Class VI: Introduction to Aerospace Engineering

M. G. Bharath and M. Ramakrishna,
Department of Aerospace Engineering,
Indian Institute of Technology Madras

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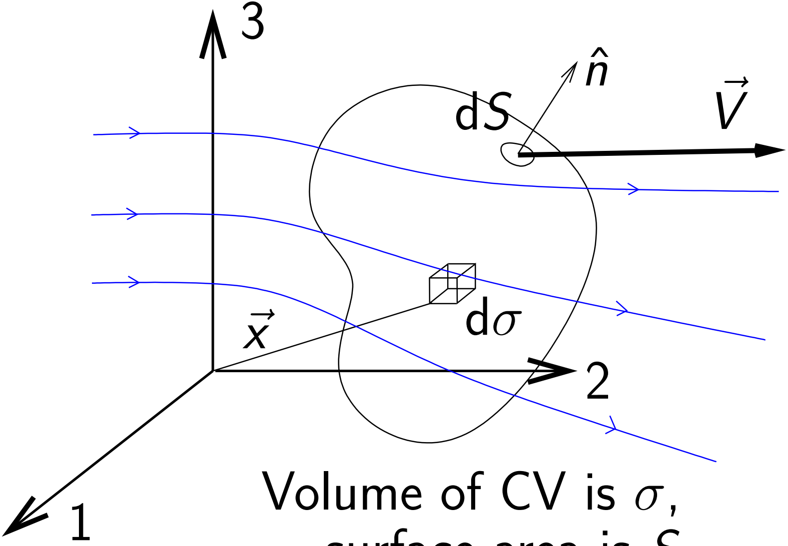
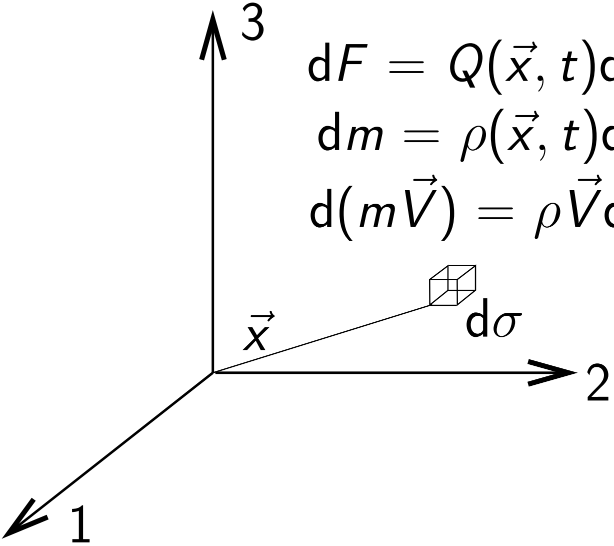
Control Volume in Region of Interest



Volume of CV is σ,
surface area is S

How much in an elemental volume?

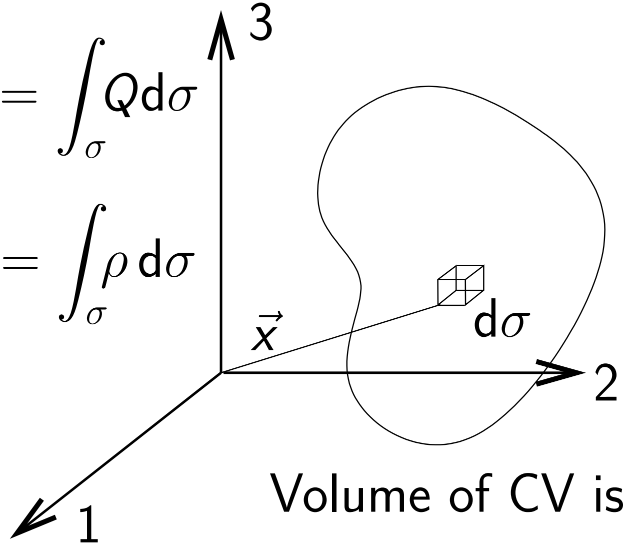
$$\mathrm{d}F = Q(\vec{x}, t)\mathrm{d}\sigma$$
$$\mathrm{d}m = \rho(\vec{x}, t)\mathrm{d}\sigma$$
$$\mathrm{d}(m\vec{V}) = \rho\vec{V}\mathrm{d}\sigma$$



How much in the Control Volume?

$$F(t) = \int_\sigma Q \mathrm{d}\sigma$$

$$m(t) = \int_\sigma \rho \, \mathrm{d}\sigma$$



Volume of CV is σ,

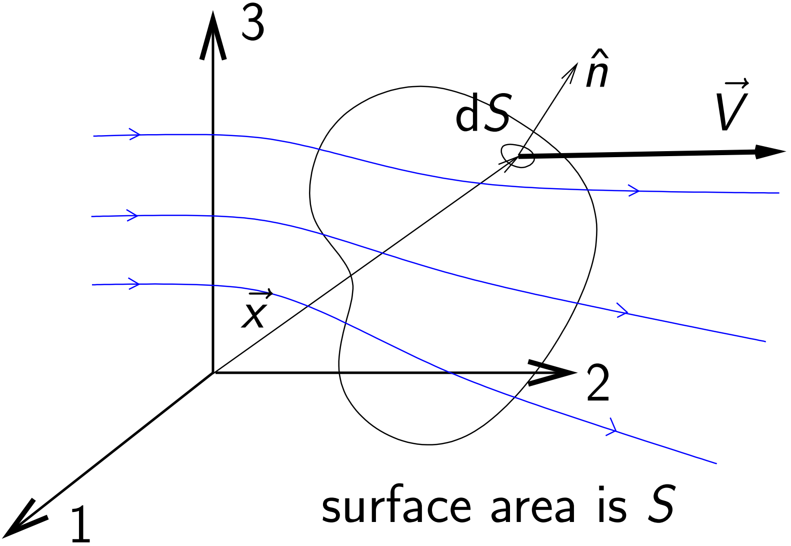
Time rate of change

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\sigma} Q\mathrm{d}\sigma$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\sigma} \rho\,\mathrm{d}\sigma$$

