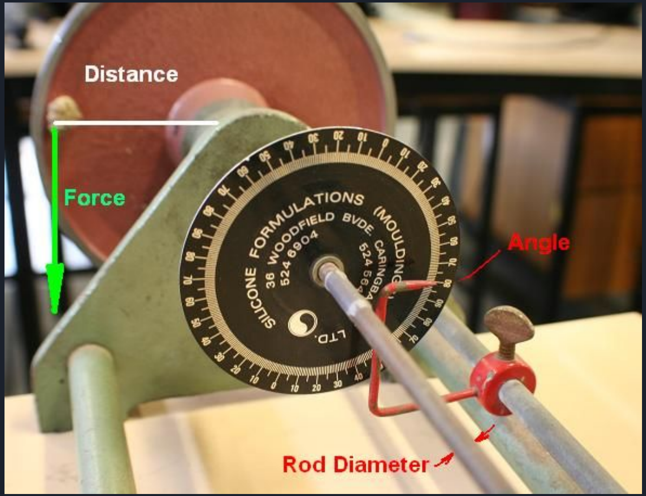
Determination of Shear Modulus of an aluminium rod using Torsion Test

Introduction



From the torsion test experiment we are able to obtain the general Shear-Strain behavior along with the shear elastic modulus (G), shear proportional stress (τ p), shear yield stress (τ y). In contrast to uniaxial tension tests which torsion test parallel to, the stresses are not distributed uniformly over the cross section.

Torsion loading results in twisting of one section of a body with respect to a contiguous section. For a circular cross-section pure shear stress state exists at each point. Torsional elastic shear stresses vary linearly from zero at the axis to a maximum at the extreme fibers.

When the surface fibers reach the yield shear stress they are, in a sense, supported by elastic interior fibers. So in a sense elastic resistance of the section masks the effect of yielding of the surface fibers during their early stage of yielding. Thus non-linearity is difficult to detect in early stages

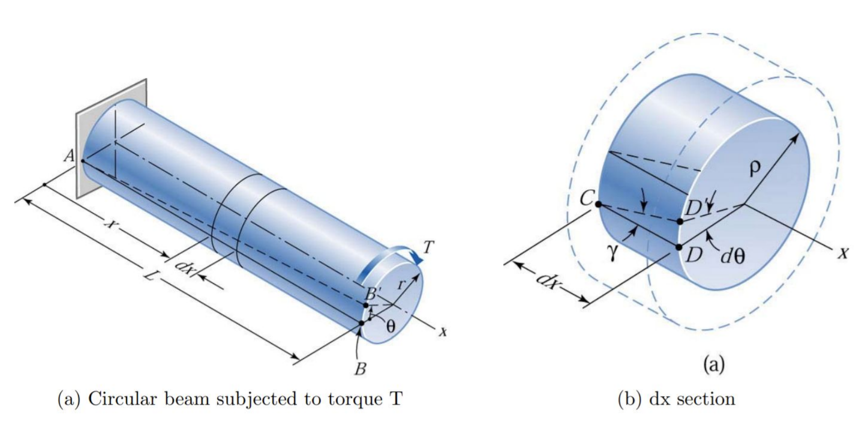
Apparatus

1. Caliper
2. Troptometer
3. Torsion Testing Machine
4. Measuring Tape
5. Testing Sample

Key assumptions made during the experiment

1. The torque is applied along the center of axis of the shaft.
2. The material is tested at steady state (absence of strain rate effects).
3. Plane sections remain plane after twisting (the circular section conforms to this condition).

1. Torsion Test : Theory and Experimental Procedure



(a) Circular beam subjected to torque T (b) dx section

The torsion equation is given as follows:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

The given equation holds for the condition of

Pure Torsion

The Assumptions made in Theory of Pure Torsion:

- The material is homogeneous and isotropic
- Hooke's law is obeyed by the material.i.e deformation are within proportionality limit (Important)
- The shaft is circular in section and remains planar even after torque is applied on it
- The cross-section of the shaft remains uniform throughout.
- The shaft is subjected to pure torque only.
- The shaft is not subjected to any initial torque. i.e initial deflection is zero
- The stress of the material should not exceed the proportionality limit i.e no permanent deformation is caused due to loading

The shear strain γ varies linearly in the radial direction

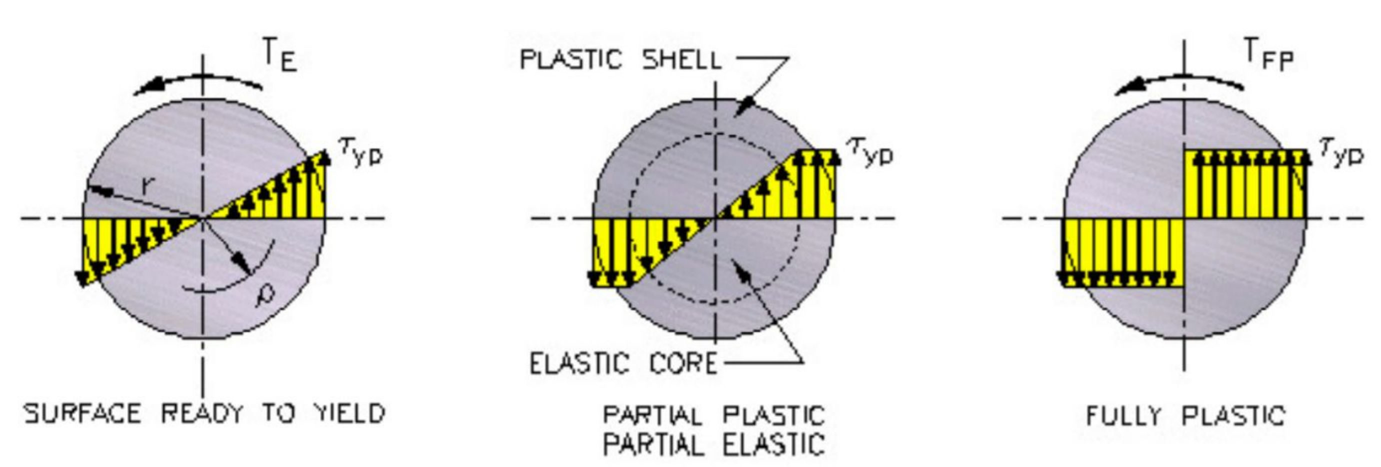


Figure 2: Types of Torques acting on a cross section:(a)Pure Torsion, (b) Partially Elastic Torsion, (c) Fully Plastic Torsion

