a. The "LIT equation"

$$\left(\hat{H}_{\text{nuclear}}\underbrace{-E_0-\mathcal{R}\left[\sigma\right]-i\;\mathcal{I}\left[\sigma\right]}_{:=-\mathcal{E}}\right)\Psi^{J^\pi m_j}_{\text{LIT}}=\left[\hat{\mathcal{O}}_{Lm_L}\left\{\left|\boldsymbol{k}\right|,\boldsymbol{j}_v\right\}\otimes\Psi_0^{J_0^{\pi 0}}\right]^{J^\pi m_j}\quad. \tag{1}$$

with

$$v(\text{ertex})\in\{\;\boldsymbol{j}_o(\boldsymbol{x})=\ldots\;,\;\boldsymbol{j}_s(\boldsymbol{x})=\ldots\;,\;\boldsymbol{j}_{mec}(\boldsymbol{x})=\ldots\;,\;\ldots\}\quad; \tag{2}$$

b. The variational basis

$$\Psi^{J^\pi m_j}_{\text{LIT}}=\sum_n u_n\;\phi_n^{J^\pi m_j}\quad. \tag{3}$$

with

$$\phi_n^{J^\pi m_j}\in\left\{\;\left[\xi_{S_n}\otimes\mathcal{Y}_{l_n}(\boldsymbol{\rho})\right]^{Jm_j}\;e^{-\gamma_n\boldsymbol{\rho}^2}\;\middle|\;\gamma\in\mathbb{R}_+\;,\;s\in\mathbb{N}^{A-1}+\left(\frac{\mathbb{N}}{2}\right)^{A-2}\;,\;l\in\mathbb{N}^{A-1}+\mathbb{N}^{A-2}\right\} \tag{4}$$

c. The matrix form of the "LIT equation"

$$\sum_{s=1}^{N_{\text{LIT}}}\phi_r^{J^\pi m_j}\left(\hat{H}_{\text{nuclear}}-\mathcal{E}\right)\phi_s^{J^\pi m_j}\;u_s=\sum_{n=1}^{N_0}\sum_{m_L}c_n\;\underbrace{\langle\,LJ_0\,;\,m_Lm_j-m_L\,|\;\;Jm_j\,\rangle}_{\text{Eq.(17)}}\;\phi_r^{J^\pi m_j}\hat{\mathcal{O}}_{Lm_L}\,\phi_n^{J_0^{\pi 0}(m_j-m_L)}\quad. \tag{5}$$

with

$$N_{\text{LIT}}\;\;:\text{number of basis states used to expand the LIT state, }e.g.\;,\Psi^{2^-}_{\text{LIT}}\quad; \tag{6}$$

$$N_0\leq N_{\text{LIT}}\;\;:\text{number of basis states used to expand the target, }e.g.\;,\text{the deuteron}; \tag{7}$$

$$\tag{8}$$

d. The matrix element Reduced matrix elements are used where possible in order to reduce the number of numerical evaluations. The composite structure of the operators (spatial and spin form a scalar) demands a more general version of the Wigner-Eckart theorem (*cf.* [1], Eq.(7.1.5)).

$$\phi_m^{J^\pi m_j}\;\hat{\mathcal{O}}_{Lm_L}\;\phi_n^{J_0^{\pi 0}m_{j0}}:=\langle\,m;l_lS_lJ_lm_{j_l}\;|\;\mathcal{A}\;\mathcal{O}_{Lm_L}\;|\;l_rS_rJ_rm_{j_r};n\,\rangle \tag{9}$$

$$=\underbrace{(-1)^{L-J_r+J_l}\frac{\langle\,LJ_r\,;\,m_Lm_{j_r}\,|\;\;J_lm_{j_l}\,\rangle}{\hat{J}_l}}_{\texttt{enemb:600ff}}\underbrace{\langle\,m;l_lS_lJ_l\,||\;\mathcal{A}\;\mathcal{O}_L\;||\,l_rS_rJ_r;n\,\rangle}_{\downarrow} \tag{10}$$

