$\psi_1 \to i$, $\psi_1^* \to j$ $\times\left[\begin{array}{l} b_i = c_1 \\ b_j = c_2 \end{array}\right.$ $a_i = b_1$

$\psi_2 \to i$, $\psi_2^* \to j$ $a_j = b_2$

$$\psi = \psi_1 + \psi_2$$

$$\psi_1 = \sum_i c_i \, e^{-a_i(\bar{r}_1^2+\bar{r}_2^2)} \quad , \quad \psi_2 = \sum_j d_j \, (\bar{r}_1^2+\bar{r}_2^2)\, e^{-b_j(\bar{r}_1^2+\bar{r}_2^2)}$$

Norm $\int \psi^* \psi \, d^3\bar{r}_1 = \int \left(\psi_1^2 + \psi_2^2 + \psi_1^*\psi_2 + \psi_2^*\psi_1\right) d^3\bar{r}_1$

$$\psi_1^2 = \int d^3\bar{r}_1 \; e^{-2(a_i+a_j)\bar{r}_1^2}$$

$$= \left[\frac{\pi}{2(a_i+a_j)}\right]^{3/2}$$

$$\psi_2^2 = \int d^3\bar{r}_1 \;\; 2\bar{r}_1^2 \, e^{-2(b_i+b_j)\bar{r}_1^2} \, 2\bar{r}_1^2$$

$$= 4\int d^3\bar{r}_1 \; (\bar{r}_{1x}^2 + \bar{r}_{1y}^2 + \bar{r}_{1z}^2)^2 \; e^{-2(b_i+b_j)\bar{r}_1^2}$$

$$= 4\int d^3\bar{r}_1 \left(\bar{r}_{1x}^4 + \bar{r}_{1y}^4 + \bar{r}_{1z}^4 + 2\bar{r}_{1x}^2\bar{r}_{1y}^2 + 2\bar{r}_{1y}^2\bar{r}_{1z}^2 + 2\bar{r}_{1x}^2\bar{r}_{1z}^2\right) e^{-2(b_i+b_j)\bar{r}_{1x}^2} \, e^{-2(b_i+b_j)\bar{r}_{1y}^2} \, e^{-2(b_i+b_j)\bar{r}_{1z}^2}$$

$$= 4\int d^3\bar{r}_1 \left[\sqrt{\frac{\pi}{2(b_i+b_j)}} \, \frac{3!}{[4(b_i+b_j)]^2} + \frac{2(\bar{r}_{1y}^2+\bar{r}_{1z}^2)}{4(b_i+b_j)} \sqrt{\frac{\pi}{2(b_i+b_j)}} + \left(\bar{r}_{1y}^4 + \bar{r}_{1z}^4 + 2\bar{r}_{1y}^2\bar{r}_{1z}^2\right)\sqrt{\frac{\pi}{2(b_i+b_j)}}\right]$$

$$= 4\sqrt{\frac{\pi}{2(b_i+b_j)}}\int\Big[\frac{6}{16(b_i+b_j)^2}+\frac{(\bar{r}_{1y}^2+\bar{r}_{1z}^2)}{2(b_i+b_j)}$$

$$+\bar{r}_{1y}^4+\bar{r}_{1z}^4+2\bar{r}_{1y}^2\,\bar{r}_{1z}^2\Big]\,d^3\bar{r}_1\; e^{-2(b_i+b_j)\bar{r}_{1y}^2}\, e^{-2(b_i+b_j)\bar{r}_{1z}^2}$$

$$= 4\sqrt{\frac{\pi}{2(b_i+b_j)}}\int\Big[\Big(\frac{3}{8(b_i+b_j)^2}+\frac{\bar{r}_{1z}^2}{2(b_i+b_j)}+\bar{r}_{1z}^4\Big)$$

