Universality of Few-body Systems in the Framework of Effective Field Theory

1st Annual Progress Report

by

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Chapter 1

Introduction

Numerous experimental and theoretical investigations have confirmed that the nuclei of an atom comprise neutrons and protons and their quantity in every atom contrasts [1] [2]. For instance, an isotope of a Hydrogen(H) atom, Deuterium comprises one proton and one neutron while Triton, another isotope of a Hydrogen atom comprises one proton and two neutrons. Each nucleus has a wide range of cluster structures and its inclination relies upon different factors such as structure and strength of connection powers, the kind of nucleus and excitation levels of this nucleus, and so forth [3]. To concentrate on different levels of nuclei, various models are utilized. For example, the shell model and the α -particle the model which portrays the ground state of a nucleus having a whole number of nucleons of α-particle ($Be^8, C^{12} and O^{16}$). In this work, the various models in the casing of a summed up shell model, the "cluster model" is perceived by which numerous significant properties of a nucleus can be determined.

In few-body physics, the scattering of particles is a fundamental process that implies many of the properties of atomic nuclei, and understanding this process is crucial for developing and comprehensive understanding of nuclear structure and reactions. The Resonating -Group Method (RGM) is particularly well-suited for studying scattering in few-body physics because it allows for the simulation of systems with multiple inter-

acting particles. Each particle is represented by a resonating group, which is characterized by a set of wave functions that describe its internal state. The interactions between particles are modeled by a set of coupling parameters, which are determined by the underlying nuclear forces.

To study the universality of few-body systems, we are dealing with two clusters system with an identical number of particles in an energy regime where the relative motion between the clusters is small in comparison to the excitation scale of each cluster. We employ the model-independent framework of pionless Effective Field Theory ($\not{\pi}$EFT) via the computational framework of a Resonating Group Method (RGM) to extract possible universal signatures of observables in systems with three or more constituent particles [4][5]. The ultimate goal of such an endeavor is to understand the underlying binding mechanisms of light nuclei ($A \geq 4$), attributing either to specific consequences of their substructure or emergent universal characteristics that follow from the certain separation of scales. In order to achieve this goal we began by first studying the behavior of dimer-dimer scattering, each consisting of two-component fermions. Here we obtained the universal result that the dimer-dimer scattering length is 0.6 times the fermion-fermion scattering length, irrespective of the short-distance nature of the fermion-fermion interaction potential [6]. As the next step, we aim to extend this study to trimer-trimer, as well as tetramer-tetramer scattering. According to a proper viewpoint, the consequences of this work will refine and extend the framework which connects efficiently the *Standard Model* (SM) of molecular physics with atomic/nuclear theories. This framework has been successful in relating two-, three-, and four-body properties to SM parameters like the quark masses.

Chapter 2

Methodology : Resonating-Group Formalism

2.1 Introduction

Resonating-group method is a microscopic method in which cluster correlations are taken into account. The RGM method treats a nucleus as a collection of overlapping clusters or "resonating groups", each of which has a specific energy and angular momentum [7]. These clusters are allowed to interact with one another through the exchange of particles which gives symmetric or anti-symmetric wave function based on the constituents particles and also leads to the formation of new, more complex clusters. Its feature of using fully anti-symmetric functions, treating the motion of the total center of mass correctly, and capability of dealing with both bound and scattering states, distinguish it from other methods used to study nuclear physics.

The fundamental idea behind this formalism is that nucleons spend a fraction of their time in various substructures or clusters. The RGM formalism provides a way to calculate the properties of a nucleus, such as its energy levels, decay modes, and reaction cross-sections, based on the interactions between its constituent particles.

2.2 Formulation of Resonating-Group Method

The time-independent Schrodinger equation is

$$(H - E)\psi = 0 \tag{2.1}$$

which can be represented in the form of a projection equation as

$$\langle \delta\psi | H - E | \psi \rangle = 0 \tag{2.2}$$

where E is the total energy of the system, $\delta\psi$ represents arbitrary variation in the space of all many-nucleon functions, and H is a Hamiltonian operator.

$$H = \sum_{i=1}^{N} \frac{1}{2M} p_i^2 + \sum_{i<j=1}^{N} V_{ij} - T_{cm} \tag{2.3}$$

where N represents the total number of nucleons, T_{cm} being the kinetic energy operator, V_{ij} is nucleon-nucleon potential.

