

# AS5212 Design of Supersonic Aircraft

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## Supersonic Transatlantic Flight

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### Group 6

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# Chapter 1

## Introduction

### 1.1 Introduction

The fundamental advantage which air travel has over other modes is the reduced time of transit. There have been efforts to push the boundaries of this capability to its limits ever since the supersonic barrier was broken in 1974. Significant historical implementation of commercial supersonic travel is limited to two airplanes : The Aérospatiale/British Aircraft Corporation Concorde and The Tupolev Tu-144.

Despite the reduction in transit time that supersonic aircrafts offer, there are multiple challenges in the operation of these aircrafts including :

- **Lack of Comfort** : Thinner wings called for very high jet velocities during take-off. Even during supersonic cruise, SST engines need a fairly high specific thrust (net thrust/airflow) to minimize engine cross-sectional area (and thus drag). The sonic boom is also a concern as it brings negative impact to humans and animal populations below.
- **Lack of Structural Integrity** : Supersonic vehicle speeds necessitate thinner wing and fuselage designs, which are also exposed to higher stresses and temperatures. Due to the resulting aeroelasticity issues, heavier structures are needed to reduce undesired flexing. Additionally, because supersonic aircraft must operate at higher altitudes than subsonic aircraft, they need a significantly stronger (and heavier) structure.
- **Aerodynamics** : Since drag rises rapidly with speed, a key priority of supersonic aircraft design is to minimize this force by lowering the coefficient of drag. To some extent, supersonic aircraft also manage drag by flying at higher altitudes than subsonic aircraft, where the air density is lower.
- **Powerplant** : Jet engine design shifts significantly between supersonic and subsonic aircraft. Since the aircraft needs to operate over a wide range of speeds, the selection and design of powerplant for supersonic airplanes is crucial.
- **Economic Inefficiency** : Supersonic planes are expensive to develop and are also extremely inefficient in air since the high per-passenger takeoff weight owing to Higher fuel costs and lower passenger capacities due to the aerodynamic requirement for a narrow fuselage. This makes it difficult to obtain a good fuel fraction.

Furthermore, at supersonic speeds an aircraft adiabatically compresses the air in front of it. The increased temperature of the air heats the aircraft. Implementing temperature-resistant materials further leads to increase in expense due to manufacturing complexity.

- **Environmental Impact** : The International Council on Clean Transportation (ICCT) estimates a SST would burn 5 to 7 times as much fuel per passenger. Due to the low fuel efficiency of supersonic aircrafts, it is essential to either meet existing environmental standards with advanced technology or bring changes in the policy to green-flag supersonic transit.

These challenges govern the design of a feasible commercial supersonic aircraft. The need to eliminate limitations on transit speed while taking into account these challenges provides the motivation to carry out this design elective. To avoid dealing with the effect of supersonic travel over residential areas, the design objective has been confined to supersonic design for trans-atlantic flights over the ocean.

## 1.2 Transatlantic Flight

The annual world airport traffic forecast predicts that flight passenger traffic growth will continue to rise at a consistent, average rate of 4.5% year-on-year.[7]

Aircraft	Seats	Capacity	Range <sub>max</sub>	V <sub>cruise</sub>	Mach	Ceiling
<b>B787-9</b>	8.2 mil	290 pax	14,140 km	900 km/h	0.846	43,100 ft
<b>A330-300</b>	8.1 mil	300 pax	11,750 km	880 km/h	0.827	-
<b>B777-300ER</b>	7.4 mil	365 pax	13,649 km	935 km/h	0.879	43,100 ft
<b>B777-200ER</b>	6.6 mil	305 pax	13,080 km	935 km/h	0.879	43,100 ft
<b>A350-900</b>	3.8 mil	315 pax	15,372 km	900 km/h	0.846	43,100 ft
<b>B767-300ER</b>	3.5 mil	250 pax	11,070 km	875 km/h	0.822	43,100 ft
<b>A330-200</b>	3.4 mil	246 pax	13,450 km	870 km/h	0.818	-
<b>B787-8</b>	2.9 mil	242 pax	13,620 km	900 km/h	0.846	43,100 ft
<b>B787-10</b>	2.7 mil	330 pax	11,910 km	900 km/h	0.846	41,100 ft

Table 1.1: Major Trans-Atlantic Flight Aircraft in 2021[8]

Most transatlantic flights fly at 36,000 ft, following the North Atlantic Tracks, along the existing jet stream (20,000ft - 50,000ft) for higher fuel efficiency. Too high and the oxygen becomes too sparse to fuel the engines, too low and the air resistance is greater. Climate change is having a greater impact on the jet stream, which has led to increasing turbulence.[6] This, along with other climate changes, largely affects transatlantic commercial aviation.[12]

Since Supersonic planes fly at a higher altitude and enable faster travel, they can be a viable solution to the above stated challenges in the future of aviation. The London-Heathrow (LHR) — New York (JFK) is the busiest route of all. Thus, the Mission Profile is based on the ability to cover the JFK-LHR route (5555 km[1]) using the designed aircraft.



### 1.3 Mission Specifications

The mission objective is to enable travel at a cruise speed of 2.25 Mach. The cost of cruising faster than Mach 2.5 can be large since it complicates the airframe (due to high structural material costs and structural stresses) and systems development (like cooling and air-conditioning) effort due to kinetic heating of the airframe skin.[11] The heating effects can be mitigated to an extent by replacing the material of structure. This is expected to cut the total cruise time from 5.5 hrs to around 2 hours, bringing the total transit time down to 3 hours (20 min climb and descent each)

*Cruise Speed : Mach 2.25*

The weighted mean approximation of Flight Data presented in Table 1.1 tells that the average trans-atlantic flight has capacity of 300 passengers and flies at a cruise speed of 0.85 Mach, with the block-time 7 hours (estimating 5.5 hours of cruise with 30 min of taxi time and 30 min of climb and descent each). Hence, currently, each aircraft transports  $\approx 43$  passengers/hour. The calculation is based on accomodating the increase in the number of passengers 10 years down the line with the same number of aircrafts operating. This assumption has been made since it is impractical to keep increasing the number of aircrafts due to over-crowding in the airspace. Thus the capacity must be increased to  $\approx 65$  passengers/hour which amounts to 455 pax. per aircraft if flying subsonic.

To achieve the transit capacity of 65 passengers/hour flying supersonic, the capacity requirement is 193 passengers. Hence the design capacity can be concluded.

*Capacity : 200 (195 Passengers + 5 Crew Members)*

The cruise altitude has been specified to avoid disturbances due to the turbulent jet streams. It has also not been kept too high due to scarcity of oxygen and thinning of air at higher altitudes. The pressure also reduces substantially as we go higher and hence going too high would translate to the need for a heavier fuselage structure.

*Cruise Altitude : 60,000ft*

The mission is to successfully fly between Heathrow Airport (LHR), London and John F. Kennedy International Airport (JFK), New York City. While both these airports have all-weather landing systems, the flight can be possibly diverted to the nearest international airports ; Gatwick Airport (LGW), London or to Newark Liberty International Airport (EWR), New York City respectively. Considering the runway lengths of the these, the take-off and landing distances have been decided

*Take – off Distance : 3500m*

 ; 

*Landing Distance : 2000m*

A realistic flight planning requires an optimisation of the route (track, vertical profile), Mach number and altitude, with the constraints imposed from the air traffic control and the aviation regulators. The Mission Profile has been designed to enable 50 minutes of loiter, split into 2 intervals of 25 minutes each, and the total range of the mission profile is 6000 km.

# Mission Profile

The aircraft mission profile has the following elements

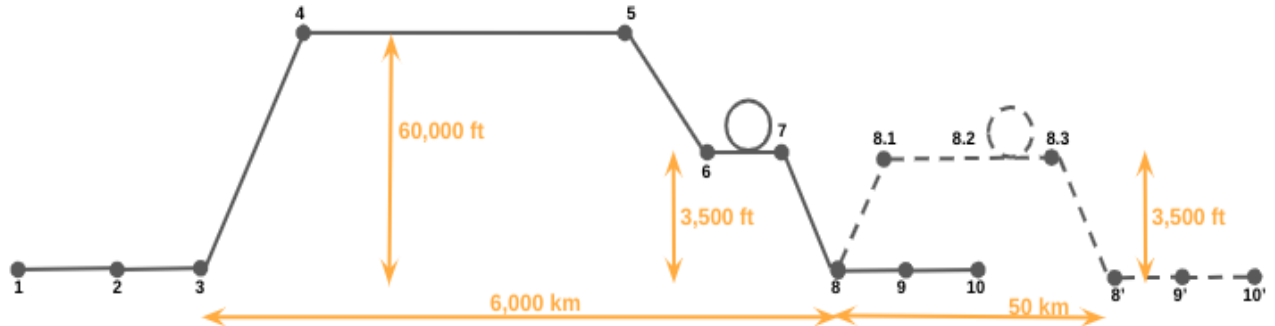


Figure 1.1: Mission Profile

- 1 : Engine Start, Warm-up
- 1 - 2 : Taxi
- 2 - 3 : Take off
- 3 - 4 : Climb
- 4 - 5 : Supersonic Cruise
- 5 - 6 : Descent
- 6 - 7 : Loiter
- 8 - 9 : Land
- 9 - 10 : Taxi, Shutdown
- 8 - 10' : Diversion Flight

## 1.4 Historic Data

	Concorde	Tu-144 D	L-2000 7A	B 2707
<b>Crew</b>	3	3	3	3
<b>Capacity</b>	120 pax.	150 pax.	270 pax.	280 pax.
<b>Length</b>	202 ft	215.58 ft	273.17 ft	306 ft
<b>Wingspan</b>	84 ft	94.6 ft	116 ft	180 ft
<b>Height</b>	40 ft	41.17 ft	46 ft	46.25 ft
<b>Wing Area</b>	3,856 ft <sup>2</sup>	5,450.3 ft <sup>2</sup>	9,424 ft <sup>2</sup>	-
<b>Max Speed</b>	Mach 2.04	Mach 2.15	Mach 3	Mach 2.7
<b>Range</b>	6,580 km	6,500 km	6,440 km	6,400 km
<b>Service Ceiling</b>	60,000 ft	66,000 ft	76,500 ft	72,200 ft
<b>OEW</b>	78,700 kg	84,200 kg	107,955 kg	119,400 kg
• <b>PP</b>	12,700 kg	15,600 kg	20,400 kg	20,400 kg
• <b>STR</b>	66,000 kg	68,600 kg	87,555 kg	99,000 kg
<b>Useful Load</b>	111,130 kg	122,800 kg	159,664 kg	186,847 kg
• <b>TFW</b>	95,680 kg	99,200 kg	115,867 kg	141,847 kg
• <b>DPL</b>	15,450 kg	23,600 kg	43,797 kg	34,000 kg
<b>MTOW</b>	189,830 kg	207,000 kg	267,619 kg	306,247 kg

