

**Department of Power Engineering and Turbomachinery
Silesian University of Technology, Gliwice**



**INSTRUCTIONS FOR LABORATORY EXERCISES OF
STRENGTH OF MATERIALS**

TORSION TEST

Dr inż. Katarzyna Stolecka

Gliwice, 2018

1) *The aim of the laboratory*

The purpose of the exercise is to find the modulus of elasticity in shear G (rigidity modulus) of the material of which a bar with a cross-section is made, by measuring the angle of twist. The second purpose of the exercise is determination of the characteristics of $M_s=f(\phi)$ (dependence of the torque (twist moment) versus angle of twist).

2) *Introduction*

Torsion occurs when the twisting moments are applied to the bar.

Torsion is induced by two pairs of forces. These two forces act in two different planes perpendicular to the axis of the bar. Moments of these two forces are called twisting moments.

If the bar is loaded by two pairs of forces of equally magnitudes moments but opposite directions in two planes which are perpendicular to the axis then we obtained the state of strain and stress called pure torsion – fig. 1.

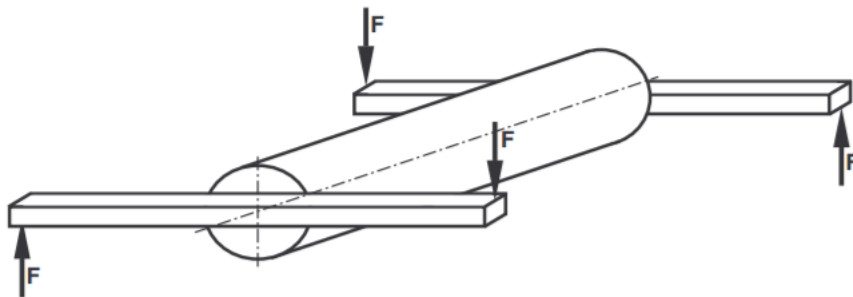


Fig. 1. Models of pure torsion

Considering a simplified model of the bar. And then, assuming a three-dimensional grid and subjecting to a pure torsion, it can be seen that (Fig. 2):

- the axis of the bar does not change its form and after deformation still remains straight line
- the circles remain wheels, and the cross-sectional ends are flat
- the horizontal lines of the grid will change its initial shape. This means that these lines are deflected from their initial position by an angle γ
- the surfaces of the front and end of the bar will rotate relative to each other by an angle of twist ϕ

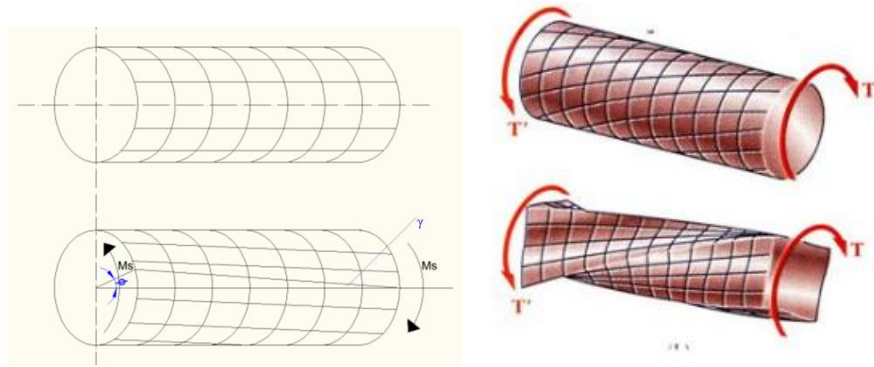


Fig. 2. Deformation of the twisted bar of circular/square cross-section

3) Calculations

If we cut a segment of the bar with height dx and we assume some certain simplifications we can evaluate the angle of twist of the bar section with height dx and radius r as:

$$d\varphi = \frac{M_s dx}{GJ_0}$$

The angle of twist for entire length of the bar can be expressed as:

$$\varphi = \frac{M_s L}{GJ_0}$$

$$J_0 = \frac{\pi d^4}{32}$$

Where:

M_s – torsion moment (torque)

dx – the height of the section of the bar

L – length of the bar

G – shear modulus, also called the rigidity modulus

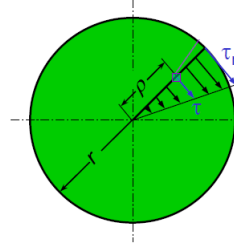
J_0 – polar moment of inertia

d – diameter of the bar

Shear stress

Shear stresses are produced by the application the twisting moments to the end of the bar. Shear stress in sections orthogonal to the axis of the torsional bar can be expressed as:

$$\tau_{\rho} = \frac{M_s}{J_0} \rho$$



The highest shear stress occurs on the surface of the bar, where the radius is maximum.

