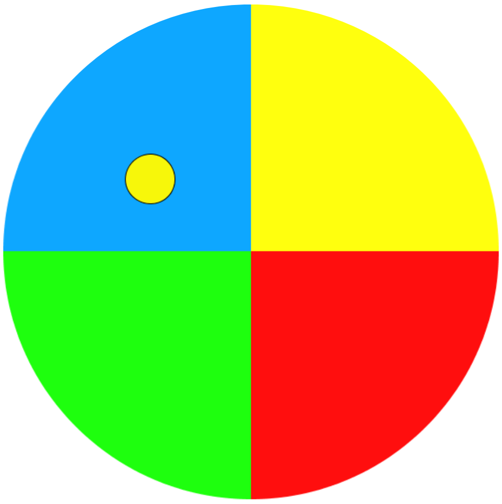
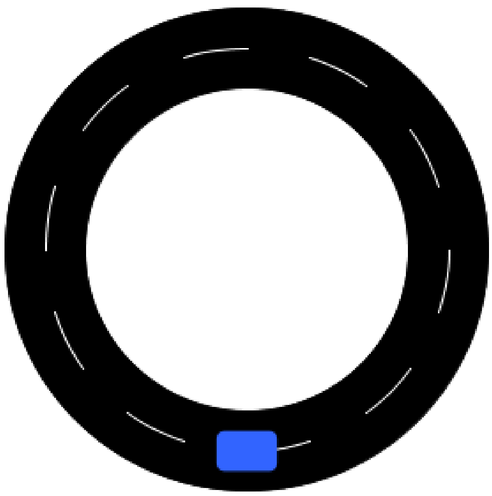


M3 Simulations

USER MANUAL



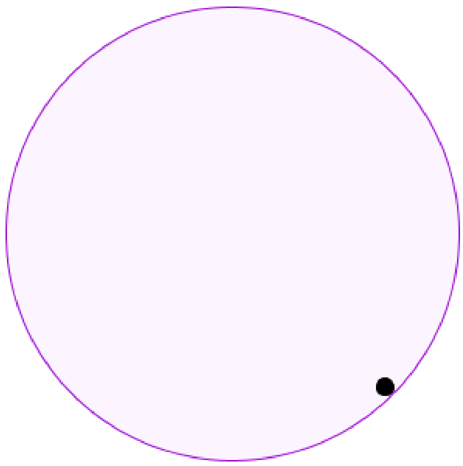


TABLE OF CONTENTS

1 INTRODUCTION

This is a simulation program, designed to accompany the Mechanics 3 course as part of a Mathematics or Further Mathematics A level with MEI. Some simulations may also be appropriate for Mathematics with other exam boards or as part of a Physics A level. It can be used by teachers to demonstrate concepts to a class, or by students to investigate topics further during their own study time.

The program consists of four different simulations covering different parts of the M3 module. These are “Coins on roundabouts”, “Cars on a Banked Racetrack”, “Vertical Circles” and “Simple Harmonic Springs”. All simulations include a moving image of the situation, graphs and force diagrams and sliders to change the relevant parameters. This guide will explain how to install the program from a USB and will explain how to use the simulation, as well as giving some background and explanation of the mechanics of each simulation.

If, while using or setting up the program, an error occurs that is not handled in this booklet, please let me know by emailing kirsten.land@kcl.ac.uk and I will be able to find out what is wrong and sort it out.

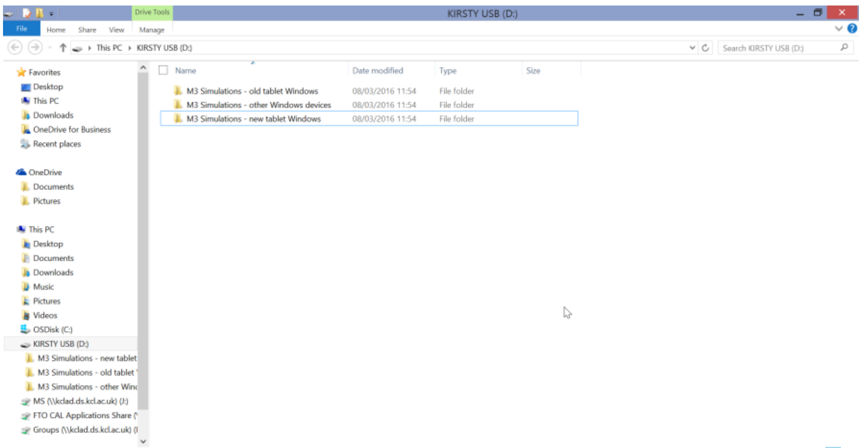
2 INSTALLATION INSTRUCTIONS

To use this simulation, you need:

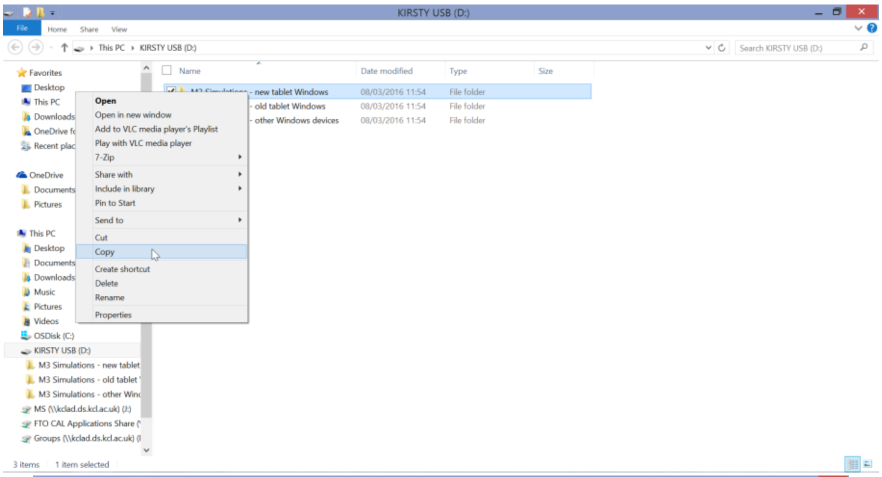
- A USB stick with a copy of the program files.
- A computer or tablet running Windows (if you use another operating system, let me know).
- At least 45MB available memory to store the program files.
- Java to be installed on your computer.

