Photodisintegration of the Three–Nucleon Systems and their Polarizabilities

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Abstract

The total photodisintegration cross sections of three–body nuclei are calculated with semirealistic NN potentials below pion threshold. Full final state interaction with Coulomb force is taken into account via the Lorentz integral transform method. The experimental total cross sections are well described and the sum rule $\sigma_{-1}(^{3}\text{H})$ agrees with elastic electron scattering data. The calculated ^{3}He polarizability is 0.15 fm 3 .

The photodisintegration cross sections of the three–nucleon systems are important quantities for understanding the physics of few-body systems. As reported in Ref. [1] in the past experimental and theoretical work have mostly concentrated either on the low-energy two-body or on the low-energy three-body breakup. On the theoretical side several calculations of the process have been performed with separable NN potentials [2,3]. In this framework quantitatively accurate results have been obtained in Ref. [3]. However, mixed symmetry states as well as noncentral and Coulomb NN forces have not been considered. The results compare rather well with experiment in the peak region, but underestimate the experimental data in the threshold region. Approximate three-body breakup calculations have also been performed with low-energy local NN potentials in the framework of the hyperspherical approach [4]. The only calculation with a realistic NN force has been carried out in Ref. [5] for the ³H case. It has been a substantial progress, but the calculation still has some shortcomings: the used NN force does not provide a sufficiently good fit to NN data; the hyperspherical expansion employed to describe the interior part of the continuum wave functions retains only the three lowest terms and the error in the truncation has not been fully estimated. The results of the two calculations of Refs. [4,5] are only in qualitative agreement with the data.

An important observable connected with the total photodisintegration cross section is the electric polarizability α_{E1} . It has to be mentioned that the experimental α_{E1} value of ³He determined from elastic scattering on ²⁰⁸Pb [6] is at variance with the value extracted from photoabsorption experiments.

In the present work total photodisintegration cross sections of the three–nucleon systems are calculated accurately with NN forces of a type different from separable potentials. The polarizabilities of the three–nucleon systems are calculated as well, both directly, by means of sum–rules techniques, and integrating the properly weighted total photodisintegration cross sections. We use local even central NN potentials that reproduce the low–energy NN properties and provide a realistic description of the 1S_0 and 3S_1 NN phase shifts up to the pion threshold. Specifically, we use the same NN potential models, Malfliet–Tjon I+III (MT) [7] and Trento (TN) potentials, as in our works on electro– [8] and photodisintegration [9] of the 4 He nucleus. The MT parameters are taken from Ref. [10]. The description of the NN scattering data given by these potentials has been shown in Ref. [9]. Besides the strong interaction also the Coulomb force is included in our nuclear hamiltonian. This leads to differences between the 3 He and 3 H cross sections, which will also be discussed in the following.

The photodisintegration cross sections are calculated within the method of the Lorentz integral transform [11]. This approach allows the inclusion of the full final state interaction without explicit calculation of the continuum states. The longitudinal (e, e') form factors of the nuclear two– [11], three– [12] and four–body systems [8] have already been calculated with this method.

Recently we have also studied in this way the total cross section of the process $\gamma + {}^{4}\text{He} \to X$ below pion threshold [9]. We have predicted a pronounced giant dipole resonance. Since the rather flat cross sections for the two-body breakup channels (${}^{3}\text{H-}p$, ${}^{3}\text{He-}n$) seem to be settled, the strong peak has to be attributed to an additional cross section from the not yet measured (γ , np) channel. Contrary to the four-body system the peak of the giant resonance is rather well determined in experiment for the three-body systems. Thus in this

case we have the possibility to get an experimental confirmation of the results obtained with our new approach.

