## Strong magnetic fields and contact interactions in few-fermion systems

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Technical manual detailing the implementation of a variational solution of the non-relativistic few-body problem in an external, i.e., static magnetic field.

The symmetric Gauge

$$oldsymbol{A}_i = rac{B_0}{2}(-y_i, x_i, 0)$$

(1)

$$\hat{H} = -\frac{\hbar^2}{2m} \sum_{i=1}^{N} \left\{ \nabla_i^2 + i \left( \frac{\hbar^2}{2m} \right) \left( \frac{q_i B_0}{\hbar} \right) L_i^z + \left( \frac{\hbar^2}{2m} \right) \left( \frac{q_i B_0}{\hbar} \right)^2 \frac{1}{4} \left( x_i^2 + y_i^2 \right) - g_i \left( \frac{\hbar^2}{2m} \right) \left( \frac{q_i B_0}{\hbar} \right) \sigma_{z_i} \right\}$$
(2)

$$+ \sum_{i < j}^{N} \left[ C_a + C_b (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+ - \sigma_i^z \sigma_j^z) \right] e^{-\frac{\Lambda^2}{4} (\mathbf{r}_i - \mathbf{r}_j)^2} + \sum_{\text{cyc. } i < j < k} D \cdot e^{-\frac{\Lambda^2}{4} ((\mathbf{r}_i - \mathbf{r}_j)^2 + (\mathbf{r}_i - \mathbf{r}_k)^2)}$$
(3)

The variational basis

input together with 
$$|A, \lambda\rangle := e^{-\frac{1}{2}\boldsymbol{x}^T A_{\boldsymbol{x}} \boldsymbol{x}} e^{-\frac{1}{2}\boldsymbol{y}^T A_{\boldsymbol{y}} \boldsymbol{y}} e^{-\frac{1}{2}\boldsymbol{z}^T A_{\boldsymbol{z}} \boldsymbol{z}} \cdot \sum_{\alpha}^{|\lambda|} \lambda_{\alpha} \sum_{n=1}^{N_{\text{comp}}} C_{\alpha}^n | s_1^n, \dots, s_N^n ; t_1^n, \dots, t_N^n \rangle$$

$$(4)$$

The generic matrix element

$$I_{\emptyset}(A', \lambda', A, \lambda; P) := \left\langle A', \lambda' \middle| \hat{\emptyset} \otimes \hat{g} \middle| \hat{P}(A), \hat{P}(\lambda) \middle\rangle = \left\langle A' \middle| \hat{\emptyset} \middle| \hat{P}(A) \middle\rangle \cdot \left\langle \lambda' \middle| \hat{g} \middle| \hat{P}(\lambda) \middle\rangle \right.$$
(5)

$$\hat{P}(A) = T_P^{\mathsf{T}} A T_P := A^P \quad . \tag{6}$$

$$\hat{\mathcal{O}} \in \left\{ \mathbb{1} \; ; \; \boldsymbol{p}^{\mathsf{T}} \mathbb{1}_{(3N \times 3N)} \boldsymbol{p} \; ; \; \sum_{i=1}^{N} q_i L_i^z \; ; \; \sum_{i=1}^{N} q_i (x_i^2 + y_i^2 + z_i^2) \; ; \; \sum_{i=1}^{N} q_i \sigma_i^z \; ; \; \sum_{i < j}^{N} e^{-\frac{\Lambda^2}{4} (\boldsymbol{r}_i - \boldsymbol{r}_j)^2} \right\}$$
(7)

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-	matrix
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<u> </u>	$\left\langle egin{array}{c c} A' & \hat{O} & \hat{P}(A) \end{array}  ight angle$	$\left\langle egin{array}{c c} oldsymbol{x} & \hat{g} & \hat{P}(oldsymbol{\lambda}) \end{array}  ight angle$
$\mathbb{1}:=\mathbb{1}^P_r\cdot\mathbb{1}^P_s$	$\left(\frac{(2\pi)^{3N}}{\det \mathbb{A}_x \det \mathbb{A}_y \det \mathbb{A}_z}\right)^{\frac{1}{2}}$	$\left  rac{ \lambda ,N_{ ext{comp}}}{\sum_{lpha,lpha',n,n'} \lambda_lpha \lambda'_{lpha'} C''_lpha C''_{lpha'} \left\langle m{s}^{n'};m{t}^{n'} \mid \hat{P}(m{s}^n);\hat{P}(m{t}^n)  ight.  ight angle$
$rac{1}{2}oldsymbol{p}^{\intercal}\mathbb{1}_{3N}oldsymbol{p}=-rac{\hbar^2}{2}oldsymbol{ abla}^{\intercal}\mathbb{1}_{3N}oldsymbol{ abla}$	$\frac{\hbar^2}{2}   \mathbb{1}^P_r  \prod_{c=x,y,z} (A_c)_{im} (\mathbb{A}_c^{-1})_{mn} (A_c^P)_{ni}$	$\mathbb{I}_{s}^{P}$
$\sum_{i=1}^{N} q_i L_i^z = q_i \left( x_i \partial_{y_i} - y_i \partial_{x_i} \right)$	0	$\mathbb{Q}^P_{\boldsymbol{s}} \coloneqq \sum_{i=1}^{N} \sum_{\alpha,n}^{ \lambda ,N_{\text{comp}}} \underbrace{(\hat{P}[t^n])_i}_{=q_{P(i)}} \lambda_{\alpha} C^n_{\alpha} \left\langle \begin{array}{c} \boldsymbol{s}^n, t^n \\ \end{array} \right  \hat{P}(\boldsymbol{s}^n); \hat{P}(t^n) \end{array} \right\rangle$
$\sum_{i=1}^{N} q_i (\omega_x x_i^2 + \omega_y y_i^2 + \omega_z z_i^2)$	$\mathbb{1}_{\boldsymbol{r}}^{P} \prod_{c=x,y,z} \omega_{c} \sum_{i=1}^{N} (\mathbb{A}_{c}^{-1})_{ii}$	$\mathbb{Q}_{s}^{P}$
$\sum_{i=1}^N q_i \sigma_i^z$	$\mathbb{I}_{\boldsymbol{r}}^{P}$	$\sum_{i=1}^{N} \sum_{lpha,n}^{ \lambda ,N_{ ext{comp}}} \widehat{\left(\hat{P}[m{t}^n] ight)_i} \ \widehat{\left(\hat{P}[m{s}^n] ight)_i}^{\lambda_lpha} C_lpha^n \left\langle m{s}^n;m{t}^n \ \middle  \ \hat{P}(m{s}^n);\hat{P}(m{t}^n) \  ight angle$
$\sum_{i < j}^N e^{-\frac{\Lambda^2}{4}(r_i - r_j)^2}$		$=q_{P(i)} $ $=s_{P(i)}^z$
$\sum_{\text{cyc.}} \sum_{i < j < k} e^{-\frac{\Lambda^2}{4} ((\boldsymbol{r}_i - \boldsymbol{r}_j)^2 + (\boldsymbol{r}_i - \boldsymbol{r}_k)^2)}$		

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$$\mathbb{A}_x = A_x' + A_x^P$$

(6)

with