Strong magnetic fields and contact interactions in few-fermion systems

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The effect of a magnetic background on few-nucleon systems is analyzed. In particular, the spectra of pure proton systems with up to 5 constituents are calculated as functions of the strength of the magnetic field. From these spectra, we conjecture the critical field strength which stabilizes the general N-proton system. For $N \gtrsim 10$, the magnetic field in the vicinity of a typical neutron star is found sufficient to form a bound structure.

Furthermore, the dependence of the tri-nucleon binding energy splitting is calculated in the presence of the magnetic field. Combining this result with the effect of the background on phase space available to the β-decay products of the triton, we predict its β-stabilization – and thereby the instability of 3-helium – at field strengths of $B = \mathcal{O}(10^{10}\mathrm{T})$.

For particle-stable systems in vacuum, field strengths are calculated which preclude this stability. Qualitatively, this result is found consistent with LQCD simulations, employing an unphysically large pion mass.

All results are based on numerical solutions of the Schrödinger equation with a nuclear-interaction as approximated by the leading order of the EFT($\not\pi$).

I. SUMMARY OF ⟨מותי|יוהנס⟩

1. We realized the need for a complete SVM basis $\phi_{AsB}(\underbrace{x_1,\ldots,x_N}_{:=\boldsymbol{x}}) = e^{-\frac{1}{2}\boldsymbol{x}^T A\boldsymbol{x}+\frac{1}{2}(\boldsymbol{x}-\boldsymbol{s})^T B(\boldsymbol{x}-\boldsymbol{s})}$

II. BENCHMARKING

The Hamiltonian which describes the nuclear few-body problem in leading-order EFT($\not\pi$), in a magnetic background field which interacts via minimal, one-nucleon coupling, only, is given by Eq.(2). The correct implementation of this operator is verified as follows.

: Nuclear two- and three-body interaction We reproduce the deuteron and triton binding energies for sets of pre-calibrated low-energy constants in the absence of the magnetic field. Furthermore, cutoff-independent postdictions for the binding energy of the α particle are compared with those of RGM, MC, and EIHH techniques.

Finally, we detail the evaluation of the matrix elements of a generic pair/triplet of interacting particles via Gaussian-regulated two-/three-body contact interactions between one representative of the SVM basis both, analytically and numerically. This step serves educational and benchmarking purposes.

$$\langle\ A'B's' \mid \hat{V}_{2(3)} \mid A,B,s\ \rangle = \ldots \overset{N=4}{=} \tag{1}$$

: Magnetic xy trap For non-interacting particles, we obtain an energy spectrum whose level splitting matches the Landau spectrum to be expected if all, or a subset of the systems carries charges. For the two-body system, we show explicitly that for $\lim\limits_{q_1/q_2\to 0}$ the single-particle Landau spectrum is attained.

: Angular z momentum (orbit)

: Angular z momentum (spin)

III. THEORETICAL DESCRIPTION OF STRONGLY INTERACTING FERMIONS IN A MAGNETIC BACKGROUND FIELD

In order to describe the effect of a static magnetic field, which is not necessarily perturbatively small, on the low-energy spectra of small nuclei, the effective-field-theory Hamiltonian of the latter is augmented by, first, minimally coupling the derivatives, and second, by the magnetic-moment 1-body interaction (a remnant of the non-relativistic reduction of the Dirac equation) and the coupling of the $\boldsymbol{B}$ field to the four-nucleon leading-order momentum-independent vertex.

As of now, we do consider minimal coupling, only. In addition, we assume the nuclear interaction to be comprised of a two-body contact interaction which yields a shallow spin-1 deuteron. The interaction in the spin-0 (iso-spin-1) channel is also attractive but not strong enough to sustain a bound di-neutron/proton. In order to stabilize the three-nucleon system in the spin-$\frac{1}{2}$ (triton) channel, a three-nucleon contact term must be included. Using the so-called symmetric gauge, $\boldsymbol{A}_i = \frac{B_0}{2}(-y_i, x_i, 0)$, and regulating the contact interactions with Gaussian functions, the dynamics of an A-particle system are dictated by the following Hamilton operator:

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \left\{ \boldsymbol{\nabla}_i^2 + i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q_i B_0}{\hbar}\right)(x_i\partial_{y_i} - y_i\partial_{x_i}) + \left(\frac{\hbar^2}{2m}\right)\left(\frac{q_i B_0}{\hbar}\right)^2 \frac{1}{4}\left(x_i^2 + y_i^2\right) - g_i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q_i B_0}{\hbar}\right)\sigma_{z_i} \right\} \tag{2}$$

$$+ \sum_{i<j}\left[C_1^\Lambda\ \hat{P}(^1S_0) + C_2^\Lambda\ \hat{P}(^3S_1)\right]\ e^{-\frac{\Lambda^2}{4}(\boldsymbol{r}_i - \boldsymbol{r}_j)^2}$$

$$+ \sum_{\substack{i<j<j \\ \text{cyc.}}} D_0^\Lambda\ \hat{P}(S = 1/2)\ e^{-\frac{\Lambda^2}{4}\left((\boldsymbol{r}_i - \boldsymbol{r}_j)^2 + (\boldsymbol{r}_i - \boldsymbol{r}_k)^2\right)} \tag{3}$$

A. Notable features

1. Total Spin in not conserved, which means that if we consider a 2-nucleon system, it resides in a superposition of spin-1 and spin-0 states.
2. Bound states might not have negative eigenvalues because the non-interacting theory still contains the effect of the magnetic field on the charged particles and shifts their ground state by, at least, the energy in the lowest Landau level.
3.

IV. TWO-NUCLEON SYSTEMS

First, we analyse the predictions in Ref. [1], *i.e.* , what is the dependence of the lowest eigenvalues of the 2-proton system on the strength of the magnetic field?

V. THREE NUCLEONS

How does the critical field strength at which the 3-proton system becomes bound compare with the corresponding strength which is necessary to bind two protons?

VI. FOUR NUCLEONS

Same critical-strength ratio? Now we extrapolate and predict that no more than $|\boldsymbol{B}| \sim 4$ T is needed to bind an A-proton cluster.

Appendix: Single-particle stochastic-variational method

We expand an N-body wave function in one Cartesian dimension x in a correlated Gaussian basis with basis vectors

$$\phi_A(\underbrace{x_1, \ldots, x_N}_{:=\boldsymbol{x}}) = e^{\boldsymbol{x}^T A \boldsymbol{x}} \tag{A.1}$$

parametrized by a symmetric $N \times N$ matrix $A = A^T$.

[1] W. Detmold, K. Orginos, A. Parreño, M. J. Savage, B. C. Tiburzi, S. R. Beane, and E. Chang (NPLQCD Collaboration), Phys. Rev. Lett. **116**, 112301 (2016).