In the general form, the total trial wave function in resonating group method can be expressed in the following form

$$\psi = \mathcal{A}\Big\{\Sigma_i \phi(A_i)\phi(B_i)F_i(R_i) + \Sigma_j \phi(A_j)\phi(B_j)\phi(C_j)F_j(R_{j1}, R_{j2}) + ... + \Sigma_m c_m \eta_m\Big\} F(R_{cm}) \tag{2.4}$$

$\mathcal{A}$ is an anti-symmetrization operator given by

$$\mathcal{A} = \sum_p (-1)^p P \tag{2.5}$$

where the sum extends over all permutations P which can be carried out on the nucleon coordinates, and p is the number of interchanges that make up the permutation P. Each term corresponds to a particular fragmentation of the particles into two, three, four,... cluster. The internal motion of such a cluster of A particles is expanded with a complete set of functions $\phi(A_i)$, and the relative motion between of the clusters is encoded in the $F_i's$.

2.2.1 Two-Cluster System

For instance, let us take the simplest case of two cluster system with no specific distortion having total A no. of particles. The wave function for two cluster system can be written as

$$\psi_2 = \mathcal{A}[\phi(A)\phi(B)F(R)Z(R_{(cm)})] \tag{2.6}$$

In parameter representation

$$\psi_2 = \mathcal{A}[\phi(A)\phi(B)\delta(R - R'')Z(R_{(cm)})]F(R'')dR'' \tag{2.7}$$

where R'' is a parameter coordinate.

$$\delta\psi = \int [\mathcal{A}[\phi(A)\phi(B)\delta(R - R'')Z(R_{(cm)})]\delta F(R'')dR''] \tag{2.8}$$

which gives the following equation

$$\int [\mathcal{H}(R', R'') - E\mathcal{N}(R', R'')]F(R'')dR'' = 0 \tag{2.9}$$

where,

$$\mathcal{H}(R', R'') = \langle \phi(A)\phi(B)\delta(R-R'')Z(R_{(cm)})|H|\mathcal{A}[\phi(A)\phi(B)\delta(R-R'')Z(R_{(cm)})]\rangle \tag{2.10}$$

$$\mathcal{N}(R', R'') = \langle \phi(A)\phi(B)\delta(R - R'')Z(R_{(cm)})|\mathcal{A}[\phi(A)\phi(B)\delta(R - R'')Z(R_{(cm)})]\rangle \tag{2.11}$$

Anti-symmetrization operator can be redefined as

$$\mathcal{A} = \mathcal{A}'\mathcal{A}_{\mathcal{A}}\mathcal{A}_{\mathcal{B}} \tag{2.12}$$

where $\mathcal{A}_{\mathcal{A}}$ and $\mathcal{A}_{\mathcal{B}}$ are anti-symmetrization operators for the nucleons in clusters A and B and $\mathcal{A}'$ is anti-symmetrization operator which interchanges nucleons in different clusters. defining further

$$\mathcal{A}' = 1 + \mathcal{A}'' \tag{2.13}$$

separates $\mathcal{H}(R', R'')$ and $\mathcal{N}(R', R'')$ into two parts

$$\mathcal{H}(R', R'') = \mathcal{H}_D(R', R'') + \mathcal{H}_E(R', R'') \tag{2.14}$$

$$\mathcal{N}(R', R'') = \mathcal{N}_D(R', R'') + \mathcal{N}_E(R', R'') \tag{2.15}$$

where $\mathcal{H}_D(R', R'')$ and $\mathcal{N}_D(R', R'')$ are direct parts and $\mathcal{H}_E(R', R'')$ and $\mathcal{N}_E(R', R'')$ represents exchange parts respectively.