$$\underbrace{\hat{J}_r\hat{J}_l\hat{L}\begin{Bmatrix}l_l^m & l_r^n & p\\ S_l^m & S_r^n & q\\ J_l & J_r & L\end{Bmatrix}}_{\texttt{enemb:ecce}}\sum_{\text{dc}}\sum_{\mathfrak{p}\in\text{dc}}\underbrace{\langle\,m;l_l^m\,||\;\mathcal{O}_p^o\;||\,\mathcal{A}_{\text{dc}}l_r^n;n\,\rangle}_{\texttt{luise}}\cdot\underbrace{\langle\,m;S_l^m\,||\;\mathcal{O}_q^s\;||\,\mathcal{A}_{\mathfrak{p}}S_r^n;n\,\rangle}_{\texttt{obem}} \tag{11}$$

$$=\hat{J}_r\hat{L}\,\langle\,J_rL\,;\,m_{j_r}m_L\,|\;\;J_lm_{j_l}\,\rangle\cdot\begin{Bmatrix}l_l^m & l_r^n & p\\ S_l^m & S_r^n & q\\ J_l & J_r & L\end{Bmatrix} \tag{12}$$

$$\cdot\sum_{\text{dc}}\sum_{\mathfrak{p}\in\text{dc}}\langle\,m;l_l^m\,||\;\mathcal{O}_p^o\;||\,\mathcal{A}_{\text{dc}}l_r^n;n\,\rangle\cdot\langle\,m;S_l^m\,||\;\mathcal{O}_q^s\;||\,\mathcal{A}_{\mathfrak{p}}S_r^n;n\,\rangle \tag{13}$$

with

$$\hat{a}:=\sqrt{2a+1}\quad; \tag{14}$$

$$\mathcal{A}=\sum_{\mathfrak{p}\in\mathcal{S}_{A-1}}(-1)^{\text{sgn}(\mathfrak{p})}\hat{\mathfrak{p}}=\oplus_{\text{dc}} \tag{15}$$

$$\text{dc}\;\;:\;\;\text{double co-set} \tag{16}$$

e. *The calculation*

(i) Solve

$$\hat{H}_{\text{nuclear}}\ \Psi^{J_0^{\pi 0}} = E_0\ \Psi^{J_0^{\pi 0}}$$

with the ansatz

$$\Psi^{J_0^{\pi 0}} = \sum_n c_n\ \phi_n^{J_0^{\pi 0}} \quad .$$

If $\hat{H}_{\text{nuclear}}$ is a spherical rank-0 operator – a condition which most practical nuclear potentials satisfy – $\Psi^{J_0^{\pi 0}} \neq f(m_{j_0})$. We obtain $\Psi^{J_0 J_0}$, in practice.

(ii) Calculate

$$\mathbb{H}_{rs} := \left\langle\ \phi_r^{J^\pi}\ \middle|\ \hat{H}_{\text{nuclear}}\ \middle|\ \phi_s^{J^\pi}\ \right\rangle \ \text{ and }\ \mathbb{N}_{rs} := \left\langle\ \phi_r^{J^\pi}\ \middle|\ \phi_s^{J^\pi}\ \right\rangle \quad \forall\ |L - J_0| \leq J \leq |L + J_0|$$

(iii) $\forall m_j \,\&\, m_L$, calculate

$$S^{Jm_jJ_0}_{rs,m_L} := \left\langle\ \phi_r^{J^\pi m_j}\ \middle|\ \hat{\mathcal{O}}_{Lm_L}\ \middle|\ \phi_s^{J_0^{\pi 0}\,J_0}\ \right\rangle \quad ,$$

and superimpose these matrix elements according to Eq.(5)

$$S_r^{Jm_j} := \sum_{m_L}\ \langle\ LJ_0\,;\, m_L m_j{-}m_L\ |\ \ Jm_j\,\rangle\ \underbrace{\sum_n^{N_0}\ c_n\ S^{Jm_jJ_0}_{rn,m_L}}_{\textbf{enemb.f OUT}} \quad . \tag{17}$$

Ecce, $\Psi^{J_0^{\pi 0}} \neq f(m_{j_0})$ does not allow for an elimination of m_j from this equation!

