Universality of the multi-channel 4-body scattering system

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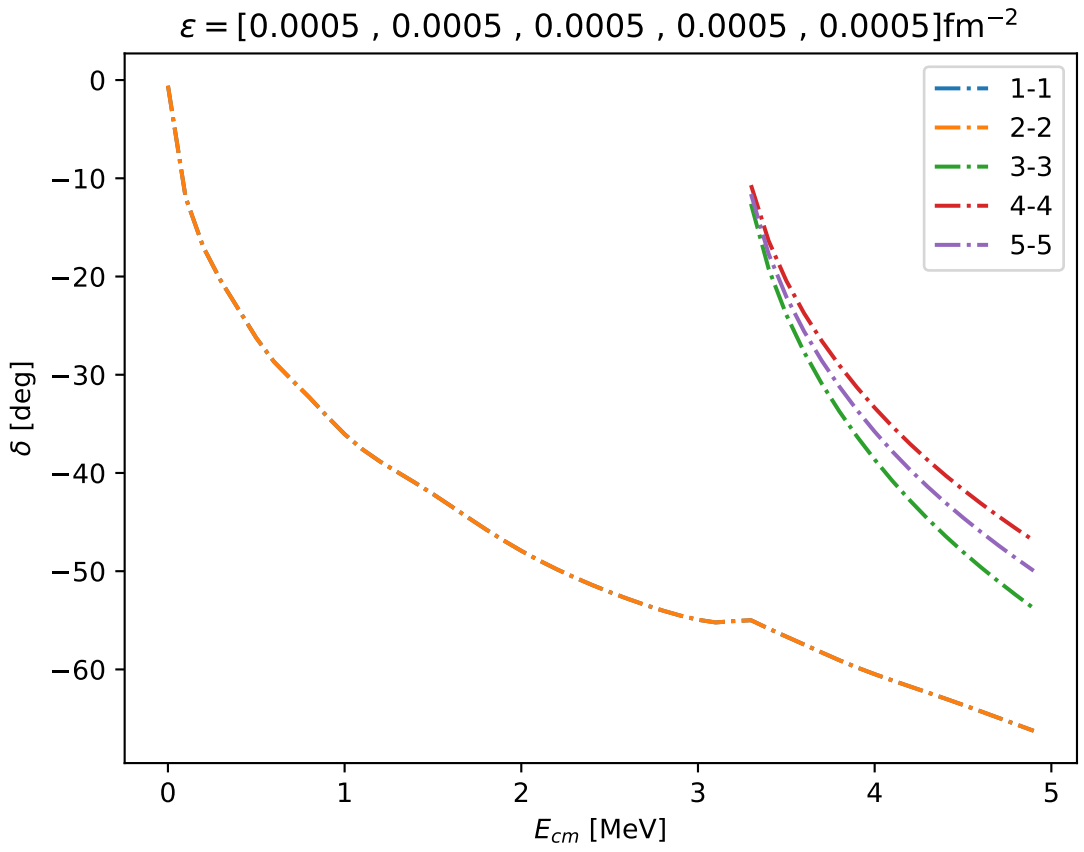

FIG. 1: Energy dependence of phase shifts which parameterize the coupled channel nnpp system in the $^1S_0$ α channel (1).

Abstract

We investigate the scattering system of 4 equal-mass quantum particles at energies where rearrangement channels are open. The interactions are renormalized to capture the essence of the pertinent nuclear 2-neutron, 2-proton system. A full treatment of the Coulomb interaction is included.

The quantity of most practical interest, namely the coupling between the deuteron-deuteron and the ^{3}H-proton/^{3}He-neutron channels, is subjected to a sensitivity analysis with respect to distorted, *i.e.* , screened Coulomb repulsion between the two protons.

a. Few is more The fundamental question of interest is on the behaviour of 4 identical spin-1/2 particles which can occupy 2 different isospin – to stress the significance to systems other than nuclei and mesons, we will use the more widely used notion of a flavour to discriminate internal states of a fermion – states. We limit the investigation to the experimental most relevant 2-fragment asymptotic configurations. These are defined by all partitions of the $N = 4$ particles into 2 clusters whose spectrum contains bound states.

Assuming zero-range, flavour[†] and spin-independent interactions

the scattering process is parametrized via a 3-channel S-matrix:

$$S_{ij} = \left\langle a\, L_a\, S_a \,\middle|\, \hat{S}^{J^\pi} \,\middle|\, b\, L_b\, S_b \right\rangle = \eta_{ij}\, e^{2i\delta} \quad , \tag{1}$$

and the almost decoupled d-d channel is encoded in $\eta_{dd} \approx 1$. Although, the Coulomb repulsion between protons

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[†]In nuclear physics the term isospin is more common to discriminate between internal states of a particle, *e.g.* , the neutron and proton, or the three charge states of a π meson.

provides a heuristic argument for this weak coupling, the comparison to the relatively strong coupling between the two 3-1 fragmentations seems to defy the argument as an equally strong force keeps the proton out of 3-helium.