Table 1.2: Comparative Data of Similar Aircrafts

## 1.5 Similar Aircrafts

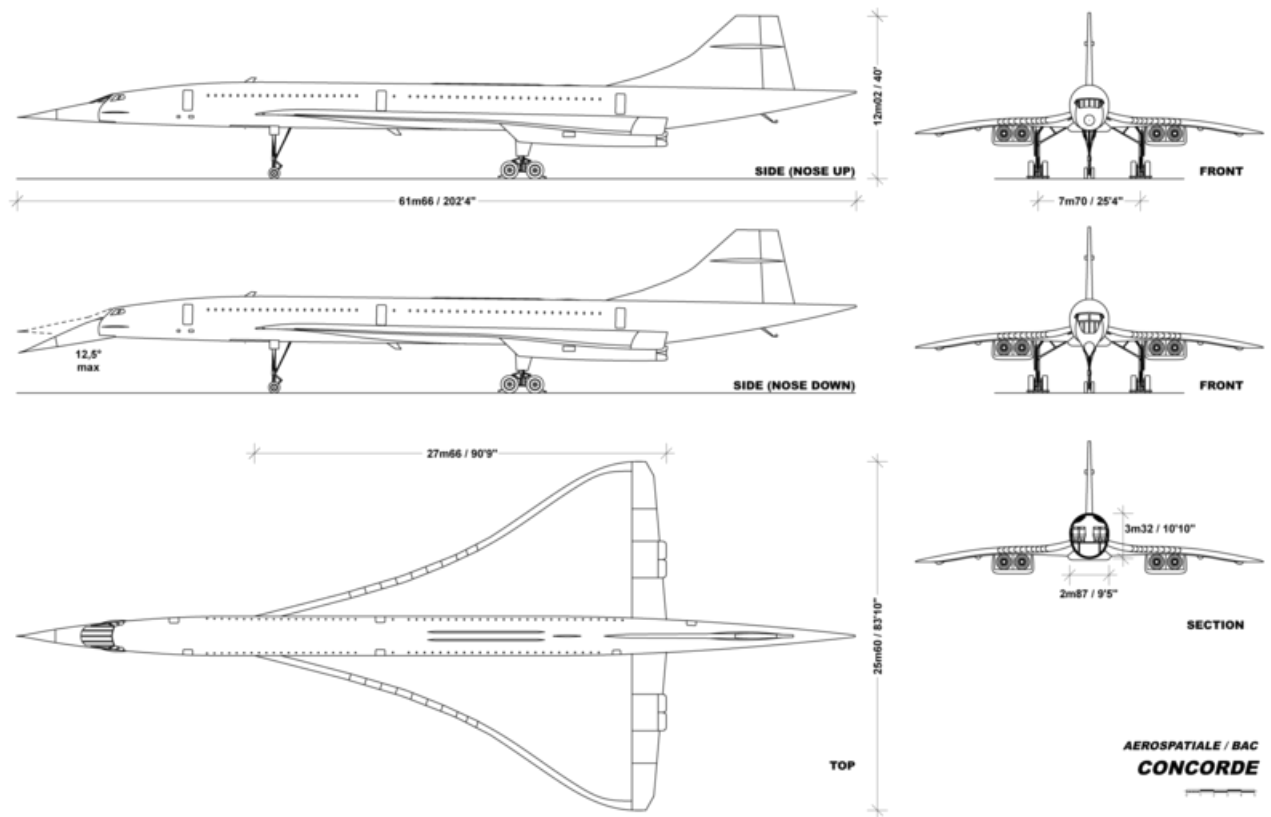


Figure 1.2: Concorde

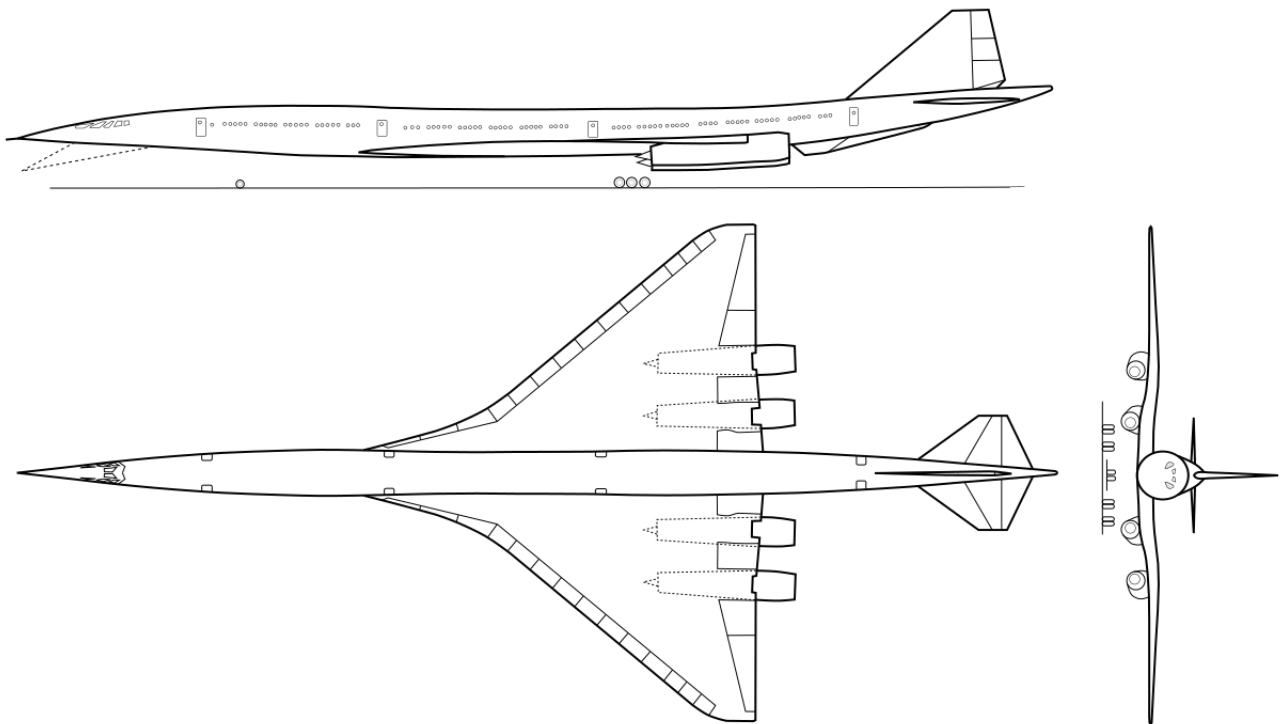


Figure 1.3: Boeing-2707

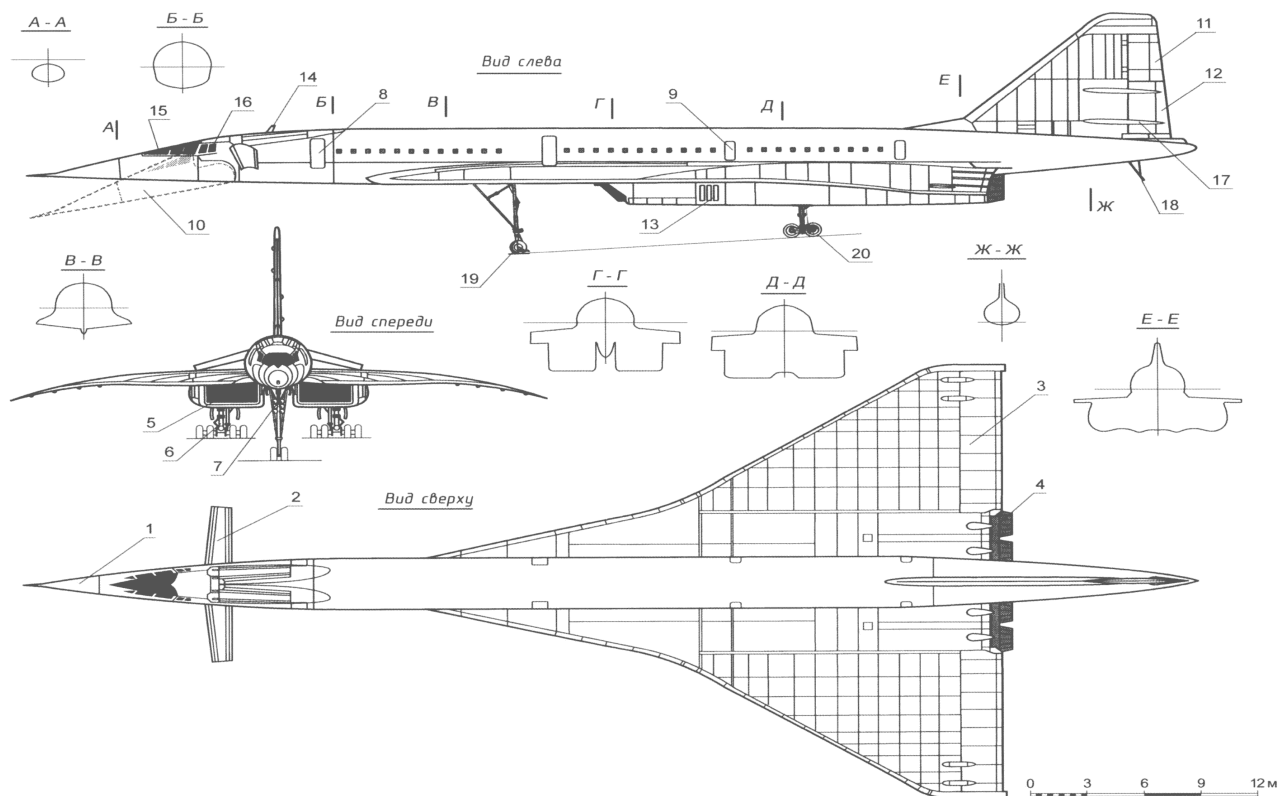


Figure 1.4: Tu-144

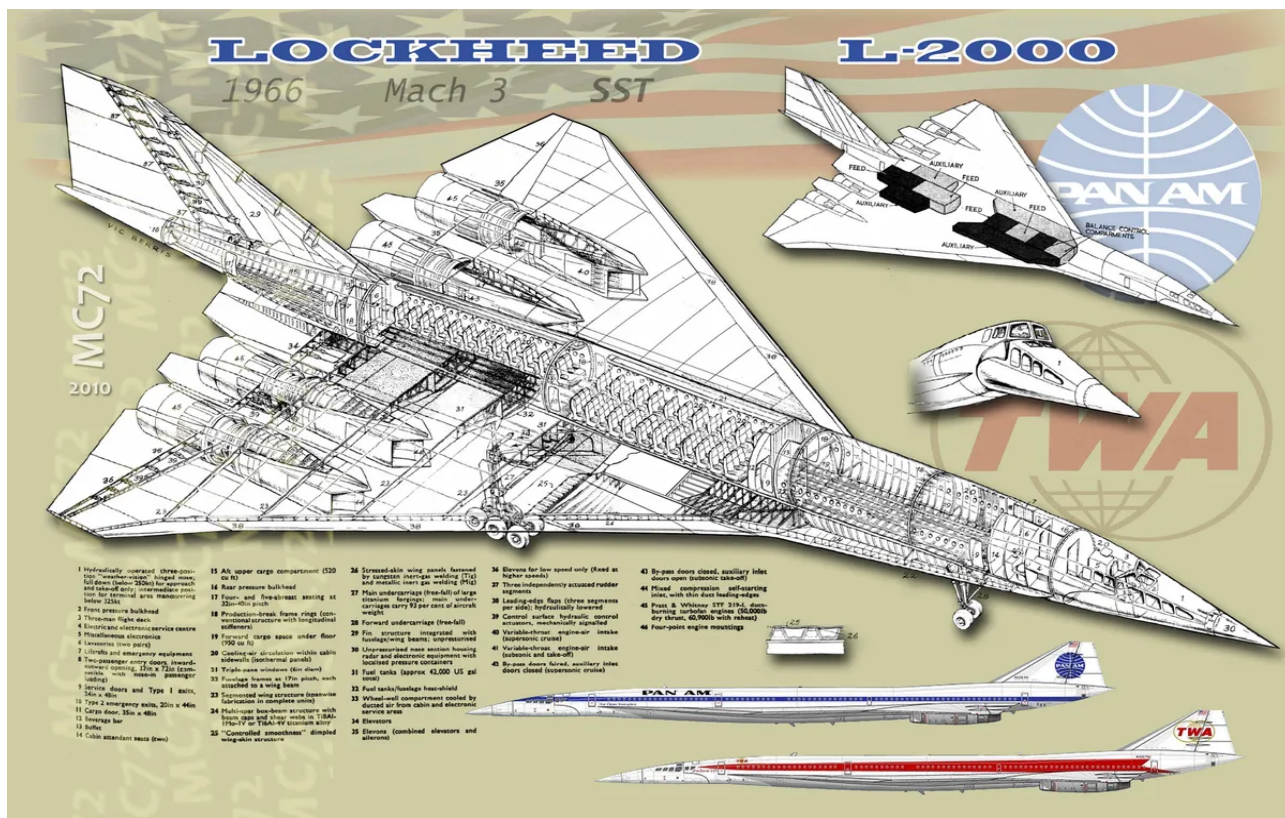


Figure 1.5: Lockheed L-2000

## 1.6 Supersonic Powerplant Data

	<b>Olympus 593</b>	<b>RD-36-51A</b>	<b>GE4-J5M/J5P</b>	<b>J58 (JT11D-20)</b>
<b>Engine Type</b>	Turbojet	Turbojet	Turbojet	Turbojet
<b>Manufacturer</b>	Rolls-Royce	Kolesov	General Electric	Pratt & Whitney
<b>Engine Diameter</b>	1,212 mm	1,486 mm	1,803 mm	1,300 mm
<b>Engine Length</b>	4,039 mm	5,976 mm	8,331 mm	4,600 mm
<b>Engine Weight</b>	3,175 kg	3,900 kg	5,100 kg	2,700 kg
<b>Max. Dry Thrust</b>	139.4 kN	-	220 kN	111.21 kN
<b>Max. Wet Thrust</b>	169.2 kN	196.1 kN	281 kN	151.24 kN
<b>T/W</b>	5.4	5.1	6.02	5.23
<b>Cruise TSFC</b>	33.8 g/kN.s	35 g/kN.s	41.8 g/kN.s	-
<b>SL TSFC</b>	39 g/kN.s	40.4* g/kN.s	48.2* g/kN.s	-
<b>Fuel Type</b>	Jet A1	-	Special JP-6	JP-7, JP-4 or JP-5

Table 1.3: Powerplant Characteristics of Supersonic Transatlantic Aircrafts

\* Estimated on the basis of available values

## 1.7 Subsonic Powerplant Data

	<b>GE90</b>	<b>CF6-80C2</b>	<b>RB-211-524</b>	<b>Trent-882</b>
<b>Engine Type</b>	High BP Turbofan		Three-shaft Turbofan	
<b>Manufacturer</b>	General Electric		Rolls Royce	
<b>Intake/Fan Diameter</b>	3,124 mm	2,362 mm	2,192	2,794 mm
<b>Engine Length</b>	4,775 mm	4,270 mm	3,175	4,369 mm
<b>Engine Weight</b>	7,893 kg	4,144 kg	4,479 kg	5,447 kg
<b>Take-off Thrust</b>	388.8 kN	276 kN	269.4 kN	366.1 kN
<b>Cruise Thrust</b>	70 kN	50.4 kN	52.1 kN	72.2 kN
<b>TSFC (SLS)</b>	8.3 g/kN.s	9.32 g/kN.s	15.95 g/kN.s	15.66 g/kN.s

Table 1.4: Powerplant Characteristics of Subsonic Transatlantic Aircrafts

# Chapter 2

## Sizing and Weight Estimation

### 2.1 Analytical Approach

The components of the weight of an aircraft can be estimated as

$$W_{MTOW} = W_{OEW} + W_{TFW} + W_{DPL} \quad (2.1)$$

where

$W_{MTOW}$  : Maximum Take-off Weight (or correspondingly Maximum Taxi Weight)

$W_{OEW}$  : Operating Empty Weight

$W_{TFW}$  : Total Fuel Weight ( or correspondingly Block Fuel Weight)

$W_{DPL}$  : Design Payload Weight

Tweaking the equation by Torenbeek[11], we get the unity equation as

$$1 = \frac{W_{OEW}}{W_{MTOW}} + \frac{W_{TFW}}{W_{MTOW}} + \frac{W_{PDL}}{W_{MTOW}} \quad (2.2)$$

During the initial phase of design of an aircraft, it is developed for a decided payload. Approximating and calculating the weight fractions of Empty Weight and Fuel Weight enables us to arrive to the first estimation of the aircraft weight.

$$W_{MTOW} = \frac{W_{PDL}}{1 - (W_{OEW}/W_{MTOW}) - (W_{TFW}/W_{MTOW})} \quad (2.3)$$

The Design Payload Weight includes the weight of the payload (passengers, luggage) and the crew.

$$W_{PDL} = W_{Crew} + W_{Payload}$$

The Empty Weight Fraction can be estimated based on historical data/tables and refined sizing data/tables.