(iv) Solve the (complex) linear matrix equation

$$\left(\mathbb{H}_{rs} - \mathcal{E}\mathbb{N}_{rs}\right) u_s^{Jm_j} = S_r^{Jm_j} \tag{18}$$

to obtain the LIT state

$$\psi^{v(\text{ertex}),(\text{mu})L(\text{tipolarity})}_{J_{i(\text{nitial})/f(\text{inal})};J_{(\text{i})n(\text{termediate})}m_n}(k,\sigma) = \psi^{v,L}_{J_0;Jm_j}(k,\sigma) := \Psi^{J^\pi m_j}_{\text{LIT}}\big(\ \underbrace{|\boldsymbol{k}|\,,v,L}_{\substack{\text{vertex}\\ \text{quantum numbers}}}\ ;\ \underbrace{E_0,J_0}_{\substack{\text{initial/final-state}\\ \text{quantum numbers}}}\ ;\mathcal{R}\left[\sigma\right],\mathcal{I}\left[\sigma\right]\big) \quad . \tag{19}$$

(v) Units exemplified for the $E1$ calculation:

The units of the LIT (ψ) and the ground state (ϕ) are not identical, in general:

$$\hat{\mathcal{O}}_{\text{nucl}}(\text{MeV})\psi(x) = \hat{\mathcal{O}}_{\text{E1}}(0)\phi(\text{MeV}^{3/2}) \tag{20}$$

$E1$ is implemented as j_1Y_{1M} without any dimensionful prefactors, and hence

$$x = \text{MeV}^{1/2} \quad \Rightarrow \quad [L] = \text{MeV}^{-2} \tag{21}$$

To recover the physical $E1$, a factor of

$$\frac{2}{\hbar k} \tag{22}$$

multiplies the RHS.

$$\tag{23}$$

(vi) The inner product

$$\mathcal{L}^{J_f,J_i;J}_{v'L',vL}(k',k,\sigma) =(-1)^{J-J_i+L-L'+v'} N_{J,\sigma} \sum_{m_j} \underbrace{\left\langle\, \psi^{v',L'}_{J_f;Jm_j}(k',\sigma) \,\middle|\, \psi^{v,L}_{J_i;Jm_j}(k,\sigma) \,\right\rangle}_{=\sum_{r,s}(u_r^{Jm_j})^* u_s^{Jm_j}\ \mathbb{N}_{rs}} \tag{24}$$

$$= \int_{e_{\text{th}}}^{\infty} \frac{\mathcal{F}^{J_f,J_i;J}_{v'L',vL}(k',k,E)}{(E-\sigma)(E-\sigma^*)}\, dE \tag{25}$$

with $N_{J,\sigma}$ being the multiplicity of Lorentz states for given J and σ.

$$\tag{26}$$

(vii) The recovery of the (partial) strength functions – The inverse Lorentz-integral transformation

At this stage, the problem is to recover $\mathcal{F}$, given $\mathcal{L}$ by inverting the integral transformation Eq. (25). The *discrete essence* of that can be written as

$$\boldsymbol{L}_s = \sum_e A_{se} \boldsymbol{F}^e = \sum_n c_n \sum_e A_{se} f_n^e \quad . \tag{27}$$

If the energy sum/integral can be evaluated efficiently for a set of basis vectors f_n which is suitable for a proper representation of the vector $\boldsymbol{L}$, it remains to solve a linear optimization problem. This is detailed below from Eq. (32) onwards.

The choice for the basis f_n is based on two ideas. Firstly, from the definition

$$\mathcal{F}^{J_f,J_i;J}_{v'L',vL}(k',k,E) = \underbrace{N_{J,E}}_{\#J,E-\text{states}} \quad \langle\, J_f\ E_f \,||\, M^{\nu',L'}(k') \,||\, J\ E \,\rangle\, \langle\, J\ E \,||\, M^{\nu,L}(k) \,||\, J_i\ E_i \,\rangle \quad , \tag{28}$$

the joint probability to, first, induce via $M^{\nu,L}(k)$ the transition from one eigenstate of a system to another, which, second is perturbed via $M^{\nu',L'}(k')$ into some final eigenstate. Let me imagine a system with one localized bound state. Its wave function is folded with the perturbation, which is a polynomial in k of order L – the multipolarity – and the wave function of another eigenstate of the system, constrained by energy conservation. I think of k large enough such that the latter state contains a free wave on some coordinate. Hence the matrix element retains a Fourier-transform character of a localized wave packet multiplied by a polynomial. The latter can be expanded well in symmetrical, peaked functions – *e.g.* , Gaussians – and the polynomial skews those. A Fourier transform of such a skewed Gaussian will retain its shape but become broader/narrower. A skewed Gaussian is expected to be amenable to an expansion in Lorentz functions.

