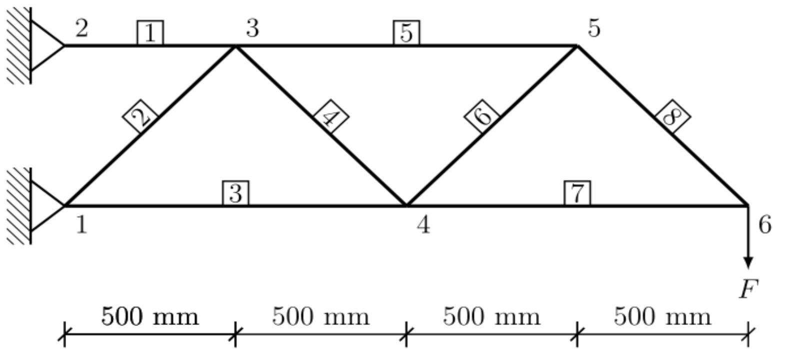


Figure 1.1: Truss framework

E = 210000 N/mm^2

Cross Section :
[1], [3], [5], [7] = 50mm^2
[2], [4], [6], [8] = 30mm^2

F = 10kN = 10,000N

The assignment is to use the finite element method to calculate the nodal displacements and the forces in the trusses. The starting point has to be the [B]-matrix, which containes the derivatives of the local interpolation functions

$$[B] = \frac{1}{L}[-1, 1]$$

Given this equation, the element stiffness matrix $[K_{el}]$ can be easily derived for the displacements and forces in the local coordinate system (equation 2.8 of the reader). The local element stiffness matrix has to be rewritten to the global coordinate system (equation 2.21 of the reader). Next, the element stiffness matrices need to be assembled in the global stiffness matrix. To solve the equilibrium equation [K]{U} = {F} the reduced stiffness matrix $[K^{red}]$ has to be selected corresponding to the set of unknown displacements. The equation can be solved using the loading on the unknown degrees of freedom $\{F^{red}\}$. Solving the set of equations using MATLAB will give the nodal displacements. In §1.3 the steps needed to solve the truss problem are further elaborated. First an introduction to MATLAB is given in the following section.

For this system $\boxed{F_e = ku}$ — (1)

Internal energy $\Lambda = \int_0^{u_1} F_e \, du \xrightarrow[(1)]{by} \Lambda = \frac{1}{2} k u_1^2$

check external work $W = F u_1$

$\boxed{\text{Assuming all external work is converted to internal energy}}$

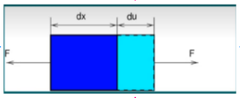
Total potential energy $\Pi = \Lambda - W$

@ equilibrium, $\pi = \pi_{min}$ $\therefore \frac{d\pi}{du} = 0$, $\frac{d^2\pi}{du^2} > 0$ $\boxed{ku = F}$

Thus $F = ku$ works for systems that obey $F_e = ku$.

- constant E, A
- linear elastic

$k = \frac{AE}{dx}$; $\sigma = E\varepsilon$; $\varepsilon = \frac{du(x)}{dx}$



$\Lambda = \frac{1}{2} k (du)^2 \longrightarrow \Lambda = \frac{1}{2} AE \int_0^L \varepsilon_{xx} \varepsilon_{xx} \, dx$

$\boxed{\Lambda = 1/2 \int_V \varepsilon_{xx} \cdot E \cdot \varepsilon_{xx} \, dV \quad \text{if } E, A \text{ not constant}}$

$\therefore$ we need ε $\therefore u(x)$ ✓

assuming linear field $\because$ we have 2 B.C. u_i, u_j $\therefore$ we can find 2 variables

$u(x) = a_1 + a_2 x \xrightarrow{BC.} \boxed{u(x) = (1 - \frac{x}{L}) u_i + (\frac{x}{L}) u_j}$ ✓

$\begin{cases} u(x=0) = u_i \\ u(x=L) = u_j \end{cases}$

$\boxed{u(x) = [N]\{u\}}$ interpolation/shape functions. $\longrightarrow$ $u(x) = [\underbrace{1 - \frac{x}{L}}_{N_i} \quad \underbrace{\frac{x}{L}}_{N_j}] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$

$\varepsilon_{xx} = \frac{du(x)}{dx} = \underbrace{\frac{d[N]}{dx}}_{[B]} \cdot \{u\}$ $\boxed{\varepsilon_{xx} = [B]\{u\}}$

$\Lambda = \frac{1}{2} AE \int_0^L \{u\}^T [B]^T [B] \{u\} dx \ldots \Lambda = \frac{1}{2} AE \{u\}^T \int_0^L [B]^T [B] dx \cdot \{u\}$

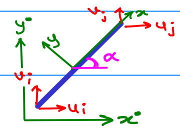
comparing with $\Lambda = \frac{1}{2} k u^2 \ldots \Lambda = \frac{1}{2} \{u\}^T [K] \{u\}$ $\ldots$ $\boxed{[K] = AE \int_0^L [B]^T [B]}$

in general for 3D configuration

$[K] = \int_V [B]^T [D] [B] dV \ldots [D]_{6\times6} \ldots$ for isotropic material $[D] \rightarrow E$.

for 1D bar element $[B] = [-1/L \;\; 1/L]$ $\ldots [K] = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

for 2D bar element $[K^\circ] = [R]^T [K] [R]$



$L = \sqrt{(x_j^\circ - x_i^\circ)^2 + (y_j^\circ - y_i^\circ)^2}$

$\cos\alpha = \frac{1}{L}(x_j^\circ - x_i^\circ)$

$\sin\alpha = \frac{1}{L}(y_j^\circ - y_i^\circ)$

$$\boxed{[K^\circ] = \frac{AE}{L} \begin{bmatrix} cc & cs & -cc & -cs \\ cs & ss & -cs & -ss \\ -cc & -cs & cc & cs \\ -cs & -ss & cs & ss \end{bmatrix}}$$

such that $\boxed{[K^\circ]\{u^\circ\} = \{F^\circ\}}$

then use direct stiffness method for global system.