The aircrafts fuel supply is available for performing the mission. The other fuel includes reserve fuel, trapped fuel (which is the fuel which cannot be pumped out of the tanks). Fuel fraction is approximately independently of aircraft weight and hence it is estimated based on the mission to be flown.

## 2.2 First Weight Estimation

### 2.2.1 Theoretical Background

Assuming that the fuel tanks are completely empty at the end of the flight.

$$W_{TFW} = W_{MOTW} - W_{end} \quad (2.4)$$

$$\frac{W_{TFW}}{W_{MTOW}} = 1 - \frac{W_{end}}{W_{MTOW}} \quad (2.5)$$

Allowing for 5% reserve and 1% trapped fuel.

$$\frac{W_{TFW}}{W_{MTOW}} = (1 + 6\%)(1 - \frac{W_{end}}{W_{MTOW}}) = 1.06(1 - \frac{W_{end}}{W_{MTOW}}) \quad (2.6)$$

We calculate  $W_{end}/W_{MTOW}$  by calculate each mission segment weight fraction[3]. This method does not include mission segments involving weight drops, aerial refuelling and combat. The weight fraction of different type of mission segments is given as follows :

#### Climb

$$\frac{W_i}{W_{i-1}} = e^{-\frac{\delta h \times TSFC \times g}{Rate_{climb} \times (L/D)_{Climb}}} \quad (2.7)$$

#### Accelerate[Daniel]

$$\frac{W_i}{W_{i-1}} = \frac{(\frac{W_f}{W_i})_{M_f}}{(\frac{W_f}{W_i})_{M_i}} \quad (2.8)$$

where  $M_f$  : Final Mach Number,  $M_i$  : Initial Mach Number and

$$(\frac{W_f}{W_i})_M = 0.991 - 0.07M - 0.01M^2 \quad (2.9)$$

which is positive in the range (-14.052, 7.052)

#### Cruise : using Breguet Range Equation

$$\frac{W_i}{W_{i-1}} = e^{-\frac{(Range) \times TSFC \times g}{(V_{Cruise}) \times (L/D)_{Cruise}}} \quad (2.10)$$

#### Descent and Deceleration : considering Gliding Descent

Considering gliding descent, and ignoring the distance travelled during descent, we can take  $W_i/W_{i-1} = 1$ ,

#### Loiter: using Endurance Equation

$$\frac{W_i}{W_{i-1}} = e^{-\frac{(Endurance) \times TSFC \times g}{(L/D)_{Loiter}}} \quad (2.11)$$

The Concorde had (L/D) at takeoff and landing 4:1, increasing to 12:1 at Mach 0.95 and 7.5:1 at Mach 2. This will be used as reference.  $(L/D)_{cruise} = 0.866 (L/D)_{loiter}$

## 2.2.2 Fuel Weight Fraction Calculation

Phase		Mission Segment	$W_i / W_{i-1}$
P1	1-3	Engine Start, Warm-up Taxi & Take-off	0.970*
P2	3-4	Climb	0.6733
P3	4-5	Supersonic Cruise	0.2712
P4	5-6	Descent	1
P5	6-7	Loiter	0.8473
P6	7-8	Descent	1
P7	8-8.1	Climb	0.9947
P8	8.1-8.2	Subsonic Cruise (not needed for 50km**)	1
P9	8.2-8.3	Loiter	0.8473
P10	8.3-8'	Descent	1
P11	8'-10'/8-10	Landing, Taxi, and Engine Shutdown	0.995*

Table 2.1: First Estimation Weight Ratio for Mission Phases

\* figures taken from reference of historic data

\*\* For a short diversion of 50km, there is no need for subsonic cruise since the distance will be covered during climb and descent.

### Phase P1

We will not be considering the fuel consumption in Phase 1, since the comparative data contains Maximum Take-off Weight and not Maximum Taxi-weight.