$$\mathcal{N}_D(R', R'') = \langle \phi(A)\phi(B)\delta(R - R')Z(R_{(cm)})|\phi(\hat{A})\phi(\hat{B})\delta(R - R'')Z(R_{(cm)})\rangle \tag{2.16}$$

$$\mathcal{N}_E(R', R'') = \langle \phi(A)\phi(B)\delta(R-R')Z(R_{(cm)})|\mathcal{A}''[\phi(\hat{A})\phi(\hat{B})\delta(R-R'')Z(R_{(cm)})]\rangle \tag{2.17}$$

$$\mathcal{H}_D(R', R'') = \langle \phi(A)\phi(B)\delta(R - R')Z(R_{(cm)})|H|\phi(\hat{A})\phi(\hat{B})\delta(R - R'')Z(R_{(cm)})\rangle \tag{2.18}$$

$$\mathcal{H}_E(R', R'') = \langle \phi(A)\phi(B)\delta(R-R')Z(R_{(cm)})|H|\mathcal{A}''[\phi(\hat{A})\phi(\hat{B})\delta(R-R'')Z(R_{(cm)})]\rangle \tag{2.19}$$

so this will take form

$$\left[-\frac{\hbar^2}{2\mu}\Delta^2_{R'} + V_D(R') - E_R \right] F(R') + \int K(R', R'')F(R'')dR'' = 0 \tag{2.20}$$

where E is the relative energy of the two clusters in c.o.m. system

$$E_R = E - E_A - E_B \tag{2.21}$$

energy dependent kernel K(R',R") is given by

$$K(R', R'') = \mathcal{H}_E(R', R'') - E\mathcal{N}_E(R', R'') \tag{2.22}$$

Resonating-group calculations are generally rather difficult to perform when the number of nucleons involved in the system is large. Thus, even though there was a substantial number of such calculations done in the sixties [8], these were mostly simple extensions and modifications of previously reported works. Indeed, it was frequently remarked [9] that the requirement of anti-symmetrization causes insurmountable computational

difficulties and, therefore, the resonating-group method cannot be expected to be useful in studying systems containing more than eight nucleons. This is, of course, no longer true. With the development of generator-coordinate techniques for continuum studies, it now becomes possible to perform scattering calculations for rather large systems, such as the scattering of ^{16}O by ^{40}Ca [10]and so on.

2.2.2 Complex-Generator-Coordinate Technique

To compute the kernel functions complex-generator-coordinate technique is introduced. For the sake of simplicity, let us take two-cluster A and B having number of particles N_A and N_B respectively. The form of a two-cluster system

$$\psi = \mathcal{A}[\phi(A)\phi(B)F(R_A - R_B)Z(R_{cm})] \tag{2.23}$$

where,

$$\phi(K) = \bar{\phi}(K)\xi_k(s_k, t_k)$$

where K represents clusters (K= A and B), ξ_k is spin-isospin function and $\bar{\phi}(K)$ are spatial parts of the cluster internal functions

$$\bar{\phi}(A) = \Pi_{j=1}^{N_A}\phi_j(r_j - R_A) \tag{2.24}$$

$$\bar{\phi}(B) = \Pi_{k=N_A+1}^{N_A+N_B}\phi_k(r_k - R_B) \tag{2.25}$$

with CM of both clusters are;

$$R_A = \frac{1}{N_A}\sum_{j=1}^{N_A} r_j \quad R_B = \frac{1}{N_B}\sum_{k=N_A+1}^{N_A+N_B} r_k \tag{2.26}$$

Solution of the resonating-group equation:

$$\sum_{n=1}^{N_{loc}} \hat{\eta}_n e^{-w_n R^2}\chi(R) - \sum_{n=1}^{N_{n-loc}} \int \left\{\hat{\zeta}_n e^{-a_n R^2 - b_n R\cdot R' - c_n R^2}\right\}\chi(R')dR' = 0 \tag{2.27}$$

where $\hat{\eta}_n, \hat{\zeta}_n, w_n, a_n, b_n, c_n$ depends upon $C(\lambda), D(\lambda), \alpha, \lambda, E, A, B$. Partial wave l,