$$+\bar{r}_{1y}^4+\bar{r}_{1y}^2\Big(\frac{1}{2(b_i+b_j)}+2\bar{r}_{1z}^2\Big)\Big]\, e^{-2(b_i+b_j)\bar{r}_{1y}^2}\, e^{-2(b_i+b_j)\bar{r}_{1z}^2}$$

$$= 4\left(\frac{\pi}{2(b_i+b_j)}\right)\Big[\frac{3}{8(b_i+b_j)^2}+\frac{\bar{r}_{1z}^2}{2(b_i+b_j)}+\bar{r}_{1z}^4$$

$$+\frac{3}{8(b_i+b_j)^2}+\frac{1}{8(b_i+b_j)^2}+\frac{\cancel{2}\,\bar{r}_{1z}^2}{2(b_i+b_j)}\Big]\, e^{-2(b_i+b_j)\bar{r}_{1z}^2}$$

$$= 4\left(\frac{\pi}{2(b_i+b_j)}\right)\Big[\frac{7}{8(b_i+b_j)^2}+\bar{r}_{1z}^4+\frac{\bar{r}_{1z}^2}{(b_i+b_j)}\Big]\, e^{-2(b_i+b_j)\bar{r}_{1z}^2}$$

$$= 4\left(\frac{\pi}{2b_i+2b_j}\right)^{3/2}\Big[\frac{7}{8(b_i+b_j)^2}+\frac{1}{\cancel{8}4(b_i+b_j)^2}+\frac{3}{8(b_i+b_j)^2}\Big]$$

$$= \frac{3.\cancel{4}^{2}}{\cancel{2}}\left(\frac{\pi}{2b_i+2b_j}\right)^{3/2}\frac{1}{(b_i+b_j)^2} = \frac{6}{2\sqrt{2}}\left(\frac{\pi}{b_i+b_j}\right)^{3/2}\frac{1}{(b_i+b_j)^2}$$

$$= \frac{3}{\sqrt{2}}\left(\frac{\pi^{3/2}}{(b_i+b_j)^{7/2}}\right)$$

$\frac{\cancel{10}^{5}}{\cancel{8}4}+\frac{1}{4}$ $\frac{\cancel{6}^{3}}{\cancel{4}2}$

2

$$\int \psi_1^* \psi_2 \, d^3\bar{r}_1 = \int (\bar{r}_1^2 + \bar{r}_2^2) \, e^{-(a_i+b_j)(r_1 + r_2^-)} d^3\bar{r}_1$$

$$= 2 \int \bar{r}_1^2 \, e^{-2(a_i+b_j)\bar{r}_1^2} d\bar{r}_{1x} \, d\bar{r}_{1y} \, d\bar{r}_{1z}$$

$$= 2 \int (\bar{r}_{1x}^2 + \bar{r}_{1y}^2 + \bar{r}_{1z}^2) \, e^{-2(a_i+b_j)\bar{r}_{1x}^2} e^{-2(a_i+b_j)\bar{r}_{1y}^2} e^{-2(a_i+b_j)\bar{r}_{1z}^2} d^3\bar{r}_1$$

$$= 2 \left[\sqrt{\frac{\pi}{2a_i+2b_j}} \frac{1}{4(a_i+b_j)} + (\bar{r}_{1y}^2 + \bar{r}_{1z}^2) \sqrt{\frac{\pi}{2a_i+2b_j}} \right] e^{-2(a_i+b_j)\bar{r}_{1y}^2} e^{-2(a_i+b_j)\bar{r}_{1z}^2}$$

$$= 2 \sqrt{\frac{\pi}{2(a_i+b_j)}} \left[\frac{1}{4(a_i+b_j)} \sqrt{\frac{\pi}{2a_i+2b_j}} + \bar{r}_{1z}^2 \sqrt{\frac{\pi}{2a_i+2b_j}} + \sqrt{\frac{\pi}{2a_i+2b_j}} \frac{1}{4(a_i+b_j)} \right]$$

