

Nucleons in magnetic field with Stochastic Variational Method

Moti Eliyahu

1 LO EFT

We need to solve the equation

$$H\Psi = E\Psi$$

The LO EFT Hamiltonian H , contain magnetic field strength B , point in the z direction, can be written as, see [1].

$$H = -\frac{\hbar^2}{2m_N} \sum_i \nabla_i^2 + \frac{\hbar^2}{2m_N} \left(\frac{eB}{\hbar}\right)^2 \sum_i y_i^2 - \frac{\hbar^2}{2m_N} \left(\frac{eB}{\hbar}\right) \sum_i g_i \sigma_{zi} \\ + \sum_{i<j} (C_1 + C_3 P_{ij}^\sigma) e^{-a r_{ij}^2} + \sum_{i<j<k} \sum_{cyc} D_1 e^{-a(r_{ij}^2 + r_{jk}^2)}$$

Where $\frac{\hbar^2}{m_N} = 41.47 \text{ Mev} \cdot f m^2$ and $\frac{eB}{\hbar}$ is input parameter for the magnetic field. $r_{ij}^2 = x_{ij}^2 +$

$y_{ij}^2 + z_{ij}^2$ are the distance between the pair ij , $x_{ij} = x_i - x_j$, and $P_{ij}^\sigma = \frac{1+\sigma_i \cdot \sigma_j}{2}$. The LEC

C_1, C_3, D [2] and the cutoff a are from our Effective field theory without magnetic field.

The basis function are $\Psi = \psi^x \psi^y \psi^z$ where $\psi^x = e^{-\frac{1}{2} \mathbf{x}^T A^x \mathbf{x}}$ etc. and $\mathbf{x} = (x_1 \dots x_N)$.

In SVM we choose randomly distance between particles d_{ij} and A^x is symmetrical matrix

translated from $\psi^x = \exp\left(-\sum_{i<j}^N \frac{(x_i - x_j)^2}{2d_{ij}^2} - \sum_i^N \varepsilon_i x_i^2\right)$.

The analytical expressions of the matrix element of the spatial basis function of all part of the Hamiltonian are: (same expression for y, z coordinats of course) [3][4][5][6][7]

$$\langle \psi'^x | \psi^x \rangle = \sqrt{\frac{(2\pi)^N}{\det(A^x + A'^x)}}$$

$$\left\langle \psi'^x \left| \frac{\partial^2}{\partial x_i^2} \right| \psi^x \right\rangle = -\langle \psi'^x | \psi^x \rangle \sum_k^N A'_{ik} A_{ik}^x (A^x + A'^x)^{-1}_{kk}$$

$$\langle \psi'^x | x_i^2 | \psi^x \rangle = \langle \psi'^x | \psi^x \rangle (A^x + A'^x)^{-1}_{ii}$$

$$\left\langle \psi'^x \left| e^{-a x_{ij}^2} \right| \psi^x \right\rangle = \langle \psi'^x | \psi^x \rangle (2as + 1)^{-1/2}$$

$$\left\langle \psi'^x \left| e^{-a(x_{ik}^2 + x_{jk}^2)} \right| \psi^x \right\rangle = \langle \psi'^x | \psi^x \rangle (2aB + I)^{-1/2}$$

Where $s = \mathbf{C}_{ij}^T (A^x + A'^x)^{-1} \mathbf{C}_{ij}$ and $B = \begin{pmatrix} \mathbf{C}_{ik}^T (A^x + A'^x)^{-1} \mathbf{C}_{ik} & \mathbf{C}_{ik}^T (A^x + A'^x)^{-1} \mathbf{C}_{jk} \\ \mathbf{C}_{jk}^T (A^x + A'^x)^{-1} \mathbf{C}_{ik} & \mathbf{C}_{jk}^T (A^x + A'^x)^{-1} \mathbf{C}_{jk} \end{pmatrix}$

I is 2×2 unit matrix and $\mathbf{C}_{ij}^T = (0, \dots, \underbrace{1}_i, \dots, 0, \dots, \underbrace{-1}_j, \dots, 0)$.

[1] Let us assume the field, of strength B , point in the z direction. There are various choices for \mathbf{A} , we choose here $\mathbf{A} = \left(-\frac{1}{2}By, \frac{1}{2}Bx, 0\right)$ we get

$$\frac{\hbar^2}{2m_N} \left(-i\nabla - \frac{e}{\hbar}\mathbf{A}\right)^2 = -\frac{\hbar^2}{2m_N} \nabla^2 + i \left(\frac{\hbar^2}{2m_N}\right) \left(\frac{eB}{\hbar}\right) \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right) + \left(\frac{\hbar^2}{2m_N}\right) \left(\frac{eB}{\hbar}\right)^2 \frac{1}{4} (x^2 + y^2)$$

The matrix element of the $y \frac{\partial}{\partial x}$ are zero because Gaussian function are symmetric.

$$\left\langle \Psi' \left| y \frac{\partial}{\partial x} \right| \Psi \right\rangle = \left\langle \psi'^x \psi'^y \psi'^z \left| y \frac{\partial}{\partial x} \right| \psi^x \psi^y \psi^z \right\rangle = \left\langle \psi'^x \left| \frac{\partial}{\partial x} \right| \psi^x \right\rangle \underbrace{\langle \psi'^y | y | \psi^y \rangle}_{=0} \langle \psi'^z | \psi^z \rangle = 0$$

The energy of spin magnetic term is

$$E = -\gamma \mathbf{B} \cdot \mathbf{S} = -\gamma B \hbar \sigma_z$$

Where $\sigma_z = \pm \frac{1}{2}$ and $\gamma = \frac{g\mu}{\hbar}$ and $\mu = \frac{e\hbar}{2m_p}$ and $g_p = 5.586$, $g_n = -3.826$.

