

# Strong magnetic fields and contact interactions in few-fermion systems

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Technical manual detailing the implementation of a variational solution of the non-relativistic few-body problem in an external, *i.e.*, static magnetic field.

a. *The symmetric Gauge*

$$\mathbf{A}_i = \frac{B_0}{2}(-y(i), x(i), 0) \quad (1)$$

b. *The Hamiltonian*

$$\hat{H} = -\frac{\hbar^2}{2m} \sum_{i=1}^N \left\{ \nabla(i)^2 + i \left( \frac{\hbar^2}{2m} \right) \left( \frac{q(i)B_0}{\hbar} \right) L_z(i) + \left( \frac{\hbar^2}{2m} \right) \left( \frac{q(i)B_0}{\hbar} \right)^2 \frac{1}{4} (x^2(i) + y^2(i)) - g(i) \left( \frac{\hbar^2}{2m} \right) \left( \frac{q(i)B_0}{\hbar} \right) \sigma_z(i) \right\} \quad (2)$$

$$+ \sum_{i < j}^N (C_a + C_b [\sigma^+(i)\sigma^-(j) + \sigma^-(i)\sigma^+(j) - \sigma^z(i)\sigma^z(j)]) e^{-\frac{\Lambda^2}{4}[\mathbf{r}(i)-\mathbf{r}(j)]^2} + \sum_{\text{cyc. } i < j < k} D \cdot e^{-\frac{\Lambda^2}{4}[\mathbf{r}(i)-\mathbf{r}(j)]^2} e^{-\frac{\Lambda^2}{4}[\mathbf{r}(i)-\mathbf{r}(k)]^2} \quad (3)$$

c. *The variational basis*

$$|A, \alpha\rangle := e^{-\frac{1}{2}\mathbf{x}^T A_x \mathbf{x}} e^{-\frac{1}{2}\mathbf{y}^T A_y \mathbf{y}} e^{-\frac{1}{2}\mathbf{z}^T A_z \mathbf{z}} \cdot \sum_{n=1}^{\text{ncmp}} C_\alpha^n |s_1^n, \dots, s_N^n; t_1^n, \dots, t_N^n\rangle \quad (4)$$

with

$$\alpha = 1, \dots, \text{nts\_states}$$

d. *The generic matrix element*

$$I_{\mathcal{O}}(A', \alpha', A, \alpha; P) := \langle A', \alpha' \mid \hat{\mathcal{O}} \otimes \hat{g} \mid \hat{P}(A), \hat{P}(\alpha) \rangle = \langle A' \mid \hat{\mathcal{O}} \mid \hat{P}(A) \rangle \cdot \langle \alpha' \mid \hat{g} \mid \hat{P}(\alpha) \rangle \quad (5)$$

with  $\hat{P} \in \mathcal{A}$ , hence,

$$\hat{P}(A) = T_P^{\mathsf{T}} A T_P := A^P \quad . \quad (6)$$

$$\hat{\mathcal{O}} \in \left\{ \mathbb{1} ; -\sum_{i=1}^N \prod_{c=x,y,z} \partial_c^{\mathsf{T}}(i) \mathbb{1} \partial_c(i) ; \sum_{i=1}^N q(i) L^z(i) ; \sum_{i=1}^N q(i) (\omega_x x^2(i) + \omega_y y^2(i) + \omega_z z^2(i)) ; \sum_{i=1}^N q(i) \sigma^z(i) ; \sum_{i < j}^N e^{-\frac{\Lambda^2}{4}[\mathbf{r}(i)-\mathbf{r}(j)]^2} \right\} \quad (7)$$