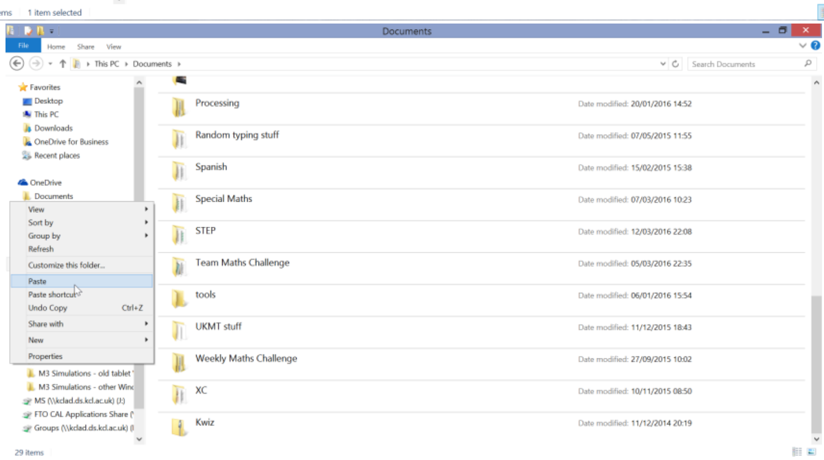
To set up the program, follow these steps.

1. Insert the USB with the program installed, and open up the USB folder.

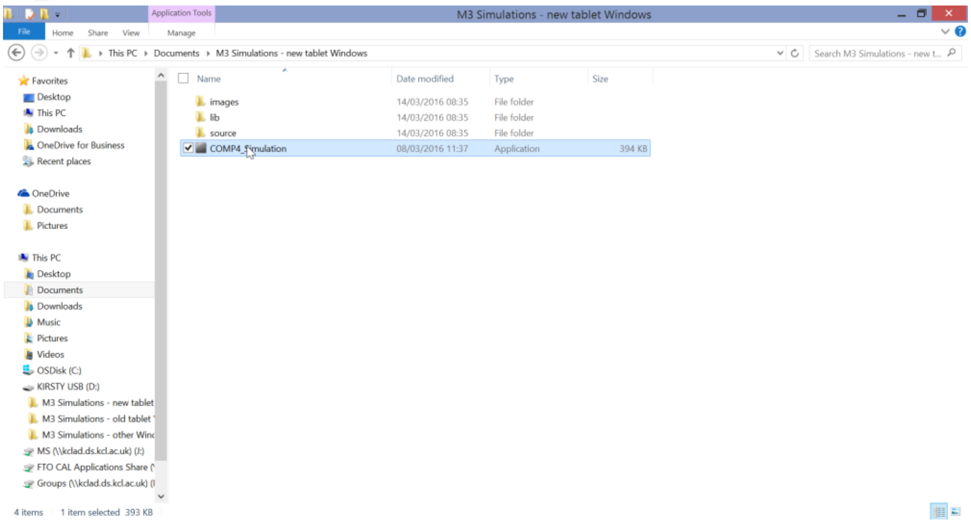


2. Copy the relevant folder, depending on what system you are using. Paste this to a folder on your computer. You can now remove the USB stick.

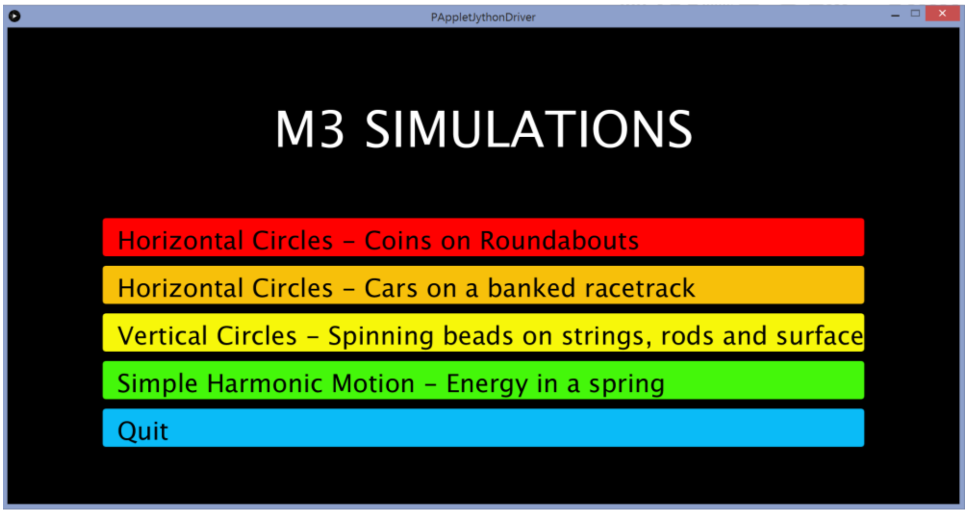




3. Open the folder you have just copied, and double click on the file called "COMP4_Simulation".



4. After about 30 seconds, the application will load. The window should look like the image below. Now you can start using the simulation.



