Strong magnetic fields and contact interactions in few-fermion systems

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Technical manual detailing the implementation of a variational solution of the non-relativistic few-body problem in an external, *i.e.* , static magnetic field.

a. The symmetric Gauge

$$\boldsymbol{A}_i = \frac{B_0}{2}(-y(i), x(i), 0) \tag{1}$$

b. The Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{\mathtt{N}}\left\{\boldsymbol{\nabla}(i)^2 + i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q(i)B_0}{\hbar}\right)L_z(i) + \left(\frac{\hbar^2}{2m}\right)\left(\frac{q(i)B_0}{\hbar}\right)^2\frac{1}{4}\left(x^2(i)+y^2(i)\right) - g(i)\left(\frac{\hbar^2}{2m}\right)\left(\frac{q(i)B_0}{\hbar}\right)\sigma_z(i)\right\} \tag{2}$$

$$+ \sum_{i<j}^{\mathtt{N}}\left(C_a + C_b\left[\sigma^+(i)\sigma^-(j) + \sigma^-(i)\sigma^+(j) - \sigma^z(i)\sigma^z(j)\right]\right)\ e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2} + \sum_{\text{cyc.}}\sum_{i<j<k} D\cdot e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2}e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(k)]^2} \tag{3}$$

c. The variational basis

$$\mid A,\alpha\,\rangle := e^{-\frac{1}{2}\boldsymbol{x}^T A_x \boldsymbol{x}}\ e^{-\frac{1}{2}\boldsymbol{y}^T A_y \boldsymbol{y}}\ e^{-\frac{1}{2}\boldsymbol{z}^T A_z \boldsymbol{z}}\cdot\sum_{n=1}^{\mathtt{ncmp}} C_\alpha^n \mid s_1^n,\ldots,s_{\mathtt{N}}^n\ ;\ t_1^n,\ldots,t_{\mathtt{N}}^n\ \rangle \tag{4}$$

with

$$\alpha = 1,\ \ldots,\ \mathtt{nts\_states}$$

d. The generic matrix element

$$I_{\mathcal{O}}(A',\alpha',A,\alpha;P) := \left\langle\ A',\alpha'\ \middle|\ \hat{\mathcal{O}}\otimes\hat{\mathcal{G}}\ \middle|\ \hat{P}(A),\hat{P}(\alpha)\ \right\rangle = \left\langle\ A'\ \middle|\ \hat{\mathcal{O}}\ \middle|\ \hat{P}(A)\ \right\rangle\cdot\left\langle\ \alpha'\ \middle|\ \hat{\mathcal{G}}\ \middle|\ \hat{P}(\alpha)\ \right\rangle \tag{5}$$

with $\hat{P}\in\mathcal{A}$, hence,

$$\hat{P}(A) = T_P^{\intercal}\,A\,T_P := A^P \quad . \tag{6}$$

$$\hat{\mathcal{O}} \in \left\{\mathbb{1}\ ;\ -\sum_{i=1}^{\mathtt{N}}\prod_{c=x,y,z}\boldsymbol{\partial}_c^{\intercal}(i)\mathbb{1}\boldsymbol{\partial}_c(i)\ ;\ \sum_{i=1}^{\mathtt{N}}q(i)L^z(i)\ ;\ \sum_{i=1}^{\mathtt{N}}q(i)(\omega_x x^2(i)+\omega_y y^2(i)+\omega_z z^2(i))\ ;\ \sum_{i=1}^{\mathtt{N}}q(i)\sigma^z(i)\ ;\ \sum_{i<j}^{\mathtt{N}}e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2}\right\} \tag{7}$$

