the incoming and scattered photons by $\mathbf{k},\lambda$ and $\mathbf{k}',\lambda'$, respectively. We would like to emphasize, that splitting the intrinsic amplitude into a resonance amplitude and a two-photon amplitude is purely formal, since both terms separately are not gauge invariant. Moreover, the separation into an intrinsic amplitude (i.e., $R+B^{\rm in}$) and a "center-of-mass" amplitude ($B^{\rm c.m.}$) is directly related to the separation of the (strong interaction) Hamiltonian of the NN system into an intrinsic and a c.m. part,

$$H=H^{\rm in}+H^{\rm c.m.},\quad H^{\rm in}=T+V\ , \tag{4}$$

with

$$H^{\rm c.m.}=\frac{\mathbf{P}^2}{2M} \tag{5}$$

and $\mathbf{P}$ the total c.m. momentum operator of the NN system and M its mass. Of course, the c.m. amplitude is of no particular dynamical interest in our context, since it describes pure c.m. excitations. But it contributes an unavoidable background, which is large at the energies considered here. The c.m. two-photon amplitude is simply the Thomson amplitude off the deuteron as a whole and is explicitly given by

$$B^{\rm c.m.}_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=\frac{\boldsymbol{\epsilon}'^{*}_{\lambda'}\cdot\boldsymbol{\epsilon}_{\lambda}}{M}\langle 1m_d'\mid\int d^3x'\int d^3x e^{-i\mathbf{k}'\cdot\mathbf{x}'}e^{i\mathbf{k}\cdot\mathbf{x}}\rho(\mathbf{x}')\rho(\mathbf{x})\mid 1m_d\rangle\ , \tag{6}$$

with $\rho(\mathbf{x})$ the charge density operator. Equation (6) can easily be obtained by minimal substitution from the c.m. Hamiltonian (5). The initial and final deuteron states have been characterized by the spin of the deuteron and its projection m_d.

The resonance amplitude R describes intermediate excitations of the deuteron and is easily written down by considering Figs. 1(a)–1(d),

$$R_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=A_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k},E)+A_{\lambda'\lambda}(\mathbf{k},-\mathbf{k}',\bar{E})+C_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k},E)+C_{\lambda'\lambda}(\mathbf{k},-\mathbf{k}',\bar{E})\ . \tag{7}$$

While the Born terms A involve free intermediate NN propagation only,

$$A_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k},E)=\sum_{s\nu}\int\frac{d^3q}{(2\pi)^3}\langle 1m_d'\mid\epsilon_{\lambda'}'^{*}\cdot\mathbf{j}(-\mathbf{k}')\mid\mathbf{q},s\nu\rangle G_0(E,q)\langle\mathbf{q},s\nu\mid\epsilon_\lambda\cdot\mathbf{j}(\mathbf{k})\mid 1m_d\rangle\ , \tag{8}$$

the rescattering terms C are determined by the full off-shell NN scattering T matrix:

$$C_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k},E)=\sum_{s'\nu's\nu}\int\frac{d^3q'}{(2\pi)^{3/2}}\int\frac{d^3q}{(2\pi)^{3/2}}\langle 1m_d'\mid\epsilon_{\lambda'}'^{*}\cdot\mathbf{j}(-\mathbf{k}')\mid q',s'\nu'\rangle$$

$$\times G_0(E,q')\langle\mathbf{q}',s'\nu'\mid T_{NN}\mid\mathbf{q},s\nu\rangle G_0(E,q)\langle\mathbf{q},s\nu\mid\epsilon_\lambda\cdot\mathbf{j}(\mathbf{k})\mid 1m_d\rangle\ , \tag{9}$$

and $\mid\mathbf{q},s\nu\rangle$ are intermediate plane wave states with momentum $\mathbf{q}$, spin s and spinprojection ν.

To specify the energies E and $\bar{E}$ entering the above formulas, we need to discuss some kinematics. To be specific, E and $\bar{E}$ are the on-shell energies available for the intermediate NN propagation for the direct and exchange diagrams, respectively. Of course, these parameters are in the γ-deuteron c.m. frame completely determined by the momenta of the incoming and outgoing photons. For the direct process we find

$$E=\frac{k^2}{2M}+k-\epsilon_d \tag{10}$$

and for the crossed diagrams

$$\bar{E}=\frac{k^2}{2M}-k-\epsilon_d-\frac{(\mathbf{k}'-\mathbf{k})^2}{2M}$$

$$=-\frac{k^2}{2M}-k-\epsilon_d+\frac{\mathbf{k}'\cdot\mathbf{k}}{2M}\ . \tag{11}$$

Here we have used $k=\mid\mathbf{k}\mid$, $k=k'$ for elastic scattering, and ϵ_d is the deuteron binding energy. Unfortunately $\bar{E}$ for the crossed diagram depends on the scattering angle θ explicitly. This term is small for energies $k\ll 2M=4m$ (m is the nucleon mass). Therefore, to simplify our calculations we will drop the term $\mathbf{k}\cdot\mathbf{k}'/2M$ in the above formula for $\bar{E}$.

We want to mention here that some other small terms have implicitly been neglected in writing down Eqs. (8) and (9). First we do not consider the deuteron state itself as a possible intermediate state. Such a contribution is only important at very low photon energies. Secondly we have neglected the recoil current of the whole NN system (cf., e.g., Ref. 9). This current is proportional to k/M and can safely be neglected at the energies considered here.

We finally want to show, that the above formalism is equivalent to more conventional formulations of Compton scattering on nuclei as, e.g., given in Ref. 9. To this end we use the well-known relation between the full Green's function $G(E)$, the free Green's function $G_0(E)$, and the off-shell NN scattering T matrix

$$G(E)=G_0(E)+G_0(E)T_{NN}(E)G_0(E)\ . \tag{12}$$

We can take advantage of this relation after taking out the complete set of intermediate plane wave states in Eqs. (8) and (9). By inserting then a complete set of eigenfunctions (eigendistributions) of the full Hamiltonian H^{in}, we arrive at the more conventional formula

$$R_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=\sum_n\frac{\langle 1m_d'\mid\epsilon_{\lambda'}'^{*}\cdot\mathbf{j}(-\mathbf{k}')\mid n\rangle\langle n\mid\epsilon_\lambda\cdot\mathbf{j}(\mathbf{k})\mid 1m_d\rangle}{k^2/2M+k-\epsilon_d-\epsilon_n+i\epsilon}+\begin{bmatrix}\mathbf{k}\leftrightarrow-\mathbf{k}'\\ \epsilon_{\lambda'}'^{*}\leftrightarrow\epsilon_\lambda\end{bmatrix}\ . \tag{13}$$

B. *NN* scattering *T* matrix

We will now construct the off-shell NN scattering T matrix in a separable potential model. This in turn makes an explicit calculation of the complete Compton scattering amplitude feasable. We here use the formalism and notation of Goldberger and Watson,[12] where a more detailed exposition of the pertinent theory can be found.