Point to take care of while performing the experiment:

- Ensure zero error is accounted for while measuring testing sample
- Check for zero error in the apparatus before the experiment
- When adding and removing weights be careful not to pull on the weights or hold them up.
- Perform 20 cycle of loading and unloading.
- The slope of the best fit line gives us the value of the Shear Modulus (G) in the plot of Shear Stress vs Shear Strain
- Use the following relations for calculation of Shear Stress and Shear Strain:
  - Radial distance is from the centroidal longitudinal axis to the outer surface
  - d is the length of the lever.

$$ShearStress = Torque \cdot \frac{RadialDistance}{J} = \frac{(mgd)\cdot r}{J}$$

$$ShearStrain = Deflection\,(radian) \cdot \frac{RadialDistance}{L} = \frac{\theta r}{L}$$

2. Sources and minimization of errors in the experiment

- Systematic Error
  - Zero error in measuring the dimensions of testing sample
  - Error in the masses used during the experiment
- Random Error
  - Observational errors by the experimenter when reading the troptometer
  - Environmental Error
- Human Error
  - Take care when adding new weights
  - The rod may be subject to fatigue as the experiment setup is old and has been performed multiple times

- The variation in shear modulus ΔG due to variations in diameter ΔD is given by the following relation:

$$\frac{\Delta G}{G} = -\frac{4\Delta D}{D}$$

- The error in shear modulus ΔG due to errors in torque ΔT is given by:

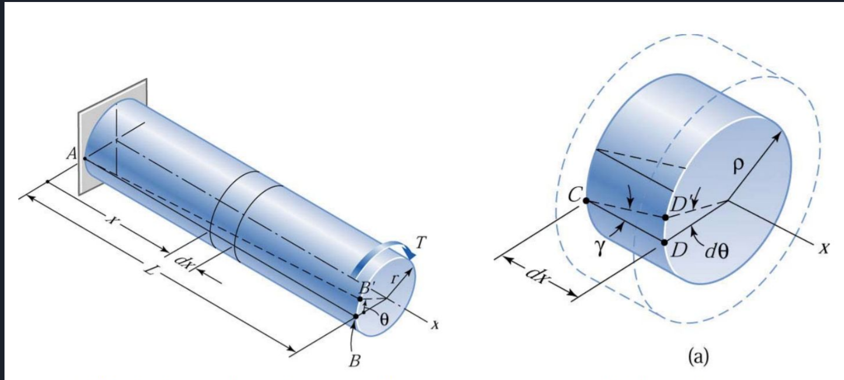
$$\frac{\Delta G}{G} = \frac{\Delta T}{T}$$

- According to the above, the error in shear modulus ΔG due to errors in angle of twist $\Delta \theta$ are given by:

$$\frac{\Delta G}{G} = \frac{\Delta \theta}{\theta}$$

- The least count of the twist gage should always be smaller than the minimum acceptable value of $\Delta \theta$

3. Relation between applied mass m and angle of deflection for a simply supported beam.



(a) Circular beam subjected to torque T

(b) dx section

For small angles of twist the shear strain γ that is developed in the shaft of radius r and length L when subjected to a Torque T is:

$$\gamma = \frac{BB'}{L} \qquad (1)$$

but arc BB' = rθ , therefore

$$\gamma = \frac{r\theta}{L} \qquad (2)$$

- Within the elastic limit, the ratio of stress to strain is constant (G=Shear Modulus)

$$\frac{\tau}{\gamma} = G \qquad (3)$$

- Substituting for γ from (2) in (3),and rearranging we have:

$$\frac{\tau}{r} = \frac{G\theta}{L} \qquad (4)$$

Consider a circular element with radius r and thickness dr centred at centre of the shaft, thus area on which the shear stress τ acts = 2πr dr.

The total shear force acting on this element = τ*(2πr dr).

The torque acting on the element is the moment of this force = τ*($2\pi r^2 dr$).

- The net torque acting on the system will be the sum of the moments of the tangential shear forces acting on all the elements:

$$T = \int_0^{\frac{D}{2}} \tau 2\pi r^2 \, dr$$

Substituting for τ from eqn(4), we have

$$T = \frac{G\theta}{L} \int_0^{\frac{D}{2}} 2\pi r^3 \, dr$$

On solving we get

$$T = \frac{G\theta}{L} \cdot \frac{\pi D^4}{32}$$

We know that for a circular cross-section

$$\frac{\pi D^4}{32} = J$$

In our experiment we produce the torque by adding mass on a lever attached to one end of the rod.

Therefore torque is:

$$T = mgd$$

m : mass of load applied

g : acceleration due to gravity; 9.81 m/s^2

d : length of lever

We get the relation between applied mass m and angle of deflection θ_0 as follows:

$$m = \frac{G\theta 0}{gdL} \cdot \frac{\pi D^4}{32}$$

4. Effect of changing the number of cycles per experiment in the mean value of G and Effect of mass increment value Δm in accuracy or precision of the G for a given m_{max}

- Precision is sometimes separated into :
  - Repeatability
  - Reproducibility

In this experiment we are determining the value of Shear Modulus of Aluminium 6061 by performing the torsion test for multiple cycle. The experiment is such that even if we perform replicate trails we will obtain scattered result.