The second, related idea for choosing a Lorentz basis is the ability to calculate the integral transform with a Lorentz kernel analytically (viz. Eq. (34)).

I. FORMULAS AND CONSTANTS

$$\text{(Wigner) 3-}j\text{ symbol:} \qquad \begin{pmatrix} L & S & J \\ m_l & m_s & -m_j \end{pmatrix} = (-1)^{L-S+m_j}\ (2J+1)^{-\frac{1}{2}}\ \langle\, LS\,;\, m_l m_s \,|\ \ J m_j \,\rangle \tag{29}$$

$$\begin{aligned} \text{Matrix for single-axis rotation:} \qquad \mathscr{D}^{(j)}_{m',m}(0\ \beta\ 0) &\equiv d^{(j)}_{m',m}(\beta) \\ &= \left[\frac{(j+m')!(j-m')!}{(j+m)!(j-m)!}\right]^{\frac{1}{2}} \\ &\quad \cdot \sum_\sigma \begin{pmatrix} j+m \\ j-m'-\sigma \end{pmatrix} \begin{pmatrix} j-m \\ \sigma \end{pmatrix} (-1)^{j-m'-\sigma} \\ &\quad \cdot \left(\cos\frac{\beta}{2}\right)^{2\sigma+m+m'} \left(\sin\frac{\beta}{2}\right)^{2j-2\sigma-m-m'} \end{aligned} \tag{30}$$

$$\tag{31}$$

a. The Lorentz transformation of a Lorentzian

The strength functions which constitute the Compton amplitudes are themselves composed of scalar functions of an energy parameter. I assume that these functions can be expanded to any desired accuracy in a Lorentzian/-Cauchy basis as follows:

$$r(e) = \sum_n c_n \frac{\theta(e-e_{\rm th})}{(a_n-e)^2+b_n^2} := \sum_n c_n \theta(e-e_{\rm th}) f_n\ . \tag{32}$$

As the process is non-trivial only if the photon's energy exceeds a minimum threshold energy $e_{\rm th}$, it is in order to include a system-characteristic step function.

In practice, we obtain an integral transformation of this quantity with a Lorentzian kernel,

$$L(\sigma) := \int_{-\infty}^{\infty} \frac{r(e)}{(\sigma_r-e)^2+\sigma_i^2}\ de = \sum_n c_n \int_{e_{\rm th}}^{\infty} \frac{f_n(e)}{(\sigma_r-e)^2+\sigma_i^2}\ de\ . \tag{33}$$

If basis basis functions[1] which we define through the integral

$$\begin{aligned} \int_{e_{\rm th}}^{\infty} \frac{f_n(e)}{(\sigma_r-e)^2+\sigma_i^2}\ de\ &=\ \int_{e_{\rm th}}^{\infty} \frac{1}{(\sigma_r-e)^2+\sigma_i^2} \cdot \frac{1}{(a_n-e)^2+b_n^2}\ de \\ &\ \ \vdots \\ &=\ \left[(\sigma_r-a_n)^2+(\sigma_i-b_n)^2\right]^{-1} \cdot \left[(\sigma_r-a_n)^2+(\sigma_i+b_n)^2\right]^{-1} \\ &\quad \cdot \Bigg\{ \sigma_i^{-1}\left((\sigma_r-a_n)^2+b_n^2-\sigma_i^2\right)\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{\sigma_r'}{\sigma_i}\right)\right) \\ &\quad + b_n^{-1}\left((\sigma_r-a_n)^2-b_n^2+\sigma_i^2\right)\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{a_n'}{b_n}\right)\right) \\ &\quad + (\sigma_r-a_n)\ \ln\left(\frac{\sigma_r'^2+\sigma_i^2}{a_n'^2+b_n^2}\right) \Bigg\} \\ &:= L_n(\sigma, e_{\rm th}) \end{aligned} \tag{34}$$

---

[1] $x' := x - e_{\rm th}$

$\alpha = \frac{e^2}{4\pi}$	dimensionless	$\frac{1}{137.03604}$
$\hbar c$		$197.32858\ \mathrm{MeV} \cdot \mathrm{fm}^2$

(36)

TABLE I. Implemented numerical values.

allow for an accurate expansion of a given, physical function $L(\sigma)$, namely for a set $\mathfrak{B} = \{(a_n, b_n) \in \mathbb{R}_{0+}\}_{n=1,\dots,d}$, we seek

$$\min_{\boldsymbol{c}} \left| L(\sigma) - \sum_n c_n L_n(\sigma, e_{\mathrm{th}}) \right| := \boldsymbol{c}^*. \tag{35}$$

The set of optimal parameters $\boldsymbol{c}^*$ represents via Eq. (32) an expansion of the response function in Lorentzians. If $\mathfrak{B}$ is "numerically" complete, the dependence of $r(e)$ on changes in this basis should be negligible, *i.e.* , the problem is **not** ill-posed. This shall now be demonstrated for an exemplary process.