One starts out by expanding the Lippmann-Schwinger Eq. (2) into partial waves using[12]

$$\langle\lambda',s'\nu'\mid T_{NN}\mid\lambda,s\nu\rangle=\sum_{l'lj}\langle\hat{\lambda}',l's'\nu'\mid\mathcal{Y}^j\mid\hat{\lambda},ls\nu\rangle T^{js}_{l'l}(\lambda',\lambda)\delta_{s's} \tag{14}$$

and an analogous decomposition for V. The NN scattering T matrix is diagonal in the spin quantum number s ($s=0$ for the singlet terms and $s=1$ for the triplet terms). The angular dependence is contained in the expression

$$\langle\hat{\lambda}',l's'\nu'\mid\mathcal{Y}^j\mid\hat{\lambda},ls\nu\rangle=\sum_{m'mM}Y_{l'm'}(\hat{\lambda}')Y^*_{lm}(\hat{\lambda})(l'm's'\nu'\mid jM)(lms\nu\mid jM)\ , \tag{15}$$

with $(lms\nu\mid jM)$ a Clebsch-Gordan coefficient. From this we get the partial wave Lippmann-Schwinger equations for NN scattering

$$T^{js}_{l'l}(\lambda',\lambda)=V^{js}_{l'l}(\lambda',\lambda)+\sum_{\bar{l}}\int_0^\alpha\frac{\bar{\lambda}^2d\bar{\lambda}}{(2\pi)^3}V^{js}_{l'\bar{l}}(\lambda',\bar{\lambda})G_0(\bar{\lambda},E)T^{js}_{\bar{l}l}(\bar{\lambda},\lambda)\ , \tag{16}$$

which can be solved analytically, if the $V_{l'l}^{js}$ have a separable structure. In particular we will take a rank two separable potential given by[13]

$$V_{l'l}^{js}(\lambda',\lambda)=i^{l'-l}(g_{l's}(\lambda')g_{ls}(\lambda)-h_{l's}(\lambda')h_{ls}(\lambda)) \ . \tag{17}$$

For $s=1$ the potential is diagonal if $j=l$ (uncoupled triplet), i.e., the potential contains only off-diagonal terms for $s=1$ and $l\neq j$. Under these conditions the solution of (16) is easily obtained analytically:[13]

$$T_{l'l}^{js}(\lambda',\lambda)=\frac{i^{l'-l}}{D^{js}(E)}\{g_{l's}(\lambda')g_{ls}(\lambda)D_1^{js}(E)-h_{l's}(\lambda')h_{ls}(\lambda)D_2^{js}(E)-[g_{l's}(\lambda')h_{ls}(\lambda)+h_{l's}(\lambda')g_{ls}(\lambda)]D_3^{js}(E)\} \ , \tag{18}$$

with the propagators

$$\begin{aligned}
D_1^{js}(E)&=1+\sum_{i=|j-t|}^{j+t}\int_0^\infty d\lambda\,\lambda^2 h_{is}^2(\lambda)G_0(E,\lambda) \ , \\
D_2^{js}(E)&=1-\sum_{i=|j-t|}^{j+t}\int_0^\infty d\lambda\,\lambda^2 g_{is}^2(\lambda)G_0(E,\lambda) \ , \\
D_3^{js}(E)&=\sum_{i=|j-t|}^{j+t}\int_0^\infty d\lambda\,\lambda^2 g_{is}(\lambda)h_{is}(\lambda)G_0(E,\lambda) \ , \\
D^{js}(E)&=D_1^{js}(E)D_2^{js}(E)+[D_3^{js}(E)]^2 \ ,
\end{aligned} \tag{19}$$

with $t=0$ for the uncouplet triplet terms, and $t=s$ in all other cases.

The advantage of formulating NN scattering in a separable potential model is that from the separable form (18) we obtain a separable structure for the rescattering terms of the Compton amplitude. This we will study in detail in the next section. For the form factors g and h in Eq. (17) one chooses a particular analytic form and fits some parameters in such a way that the model reproduces the experimental NN scattering phase shifts. Details can be found in Ref. 13.

III. POLARIZABILITIES

It is now useful to expand the Compton amplitude (7) in terms of the polarizabilities P_J, which are implicitly defined by[14] ($\hat{J}=\sqrt{2J+1}$)

$$T_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=(-)^{1-m_d}\sum_{\substack{L'M'LMJ\\ \nu',\nu=0,1}}(-)^{L'+L}\hat{J}^2\begin{pmatrix}1 & J & 1\\ -m_d' & m_J & m_d\end{pmatrix}\begin{pmatrix}L & L' & J\\ M & M' & -m_J\end{pmatrix}$$

$$\times\lambda'^{\nu'}\lambda^{\nu}P_J(M'^{\nu'}L',M^{\nu}L,k)D_{M',-\lambda'}^{(L')}(R')D_{M,\lambda}^{(L)}(R) \ . \tag{20}$$

Another convenient partial wave decomposition of the Compton amplitude is in terms of partial wave helicity amplitudes. These can be easily obtained as linear superpositions of different polarizabilities as has been shown in Ref. 14. The polarizabilities are functions of the photonenergy k only, since we only consider elastic scattering here. They contain terms related to intrinsic excitation of the system, R_J (resonance polarizabilities), and terms B_J^{in} and $B_J^{\text{c.m.}}$ (two-photon polarizabilities) corresponding to the intrinsic two-photon amplitude and the c.m. two-photon amplitude, respectively [cf. Eq. (3)],

$$P_J=R_J+B_J^{\text{in}}+B_J^{\text{c.m.}} \ . \tag{21}$$

The polarizabilities are complex-valued functions, and it holds in particular that

$$\text{Im}(P_J)=\text{Im}(R_J) \ , \tag{22}$$

since B^{in} and $B^{\text{c.m.}}$ contribute to the real part only.

Explicit formulas for the polarizabilities are easily derived by employing the following partial wave expansion of the current matrix element

$$\langle 1m_d\,|\,\boldsymbol{\epsilon}_\lambda\cdot\mathbf{j}(\mathbf{k})\,|\,\mathbf{q},s\nu\rangle=\sum_{\substack{lm\\ jm\hat{j}}}\langle 1m_d\,|\,\boldsymbol{\epsilon}_\lambda\cdot\mathbf{j}(\mathbf{k})\,|\,R_l(q);(ls)jm\hat{j}\rangle(lms\nu\,|\,jm\hat{j})Y_{lm}^*(\hat{q}) \ . \tag{23}$$

The radial part of the wave function is in coordinate space given by

$$R_l(q,r)=i^l 4\pi j_l(qr) \ . \tag{24}$$

Furthermore, we use the well-known expansion of the current operator in terms of electric and magnetic multipole com-

ponents[15]

$$\epsilon_\lambda \cdot \mathbf{j}(\mathbf{k}) = -\sqrt{2\pi} \sum_{\substack{LM \\ L\geq 1 \\ \nu=0,1}} \hat{L}\lambda^\nu T^{[L]}_{\nu,M}(k) D^{(L)}_{M,\lambda}(R) \ , \tag{25}$$

with

$$\begin{aligned} T^{[L]}_0(k) &= E^{[L]}(k) = \int d^3x \ \mathbf{j}(\mathbf{x}) \cdot \mathbf{A}^{[L]}(E;\mathbf{x},k) \ , \\ T^{[L]}_1(k) &= M^{[L]}(k) = \int d^3x \ \mathbf{j}(\mathbf{x}) \cdot \mathbf{A}^{[L]}(M;\mathbf{x},k) \ . \end{aligned} \tag{26}$$

For the rotation matrices $D^{(L)}(R)$ we use the convention of Rose;[16] R denotes the rotation of $\mathbf{k}$ into the quantization axis. Explicit expressions for the electric and magnetic multipole components of the photon field [$\mathbf{A}^{[L]}(E)$ and $\mathbf{A}^{[L]}(M)$] are given in Refs. 15 and 17 and are for convenience reproduced in the Appendix.

