The product of a and $a^\dagger$ is

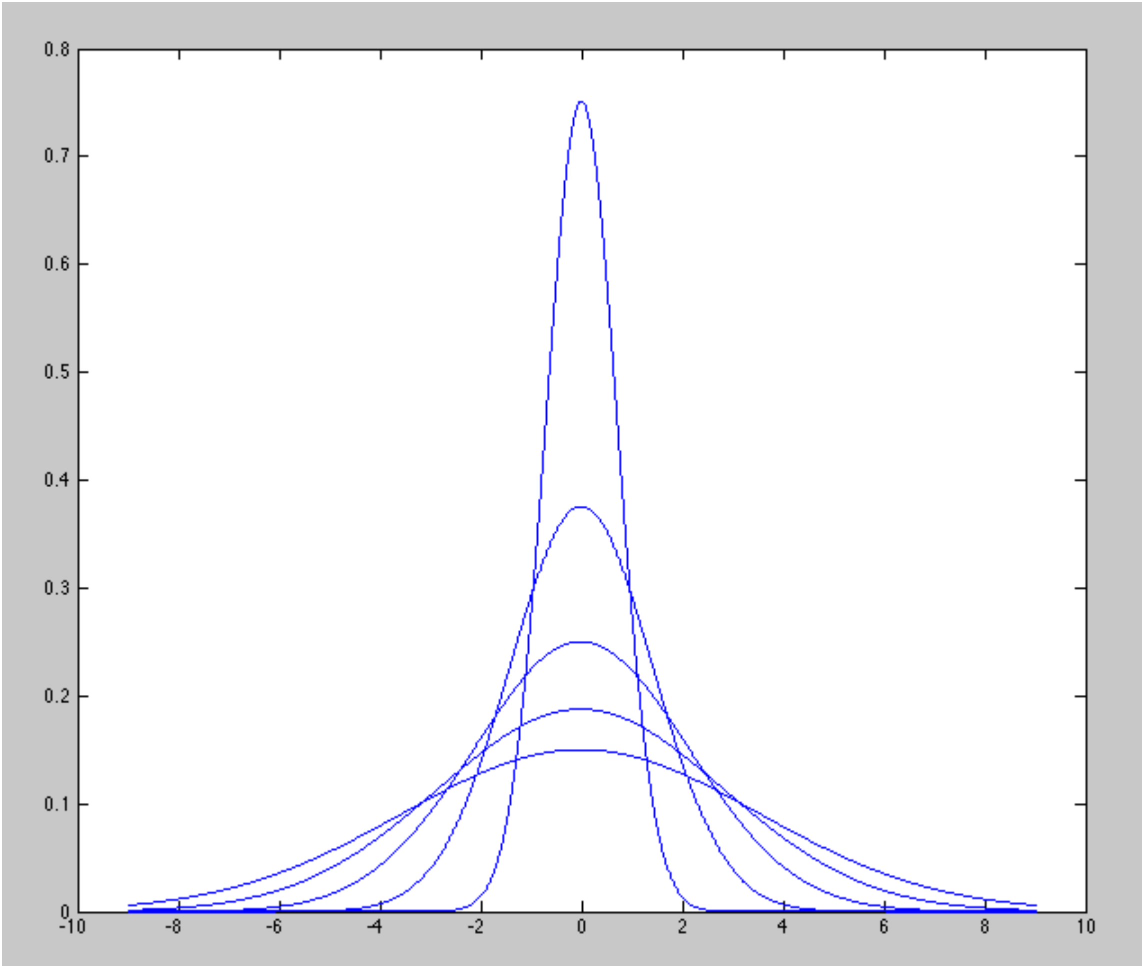
$$aa^\dagger = \frac{m\omega}{2\hbar}\left(\hat{x} + i\frac{\hat{p}}{m\omega}\right)\left(\hat{x} - i\frac{\hat{p}}{m\omega}\right) = \frac{m\omega}{2\hbar}\left(\hat{x}^2 + \frac{\hat{p}^2}{m^2\omega^2} + \frac{i}{m\omega}\left[\hat{p}\hat{x} - \hat{x}\hat{p}\right]\right)$$

$$= \frac{m\omega}{2\hbar}\left(\hat{x}^2 + \frac{\hat{p}^2}{m^2\omega^2} + \frac{i}{m\omega}\left[\hat{p},\hat{x}\right]\right) = \frac{m\omega}{2\hbar}\left(\hat{x}^2 + \frac{\hat{p}^2}{m^2\omega^2} - \frac{i}{m\omega}\left[\hat{x},\hat{p}\right]\right)$$

The commutator of $\hat{x}$ and $\hat{p}$ is $\left[\hat{x},\hat{p}\right] = i\hbar$, giving

$$aa^\dagger = \frac{m\omega}{2\hbar}\left(\hat{x}^2 + \frac{\hat{p}^2}{m^2\omega^2}\right) + \frac{1}{2}$$

```
%Gaussian Plot-----A-----A-----A-----A-----A-----A-----A-----A-----A-----A-----A
clear
%Parameters for solving the problem
% PARAMETERS:
Xzero=1;
%Prepares the increment, and the range of values for the position that
%we will analyze
dp = 0.001;
Range = -9:dp:9; %range from -4 to 4 in increments of .001
X(1,:)=Range; %-4 to 4, (1,:) means first row, all elements
%-5 to 3 (2:) means second row, all elements
%X(3,:)=Range-2*Xzero; %-6 to 2
%Here is the function that we calculated
%note that loop covers gnd k=1 and offset k=2
for k=1:5;
Phi_x=(1/k)*(pi^(-0.25))*exp(-((X(1,:)/k).^2)); % first time it's row 1, 2nd time it's
row 2
%
%Now plot the wavefunction
%
figure(1)
plot(Range,Phi_x);
hold on
end
```




```
%**************************************************************
% Program 2: Calculate first and second derivative numerically
% showing how to write differential operator as a matrix
%************************************************************** % Parameters for
solving problem in the interval 0 < x < L
L = 2*pi;
N = 5;
x = linspace(0,L,N)';
dx = x(2) - x(1);
% Interval Length
% No. of coordinate points
% Coordinate vector
% Coordinate step
% Two-point finite-difference representation of Derivative
D=(diag(ones((N-1),1),1)-diag(ones((N-1),1),-1))/(2*dx);
% Next modify D so that it is consistent with f(0) = f(L) = 0
D(1,1) = 0; D(1,2) = 0; D(2,1) = 0;
% So that f(0) = 0
D(N,N-1) = 0; D(N-1,N) = 0; D(N,N) = 0;
% So that f(L) = 0
```

```
% Three-point finite-difference representation of Laplacian
Lap = (-2*diag(ones(N,1),0) + diag(ones((N-1),1),1) + diag(ones((N-1),1),-
1))/(dx^2);
% Next modify Lap so that it is consistent with f(0) = f(L) = 0
Lap(1,1) = 0; Lap(1,2) = 0; Lap(2,1) = 0;
% So that f(0) = 0
Lap(N,N-1) = 0; Lap(N-1,N) = 0; Lap(N,N) = 0;
% So that f(L) = 0
% To verify that D*f corresponds to taking the derivative of f % and Lap*f
corresponds to taking a second derviative of f,
% define f = sin(x) or choose your own f
f = sin(x);
% And try the following:
Df = D*f; Lapf = Lap*f;
plot(x,f,'b',x,Df,'r', x,Lapf,'g');
axis([0 5 -1.1 1.1]); % Optimized axis parameters
% To display the matrix D on screen, simply type D and return ... D % Displays the
matrix D in the workspace
Lap % Displays the matrix Lap
```