$$
\begin{aligned}
0 = & \left(\frac{\hbar^2}{2\mu}\Big[- \partial_R^2 + \frac{l(l+1)}{R^2}\Big] - E\right)\phi_{lm}(R) + \sum_{n=2}^{N_{loc}} \eta_n e^{-w_n R^2}\phi_{lm}(R) \\
& - \int dR' \phi_{lm}(R')(4\pi RR')\Big[\zeta_1 e^{-a_1 R^2 - c_1 R'^2}\Big\{\Big[- (4a_1^2 R^2 + b_1^2 R'^2 - 2a_1) + \frac{l(l+1)}{R^2} \\
& - \frac{2\mu}{\hbar^2}E\Big] i^l j_l(ib_1 RR') - (4a_1 b_1)RR' \sum_L i^L j_L(ib_1 RR')\hat{L}\hat{l}\begin{pmatrix} L & 1 & l \\ 0 & 0 & \end{pmatrix}\Delta_{L1l}\Big\} \\
& + \sum_{n=2}^{N_{n-loc}} \zeta_n i^l j_l(ib_n RR') e^{-a_n R^2 - c_n R'^2}\Big]
\end{aligned}
\tag{2.28}
$$

Chapter 3

Pionless Effective Field Theory

3.1 Introduction

Effective field theories (EFTs) provide a thorough framework for the study of low-energy physics. In this context, low energy is defined in relation to a particular energy scale, denoted by Λ. EFTs utilize a subset of the full range of degrees of freedom available in a given physical system, focusing only on those that are relevant at low energies. Within the EFT framework, the interactions between the relevant degrees of freedom are described using a full field-theoretic approach. However, only states with energies much lower than the energy scale Λ (i.e., with $\Lambda \gg m$) are taken into account, while the contributions of heavier excitations with energies much larger than Λ (i.e., with $M \gg \Lambda$) are integrated out from the action. This allows for a more efficient and accurate description of physical systems at low energies [4] [5].

In nuclear physics, EFTs are low-energy manifestations of the underlying theory, QCD. As such, they are formulated in terms of baryons and mesons as the fundamental degrees of freedom, rather than quarks and gluons.

On particular interest is pionless EFT ($\not{\pi}$ EFT), which is the simplest possible nuclear EFT, having the mesons integrated out leaving the nucle-

ons as the only degrees of freedom. Pionless EFT is a standard EFT for low-energy nuclear systems which are insensitive to the details of mechanisms associated with the exchange of pions. At very low energies-i.e., for momenta $Q \ll m_\pi$ few-nucleon systems are not sensitive to the details associated with pion (or another meson) exchange. This fact makes it best suited to explore universal physics [11].

Two very-low-energy particles, represented by a field ψ, can be described by an effective Lagrangian[12]

$$\mathcal{L} = \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 + \frac{C_2}{16}[(\psi\psi)^\dagger(\psi \overleftrightarrow{\nabla}^2\psi) + H.c.] + ... \quad (3.1)$$

where $\overleftrightarrow{\nabla} = \overleftarrow{\nabla} - \overrightarrow{\nabla}$ is the Galilei- invariant derivative and H.c. denotes the Hermitian conjugate. The "..." represents local operators with other combinations of derivatives, including relativistic corrections. C_0, C_2 represents low energy constants.

3.2 RGM formalism in the framework of pionless EFT

Pionless effective field theory is a powerful tool used in nuclear physics to describe low energy nuclear physics in a systematic way without explicitly including the pion degrees of freedom which are typically associated with the strong force between nucleons. One important application of pionless EFT is in the study of few-body systems where the interactions between the nucleons are strong and complex. The combination of pionless EFT and RGM provides a powerful theoretical framework for studying few-body nuclear systems [13]. In this approach, the nucleon-nucleon interaction is described in terms of contact interaction that depend on the relative distance of the clusters and the spin state of the interacting nucleon cluster. The use of contact interaction is justified by the fact that the inter-

action between the nucleons at low energies is dominated by short-range forces only.

The power counting scheme suggests only two-body s-wave interactions at leading order LO. The relevant Lagrangian can be represented as follows:

$$\mathcal{L}_{LO} = N^{\dagger}\left(i\partial_0 + \frac{\nabla^2}{2m}\right)N - \frac{1}{2}C_0^{(0)}(N^{\dagger}N)^2 - \frac{1}{2}C_1^{(0)}(N^{\dagger}N)^2 \tag{3.2}$$

where $N^{\dagger}$ and N represents nucleon fields while $C_0^{(0)}$ and $C_1^{(0)}$ are the two-body low-energy constants.