e. The matrix elements

$\hat{\mathcal{O}}$	$\left\langle A' \,\middle|\, \hat{\mathcal{O}} \,\middle|\, \hat{P}(A) \right\rangle$	$\left\langle \alpha' \,\middle|\, \hat{\mathcal{G}} \,\middle|\, \hat{P}(\alpha) \right\rangle$
$\mathbb{1} := \mathbb{1}_{\boldsymbol{r}}^P \cdot \mathbb{1}_{\boldsymbol{s}}^P$	$\left(\frac{(2\pi)^{3\mathtt{N}}}{\det \mathbb{A}_x \det \mathbb{A}_y \det \mathbb{A}_z}\right)^{\frac{1}{2}}$	$\sum_{n,n'}^{\mathtt{ncmp}(\alpha),\mathtt{ncmp}(\alpha')} C_\alpha^n C_{\alpha'}^{n'} \left\langle \boldsymbol{s}^{n'}; \boldsymbol{t}^{n'} \,\middle|\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$
$-\sum_{i=1}^{\mathtt{N}} \prod_{c=x,y,z} \boldsymbol{\partial}_c^{\intercal}(i) \mathbb{1} \boldsymbol{\partial}_c(i) \cdot \mathbb{1}_{\boldsymbol{s}}^P$	$\mathbb{1}_{\boldsymbol{r}}^P \cdot \prod_{c=x,y,z} (A_c)_{im} (\mathbb{A}_c^{-1})_{mn} (A_c^P)_{ni}$	$\mathbb{1}_{\boldsymbol{s}}^P$
$\sum_{i=1}^{\mathtt{N}} q(i) L_z(i) = q(i) \left[x(i)\partial_y(i) - y(i)\partial_x(i)\right]$	0	$\mathbb{Q}_{\boldsymbol{s}}^P := \sum_{i=1}^{\mathtt{N}} \sum_{n,n'}^{\mathtt{ncmp}(\alpha),\mathtt{ncmp}(\alpha')} \left[2 - \boldsymbol{t}^n(\hat{P}(i))\right] C_\alpha^n C_{\alpha'}^{n'} \left\langle \boldsymbol{s}^{n'}; \boldsymbol{t}^{n'} \,\middle|\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$
$\sum_{i=1}^{\mathtt{N}} q(i)(\omega_x x^2(i) + \omega_y y^2(i) + \omega_z z^2(i))$	$\mathbb{1}_{\boldsymbol{r}}^P \cdot \prod_{c=x,y,z} \omega_c \sum_{i=1}^{\mathtt{N}} (\mathbb{A}_c^{-1})_{ii}$	$\mathbb{Q}_{\boldsymbol{s}}^P$
$\sum_{i=1}^{\mathtt{N}} q(i)\sigma^z(i)$	$\mathbb{1}_{\boldsymbol{r}}^P$	$\sum_{i=1}^{\mathtt{N}} \sum_{n,n'}^{\mathtt{ncmp}(\alpha),\mathtt{ncmp}(\alpha')} \left[2 - \boldsymbol{t}^n(\hat{P}(i))\right] \left[3 - 2\boldsymbol{s}^n(\hat{P}(i))\right] C_\alpha^n C_{\alpha'}^{n'} \left\langle \boldsymbol{s}^{n'}; \boldsymbol{t}^{n'} \,\middle|\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$
$\sum_{i<j}^{\mathtt{N}} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2}$		
$\sum_{\text{cyc.}} \sum_{i<j<k} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(k)]^2}$		

(8)

with

$$\mathbb{A}_x = A'_x + A_x^P \tag{9}$$

$$\frac{\Lambda^2}{4} = \mathtt{apot} \tag{10}$$

Spin-isospin wave function for the deuteron:

$$\left|\ \alpha \hat{=} {}^3S_1 \right\rangle = \Big[\left|\ \mathtt{12} \right\rangle + \left|\ \mathtt{21} \right\rangle\Big] \otimes \Big[\left|\ \mathtt{pn} \right\rangle - \left|\ \mathtt{np} \right\rangle\Big] \tag{11}$$

$$\boldsymbol{s}^n(i) = \begin{cases} 1 \ \Rightarrow \ \sigma^z(i) = \uparrow \\ 2 \ \Rightarrow \ \sigma^z(i) = \downarrow \end{cases} \quad \text{and} \quad \boldsymbol{t}^n(i) = \begin{cases} 1 \ \Rightarrow \ \tau^z(i) = \mathtt{p} \\ 2 \ \Rightarrow \ \tau^z(i) = \mathtt{n} \end{cases} \tag{12}$$

#	01b_#	02b_#
0	$\left\langle\, \boldsymbol{\xi}_1^{n'}, \ldots, \boldsymbol{\xi}_N^{n'} \;\middle|\; \boldsymbol{\xi}_{\mathfrak{P}_1}^{n}, \ldots, \boldsymbol{\xi}_{\mathfrak{P}_N}^{n} \,\right\rangle$	$\left\langle\, \boldsymbol{\xi}_1^{n'}, \ldots, \boldsymbol{\xi}_N^{n'} \;\middle|\; \boldsymbol{\xi}_{\mathfrak{P}_1}^{n}, \ldots, \boldsymbol{\xi}_{\mathfrak{P}_N}^{n} \,\right\rangle$
1	$--$	$\left\langle\, \boldsymbol{\xi}_1^{n'}, \ldots, \boldsymbol{\xi}_i^{n'}, \ldots, \boldsymbol{\xi}_j^{n'}, \ldots, \boldsymbol{\xi}_N^{n'} \;\middle|\; \boldsymbol{\xi}_{\mathfrak{P}_1}^{n}, \ldots, \boldsymbol{\xi}_{\mathfrak{P}_j}^{n}, \ldots, \boldsymbol{\xi}_{\mathfrak{P}_i}^{n}, \ldots, \boldsymbol{\xi}_{\mathfrak{P}_N}^{n} \,\right\rangle$

TABLE I. Two basic operations are performed by the interaction Hamiltinian Eq.(2), namely, the identity, and the exchange.