Strong magnetic fields and contact interactions in few-fermion systems

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Technical manual detailing the implementation of a variational solution of the non-relativistic few-body problem in an external, *i.e.* , static magnetic field.

a. The symmetric Gauge

$$\boldsymbol{A}_i = \frac{B_0}{2}(-y_i, x_i, 0) \tag{1}$$

b. The Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\left\{\boldsymbol{\nabla}_i^2 + i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q_iB_0}{\hbar}\right)L_i^z + \left(\frac{\hbar^2}{2m}\right)\left(\frac{q_iB_0}{\hbar}\right)^2\frac{1}{4}\left(x_i^2+y_i^2\right) - g_i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q_iB_0}{\hbar}\right)\sigma_{z_i}\right\} \tag{2}$$

$$+ \sum_{i<j}^{N}\left[C_a + C_b(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+ - \sigma_i^z\sigma_j^z)\right]\; e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i-\boldsymbol{r}_j)^2} + \sum_{\text{cyc.}}\sum_{i<j<k} D\cdot e^{-\frac{\Lambda^2}{4}\left((\boldsymbol{r}_i-\boldsymbol{r}_j)^2+(\boldsymbol{r}_i-\boldsymbol{r}_k)^2\right)} \tag{3}$$

c. The variational basis

$$\mid A, \boldsymbol{\lambda}, \boldsymbol{\theta}\,\rangle := e^{-\frac{1}{2}\boldsymbol{x}^TA_x\boldsymbol{x}}\; e^{-\frac{1}{2}\boldsymbol{y}^TA_y\boldsymbol{y}}\; e^{-\frac{1}{2}\boldsymbol{z}^TA_z\boldsymbol{z}}\cdot\sum_\alpha\lambda_\alpha\sum_{n=1}^{N_{\text{int}}}C_\alpha^n\mid s_1^n,\ldots,s_N^n\;;\;t_1^n,\ldots,t_N^n\,\rangle \tag{4}$$

d. The generic matrix element

$$I_{\mathcal{O}}(A',\boldsymbol{\lambda}',\boldsymbol{\theta}',A,\boldsymbol{\lambda},\boldsymbol{\theta};P) := \langle\, A',\boldsymbol{\lambda}',\boldsymbol{\theta}' \mid \hat{\mathcal{O}}\;\hat{P}\mid A,\boldsymbol{\lambda},\boldsymbol{\theta}\,\rangle \tag{5}$$

with $\hat{P}\in\mathcal{A}$ and

$$\hat{\mathcal{O}} \in \left\{\mathbb{1}\;;\;\boldsymbol{p}^{\mathsf{T}}\mathbb{1}_{(3N\times3N)}\boldsymbol{p}\;;\;\sum_{i=1}^{N}q_iL_i^z\;;\;\sum_{i=1}^{N}q_i(x_i^2+y_i^2+z_i^2)\;;\;\sum_{i=1}^{N}q_i\sigma_i^z\;;\;\sum_{i<j}^{N}e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i-\boldsymbol{r}_j)^2}\right\} \tag{6}$$

e. The matrix elements

$\hat{\mathcal{O}}$	$I_{\mathcal{O}}(A',\boldsymbol{\lambda}',\boldsymbol{\theta}',A,\boldsymbol{\lambda},\boldsymbol{\theta};P)$
$\mathbb{1}$	
$\boldsymbol{p}^{\mathsf{T}}\mathbb{1}_{(3N\times3N)}\boldsymbol{p}$	
$\sum_{i=1}^{N}q_iL_i^z = q_i\;(x_i\partial_{y_i} - y_i\partial_{x_i})$	0
$\sum_{i=1}^{N}q_i(x_i^2+y_i^2+z_i^2)$	
$\sum_{i=1}^{N}q_i\sigma_i^z$	
$\sum_{i<j}^{N}e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i-\boldsymbol{r}_j)^2}$	

(7)

with

$$\mathbb{A}_x = \qquad A'_x \tag{8}$$