A three-body contact term in the Lagrangian is the form of [14]

$$\mathcal{L}_{LO}^{(3b)} = -\frac{1}{6}D^{(0)}(N^{\dagger}N)^3 \tag{3.3}$$

where $D^{(0)}$ is the three-body LEC.

At LO, Hamiltonian for non-relativistic Schrodinger equation takes the form

$$H_{LO} = -\frac{1}{2m}\sum_i \nabla_i^2 + V_2^{(0)} + V_3^{(0)} \tag{3.4}$$

where $V_2^{(0)}$ and $V_3^{(0)}$ are LO two- and three-body contact potentials, respectively.

For a two-cluster system, contact interaction potential is

$$\mathcal{V}_{AB} = C(\lambda)\sum_{i\epsilon A, j\epsilon B}\delta_\lambda^{(3)}(r_i - r_j) + D(\lambda)\sum_{i,j,k,i\epsilon A => jvk\epsilon B}\delta_\lambda^{(3)}(r_i - r_j)\delta_\lambda^{(3)}(r_i - r_k) \tag{3.5}$$

The singular nature of contact interactions requires regularization, which is performed by applying a local Gaussian regulator that suppresses momenta above an ultraviolet cutoff. Physical quantities predicted by the theory have to be independent of the cutoff since it is not physical quantity. This is achieved via renormalization, i.e. by fitting the values of the LECs to run with the cutoff in such a way that a chosen set of physical observables is reproduced. Regularization for one Cartesian dimension

$$\delta^{(1)}(x) = \lim_{\lambda\to\infty}\sqrt{\frac{\lambda}{\pi}}e^{-\lambda x^2} \tag{3.6}$$

The regularised LO two-body potential is

$$V_2^{(0)} = \sum_{i<j} C(\lambda) g_\lambda(r_{ij}) \tag{3.7}$$

and three-body potential

$$V_3^{(0)} = \sum_{i<j<k} \sum_{cyc} D(\lambda) g_\lambda(r_{ij}) g_\lambda(r_{ik}) \tag{3.8}$$

where $g_\lambda(r) \propto exp(-\frac{\lambda^2 r^2}{4})$ is the chosen regulator, cyc denotes the cyclic sum, and $r_{ij} = |r_i - r_j|$ is a relative distance between nucleons i and j. As $\lambda \to \infty$, the contact nature of the interaction is recovered.

Chapter 4

Dimer-Dimer Scattering

The RGM method can be used to investigate the universality in the case of dimer-dimer scattering where the term 'dimer' alludes to a cluster having two atoms and it is the statistics, specifically Bose and Fermi dynamics, followed by the particles that choose the way of behaving of the dimer. Consider a framework having two dimers and every dimer has atoms of the same mass. All possibilities of this kind of system can be represented as follows:

$$(ab) : (cd) \tag{4.1}$$

$$(ab) : (ab) \tag{4.2}$$

$$(ab) : (ac) \tag{4.3}$$

where a,b,c, and d represent the atoms of different kinds.

For the scattering of two dimers A and B with atoms N_A and N_B respectively, the two-fragment resonating group equation is

$$\int \{\phi_A^* \phi_B^* (-\frac{\hbar^2}{2\mu}\Delta_R - E + \mathcal{V}_{AB})\mathcal{A}_{AB}[\phi_A \phi_B \chi(R)]\} dr_{A,B}^{internal} = 0 \tag{4.4}$$

where the fragment-internal wave function $\phi_{A,B}$ with a Gaussian basis:

$$\phi_A = \sum_n^{N_{f_1}} c_n Exp[-\alpha_n \sum_{i=1}^{N_A} (r_i - R_A)^2] \tag{4.5}$$

$$\phi_B = \sum_{n}^{N_{f_2}} c_n Exp[-\alpha_n \sum_{i=1}^{N_B} (r_i - R_B)^2] \tag{4.6}$$

with,

$N_{A,B}$ = Number of particles in fragment A

$N_{N_{f_{1/2}}}$ = Basis dimension

r_i = Single-particle coordinates

$R_{A/B}$ = Center of mass of fragments A and B respectively

The norm of this function

$$\langle \phi_A | \phi_A \rangle = \sum_{i=1}^{N_f} C_i C_j \left(\frac{\pi}{\alpha_i + \alpha_j} \right)^{\frac{3}{2}} \tag{4.7}$$

If the two fragments contain identical particle species, antisymmetrization is required between these,

$$\mathcal{A}_{AB} = \mathbf{1} + \sum_{p} (-1)^p P \tag{4.8}$$

The inter-cluster antisymmetrizer comprises elements P of the symmetric group of the particle labels in clusters A and B.

