Class 02, July 10, 2021

"This is shown ...."

Fig ~10
Table ~4
100~m/s

1 .... figure
Figure 1

x^2 $x**2$ $x*x$ $x/4 \rightarrow 0.25 * x$
         $x^3 : x*x*x$

- Let us assume we are doing an experiment

$V_\infty \rightarrow$

or

How to find velocity?
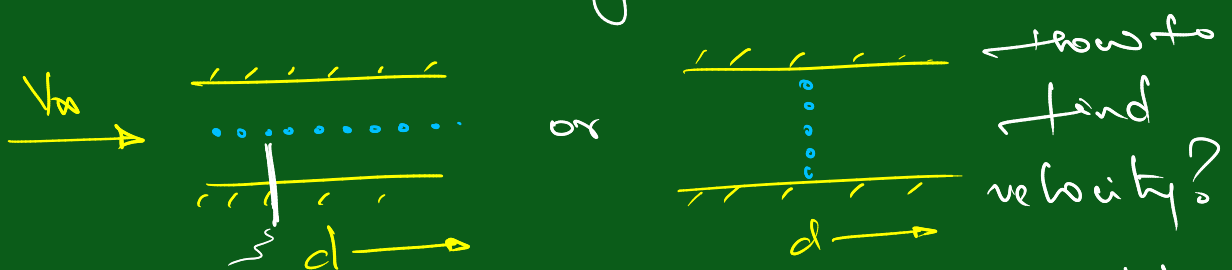
. Hot - wire anemometer
  ↳ Thermal transducer
  ↳ Single point
  ↳ Velocity - resistance
              - voltage

. pitot static tube
. Venturi meter
. PIV - ....

?
Eqn
↓
Tweak
↓
Least error

model
↓
— data
Velocity
V
|
|_____
        d →

Velocity
V
| ....
|_____
        d →
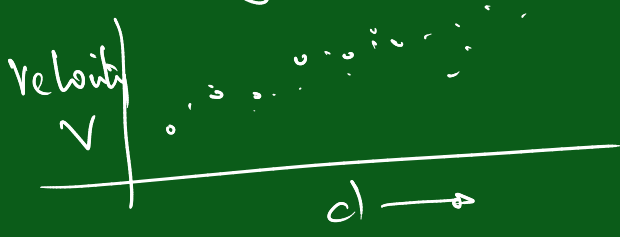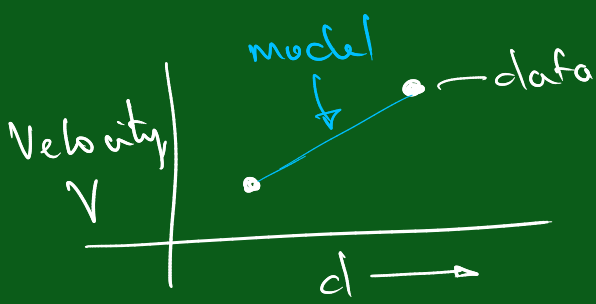
- linear eqn
- polynomial
- exponential

× Let us assume

$$V = md + c$$

to be estimated

$V_1 = md_1 + c$
$V_2 = md_2 + c$

N observations $V_N = md_N + c$

$$\begin{bmatrix} d_1 & 1 \\ d_2 & 1 \\ \vdots & \vdots \\ d_N & 1 \end{bmatrix}_{N \times 2} \begin{Bmatrix} m \\ c \end{Bmatrix}_{2 \times 1} = \begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix}_{N \times 1}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ a_N \end{bmatrix} \begin{Bmatrix} m \\ C \end{Bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ \cdot \\ v_N \end{Bmatrix}$$

$$A \qquad x \quad = \quad b$$

* What does $Ax = b$ mean in this context?