II. "HOW TO DO AN INTEGRAL"

a. The multipole operators

- convection current[①]

$$\boldsymbol{j}_o(\boldsymbol{x}) = \frac{e}{2m} \sum_i^A \frac{1}{2}\left(1 + \tau_z(i)\right) \left\{ \boldsymbol{p}(i)\ ,\ \delta^{(3)}(\boldsymbol{x} - \boldsymbol{r}(i)) \right\} \tag{37a}$$

$$\textbf{(electric)}\ \mathcal{O}^{o^{\mathrm{el}}}_{Lm_L} = \frac{e\hbar}{mc} \hat{L}^{-1} \sum_i^A g_l(i) \left[\sqrt{L}\Delta^{L+1}_{LM}(\boldsymbol{r}(i)) - \sqrt{L+1}\Delta^{L-1}_{LM}(\boldsymbol{r}(i)) \right] \tag{37b}$$

$$\textbf{(magnetic)}\ \mathcal{O}^{o^{\mathrm{mag}}}_{Lm_L} = i\frac{e\hbar}{mc} \sum_i^A g_l(i) \Delta^L_{LM}(\boldsymbol{r}(i)) \tag{37c}$$

- spin current

$$\boldsymbol{j}_s(\boldsymbol{x}) = \frac{e\hbar}{2m} \sum_i^A \frac{1}{2} \left(g_{s_p}\left(1 + \tau_z(i)\right) + g_{s_n}\left(1 - \tau_z(i)\right) \right) \boldsymbol{\sigma}(i) \times \boldsymbol{\nabla}(i) \delta^{(3)}(\boldsymbol{x} - \boldsymbol{r}(i)) \tag{38a}$$

$$\textbf{(electric)}\ \mathcal{O}^{s^{\mathrm{el}}}_{Lm_L} = -\frac{e\hbar|\boldsymbol{k}|}{2mc} \sum_i^A g_s(i) \sum_{M,\nu} \langle\, L1\,;\ M\nu \,|\ \ Lm_L \,\rangle \cdot \boldsymbol{\sigma}_\nu(i) \Phi_{LM}(\boldsymbol{r}(i)) \tag{38b}$$

$$\textbf{(magnetic)}\ \mathcal{O}^{s^{\mathrm{mag}}}_{Lm_L} = i\frac{e\hbar|\boldsymbol{k}|}{2mc} \hat{L}^{-1} \sum_i^A g_s(i) \sum_{M,\nu} \Big[\sqrt{L}\, \langle\, L+11\,;\ M\nu \,|\ \ Lm_L \,\rangle\, \boldsymbol{\sigma}_\nu(i) \Phi_{L+1M}(\boldsymbol{r}(i)) \tag{38c}$$

$$-\sqrt{L+1}\, \langle\, L-11\,;\ M\nu \,|\ \ Lm_L \,\rangle\, \boldsymbol{\sigma}_\nu(i) \Phi_{L-1M}(\boldsymbol{r}(i)) \Big] \tag{38d}$$

[①]Indices referring to particles are put in brackets.

with

$$\Phi_{Lm_L}(\boldsymbol{r}) = j_L(kr)\; Y_{Lm_L}(\Omega_r) \qquad ① \tag{39}$$

$$\Delta^J_{Lm_L}(\boldsymbol{r}) = \sum_{M,\nu} \langle\, L1\,;\, M\nu \,|\;\; Jm_L \,\rangle\, \Phi_{Lm_L}(\boldsymbol{r})\; \boldsymbol{p}_\nu \quad . \qquad ② \tag{40}$$

