# Strong magnetic fields and contact interactions in few-fermion systems

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Technical manual detailing the implementation of a variational solution of the non-relativistic few-body problem in an external, *i.e.* , static magnetic field.

*a. The symmetric Gauge*

$$\boldsymbol{A}_i = \frac{B_0}{2}(-y(i), x(i), 0) \tag{1}$$

*b. The Hamiltonian*

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{\mathtt{N}}\left\{\boldsymbol{\nabla}(i)^2 + i\left(\frac{\hbar^2}{2m}\right)\left(\frac{q(i)B_0}{\hbar}\right)L_z(i) + \left(\frac{\hbar^2}{2m}\right)\left(\frac{q(i)B_0}{\hbar}\right)^2\frac{1}{4}\left(x^2(i)+y^2(i)\right) - g(i)\left(\frac{\hbar^2}{2m}\right)\left(\frac{q(i)B_0}{\hbar}\right)\sigma_z(i)\right\} \tag{2}$$

$$+ \sum_{i<j}^{\mathtt{N}}\left(C_a + C_b\left[\sigma^+(i)\sigma^-(j) + \sigma^-(i)\sigma^+(j) - \sigma^z(i)\sigma^z(j)\right]\right)\; e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2} + \sum_{\text{cyc.}}\sum_{i<j<k} D\cdot e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(k)]^2} \tag{3}$$

*c. The variational basis*

$$|\, A, \alpha \,\rangle := e^{-\frac{1}{2}\boldsymbol{x}^T A_x \boldsymbol{x}}\; e^{-\frac{1}{2}\boldsymbol{y}^T A_y \boldsymbol{y}}\; e^{-\frac{1}{2}\boldsymbol{z}^T A_z \boldsymbol{z}} \cdot \sum_{n=1}^{\mathtt{ncmp}} C_\alpha^n \,|\, s_1^n, \ldots, s_{\mathtt{N}}^n \;;\; t_1^n, \ldots, t_{\mathtt{N}}^n \,\rangle \tag{4}$$

with

$$\alpha = 1,\; \ldots,\; \mathtt{nts\_states}$$

*d. The generic matrix element*

$$I_{\mathcal{O}}(A', \alpha', A, \alpha; P) := \left\langle\, A', \alpha' \,\middle|\, \hat{\mathcal{O}} \otimes \hat{\mathcal{G}} \,\middle|\, \hat{P}(A), \hat{P}(\alpha) \,\right\rangle = \left\langle\, A' \,\middle|\, \hat{\mathcal{O}} \,\middle|\, \hat{P}(A) \,\right\rangle \cdot \left\langle\, \alpha' \,\middle|\, \hat{\mathcal{G}} \,\middle|\, \hat{P}(\alpha) \,\right\rangle \tag{5}$$

with $\hat{P} \in \mathcal{A}$, hence,

$$\hat{P}(A) = T_P^{\intercal}\, A\, T_P := A^P \quad . \tag{6}$$

$$\hat{\mathcal{O}} \in \left\{\mathbb{1}\;;\; -\sum_{i=1}^{\mathtt{N}}\prod_{c=x,y,z}\boldsymbol{\partial}_c^{\intercal}(i)\mathbb{1}\boldsymbol{\partial}_c(i)\;;\; \sum_{i=1}^{\mathtt{N}} q(i)L^z(i)\;;\; \sum_{i=1}^{\mathtt{N}} q(i)(\omega_x x^2(i) + \omega_y y^2(i) + \omega_z z^2(i))\;;\; \sum_{i=1}^{\mathtt{N}} q(i)\sigma^z(i)\;;\; \sum_{i<j}^{\mathtt{N}} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2}\right\} \tag{7}$$