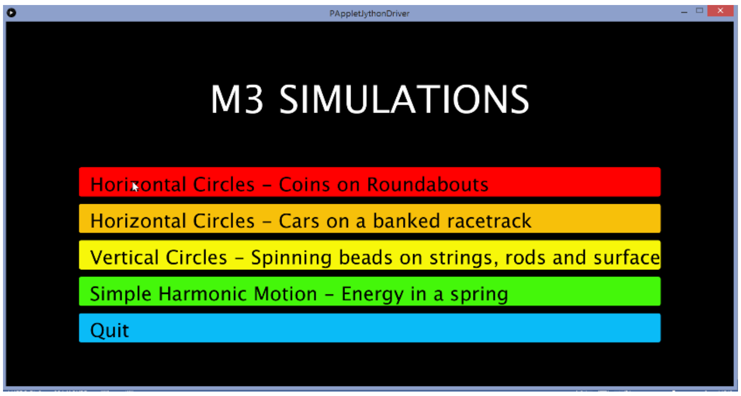
2.1 Possible Problems

During setup, the following issues may lead to problems with setting it up.

1. The program prompts you to download Java - You may not have Java installed.
   In this case, Processing will inform you that 32 bit Java is required to run this program. It will prompt you to download Java and will take you to a page where you can download Java. Choose the 32 bit version and download it. The program can then be reopened, and should work as expected. If you are using a school laptop, you will need an administrator's password to do this, which you will need to ask a teacher to enter.
2. When opened, the simulation screen does not scale correctly or only part of the simulation is visible – You may have copied the wrong folder for your computer.
   If you have a new school tablet (you received it in 2015 or more recently), an old tablet (received before 2015) or some other Windows device, make sure you have copied the appropriate files. When the right file is chosen, the entire screen should be visible within the window. If you are using the version for any Windows device, there may be some white space at the bottom of the screen, and to exit the program you will need to use the Quit button on the home menu.

3 Navigating Between Simulations

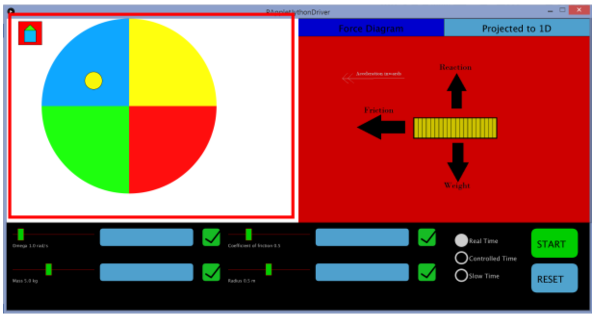
From the home menu, you can choose between any of the four simulations, or you can quit. Clicking on a simulation will launch it. If you are using the general version for any Windows device, you may only be able to quit by clicking on that option in the menu. Otherwise, there should also be a button in the top right corner that allows you to exit the simulation.



From any simulation, you can get back to the home menu by clicking on the home button in the top left corner of the page. You can do this whenever you would like to look at a different simulation or quit the program.

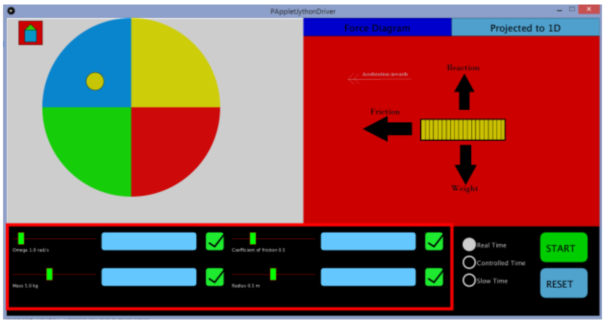
4 USING THE SIMULATIONS

Once you have chosen a simulation, you can then use it. Each simulation is set out in a similar way, with the screen split into three main sections. This section will explain what each section is for, using the first simulation for illustrations. The specifics of each simulation will be outlined later on in this booklet.



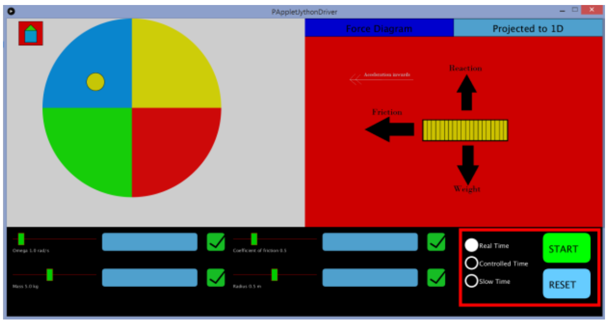
The top left panel of the screen shows an animation detailing what is going on in the simulation. When the simulation breaks down (for example, the coin falls off the roundabout), you will be able to see this in the top left panel.

The top right panel of the screen displays any relevant graphs or force diagrams. You can switch between these features using the tabs at the top of the right panel. The selected tab will be displayed in a darker shade of blue.



The bottom left of the screen has all the controls for changing the parameters. Each parameter can be changed by a slider or an input box. For more information on this, see the next section.