### Phase P2

This phase can be split into 2 climbs : subsonic climb (upto 40,000 ft) followed by a supersonic climb (40,000 - 60,000 ft). The aircraft accelerates to supersonic between these climbs.

$$\left(\frac{W_i}{W_{i-1}}\right)_{P2} = \left(\frac{W_i}{W_{i-1}}\right)_{subsonic\ climb} \times \left(\frac{W_i}{W_{i-1}}\right)_{accelerate} \times \left(\frac{W_i}{W_{i-1}}\right)_{supersonic\ climb} \quad (2.12)$$

calculating these components individually,

$$\left(\frac{W_i}{W_{i-1}}\right)_{subsonic\ climb} = e^{-\frac{\delta h \times TSFC \times g}{Rate_{climb} \times (L/D)_{subsonic}}} = 0.9264 \quad (2.13)$$

$$\left(\frac{W_i}{W_{i-1}}\right)_{accelerate} = \frac{\left(\frac{W_f}{W_i}\right)^{2.25}}{\left(\frac{W_f}{W_i}\right)^{0.85}} = 0.847 \quad (2.14)$$

$$\left(\frac{W_i}{W_{i-1}}\right)_{supersonic\ climb} = e^{-\frac{\delta h \times TSFC \times g}{Rate_{climb} \times (L/D)_{supersonic}}} = 0.94068 \quad (2.15)$$

The values taken are justified as commercial planes typically climb 1,000 to 2,000 feet per minute (can go up to 3,000 feet per minute) as recommended by the Aeronautical Information Manual. Taking the subsonic cruise speed to be around 0.85 Mach is also justified. With data of previous supersonic aircrafts, assuming the climb rate of 3000 feet per minute throughout, it covers 500 km during climb.

$$\left(\frac{W_i}{W_{i-1}}\right)_{P2} = 0.738135 \quad (2.16)$$



### Phase P3

Supersonic Cruise at Mach 2.25 is the core part of the journey. The distance to be travelled is 5,000 km. At 60,000 ft, Mach 2.25 is 663.75 m/s. (take g at 60,000 feet?)

$$\left(\frac{W_i}{W_{i-1}}\right)_{P3} = e^{-\frac{(Range) \times TSFC \times g_{60,000ft}}{(V_{Cruise}) \times (L/D)_{supersonic}}} = 0.2648 \quad (2.17)$$

### Phase P4

The weight fraction for gliding descent is 1. According to the Rule of Three (aeronautics), 3 nautical miles (5.6 km) of travel should be allowed for every 1,000 feet (300 m) of descent. The aircraft can be approximated to cover 350km during the descent.

### Phase P5

Enabling subsonic loiter for 25 minutes, as per FAA Guidelines

$$\left(\frac{W_i}{W_{i-1}}\right)_{P5} = e^{-\frac{(Endurance) \times TSFC \times g}{(L/D)}} = 0.8664 \quad (2.18)$$

### Phase P7

The aircraft climbs to subsonic loiter altitude of 1,000m.

$$\left(\frac{W_i}{W_{i-1}}\right)_{P7} = e^{-\frac{\delta h \times TSFC \times g}{Rate_{climb} \times (L/D)}} = 0.9947 \quad (2.19)$$

### Phase P9

Enabling subsonic loiter for 25 minutes, as per FAA Guidelines,

$$\left(\frac{W_i}{W_{i-1}}\right)_{P9} = e^{-\frac{(Endurance) \times TSFC \times g}{(L/D)}} = 0.8664 \quad (2.20)$$

### Weight Fraction Calculation

$$\frac{W_{end}}{W_{MTOW}} = \prod_{k=2}^{11} \left(\frac{W_i}{W_{i-1}}\right)_{Pk} \approx 0.1465 \quad (2.21)$$

$$\frac{W_{FTW}}{W_{MTOW}} = 1.06 \left(1 - \frac{W_{end}}{W_{MTOW}}\right) = 0.9046 \quad (2.22)$$

### 2.2.3 Design Payload Weight

For a capacity of 195 including crew, assuming 90kg of total weight per individual(80 kg avg. weight + 10kg carry on), the Design Payload must be

$$W_{DPL} = 195 \times 90kg = 17,550kg \quad (2.23)$$

### 2.2.4 Empty Weight Calculation

$W_{OEW}$  is known to have an empirical relation in the form

$$W_{OEW} = aW_{MTOW}^c \times K \quad (2.24)$$

where  $a$  and  $c$  depend on the class of aircrafts and was given by the data interpolation of historical aircraft weight parameters. The factor  $K$  is a factor depending on the aircraft sweep.

$$K = 1 \text{ for conventional fixed-wing aircrafts}$$

$$K = 1.04 \text{ for aircrafts with variable sweep}$$

Considering the aircraft to be fixed wing for simpler structural approximations and taking the log, enables us to find the parameters using linear curve fit to the weight data??.

$$\log(W_{OEW}) = \log(a) + c \times \log(W_{MTOW}) \quad (2.25)$$

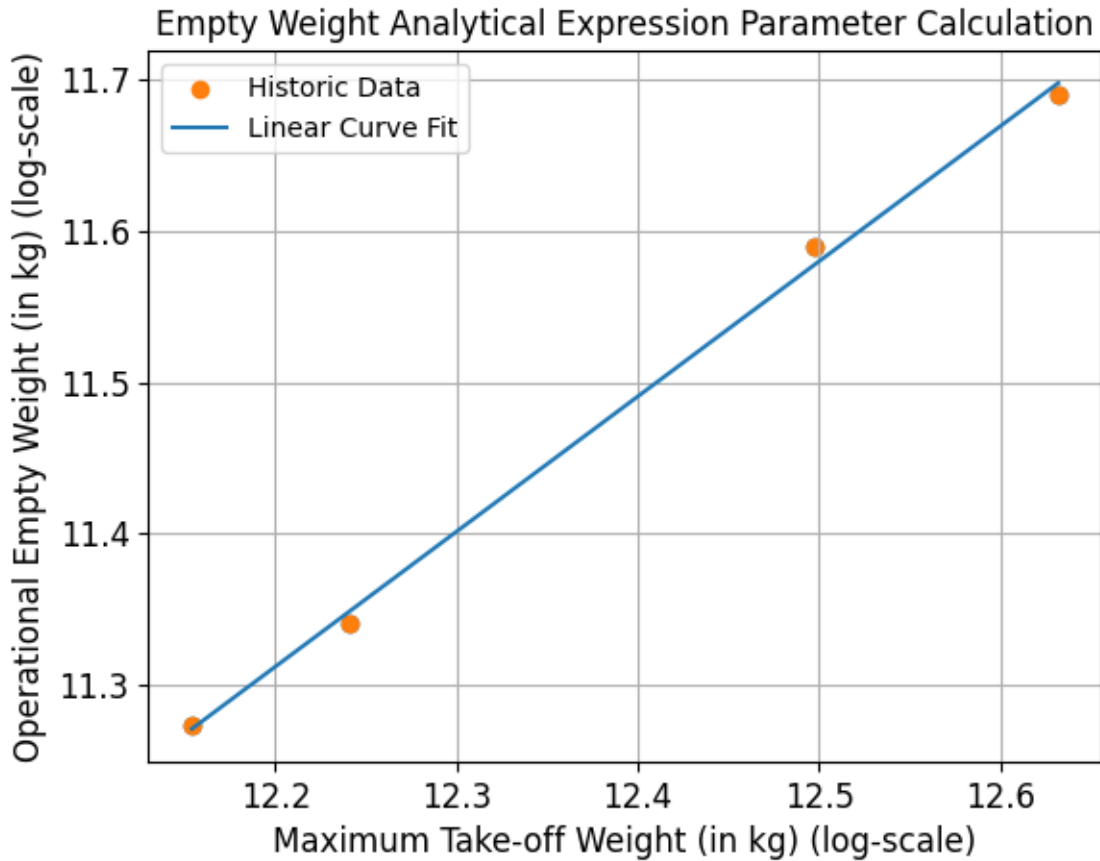


Figure 2.1: OEW Expression Parameter Calculation : Linear Curve Fit

The linear curve fit parameters for the log-scale plot are given as

$$a = 1.5320017025488337$$

$$c = 0.8922524060747525$$

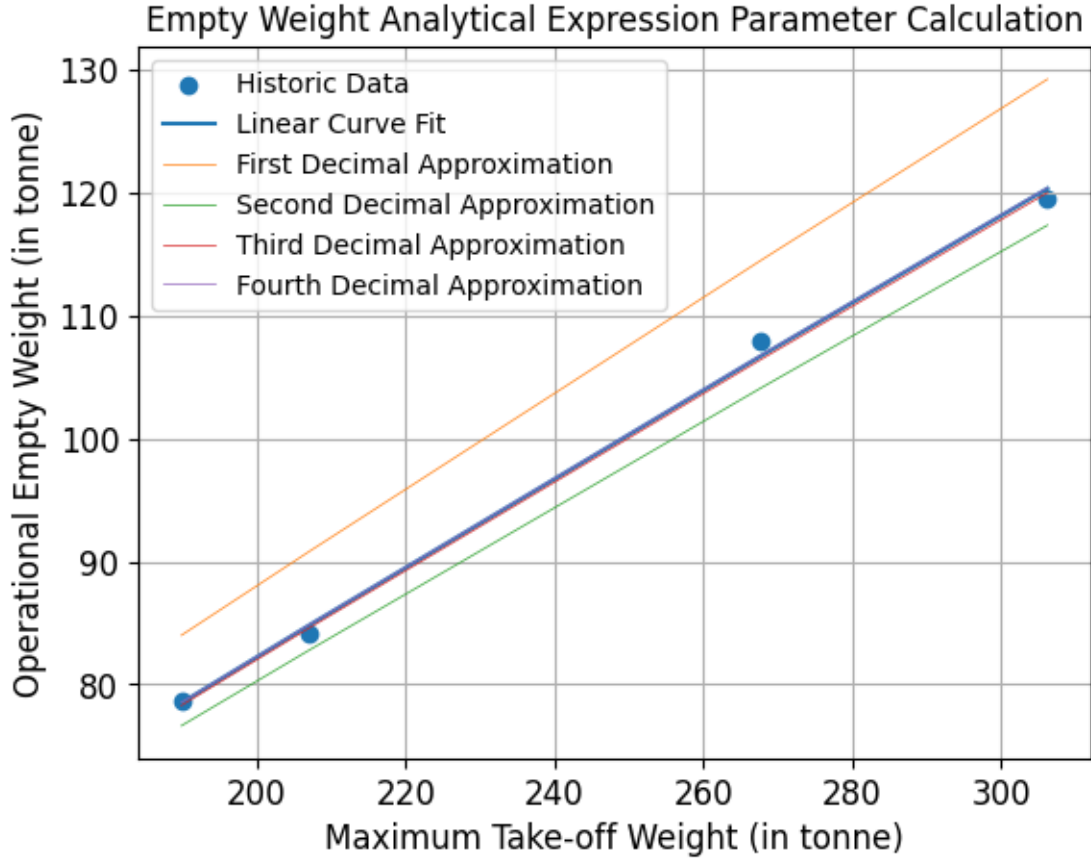


Figure 2.2: OEW Expression Parameter Calculation : Decimal Approximation

We can round the parameters off to the 4th decimal with sufficient accuracy. Thus, for further calculations, we take

$$a = 1.5320$$

$$c = 0.8923$$

Thus, we have

$$\left(\frac{W_{OEW}}{W_{MTOW}}\right) = 0.8923 * W_{MTOW}^{0.5320} \quad (2.26)$$

### 2.2.5 Empty Weight Calculation : Updated Weights

Modern material can reduce the structural weight of the aircrafts. Early planes used metals the aircraft because of their familiarity, cost and ease of construction. With the advent of Aviation Grade Carbon Fiber- Reinforced Plastics, there can be significant reduction in the structural weight of the aircraft. The use of Carbon Composites has been shown to have a 20% - 50%[4] reduction in the structural weight. Taking a low-ball figure of 20%, and re-calculating the Structural weights, we get a better estimation of the Operational Empty Weight of the aircraft.

