$d = {}^3S_1 = (n+p)$

$(d-d)\ ^0J=0, m_J=0$ configuration

$[[S_1 \otimes S_2]^{S=1} \otimes [S_3 \times S_4]^{S=1}]^{J=0}$ $[S=1, T=0]$

$\frac{1}{p\uparrow}\ \frac{2}{p\downarrow}\ \frac{3}{n\uparrow}\ \frac{4}{n\downarrow}$

$$\otimes|dd\rangle = \Big[\tfrac{1}{\sqrt{12}}|4231\rangle - \tfrac{1}{\sqrt{12}}|4213\rangle - \tfrac{1}{\sqrt{48}}|4141\rangle + \tfrac{1}{\sqrt{48}}|4123\rangle - \tfrac{1}{\sqrt{48}}|4132\rangle$$
$$+\tfrac{1}{\sqrt{48}}|4114\rangle - \tfrac{1}{\sqrt{12}}|2431\rangle + \tfrac{1}{\sqrt{12}}|2413\rangle + \tfrac{1}{\sqrt{48}}|2341\rangle - \tfrac{1}{\sqrt{48}}|2323\rangle + \tfrac{1}{\sqrt{48}}|2332\rangle$$
$$-\tfrac{1}{\sqrt{48}}|2314\rangle - \tfrac{1}{\sqrt{48}}|3241\rangle + \tfrac{1}{\sqrt{48}}|3223\rangle - \tfrac{1}{\sqrt{48}}|3232\rangle + \tfrac{1}{\sqrt{48}}|3214\rangle + \tfrac{1}{\sqrt{12}}|3142\rangle$$
$$-\tfrac{1}{\sqrt{12}}|3124\rangle + \tfrac{1}{\sqrt{48}}|1441\rangle - \tfrac{1}{\sqrt{48}}|1423\rangle + \tfrac{1}{\sqrt{48}}|1432\rangle - \tfrac{1}{\sqrt{48}}|1414\rangle - \tfrac{1}{\sqrt{12}}|1342\rangle$$
$$+\tfrac{1}{\sqrt{12}}|1324\rangle\Big] +$$

$t+p$ $[S=0 \otimes T=1]^{J=0}$ & $[S=1 \otimes T=0]^{J=0}$

$$|tp\rangle = \Big[\tfrac{1}{\sqrt{12}}|4231\rangle - \tfrac{1}{\sqrt{48}}|4141\rangle - \tfrac{1}{\sqrt{48}}|4132\rangle - \tfrac{1}{\sqrt{12}}|2431\rangle + \tfrac{1}{\sqrt{48}}|2341\rangle$$
$$+\tfrac{1}{\sqrt{48}}|2332\rangle - \tfrac{1}{\sqrt{48}}|3241\rangle - \tfrac{1}{\sqrt{48}}|3232\rangle + \tfrac{1}{\sqrt{12}}|3142\rangle + \tfrac{1}{\sqrt{48}}|1441\rangle$$
$$+\tfrac{1}{\sqrt{48}}|1432\rangle - \tfrac{1}{\sqrt{12}}|1342\rangle\Big] +$$
$$\Big[\tfrac{1}{\sqrt{12}}|4321\rangle - \tfrac{1}{\sqrt{12}}|4312\rangle - \tfrac{1}{\sqrt{48}}|4141\rangle + \tfrac{1}{\sqrt{48}}|4132\rangle - \tfrac{1}{\sqrt{48}}|2341\rangle$$
$$+\tfrac{1}{\sqrt{48}}|2332\rangle - \tfrac{1}{\sqrt{12}}|3421\rangle + \tfrac{1}{\sqrt{12}}|3412\rangle + \tfrac{1}{\sqrt{48}}|3241\rangle - \tfrac{1}{\sqrt{48}}|3232\rangle$$
$$+\tfrac{1}{\sqrt{48}}|1441\rangle - \tfrac{1}{\sqrt{48}}|1432\rangle\Big]$$