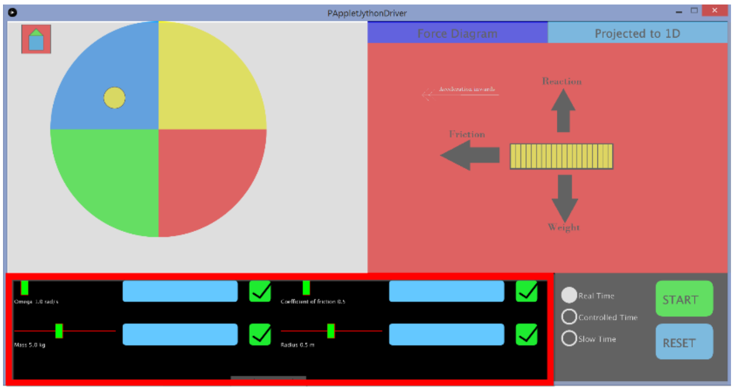
The bottom right of the screen contains the controls for the simulation. The start button will start the animation and graphs with the current parameters, and the stop button (replaces the start button when the simulation is playing) will keep the parameters but will stop the simulation. The reset button resets all parameters to their original value and resets all objects so the simulation breakdown is



The different time mode settings are chosen by the radio buttons. In real time, the motion of the simulation is controlled by the actual time you have been playing the simulation. Controlled time looks the same, but the time steps are constant and cannot be guaranteed to be accurate. When using slow time, each time step is significantly shorter than the actual time elapsed. Increasing the time setting makes the simulation more accurate.

5 Using the Inputs

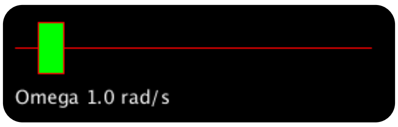
In each simulation, every parameter can be changed using either a slider or an input box. This allows you to investigate different outcomes and see the effect of changing lots of different parameters on the situation.



The controls for all of the parameters are in the bottom left of the screen, as shown in the image to the left. For each parameter, the name of the parameter, its current value and its units are shown, as well as a slider and input box for changing its values.

5.1 Sliders

Each parameter can be changed using the slider above its label, as shown below. The slider can be moved by clicking on it and moving the mouse backwards and forwards, while holding down the mouse. The line that the slider moves along limits the sliders to a range, and attempting to drag the slider outside of this range will move it to the maximum or minimum, depending on which side you are dragging it.



5.2 Input Boxes

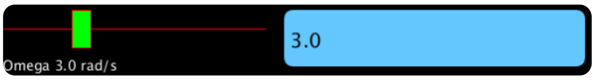
The values of the parameter can also be entered using the input box to the left of its label and slider, as shown below. Integer or decimal values can be entered into these input boxes, as well as more complex expressions.

To enter a value, click on the input box to select it, and then type in your value using the keyboard. A selected input box is displayed in a darker shade of blue. Once you have entered your value, press the Enter key or click the green tick next to the input box to change the value of the parameter. You can unselect the box by clicking on the green tick (and submitting your value) or choosing another box.

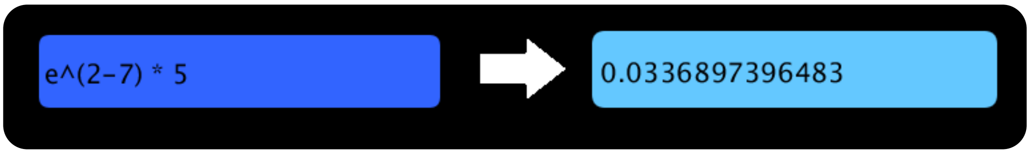
Whenever you reselect the box, anything you have typed before will disappear and the input box will be empty.

Each input box has an allowable range of values, which match up with the limits of the sliders. These ranges are listed in the pages in this guide for each simulation.

Once you have submitted a value, if the value is in the allowable range, the sliders value will change, the parameters value will change, the input box will be unselected and the new value will be displayed in the input box. If the value is not in the correct range, then a message will be displayed inside the box. If this happens, then reselect the box and submit a value in the correct range (see above). Images of these cases are shown below.



More complex expressions can also be entered, such as 3+4 or sin(2), in exactly the same way. These will be evaluated by a parser, and then the result will be used in the same way as if you had just entered it. Range checking applies as before. The evaluated expression will be displayed in the input box, so you can check if the value seems reasonable. An example is shown below.



This can be used when you have an exact answer that you have calculated and you wish to check, but you don't want to have to evaluate it first using a calculator or type in lots of digits to keep the precision.

If you enter an invalid expression (including if you do not enter anything before pressing submit), the following error message will be displayed inside the box.

If this happens, first try clicking on the box and re-entering the expression, in case you had made a small error, for example by missing a bracket in your original expression. If the error message displays again, then you can try to enter the expression in a less ambiguous way by adding more brackets, or use a normal calculator to evaluate it and type in the result. You might also want to check that the operations you are attempting are supported by the parser. The table below shows which operations, functions and constants are recognised by the parser.

Operations (symbols)	Notes
Addition (+)	
Subtraction (-)	
Multiplication (* or x)	Implicit multiplication (eg 2pi) is also supported.
Division (/)	This is always carried out as float division.
Exponentiation (^)	Note that exponentiation is always carried out before multiplication, which may not always be what you expect. Use brackets when multiplying inside the exponent.

Function	Notes
sqrt()	
cos()	The value given assumes input in radians.
sin()	The value given assumes input in radians.
tan()	The value given assumes input in radians.
Constants	**Notes**
pi	Value is 3.141592653589793.
e	Value is 2.718281828459045.

If you regularly need other functions or constants, please let me know using the contact details at the start of this guide, as functions and constants can easily added when they are required.

6 Individual Simulations

There are four simulations in this program. For each section, this manual explains what the simulation shows, the mechanics behind it, the parameters that can be changed and highlights the main features of the simulation.

6.1 Simulation 1: Coins on Roundabouts

This simulation is about modelling horizontal circular motion with constant speed. According to the MEI specification, for M3 you need to be able to:

1. Identify the force(s) acting on a body in circular motion.
2. Be able to calculate acceleration towards the centre of circular motion.
3. Be able to solve problems involving circular motion with uniform speed.

The situation involves a coin on a roundabout, which is spinning. The coin is held on to the roundabout by friction. When you increase the speed of the roundabout, the friction has to increase, until eventually the friction cannot be provided and the coin flies off the roundabout.