A. The resonance polarizabilities R_J

With Eqs. (23)–(26) and the separable form (18) for the off-shell NN T matrix, it is not difficult to obtain explicit expressions for the resonance polarizabilities, viz.,

$$\begin{aligned} R_J(M^{\nu'}L',M^\nu L,k) = 2\pi \hat{L}'\hat{L} \sum_{l'ljs} (-)^{L'+L+j} \begin{Bmatrix} 1 & L' & j \\ L & 1 & J \end{Bmatrix} \frac{i^{l'-l}}{D^{js}(E)} \\ \times \{ F^{L'\nu'L\nu}_{lsj}(k,E) D^{js}(E)\delta_{l'l} + G^{L'\nu'}_{l'sj}(k,E) G^{L\nu}_{lsj}(k,E) D^{js}_1(k,E) - H^{L'\nu'}_{l'sj}(k,E) H^{L\nu}_{lsj}(k,E) D^{js}_2(E) \\ - [G^{L'\nu'}_{l'sj}(k,E) H^{L\nu}_{lsj}(k,E) + H^{L'\nu'}_{l'sj}(k,E) G^{L\nu}_{lsj}(k,E)] D^{js}_3(E) + (-)^{L'+L+J}(E \leftrightarrow \bar{E}) \} \ , \end{aligned} \tag{27}$$

where the last term with $(E \leftrightarrow \bar{E})$ takes care of the exchange diagrams shown in Figs. 1(b) and 1(d). The functions G and H are reduced matrix elements of the multipole operators (26)

$$\begin{aligned} G^{L\nu}_{ljs}(k,E) &= \langle (ls)j; \phi^{ls}_g(E) \| T^{[L]}_\nu(k) \| 1 \rangle \ , \\ H^{L\nu}_{ljs}(k,E) &= \langle (ls)j; \phi^{ls}_h(E) \| T^{[L]}_\nu(k) \| 1 \rangle \ . \end{aligned} \tag{28}$$

They are given in terms of the functions Φ, whose coordinate space representation reads

$$\phi^{ls}_g(E,r) = \frac{4\pi i^l}{(2\pi)^{3/2}} \int_0^\infty d\lambda \, \lambda^2 g_{ls}(\lambda) G_0(E,\lambda) j_l(\lambda r) \ . \tag{29}$$

For suitably defined form factors g_{ls} these functions can be calculated analytically. In other cases they have to be evaluated numerically, and it proved convenient (and very efficient) to first rotate the integration contour into the complex plane and then use a Gauss-Laguerre integration.

The functions $F^{L'\nu'L\nu}_{lsj}$ contain the contribution of the Born amplitude [Fig. 1(a)] and are unfortunately not separable as are all the other terms in Eq. (27). Explicitly they are given by

$$F^{L'\nu'L\nu}_{lsj}(k,E) = (2\pi)^{-3/2} \int_0^\infty d\lambda \, \lambda^2 \langle 1 \| T^{[L']}_{\nu'}(k) \| R_l(\lambda); (ls); \rangle G_0(E,\lambda) \langle (ls)j; R_l(\lambda) \| T^{[L]}_\nu(k) \| 1 \rangle \ . \tag{30}$$

It is possible to perform the λ integration analytically

$$\int_0^\infty d\lambda \, \lambda^2 j_l(\lambda r') j_l(\lambda r) G_0(E,\lambda) = \begin{cases} -\pi i q \mu j_l(qr_<) h^{(1)}_l(qr_>), & E>0 \ , \\ 2q\mu i_l(qr_<) k_l(qr_>), & E<0 \ , \end{cases} \tag{31}$$

with $q^2 = 2\mu E$, $r_< = \min(r',r)$, $r_> = \max(r',r)$, and $h^{(1)}_l(x) = j_l(x) + iy_l(x)$ the well-known Hankel functions of the first kind. Furthermore, i_l and k_l are modified spherical Bessel functions. The imaginary part of $F^{L'\nu'L\nu}_{lsj}$ is then given by

$$\mathrm{Im} F^{L'\nu'L\nu}_{lsj}(k,E) = 2(-)^{l+1} q\mu \langle 1 \| T^{[L']}_{\nu'}(k) \| j_l(qr'); (ls)j \rangle \langle j_l(qr); (ls)j \| T^{[L]}_\nu(k) \| 1 \rangle \ , \tag{32}$$

i.e., the imaginary part is separable. Explicit expressions for the real part are more involved and given in the Appendix.

B. The two-photon polarizabilities

Since for the c.m. two-photon amplitude we have the explicit result (6), the c.m. polarizabilities can be calculated easily. For the deuteron we find after some algebra

$$B_J^{\text{c.m.}}(M^{\nu'}L',M^{\nu}L,k)=\frac{e^2}{M}\frac{\sqrt{\pi}}{2}\frac{(\hat{L}'\hat{L})^2}{\hat{J}}(-)^{L'}\sum_{\substack{l'l\\ \lambda'\lambda}}\lambda'^{\nu'}\lambda^{\nu}\begin{pmatrix}1 & l' & L'\\ -\lambda' & 0 & \lambda'\end{pmatrix}\begin{Bmatrix}1 & l & L\\ \lambda & 0 & -\lambda\end{Bmatrix}\begin{Bmatrix}l' & l & J\\ 0 & 0 & 0\end{Bmatrix}\begin{Bmatrix}L' & L & J\\ l' & l & 1\end{Bmatrix}(-)^{(l+l')/2}$$

$$\times\langle 1\|j_{l'}(kr/2)j_l(kr/2)Y^{[J]}(\hat{r})\|1\rangle\ . \tag{33}$$

The polarizabilities of the intrinsic two-photon amplitude have to be calculated from the general expression[11,14]

$$B_J^{\text{in}}(M^{\nu'}L',M^{\nu}L,k)=2\pi(-)^{L+J}\frac{\hat{L}'\hat{L}}{\hat{J}}\left\langle 1\left|\left|\sum_{l'l}\int d^3x'\int d^3x[A_{l'}^{[L']}(M^{\nu'};\mathbf{x}',k)\times A_l^{[L]}(M^{\nu};\mathbf{x},k)]^{[J]}B_{l'l}^{\text{in}}(\mathbf{x}',\mathbf{x})\right|\right|1\right\rangle \tag{34}$$

in terms of the electric and magnetic multipole components of the photon field as defined in Ref. 14 and the Appendix. More specific formulas can only be derived after a model for B^{in} has been specified. We will do this in the next section after the question of gauge invariance has been addressed.

The results (27), (33), and (34) for the polarizabilities form the basic incredients of our numerical analysis. We only need to specify the explicit structure of the current and intrinsic two-photon operators. From the polarizabilities we then can calculate all observables of interest as is described in detail in Sec. V.

IV. GAUGE INVARIANCE AND LOW-ENERGY BEHAVIOR

Up to now we did not need to specify the explicit structure of the current and two-photon operators. But, of course, in order to explicitly calculate the scattering amplitude we need to have a model for these operators. In particular, they have to be constructed in such a way, that gauge invariance (and as a consequence the low energy theorem) is fulfilled.

It is immediately obvious, that the single-body current of two particles with charge e_i and magnetic moment μ_i and momentum π_i $(i=1,2)$

$$\mathbf{j}^{(1)}(\mathbf{x})=\frac{e}{2m}\sum_{i=1}^{2}\{e_i[\delta(\mathbf{x}-\mathbf{r}_i),\boldsymbol{\pi}_i]_+ +\mu_i\nabla_x\times\boldsymbol{\sigma}_i\delta(\mathbf{x}-\mathbf{r}_i)\} \tag{35}$$

is *not* gauge invariant within the framework considered here. The reason for this is that the *NN* potential we use is nonlocal, and as a consequence the current (35) is not conserved, i.e.,

$$\nabla\cdot\mathbf{j}^{(1)}(\mathbf{x})\neq -i[H^{\text{in}},\rho(\mathbf{x})]\ . \tag{36}$$

Here we have introduced the charge density operator

$$\rho(\mathbf{x})=e\sum_{i=1}^{2}e_i\delta(\mathbf{x}-\mathbf{r}_i)\ . \tag{37}$$

To remedy this nonconservation, we, in principle, would have to construct a nonlocal current $\mathbf{j}^{(2)}(\mathbf{x},\mathbf{r}_1,\mathbf{r}_2)$ in order to fulfill the continuity equation with the given potential $V(\mathbf{r}_1,\mathbf{r}_2)$,

$$\nabla\cdot\mathbf{j}^{(2)}(\mathbf{x},\mathbf{r}_1,\mathbf{r}_2)=-i[V(\mathbf{r}_1,\mathbf{r}_2),\rho(\mathbf{x})]\ . \tag{38}$$