hence

$$\langle tp|dd\rangle = \tfrac{1}{12} + \tfrac{1}{48} + \tfrac{1}{48} + \tfrac{1}{12} + \tfrac{1}{48} + \tfrac{1}{48} + \tfrac{1}{48} + \tfrac{1}{48} + \tfrac{1}{12} + \tfrac{1}{48} + \tfrac{1}{48} + \tfrac{1}{12} = \frac{4+1+1+4+1+1+1+1+4+1+1+4}{48} = \frac{24}{48} = \frac{1}{2}$$

$$|tp\rangle = \Big[\tfrac{1}{\sqrt{12}}|4231\rangle - \tfrac{2}{\sqrt{48}}|4141\rangle + 0\,|4132\rangle - \tfrac{1}{\sqrt{12}}|2431\rangle$$
$$+0\,|2341\rangle + \tfrac{2}{\sqrt{48}}|2332\rangle + 0\,|3241\rangle - \tfrac{2}{\sqrt{48}}|3232\rangle + \tfrac{1}{\sqrt{12}}|3142\rangle$$
$$+\tfrac{2}{\sqrt{48}}|1441\rangle + 0\,|1432\rangle - \tfrac{1}{\sqrt{12}}|1342\rangle + \tfrac{1}{\sqrt{12}}|4321\rangle - \tfrac{1}{\sqrt{12}}|4312\rangle$$
$$-\tfrac{1}{\sqrt{12}}|3421\rangle + \tfrac{1}{\sqrt{12}}|3412\rangle\Big]$$

$$\langle tp|dd\rangle = \tfrac{1}{12} + \tfrac{2}{48} + 0 + \tfrac{1}{12} + 0 + \tfrac{2}{48} + 0 + \tfrac{2}{48} + \tfrac{1}{12} + \tfrac{2}{48} + 0 + \tfrac{1}{12} = \frac{4+2+4+2+2+4+2+4}{48} = \frac{24}{48} = \frac{1}{2}$$

$$|nnpp\rangle = \Big[\tfrac{1}{\sqrt{12}}|2143\rangle - \tfrac{1}{\sqrt{12}}|2134\rangle - \tfrac{1}{\sqrt{12}}|1243\rangle + \tfrac{1}{\sqrt{12}}|1234\rangle\Big]$$

$\therefore \langle nnpp|tp\rangle = 0$

$\langle nnpp|dd\rangle = 0$

$$|He\text{-}n\rangle = \Big[-\tfrac{2}{\sqrt{12}}|4213\rangle + \tfrac{1}{\sqrt{48}}|4123\rangle + \tfrac{1}{\sqrt{48}}|4114\rangle + \tfrac{1}{\sqrt{12}}|2413\rangle - \tfrac{1}{\sqrt{48}}|2323\rangle$$
$$-\tfrac{1}{\sqrt{48}}|2314\rangle + \tfrac{1}{\sqrt{48}}|3223\rangle + \tfrac{1}{\sqrt{48}}|3214\rangle - \tfrac{1}{\sqrt{12}}|3124\rangle - \tfrac{1}{\sqrt{48}}|1423\rangle$$
$$-\tfrac{1}{\sqrt{48}}|1414\rangle + \tfrac{1}{\sqrt{12}}|1324\rangle\Big] +$$
$$\Big[-\tfrac{1}{\sqrt{48}}|4123\rangle + \tfrac{1}{\sqrt{48}}|4114\rangle - \tfrac{1}{\sqrt{48}}|2323\rangle + \tfrac{1}{\sqrt{48}}|2314\rangle + \tfrac{1}{\sqrt{12}}|2143\rangle - \tfrac{1}{\sqrt{12}}|2134\rangle$$
$$+\tfrac{1}{\sqrt{48}}|3223\rangle - \tfrac{1}{\sqrt{48}}|3214\rangle + \tfrac{1}{\sqrt{48}}|1423\rangle - \tfrac{1}{\sqrt{48}}|1414\rangle + \tfrac{1}{\sqrt{12}}|1243\rangle + \tfrac{1}{\sqrt{12}}|1234\rangle\Big]$$