42 *b. Siegert form* Linear-in-α coupling of a (background) photon (momentum $\boldsymbol{k}$) to a system (j^ν) at $\boldsymbol{x}$:

$$\left\langle\, f \,\middle|\, -\frac{1}{c}\int d\boldsymbol{x} A_\nu(\boldsymbol{x}) j^\nu(\boldsymbol{x}) \,\middle|\, i \,\right\rangle \xrightarrow{\text{transverse}} \left\langle\, f \,\middle|\, -\sum_{LM} i^L \hat{L}\sqrt{2\pi}\left(\mathbb{O}^{\text{el}}_{Lm_L} + \mu \mathbb{O}^{\text{mag}}_{Lm_L}\right) \,\middle|\, i \,\right\rangle \tag{41}$$

$$\xrightarrow{\text{el. only}} \quad -\sum_{LM} i^L \hat{L}\sqrt{2\pi} \left\langle\, f \,\middle|\, \left(\frac{1}{ck}\int d\boldsymbol{x}\; \boldsymbol{j}(\boldsymbol{x}) \cdot \boldsymbol{\nabla}_x \times \boldsymbol{L}\left[j_L(kx) Y_{LM}(\Omega_x)\right]\right) \,\middle|\, i \,\right\rangle \tag{42}$$

$$\xrightarrow{k\to 0} \quad -\sum_{LM} \frac{i^{L+1}\hat{L}\sqrt{2\pi}}{k} \cdot \frac{L+1}{L} \left\langle\, f \,\middle|\, \left(\int d\boldsymbol{x}\; \boldsymbol{j}(\boldsymbol{x}) \cdot \boldsymbol{\nabla}_x \left[j_L(kx) Y_{LM}(\Omega_x)\right]\right) \,\middle|\, i \,\right\rangle \tag{43}$$

$$= \quad \sum_{LM} i^L \cdot \frac{\hat{L}\sqrt{2\pi}}{\hbar k} \cdot \frac{L+1}{L} \left\langle\, f \,\middle|\, \int d\boldsymbol{x}\; \left[\rho(\boldsymbol{x})\,,\, \hat{H}_{\text{nuclear}}\right]\; j_L(kx) Y_{LM}(\Omega_x) \,\middle|\, i \,\right\rangle \tag{44}$$

$$\underset{\rho=\rho^{(1)}}{\overset{L=1}{=}} \quad \frac{2}{\hbar k} \left\langle\, f \,\middle|\, \sum_i^A \; q(i)\; \left[j_1(kr(i)) Y_{1M}(\Omega_{\boldsymbol{r}(i)})\,,\, \hat{H}_{\text{nuclear}}\right] \,\middle|\, i \,\right\rangle \tag{45}$$

with

$$\boldsymbol{L} = -\, i\hbar\, (\boldsymbol{x} \times \boldsymbol{\nabla}_x) \tag{46}$$

$$\rho^{(1)}(\boldsymbol{x}) = \sum_i^A \underbrace{\frac{e}{2}(1 + \tau_z(i))}_{:=q(i)} \delta^{(3)}(\boldsymbol{x} - \boldsymbol{r}_i) \tag{47}$$

$$\lim_{x\to 0} j_l(x) = \frac{x^l}{(2l+1)!!} \quad . \tag{48}$$

43 We abstain

c. The non-trivial matrix element (which serves **2-body** *currents, too)*

$$\left\langle\, m; l_l m_{l_l} \,\middle|\, \Phi_{Lm_L}(\boldsymbol{\rho_\nu})\; e^{-\beta \boldsymbol{r}_{ij}} \prod_N^{N_{\text{op}}} \mathcal{Y}_{L_N M_N}(\boldsymbol{r}_{ij}) \,\middle|\, l_r m_{l_r}; n \,\right\rangle \tag{49}$$

---

②rank-L spherical $\boldsymbol{L}^2$ tensor

④linear combination of spherical $\boldsymbol{L}^2$ rank-L tensors, hence, itself a rank-L spherical $\boldsymbol{L}^2$ tensor and not rank-J spherical $\boldsymbol{L}^2$.

III. OBTAINING A CROSS SECTION

If one is interested in a quantification of the reaction of a nucleus being irradiated with an electromagnetic wave, the following formulae might help.