We commence with the four-body system, dimer-dimer scattering therein. There are two scenarios that embody the transition between Bose and Fermi dynamics, namely a system of two dimers, each one composed of the same two species of a fermion, denoted by (ab:ab). Such a system is by all, we know scale-invariant, which means that its low-energy behavior is independent of λ, and does not require the additional three-body renormalization constraint manifests in $D(\lambda)$. These dimers are weakly two-body s-wave bound states which is parameterized by scattering length a and the normalized wave function $\phi(r)$ for the relative motion of the two particles forming the dimers is given by

$$\phi(r) = \frac{e^{-\frac{r}{a}}}{\sqrt{2\pi a r}} \tag{4.9}$$

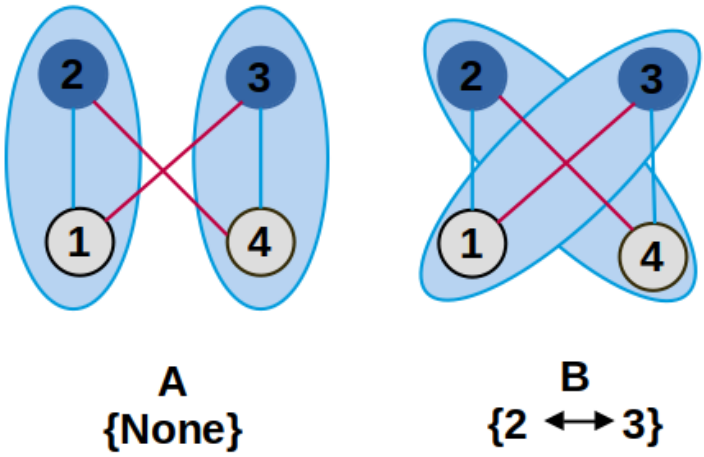

Figure 4.1: Schematic representation of the two permutations of identical fermions between two dimers.

Assuming that there is no interaction between the two identical fermions and also exchange only possible for identical fermions. The antisymmetrization operator in the calculation is obtained by considering all possible permutations of identical fermions but for this case only two permutations is possible as shown in figure (4.1).

This arrangement has been studied very thoroughly and indeed, the remarkable universality of the ratio between dimer-dimer and fermion-fermion attraction measured by scattering lengths $a > 0$ – has been numerically discovered [6].

$$\frac{a_{dd}}{a_{ff}} \approx 0.6 \tag{4.10}$$

Thus, the interaction (3.5) takes the form

$$\mathcal{V}_{AB} = C(\lambda)\delta^{(3)}_{\lambda}(r_1 - r_4) + C(\lambda)\delta^{(3)}_{\lambda}(r_2 - r_3) \tag{4.11}$$

The parameter representation $\chi(R) = \int dR'\delta^{(3)}(R - R')\chi(R')$ allows for a translation of the inter-cluster antisymmetrizer $\mathcal{A}_{AB}$ into a non-local integrodifferential equation which, in general, assumes the form

$$(\hat{T} - E)\chi(r) + \mathcal{V}^{(1)}(r)\chi(r) + \int d^{(3)}r'\mathcal{V}^{(2)}(r, r', E)\chi(r') = 0 \tag{4.12}$$

with the radial coordinates denoting the spatial separation between the two fragment's mass centers.

If these fragments are bound states of the spectrum of the single-particle interaction and if the energy of the relative motion between these fragments E is small relative to their binding and excitation energies, it is in order to attach physical meaning to the equation of motion which follows after the averaging over internal degrees of freedom. That being, an approximation of the scattering characteristics of the two clusters. The coefficients in (2.27) are listed in Table-4.1.