Keeping the same Maximum Take-off Weight (MTOW), the reduction in the operational empty weight (OEW) in-turn increases the maximum useful payload and reduces the fuel consumption per kilogram of design payload (DPL).

	Concorde	Tu-144 D	L-2000 7A	B 2707
<b>OEW</b>	78,700 kg	84,200 kg	107,955 kg	119,400 kg
• <b>PP</b>	12,700 kg	15,600 kg	20,400 kg	20,400 kg
• <b>STR</b>	66,000 kg	68,600 kg	87,555 kg	99,000 kg
<b>New OEW</b>	65,500 kg	70,480 kg	90,444 kg	99,600 kg
• <b>PP</b>	12,700 kg	15,600 kg	20,400 kg	20,400 kg
• <b>New STR</b>	52,800 kg	54,880 kg	70,044 kg	79,200 kg
<b>MTOW</b>	189,830 kg	207,000 kg	267,619 kg	306,247 kg

Table 2.2: Updated Weight Distribution of Supersonic Transatlantic Aircrafts

With the updated data, owing to the reduction in the structural weight by the use of carbon-composites, we can re-do the parameter calculation.

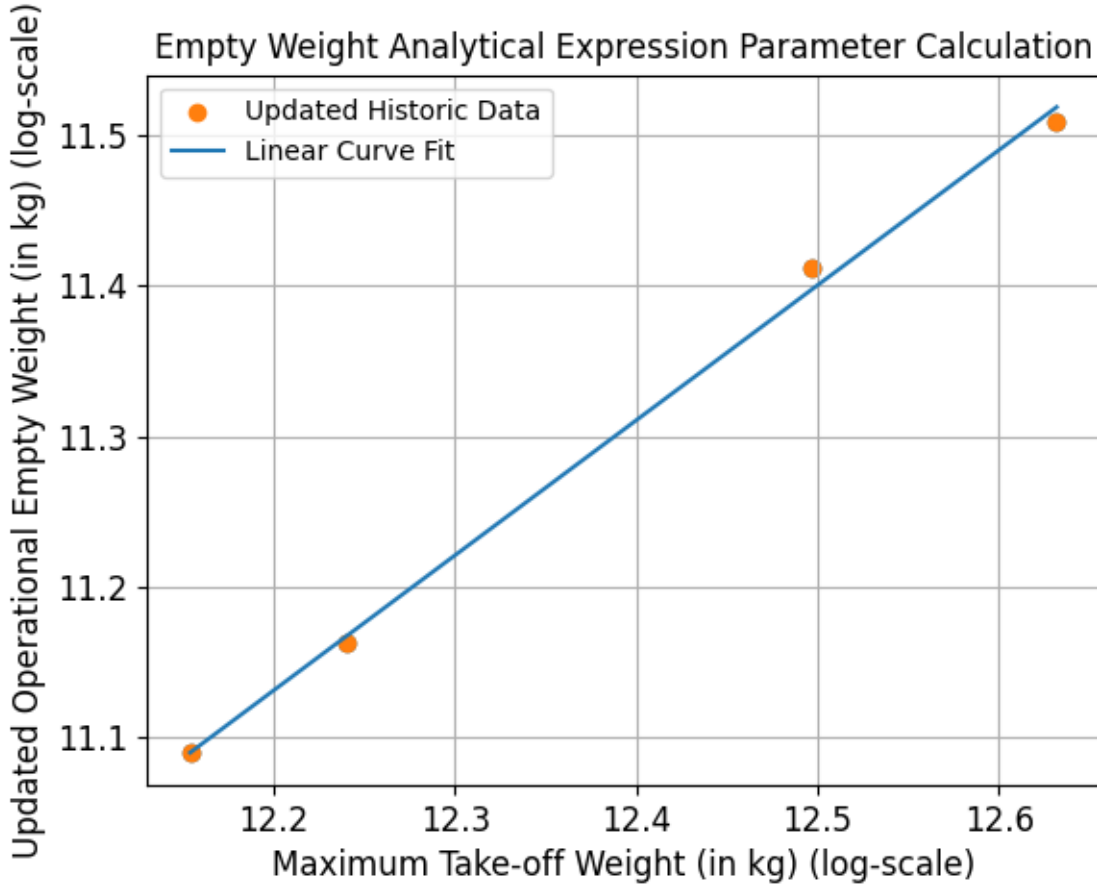


Figure 2.3: OEW Expression New Parameter Calculation : Linear Curve Fit

An analysis similar to the above subsection gives the parameters :

$$a = 1.2236$$

$$c = 0.8959$$

Hence the new ratio of OEW to MTOW is given as :

$$\left(\frac{W_{OEW}}{W_{MTOW}}\right) = 0.8959 * W_{MTOW}^{0.2236} \quad (2.27)$$

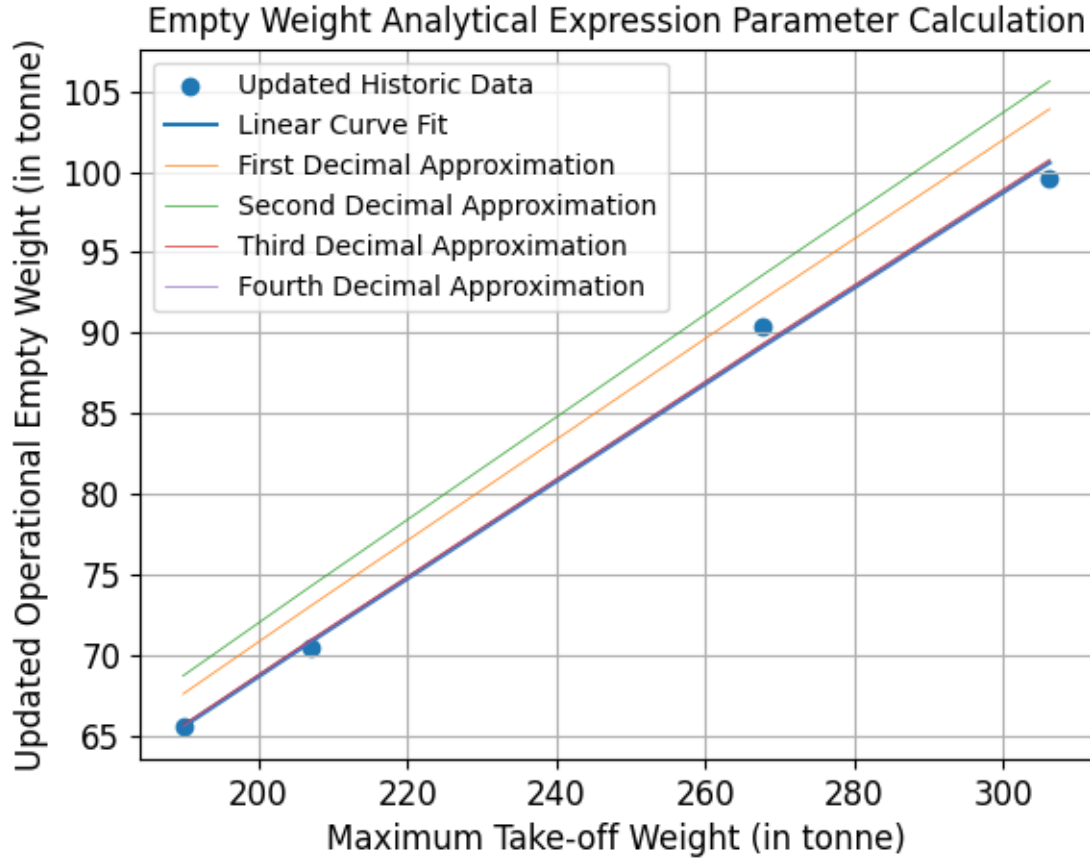


Figure 2.4: OEW Expression New Parameter Calculation : Decimal Approximation

### 2.2.6 MTOW Calculation

Running the above obtained value of Fuel Weight Fraction and Operational Empty Weight, for the stated Design Payload, a value of  $W_{MTOW}$  is not found.

The calculated OEW parameters and the design payload only converge to a reasonably appropriate value MTOW for Fuel Ratio ( $W_{end}/W_{MTOW}$ ) in the range of 0.425 - 0.55. The fuel weight ratio of Phase P3 seems relatively low for the parameters used. A possible work-around would be to select powerplants such that the fuel weight ratio is around 50% of the MTOW.

The current calculation is based on using 4 X RR Olympus 593 Engines throughout the mission. The maximum wet thrust that each engine can provide is 169.2 kN at a Cruise TSFC of 33.8 g/(kN.s). Using 4 such engines amounts to a maximum wet thrust of 676.8 kN at a cruise TSFC of 135.2 g/(kN.s).

With the replacement of metals in the aircraft structure with carbon composites, the thrust requirement at supersonic travel can be employed to achieve using 2 X GE4 Engines, which provide 562 kN of thrust at a cruise TSFC of 83.6 g/(kN.s). The subsonic part of the mission will be carried out using engines which provide the necessary subsonic thrust at a lesser TSFC than the Olympus 593. This configuration could provide a work-around to the issue faced during the first weight estimation.

## 2.3 Powerplant Selection

Considering using 2X GE4 for Supersonic flight and 2X GE CF6-80C2 for Subsonic flight.

## 2.4 Second Weight Estimation

### 2.4.1 Fuel Weight Fraction Calculation

Phase		Mission Segment	$W_i / W_{i-1}$
P1	1-3	Engine Start, Warm-up Taxi & Take-off	0.970*
P2	3-4	Climb	0.9256
P3	4-5	Supersonic Cruise	0.4776
P4	5-6	Descent	1
P5	6-7	Loiter	0.9999
P6	7-8	Descent	1
P7	8-8.1	Climb	0.9978
P8	8.1-8.2	Subsonic Cruise (not needed for 50km**)	1
P9	8.2-8.3	Loiter	0.9999
P10	8.3-8'	Descent	1
P11	8'-10'/8-10	Landing, Taxi, and Engine Shutdown	0.995*

Table 2.3: Second Estimation Weight Ratio for Mission Phases

\* figures taken from reference of historic data

\*\* For a short diversion of 50km, there is no need for subsonic cruise since the distance will be covered during climb and descent.

#### Phase P1

We will not be considering the fuel consumption in Phase 1, since the comparative data contains Maximum Take-off Weight and not Maximum Taxi-weight.