All together gives

$$-\frac{\hbar^2}{2m_N} \left(\frac{eB}{\hbar}\right) g \sigma_z$$

[2] The connection between our LEC to $C_{S,T}$ LEC are $C_1 = \frac{C_{1,0} + C_{0,1}}{2}$, $C_3 = \frac{C_{1,0} - C_{0,1}}{2}$.

[3]

[4]

[5]

[6]

[7]

[8] The dimension of the parameter $\frac{eB}{\hbar}$ is $length^{-2}$. So if $B = 1T$, the numerical value of $\frac{e}{\hbar}B$ in

unit of fm^{-2} will be $\frac{1 \cdot 1.6 \cdot 10^{-19} \cdot 1}{6.6 \cdot 10^{-34}} (10^{-15})^2 = 2.4 \cdot 10^{-16} fm^{-2}$ so just large magnetic field like

$10^{15}T$ will be significant. **eB/\hbar is eB in the input files**

[9]

Identity

$$\sum_i^n y_i^2 = \frac{1}{n} \left[\left(\sum_i^n y_i \right)^2 + \sum_{i < j} (y_j - y_i)^2 \right]$$

Or

$$\sum_i^n y_i^2 = nY^2 + \frac{1}{n} \sum_{i < j} y_{ij}^2$$

Where $y_{ij} = y_j - y_i$ and $Y = \frac{1}{n} \sum_i^n y_i$

In the same way

$$\sum_i (p_y)_i^2 = \frac{1}{n} P_y^2 + \frac{4}{n} \sum_{i < j} p_{ij}^2$$

Where $p_{ij} = \frac{p_j - p_i}{2}$ and $P_y = \sum_i^n p_i$

Such that $[Y, P_y] = i\hbar$ and $[y_{ij}, p_{ij}] = i\hbar$

Write $\frac{\hbar^2}{2m} \left(\frac{eB}{\hbar} \right)^2 = \frac{1}{2} m \omega^2$ where $\omega = \frac{eB}{m}$

We get for our Hamiltonian,

$$\begin{aligned} \frac{1}{2m} \sum_i (p_y)_i^2 + \frac{\hbar^2}{2m} \left(\frac{eB}{\hbar} \right)^2 \sum_i y_i^2 &= \frac{1}{2m} \left(\frac{1}{n} P_y^2 + \frac{4}{n} \sum_{i < j} p_{ij}^2 \right) + \frac{1}{2} m \omega^2 \left(nY^2 + \frac{1}{n} \sum_{i < j} y_{ij}^2 \right) \\ &= \frac{1}{2M} P_y^2 + \frac{1}{2} M \omega^2 Y^2 + \sum_{i < j} \left(\frac{1}{2 \left(\frac{M}{4} \right)} p_{ij}^2 + \frac{1}{2} \left(\frac{M}{4} \right) \left(\frac{2\omega}{n} \right)^2 y_{ij}^2 \right) \end{aligned}$$

The ground state energy of the $\frac{1}{2M} P_y^2 + \frac{1}{2} M \omega^2 Y^2$ is $\frac{1}{2} \hbar \omega$ and for each $\frac{1}{2 \left(\frac{M}{4} \right)} p_{ij}^2 + \frac{1}{2} \left(\frac{M}{4} \right) \left(\frac{2\omega}{n} \right)^2 y_{ij}^2$

is $\frac{1}{2} \hbar \left(\frac{2\omega}{n} \right)$ and for all pair is $(n-1) \frac{1}{2} \hbar \omega$ or $\left(\frac{n-1}{2} \right) \left(\frac{\hbar^2}{m} \right) \left(\frac{eB}{\hbar} \right)$ in terms of our parametes.

So if we calculate the ground state for two, three, and four systems without the term $\frac{1}{2} M \omega^2 Y^2$ (the center of mass vanish..) in magnetic field along z direction and with zero contact interaction, if we choose $\left(\frac{eB}{\hbar} \right) = \left(\frac{\hbar^2}{m} \right)^{-1}$ we need to get 0.5, 1, 1.5 for deuteron triton and helium.

If our Hamiltonian will be just

$$\frac{1}{2m} \sum_i (p_x)_i^2 + \frac{1}{2m} \sum_i (p_z)_i^2 + \frac{1}{2M} P_y^2 + \sum_{i < j} \left(\frac{1}{2 \left(\frac{M}{4} \right)} p_{ij}^2 + \frac{1}{2} \left(\frac{M}{4} \right) \left(\frac{2\omega}{n} \right)^2 y_{ij}^2 \right)$$