Therefore as we need to make more measurements to get closer to the true value of G. Increasing the number of cycles per experiment will cause the mean of G to be closer to the true value of G.

Yes,mass increment value Δm effects the accuracy and precision of the G for a given m_{max}.

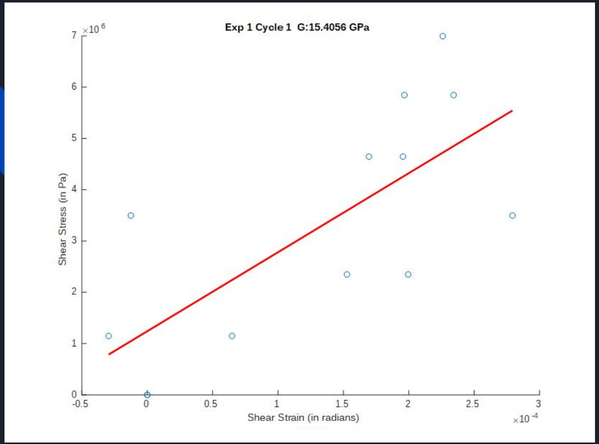
1. Smaller the Δm better will be our plot for Shear Stress vs Shear Strain giving us greater flexibility to drop data.
2. Taking smaller values of Δm reduces random error.

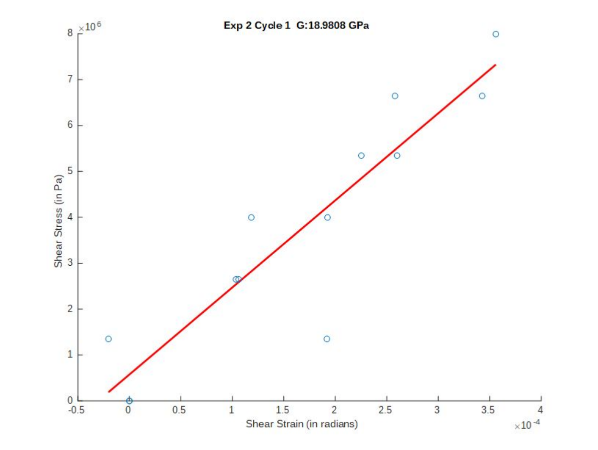
Therefore Δm will always affects the precision of Shear Modulus G.

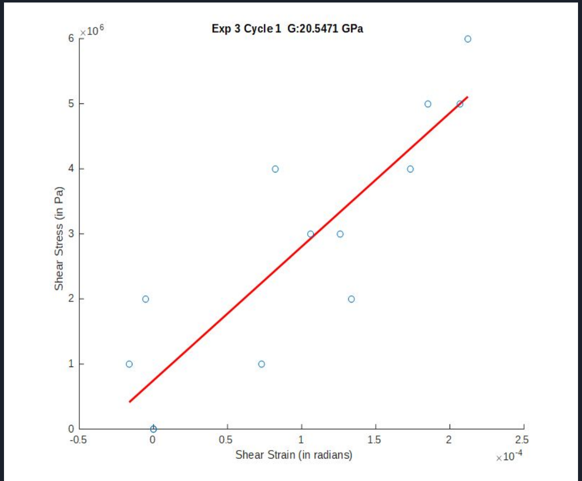
5. Shear Stress vs Shear Strain for the first cycle of each experiment using the best linear fit

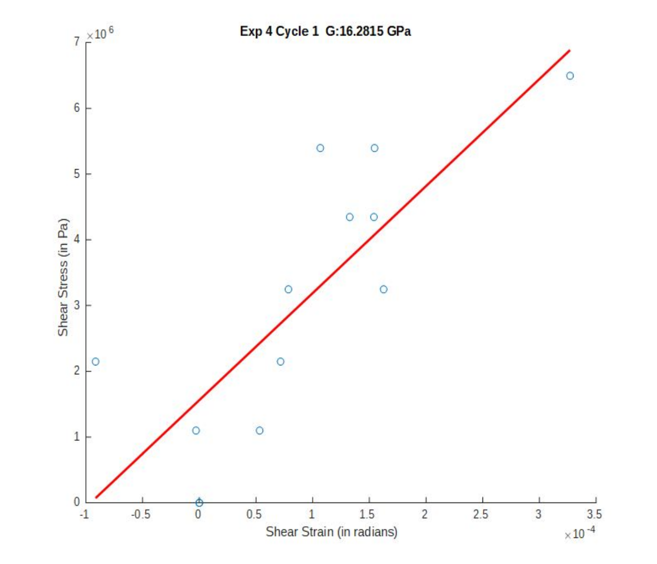
Observations Recorded:

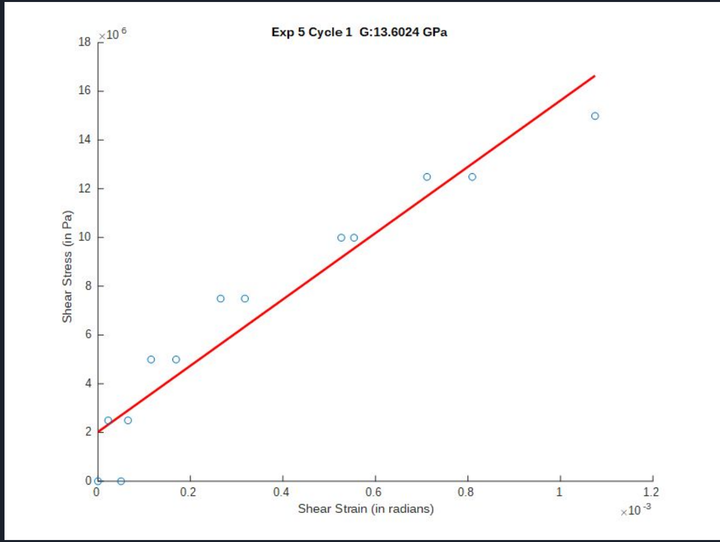
Best Estimate of Shear Modulus of Aluminum 6061			
Experiment Number	Cycle	Slope (Pa/rad)	G (GPa)
1 (Group 1)	1	15405632012.205857	15.4056
2 (Group 2)	1	18980810078.480890	18.9808
3 (Group 3)	1	20547119551.370123	20.5471
4 (Group 4)	1	16281510793.680640	16.2815
5 (Group 5)	1	13602448487.800990	13.6024


Exp 1 Cycle 1 G:15.4056 GPa
×10^6
Shear Stress (in Pa)
Shear Strain (in radians)
×10^-4


Exp 2 Cycle 1 G:18.9808 GPa
×10^6
Shear Stress (in Pa)
Shear Strain (in radians)
×10^-4


Exp 3 Cycle 1 G:20.5471 GPa
×10^6
Shear Stress (in Pa)
Shear Strain (in radians)
×10^-4

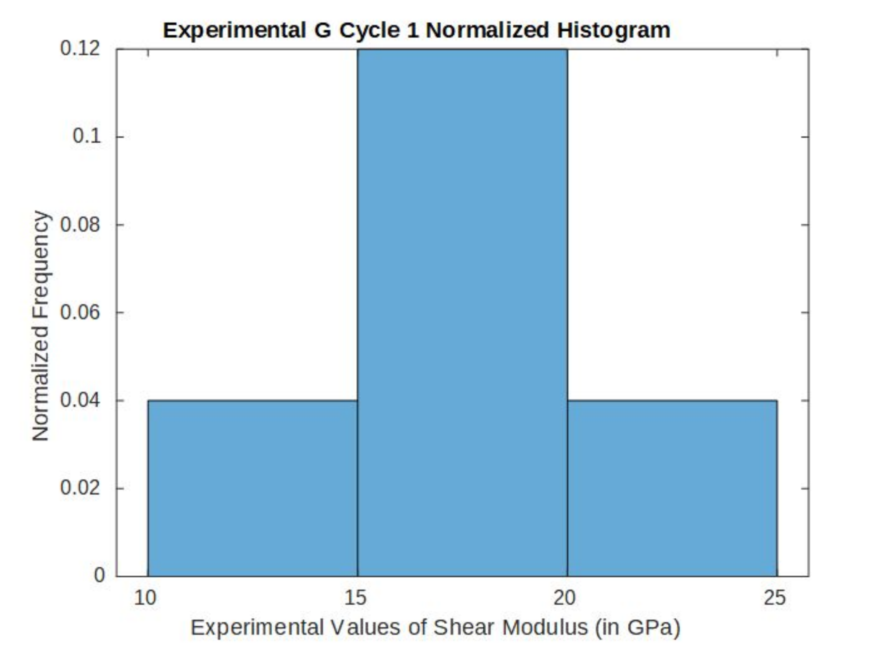

Exp 4 Cycle 1 G:16.2815 GPa
×10^6
Shear Stress (in Pa)
Shear Strain (in radians)
×10^-4

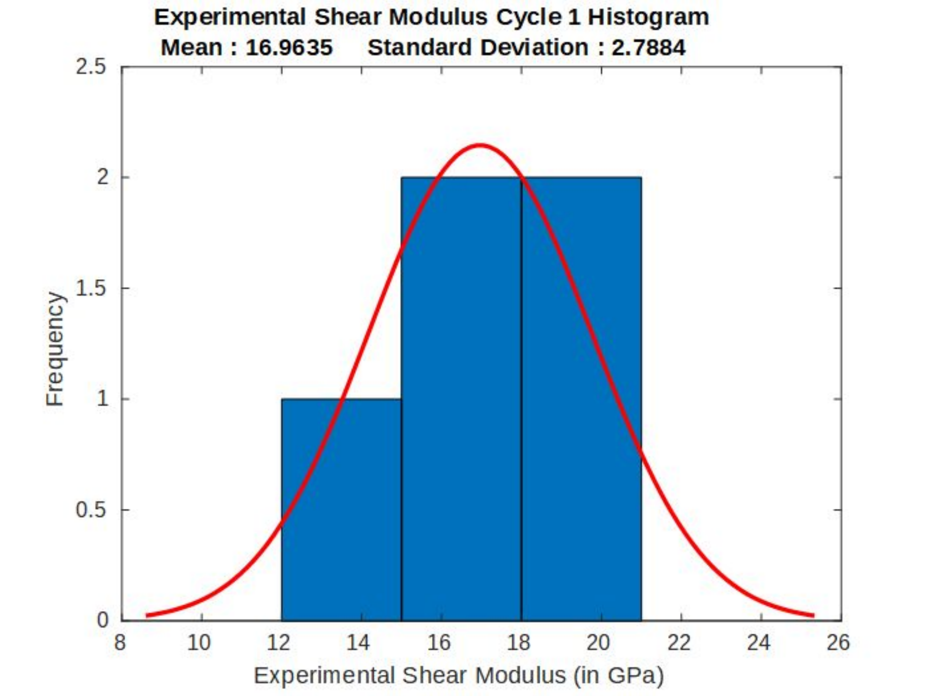

Exp 5 Cycle 1 G:13.6024 GPa
×10^6
Shear Stress (in Pa)
Shear Strain (in radians)
×10^-3