It is easily possible to construct such a nonlocal current, and the method is outlined by Osborne and Foldy.[18] Similarly a nonlocal contribution to the two-photon operator has to be constructed, in order to have the Compton amplitude gauge invariant. The nonlocal two-photon operator has to satisfy the following gauge condition:

$$\sum_{l'}\frac{\partial}{\partial x_{l'}}B_{l'l}^{\text{in},(2)}(\mathbf{x}',\mathbf{x})=i[\rho(\mathbf{x}'),\mathbf{j}^{(2)}(\mathbf{x})]\ . \tag{39}$$

Explicit construction of this operator is not difficult along the lines outlined by Osborne and Foldy. Unfortunately this method does not allow a *unique* construction of the nonlocal current and two-photon operators, because the gauge conditions (38) and (39) only determine the divergence of these operators. Therefore, in principle, we obtain an infinite set of nonlocal operators, which all satisfy the relevant gauge conditions.

For this reason we want to follow here a different procedure, namely to use the gauge conditions (38) and (39), in order to reformulate the formulas for the polarizabilities by using a generalized "Siegert's theorem" for Compton scattering, a derivation of which has been presented already previously.[11,14] This procedure (to be briefly outlined in the Appendix) leads to significant cancellations between resonance and intrinsic two-photon polarizabilities, as a consequence of which the low-energy theorem is fulfilled. As is shown in the Appendix, the main structure of the formulas (27) and (34) for the polarizabilities is retained, but some replacements have to be made for the various *electric* multipole operators. Explicitly we find, that the electric multipoles of the current operator have to be replaced by

$$E^{[L]}(k)=\frac{1}{k}[E-H^{\text{in}}]C^{[L]}(k)+\mathcal{T}^{[L]}(k)\ , \tag{40}$$

with

$$C^{[L]}(k)=\frac{i^L}{\sqrt{L(L+1)}}\int d^3x\,\rho(\mathbf{x})\left[1+x\frac{d}{dx}\right]\times j_L(kx)Y^{[L]}(\hat{x})\,, \tag{41}$$

$$\mathcal{T}^{[L]}(k)=\frac{i^{L+1}}{\sqrt{L(L+1)}}\int d^3x\,\mathbf{j}(\mathbf{x})\cdot\mathbf{x}\times j_L(kx)Y^{[L]}(\hat{x})\,. \tag{42}$$

We observe that to leading order in k the electric multipoles are determined by Coulomb multipoles alone. Only to order k^{L+1} the multipoles are influenced by matrix elements explicitly involving the current operator.

Besides the above replacements in calculating the electric current multipoles, the following changes in determining the contributions of the intrinsic two-photon amplitude have to be made

$$\mathbf{A}^{[L]}(E;\mathbf{x},k)\rightarrow\mathbf{a}^{[L]}(E;\mathbf{x},k)=\frac{i^{L+1}}{\sqrt{L(L+1)}}\mathbf{x}j_L(kx)Y^{[L]}(\hat{x})\,. \tag{43}$$

Note, that the lowest-order term of the above expression is of order $k^{L'+L+2}$. The reason for this is that all lower-order terms appearing in (34) have exactly canceled against terms arising in the resonance amplitude. It is important to realize that the above result holds to all orders in k and it is *not* the consequence of a low-energy expansion, as a common misunderstanding of Siegert's theorem might suggest. However, we now proceed by calculating the higher-order terms $\mathcal{T}^{[L]}$ (which depend on the current operator explicitly) by simply evaluating them with the single-body operator (35). At this point an approximation is made and—strictly speaking—gauge invariance is slightly violated. But this violation only concerns terms of at least order $k^{L'+L+2}$ in all electric polarizabilities. Furthermore, these contributions become important only at large NN distances [cf. Eq. (42)], and there they will be suppressed by the falloff of the nuclear wave function. Therefore, we can safely assume that the above-mentioned violation of gauge invariance is not very serious for the matrix elements to be calculated.

In practice our calculation now proceeds as follows: The Coulomb multipoles (41) we calculate with charge operator (37). The contributions to the electric multipoles $\mathcal{T}^{[L]}$ as well as the magnetic multipoles we determine with the single body current (35) and the remaining terms of the intrinsic two-photon operator we evaluate from the single-body two-photon operator[10]

$$B^{\rm in}_{l'l}(\mathbf{x}',\mathbf{x})=\frac{\delta_{l'l}}{2m}[e_1\delta(\mathbf{x}'-\mathbf{r}_1)-e_2\delta(\mathbf{x}'-\mathbf{r}_2)]\times[e_1\delta(\mathbf{x}-\mathbf{r}_1)-e_2\delta(\mathbf{x}-\mathbf{r}_2)]\,, \tag{44}$$

which corresponds to Thomson scattering off the nucleons within the deuteron.

From the operator (44) the electric intrinsic two-photon polarizabilities are easily calculated using Eq. (34), and we find

$$B^{\rm in}_J(EL',EL,k)=-\frac{k^2e^2}{8m}\sqrt{\pi}(-)^{L+J+(L'+L)/2}\frac{(\hat{L}'\hat{L})^2}{\sqrt{L'L(L'+1)(L+1)}\hat{J}}\times\begin{bmatrix}L' & L & J\\ 0 & 0 & 0\end{bmatrix}\langle 1\|r^2j_{L'}(kr/2)j_L(kr/2)Y^{[J]}(\hat{r})\|1\rangle \tag{45}$$

and similar formulas for the mixed and pure magnetic terms. With this result our formalism is completely defined and we can proceed to any analysis of our numerical results.

Before we do so we briefly want to discuss the low-energy behavior of the polarizabilities. It is obvious, that after the above mentioned replacements both R_J and $B^{\rm in}_J$ have no contribution to order k^0, so that the $B^{\rm c.m.}_J$ determines the behavior of the scattering amplitude to this order, and therefore the low energy theorem is explicitly fulfilled. From Eq. (33) one gets

$$P_0(E1,E1,k=0)=\frac{3e^2}{M} \tag{46}$$

as the low-energy theorem requires, and all other polarizabilities are zero to this order. For the polarizabilities to order k and k^2 at energies *below* deuteron breakup, we do not fulfill Friar's low-energy theorem[9] exactly, since we have neglected some recoil terms [cf. the remarks in Sec. II after Eq. (11)]. But since these corrections are small, we do not need to consider them here.

V. NUMERICAL RESULTS AND DISCUSSION

The basic dynamical input into our calculation is the separable potential V as defined in Eq. (17), whose parameters have to be fitted in such a way as to reproduce the experimental nucleon-nucleon phase shifts. Here we take the parameters C_R, C_A, a_R, and a_A of a fit obtained by Mongan[13] with the form factors parametrized by

$$g_{ls}(p)=C_Rp^{l+i}/(p^2+a_R^2)^{(l+2+i)/2}\,,\qquad h_{ls}(p)=C_Ap^l/(p^2+a_A^2)^{(l+2)/2}\,, \tag{47}$$

where $i=2$ for the $^1S_0$ and $^3P_0$ partial waves, and $i=0$ in all other cases. (To be specific, we report here results obtained with Mongan's "Case II" fits, but all other fits we tried give basically identical results.) With this input we calculate the polarizabilities P_J from Eqs. (27), (33), and

(45).