$$\tau_{max} = \tau_{\rho=r} = \frac{M_s}{J_0} r$$

$$\frac{J_0}{r} = W_0$$

$$\tau_{max} = \frac{M_s}{W_0} \leq \tau_{dop}$$

W_0 – section modulus of torsion $W_0 = \frac{\pi d^3}{16}$

τ_{dop} – permissible stress

NOTE: Above formulas are valid for the bar of circular cross-section in the area of stress falling below the *proportional limit* (Fig. 3).

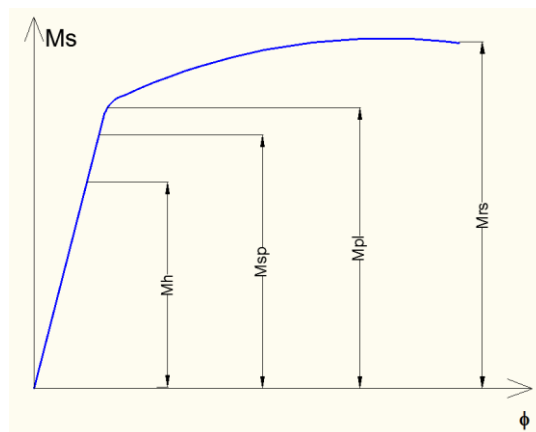


Fig. 3. The torque-twist plot of a torsion specimen

M_h – torque corresponding to the proportional limit stress

M_{sp} – torque corresponding to the elastic limit stress

M_{pl} – torque corresponding to the yield stress

M_{rs} – torque corresponding to the fracture (rupture strength)

In the torsion bar, a change of stress in the circular cross-section can be observed. Graph of shear stress to the proportional limit takes the form of a right triangle. After crossing the proportional limit, the triangular form of a graph passes in a rectangular form, where the shear stress corresponding to the yield stress already exists in the entire cross-section of the bar (see Fig. 4). The change of distribution of stresses (from triangular to rectangular) results from shifting the first layers of the external cross-section of the specimen beyond its yield stress. While the layers closer to the axis still remain in the elastic region.

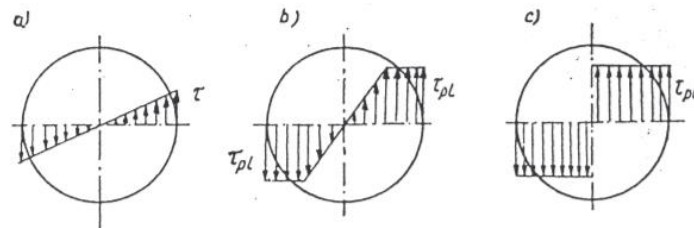


Fig. 4. Shear stress in the twisted bar of circular cross-section: a) elastic region, b) elastic-plastic region, c) plastic region

4) Torsion test

Torsion test is usually carried out in the specimens of the circular cross-section for which determine the state of stress and strain is relatively simple. Specimens with different cross-sections are used for special tests. In this time, it is required to use different formulas than in the presented above. In the case of solid specimens of circular cross-section the gauge length of the bar is equal to $(5-20)d_0$, generally $l=10d_0$ where $d_0=(10-30)mm$ is a gauge diameter of specimen.

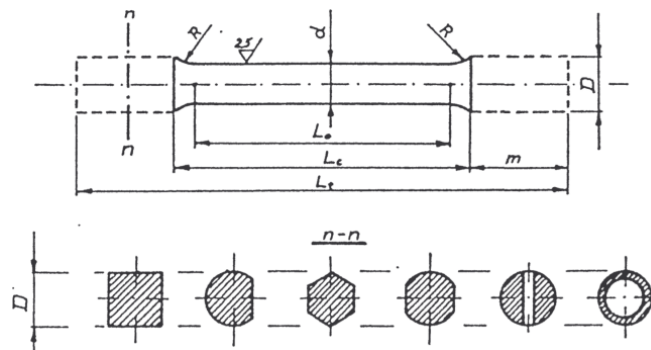
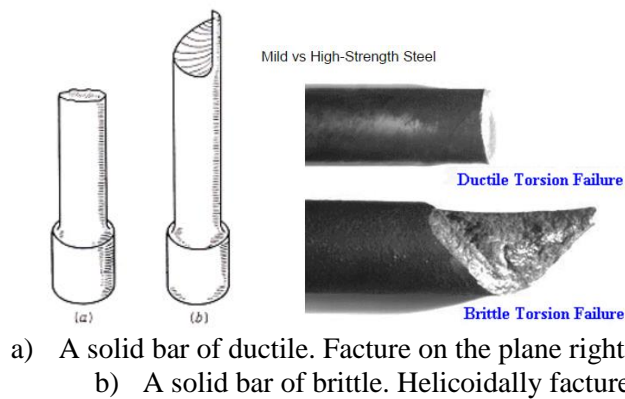