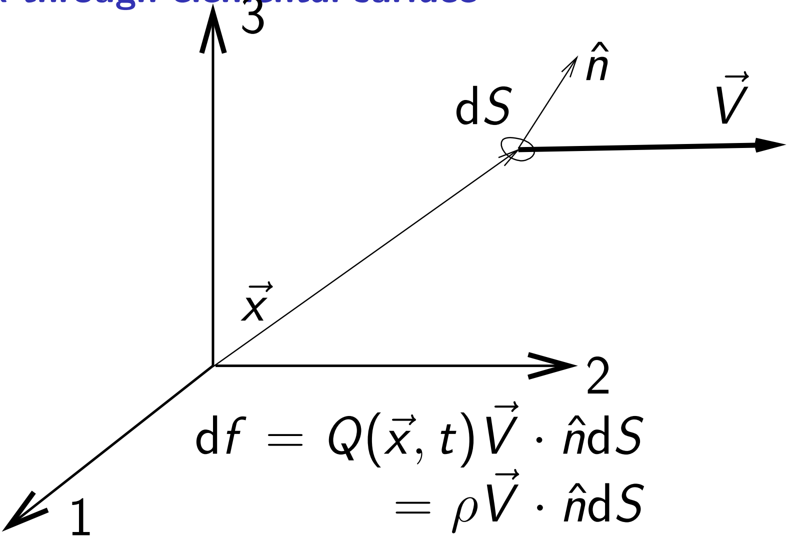
$$\frac{\mathrm{d}m\vec{V}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\sigma} \rho\vec{V}\,\mathrm{d}\sigma$$

Control Surface



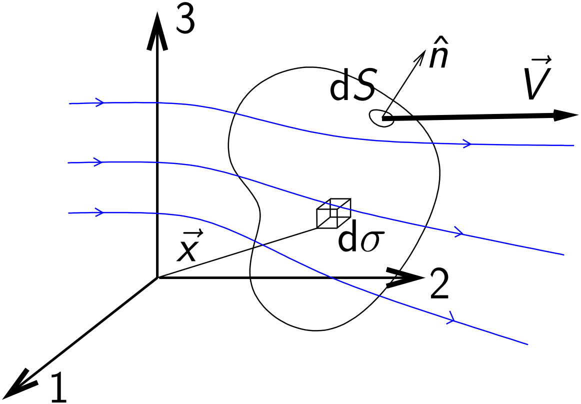
surface area is S

Flux through elemental surface



$$
\begin{aligned}
\mathrm{d}f &= Q(\vec{x}, t)\vec{V} \cdot \hat{n}\mathrm{d}S \\
&= \rho\vec{V} \cdot \hat{n}\mathrm{d}S
\end{aligned}
$$

Generalised Conservation Principle (FRP)



$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\sigma} Q\,\mathrm{d}\sigma = -\int_{S} Q\vec{V}\cdot\hat{n}\,\mathrm{d}S + \underset{\text{terms}}{\text{Production}}$$

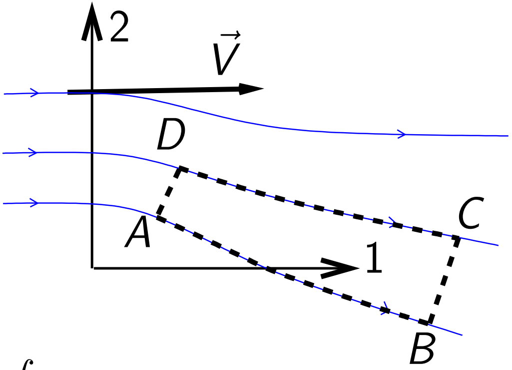
CM

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\sigma}\rho\,\mathrm{d}\sigma = -\int_{S}\rho\vec{V}\cdot\hat{n}\,\mathrm{d}S$$

for steady flow

$$\int_{S}\rho\vec{V}\cdot\hat{n}\,\mathrm{d}S = 0$$

CM - 2D Application - Streamlines



$$\int_S \rho \vec{V} \cdot \hat{n} \, \mathrm{d}S =$$

$$\int_{S_{AB}} \rho \vec{V} \cdot \hat{n} \, \mathrm{d}S + \int_{S_{BC}} \rho \vec{V} \cdot \hat{n} \, \mathrm{d}S + \int_{S_{CD}} \rho \vec{V} \cdot \hat{n} \, \mathrm{d}S + \int_{S_{DA}} \rho \vec{V} \cdot \hat{n} \, \mathrm{d}S$$

CLM

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{\sigma}\rho\vec{V}\,\mathrm{d}\sigma = -\int\limits_{S}\rho\vec{V}\vec{V}\cdot\hat{n}\,\mathrm{d}S + \int\limits_{\sigma}\vec{f}\,\mathrm{d}\sigma + \int\limits_{S}\vec{T}\,\mathrm{d}S$$