For the total cross section for the reaction of a system initially in state α induced by a perturbation which transforms α into a set of final states (labeled α') Taylor (*cf.* [2] Eq.(17.17)) writes

$$\begin{aligned} \sigma(\star \leftarrow \alpha) &= \sum_{\alpha'} \sigma(\alpha' \leftarrow \alpha) \\ &= (2\pi)^4 \, \frac{m}{k} \, \sum_{\alpha'} \int d\boldsymbol{k}' \, \delta(E' - E) \left| t(\boldsymbol{k}', \alpha' \leftarrow \boldsymbol{k}, \alpha) \right|^2 \quad . \end{aligned} \tag{50}$$

The perturbation is implicit in the transition matrix t. For an electromagnetic current,

$$\tag{51}$$

IV. NOTES ON THE RRGM IMPLEMENTATION

The expansion of the 3-helium ground state

We approximate the antisymmetric nuclear three-body state with the expansion

$$\begin{aligned} \left\langle \boldsymbol{\rho}_1, \boldsymbol{\rho}_2 \,\middle|\, {}^3\mathrm{He} \right\rangle &= \sum_i \; c_i \, e^{-\gamma_i \boldsymbol{\rho}_1^2} \, e^{-\delta_i \boldsymbol{\rho}_2^2} \cdot \left[\left[\mathcal{Y}_{l_{1,i}}(\boldsymbol{\rho}_1) \otimes \mathcal{Y}_{l_{2,i}}(\boldsymbol{\rho}_2) \right]^{L_i} \otimes \left[[\xi_1 \otimes \xi_2]^{\mathscr{S}_i} \otimes \xi_3 \right]^{S_i} \right]^J \\ &\equiv \sum_i \; c_i \; \phi_i(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \;\; . \end{aligned} \tag{52}$$

A single basis vector is parametrized by two width parameters γ and δ, and a (spin) angular-momentum coupling scheme which specifies two angular-momentum quanta $l_{1,2}$ (one for each Jacobi coordinate $\boldsymbol{\rho}_{1,2}$), a total orbital angular momentum L, an intermediate spin coupling $\mathscr{S}$, and the total spin S.

In practise, we proceed as follows:

1. Fix an orbital-angular-momentum cutoff $l_{\max}$ and add **all** spin- and orbital coupling schemes to the basis with $l_{1,2} \leq l_{\max}$ and intermediate/total angular momenta which contribute to a total J and parity.

   The structure $\left[\left[\mathcal{Y}_1(\boldsymbol{\rho}_1) \otimes \mathcal{Y}_1(\boldsymbol{\rho}_2) \right]^2 \otimes \left[\left[\xi_1 \otimes \xi_2 \right]^1 \otimes \xi_3 \right]^{3/2} \right]^{1/2^+}$, for example, would be included if $l_{\max} = 2$ and combines two negative-parity solid harmonics to form a of positive parity.

2. For each of the above blocks, two sets of width parameters are chosen. At present, this means selecting upper and lower bounds $\gamma/\delta_{\min,\max}$ of geometric grids with a fixed number of nodes n.

   For a particular coupling block, one uses $\gamma \in \{\gamma_{\min}, \gamma_{\max}\}_{\mathrm{geom}}^{n(l_{1,2,i}, L_i, \mathscr{S}_i, S_i)} \equiv w_\gamma$ and $\delta \in \{\delta_{\min}, \delta_{\max}\}_{\mathrm{geom}}^{m(l_{1,2,i}, L_i, \mathscr{S}_i, S_i)} \equiv w_\delta$. Hence, for each coupling block there are $m \cdot n$ ("in principle") independent Gaussian prefactors. To select those combinations which form a numerically stable and yet complete-for-its-purpose set is an unsolved problem, if the state should be fed into a LIT calculation.