$$= 2 \left(\frac{\pi}{2(a_i+b_j)} \right) \int \left[\frac{1}{2(a_i+b_j)} + \bar{r}_{1z}^2 \right] e^{-2(a_i+b_j)\bar{r}_{1z}^2}$$

$$= 2 \left(\frac{\pi}{2(a_i+b_j)} \right)^{3/2} \frac{3}{\cancel{4}_2 (a_i+b_j)} \qquad \frac{\frac{1}{2}+\frac{1}{4}}{} \quad \frac{2+1}{4}$$

$$= \frac{3}{4\sqrt{2}} \frac{\pi^{3/2}}{(a_j+b_j)^{5/2}}$$

$$\int \psi_2^* \psi_1 \, d^3\bar{r}_1 = \int (\bar{r}_1^2 + \bar{r}_2^2) \, e^{-(a_i + b_j)(\bar{r}_1^2 + \bar{r}_2^2)} \, d^3\bar{r}_1$$

$$= \frac{3}{4\sqrt{2}} \frac{\pi^{3/2}}{(a_i + b_j)^{5/2}}$$

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Two Body Potential

$$\int \psi^* V \psi \, d^3\bar{r}_1 = \int \left(\psi_1^* V \psi_1 + \psi_2^* V \psi_2 + \psi_1^* V \psi_2 + \psi_2^* V \psi_1 \right) d^3\bar{r}_1$$

$$\int \psi_1^* V \psi_1 = \int d^3\bar{r}_1 \; e^{-2(a_i+a_j)\bar{r}_1^2} e^{-4\lambda \bar{r}_1^2}$$

$$= \int d^3\bar{r}_1 \; e^{-(2a_i+2a_j+4\lambda)\bar{r}_1^2}$$

$$= \left(\frac{\pi}{2a_i+2a_j+4\lambda} \right)^{3/2}$$

$$\int \psi_2^* V \psi_2 = 4 \int d^3\bar{r}_1 \left(\bar{r}_{1x}^2 + \bar{r}_{1y}^2 + \bar{r}_{1z}^2 \right)^2 e^{-2(b_i+b_j)\bar{r}_1^2} e^{-4\lambda \bar{r}_1^2}$$

$$= \frac{3}{\sqrt{2}} \frac{\pi^{3/2}}{(b_i+b_j+2\lambda)^{7/2}}$$

$$\int \psi_1^* V \psi_2 = 2 \int d^3\bar{r}_1 \; \bar{r}_1^2 \; e^{-2(a_j+b_i)\bar{r}_1^2} e^{-4\lambda \bar{r}_1^2}$$

$$= 2 \int d^3\bar{r}_1 \left(\bar{r}_{1x}^2 + \bar{r}_{1y}^2 + \bar{r}_{1z}^2 \right) e^{-2(a_j+b_i+2\lambda)\bar{r}_1^2}$$

$$= 2 \int \left[\frac{1}{4(a_j+b_i+2\lambda)} + \bar{r}_{1y}^2 + \bar{r}_{1z}^2 \right] \sqrt{\frac{\pi}{2a_j+2b_i+4\lambda}} \; e^{-2(a_j+b_i+2\lambda)\bar{r}_{1y}^2} e^{-2(a_j+b_i+2\lambda)\bar{r}_{1z}^2}$$

$$2\left(\frac{\pi}{2a_j+2b_i+4\lambda}\right)\int\left[\frac{1}{4(a_j+b_i+2\lambda)}+r_{12}^2+\frac{1}{4(a_j+b_i+2\lambda)}\right]dr_{12}$$

$$=2\left(\frac{\pi}{2a_j+2b_i+4\lambda}\right)^{3/2}\left[\frac{1}{2(a_j+b_i+2\lambda)}+\frac{1}{4(a_j+b_i+2\lambda)}\right]$$

$$=2\left(\frac{\pi}{2a_j+2b_i+4\lambda}\right)^{3/2}\frac{3}{4(a_j+b_i+2\lambda)}$$

$$=\frac{3}{4\sqrt{2}}\frac{\pi^{3/2}}{(a_j+b_i+2\lambda)^{5/2}}$$

$$\int\psi_2^*V\psi_1=\frac{3}{4\sqrt{2}}\frac{\pi^{3/2}}{(a_i+b_j+2\lambda)^{5/2}}$$

Multiply ½ factor in overall K.E.