#### Phase P2

Similar to the first weight estimation

$$\left(\frac{W_i}{W_{i-1}}\right)_{P2} = \left(\frac{W_i}{W_{i-1}}\right)_{subsonic\ climb} \times \left(\frac{W_i}{W_{i-1}}\right)_{accelerate} \times \left(\frac{W_i}{W_{i-1}}\right)_{supersonic\ climb} \quad (2.28)$$

calculating these components individually, rounded off to 4th decimal

$$\left(\frac{W_i}{W_{i-1}}\right)_{subsonic\ climb} = e^{-\frac{\delta h \times TSFC \times g}{Rate_{climb} \times (L/D)_{subsonic}}} = 0.9757 \quad (2.29)$$

$$\left(\frac{W_i}{W_{i-1}}\right)_{accelerate} = \frac{\left(\frac{W_f}{W_i}\right)^{2.25}}{\left(\frac{W_f}{W_i}\right)^{0.85}} = 0.8552 \quad (2.30)$$

$$\left(\frac{W_i}{W_{i-1}}\right)_{supersonic\ climb} = e^{-\frac{\delta h \times TSFC \times g}{Rate_{climb} \times (L/D)_{supersonic}}} = 0.9486 \quad (2.31)$$

The values taken are justified as commercial planes typically climb 1,000 to 2,000 feet per minute (can go up to 3,000 feet per minute) as recommended by the Aeronautical Information Manual. Taking the subsonic cruise speed to be around 0.85 Mach is also justified. With data of previous supersonic aircrafts, assuming the climb rate of 3000 feet per minute throughout, it covers 500 km during climb.

$$\left(\frac{W_i}{W_{i-1}}\right)_{P2} = 0.9256 \quad (2.32)$$

### Phase P3

Supersonic Cruise at Mach 2.25 is the core part of the journey. The distance to be travelled is 5,000 km. At 60,000 ft, Mach 2.25 is 663.75 m/s.

$$\left(\frac{W_i}{W_{i-1}}\right)_{P3} = e^{-\frac{(Range) \times TSFC \times g_{60,000ft}}{(V_{Cruise}) \times (L/D)_{supersonic}}} = 0.4776 \quad (2.33)$$

### Phase P4

The weight fraction for gliding descent is 1. According to the Rule of Three (aeronautics), 3 nautical miles (5.6 km) of travel should be allowed for every 1,000 feet (300 m) of descent. The aircraft can be approximated to cover 350km during the descent.

### Phase P5

Enabling subsonic loiter for 25 minutes, as per FAA Guidelines, the aircraft loiter covers atleast 5km

$$\left(\frac{W_i}{W_{i-1}}\right)_{P5} = e^{-\frac{(Endurance) \times TSFC \times g}{(L/D)}} = 0.9999 \quad (2.34)$$

### Phase P7

The aircraft climbs to subsonic loiter altitude of 1,000m.

$$\left(\frac{W_i}{W_{i-1}}\right)_{P7} = e^{-\frac{\delta h \times TSFC \times g}{Rate_{climb} \times (L/D)}} = 0.9978 \quad (2.35)$$

### Phase P9

Enabling subsonic loiter for 25 minutes, as per FAA Guidelines

$$\left(\frac{W_i}{W_{i-1}}\right)_{P9} = e^{-\frac{(Endurance) \times TSFC \times g}{(L/D)}} = 0.9999 \quad (2.36)$$

### Weight Fraction Calculation

$$\frac{W_{end}}{W_{MTOW}} = \prod_{k=2}^{11} \left(\frac{W_i}{W_{i-1}}\right)_{Pk} \approx 0.4389 \quad (2.37)$$

$$\frac{W_{FTW}}{W_{MTOW}} = 1.06 \left(1 - \frac{W_{end}}{W_{MTOW}}\right) \approx 0.5947 \quad (2.38)$$

### 2.4.2 MTOW Calculation

Substituting the ratio  $W_{end}/W_{MTOW}$  obtained using the code in Appendix A.2 in the calculation above :

$$\frac{W_{end}}{W_{MTOW}} = 0.4389 \quad (2.39)$$

in the code in Appendix A.4 , we iteratively get the value of  $W_{MTOW}$

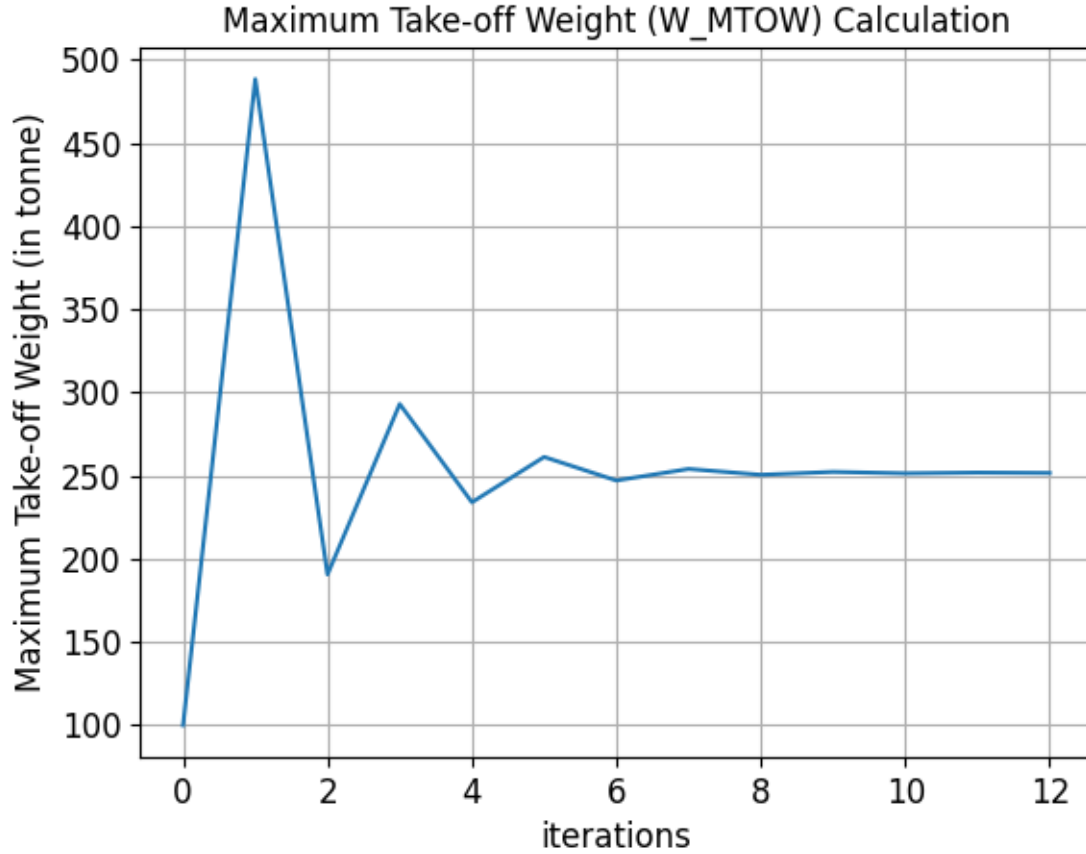


Figure 2.5: MTOW Calculation Convergence Plot

$$W_{MTOW} = 250.998 \text{ tonnes} \quad (2.40)$$

Thus, the final weight and its distributions can be taken to be as follows :

	Estimated Weight
<b>OEW</b>	84,175 kg
• <b>PP</b>	18,488 kg
• <b>STR</b>	65,687 kg
<b>Useful Load</b>	166,825 kg
• <b>TFW</b>	149,275 kg
• <b>DPL</b>	17,550 kg
<b>MTOW</b>	251,000 kg

Table 2.4: Estimated MTOW Weight Distribution



# Chapter 3

## Constraint Analysis : Thrust-Loading and Wing Loading

### 3.1 Analytical Approach

It is important to note that T/W keeps changing during the mission since W keeps changing during the mission, and hence we decide a norm which we use during design. : Design T/W ratio. In the design T/W we assume International Standard Atmosphere, Thrust at Sea-Level Static condition,  $W = W$  gross, and maximum throttle setting. Thus the T/W required for different mission segments has to be adjusted to the design T/W ratio.

### 3.2 Correction Factors

Since all the factors need to be compared we need to apply a few correction factors to set all weights and thrusts at sea level take-off standard.

#### 3.2.1 Weight Correction

To correct for the difference in weight across different mission segments such as landing, approach, climbs and cruise, a correction factor was applied. This was obtained by calculating the maximum weight at the beginning of the concerned segment, and dividing it by the maximum take-off weight(MTOW).

#### 3.2.2 Thrust Correction

##### Density Correction

The density is known to decrease with altitude and so does the thrust, this effect also depends on the bypass ratio of the engine. This effect can be captured by defining the density correction factor , given as :

$$\tau_p = \left( \frac{\rho_{11km}}{\rho_0} \right)^{0.7} \times \frac{\rho_h}{\rho_{11km}} \quad (3.1)$$

The above equation is applicable for altitudes above 11km and the factor 0.7 is used to take into account the by-pass effect.

## Pressure Correction

To consider effects due to flight speed, a pressure effect is also considered. This describes the ram compression at the engine inlet based on the Mach number. This effect can be captured by defining the pressure correction factor, given as :

$$\tau_P = (1 + \eta_{inlet} \times \frac{\gamma_{air} - 1}{2} \times M^2)^{\frac{\gamma_{air}}{\gamma_{air} - 1}} \quad (3.2)$$

## 3.3 Constraint Calculation

### 3.3.1 Climb Gradient Considerations

Since climbs are meant to be done by the subsonic engines, this thrust loading (AEO) is primarily for the subsonic engines. There are two major Climb Gradient Rate (CGR) considerations that need to be taken into account as per the regulatory bodies.