genetic basis selection

initial basis – *the first generation of parents* **:**

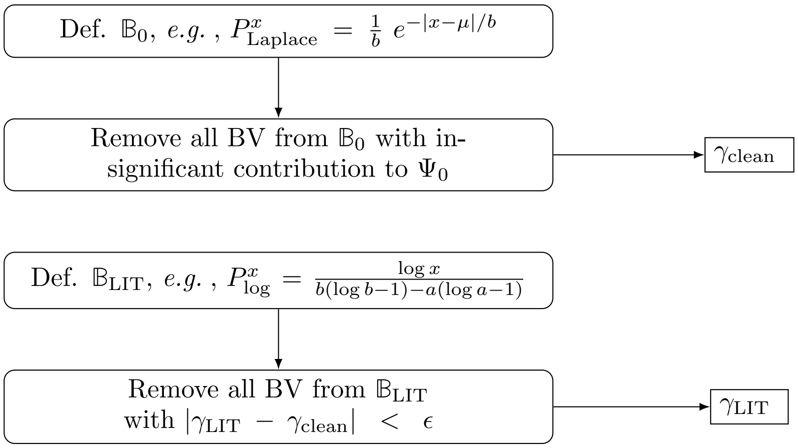
A single basis vector is parametrized by a particular orbital- and (iso)spin angular-momentum coupling scheme, $\mathtt{cfg}_j := \{l_{1,2}^{(j)}, L^{(j)}, \mathscr{s}^{(j)}, S^{(j)}, \mathscr{l}^{(j)}, T^{(j)}\}$, **and** a pair of width parameters, $\mathtt{bv}_i := \{\gamma^{(i)}, \delta^{(i)}\}$.

We obtain an initial set of vectors by defining **upper** and **lower** bounds within which logarithmically spaced values are selected **for each** `cfg`. As

$$\langle\, \mathtt{cfg}_n \mathtt{bv}_i \,|\, \mathtt{cfg}_m \mathtt{bv}_i \,\rangle \propto \delta_{mn} \qquad ,$$

i.e. , choosing a basis which contains elements which differ solely in their discrete quantum numbers but have **identical** widths, appears not to introduce linearly dependent vectors, the (anti)symmetrization of the basis,

$$\mathbb{1} := \left\langle\, \mathtt{cfg}_n \mathtt{bv}_i \,\middle|\, \hat{\mathcal{A}} \,\middle|\, \mathtt{cfg}_m \mathtt{bv}_j \,\right\rangle \qquad ,$$



luise.f → qual.f

P_{dc} and width-independent quantities of $\Gamma_{l_1m_1,\ldots,l_zm_z}$ with $z = n_{c_l} - 1 + n_{c_r} - 1 + n_{ww}$.

obem.f → qual.f

qual.f → enemb.f

```
 WRITE(NBAND1) NZF,MUL,(LREG(K),K=1,NZOPER),I,(NZRHO(K),K=1,NZF)

 WRITE(NBAND1) N3,MMASSE(1,N1,K),MMASSE(2,N1,K),MLAD(1,N1,K),
1 MLAD(2,N1,K),MSS(1,N1,K),MSS(2,N1,K),MS(N1,K),
2 (LZWERT(L,N2,K),L=1,5),(RPAR(L,K),L=1,N3),KP(MC1,N4,K)

 WRITE(NBAND1) NTE,NC,ND,ITV2

 WRITE(NBAND1) ((IND(MM,NN), NN=1, JRHO), MM=1, IRHO)

 WRITE(NBAND1) NUML, NUMR, IK1H, JK1H, LL1,
*                 ((F(K,L),(J-1,DM(K,L,J),J=1,LL1),L=1,JK1),
*                                                          K=1,IK1)
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[1] A. R. EDMONDS, *Angular Momentum in Quantum Mechanics* (Princeton University Press, 1985).
[2] J. Taylor, *Scattering Theory: The Quantum Theory of Nonrelativistic Collisions*, Dover Books on Engineering (Dover Publications, 2006).
[3] V. D. Efros, W. Leidemann, G. Orlandini, and N. Barnea, Journal of Physics G: Nuclear and Particle Physics **34**, R459 (2007).
[4] G. Bampa, W. Leidemann, and H. Arenhovel, Phys. Rev. **C84**, 034005 (2011), arXiv:1107.2320 [nucl-th].
[5] R. P. Hildebrandt, *Elastic Compton Scattering from the Nucleon and Deuteron*, Ph.D. thesis, Munich, Tech. U. (2005), arXiv:nucl-th/0512064 [nucl-th].
[6] J. Golak, R. Skibiński, W. Glöckle, H. Kamada, A. Nogga, H. Witała, V. Efros, W. Leidemann, G. Orlandini, and E. Tomusiak, Nuclear Physics A **707**, 365 (2002).
[7] B. F. Gibson and D. R. Lehman, Phys. Rev. C **13**, 477 (1976).
[8] T. Mertelmeier and H. Hofmann, Nuclear Physics A **459**, 387 (1986).

[9] I. M. Barbour and A. C. Phillips, Phys. Rev. C **1**, 165 (1970).