*e. The matrix elements*

| $\hat{\mathcal{O}}$ | $\left\langle A' \,\middle\vert\, \hat{\mathcal{O}} \,\middle\vert\, \hat{P}(A) \right\rangle$ | $\left\langle \alpha' \,\middle\vert\, \hat{\mathcal{I}} \,\middle\vert\, \hat{P}(\alpha) \right\rangle$ |
|---|---|---|
| $\mathbb{1} := \mathbb{1}_{\boldsymbol{r}}^P \cdot \mathbb{1}_{\boldsymbol{s}}^P$ | $\left(\frac{(2\pi)^{3\mathtt{N}}}{\det \mathbb{A}_x \det \mathbb{A}_y \det \mathbb{A}_z}\right)^{\frac{1}{2}}$ | $\sum_{n,n'}^{\mathtt{ncmp}(\alpha),\mathtt{ncmp}(\alpha')} C_\alpha^n C_{\alpha'}^{n'} \left\langle \boldsymbol{s}^{n'}; \boldsymbol{t}^{n'} \,\middle\vert\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$ |
| $-\sum_{i=1}^{\mathtt{N}} \prod_{c=x,y,z} \boldsymbol{\partial}_c^{\intercal}(i) \mathbb{1} \boldsymbol{\partial}_c(i) \cdot \mathbb{1}_{\boldsymbol{s}}^P$ | $\mathbb{1}_{\boldsymbol{r}}^P \cdot \prod_{c=x,y,z} (A_c)_{im} (\mathbb{A}_c^{-1})_{mn} (A_c^P)_{ni}$ | $\mathbb{1}_{\boldsymbol{s}}^P$ |
| $\sum_{i=1}^{\mathtt{N}} q(i) L_z(i) = q(i) \left[x(i)\partial_y(i) - y(i)\partial_x(i)\right]$ | $0$ | $\mathbb{Q}_{\boldsymbol{s}}^P := \sum_{i=1}^{\mathtt{N}} \sum_{n,n'}^{\mathtt{ncmp}(\alpha),\mathtt{ncmp}(\alpha')} \left[2 - \boldsymbol{t}^n(\hat{P}(i))\right] C_\alpha^n C_{\alpha'}^{n'} \left\langle \boldsymbol{s}^{n'}; \boldsymbol{t}^{n'} \,\middle\vert\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$ |
| $\sum_{i=1}^{\mathtt{N}} q(i)(\omega_x x^2(i) + \omega_y y^2(i) + \omega_z z^2(i))$ | $\mathbb{1}_{\boldsymbol{r}}^P \cdot \prod_{c=x,y,z} \omega_c \sum_{i=1}^{\mathtt{N}} (\mathbb{A}_c^{-1})_{ii}$ | $\mathbb{Q}_{\boldsymbol{s}}^P$ |
| $\sum_{i=1}^{\mathtt{N}} q(i) \sigma^z(i)$ | $\mathbb{1}_{\boldsymbol{r}}^P$ | $\sum_{i=1}^{\mathtt{N}} \sum_{n,n'}^{\mathtt{ncmp}(\alpha),\mathtt{ncmp}(\alpha')} \left[2 - \boldsymbol{t}^n(\hat{P}(i))\right] \left[3 - 2\boldsymbol{s}^n(\hat{P}(i))\right] C_\alpha^n C_{\alpha'}^{n'} \left\langle \boldsymbol{s}^{n'}; \boldsymbol{t}^{n'} \,\middle\vert\, \hat{P}(\boldsymbol{s}^n); \hat{P}(\boldsymbol{t}^n) \right\rangle$ |
| $\sum_{i<j}^{\mathtt{N}} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2}$ | | |
| $\sum_{\text{cyc.}} \sum_{i<j<k} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(j)]^2} e^{-\frac{\Lambda^2}{4}[\boldsymbol{r}(i)-\boldsymbol{r}(k)]^2}$ | | |

(8)

with

$$\mathbb{A}_x = A'_x + A_x^P \tag{9}$$

$$\frac{\Lambda^2}{4} = \mathtt{apot} \tag{10}$$