*e. The matrix elements*

$\hat{\odot}$	$\langle A' \mid \hat{\odot} \mid \hat{P}(A) \rangle$	$\langle \alpha' \mid \hat{g} \mid \hat{P}(\alpha) \rangle$
$\mathbb{1} := \mathbb{1}_{\mathbf{r}}^P \cdot \mathbb{1}_{\mathbf{s}}^P$ $-\sum_{i=1}^{\mathbb{N}} \prod_{c=x,y,z} \partial_c^\top(i) \mathbb{1} \partial_c(i) \cdot \mathbb{1}_{\mathbf{s}}^P$ $\sum_{i=1}^{\mathbb{N}} q(i) L_z(i) = q(i) [x(i) \partial_y(i) - y(i) \partial_x(i)]$ $\sum_{i=1}^{\mathbb{N}} q(i) (\omega_x x^2(i) + \omega_y y^2(i) + \omega_z z^2(i))$ $\sum_{i=1}^{\mathbb{N}} q(i) \sigma^z(i)$ $\sum_{i < j}^{\mathbb{N}} e^{-\frac{\Lambda^2}{4} [\mathbf{r}(i) - \mathbf{r}(j)]^2}$ $\sum_{\text{cyc. } i < j < k} e^{-\frac{\Lambda^2}{4} [\mathbf{r}(i) - \mathbf{r}(j)]^2} e^{-\frac{\Lambda^2}{4} [\mathbf{r}(i) - \mathbf{r}(k)]^2}$	$\left( \frac{(2\pi)^{3\mathbb{N}}}{\det A_x \det A_y \det A_z} \right)^{\frac{1}{2}}$ $\mathbb{1}_{\mathbf{r}}^P \cdot \prod_{c=x,y,z} (A_c)_{im} (\Lambda_c^{-1})_{mn} (A_c^P)_{ni}$ $0$ $\mathbb{1}_{\mathbf{r}}^P \cdot \prod_{c=x,y,z} \omega_c \sum_{i=1}^{\mathbb{N}} (\Lambda_c^{-1})_{ii}$ $\mathbb{1}_{\mathbf{r}}^P$	$\sum_{n,n'}^{\text{ncmp}(\alpha), \text{ncmp}(\alpha')} C_{\alpha}^n C_{\alpha'}^{n'} \langle \mathbf{s}^{n'}; \mathbf{t}^{n'} \mid \hat{P}(\mathbf{s}^n); \hat{P}(\mathbf{t}^n) \rangle$ $\mathbb{1}_{\mathbf{s}}^P$ $\mathbb{Q}_{\mathbf{s}}^P := \sum_{i=1}^{\mathbb{N}} \sum_{n,n'}^{\text{ncmp}(\alpha), \text{ncmp}(\alpha')} [2 - \mathbf{t}^n(\hat{P}(i))] C_{\alpha}^n C_{\alpha'}^{n'} \langle \mathbf{s}^{n'}; \mathbf{t}^{n'} \mid \hat{P}(\mathbf{s}^n); \hat{P}(\mathbf{t}^n) \rangle$ $\mathbb{Q}_{\mathbf{s}}^P$ $\sum_{i=1}^{\mathbb{N}} \sum_{n,n'}^{\text{ncmp}(\alpha), \text{ncmp}(\alpha')} [2 - \mathbf{t}^n(\hat{P}(i))] [3 - 2\mathbf{s}^n(\hat{P}(i))] C_{\alpha}^n C_{\alpha'}^{n'} \langle \mathbf{s}^{n'}; \mathbf{t}^{n'} \mid \hat{P}(\mathbf{s}^n); \hat{P}(\mathbf{t}^n) \rangle$

with

$$(8)$$

$$(9)$$

$$(10)$$

$$A_x = A'_x + A_x^P$$

$$\frac{\Lambda^2}{4} = \text{apot}$$

Spin-isospin wave function for the deuteron:

$$(11)$$

$$(12)$$

$$\mid \alpha \hat{=^3} S_1 \rangle = \left[ \mid 12 \rangle + \mid 21 \rangle \right] \otimes \left[ \mid \text{pn} \rangle - \mid \text{np} \rangle \right]$$

$$\mathbf{s}^n(i) = \begin{cases} 1 \Rightarrow \sigma^z(i) = \uparrow \\ 2 \Rightarrow \sigma^z(i) = \downarrow \end{cases} \quad \text{and} \quad \mathbf{t}^n(i) = \begin{cases} 1 \Rightarrow \tau^z(i) = \mathbf{p} \\ 2 \Rightarrow \tau^z(i) = \mathbf{n} \end{cases}$$