It is well known that the total photoabsorption cross section can be reliably calculated in the dipole approximation

$$\sigma_T(E_\gamma) = 4\pi^2 (e^2/\hbar c) E_\gamma R(E_\gamma) \,, \tag{1}$$

with

$$R(E_{\gamma}) = \int df |\langle \Psi_f | D_z | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E_{\gamma}), \qquad (2)$$

where

$$\vec{D} = \sum_{i=1}^{Z} (\vec{r}_i - \vec{R}_{cm}). \tag{3}$$

In Eq.(2) Ψ_0 is the three-body bound state wave function with energy E_0 , and Ψ_f are final state wave functions normalized as $\langle \Psi_f | \Psi_{f'} \rangle = \delta(f - f')$. In the delta function in Eq.(2) we neglect the very small nuclear recoil energy. We calculate the response function R via evaluation and subsequent inversion of its Lorentz integral transform

$$\mathcal{L}(\sigma = -\sigma_R + i\sigma_I) = \int_{E_{th}}^{\infty} dE_{\gamma} \frac{R(E_{\gamma})}{(E_{\gamma} - \sigma_R)^2 + \sigma_I^2}.$$
 (4)

The transform $\mathcal{L}(\sigma)$ of the response R is found as

$$\mathcal{L}(\sigma) = \langle \tilde{\Psi}(\sigma) | \tilde{\Psi}(\sigma) \rangle, \tag{5}$$

 $\tilde{\Psi}$ being the localized solution of the Schrödinger–like equation

$$(\hat{H} - E_0 + \sigma)\tilde{\Psi}(\sigma) = Q \tag{6}$$

with the source term $Q = D_z \Psi_0$.

The wave function Ψ_0 is the ground state solution for the same hamiltonian. The binding energies for both our potential models are 8.7 MeV (³H) and 8.0 MeV (³He). Compared to the experimental values we have a very small overbinding of 0.2 MeV. For the charge radii (with finite size nucleons) we obtain the values $r_{ch}(^3\text{H}) = 1.76$ fm, $r_{ch}(^3\text{He}) = 1.94$ fm, and $r_{ch}(^3\text{H}) = 1.74$ fm, $r_{ch}(^3\text{He}) = 1.92$ fm for the MT and TN potentials, respectively, which are in a good agreement with the experimental radii $r_{ch}(^3\text{H}) = 1.76 \pm 0.04$ fm [13], $r_{ch}(^3\text{He}) = 1.976 \pm 0.015$ fm [14]. We have checked the reliability of our ground state calculation by comparison with the MT potential results of Ref. [15]. For this check we have used the same MT potential parameters as given in Ref. [15]. The agreement for binding energies, radii and mixed symmetry state probabilities is very good.

We solve Eq. (6) for the multipolarity L=1, spin $S=\frac{1}{2}$, and isospin $T=\frac{1}{2}$ and $\frac{3}{2}$ with the help of the correlated hyperspherical expansion and the hyperradial expansion of the same form as in Refs. [8,9]. The K_{max} value equals to 7. The convergence of the transform is well attained with this K_{max} value and the convergence rate is similar to that shown in Ref. [9] for the ⁴He case. The values of $\sigma_I=20$ MeV and 5 MeV have been employed.

As for ⁴He the inversion has been performed both for $\sigma_I = 20$ MeV and for a combination of the transforms with $\sigma_I = 5$ and 20 MeV, chosen so that the former transform gives a predominant contribution to the very steeply rising low–energy wing of the response and the latter to its high–energy wing. While inverting the transform the n-d low–energy behaviour $[E_{\gamma} - (E_{\gamma})_{min}]^{3/2}$ has been incorporated into our trial response. Of course for ³He this is not the correct breakup behaviour at the very threshold, but already 1 MeV above it the assumed low–energy response should be a rather good approximation. This is also confirmed by our sum rule checks (see below).

