On the effect of particle identity on inter-cluster interactions: dimer-dimer

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Abstract

The effects of particle statistics, proximity to the two-particle unitary limit, and short-distance structure of the particle-particle interaction on elastic scattering between two composite dimers is analyzed. Specifically, we obtain a relative S-wave scattering length for four distinguishable particles of $a_{(\mathrm{abcd})} = 3.1(2)(2)$ fm, and if three of four fermions are distinguishable and the fourth is identical to one of them, $a_{(\mathrm{acbc})} = 2.7(2)(2)$ fm. For two-component fermions, we recover the well-known universal ratio between dimer-dimer and particle-particle scattering lengths $a_{(\mathrm{abab})} \approx 0.6\, a$. The dependence of all dimer-dimer scattering lengths on short-distance interactions (two- and three-body contacts) for which the employed one-channel resonating-group technique is appropriate is finite and unaffected by a change in the number of four-body bound states as induced by the renormalization-group transformation. We provide an intuitive interpretation of the increasing attraction/repulsion between the dimers as an effect of particle statistics by visualizing the ensuing effective local and non-local energy-dependent dimer-dimer potentials in coordinate space.

I. INTRODUCTION

This is the first in a series of four articles in which we employ the resonating-group technique – also referred to as folding-model[1] – in order to parametrize effective interactions between composites which are bound in a minimal theory for two- and three-body systems with the parameters of the latter. In this first part of the series, we introduce the technique as it yields an interaction between the centers of mass of two composites (henceforth referred to as dimers), each of which forming a state of two assumed-fundamental particles with identical binding energy B_2. We will justify the seemingly drastic assumption of dimers being unaffected in the course of a low-energy scattering event described by this effective potential through a comparison with numerical solutions of the full four-body problem [6] [2]. In the second and third parts we will apply the method to the trimer-trimer and tetramer-tetramer systems, respectively. In the fourth article, we treat the number of particles as a parameter and present the dependence of an effective (A-body)-(B-body) potential on the two- and three-body coupling strengths, and we extend the framework to include three-fragment channels.

With this work, we attempt to contribute to all earlier and ongoing efforts to analyze the features of few- and many-body quantum systems in terms of a small set of fundamental parameters. A set of (effective) field theories (EFTs) – crudely understood as the description of a system by an appropriate set of degrees of freedom (DoF), symmetries, interaction structure, and with a defined, so-called breakdown scale beyond which typically other DoFs are resolved – replacing a single theory from which everything can be derived has been successful especially in identifying universal behavior. In principle, the EFT framework prescribes how to express the parameters of one theory in terms of its underlying one, e.g. the nuclear coupling strengths in terms of Standard-model parameters, or the strength of the attraction between α clusters in terms of nuclear masses and couplings. In practice, the change from one set of Dofs to another precludes a rigorous derivation of such relations.

Here, we advance the folding/resonating-group potentials to provide such relations when dimer, trimer, tetramer, &c DoFs are composites of a certain class of point fermions/bosons whose pair interaction is vanishingly small compared with the two-body scale it produces.

[1] Refs. [1, 2] review the original ideas and the relation to the optical model. Refs. [3–5] are recent refinements/applications of the methodology to other than nuclear systems.

[2] The problem was analyzed in Refs. [7–9] (each of which represents a particular numerical approach) in a regime of

Nucleons, helium-isotope atoms, and a set of trapped cold atoms fall into this category. For nucleons, in particular, the application of such zero-range interactions to few-body systems in combination with renormalization-group techniques allowed to discriminate between universal and characteristic properties of larger-in-number systems; prominently, the sheer existence of an excited, resonant state and a deeper ground state of the $J^\pi = 0^+$ α nucleus are universal consequences of a large nucleon-nucleon scattering length compared with the nuclear force range ($\approx$ inverse π-meson mass). The magnitude of the binding energy and the location of the resonance relative to the 3-helium-neutron and 3-hydrogen-proton thresholds is a characteristic of the nuclear substructure with is encoded in the binding energy of the nuclear trimers (with B_3, we denote the ground-state energy of the generic trimer and not a specific datum). To classify observables involving more than four nucleons – in general, systems comprised of more particles than accessible internal degrees of freedom – analogously as universal or characteristic is desirable in order to, first, understand features like mass gaps and shell structure from a minimal set of underlying parameters, and subsequently, to employ this insight for a reliable, model-independent prediction of unknown and hard-to-measure quantities like, e.g., few-neutron states.

While keeping these features of nuclei in mind, we analyze the essence of the problem: the effect of particle statistics on the effective composite-composite interaction with the bound composites being universally correlated to an approximately scale-free two-body system and, for systems with more than three distinguishable particles, a finite three-body scale.

A pioneering study [6] found a ratio between dimer-dimer and atom-atom scattering lengths of $a_{(\mathrm{abab})}/a_{aa} \approx 0.6$ if the atom is a fermion with two internal states. Before putting the RGM approach to a test in comparison with this result in sec. (III A), we summarize the RGM transcription of two- and three-particle interaction parameters into the pertinent inter-dimer potential in sec. (II). Dimer-dimer scattering with three- and four-component-fermion statistics as a function of the present three-body scale is presented in sec. (III B) and sec. (III C). The de- or independence of cluster-cluster scattering on this scale - which sets a characteristic size for a fully spatially symmetric three-body subsystem - and how its existence weakens or strengthens the dimer-dimer interaction compared with the two-component-fermion system is one main result of this article summarized in sec. (IV). We deem the presentations of the folding technique as part of the EFT framework to obtain model-independent inter-cluster interactions of even more importance. We derive these an-

alytically as potentials which depend solely on the separation of the centers of mass of the two dimers, their binding energy B_2, and a regularization parameter (henceforth denoted as λ) that is introduced when the EFT is renormalized using B_2 (and B_3 when appropriate) as *the* constraint(s) in sec. (V), including graphical illustrations.

II. CONTACT EFT → INTER-CLUSTER EFT

A non-relativistic scalar ϕ^4 theory is arguably the simplest non-trivial interacting quantum field theory. As the non-relativistic reduction of a chiral field theory whose meson DoF were integrated out, it constitutes the leading order (LO) of another EFT with only 'ϕ' DoFs.

III. LEADING-ORDER DESCRIPTION OF THE DIMER-DIMER INTERACTION

A. (abab)

B. (abcd)

C. (abcc)

IV. SUMMARY

V. APPENDIX: FOLDING CONTACT POTENTIALS

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