To give a first impression of our results we calculate the total photoabsorption cross section, which is related to the polarizabilities via the optical theorem,

$$\bar{\sigma}_{\rm abs}(k)=\frac{4\pi}{k}\frac{1}{\sqrt{3}}\sum_L\frac{(-)^L}{\hat{L}}{\rm Im}[P_0(EL,EL)+P_0(ML,ML)]. \tag{48}$$

We would like to remind the reader, that only the direct graphs [Figs. 1(a) and 1(c)] contribute to the imaginary part of the polarizabilities and, therefore, to the photoabsorption cross section. Furthermore, only the scalar ($J=0$) polarizabilities are involved. In Fig. 2 we show results calculated with the Born term only and compare them with the full calculation. As is obvious (and well known since a long time[19]) the Born term dominates the total photo absorption cross section. Our result compares well with results obtained from local potentials and the experimental data (cf., e.g., Ref. 17). Of course, with this calculation we do not want to compete with much more elaborate calculations of the deuteron photoabsorption cross section starting from realistic potentials and including all sorts of non-nucleonic degrees of freedom.[2] We show this result here for completeness and comparison only.

We now present in detail our results of the real and imaginary parts of the polarizabilities $P_J(M'L,M^\nu L)$. These are presented in Figs. 3–5. Before we discuss these results, we would like to remind the reader that due to angular momentum conservation $|L-J|\le L'\le L+J$ and parity conservation $L'+\nu'+L+\nu=$even only the following multipole combinations have to be considered for the deuteron:

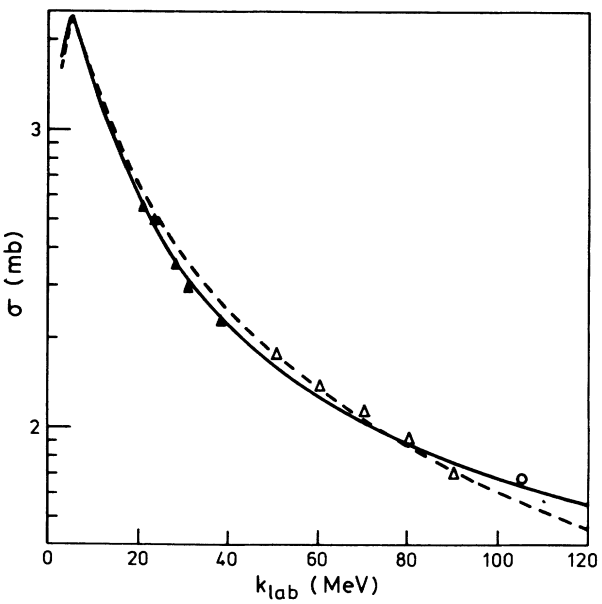

FIG. 2. The total photoabsorption cross-section: Born terms only (dashed) and complete calculation (solid line). The experimental data are from Ref. 23 (solid triangle), Ref. 24 (open triangle), and Ref. 25 (open circle).

scalar ($J=0$)

$$P_0(EL,EL),P_0(ML,ML)\ ,$$

vector ($J=1$)

$$P_1(EL,EL),P_1(ML,ML)\ ,$$

$$P_1(EL,M(L+1)),P_1(ML,E(L+1))\ ,$$

tensor ($J=2$)

$$P_2(EL,EL),P_2(ML,ML)\ ,$$

$$P_2(EL,M(L+1)),P_2(ML,E(L+1))\ ,$$

$$P_2(EL,E(L+2)),P_2(ML,M(L+2))\ .$$

The polarizabilities with $L'>L$ are obtained with the help of a symmetry relation derived from time reversal invariance[20]

$$P_J(M^{\nu'}L',M^\nu L)=P_J(M^\nu L,M^{\nu'}L')\ . \tag{49}$$

We note in passing that the two-photon polarizabilities vanish for all vector polarizabilities because of the following symmetry relation for the two-photon amplitude[11]

$$B_J^{\rm in/c.m.}(M^{\nu'}L',M^\nu L)=(-)^J B_J^{\rm in/c.m.}(M^\nu L,M^{\nu'}L')\ . \tag{50}$$

The scalar $E1E1$ polarizability is shown in Fig. 3(a). This is the dominant multipole in the energy region considered here, and it is obviously mainly determined by the Born term [Fig. 1(a)]. The imaginary part is consistent with a privious calculation utilizing the Reid soft-core potential.[11] However, the result for the real part of the $E1E1$ polarizability is very different from what was obtained previously with a dispersion theoretical method. Let us discuss the different contributions in detail. Henceforth, we add the intrinsic two-photon amplitude to the different terms of the resonance part without always mentioning it explicitly. Therefore, what we show and discuss are the gauge invariant polarizabilities $R_J+B_J^{\rm in}$. The dashed curve in Fig. 3(a) (labeled with Re direct) shows the Born term [Fig. 1(a)] of the direct process. This contribution alone is already larger than the dispersion theoretical result. Figure 3(a) also shows a very sizable effect coming from the crossed Born term [Fig. 1(b)]. On the other hand, the contributions of the rescattering terms [Figs. 1(c) and 1(d)] are small in the real part of this polarizability. Much more important than the rescattering terms is the contribution of the c.m. two-photon amplitude. As already remarked before this term is the only one which survives for $k=0$ and is numerically given by

$$P_0(E1,E1,k=0)=\frac{3e^2}{M}=2.3\times10^{-3}\ {\rm fm}\ .$$

Obviously this term is of the same order of magnitude as the Born terms, and, therefore, the c.m. two-photon am-

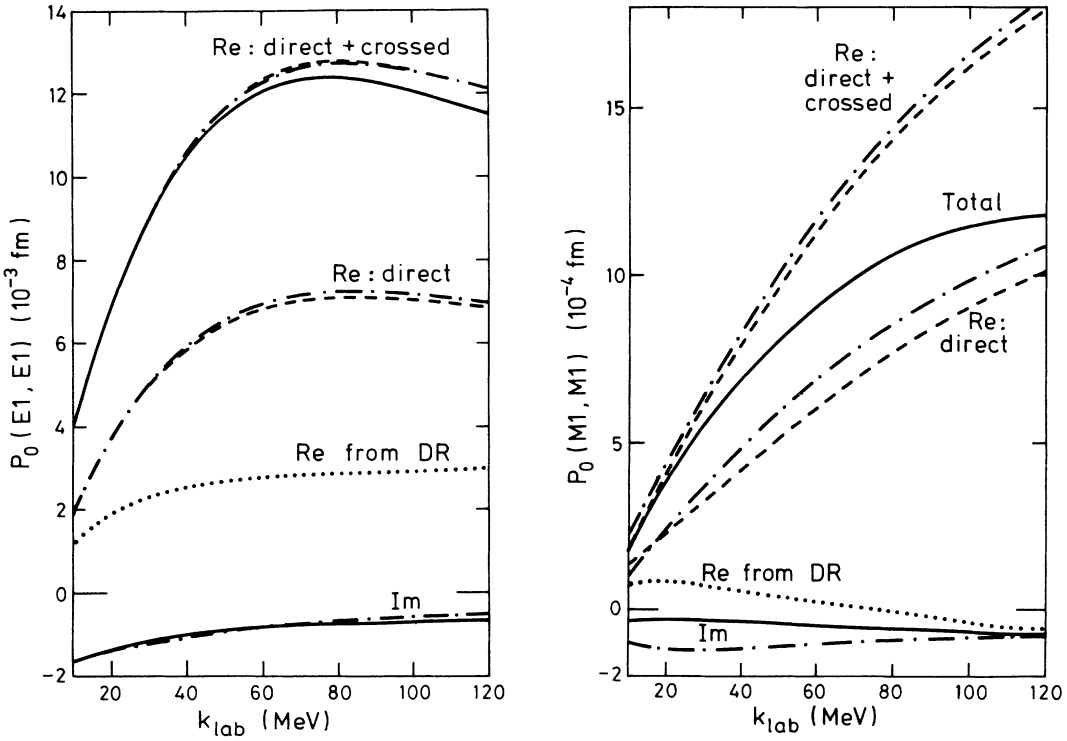

FIG. 3. The scalar $E1E1$ and $M1M1$ polarizabilities. Imaginary part (Im): Born term only (dashed), rescattering included (solid). Real part (Re): Born term only (dash-dotted), rescattering included (dashed), complete calculation with c.m. two-photon contribution (solid). Dotted line represents the previous dispersion relation result. The constant $3e^2/M$ has been subtracted for the electric polarizability in this figure. For further explanations see text.