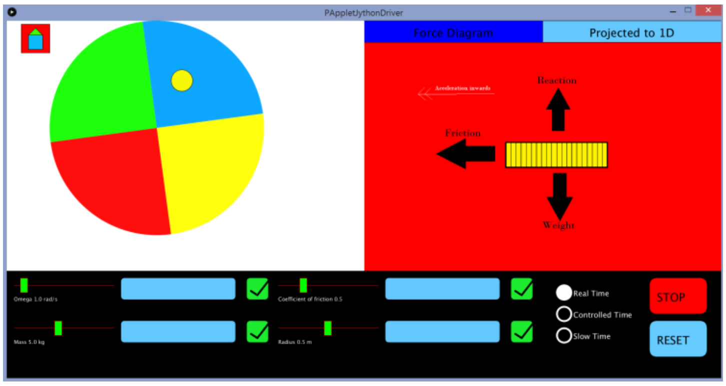
This particular situation is explained in example 1.3, on page 10 of your M3 textbook.

6.1.1 Using the Simulation

To use this simulation, choose some values for the mass, the coefficient of friction and the radius, and set them using the sliders or input boxes in the bottom of the screen. Then you can use the mechanics explained below to find out what value of omega the coin should fall off the roundabout at.

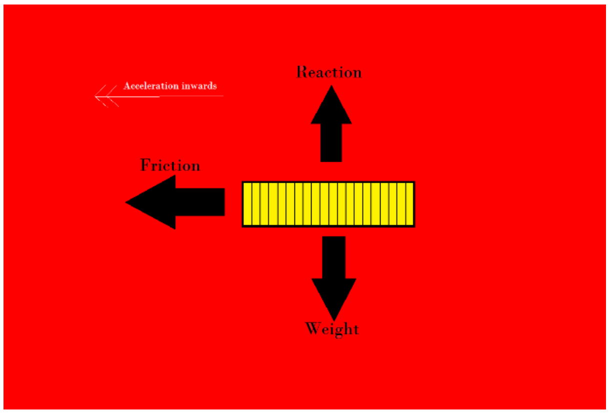
Now you can test your predictions using the simulation. Press on the start button in the bottom right of the screen. Slowly increase the omega slider until the coin starts to move outwards. Check the current value of omega against your predictions. To do this again or to test different values, click on the reset button that is below the start/stop button in the bottom right of the screen. This will reset all the parameters and place the coin back on the roundabout.

The image below shows what this simulation looks like.



6.1.2 Mechanics

The force diagram to the left explains which forces are acting on the coin. By resolving vertically, you can find the reaction R:



$$R = Weight = mg$$

As the coin is in circular motion with an angular speed of ω, the centripetal acceleration is $\omega^2 r$.

The only centripetal force is the friction Fr, so we can resolve inwards to find that:

$$Fr = m\omega^2 r$$

When the coin is on the point of slipping, $Fr = R\mu = mg\mu$, where μ is the coefficient of friction. Therefore, the maximum angular speed is given by:

$$mg\mu = m\omega^2 r$$

$$\Rightarrow \omega = \sqrt{\frac{g\mu}{r}}$$

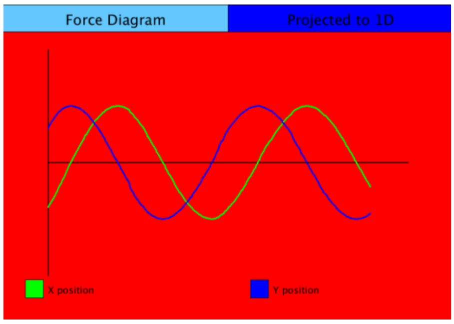
6.1.3 Parameters

The parameters that you can change in this simulation and the allowed ranges for each parameter is shown in the table below. All parameters in this simulation are changed in the bottom panel of the screen.

Parameter Name	Allowable inputs	Description
Omega	0 – 10	The current angular velocity of the roundabout, representing how fast it is rotating.
Mass	1 – 10	The mass of the coin on the roundabout.
Mu	0 – 2	The coefficient of friction between the coin and the roundabout, representing how rough the roundabout is.
Radius	0.05 – 1	The distance between the coin and the centre of the roundabout.

6.1.4 Features

This simulation has a force diagram image of the coin on the roundabout, and a graph of the x and y components of the coin's position over time. As the coin is moving at a constant speed, the x and y position will be two out of phase sine waves. The force diagram was shown above in the mechanics section. A sample graph is shown below.



6.2 Simulation 2: Cars on a Banked Racetrack

Like simulation 1, this simulation is about modelling horizontal circular motion with constant speed. In this simulation, you will need to resolve forces to find the resultant force. According to the MEI specification, for M3 you need to be able to:

1. Identify the force(s) acting on a body in circular motion.
2. Be able to calculate acceleration towards the centre of circular motion.
3. Be able to solve problems involving circular motion with uniform speed.

In this simulation, there is a car travelling on a circular banked racetrack, with friction. If the car travels too fast, then it will fly off the top of the racetrack, and if the car travels too slowly, then the car will fall into the centre. This problem is about investigating the speeds at which this happens.