b. Spin-wave-function overlap

$$
\begin{array}{r c}
 & \begin{array}{ccccccc} \mid \text{t-p}_1 \rangle & \mid \text{t-p}_6 \rangle & \mid \text{he-n}_1 \rangle & \mid \text{he-n}_6 \rangle & \mid \text{d-d} \rangle & \mid \text{d}_q - \text{d}_q \rangle & \mid \text{nn-pp} \rangle \end{array} \\
\begin{array}{r} \langle \text{t-p}_1 \mid \\ \langle \text{t-p}_6 \mid \\ \langle \text{he-n}_1 \mid \\ \langle \text{he-n}_6 \mid \\ \langle \text{d-d} \mid \\ \langle \text{d}_q - \text{d}_q \mid \\ \langle \text{nn-pp} \mid \end{array} &
\left(\begin{array}{ccccccc}
6 & & & & & & \\
-6 & 6 & & & & & \\
-2 & +2 & 0.66 & & & & \\
-6 & +6 & +2 & 6 & & & \\
+8.5 & -8.5 & -2.8 & -8.5 & 12 & & \\
-4.9 & +4.9 & +1.6 & +4.9 & -6.9 & 4 & \\
-4.9 & +4.9 & +1.6 & +4.9 & -6.9 & +4 & 4
\end{array} \right)
\end{array}
$$

c. Two-fragment approximation We expand a fragment state, $\langle\, \bar{\boldsymbol{r}}_1 \ldots \bar{\boldsymbol{r}}_\text{A} \mid \phi \,\rangle$, which binds A particles in a Gaussian basis whose elements, $\langle\, \bar{\boldsymbol{r}}_1 \ldots \bar{\boldsymbol{r}}_\text{A} \mid m \,\rangle$, are parametrized by A width parameters $\alpha^m_{1\ldots A}$:

$$
\langle\, \bar{\boldsymbol{r}}_1 \ldots \bar{\boldsymbol{r}}_\text{A} \mid \phi \,\rangle = \sum_m c_m \cdot \langle\, \bar{\boldsymbol{r}}_1 \ldots \bar{\boldsymbol{r}}_\text{A} \mid m \,\rangle = \sum_m c_m \cdot e^{-\sum_i^A \alpha_i^m \bar{\boldsymbol{r}}_\text{i}} \quad .
$$

The basis is neither orthogonal nor normalized with a **norm-matrix** element given by:

$$
\begin{aligned}
\langle\, m \mid n \,\rangle \;&:=\; \underbrace{\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty}}_{6\times} d^3(\bar{\boldsymbol{r}}_1, \bar{\boldsymbol{r}}_2) e^{-\frac{1}{2} \begin{bmatrix} \bar{\boldsymbol{r}}_1 \\ \bar{\boldsymbol{r}}_2 \end{bmatrix}^T \cdot \begin{bmatrix} 2(\alpha_1^m + \alpha_3^m) & 2\alpha_3^m \\ 2\alpha_3^m & 2(\alpha_2^m + \alpha_3^m) \end{bmatrix} \cdot \begin{bmatrix} \bar{\boldsymbol{r}}_1 \\ \bar{\boldsymbol{r}}_2 \end{bmatrix}} \cdot e^{(m \leftrightarrow n)} \\
&= \frac{\pi^3}{8\left[(\alpha_1^m + \alpha_1^n + \alpha_3^m + \alpha_3^n)(\alpha_2^m + \alpha_2^n + \alpha_3^m + \alpha_3^n) - (\alpha_3^m + \alpha_3^n)^2\right]^{3/2}} \quad . \qquad (2)
\end{aligned}
$$

To obtain the expansion coefficients c_i via a variational principle, we need to express the Hamilton operator in the given basis. The leading-order theory comprises a kinetic energy, a 2-, and a 3-body interaction term:

$$
\hat{H} \mid \psi \,\rangle = \left[(2m)^{-1} \sum_{i=1}^{A} \hat{p}_i^2 + \sum_{i<j}^{A} \delta_{ij}^\lambda + \sum_{i<j<k}^{A} \delta_{ij}^\lambda \delta_{ik}^\lambda \right] \mid \psi \,\rangle \quad .
$$

In coordinate representation, we obtain the kinetic-energy matrix elements in the Gaussian basis:

$$
\left\langle m \left| \sum_i \hat{p}_i^2 \right| n \right\rangle = \underset{\boldsymbol{r}_\text{i} \to \bar{\boldsymbol{r}}_\text{i}}{=} \left\langle m \left| \sum_i \hat{p}_i \, \hat{p}_i \right| n \right\rangle = \sum_i \int d^3(\boldsymbol{r}_{1\ldots\text{A}}) \, \langle\, m \mid \hat{p}_i \mid \boldsymbol{r}_{1\ldots\text{A}} \,\rangle \, \langle\, \boldsymbol{r}_{1\ldots\text{A}} \mid \hat{p}_i \mid n \,\rangle \qquad (3)
$$

$$
\qquad (4)
$$