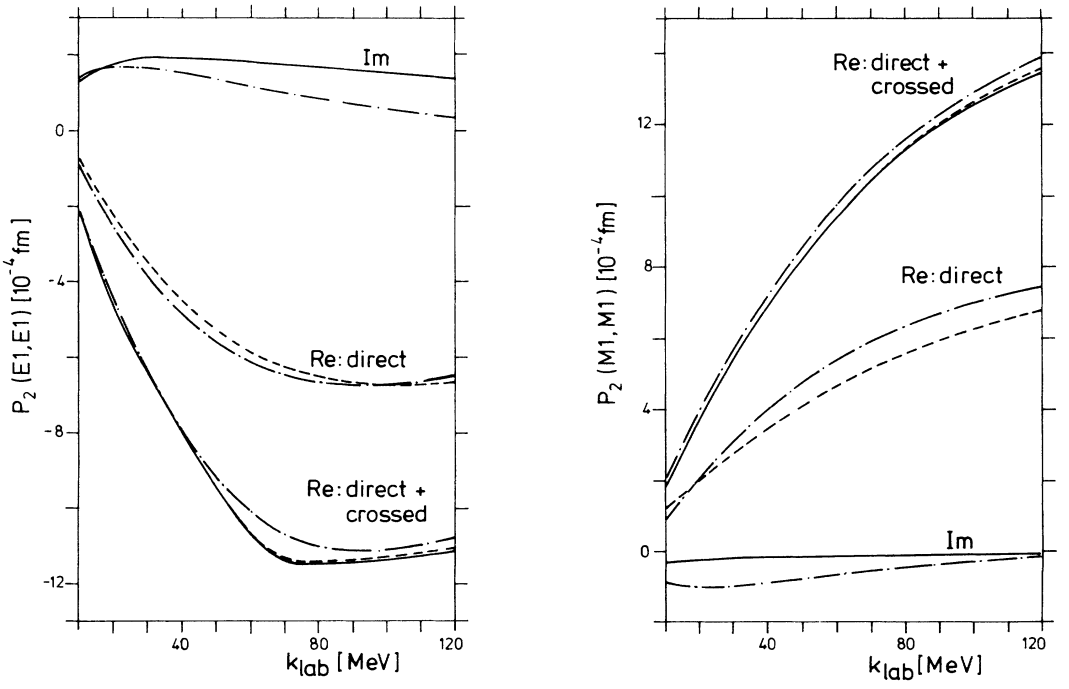

FIG. 4. The tensor $E1E1$ and $M1M1$ polarizabilities. Notations as in Fig. 3.

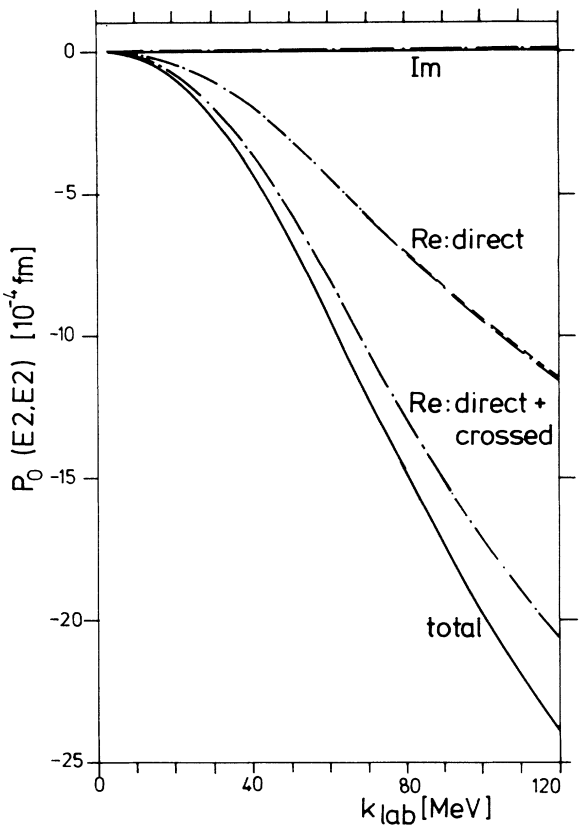

FIG. 5. The scalar $E2E2$ polarizability. Notation as in Fig. 3.

plitude is an important background contribution. The energy dependence of the c.m. two-photon amplitude is also quite sizable, whereas the intrinsic two-photon contribution is negligible for this polarizability. This is no surprise, since to a large extent the intrinsic two-photon contributions have exactly canceled against terms of the resonance polarizability by applying Siegert's theorem as was explained in the previous section. Overall we note, that the complete result for the real part [solid curve in Fig. 3(b)] is much larger than the result obtained previously by a dispersion theoretical method (cf. Ref. 11). The strong dominance of the Born terms (if we disregard the c.m. background contribution) is remarkable.

Here a brief remark on our calculation of the rescattering terms is in order. As mentioned before we have calculated the NN propagator in a separable potential model, whose parameters are fixed to describe on-shell NN scattering. Compton scattering, however, probes off-shell NN scattering and we do not know how reliable the model is in predicting the off-shell structure of the NN propagator. Therefore, calculating, e.g., the crossed diagrams [Fig. 1(d)] with this propagator involves some uncertainties. However, our conclusion that the Born terms strongly dominate Compton scattering should be unaffected by this reservation.

What are the reasons for the strong disagreement with the dispersion theoretical method? Several aspects have to be considered: First of all, in order to apply dispersion relations, one in principle has to know the imaginary part of the amplitude up to infinity. This, of course, is not the case. To practically perform the Hilbert transform one has to *assume* the high-energy behavior as well as the number of subtractions to be performed. In the previous dispersion theoretical calculation the minimal number of subtractions, which is consistent with the correct low-energy behavior of each polarizability has been applied. These assumptions seem to be incorrect in the light of the present calculation, which does not rely on assumptions on the high-energy behavior of the Compton amplitude. Furthermore, one has to admit that the specific form of the dispersion relations which have been used in Ref. 11 is debatable. These dispersion relations are derived from the forward dispersion relation for the Compton amplitude due to Gell-Mann, Goldberger, and Thirring,[21] but it is not clear, whether they are valid for partial wave amplitudes.

We will now briefly discuss the scalar $M1M1$ polarizability shown in Fig. 3(b). Here the rescattering terms play a somewhat stronger role than in the scalar $E1E1$ polarizability. In the imaginary part the rescattering term and the Born term are of comparable size, but of opposite sign leading to an overall relatively small magnetic contribution. For the real part most comments already made for the electric polarizability also apply except that here the rescattering terms are more sizable. Of course, for $k=0$ the scalar $M1M1$ vanishes.

For the vector polarizabilities ($E1E1$ and $M1M1$, $J=1$) we find that both the real and imaginary parts are small (smaller than 1% of the respective scalar polarizabilities). The small real part comes about by a cancellation of the direct and crossed processes. Both contributions separately are large, but they cancel each other due to the phase $(-1)^{L+L'+J}$ occurring in Eq. (27) for the crossed terms. In Fig. 4 we show our results for the tensor polarizability $P_2(E1,E1)$ and $P_2(M2,M2)$, and as an example for a higher multipole we display $P_0(E2,E2)$ in Fig. 5.