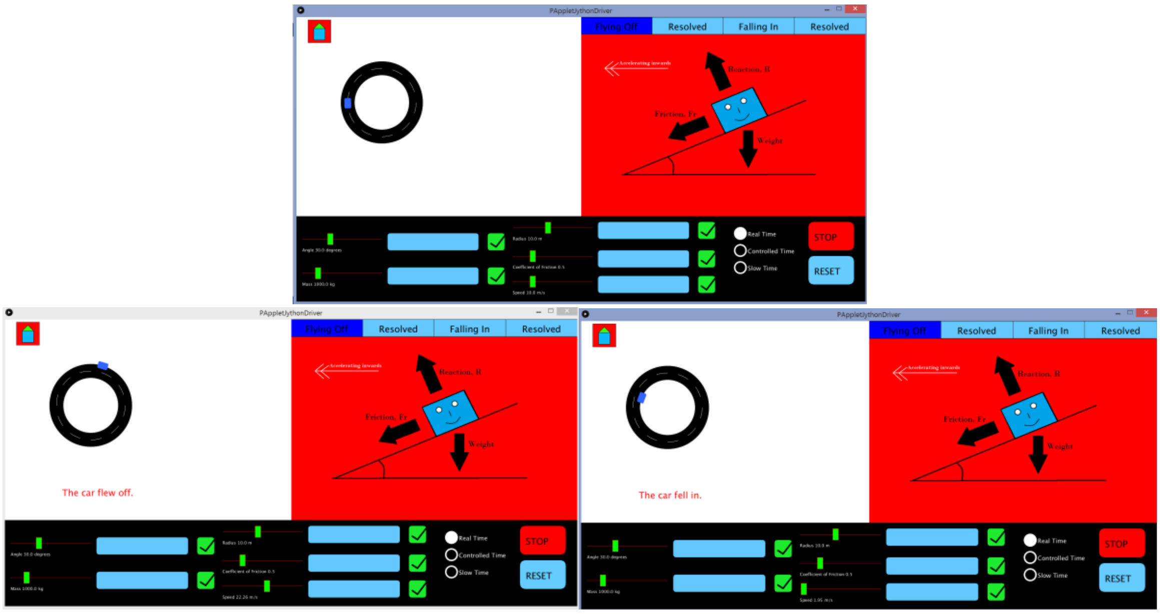
This particular situation is explained in example 1.5, on pages 16-18 of your M3 textbook.

6.2.1 Using the Simulation

To use this simulation, choose some values for the mass, the coefficient of friction and the radius and the angle at which the racetrack is inclined to the horizontal, and set them using the sliders or input boxes in the bottom of the screen. Then you can use the mechanics explained below to find the maximum speed before the car flies off the racetrack and the minimum speed before the car falls in towards the centre.

Now you can test your predictions using the simulation. Set the speed to a value between the upper and lower limit you have calculated. Press on the start button in the bottom right of the screen. Slowly increase the speed slider until the car starts to move outwards and a message is displayed. Check the current value of the speed against your predictions. Now click on the reset button, and set up the simulation as before. Press on the start button, and this time decrease the speed until the car starts to move inwards and a message is displayed. Check this value against your predictions. Now you can reset the simulation and experiment with other parameters.

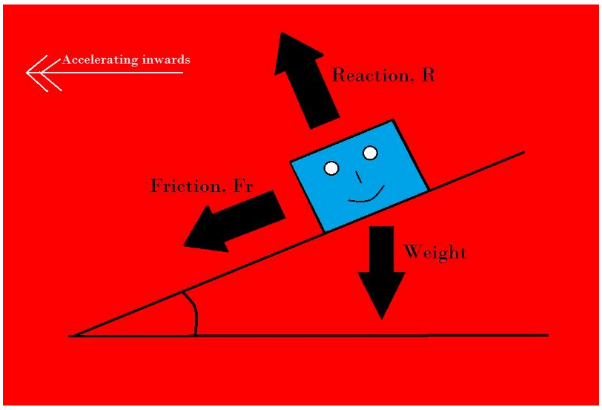
The images below show the simulation screen and messages when it is playing, after the car has flown off the track and after it has fallen into the centre.

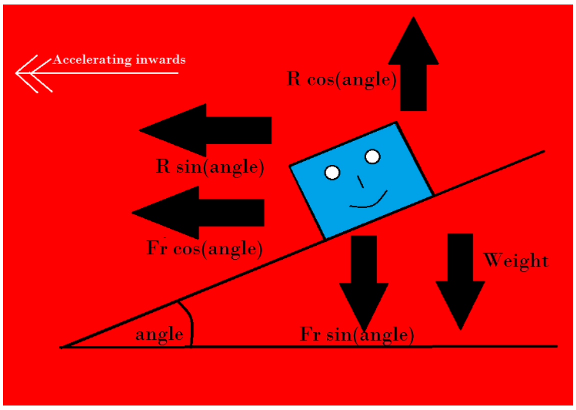


6.2.2 Mechanics

To find the flying off and falling in speeds, you can draw a clear force diagram and resolve vertically and horizontally. This booklet show the flying off case, and the falling in case works exactly the same, but with the direction of friction reversed.

The force diagram and the resolved force diagram are shown below.





By resolving vertically, you obtain the equation:

$$R\cos\theta = Fr\sin\theta + mg$$

And by resolving inwards, you obtain the equation:

$$\frac{mv^2}{r} = R\sin\theta + Fr\cos\theta$$

When the car is on the point of slipping up the slope, the friction will be at its maximum value, so $Fr = R\mu$. Substituting in and rearranging gives:

$$R(\cos\theta - \mu\sin\theta) = mg$$

$$R(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r}$$

You can then divide one equation by the other and rearrange to find that:

$$v^2 = gr\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

From this you can work out the critical speeds. The important thing is to resolve horizontally and vertically, as the circle is horizontal, rather than parallel and perpendicular to the plane.

6.2.3 Parameters

The parameters that you can change in this simulation and the allowed ranges for each parameter is shown in the table below. All parameters in this simulation are changed in the bottom panel of the screen.

Parameter Name	Allowable inputs	Description
Angle	0 – 85	The angle in degrees at which the slope is inclined to the horizontal.
Radius	2 – 20	The radius of the racetrack in metres, so the distance from the car to the centre.
Mu	0 – 2	The coefficient of friction between the car and the racetrack, representing how rough the racetrack is.
Mass	0 – 5000	The mass of the car in kg.
Speed	0 – 40	The tangential speed, in metres per second, that the car is travelling at.