1. Second Stage Climb Gradient

Sufficient thrust must be installed in the aircraft so that in the event of an engine failure the minimum gradient (2.4% for 2 engines) may be sustained, with flaps in the take-off position, but the landing gear retracted

2. Missed Approach Gradient

Sufficient thrust must be installed in the aircraft so that in the event of an engine failure the minimum gradient (2.1% for 2 engines) may be sustained, with flaps in the approach position, but the landing gear down

For the above mentioned gradients, the thrust loading for All Engines Operational (AEO) is given by

$$\frac{T}{W} = \frac{1}{L/D} + CGR \quad (3.3)$$

and the thrust loading for One-Engine Inoperative (OEI) is given by

$$\frac{T}{W} = \frac{n_{eng}}{n_{eng} - 1} \times (\frac{1}{L/D} + CGR) \quad (3.4)$$

For the AEO case,

$$\frac{T}{W} = (\frac{1}{5} + 2.1 \times 10^{-2}) \times \frac{W_{MTOW}}{W_{MTW}} \approx 0.21437 \quad (3.5)$$

$$\frac{T}{W} = (\frac{1}{5} + 2.4 \times 10^{-2}) \times \frac{W_{approach}}{W_{MTW}} \approx 0.096026 \quad (3.6)$$

Considering the OEI case since these are more stringent limit on the thrust loading. It must also be noted, that although there are two engines each preferred for subsonic and supersonic use respectively, in case of failure of one subsonic engine, one of the supersonic engines can be used despite having a higher TSFC, thus,  $n_{eng}$  is taken to be 4. The higher TSFC will reduce W faster than before and once the T/W is sufficiently low for one engine to satisfy the requirement, the other engine will be turned off. Calculating the thrust loading adjusted to the design T/W ratio by multiplying the weight ratio.

$$\frac{T}{W} = \frac{4}{4-1} \times \left(\frac{1}{5} + 2.1 \times 10^{-2}\right) \times \frac{W_{MTOW}}{W_{MTW}} \approx 0.2858 \quad (3.7)$$

$$\frac{T}{W} = \frac{4}{4-1} \times \left(\frac{1}{5} + 2.4 \times 10^{-2}\right) \times \frac{W_{approach}}{W_{MTW}} \approx 0.1281 \quad (3.8)$$

where,  $W_{MTOW}$  : Maximum Take-off Weight;  $W_{MTW}$  : Maximum Taxi Weight

### 3.3.2 Cruise Considerations

In level un-accelerating flight, the thrust must equal the drag. Likewise the weight must equal the lift (assuming the thrust is aligned with the flight path). Cruise is done at supersonic speed of 2.25 Mach, and hence the thrust loading obtained with this consideration is primarily for the Supersonic engine pair. Thus,  $T/W$  for the supersonic engine pair must equal the inverse of  $L/D$  can be given by :

$$\left(\frac{T}{W}\right)_{cruise} = \frac{1}{(L/D)_{cruise}} \quad (3.9)$$

furthermore, writing

$$\left(\frac{L}{D}\right)_{cruise} = \frac{(W/S)_{cruise}}{(D/S)_{cruise}} = \frac{(W/S)_{take-off}(W_{cruise}/W_{MTW})}{(C_{D,0} + kC_L^2) \times q_4} \quad (3.10)$$

The thrust loading adjusted to take-off conditions gives us another requirement which needs to be fulfilled, which is given by :

$$\frac{T}{W} = \left(\frac{C_{D,0} \times q}{(W/S)} + \frac{K}{q} \times (W/S)\right) \times \frac{W_{cruise}/W_{MTW}}{\tau_\rho \times \tau_P} \quad (3.11)$$

$$\frac{T}{W} = \left(\frac{0.0185 \times (0.5 \times 0.1164 \times 665^2)}{(W/S) \times (W_{cruise}/W_{MTW})} + \frac{0.052 \times (W_{cruise}/W_{MTW})}{(0.5 \times 0.1164 \times 665^2)} \times (W/S)\right) \times \frac{W_{cruise}/W_{MTW}}{\tau_\rho \times \tau_P} \quad (3.12)$$

$$\frac{T}{W} = \frac{494.54}{(W/S)} + 1.691e-6 \times (W/S) \quad (3.13)$$

### 3.3.3 Landing Distance

Landing distance is the horizontal distance the aircraft covers from the screen height till it comes to stop. According to FAR-25 :

$$x_{land} = 0.3455 \times V_{approach}^2 \quad (3.14)$$

which we use to calculate the wing loading, adjusted to the take-off wing loading as :

$$\frac{W}{S} = 0.8563 \rho_0 C_{L_{max}} x_{land} \times \frac{W_{MTW}}{W_{approach}} \quad (3.15)$$

$$\frac{W}{S} = 0.8563 \times 1.225 \times 1.2 \times 2,000 \times \frac{W_{MTW}}{W_{approach}} \approx 5871 N/m^2 \quad (3.16)$$

### 3.3.4 Take-off Distance

The take-off distance is defined as the distance between the start of the take-off path and the point where the aircraft reaches 35 ft above the runway surface (according to European Aviation Safety Agency (EASA)). The relation between the thrust loading and the wing loading given a take-off distance is given in Torenbeek [11] as:

$$\frac{T}{W} = 1.1 \sqrt{\frac{n_{eng}}{n_{eng} - 1} \times \frac{1}{AR \times \rho \times g \times x_{to}}} \times \frac{W}{S} \quad (3.17)$$

$$\frac{T}{W} = 1.1 \sqrt{\frac{4}{4 - 1} \times \frac{1}{1.69 \times 1.225 \times 9.8067 \times 3,000}} \times \frac{W}{S} \quad (3.18)$$

$$\frac{T}{W} = (5.1467e - 3) \sqrt{\frac{W}{S}} \quad (3.19)$$

### 3.3.5 Stall Speed

The maximum wing loading required that can be met with a certain stall speed is given as :

$$\frac{W}{S} = \frac{1}{2} \rho V_s^2 C_{L_{max}} \quad (3.20)$$

$$\frac{W}{S} = \frac{1}{2} \times 1.225 \times 76.25^2 \times 1.2 \approx 4275 N/m^2 \quad (3.21)$$

### 3.3.6 Maximum Speed

The thrust loading ( $T/W$ ) is a non-linear function of wing loading ( $W/S$ ) in terms of maximum speed, and may be simplified as[10]:

$$\left(\frac{T}{S}\right)_{V_{max}} = \rho_0 V_{max}^2 C_{D_0} \frac{1}{2(W/S)} + \frac{2K\rho_0}{\rho^2 V_{max}^2} \left(\frac{W}{S}\right) \quad (3.22)$$

where  $\rho$  is density of air at cruise altitude, and  $\rho_0$  is density of air at sea level, and  $\sigma = \rho/\rho_0$

### 3.3.7 Ceiling Height

The relation between the Thrust-Loading and Wing-Loading is given in terms of the Rate-of-Climb at that ceiling as :

$$\left(\frac{T}{W}\right)_{h_c} = \frac{ROC_c}{\sigma_c \sqrt{\frac{2}{\rho_c \sqrt{C_{D_0}/K} \left(\frac{W}{S}\right)}}} + \frac{1}{\sigma_c (L/D)_{max}} \quad (3.23)$$

For different ceilings, the ROC requirement is different. For absolute ceiling  $ROC_{ac} = 0$ , therefore

$$\left(\frac{T}{W}\right)_{h_{ac}} = \frac{1}{\sigma_{ac} (L/D)_{max}} \quad (3.24)$$

where  $\sigma_{ac} = \rho_{ac}/\rho_0$

Design for other Ceiling heights can be found by replacing the required ROC in the above equation

1. Absolute ceiling ( $h_{ac}$ ) :  $ROC_{ac} = 0$  ft/min
2. Service ceiling ( $h_{sc}$ ) :  $ROC_{ac} = 100$  ft/min
3. Cruise ceiling ( $h_{cc}$ ) :  $ROC_{ac} = 300$  ft/min
4. Combat ceiling ( $h_{cc}$ ) :  $ROC_{ac} = 500$  ft/min

### 3.4 Combined Constraint Diagram

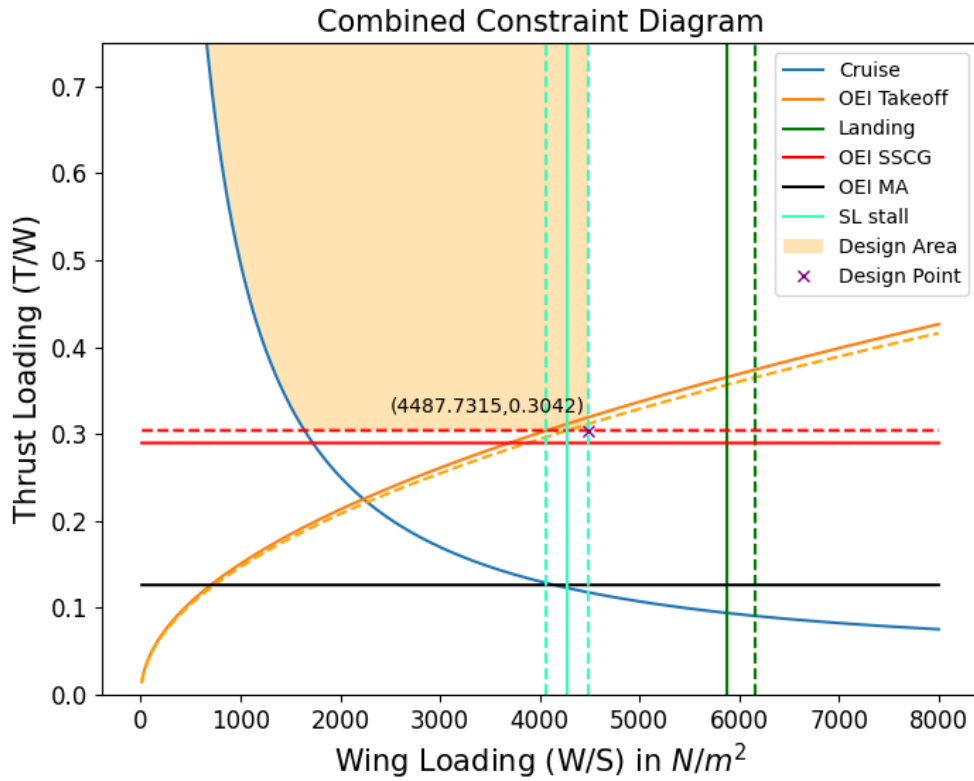


Figure 3.1: Combined Constraint Diagram

### 3.5 Design Point

In the combined constrain diagram, the cruise plot gives the required thrust loading for a given wing loading. If the thrust loading is more than the required value then the throttle can be adjusted such a way as to get required thrust loading and at the same time account for any additional thrust loading requirement. Considering the Design Point, a wing loading of  $W/S = 4487.7315$   $N/m^2$  and a thrust loading of  $T/W = 0.3042$  was obtained. From our initial MTOW estimate, this would imply that the aircraft would need a wing area of  $S = 548.49$   $m^2 \approx 548.5$   $m^2$ .

### 3.6 Historic Data

	Concorde	Tu-144 D	L-2000 7A	B 2707
<b>S</b>	358 m <sup>2</sup>	506.97 m <sup>2</sup>		
<b>Span</b>	25.6 m	28 m		
<b>t/c</b>	0.035	-		
<b>S<sub>vt</sub></b>	44.32 m <sup>2</sup>	66.68 m <sup>2</sup>		
<b>S<sub>wet</sub></b>	1,390.90 m <sup>2</sup>	2,074.91 m <sup>2</sup>		
<b>c<sub>root</sub></b>	27.66 m	33.55 m		
<b>AR</b>	1.83	1.55		
<b>W/S</b>	5,066.21 N/m <sup>2</sup>	3,481.85 N/m <sup>2</sup>		
<b>x<sub>to</sub></b>	3,400 m	3,000 m		
<b>x<sub>land</sub></b>	2,200 m	2,600 m		
<b>C<sub>D,0</sub></b>	0.017	0.02		
<b>K</b>	0.042	0.062		

Table 3.1: Comparative Data of Similar Aircrafts

# Chapter 4

## Wing Design and Airfoil Selection

### 4.1 Introduction

The design of a supersonic wing is a complex process which includes multiple factors that need to be taken into consideration. The optimal design depends on a variety of factors, including the specific mission requirements, operating conditions, and desired performance characteristics. The wing planform type chosen for a supersonic transport aircraft will depend on a range of factors, and trade-offs will need to be made between competing design considerations to arrive at an optimal design.