We now would like to comment on possible corrections, which go beyond the framework applied here. As already remarked previously the bulk part of meson exchange effects to the electric polarizabilities has been included in the present calculation by applying Siegert's theorem. The most important consequence of this procedure is that crucial cancellations between resonance and two-photon polarizabilities occur, so that the low-energy theorem is satisfied. Explicit exchange effects, i.e., those which cannot be taken care of by using the Siegert operators, are very small in the kinematic region considered here.[10] Furthermore, explicit corrections from one-pion exchange to the two-photon amplitude are completely negligible.[11] Therefore, the most important effects omitted here are contributions of the Δ resonance. Those will be studied in a forthcoming publication.

Finally we would like to show results for the spin averaged differential cross section. In the c.m. system it is given by

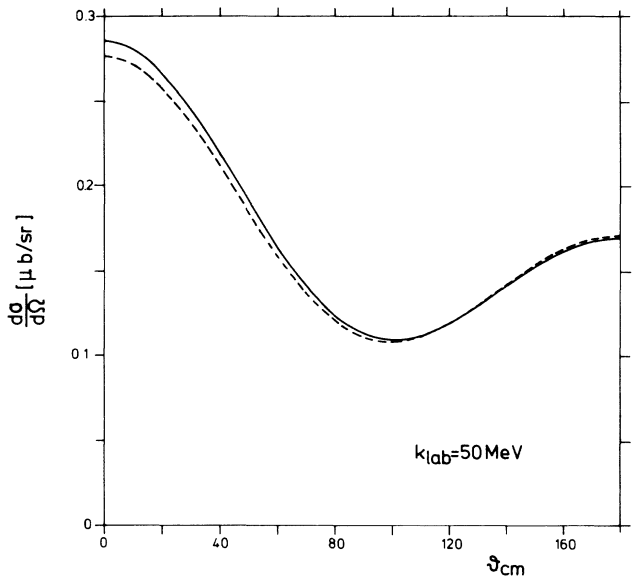

FIG. 6. Differential cross section for $k_{\text{lab}}=50$ MeV. Born terms only (dashed), complete calculation (solid).

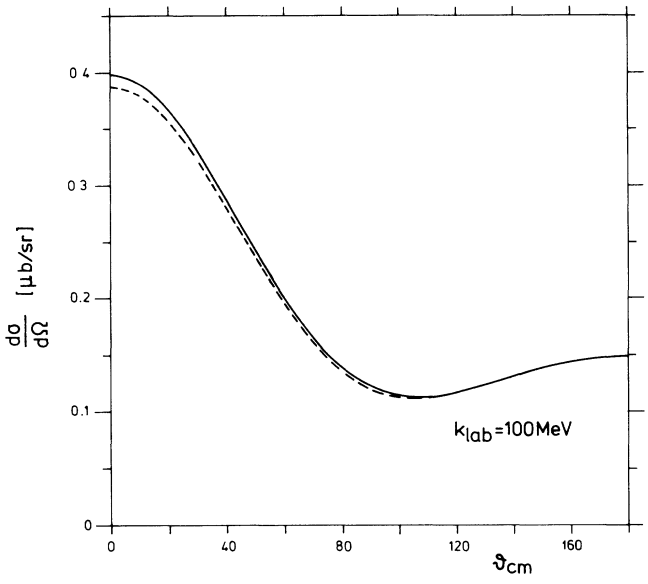

FIG. 7. Differential cross section for $k_{\text{lab}}=100$ MeV. Notation as in Fig. 6.

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \tfrac{1}{6} \sum_{\substack{\lambda'\lambda \\ m_d' m_d}} |\, T_{\lambda'\lambda}^{m_d' m_d} \,|^2$$

$$= \sum_{\substack{L'LJ\nu'\nu \\ K'Kj\mu'\mu}} P_J(M^{\nu'}L', M^{\nu}L) P_J^*(M^{\mu'}K', M^{\mu}K) a_{K'Kj}^{L'LJ} P_j(\theta)[2+(-)^{L+K+j+\nu+\mu}+(-)^{L'+K'+j+\nu'+\mu'}] , \tag{51}$$

with

$$a_{K'Kj}^{L'LJ} = \tfrac{1}{6}(-)^{L+K+J} \hat{J}^2 \hat{j}^2 \begin{Bmatrix} L & L' & J \\ K & K' & j \end{Bmatrix} \begin{bmatrix} K & L & j \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} L' & K' & j \\ -1 & 1 & 0 \end{bmatrix} \tag{52}$$

and $P_j(\theta)$ the usual Legendre polynomials depending on the c.m. scattering angle θ. If one wants to consider photon polarizations, then the square brackets in (51) have to be replaced by $3\lambda^{L+K+j+\nu+\mu}\lambda'^{L'+K'+j+\nu'+\mu'}$. From that it follows that in order to study interference terms involving polarizabilities with different J, polarized deuterons would be necessary.

In Figs. 6 and 7 we show our results for the differential cross section for laboratory photon energies $k=50$ and 100 MeV. We again compare results obtained with the Born terms only (including all two-photon contributions) with the full calculation. It is quite obvious that in order to distinguish rescattering effects from the Born terms an extremely exact measurement of this already small cross section would be necessary. But we note that because of the strong contribution of the real part of the resonance amplitude the cross sections turn out to be more than a factor of 10 larger than the pure Thomson cross section.

VI. CONCLUSION

We have studied Compton scattering off the deuteron for photon energies up to about 100 MeV. This energy limit reflects the fact that we have only included intermediate NN states. The dynamical framework to describe the process is graphically represented in Fig. 1. It turns out that the Born terms Figs. 1(a) and 1(b) are dominant, and that rescattering effects show up only as small corrections. This, of course, is a consequence of the very dilute structure of the deuteron. That means that to experimentally distinguish interaction-dependent effects in the kinematic range considered here would not be easy.

On the other hand, we make some specific assumptions concerning the electromagnetic structure of the two-nucleon system. Specifically we consider the standard nonrelativistic single-body charge and current operators for point nucleons. Two-body and nonlocal currents are taken into account in terms of the Siegert operators. It has been shown in previous calculations,[10,14] that additional meson-exchange corrections (i.e., such corrections which cannot be taken into account by using Siegert's theorem) are small. However, the electromagnetic interaction operators we consider here do neither take into account nucleon substructure nor possible relativistic corrections. Consideration of such corrections proved, e.g., to be necessary[22] in order to understand photodisin-

tegration of the deuteron at zero degrees. It is an open question how those corrections eventually influence the real part of the deuteron Compton amplitude.

The real part has contributions from the direct processes [Figs. 1(a) and 1(c)] as well as a surprisingly large part from the crossed processes [Figs. 1(b) and 1(d)], as we showed in the present calculation. Numerically the real part is much larger than the imaginary part, and therefore the Compton scattering cross section (as well as polarization variables, which we did not yet consider in detail) is essentially determined by the real part of the Compton amplitude. Therefore, Compton scattering off the deuteron offers a new and independent test of our understanding of the electromagnetic structure of the two-nucleon system, if scattering experiments can be performed with sufficiently high precision.

ACKNOWLEDGMENTS

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APPENDIX

Here we give some useful formulas necessary to explicitly calculate the polarizabilities (27) and (34). The multipole fields $A^{[L]}(E)$ and $A^{[L]}(M)$ entering the definition of the current multipoles are defined by[15]

$$\mathbf{A}^{[L]}(E;\mathbf{x},k)=\frac{1}{k}\nabla_x \mathbf{A}^{[L]}(M;\mathbf{x},k) , \tag{A1}$$

$$\mathbf{A}^{[L]}(M;\mathbf{x},k)=i^L j_L(kx)\mathbf{Y}^{[L],L}(k) , \tag{A2}$$

where $\mathbf{Y}^{[L],L}$ is a vector spherical harmonic.