6. Best estimate of G for each cycle for all the 5 experiments

Best Estimate of Shear Modulus G of Aluminium 6061(in GPa)					
Cycle	Group 1	Group 2	Group 3	Group 4	Group 5
1	15.4056	18.9808	20.5471	16.2815	13.6024
2	23.1932	16.2989	20.9568	15.3368	14.5182
3	14.4278	18.0067	19.1010	23.6965	15.3531
4	23.6029	21.3467	28.9270	21.5365	15.4221
5	24.4523	19.9375	19.6609	15.0040	13.9876
6	19.6592	18.7894	16.4957	20.3673	14.1422
7	16.0682	21.0934	17.8017	24.9917	13.5261
8	22.3908	19.9046	24.9266	20.7837	15.7506
9	19.3626	19.9448	20.2522	26.5262	14.9680
10	19.2821	23.1883	15.2823	25.6834	13.8522
11	21.3640	19.7951	21.5843	23.2311	14.1783
12	18.5747	18.8597	21.9365	20.4039	14.0114
13	18.3262	17.0349	23.8582	19.3746	15.5855
14	22.3425	19.4617	26.1568	17.6308	13.5734
15	26.1794	15.5853	18.5172	23.6005	14.4280
16	20.4599	15.9529	17.4638	27.5469	14.5544
17	19.5354	22.2001	27.0359	16.3402	14.6347
18	19.6729	21.2489	21.3897	17.1683	12.7887
19	17.2217	22.7034	17.8797	16.3786	12.6379
20	18.6922	21.4132	17.4849	20.1685	13.6559
μ	20.0107	19.5873	20.8629	20.6025	14.2585
σ	3.0582	2.2000	3.7034	3.9131	0.8667

7. Normalised Histogram of G using values from Question 5 and a Smoothed probability density function

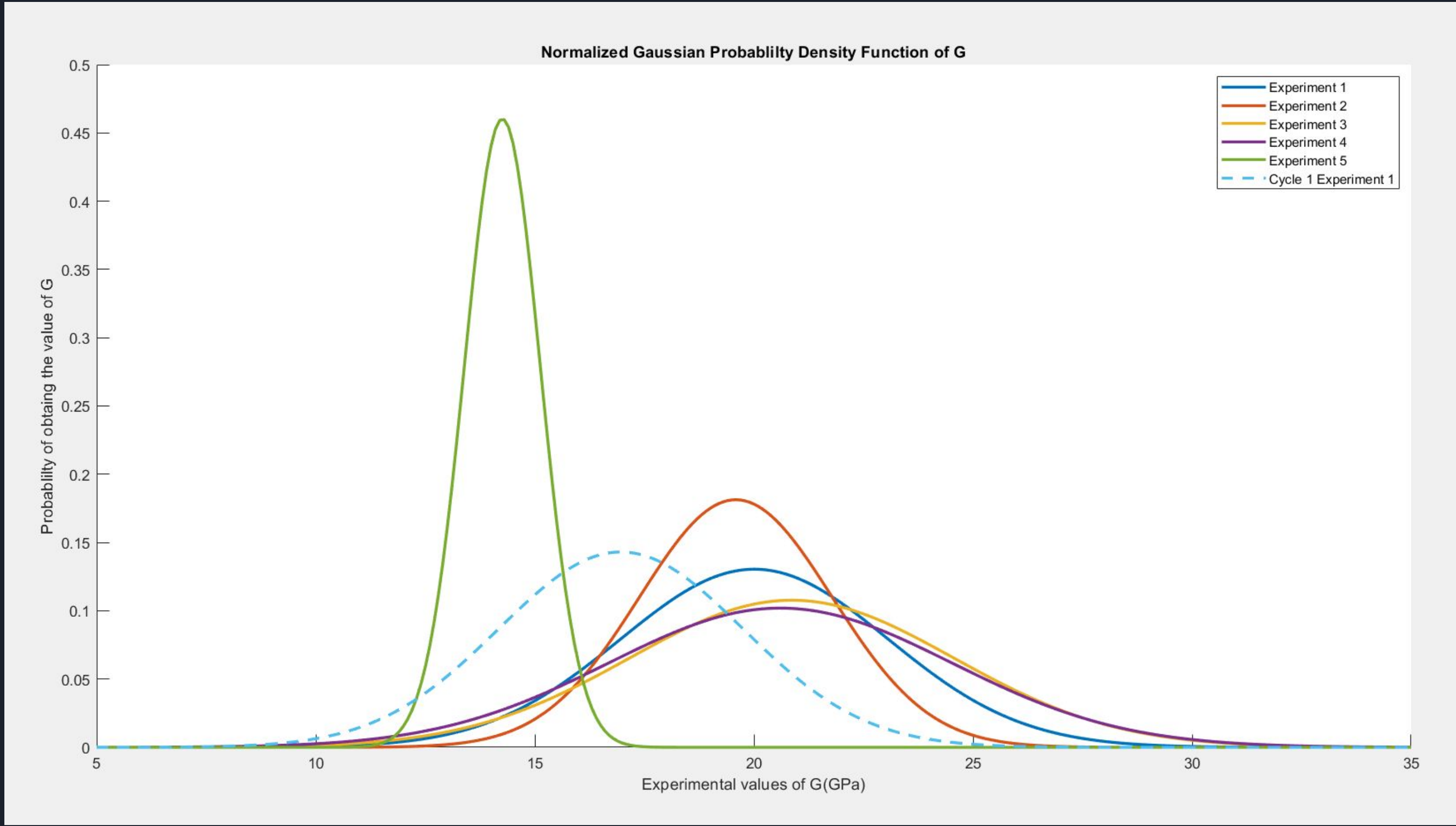




8. Statistical Quantities Comparison

Statistical Quantities		
	μ	σ
Experiment 1	20.0107	3.0582
Experiment 2	19.5873	2.2000
Experiment 3	20.8629	3.7034
Experiment 4	20.6025	3.9131
Experiment 5	14.2585	0.8667
From Question 7	16.9635	2.7884

