Experimental determination of Meta center height

(AS2100: Basic Aerospace Engineering Lab)

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Metacenter is the point at which a vertical line through the centre of buoyancy crosses the line through the original and the metacenter remains directly above the centre of buoyancy by definition. The metacentric height is the distance between the centre of gravity of the body and metacenter(G_0M as shown below in figure 4).

In this experiment the metacentric height G_0M of a cubical solid block is determined as follows. Firstly the solid is slowly immersed in a tank initially filled with water up to height H. Once, the solid floats and stabilises, the rise in height of water column ΔH is then noted down.

Then weights(W_1) are loaded on to one end of the solid in steps of $100gms$ up to $W_1 = 500gms$ and the resulting static angular displacement(θ) is measured at each step and then the loads are removed one by until $W_1 = 0$ and the average angular displacement θ for each W_1 for loading and unloading cycle is recorded in the data set described below. The whole experiment is repeated by 25 members and the data is recorded by each of them. The experimental setup is shown below in figure.

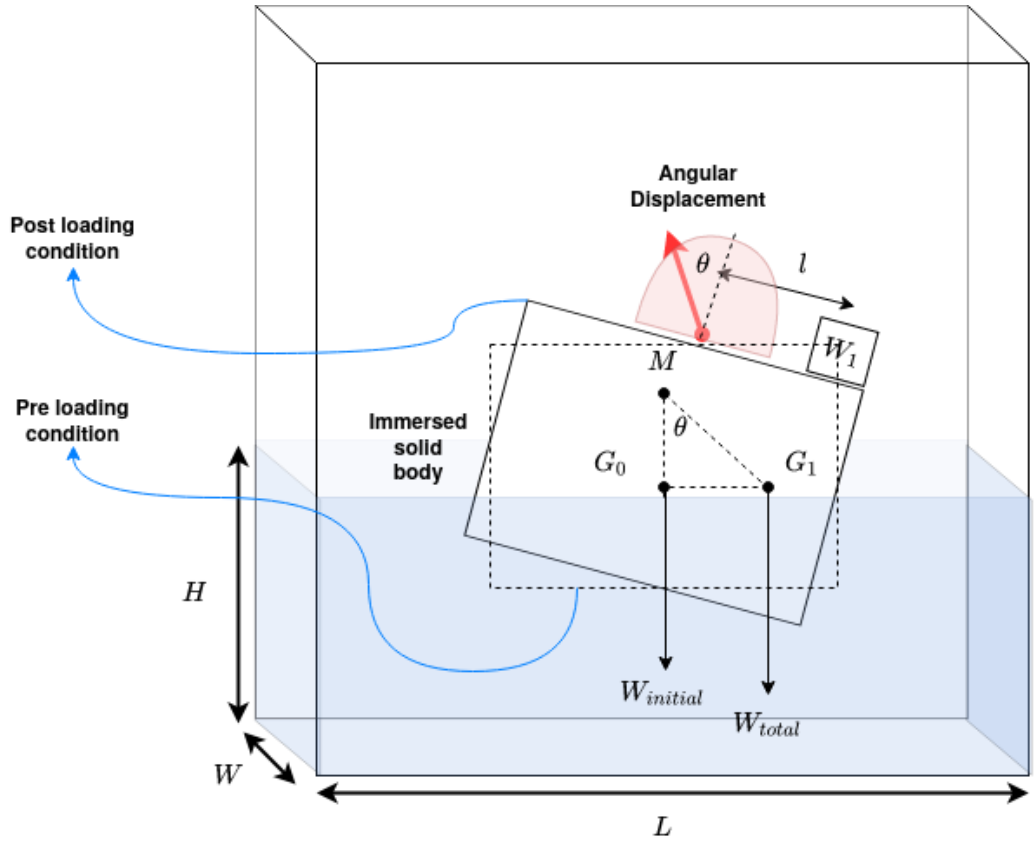

Figure 4: Schematics of the experimental setup.

Use the following parameters for your calculations: :

- Tank dimensions: $65cm \times 45cm \times 30cm$
- Dimensions of the cubical solid block: $20cm \times 20cm \times 20cm$
- Initial height of the water column, $H = 16cm$
- Rise in height of water column after immersing the solid, $\Delta H = 4cm$
- $\rho_{water} = 997kg/m^3$ at mean ambient temperature of $25°C$.
- Moment arm, $l = 10cm$
- Acceleration due to gravity, $g = 9.81m/s^2$

The data set `P4_MetaCenter.mat` contains the following:

- Loaded weights(in grams) (`"W1"`):

$$\boldsymbol{W_1} = \{W_1^{(1)}, W_1^{(2)}, W_1^{(3)}, W_1^{(4)}, W_1^{(5)}\}$$

- Angular displacements(in degrees)((`"Theta_Exp"`)) corresponding to the zero load condition and $W_1^{(j)}$ for $j = 1 \cdots 5$:

$$\boldsymbol{\Theta} = \{\theta_0^{(i)}, \theta_1^{(i)}, \theta_2^{(i)}, \theta_3^{(i)}, \theta_4^{(i)}, \theta_5^{(i)}\}_{i=1}^{N=25}$$

  Here, θ_0 is the angular displacement measured at zero load condition($W_{initial} = W_{body}$).

Using the data set, do the following:

1. Explain the theory behind this experiment and obtain the analytical relationship for metacentric height for a cubical solid using first principles. Detail all the assumptions made.

2. What are the possible sources of errors in the experiment and how to minimize them?

3. The dimensions of the immersed solid are $20cm \times 20cm \times 20cm$ and the rise of the water level as given earlier is $\Delta H = 4cm$. Using the fundamental principles as in question 1, determine the density of the solid. Also, obtain the theoretical metacentric height using the relationship obtained in question 1. (Consider the obtained metacentric height as the true value(G_0M^{True}) for further calculations).

4. Given that the empirical relationship between experimentally determined angular displacement θ, metacentric height G_0M^{Exp}, initial weight (W_{body}), loaded weight (W_1) and moment arm (l) is as follows:

$$G_0M^{Exp} = \frac{W_1\, l}{(W_{body} + W_1)\tan(\theta)},$$

   Estimate the experimental metacentric height from each observation of θ.

5. Plot the normalised histograms of the observed angular displacements (θ) for each W_1. Also, plot a smoothened probability density trend over each of the histograms. Determine if the probability density functions are Gaussian in nature. If so, using the statistical quantities μ and σ, of θ for each W_1 plot the corresponding Gaussian density functions over the histograms.

6. Using the theoretical value for metacentric height(obtained in question 3) as true value G_0M^{True}, plot the normalised histograms and corresponding smoothened probability density functions of the errors($e_{ij} = G_0M_j^{True} - G_0M_{ij}^{Exp}$ for $i = 1 \cdots 25$, and $j = 1 \cdots 5$) in the experimentally determined metacentric height G_0M^{Exp} for each W_1. Determine the accuracy($|G_0M^{True} - G_0M_{Mean}^{Exp}|$) and precision (given in terms of $\pm 1\sigma_{G_0M}$) in each case.

7. Using the given data set, obtain a linear($y = a_1x + a_0$) and polynomial($y = \Sigma_{i=1}^{n} a_i x^i + a_0$ with $n = 2, 3$) fits for the mean angular displacement($\bar{\theta}$) as a function of loaded weight(W_1) data. Calculate the mean squared error for each fit.

In addition to answers to the above questions each report should also include introduction about the experiment , schematics of the experimental setup, a results and discussion section and a conclusion section.