η	w
$\frac{C(\lambda)}{(4\lambda a_2+4a_1(\lambda+2b_1+2a_2)+4\lambda b_2 +8a_2b_2+4b_1(\lambda+2b_2))^{3/2}}$	$\frac{2\lambda(a_1+b_2)(b_1+a_2)}{(\lambda a_2+a_1(\lambda+2b_1+2a_2)+\lambda b_2+2a_2b_2+b_1(\lambda+2b_2))}$
$\frac{C(\lambda)}{(4\lambda a_2+4a_1(\lambda+2b_1+2b_2)+4\lambda b_2 +8a_2b_2+4b_1(\lambda+2a_2))^{3/2}}$	$\frac{2\lambda(a_1+a_2)(b_1+b_2)}{(\lambda a_2+b_1(\lambda+2a_2)+\lambda b_2+2a_2b_2+a_1(\lambda+2b_1+2b_2))}$

Table 4.1: Local (η, w) component of the resonating group equation for a 2-2 partition of four 2-flavour fermions.

Coefficients for non-local potential :

$$\zeta_1 = -\frac{\hbar^2}{2\mu}\frac{2}{2(a_1+a_2+b_1+b_2)^{3/2}} \tag{4.13}$$

$$a_1 = \frac{4a_2b_2+b_1a_2+b_1b_2+a_1a_2+a_1b_2}{2(a_1+a_2+b_1+b_2)} \tag{4.14}$$

$$b_1 = \frac{b_1a_2-b_1b_2-a_1a_2+a_1b_2}{(a_1+a_2+b_1+b_2)} \tag{4.15}$$

$$c_1 = \frac{b_1a_2+b_1b_2+4a_1b_1+a_1a_2+a_1b_2}{2(a_1+a_2+b_1+b_2)} \tag{4.16}$$

$$\zeta_2 = -\frac{2C(\lambda)}{(4\lambda+2a_1+2a_2+2b_1+2b_2)^{3/2}} \tag{4.17}$$

$$a_2 = \frac{4b_2(2\lambda + a_2) + b_1(2\lambda + a_2 + b_2) + a_1(2\lambda + a_2 + b_2)}{4\lambda + 2a_1 + 2a_2 + 2b_1 + 2b_2} \tag{4.18}$$

$$b_2 = \frac{b_1(4\lambda + 2a_2 - 2b_2) + a_1(-4\lambda - 2a_2 + 2b_2)}{4\lambda + 2a_1 + 2a_2 + 2b_1 + 2b_2} \tag{4.19}$$

$$c_2 = \frac{b_1(2\lambda + a_2 + b_2) + a_1(2\lambda + 4b_1 + a_2 + b_2)}{4\lambda + 2a_1 + 2a_2 + 2b_1 + 2b_2} \tag{4.20}$$

$$\zeta_3 = -\frac{2C(\lambda)}{(4\lambda + 2a_1 + 2a_2 + 2b_1 + 2b_2)^{3/2}} \tag{4.21}$$

$$a_3 = \frac{4a_2(2\lambda + b_2) + b_1(2\lambda + a_2 + b_2) + a_1(2\lambda + a_2 + b_2)}{4\lambda + 2a_1 + 2a_2 + 2b_1 + 2b_2} \tag{4.22}$$

$$b_3 = \frac{b_1(2a_2 - 4\lambda - 2b_2) + a_1(4\lambda + 2b_2 - 2a_2}{4\lambda + 2a_1 + 2a_2 + 2b_1 + 2b_2} \tag{4.23}$$

$$c_3 = \frac{b_1(2\lambda + a_2 + b_2) + a_1(2\lambda + 4b_1 + a_2 + b_2)}{4\lambda + 2a_1 + 2a_2 + 2b_1 + 2b_2} \tag{4.24}$$

After some straightforward calculations of the direct and exchange potentials as well as exchange kernel of RGM equation and taking two masses of the fermion mass same i.e M = m it will give $a_{dd} = 0.6a_{aa}$ where a_{dd} and a_{aa} are the scattering length between the two dimers and two atoms respectively . In my ongoing research work I established this result numerically at low- energy regime using different cut-off scale which looks as shown in the fig. (4.2)

4.1 Future Plan

We have studied the interaction behavior between two dimers by dimer-dimer scattering, where each dimer consists of two-component fermions and we obtained the universal result i.e. dimer-dimer scattering length is 0.6 times the fermion-fermion scattering length, irrespective of the short-distance nature of fermion-fermion interaction potential.