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Kinetic Part :- $\psi_1 = e^{-2a_i \bar{r}_1^2}$

$$\psi_2 = 2\bar{r}_1^2 e^{-2b_i \bar{r}_1^2}$$

$$\frac{\partial \psi_1}{\partial \bar{r}_1} = -4a_i \bar{r}_1 e^{-2a_i \bar{r}_1^2}$$

$$\frac{\partial \psi_2}{\partial \bar{r}_1} = 2\left[2\bar{r}_1 - 4b_i \bar{r}_1^3\right] e^{-2b_i \bar{r}_1^2}$$

$$\boxed{\psi_2^* \frac{p^2}{2m} \psi_2} = 16\left(\bar{r}_1 - 2b_i \bar{r}_1^3\right)\left(\bar{r}_1 - 2b_j \bar{r}_1^3\right) e^{-2(b_i+b_j)\bar{r}_1^2}$$

$$= 16\left(\bar{r}_1^2 - 2b_i \bar{r}_1^4 - 2b_j \bar{r}_1^4 + 4 b_i b_j \bar{r}_1^6\right) e^{-2(b_i+b_j)\bar{r}_1^2}$$

$$= 16\left(\bar{r}_1^2 - 2(b_i+b_j)\bar{r}_1^4 + 4 b_i b_j \bar{r}_1^6\right) e^{-2(b_i+b_j)\bar{r}_1^2}$$

$$I_1 = \int \bar{r}_1^6 \, e^{-2(b_i+b_j)\bar{r}_{1x}^2} e^{-2(b_i+b_j)\bar{r}_{1y}^2} e^{-2(b_i+b_j)\bar{r}_{1z}^2}$$

$$= \left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2} \left[\frac{5!}{[4(b_i+b_j)]^3} + \frac{3(\bar{r}_{1y}^2 + \bar{r}_{1z}^2)\ 3!}{[4(b_i+b_j)]^2}\right.$$

$$+ 3(\bar{r}_{1y}^4 + \bar{r}_{1z}^4 + 2\bar{r}_{1y}^2 \bar{r}_{1z}^2)\frac{1}{4(b_i+b_j)} + \bar{r}_{1y}^6 + \bar{r}_{1z}^6$$

$$+ 3\bar{r}_{1y}^4 \bar{r}_{1z}^2 + 3\bar{r}_{1z}^4 \bar{r}_{1y}^2$$

$$\left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2}\left[\frac{5!}{[4(b_i+b_j)]^3}+\frac{3.3!}{[4(b_i+b_j)]^2}\overline{r_{12}}^2\right.$$

$$+\frac{3\overline{r_{12}}^4}{4(b_i+b_j)}+\overline{r_{12}}^6+\left[\frac{3.3!}{[4(b_i+b_j)]^2}+\frac{6\overline{r_{12}}^2}{4(b_i+b_j)}+3\overline{r_{12}}^4\right]\frac{1}{4(b_i+b_j)}$$

$$\left.+\left[\frac{3}{4(b_i+b_j)}+3\overline{r_{12}}^2\right]\frac{3!}{[4(b_i+b_j)]^2}+\frac{5!}{[4(b_i+b_j)]^3}\right]$$

$$\Rightarrow\left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2}\left[\frac{5!}{64(b_i+b_j)^3}+\frac{3.3!}{64(b_i+b_j)^3}\right.$$

$$+\frac{3.3!}{64(b_i+b_j)^3}+\frac{5!}{64(b_i+b_j)^3}+\frac{3.3!}{64(b_i+b_j)^3}$$