6.2.4 Features

This simulation has four force diagrams to describe the different cases in which the model can fail and so you can check you have resolved the forces in the correct directions if you are not sure what to do.

The first two diagrams are for when the car is about to slide up the slope, so the friction is acting down the slope. The first diagram is unresolved and the second diagram is resolved vertically and horizontally.

The next two diagrams are for when the car is about to fall down the slope, so the friction is acting up the slope. The first diagram is unresolved and the second diagram is resolved vertically and horizontally.

You can change between force diagrams using the tabs, but this needs to be done manually and will not depend on what is happening in the situation.

6.3 Simulation 3: Vertical Circles

This simulation is about modelling vertical circular motion with a non-constant speed. According to the MEI specification, for M3 you need to be able to:

1. Be able to solve problems involving circular motion with non-uniform speed.
2. Be able to calculate tangential acceleration.
3. Be able to solve problems involving motion in a vertical circle.
4. Identify the conditions under which a particle departs from circular motion.

In this simulation, a small mass is made to perform vertical circular motion, either attached to a string or stiff wire, or on the inside or outside of a sphere. Under certain conditions, the mass will complete full circles, but this simulation investigates the various cases when this may not be achieved.

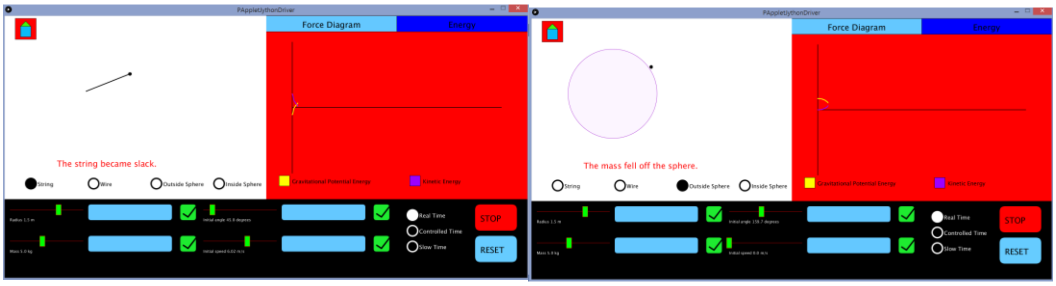
The situations covered by this simulation are described and explained from page 30 to page 42 of the M3 textbook.

6.3.1 Using the Simulation

To use this simulation, choose some values for the mass, radius, initial angle and initial speed, and set them using the sliders or input boxes in the bottom of the screen. Set the simulation mode using the radio buttons that are underneath the animation in the top left panel. You can choose between a string, wire, outside sphere and inside sphere. Changing the simulation mode changes the possible breakdown situations that can occur. These are explained in the mechanics section. Now you can use the mechanics below to predict whether it will complete full vertical circles, and if not then why it would fail.

Now you can test your predictions using the simulation. Press on the start button in the bottom right of the screen. Watch the simulation and see whether the mass completes full vertical circles, whether it falls down and carries out oscillations like a pendulum or whether some failure case of the situation occurs. You can then reset the simulation by pressing the reset button in the bottom left and investigate other parameters.

The image below shows some screenshots and possible breakdown cases, although the breakdown messages are quite similar, so not all of them are shown.



6.3.2 Mechanics

For this situation, you will often need to use energy methods to find the speed at a certain point, and then force methods to find the reaction or tension forces. You will also need to interpret the situation to see if that reaction or tension is allowed. A table showing the allowed force in different situations is shown below.

Situation	Possible Forces	Explanation
String	The force must be inwards.	The string is not rigid, so it cannot provide any force to push out on the mass.
Wire	Any force.	The wire is rigid, so it can pull the mass inwards, or it can push it outwards.
Inside sphere	The force must be inwards.	The sphere cannot “hold on” to the mass, so only an inwards reaction can be produced.

Outside sphere	The force must be outwards.	The sphere cannot “hold on” to the mass, so only an outwards reaction can be produced.

The rest of the mechanics has too many possible outcomes and questions to do generally, but the general procedure is as follows.

1. By considering gravitational potential energy (E = mgh) and kinetic energy ($E = \frac{1}{2}mv^2$), find the speed at the angle you are interested in, or at a general angle if the question wants you to find something else. If v^2 is negative, then the mass does not have enough energy to reach this angle.
2. By resolving inwards, and using the fact that centripetal acceleration is given by $\frac{v^2}{r}$, find the required force from the object to keep the mass in circular motion.
3. Check that this force is reasonable given the object that is keeping the mass in circular motion, by checking the table above. If you need to find the angle at which the simulation breaks down, set the force to the minimum or maximum value that would still work (generally 0) and solve to find the angle.

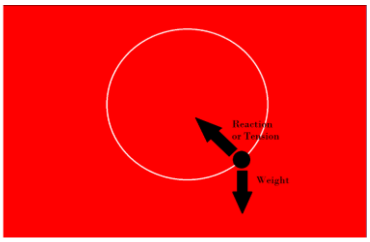
6.3.3 Parameters

The parameters that you can change in this simulation and the allowed ranges for each parameter is shown in the table below. All parameters in this simulation are changed in the bottom panel of the screen.