For ³H we can first compare the inverse energy weighted sum $\sigma_{-1} = \int_{E_{th}}^{\infty} E_{\gamma}^{-1} \sigma_T(E_{\gamma}) dE_{\gamma}$ with experiment, using the calculated value of the radius. In fact this sum is known to be entirely determined by the triton point proton radius [16]

$$\sigma_{-1}(^{3}\mathrm{H}) = \frac{4\pi}{3} \frac{e^{2}}{\hbar c} \langle r_{p}^{2}(^{3}\mathrm{H}) \rangle.$$
 (7)

Eq. (7) is exact and follows directly from the sum rule $\sigma_{-1} = (4\pi/3)(e^2/\hbar c)\langle\Psi_0|D^2|\Psi_0\rangle$ and the fact that for a single–proton nucleus $\vec{D} = \vec{r_p} - \vec{R}_{cm}$. Using the relation $\langle r_p^2(^3\mathrm{H}) \rangle = r_{ch}^2(^3\mathrm{H}) - r_{ch}^2(p) - 2r_{ch}^2(n)$ with the experimental values $r_{ch}(p) = 0.862$ fm [17], $r_{ch}^2(n) = -0.117$ fm² [18] and $r_{ch}(^3\mathrm{H})$ [13] one obtains $\sigma_{-1}(^3\mathrm{H}) = 2.485 \pm 0.135$ mb, which agrees with our sum rule values listed in Table 1. We would like to point out that the neutron charge radius is not negligible in obtaining the correct value of $\langle r_p^2(^3\mathrm{H}) \rangle$ from $r_{ch}^2(^3\mathrm{H})$ [19]. For ³He a sum rule analogous to Eq.(7) involves $\langle r_n^2(^3\mathrm{H}e) \rangle$. Attempts in the literature to predict $\sigma_{-1}(^3\mathrm{H}e)$ estimating $\langle r_n^2(^3\mathrm{H}e) \rangle$ by means of the ³He experimental magnetic radius are much more uncertain. In fact no exact relation involving only experimental observables exists between these two radii.

Now we turn to the comparison of our results with the experimental data for the total cross section. The total cross sections have been studied only in Ref. [1]. In that experiment the triton low–energy total cross section was measured as the sum of two– and three–body breakups. In the same experiment also the ³He three–body breakup cross section was determined. Combined with the two–body breakup data from Ref. [20] this led to a total cross section also for ³He. In the energy range between electric dipole peak and pion threshold there exists only one experiment. It consists in a measurement of the two– and three–body ³He photodisintegration [21]. The results for the total cross section were not listed but since two– and three–body reactions were both measured for the same photon energies we could determine the total cross section summing them.

Our results for the low–energy cross sections are shown in Fig. 1 together with the experimental data. It is seen that there is a good agreement with experiment in the threshold region. In the peak region, where one has a slight difference between the two experimental data sets for 3 He, our results agree with those of Ref. [21]. For triton the agreement with Ref. [1] is somewhat better. We would like to point out that seemingly only the upper experimental curve could lead to a fulfillment of the σ_{-1} sum rule. Both our potential models lead to rather similar cross sections. As in the 4 He photodisintegration [8] one finds a somewhat stronger cross section up to about 10 MeV above threshold for the MT potential. In Fig. 2 we show the results up to pion threshold. Again one has very similar cross sections with MT and TN potentials for both nuclei. For 3 He one sees a rather good agreement between the theoretical and experimental cross sections. Unfortunately, as mentioned above, there are no experimental total cross section data at higher energies for triton.

In Fig. 3 we show the effect of the Coulomb force on the three–nucleon photodisintegration for the case of the MT potential. Because of the Coulomb force one has different thresholds for ³He and ³H. This relative shift of the two cross sections is more and more compensated with increasing energy, and, eventually, at higher energies one finds almost identical cross sections. Furthermore, one sees a small reduction of the peak height due to the Coulomb force. For the not shown case of the TN potential one finds very similar results.

Like in our previous works for the electromagnetic breakup of ^4He [8,9] we have checked the quality of the obtained cross sections with the help of sum rules. Table 1 shows the comparison of the sum rules evaluated with the usual sum rule techniques with the results of an explicit integration of the properly weighted calculated cross sections. The differences are about 2 % for σ_{-2} , less than 1 % for σ_{-1} , and between 3 and 4 % for σ_0 . This shows the good accuracy of the obtained cross sections. From the rather good agreement of the σ_{-2} sum rule one sees that the calculated ^3He cross section is also reliable in the threshold region.