$$+\frac{6}{64(b_i+b_j)^3}+\frac{3.3!}{64(b_i+b_j)^3}+\frac{3.3!}{64(b_i+b_j)^3}$$

$$\left.+\frac{3.3!}{64(b_i+b_j)^3}+\frac{5!}{64(b_i+b_j)^3}\right]$$

$$=\left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2}\left[120+18+18+120+18+6+18+18+18+120\right]\frac{1}{64(b_i+b_j)^3}$$

$$\Rightarrow\frac{\pi^{3/2}}{(b_i+b_j)^{9/2}}\frac{474}{128\sqrt{2}}=\frac{2.61\;\pi^{3/2}}{(b_i+b_j)^{9/2}}$$

$$\cancel{\frac{\hbar^2}{2m}} I_2 = \int \bar{r}_1^4 \, e^{-2(b_i+b_j)\bar{r}_1^2}$$

$$= \int \left(\bar{r}_{1x}^4 + \bar{r}_{1y}^4 + \bar{r}_{1z}^4 + 2\bar{r}_{1x}^2\bar{r}_{1y}^2 + 2\bar{r}_{1y}^2\bar{r}_{1z}^2 + 2\bar{r}_{1z}^2\bar{r}_{1x}^2\right) e^{-2(b_i+b_j)\bar{r}_1^2}$$

$$= \left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2} \left[\frac{3!}{[4(b_i+b_j)]^2} + \frac{2(\bar{r}_{1y}^2+\bar{r}_{1z}^2)}{4(b_i+b_j)} + \bar{r}_{1y}^4 + \bar{r}_{1z}^4 + 2\bar{r}_{1y}^2\bar{r}_{1z}^2\right]$$

$$= \left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2} \left[\frac{3!}{[4(b_i+b_j)]^2} + \frac{2\bar{r}_{1z}^2}{4(b_i+b_j)} + \bar{r}_{1z}^4 + \left(\frac{2}{4(b_i+b_j)} + 2\bar{r}_{1z}^2\right)\frac{1}{4(b_i+b_j)} + \frac{3!}{[4(b_i+b_j)]^2}\right]$$

$$= \left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2} \left[\frac{3!}{[4(b_i+b_j)]^2} + \frac{2}{[4(b_i+b_j)]^2} + \frac{3!}{[4(b_i+b_j)]^2} + \frac{2}{[4(b_i+b_j)]^2} + \frac{2}{[4(b_i+b_j)]^2} + \frac{3!}{[4(b_i+b_j)]^2}\right]$$

$$= \left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2} \left[\frac{\cancel{24}\,3}{\cancel{16}_{2}(b_i+b_j)^2}\right] = \frac{3}{4\sqrt{2}} \frac{\pi^{3/2}}{(b_i+b_j)^{7/2}}$$

$\frac{3}{2}+2$

$$I_3 = \int \left(\bar{r}_{1x}^2 + \bar{r}_{1y}^2 + \bar{r}_{1z}^2\right) e^{-2(b_i+b_j)\bar{r}_1^2} \, d^3\bar{r}_1$$

$$= \left[\frac{\pi}{2(b_i+b_j)}\right]^{3/2} \left[\frac{3}{4(b_i+b_j)}\right]$$

$$= \frac{3\,\pi^{3/2}}{8\sqrt{2}\,(b_i+b_j)^{5/2}}$$

So, $$\psi_2^* \frac{p^2}{2m} \psi_2 = 16\left[\frac{3\,\pi^{3/2}}{8\sqrt{2}\,(b_i+b_j)^{5/2}} - \frac{4(b_i+b_j)\;3\,\pi^{3/2}}{4\sqrt{2}\,(b_i+b_j)^{7/2}} + \frac{16\,b_i b_j\;2.61\,\pi^{3/2}}{(b_i+b_j)^{9/2}}\right]$$