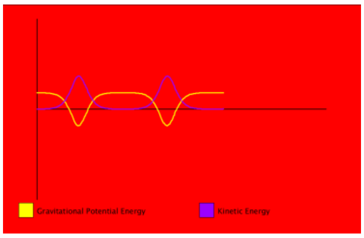
Parameter Name	Allowable inputs	Description
Mass	1 – 10	The mass of the object that is moving in a circle in kg.
Radius	0.5 – 2	The distance between the mass and the centre of its circle, or the length of string or rod if applicable in m.
Initial Angle	0 – 360	The angle in degrees between the string and the vertical when the simulation is started.
Initial Speed	0 – 10	The tangential speed of the object when the simulation is started in m/s.

6.3.4 Features

This simulation has a force diagram showing a general situation for vertical circular motion, with the force from the object keeping the mass in a circle and its weight shown on the diagram. The force diagram is fairly simple as there are many different situations, but it reminds you of the key forces to include. The diagram looks like this.



There is also a force diagram showing how gravitational potential energy and kinetic energy change over time. The total should be conserved over time, and energy is often important in solving problems with vertical circles, so it is important to be aware of this. The gravitational potential energy is measured with the zero line being at the centre of the circle, as this is often a simple place to calculate it from. The graph fills in dynamically while you are watching the simulation, and is reset whenever the simulation is reset, the tab is reselected or the curve reaches the end of the graph. An example graph is shown below.



6.4 Simulation 4: Simple Harmonic Springs

This simulation is about modelling simple harmonic motion (SHM) in springs. According to the MEI specification, for M3 you need to be able to:

1. Be able to recognise and formulate the standard equation of motion of SHM.
2. Be able to select a form of the solution of the SHM equation appropriate to the initial conditions. 7 Be able to verify solutions of the SHM equation using calculus.
3. Be able to apply standard results for SHM in context.
4. Be able to analyse motion under the action of springs as examples of SHM.
5. Be able to calculate the tension or energy in an elastic string or spring.

This simulation investigates the motion of a mass-spring system. A mass is attached to a spring and is pulled down to some initial extension, before it is released. The mass then oscillates simple harmonically. The time period, equilibrium extension, maximum velocity and maximum acceleration can be calculated theoretically using the standard results for SHM and measured from the simulation.

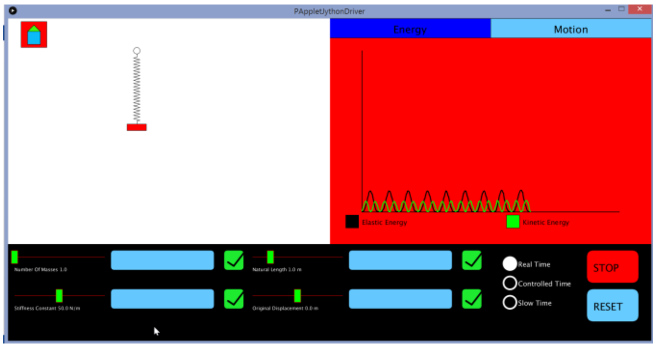
Springs are covered in the textbook throughout chapter 2, and SHM is covered in chapter 3. The specific situation in this simulation is explained on pages 119 – 120 of the M3 textbook.

6.4.1 Using the Simulation

To use this simulation, choose some initial values for the number of masses, stiffness constant, natural length and original displacement. Start the simulation to see the effect of these values. You can then stop the simulation, change one of the parameters and investigate how the time period, equilibrium and amplitude have changed.

You can also calculate the time period theoretically, and then use a timer to measure the time period of the spring. You can compare these values to your predicted values to check that you have calculated the theoretical values correctly.

The image below shows the layout of the screen when the simulation is playing.



6.4.2 Mechanics

To model an object as being in simple harmonic motion, the equation of motion $\ddot{x} = -\omega^2 x$ must apply for some value of ω.

In this case, simple harmonic motion occurs about the equilibrium position. Let e be the equilibrium extension and let x be the extension at equilibrium. By considering the forces when in equilibrium:

$$mg = ke$$

Where k is the spring constant.

When the spring is extended by some amount x from the equilibrium position, we can resolve downwards to get:

$$m\ddot{x} = mg - k(e + x) = mg - ke - kx = -kx$$

$$\Rightarrow \ddot{x} = -\frac{k}{m}x$$

Therefore, the mass is in simple harmonic motion with $\omega = \sqrt{\frac{k}{m}}$.

We can then use the standard formulae for simple harmonic motion to deduce that the extension should form a sine wave when plotted against time, with a period of $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$. We can also consider the initial conditions to find out what the amplitude of the motion is.

6.4.3 Parameters

The parameters that you can change in this simulation and the allowed ranges for each parameter is shown in the table below. All parameters in this simulation are changed in the bottom panel of the screen.

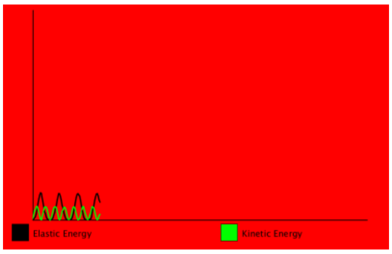
Parameter Name	Allowable inputs	Description
Stiffness Constant	1 – 100	The stiffness constant (sometimes known as the spring constant), representing the tension in the spring when the extension is 1m. It is measured in N/m.
Number of Masses	1 – 5	The number of masses currently loaded on the spring. Each mass has weighs 1kg.

Natural Length	0.5 – 3	The natural length of the spring in metres.
Original Displacement	-1 – 1	The original extension of the spring from its natural length in metres when the simulation is started.

6.4.4 Features

This simulation has two graphs – one for energy and one for motion.

The energy graph shows kinetic energy and elastic potential energy over time. These do not add up to a constant value, as they do not account for gravitational potential energy, but they give some idea of the energy transfers over time. An example graph is shown below.



The motion graph shows displacement, velocity and acceleration over time. The displacement is measured as the extension from the natural length of the string. These should all be sine waves shifted by different amounts. An example graph is shown below.