Secant Method of Numerical analysis

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Secant method is also a recursive method for finding the root for the polynomials by successive approximation. It's similar to the **Regular-falsi** method but here we don't need to check **$f(x_1)f(x_2)<0$** again and again after every approximation. In this method, the neighbourhoods roots are approximated by secant line or chord to the function **f(x)**. It's also advantageous of this method that we don't need to differentiate the given function **f(x)**, as we do in **Newton-raphson** method.

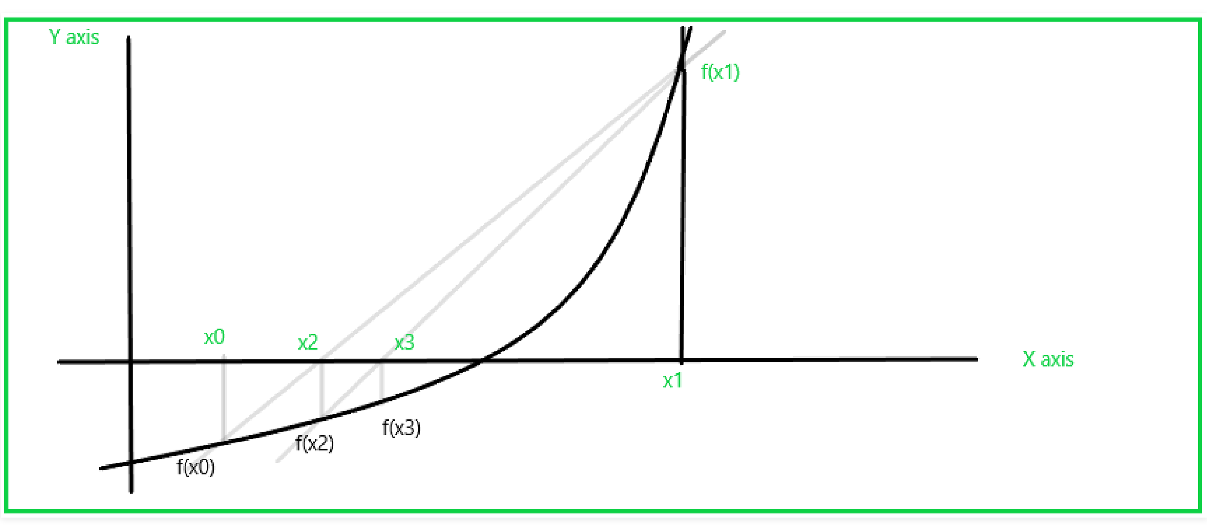

Figure – Secant Method

Now, we'll derive the formula for secant method. The equation of Secant line passing through two points is :
$Y - Y_1 = m(X - X_1)$
Here, m=slope

So, apply for **$(x_1, f(x_1))$** and **$(x_0, f(x_0))$**

$$Y - f(x_1) = [f(x_0)-f(x_1)/(x_0-x_1)]\ (x-x_1) \qquad \textbf{Equation (1)}$$

As we're finding root of function f(x) so, Y=f(x)=0 in Equation (1) and the point where the secant line cut the x-axis is,

$$x= x_1 - [(x_0 - x_1)/ (f(x_0) - f(x_1)]f(x_1) .$$

We use the above result for successive approximation for the root of function f(x). Let's say the first approximation is **$x=x_2$:**

$$x_2= x_1 - [(x_0 - x_1)/ (f(x_0)-f(x_1))]f(x_1)$$

Similarly, the second approximation would be **$x = x_3$**:

$$x_3= x_2 - [(x_1-x_2)/ (f(x_1)-f(x_2))]f(x_2)$$

And so on, **till k^{th} iteration,**,

$$x_{k+1}= x_k - [(x_{k-1} - x_k) / (f(x_{k-1}) - f(x_k))]f(x_k)$$

Note: To start the solution of the function f(x) two initial guesses are required such that **$f(x_0)<0$** and **$f(x_1)>0$.** Usually it hasn't been asked to find, that root of the polynomial f(x) at which **f(x) =0.** Mostly You would only be asked by the problem to find the root of the **f(x)** till two decimal places or three decimal places or four etc.

Example-1 :
Compute the root of the equation $x^2e^{-x/2} = 1$ in the interval [0, 2] using the secant method. The root should be correct to three decimal places.

Solution –
$x_0 = 1.42$, $x_1 = 1.43$, $f(x_0) = -0.0086$, $f(x_1) = 0.00034$.

Apply, **secant method,** The first approximation is,
$x_2 = x_1 – [(x_0 – x_1) / (f(x_0) – f(x_1)]f(x_1)$
$= 1.43 – [(1.42 – 1.43) / (0.00034 – (– 0.0086))](0.00034)$
$= 1.4296$
$f(x_2) = – 0.000011$ (–ve)

The second approximation is,
$x_3 = x_2 – [(x_1 – x_2) / (f(x_1) – f(x_2))]f(x_2)$
$= 1.4296 – [(1.42 – 1.4296) / (0.00034 – (– 0.000011](– 0.000011)$
$= 1.4292$
Since, **x_2** and **x_3** matching up to **three decimal places,** the required root is **1.429**.

Example-2 :
A real root of the equation $f(x) = x^3 – 5x + 1 = 0$ lies in the interval (0, 1). Perform four iterations of the secant method.

Solution –
We have, $x_0 = 0$, $x_1 = 1$, $f(x_0) = 1$, $f(x_1) = – 3$
$x_2 = x_1 – [(x_0 – x_1) / (f(x_0) – f(x_1))]f(x_1)$
$= 1 – [(0 – 1) / ((1-(-3))](-3)$
$= 0.25.$

$f(x_2) = – 0.234375$

The second approximation is,
$x_3 = x_2 – [(x_1 – x_2) / (f(x_1) – f(x_2))]f(x_2)$
$=(– 0.234375) – [(1 – 0.25)/(–3 – (– 0.234375))](– 0.234375)$
$= 0.186441$
$f(x_{34} = x_3 – [(x_2 – x_3) / (f(x_2) – f(x_3))]f(x_3)$
$= 0.186441 – [(0.25 – 0.186441) / (– 0.234375)$ **– (0.074276)](– 0.234375)**
= 0.201736.

$f(x_4) = – 0.000470$
The fourth approximation is,
$x_5 = x_4 – [(x_3 – x_4) / (f(x_3) – f(x_4))]f(x_4)$
$= 0.201736 – [(0.186441 – 0.201736) / (0.074276 – (– 0.000470)](– 0.000470)$
$= 0.201640$

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