Fig 5. Specimens for torsion test

During the destructive torsion test, torsion fracture geometry may take the following forms:

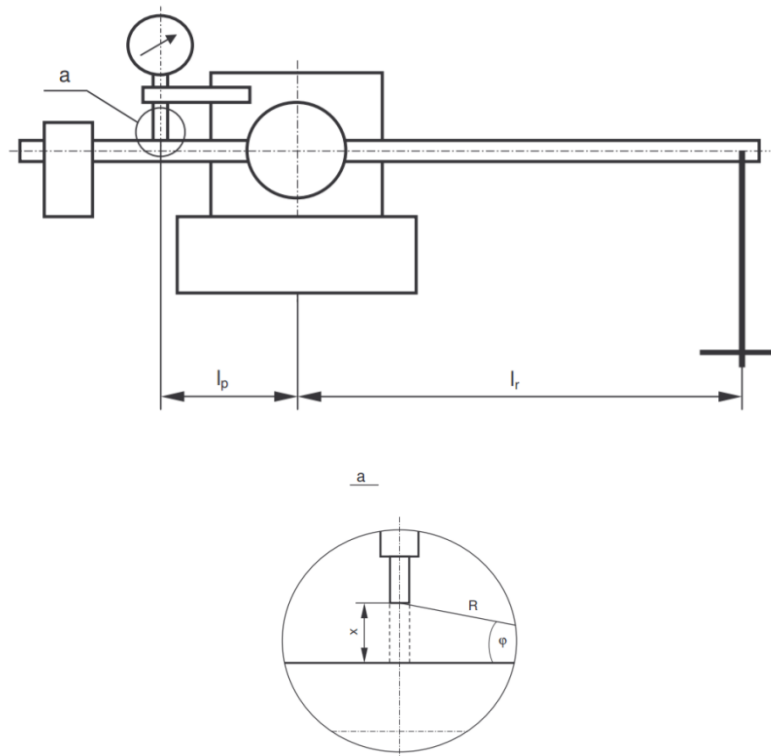
- When subjected to torsion a ductile specimen breaks along the plane perpendicular to the bar axis

- b) When subjected to torsion a brittle material breaks along the plane perpendicular to the direction in which the tension is maximum



5) Determination of shear modulus G (modulus of rigidity)

Scheme of the test stand is shown in Fig.6.



For solid circular torsion shafts:

$$J_0 = \frac{\pi d^2}{32}$$

so:

$$G = \frac{32M_s L}{\pi d^4 \varphi}$$

Where:

$$\varphi \cong \arctan \frac{x}{l_p} [^\circ]$$

φ – the angle of twist

x – the displacement of the end of the spindle sensor caused by the angle of twist

l_p – the distance between the spindle sensor and axis of the shafts

6) *Torsion test*

To perform the measurement do the following:

- 1) Set the sensor to zero.
- 2) Put the weight on the plate. The greater weights causes the increasing of the torque.
- 3) Each time make a response of sensor reading and written in measurement card.
- 4) After reaching the maximum load take off the weight and also write a response of the sensor.

7) *Laboratory report*

The laboratory's report should contain:

- a) A short introduction to torsion test
- b) Brief description of the aim of the laboratory
- c) Calculations
- d) The formula for angle of twist
- e) Plot of function $M_s = f(\varphi)$
- f) Scheme of a test stand
- g) Complete measurement card
- h) Short conclusions.

Deadline for a laboratory report is 2 weeks.

Date of the laboratory:

Section:

Name	Surname

Measurement data

$l_p = 90$ mm (see fig. 6)

$l_r = 700$ mm (see fig. 6)

length of specimen $l = 500$ mm

diameter of specimen $d = 20$ mm

Lp.	m [kg]	x [mm]	$M_s = m \cdot l_r \cdot g$ [N]	φ [°]	G_i [MPa]	$G_{av} = \frac{1}{n} \sum G_i$
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						
12.						
13.						
14.						