Figure for Question 8



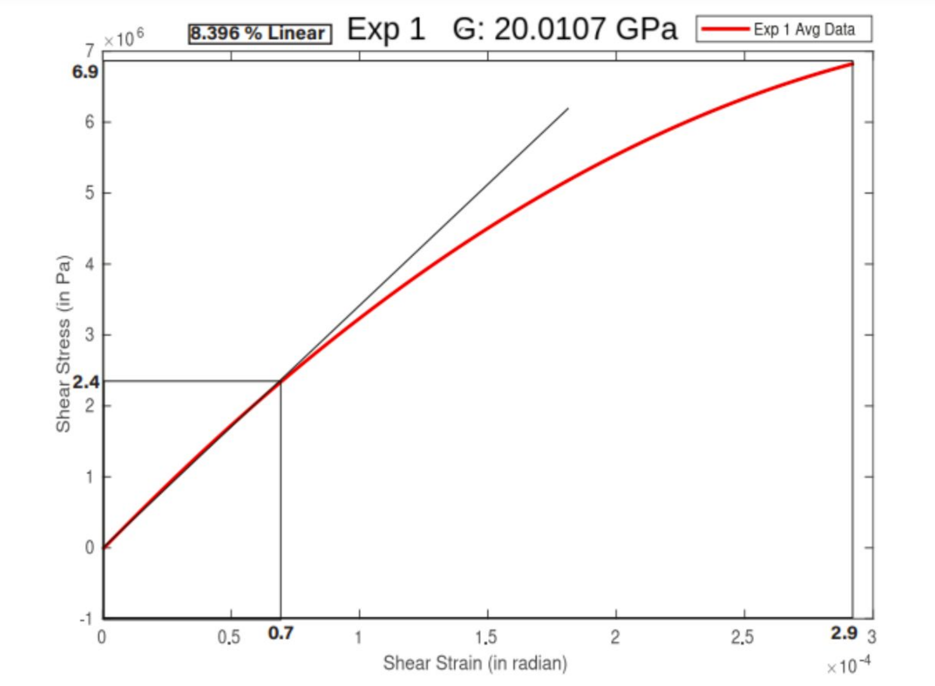
9. Anomalous Experiment

In Experiment 5, the load is too large which implies higher Stress and hence a higher value of strain.

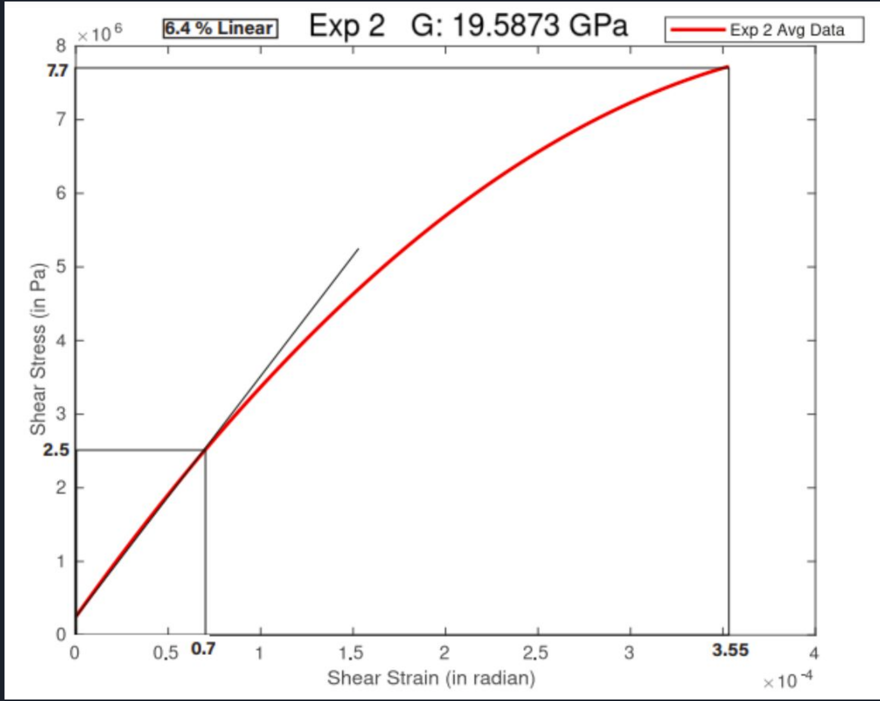
- Range of Maximum Mass for other Experiments : 1.2-1.6kg
- Maximum Mass for Experiment 5 : 3kg

Reason:

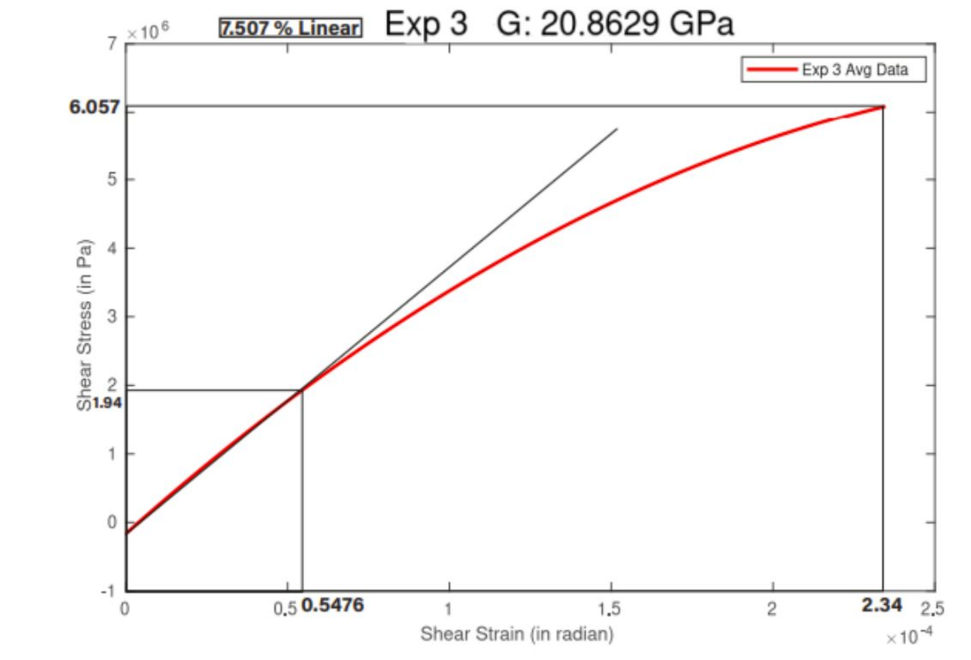
- Our assumption that the stress and strain our small and we are well within the proportionality limit.
- Non-Linearity in our experiments



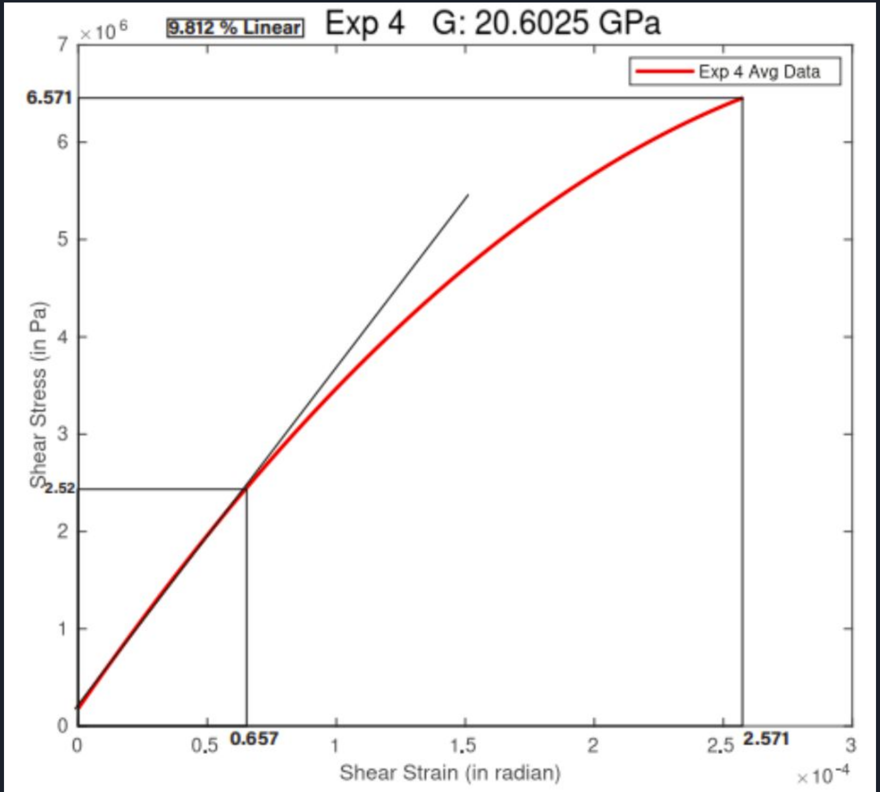
8.396 % Linear
Exp 1 G: 20.0107 GPa
Exp 1 Avg Data
Shear Stress (in Pa)
Shear Strain (in radian)
×10^6
×10^-4
6.9
2.4
0.7
2.9