Here we would like to mention that the σ_{-2} sum rule value,

$$\sigma_{-2} = 4\pi^2 (e^2/\hbar c) \langle |\Psi_0| D_z (H - E_0)^{-1} D_z |\Psi_0 \rangle, \tag{8}$$

was evaluated as $4\pi^2(e^2/\hbar c)\langle\Psi_0|D_z|\varphi\rangle$ [22] where φ is the negative parity (L=1) solution to the equation

$$(\hat{H} - E_0)\varphi = D_z \Psi_0. \tag{9}$$

This sum rule is particularly interesting because it is directly connected to the electric polarizability α_{E1} :

$$\alpha_{E1} = \frac{\hbar c}{2\pi^2} \sigma_{-2}(E1) \,. \tag{10}$$

We obtain the following values for ${}^{3}\text{He}$: 0.143 fm 3 (TN potential) and 0.151 fm 3 (MT potential). Therefore with our calculation we confirm the α_{E1} value of 0.15 \pm 0.02 fm 3 extracted from photoabsorption experiments [1,20,6]. For ${}^{3}\text{H}$ we get 0.135 fm 3 (TN potential) and 0.143 fm 3 (MT potential)

Our results for σ_0 (TRK sum rule) are about 20 % lower than those with realistic potentials [23,24]. This is mainly due to tensor correlations, which are neglected in our semirealistic potentials. In fact in Ref. [23] it is shown that 26 % of the TRK sum rule originates from tensor correlations. They should affect the photodisintegration cross sections mainly at higher energies with a considerable contribution beyond pion threshold. Thus we believe that our results should describe the three–nucleon photodisintegration fairly well in the shown energy range.

In the following we give a summary of our work. We have calculated the total photodisintegration cross section of ${}^{3}H$ and ${}^{3}He$ with semirealistic potential models using the method of the Lorentz integral transform. We confirm the value of the polarizability of ${}^{3}He$, which has been deduced from photoabsorption experiments. Since the used potential models lead to a realistic ${}^{3}H$ charge radius one has also a realistic $\sigma_{-1}({}^{3}H)$ value. We find a good agreement of the total cross section with experimental data, also in the region of the peak of the dipole resonance. This supports our prediction of a pronounced giant dipole resonance

in ⁴He, which we made in Ref. [9] calculating the total ⁴He photoabsorption cross section in analogy to the present work. Like for ⁴He it would be desirable to have better experimental determinations of the total photoabsorption cross section for the three–body nuclei. Also the theory can still be improved. Effects of multipoles others than E1, of retardation, of tensor correlations, and of subnuclear currents should have more and more influence at higher energies. Also relativistic currents could have a similar effect as in deuteron photodisintegration [25].

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TABLES

TABLE I. Comparison of sum rules from explicit integration (A) and from direct evaluation (B) with MT and TN potentials

Method	A	В	A	В	A	В	A	В
Potential	MT	MT	MT	MT	TN	TN	TN	TN
Nucleus	$^3\mathrm{H}$	$^3\mathrm{H}$	³ He	³ He	$^3\mathrm{H}$	$^3\mathrm{H}$	³ He	³ He
$\sigma_{-2} \; [\mathrm{MeV^{-1} \; mb}]$	0.146	0.143	0.154	0.151	0.138	0.135	0.147	0.143
σ_{-1} [mb]	2.49	2.48	2.54	2.53	2.42	2.42	2.49	2.47
$\sigma_0 [{ m MeV mb}]$	57.8	56.1	58.1	55.8	58.6	57.0	59.0	56.7

FIGURES

- FIG. 1. Low–energy total photoabsorption cross sections for ³He (a) and ³H (b): theoretical results with MT and TN potentials, experimental results with error range (dotted curves) from Ref. [1]. In (a) also the sum of the experimental two– and three–body cross sections from Ref. [21] is shown (filled circles). The total error is assumed to be the square root of the sum of the squared errors.
 - FIG. 2. As Fig. 1, but for an extended energy range up to pion threshold
 - FIG. 3. Cross sections for ³H (dashed curve) and ³He (full curve) with MT potential