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{N1} & a_{N2} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \cdot \\ b_N \end{Bmatrix}$$

$$\underbrace{\begin{Bmatrix} a_{11} \\ a_{21} \\ \cdot \\ a_{N1} \end{Bmatrix}}_{A_1} x_1 + \underbrace{\begin{Bmatrix} a_{12} \\ a_{22} \\ \cdot \\ a_{N2} \end{Bmatrix}}_{A_2} x_2 = \begin{Bmatrix} b_1 \\ b_2 \\ \cdot \\ b_N \end{Bmatrix}$$
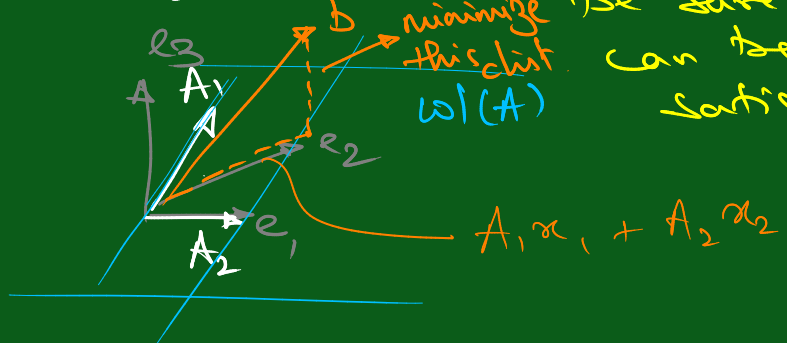
* b is a linear combination of the columns of A!

* Example.

$$\begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix} x_1 + \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} x_2 \approx b$$
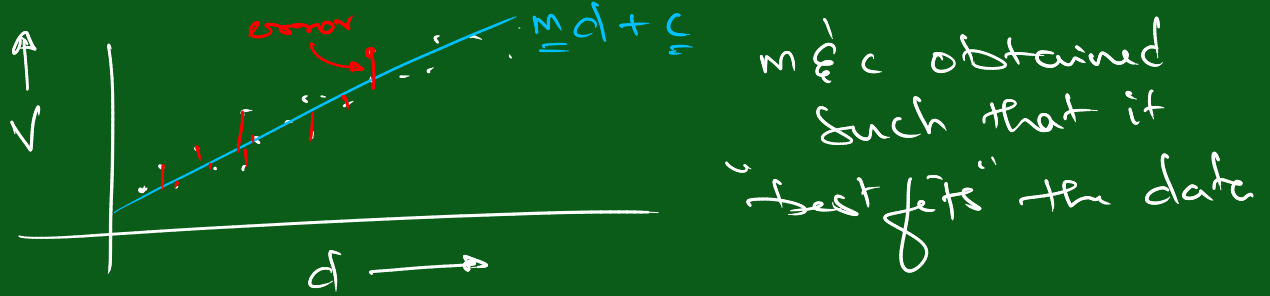
how can you be sure this can be satisfied?

minimize this dist. w/ (A)

$A_1 x_1 + A_2 x_2$

* $A_{N\times 2}\, x_{2\times 1} = b_{N\times 1}$

$$(A^T A)_{2\times 2}\, x_{2\times 1} = (A^T b)_{2\times 1}$$

how can you be sure this matrix is invertible?

$$x = (A^T A)^{-1} A^T b$$

solution for x!

error $y = md + c$

m & c obtained
such that it
"best fits" the data

Linear Regression

$Ax = b$

find $\hat{x} = \arg\min_x \|b - Ax\|$
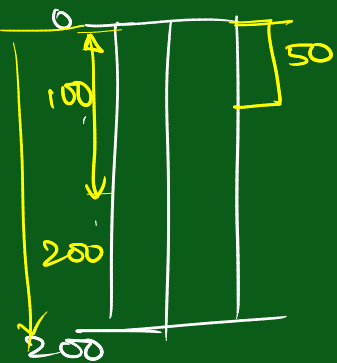
$Ax - b = \epsilon$ ← minimize this term

$\hat{x} = (A^T A)^{-1} A^T b$

x $V = md^c \longrightarrow \log V = \log m + c \log d$

$V = md + c + kd^A \longrightarrow$ valid?

x <u>Task</u> : data/scripts - class 02 - data1.txt

200(x, y)   2.txt
3.txt

- Perform linear regression using $y = mx + c$

- Using first 50, 100, 200 points

Data	m	c		D2				D3		
50										
100										
200										

- Based on the behaviour of m & c, infer the type of data.

- Do you think a better model would work instead of $y = mx + c$? What? Why?

- Plot the data a scatter plot & a line for the model. Support previous inference

- Use octave

do not use inv(A) or A\b !

- Invert matrix using Gaussian-Elimination