Strong magnetic fields and contact interactions in few-fermion systems

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Technical manual detailing the implementation of a variational solution of the non-relativistic few-body problem in an external, *i.e.* , static magnetic field.

a. The symmetric Gauge

$$\boldsymbol{A}_i = \frac{B_0}{2}(-y_i, x_i, 0) \tag{1}$$

b. The Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\left\{\boldsymbol{\nabla}_i^2 + i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q_iB_0}{\hbar}\right)L_i^z + \left(\frac{\hbar^2}{2m}\right)\left(\frac{q_iB_0}{\hbar}\right)^2\frac{1}{4}\left(x_i^2+y_i^2\right) - g_i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q_iB_0}{\hbar}\right)\sigma_{z_i}\right\} \tag{2}$$

$$+ \sum_{i<j}^{N}\left[C_a + C_b(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+ - \sigma_i^z\sigma_j^z)\right]\; e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i-\boldsymbol{r}_j)^2} + \sum_{\text{cyc.}}\sum_{i<j<k} D\cdot e^{-\frac{\Lambda^2}{4}\left((\boldsymbol{r}_i-\boldsymbol{r}_j)^2+(\boldsymbol{r}_i-\boldsymbol{r}_k)^2\right)} \tag{3}$$

c. The variational basis

$$\mid A,\boldsymbol{\lambda}\,\rangle := e^{-\frac{1}{2}\boldsymbol{x}^TA_x\boldsymbol{x}}\; e^{-\frac{1}{2}\boldsymbol{y}^TA_y\boldsymbol{y}}\; e^{-\frac{1}{2}\boldsymbol{z}^TA_z\boldsymbol{z}}\cdot\sum_{\alpha}^{|\lambda|}\lambda_\alpha\underbrace{\sum_{n=1}^{N_{\text{comp}}}C_\alpha^n\mid s_1^n,\dots,s_N^n\;;\;t_1^n,\dots,t_N^n\,\rangle}_{\text{input together with }|\lambda|} \tag{4}$$

d. The generic matrix element

$$I_{\mathcal{O}}(A',\boldsymbol{\lambda}',A,\boldsymbol{\lambda};P) := \left\langle\, A',\boldsymbol{\lambda}' \;\middle|\; \hat{\mathcal{O}}\otimes\hat{\mathcal{G}} \;\middle|\; \hat{P}(A),\hat{P}(\boldsymbol{\lambda}) \,\right\rangle = \left\langle\, A' \;\middle|\; \hat{\mathcal{O}} \;\middle|\; \hat{P}(A) \,\right\rangle\cdot\left\langle\, \boldsymbol{\lambda}' \;\middle|\; \hat{\mathcal{G}} \;\middle|\; \hat{P}(\boldsymbol{\lambda}) \,\right\rangle \tag{5}$$

with $\hat{P}\in\mathcal{A}$, hence,

$$\hat{P}(A) = T_P^{\intercal}\,A\,T_P := A^P \quad . \tag{6}$$

$$\hat{\mathcal{O}} \in \left\{\mathbb{1}\;;\;\boldsymbol{p}^{\intercal}\mathbb{1}_{(3N\times3N)}\boldsymbol{p}\;;\;\sum_{i=1}^{N}q_iL_i^z\;;\;\sum_{i=1}^{N}q_i(x_i^2+y_i^2+z_i^2)\;;\;\sum_{i=1}^{N}q_i\sigma_i^z\;;\;\sum_{i<j}^{N}e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i-\boldsymbol{r}_j)^2}\right\} \tag{7}$$

e. The matrix elements

$\hat{\mathcal{O}}$	$\left\langle A' \,\middle\vert\, \hat{\mathcal{O}} \,\middle\vert\, \hat{P}(A) \right\rangle$	$\left\langle \boldsymbol{\lambda}' \,\middle\vert\, \hat{\mathcal{G}} \,\middle\vert\, \hat{P}(\boldsymbol{\lambda}) \right\rangle$
$\mathbb{1} := \mathbb{1}^P_{\boldsymbol{r}} \cdot \mathbb{1}^P_{\boldsymbol{s}}$	$\left(\frac{(2\pi)^{3N}}{\det \mathbb{A}_x \det \mathbb{A}_y \det \mathbb{A}_z}\right)^{\frac{1}{2}}$	$\sum_{\alpha,\alpha',n,n'}^{\lvert\lambda\rvert,N_{\text{comp}}} \lambda_\alpha \lambda'_{\alpha'} C^n_\alpha C^{n'}_{\alpha'} \left\langle \boldsymbol{s}^{n'}; \boldsymbol{t}^{n'} \,\middle\vert\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$
$\frac{1}{2}\boldsymbol{p}^\intercal \mathbb{1}_{3N} \boldsymbol{p} = -\frac{\hbar^2}{2} \boldsymbol{\nabla}^\intercal \mathbb{1}_{3N} \boldsymbol{\nabla}$	$\frac{\hbar^2}{2}\ \mathbb{1}^P_{\boldsymbol{r}} \prod_{c=x,y,z} (A_c)_{im} (\mathbb{A}_c^{-1})_{mn} (A^P_c)_{ni}$	$\mathbb{1}^P_{\boldsymbol{s}}$
$\sum_{i=1}^N q_i L^z_i = q_i\ \left(x_i \partial_{y_i} - y_i \partial_{x_i}\right)$	0	$\mathbb{Q}^P_{\boldsymbol{s}} := \sum_{i=1}^N \sum_{\alpha,n}^{\lvert\lambda\rvert,N_{\text{comp}}} \underbrace{(\hat{P}[\boldsymbol{t}^n])_i}_{=q_{P(i)}} \lambda_\alpha C^n_\alpha \left\langle \boldsymbol{s}^n; \boldsymbol{t}^n \,\middle\vert\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$
$\sum_{i=1}^N q_i (\omega_x x_i^2 + \omega_y y_i^2 + \omega_z z_i^2)$	$\mathbb{1}^P_{\boldsymbol{r}} \prod_{c=x,y,z} \omega_c \sum_{i=1}^N (\mathbb{A}_c^{-1})_{ii}$	$\mathbb{Q}^P_{\boldsymbol{s}}$
$\sum_{i=1}^N q_i \sigma^z_i$	$\mathbb{1}^P_{\boldsymbol{r}}$	$\sum_{i=1}^N \sum_{\alpha,n}^{\lvert\lambda\rvert,N_{\text{comp}}} \underbrace{\left(\hat{P}[\boldsymbol{t}^n]\right)_i}_{=q_{P(i)}} \underbrace{\left(\hat{P}[\boldsymbol{s}^n]\right)_i}_{=s^z_{P(i)}} \lambda_\alpha C^n_\alpha \left\langle \boldsymbol{s}^n; \boldsymbol{t}^n \,\middle\vert\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$
$\sum_{i<j}^N e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i - \boldsymbol{r}_j)^2}$		
$\sum_{\text{cyc.}} \sum_{i<j<k} e^{-\frac{\Lambda^2}{4}((\boldsymbol{r}_i - \boldsymbol{r}_j)^2 + (\boldsymbol{r}_i - \boldsymbol{r}_k)^2)}$		

(8)

with

$$\mathbb{A}_x = A'_x + A^P_x \tag{9}$$