I will try to extend my observation to the systems with more nucleons as trimer-trimer, as well as tetramer-tetramer scattering. In addition to that, I will try to extend the same results for charged particles, including long-range coulomb forces in my calculations.

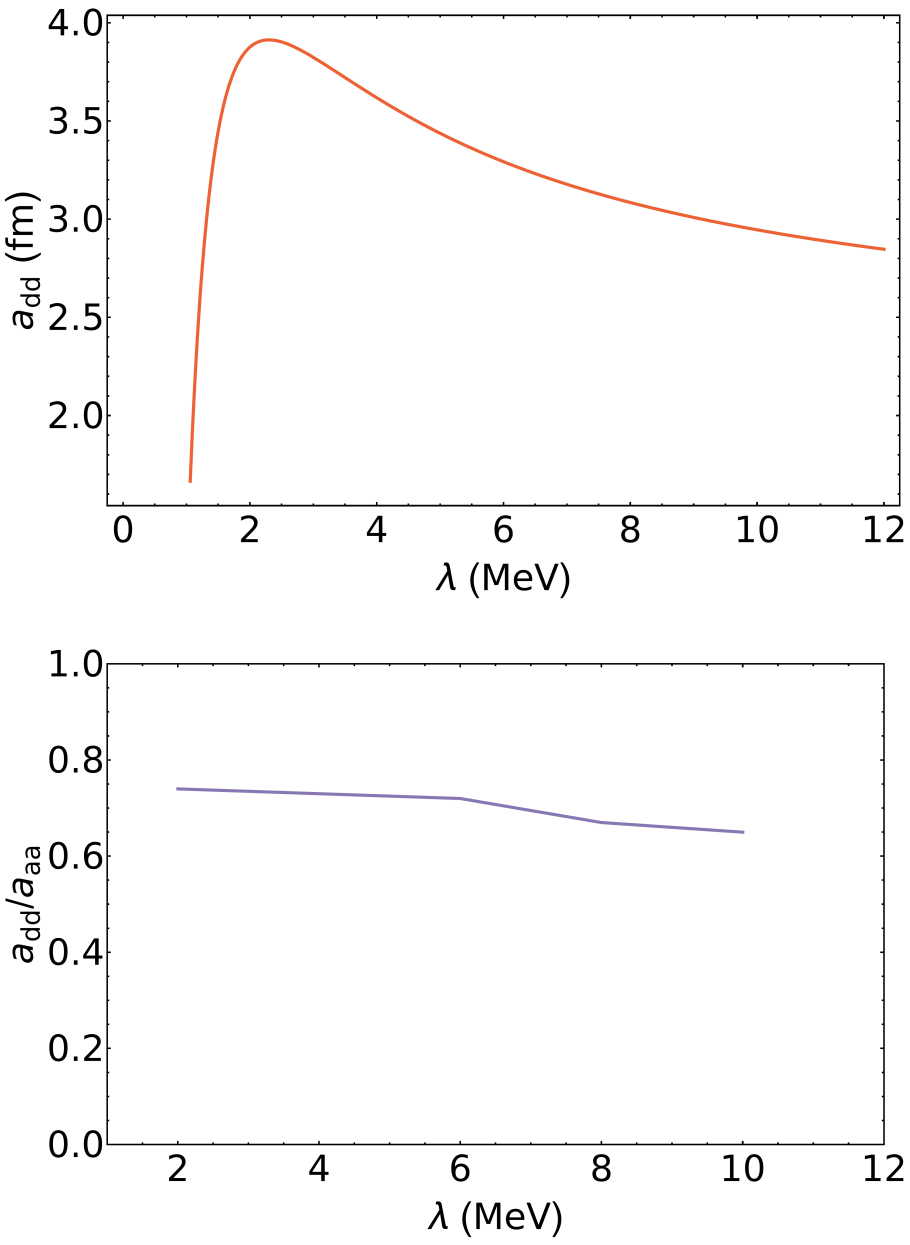

Figure 4.2: Dependence of dimer-dimer scattering length upon Cut-off scale λ.

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