$$= 16\left[\frac{3}{8\sqrt{2}}\frac{\pi^{3/2}}{(b_i+b_j)^{5/2}} - \frac{3\,\pi^{3/2}}{\sqrt{2}\,(b_i+b_j)^{5/2}} + \frac{41.76\,b_i b_j\,\pi^{3/2}}{(b_i+b_j)^{9/2}}\right]$$

$$= 16\left[\frac{3\pi^{3/2}}{\sqrt{2}\,(b_i+b_j)^{5/2}}\left(-\frac{7}{8}\right) + \frac{41.76\,b_i b_j\,\pi^{3/2}}{(b_i+b_j)^{9/2}}\right]$$

$$= \frac{\pi^{3/2}}{(b_i+b_j)^{5/2}}\left[-\frac{21\times \cancel{16}^{\,2}}{\cancel{8}\sqrt{2}} + \frac{41.76\,b_i b_j}{(b_i+b_j)^{\cancel{7/2}\,2}}\right]$$

$\frac{d}{8} - 1 \qquad \frac{1}{8} - 1 \Rightarrow -\frac{7}{8}$

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Total K.E

$$16\left[\frac{4\, b_i b_j\; 2.61\; \pi^{3/2}}{(b_i+b_j)^{9/2}} - \frac{2(b_i+b_j)\,3\quad \pi^{3/2}}{4\sqrt{2}\;(b_i+b_j)^{7/2}} + \frac{3}{8\sqrt{2}}\frac{\pi^{3/2}}{(b_i+b_j)^{5/2}}\right]$$

$$16\left[\frac{2.61\times 4\; b_i\, b_j\, \pi^{3/2}}{(b_i+b_j)^{9/2}} - \frac{6\,\pi^{3/2}}{4\sqrt{2}\,(b_i+b_j)^{5/2}} + \frac{3\,\pi^{3/2}}{8\sqrt{2}\,(b_i+b_j)^{5/2}}\right]$$

$$= 16\left[\frac{2.61\times 4\; b_i b_j}{(b_i+b_j)^2} - \frac{3}{2\sqrt{2}} + \frac{3}{8\sqrt{2}}\right]\frac{\pi^{3/2}}{(b_i+b_j)^{5/2}}$$

$$= 16\left[\frac{2.61\times 4\; b_i b_j}{(b_i+b_j)^2} - \frac{9}{8\sqrt{2}}\right]\frac{\pi^{3/2}}{(b_i+b_j)^{5/2}}$$

$$= \frac{\pi^{3/2}}{(b_i+b_j)^{5/2}}\left[\frac{64\times 2.61\; b_i b_j}{(b_i+b_j)^2} - \frac{18}{\sqrt{2}}\right]$$

$$= \frac{2\,\pi^{3/2}}{(b_i+b_j)^{5/2}}\left[\frac{2.61\times 32\; b_i b_j}{(b_i+b_j)^2} - \frac{9}{\sqrt{2}}\right] = \psi_2^* \frac{p^2}{2m}\psi_2$$

$$\psi_1^* \frac{\hbar^2}{2m} \psi_2 = -16\, a_i\, \overline{r_1}^2 \left(\cancel{1} - 2 b_i \overline{r_1}^2 \right) e^{-2(a_i+b_i)\overline{r_1}^2}$$

$$= -16 a_i \left[\overline{r_1}^2 - 2 b_i \overline{r_1}^4 \right] e^{-2(a_i+b_i)\overline{r_1}^2}$$

$$= -16 a_i \left[\frac{3 \quad \pi^{3/2}}{8\sqrt{2}\,(a_i+b_j)^{5/2}} - \frac{2 b_i \; 3 \; \pi^{3/2}}{4\sqrt{2}\,(a_i+b_j)^{7/2}} \right]$$

$$= \frac{-16\, a_i\, \pi^{3/2}}{(a_i+b_j)^{5/2}} \left[\frac{3}{8\sqrt{2}} - \frac{\overset{3}{\cancel{6}}\, b_i}{\overset{2}{\cancel{4}}\sqrt{2}\,(a_i+b_j)} \right]$$