The electric multipole field can be conveniently split into two terms[17]

$$\mathbf{A}^{[L]}(E;\mathbf{x},k)=\frac{1}{k}\nabla_x c^{[L]}(\mathbf{x},k)+\mathbf{a}^{[L]}(\mathbf{x},k) , \tag{A3}$$

with

$$c^{[L]}(\mathbf{x},k)=\frac{i^{L+1}}{\sqrt{L(L+1)}}\left[1+x\frac{d}{dx}\right]j_L(kx)Y^{[L]}(\hat{\mathbf{x}}) \tag{A3'}$$

and $\mathbf{a}^{[L]}(\mathbf{x},k)$ as defined in Eq. (43). Note, that the above separation is not unique, and we have chosen here a form slightly different from that in Refs. 11 and 14.

It is now easy but lengthy to derive the following expression for the electric intrinsic two-photon polarizabilities defined in Eq. (34) (cf. Ref. 14):

$$B_J^{\text{in}}(EL',EL)=\mathcal{A}_J(EL',EL)+\mathcal{B}_J(EL',EL) , \tag{A4}$$

with

$$\mathcal{A}_J(EL',EL)=(-)^{L+J}2\pi\frac{\hat{L}'\hat{L}}{\hat{J}}\left\langle 1\left|\left|\sum_{l'l}\int d^3x'\int d^3x[a_{l'}^{[L']}(k,\mathbf{x}')\times a_l^{[L]}(k,x)]^{[J]}B_{l'l}(\mathbf{x}',\mathbf{x})\right|\right|1\right\rangle , \tag{A5}$$

$$\mathcal{B}_J(EL',EL)=(-)^{L+J}2\pi\frac{\hat{L}'\hat{L}}{\hat{J}}\left[\frac{1}{k^2}\langle 1\|[M^{[L']}\overset{\times}{,}[H^{\text{in}}\overset{\times}{,}M^{[L]}]]_-^{[J]}\|1\rangle\right]+\frac{1}{k}\langle 1\|[C^{[L']}(k)\overset{\times}{,}\mathcal{T}^{[L]}(k)]_-^{[J]}\|1\rangle$$
$$+\frac{1}{k}\langle 1\|[C^{[L]}(k)\overset{\times}{,}\mathcal{T}^{[L']}(k)]_-^{[J]}\|1\rangle , \tag{A6}$$

with $C^{[L]}(k)$ and $\mathcal{T}^{[L]}(k)$ defined in Eqs. (41) and (42). The commutator of a tensor product is defined as

$$[A^{[L']}\overset{\times}{,}B^{[L]}]^{[J]}=[A^{[L']}\times B^{[L]}]^{[J]}-(-)^{L'+L+J}[B^{[L]}\times A^{[L']}] .$$

Similarly, $R_J(EL,EL')$ can be written as (for elastic scattering only, see Ref. 14 for the more general case)

$$R_J(EL',EL)=(-)^{L+J}2\pi\frac{\hat{L}'\hat{L}}{\hat{J}}\langle 1\|[E^{[L']}(k)\times G(E)E^{[L]}(k)]^{[J]}\|1\rangle+(E\leftrightarrow\bar{E})(-)^{L'+L+J} , \tag{A7}$$

with $E^{[L]}$ given by Eq. (26) and $G(E)$ by Eq. (12). If we now use Eq. (A3) in evaluating $E^{[L]}$, we will see that a crucial cancellation occurs if we add up Eqs. (A4) and (A7). In particular, the double commutator term of Eq. (A6) drops out of the calculation. If this would not be the case, the correct low-energy behavior would be spoiled.

We now explain, how to explicitly calculate the reduced matrix elements G_{ljs} and H_{ljs} defined in (28). The explicit formulas can in fact be directly taken over from the work of Partovi.[17] We only have to replace the function $v_{ls\lambda}$ occuring in the radial integrals defined in Eqs. (54)–(56) of Ref. 17 by the corresponding functions Φ_g and Φ_h, respectively. Furthermore, the mixing matrix defined in Eq. (18) of Ref. 17 has to be replaced by

$$M^j_{ls,\mu}=\begin{array}{c} \\ l=j-1\quad s=1 \\ l=j\quad\ s=0 \\ l=j+1\quad s=1 \\ l=j\quad\ s=1 \end{array}\begin{array}{c}\begin{array}{cccc}\mu=1 & \mu=2 & \mu=3 & \mu=4\end{array}\\ \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\end{array} . \tag{A8}$$

Moreover, we note the following relation between the reduced matrix elements $\mathcal{S}^{(L)}(\mu j)$ and $\mathcal{J}^{[L]}(\mu j)$ defined by Partovi and the reduced matrix elements used here,

$$M_\nu^{[L]}(\mu j)=\begin{cases}\mathcal{J}^{[L]}(\mu j), & \nu=0\ ,\\ \mathcal{S}^{[L]}(\mu j), & \nu=1\ ,\end{cases} \tag{A9}$$

$$\langle \mu j\|T_\nu^{[L]}\|1\rangle=(-)^{j+L}\langle 1\|T_\nu^{[L]}\|\mu j\rangle=\frac{2}{\hat{L}}\left[\frac{k}{\pi pM}\right]^{1/2}M_\nu^{[L]}(\mu j)\ , \tag{A10}$$

where the relation between the labels (ls) and μ is provided through (A8).

Since the imaginary part of the Born amplitude, Eq. (32), is a product of two current multipole matrix elements it can be calculated in a similar way using (A8) to (A10). However, the calculation of the real part of the Born amplitude involves some slight complications, because its formula is not separable into a product of two current multipoles,

$$\mathrm{Re}[F_{lsj}^{L'\nu'L\nu}(k,E)]=\frac{2}{\pi}\frac{1}{p^2}\frac{1}{L'L(L'+1)(L+1)}\mathcal{F}_{lsj}^{L'\nu'L\nu}(k,E)\ , \tag{A11}$$

with

$$\mathcal{F}_{lsj}^{L'\nu'L\nu}(k,E)=\sum_{i=1}^{N}\mathcal{L}_{lsj}^{L'\nu'L\nu(i)}(k,E) \tag{A12}$$

and $N=16$ for $\nu=\nu'=0$ and $N=36$ for $\nu=\nu'=1$. The complete expressions are too lengthy to be reproduced here, and, therefore, we only give here a typical term:

$$\begin{aligned}\mathcal{L}_{lsj}^{L'\nu'L\nu,(1)}(k,E)=\sum_{l'',\overline{l''}}&\left[1+\frac{k}{2M_d}\right]^2(e_1+(-)^Le_2)(e_1+(-)^{L'}e_2)\\ &\times\langle(ls)j\|Y^{[L]}\|(l''1)1\rangle\langle(ls)j;\|Y^{[L']}1)\|(\overline{l''}1)1\rangle I_{11}(l,L,L'',l'',\overline{l''})\ ,\end{aligned} \tag{A13}$$

with

$$\begin{aligned}&I_{11}(l,L',L'',l'',\overline{l''})=\int dr\int dr'\phi_l(r,r')a_L(kr/2)a_{L'}(kr'/2)u_{l''}(r)u_{\overline{l''}}(r')\ ,\\ &a_L(x)=j_L(x)+\frac{L}{2L+1}xj_{L-1}(x)-\frac{L+1}{2L+1}xj_{L+1}(x)\ ,\\ &\phi_l(r,r')=\mathrm{Re}\int_0^\infty d\lambda\,\lambda^2j_l(\lambda r')j_l(\lambda r)G_0(E,\lambda)\ .\end{aligned} \tag{A14}$$

All other terms can be immediately constructed from Partovi's expressions for the current multipoles with appropriate replacements of the radial matrix elements.

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*Present address: Department of Physics and Mathematical Physics, University of Adelaide, Adelaide, South Australia 5001.

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