7.6 a) The possible results of a measurement of the angular momentum component L_z are always $+1\hbar, 0\hbar, -1\hbar$ for an $\ell = 1$ system. The probabilities are

$$\mathsf{P}_1 = \left|\langle 11|\psi\rangle\right|^2 = \left|\langle 11|\left[\tfrac{1}{\sqrt{14}}|11\rangle - \tfrac{3}{\sqrt{14}}|10\rangle + i\tfrac{2}{\sqrt{14}}|1,-1\rangle\right]\right|^2$$

$$= \left|\tfrac{1}{\sqrt{14}}\langle 11|11\rangle - \tfrac{3}{\sqrt{14}}\langle 11|10\rangle + i\tfrac{2}{\sqrt{14}}\langle 11|1,-1\rangle\right|^2 = \left|\tfrac{1}{\sqrt{14}}\right|^2 = \tfrac{1}{14}$$

$$\mathsf{P}_0 = \left|\langle 10|\psi\rangle\right|^2 = \left|\langle 10|\left[\tfrac{1}{\sqrt{14}}|11\rangle - \tfrac{3}{\sqrt{14}}|10\rangle + i\tfrac{2}{\sqrt{14}}|1,-1\rangle\right]\right|^2$$

$$= \left|\tfrac{1}{\sqrt{14}}\langle 10|11\rangle - \tfrac{3}{\sqrt{14}}\langle 10|10\rangle + i\tfrac{2}{\sqrt{14}}\langle 10|1,-1\rangle\right|^2 = \left|-\tfrac{3}{\sqrt{14}}\right|^2 = \tfrac{9}{14}$$

$$\mathsf{P}_{-1} = \left|\langle 1,-1|\psi\rangle\right|^2 = \left|\langle 1,-1|\left[\tfrac{1}{\sqrt{14}}|11\rangle - \tfrac{3}{\sqrt{14}}|10\rangle + i\tfrac{2}{\sqrt{14}}|1,-1\rangle\right]\right|^2$$

$$= \left|\tfrac{1}{\sqrt{14}}\langle 1,-1|11\rangle - \tfrac{3}{\sqrt{14}}\langle 1,-1|10\rangle + i\tfrac{2}{\sqrt{14}}\langle 1,-1|1,-1\rangle\right|^2 = \left|i\tfrac{2}{\sqrt{14}}\right|^2 = \tfrac{4}{14}$$

The three probabilities add to unity, as they must.

b) Now the initial state is $|\psi\rangle = |1,-1\rangle$. The possible results of a measurement of the angular momentum component L_x are always $+1\hbar, 0\hbar, -1\hbar$ for an $\ell = 1$ system. The probabilities are

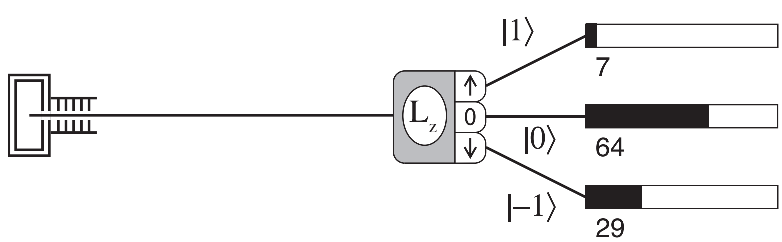
$$\mathsf{P}_{1x} = \left|{}_x\langle 11|\psi\rangle\right|^2 = \left|\left(\tfrac{1}{2}\langle 11| + \tfrac{1}{\sqrt{2}}\langle 10| + \tfrac{1}{2}\langle 1,-1|\right)|1,-1\rangle\right|^2 = \left|\tfrac{1}{2}\right|^2 = \tfrac{1}{4}$$

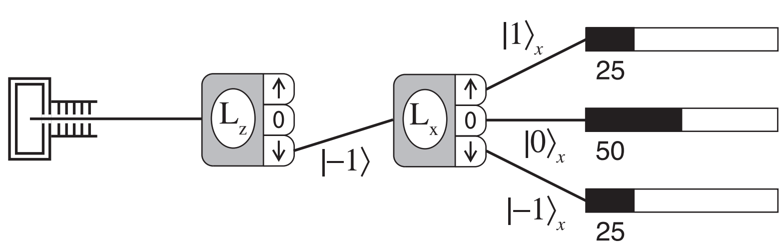
$$\mathsf{P}_{0x} = \left|{}_x\langle 10|\psi\rangle\right|^2 = \left|\left(\tfrac{1}{\sqrt{2}}\langle 11| - \tfrac{1}{\sqrt{2}}\langle 1,-1|\right)|1,-1\rangle\right|^2 = \left|-\tfrac{1}{\sqrt{2}}\right|^2 = \tfrac{1}{2}$$

$$\mathsf{P}_{-1x} = \left|{}_x\langle 1,-1|\psi\rangle\right|^2 = \left|\left(\tfrac{1}{2}\langle 11| - \tfrac{1}{\sqrt{2}}\langle 10| + \tfrac{1}{2}\langle 1,-1|\right)|1,-1\rangle\right|^2 = \left|\tfrac{1}{2}\right|^2 = \tfrac{1}{4}$$

The three probabilities add to unity, as they must.

c) The schematic diagrams of these measurements are shown below.





7.11 The eigenstates are

$$\left|m\right\rangle \doteq \Phi_m(\phi) = \frac{1}{\sqrt{2\pi}}e^{im\phi}$$

The inner product for $m \neq n$ is

$$\left\langle m\middle|n\right\rangle = \int_0^{2\pi}\Phi_m^*(\phi)\Phi_n(\phi)d\phi = \int_0^{2\pi}\frac{1}{\sqrt{2\pi}}e^{-im\phi}\frac{1}{\sqrt{2\pi}}e^{in\phi}d\phi = \frac{1}{2\pi}\int_0^{2\pi}e^{i(n-m)\phi}d\phi$$

$$= \frac{1}{2\pi}\left[\frac{e^{i(n-m)\phi}}{i(n-m)}\right]_0^{2\pi} = \frac{1}{2\pi}\left[\frac{e^{i(n-m)2\pi}-1}{i(n-m)}\right] = \frac{1}{2\pi}\left[\frac{1-1}{i(n-m)}\right] = 0$$

The inner product for $m = n$ is

$$\left\langle m\middle|m\right\rangle = \int_0^{2\pi}\Phi_m^*(\phi)\Phi_m(\phi)d\phi = \int_0^{2\pi}\frac{1}{\sqrt{2\pi}}e^{-im\phi}\frac{1}{\sqrt{2\pi}}e^{im\phi}d\phi = \frac{1}{2\pi}\int_0^{2\pi}d\phi = \frac{2\pi}{2\pi} = 1$$

Thus, we get

$$\left\langle m\middle|n\right\rangle = \delta_{mn}$$

and the states are orthonormal.