$$= \frac{-16\, a_i\, \pi^{3/2}}{(a_i+b_j)^{5/2}} \frac{3}{2\sqrt{2}} \left[\frac{1}{4} - \frac{b_i}{(a_i+b_j)} \right]$$

$$\cancel{= \frac{-\overset{2}{\cancel{16}}\, a_i\, \pi^{3/2}\, 3}{\cancel{8}\sqrt{2}\,(b_i+b_j)^{5/2}} \left[\frac{b_j - 3b_i}{4(b_i+b_j)} \right]}$$

$b_i + b_j - 4b_i$

$a_i + b_j - 4b_i$

$$\cancel{= \frac{3\sqrt{2}\; a_i\; \pi^{3/2}}{(b_i+b_j)^{7/2}} (b_j - 3b_i)}$$

$$= \frac{-\overset{2}{\cancel{8}}\, a_i\, \pi^{3/2}\, 3}{\sqrt{2}\,(a_i+b_j)^{5/2}} \left[\frac{a_i - 3b_i}{\cancel{4}(a_i+b_i)} \right]$$

$$\psi_1 \frac{p^2}{2m} \psi_1 =$$

$$= 16\, a_i a_j\, \bar{r}_1^2\, e^{-2(a_i+a_j)\bar{r}_1^2}$$

$$= \frac{16\, a_i a_j \cdot 3\, \pi^{3/2}}{8\sqrt{2}\,(a_i+a_j)^{5/2}}$$

$$= \frac{3\sqrt{2}\, a_i a_j\, \pi^{3/2}}{(a_i+a_j)^{5/2}}$$

Summary

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$$\Psi = \sum_{i=1}^{N_1} C_i \, e^{-a_i(\bar{r}_1^2+\bar{r}_2^2)} + \sum_{j=N_1+1}^{N} d_j \, (\bar{r}_1^2+\bar{r}_2^2) \, e^{-b_j(\bar{r}_1^2+\bar{r}_2^2)}$$

$$\Psi = \Psi_1 + \Psi_2 \Rightarrow b_1, \; \Psi_1 = \sum_{i=1}^{N_1} C_i \, e^{-a_i(\bar{r}_1^2+\bar{r}_2^2)}$$

$$\Psi_2 = \sum_{j=N_1+1}^{N} d_j \, (\bar{r}_1^2+\bar{r}_2^2) \, e^{-b_j(\bar{r}_1^2+\bar{r}_2^2)}$$

Norm $\int \Psi^* \Psi \, d\tau$

$$\int (\Psi_1+\Psi_2)^* (\Psi_1+\Psi_2)\, d\tau = \int \left(\Psi_1^*\Psi_1 + \Psi_1^*\Psi_2 + \Psi_2^*\Psi_1 + \Psi_2^*\Psi_2\right) d\tau$$

$$\Psi_1^* = \sum_{k=1}^{N_1} C_k \, e^{-a_k(\bar{r}_1^2+\bar{r}_2^2)}$$

$$\Psi_2^* = \sum_{l=N_1+1}^{N} d_l \, (\bar{r}_1^2+\bar{r}_2^2) \, e^{-b_l(\bar{r}_1^2+\bar{r}_2^2)}$$

$$\Psi_1^*\Psi_1 = \left[\frac{\pi}{2(b_1+b_2)}\right]^{3/2}$$

$$\left.\begin{matrix} b_1 = k \\ b_2 = i \end{matrix}\right] \text{widths [1]}$$