$$= \Big[-\tfrac{1}{\sqrt{12}}|4213\rangle + 0|4123\rangle + \tfrac{2}{\sqrt{48}}|4114\rangle + \tfrac{1}{\sqrt{12}}|2413\rangle - \tfrac{2}{\sqrt{48}}|2323\rangle$$
$$+ 0|2314\rangle + \tfrac{2}{\sqrt{48}}|3223\rangle + 0|3214\rangle + 0|1423\rangle - \tfrac{1}{\sqrt{12}}|3124\rangle$$
$$-\tfrac{2}{\sqrt{48}}|1414\rangle + \tfrac{1}{\sqrt{12}}|1324\rangle + \cancel{\tfrac{1}{\sqrt{12}}|1243\rangle} - \tfrac{1}{\sqrt{12}}0|2134\rangle - \tfrac{1}{\sqrt{12}}0|1243\rangle$$
$$\cancel{+\tfrac{1}{\sqrt{12}}|2323\rangle}\Big]$$

$$\langle He\text{-}n|pp\rangle = 0$$

$$\langle He\text{-}n|dd\rangle = \tfrac{1}{12} + \tfrac{2}{48} + \tfrac{1}{12} + \tfrac{2}{48} + \tfrac{2}{48} + \tfrac{1}{12} + \tfrac{2}{48} + \tfrac{1}{12}$$
$$= \frac{4+2+4+2+2+4+2+4}{48} = \frac{24}{48} = \frac{1}{2}.$$

$$|dqdq\rangle = \Big[-\tfrac{1}{\sqrt{48}}|1414\rangle + \tfrac{1}{\sqrt{48}}|1423\rangle - \tfrac{1}{\sqrt{48}}|1432\rangle + \tfrac{1}{\sqrt{48}}|1441\rangle + \tfrac{1}{\sqrt{48}}|2314\rangle$$
$$-\tfrac{1}{\sqrt{48}}|2323\rangle + \tfrac{1}{\sqrt{48}}|2332\rangle - \tfrac{1}{\sqrt{48}}|2341\rangle - \tfrac{1}{\sqrt{48}}|2314\rangle + \tfrac{1}{\sqrt{48}}|3223\rangle - \tfrac{1}{\sqrt{48}}|3232\rangle$$
$$+\tfrac{1}{\sqrt{48}}|3241\rangle + \tfrac{1}{\sqrt{48}}|1414\rangle - \tfrac{1}{\sqrt{48}}|1423\rangle + \tfrac{1}{\sqrt{48}}|1432\rangle - \tfrac{1}{\sqrt{48}}|1441\rangle\Big]$$

$$\langle dqdq|dd\rangle = \cancel{\tfrac{1}{48}} - \cancel{\tfrac{1}{48}} - \cancel{\tfrac{1}{48}} + \cancel{\tfrac{1}{48}} - \cancel{\tfrac{1}{48}} + \cancel{\tfrac{1}{48}} + \cancel{\tfrac{1}{48}} - \cancel{\tfrac{1}{48}} + \tfrac{1}{48} + \tfrac{1}{48}$$
$$+\cancel{\tfrac{1}{48}} - \cancel{\tfrac{1}{48}} + \cancel{\tfrac{1}{48}} - \cancel{\tfrac{1}{48}} - \cancel{\tfrac{1}{48}} + \cancel{\tfrac{1}{48}} = \tfrac{2}{48} = \boxed{\tfrac{1}{24}}$$

$$|SS\rangle = \sum_k c_k |\tfrac{1}{2}\, m_1\rangle \ldots |\tfrac{1}{2}\, m_N\rangle$$