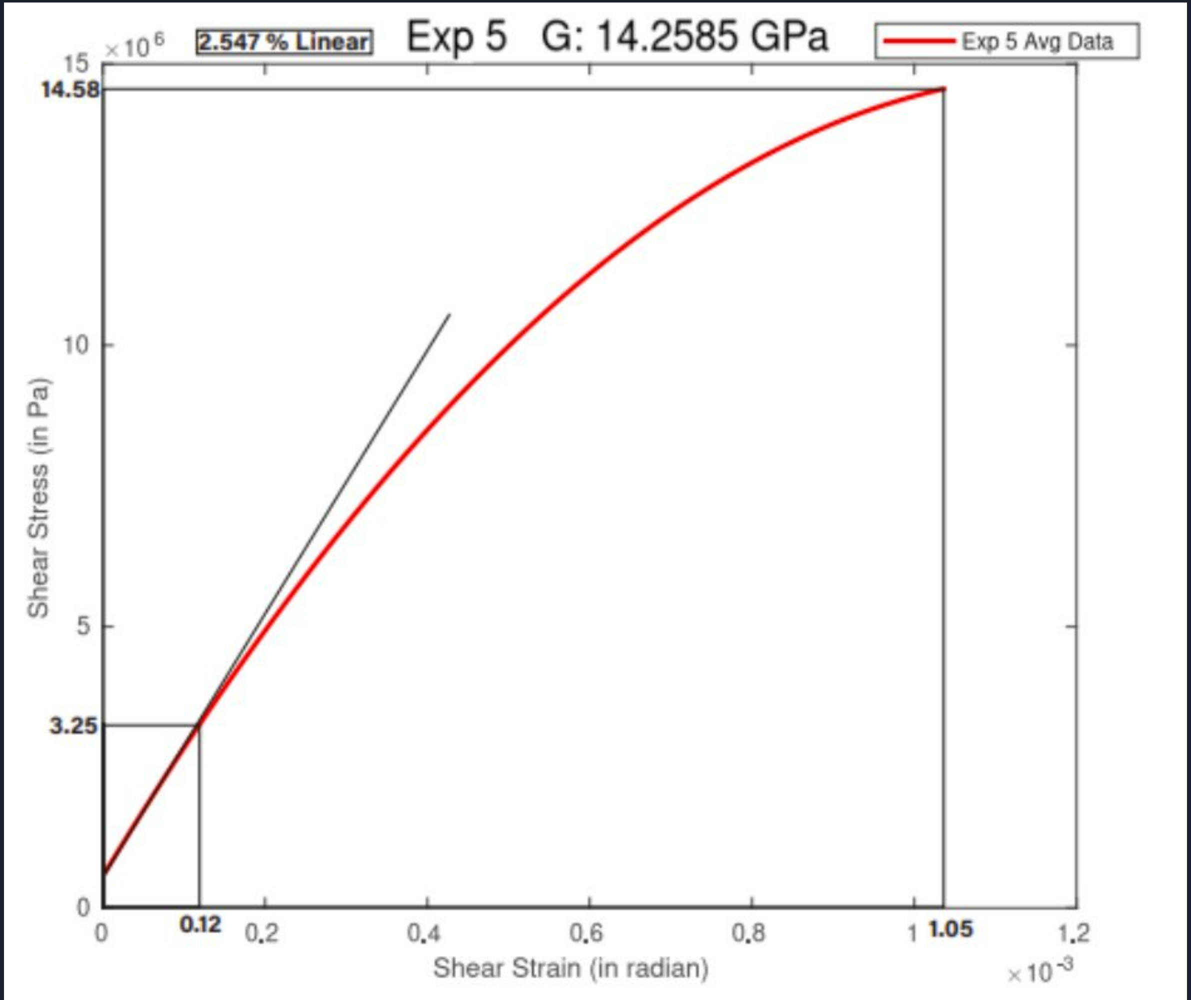
6.4 % Linear
Exp 2 G: 19.5873 GPa
Exp 2 Avg Data
Shear Stress (in Pa)
Shear Strain (in radian)
×10^6
×10^-4
7.7
2.5
0.7
3.55



7.507 % Linear
Exp 3 G: 20.8629 GPa
Exp 3 Avg Data
Shear Stress (in Pa)
$\times 10^6$
6.057
1.94
0.5476
2.34
Shear Strain (in radian)
$\times 10^{-4}$



9.812 % Linear
Exp 4 G: 20.6025 GPa
Exp 4 Avg Data
Shear Stress (in Pa)
$\times 10^6$
6.571
2.52
0.657
2.571
Shear Strain (in radian)
$\times 10^{-4}$



2.547 % Linear
Exp 5 G: 14.2585 GPa
Exp 5 Avg Data
×10^6
15
14.58
10
5
3.25
0
Shear Stress (in Pa)
0
0.12
0.2
0.4
0.6
0.8
1
1.05
1.2
×10^-3
Shear Strain (in radian)

True Value of Shear Modulus of Aluminium 6061: **26 GPa**

Best Estimate of Shear Modulus of Aluminum 6061			
Experiment Number	% Linear	G_Value (GPa)	|Error| (GPa)
Group 1	8.396%	20.0107	5.9893
Group 2	6.4%	19.5873	6.4127
Group 3	7.507%	20.8629	5.1371
Group 4	9.812%	20.6025	5.3975
Group 5	2.547%	14.2585	11.7415

Table 1. Percentage Linearity of Average Experimental Readings and corresponding Magnitude of Error in Shear Modulus .

10. Best Estimate of Shear Modulus from all Non-Anomalous Experiments

For the Best estimate of the Shear Modulus G we find mean(μ) and standard deviation(σ) from experimental values of G obtained from all Experiments (i.e Exp1, Exp2, Exp3, Exp4) except Exp5 since it is Anomalous

Experimental Result :

Mean(μ) = 20.2659 GPa

Standard Deviation(σ) = 3.2629 GPa

Experimental Value of Shear Modulus : 20.2659 ± 3.2629 GPa

Published Value of G = 26 GPa

11. Short Comings and Alternatives

- Presence of negative values of theta in the dataset.
- Error caused due to the use of masses and a manual apparatus
- Bending moment caused due to the use of levers to produce torque

Some improvement to the current setup:

- Regularly changing testing sample
- Using a machine (chuck)
- Being considerate of the proportionality limit

Alternatives experiments:-

- Rail Shear Test:
  - Two Rail Fixture
    - Another modification to this is **V-Notched Rail Shear Test**
  - Three Rail Fixture
- Four-Point Loading or Saddle Test

Results:

Experimental Shear Modulus of Aluminium 6061 :

20.2659 ± 3.2629 GPa

GPa Published Shear Modulus of Aluminium 6061 :

26 GPa

Takeaways for the Experiment

- Disagreement between Experimental values and Published Value of Aluminium 6061.
- The effect on the final result due to the consideration of values measured from beyond the proportionality limit.
- Observed discrepancies in the precision of the data given to us and that which can be obtained using the apparatus.

Conclusion

- We understood that Modulus of rigidity is the coefficient of the elasticity for a shearing force, it measure the stiffness of the particular materials in torsion test.
- Different type of material will have different elastic limit. The higher value of modulus of rigidity, G , the higher the torsion stiffness of material.
- If the material exceeds the elastic limit,permanent deformation will be occurring. On exceeding the proportionality limit the objective of this experiment will be defied.
- We learnt how Torsion Test can be used to calculate the Shear Modulus of a Material.
- Learning about the Errors that might persist during the Torsion Test and how to minimize them.
- Using tools with limited technology and precision,the Torsion Test gives a decent estimate of the Shear Modulus of a material