$$\Psi_2^*\Psi_2 = \frac{3}{\sqrt{2}} \frac{\pi^{3/2}}{(C_1+C_2)^{7/2}}$$

$$\left.\begin{matrix} C_1 = l \\ C_2 = j \end{matrix}\right] \text{widths 2}$$

$$\Psi_1^*\Psi_2 = \frac{3}{4\sqrt{2}} \frac{\pi^{3/2}}{(b_1+C_2)^{5/2}}$$

$$\Psi_2^*\Psi_1 = \frac{3}{4\sqrt{2}} \frac{\pi^{3/2}}{(C_1+b_2)^{5/2}}$$

Two body potential $\langle \Psi | V_2 | \Psi \rangle$

$$V_2 = C_0\, e^{-\lambda(\bar{r}_1 - \bar{r}_2)^2} = C_0\, e^{-4\lambda \bar{r}_1^2}$$

$$I = \int \Psi_1^* V_2 \Psi_1 + \int \Psi_2^* V_2' \Psi_2 + \int \Psi_1^* V_2 \Psi_2 + \int \Psi_2^* V_2 \Psi_1$$

$b_2 = i$
$b_1 = K$
$C_1 = l$
$C_2 = j$

$$\Psi_1^* V_2 \Psi_1 = \frac{\pi^{-3/2}}{(2b_1 + 2b_2 + \Lambda^2)^{3/2}}$$

$$\Psi_2^* V_2 \Psi_2 = \frac{3\qquad \pi^{3/2}}{\sqrt{2}\ (C_1 + C_2 + 0.5\Lambda^2)^{7/2}}$$

$$\Psi_1^* V_2 \Psi_2 = \frac{3}{4\sqrt{2}} \frac{\pi^{3/2}}{\left(b_1 + C_2 + \frac{\Lambda^2}{2}\right)^{5/2}}$$

$$\Psi_2^* V_2 \Psi_1 = \frac{3}{4\sqrt{2}} \frac{\pi^{3/2}}{\left(C_1 + b_2 + \frac{\Lambda^2}{2}\right)^{5/2}}$$

$\Psi_1 = b_2$
$\Psi_1^* = b_1$

$\Psi_2 = C_2$
$\Psi_2^* = C_1$

- Kinetic Part

$$\psi_1^* \frac{p^2}{2m} \psi_1 = \frac{3\, b_1\, b_2\, \pi^{3/2}}{\sqrt{2}\, (b_1 + b_2)^{5/2}}$$

$$\psi_2^* \frac{p^2}{2m} \psi_2 = 4 \sqrt{\frac{\pi}{2c_1 + 2c_2}} \left(\frac{-5}{4c_1 + 4c_2} + \frac{30\, c_1 c_2}{(c_1 + c_2)^3} \right)$$

$$\psi_1^* \frac{p^2}{2m} \psi_2 = \frac{3\sqrt{2}\, b_1\, \pi^{3/2}}{(b_1 + c_2)^{7/2}} (b_1 - 3c_2)$$

$$\psi_2^* \frac{p^2}{2m} \psi_1 = \frac{3\sqrt{2}\, b_2\, \pi^{3/2}}{(b_2 + c_1)^{7/2}} (b_2 - 3c_1)$$

Kinetic Part

$$\psi_1^* \frac{p^2}{2m} \psi_1 = \frac{3\, b_1\, b_2\, \pi^{3/2}}{\sqrt{2}\, (b_1 + b_2)^{5/2}}$$

$$\psi_2^* \frac{p^2}{2m} \psi_2 = \frac{\pi^{3/2}}{(c_1 + c_2)^{5/2}} \left[\frac{83.52\, c_1 c_2}{(c_1 + c_2)^2} - \frac{9}{\sqrt{2}} \right]$$

$$\psi_1^* \frac{p^2}{2m} \psi_2 = -\frac{3}{\sqrt{2}} \frac{b_1\, \pi^{3/2}}{(b_1 + c_2)^{7/2}} (b_1 - 3c_2)$$

$$\psi_2^* \frac{p^2}{2m} \psi_1 = -\frac{3}{\sqrt{2}} \frac{b_2\, \pi^{3/2}}{(b_2 + c_1)^{7/2}} (b_2 - 3c_1)$$