Some general considerations for supersonic transport aircraft wing design include:

1. Sweep angle: A high sweep angle can help reduce drag and improve aerodynamic efficiency at supersonic speeds.
2. Aspect ratio: A higher aspect ratio can improve lift-to-drag ratio and reduce induced drag.
3. Thickness ratio: A lower thickness ratio can reduce wave drag at supersonic speeds.
4. Leading edge shape: A sharp leading edge can help reduce shock wave formation and wave drag, while a rounded leading edge can improve low-speed performance.
5. Wing loading: A lower wing loading can improve takeoff and landing performance, but may require a larger wing area.

The wing design trade-offs will primarily focus on the major phases of the mission, namely : Supersonic Cruise and Subsonic Loiter ; and secondarily : Take-off, Climb and Descent, Land.

### 4.2 Wing Planform Shape

The selection of a wing planform for a supersonic commercial plane will depend on a variety of factors, such as aerodynamic performance, structural requirements, stability and control characteristics, operational requirements, and cost. To get the optimal design, a careful trade-off analysis is required.

For supersonic wing designs, a variety of forms have already been employed. These forms' advantages and disadvantages are as follows:

### 4.2.1 Variable Sweep Wing

Variable sweep wings have hinges that can swivel, changing the sweep angle. The wing form can be tailored for the actual flight speed since the sweep angle can be modified. The ability of the aircraft to fly many subsonic and supersonic phases for an extended period of time is the main advantage of this type of wing. The machinery required to alter the wing sweep makes the aircraft heavier. Variable sweep is not required because our aircraft will primarily be travelling at a constant supersonic speed.

### 4.2.2 Delta Wing

The delta wing is a sharp triangle. Its high sweep angle reduces the volume wave drag, which is prominent in supersonic aircrafts due to the presence of shock waves. The aircraft is able to fly at high subsonic, trans-sonic, or low supersonic speeds while maintaining the subsonic lifting properties of the airflow over the wing because to the rearward sweep angle's reduction in the airspeed normal to the leading edge of the wing. Because of its long root chord, the wing is more rigid and has lighter wings. Moreover, a bigger wing area with less wing loading is produced by this.

A delta wing aircraft must fly at a higher angle of attack due to the low aspect ratio and high sweep angle. Flaps are also ineffective on a delta wing without a tailplane or canard. This is due to the fact that it cannot counter the nose down pitching moment caused by the flap deflection. A highly swept delta wing planform would be a suitable choice for a supersonic commercial plane as seen by historic attempts. This is because

1. **Aerodynamic Performance:** A highly swept delta wing planform is known for its excellent aerodynamic performance, particularly at supersonic speeds. It can efficiently handle the shocks and waves generated at high speeds, reduce drag to provide high lift-to-drag ratio and also provide good maneuverability.
2. **Structural Requirements:** While a highly swept delta wing planform may require a more complex and heavier structure to handle the loads at high speeds, advances in materials and manufacturing techniques can help mitigate this issue. Additionally, the delta wing planform has been used successfully in supersonic aircraft such as the Concorde, demonstrating its viability.
3. **Stability and Control:** A delta wing planform provides good stability and control at high speeds, which is critical for safe and efficient operation of a supersonic commercial plane. The highly swept design of the delta wing also helps to reduce the effects of supersonic shock waves on stability and control.
4. **Operational Requirements:** A highly swept delta wing planform can meet the operational requirements of a supersonic commercial plane, such as range, payload capacity, and takeoff and landing performance. While a delta wing planform may require longer runways for takeoff and landing, this can be managed through careful airport planning and design.
5. **Cost:** The cost of developing and manufacturing a supersonic commercial plane is an important consideration, and a highly swept delta wing planform can help to minimize costs by providing good aerodynamic performance and stability and control characteristics without the need for a more complex design. It can provide a



cost-effective design as it is simple and easy to manufacture, reducing development and manufacturing costs.

Overall, a highly swept delta wing planform would be a suitable choice for a supersonic commercial plane, balancing the various factors that must be considered in selecting a wing planform. However, it's important to note that other wing planform types, such as the blended wing-body, may also offer advantages and should be considered as part of the overall design trade-off analysis.

### 4.2.3 Delta Wing Design Variations

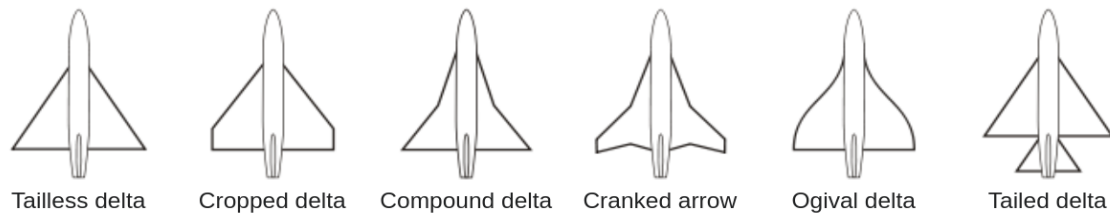


Figure 4.1: Delta Wing Design Variation

Variants of the delta wing plan aim to offer improvements to the basic configuration. Optimal selection of planform requires multi-parameter analysis of the delta wing variations. The current analysis considers only 2 constraints : Wing Area Requirement and Mach Cone Rule. Further analysis on the Aspect Ratio, Thickness ratio and leading edge shape enables design of wing planform with more than 2 parameters. Since we are working with only 2 constraints, the analysis is restricted to delta wing and its variations which can be defined in no more than 2 variables.

### 4.2.4 Planform Selection

Delta wing and ogive wing are both types of wing planforms that have been used in supersonic aircraft, but they have some differences in their characteristics and applications. These can be defined completely using 2 parameters and hence our analysis will be limited to comparing between these planforms and choosing the best among them.

#### Delta Wing [13]

A delta wing is a highly swept wing with a triangular shape that is designed to minimize drag and maximize lift at supersonic speeds. The Concorde, a supersonic transport aircraft, used a delta wing planform. The advantages of the delta wing include:

1. Excellent supersonic performance: Delta wings are designed to minimize drag and maximize lift at high speeds, making them ideal for supersonic aircraft.
2. High lift-to-drag ratio: Delta wings have a high lift-to-drag ratio, which allows the aircraft to fly at high speeds with minimal drag.
3. Good structural efficiency: Delta wings have a low aspect ratio and a straight leading edge, which makes them structurally efficient.

## Ogive Wing

An ogive wing, also known as a double-delta wing, has a delta-shaped leading edge followed by a straight or slightly curved trailing edge. The Tu-144, a supersonic transport aircraft developed by the Soviet Union, used an ogive wing planform. The advantages of the ogive wing include:

1. Good lift and control at high speeds: Ogive wings provide good lift and control at high speeds, making them ideal for supersonic aircraft.
2. Good handling characteristics: Ogive wings provide good handling characteristics and directional stability, which make them suitable for military aircraft.
3. Lower drag at low speeds: The ogive wing provides lower drag at low speeds than the delta wing, making it more suitable for aircraft that operate at both subsonic and supersonic speeds.

In summary, the delta wing is ideal for supersonic transport aircraft that require excellent supersonic performance and high lift-to-drag ratio, while the ogive wing is more suitable for aircrafts that require good handling characteristics and control at high speeds as well as lower drag at low speeds. A careful analysis is needed to decide between the two or come up with a combination of both for improved performance

## 4.3 Wing Planform Geometry

### 4.3.1 Design Constraints

The design of the wing in this project is governed by 2 constraints. The Total wing area requirement, and the Mach Cone Rule. The design is defined using 2 parameters  $a$  and  $b$ . Changing the parameters, we can get the desired wing planform geometry.

Ideal wing area required is  $548.5 \text{ m}^2$  (i.e.  $273.25 \text{ m}^2$  per wing). The design wing area must be greater than the ideal wing area, as close to it as possible. Thus, iterating over the parameters with a defined upper bound on wing area, we get a set of wing planforms which can be used. Let the bound on wing area be given as

$$Total \text{ Area} \in [548.5 \text{ m}^2, 548.5 \text{ m}^2 + e] \quad (4.1)$$

As the aircraft moves through the air at a supersonic speed, it generates shock waves, which are created by the compression of the air in front of the aircraft. These shock waves can have significant effects on the performance and stability of the aircraft.

One of the key considerations in supersonic flight is the position of the wing relative to the shock waves. In general, the wing should be located within the Mach cone, which is the cone-shaped region formed by the shock waves emanating from the nose of the aircraft. This is known as the "Mach cone rule."

If the wing is located outside of the Mach cone, it can be subject to significant and unpredictable changes in lift and drag as it moves through the shock waves. This can make the aircraft difficult to control, and can lead to loss of stability and control.

Furthermore, by positioning the wing within the Mach cone, the aircraft can take advantage of the effects of the shock waves to enhance lift and reduce drag. This can improve the aircraft's performance and efficiency, and can also help to reduce the amount of fuel needed to fly at supersonic speeds.

Overall, the Mach cone rule is an important consideration in the design and operation of supersonic aircraft, and helps to ensure that they are safe, stable, and efficient. Hence, the design selection can be narrowed down by implementing the Mach Cone Rule.

The aircraft is designed to cruise at Mach 2.25. Along with a 5% safety factor, the Mach Cone Rule must be implemented for the cone rule design mach number of Mach  $2.25 \times 1.05$ . The mach cone angle can then be calculated as

$$\mu = \sin^{-1}\left(\frac{1}{M}\right) \quad (4.2)$$

### 4.3.2 Effect of Aspect Ratio

Aspect ratio is a key parameter in the design of aircraft wings. It is defined as the ratio of the wingspan to the mean chord (the average distance from the leading edge to the trailing edge of the wing). Aspect ratio affects many aspects of wing design, including lift, drag, and stall characteristics.

1. Lift: A higher aspect ratio wing generally produces more lift than a lower aspect ratio wing, all other factors being equal. This is because a higher aspect ratio wing has a longer wingspan, which creates a larger lifting surface area. As a result, high aspect ratio wings are often used on gliders and other aircraft that need to generate a lot of lift without a lot of engine power.
2. Drag: A high aspect ratio wing generally produces less drag than a low aspect ratio wing, all other factors being equal. This is because a high aspect ratio wing has a longer, thinner wing shape that creates less drag-inducing turbulence. This is why high aspect ratio wings are often used on aircraft that need to fly at high speeds, such as commercial airliners.
3. Stall characteristics: The stall speed of a wing (the speed at which the wing loses lift and the aircraft begins to descend) is affected by its aspect ratio. A low aspect ratio wing stalls at a higher speed than a high aspect ratio wing. This is because a low aspect ratio wing has a wider chord and creates more turbulence when it stalls, while a high aspect ratio wing stalls more gradually.
4. Weight: High aspect ratio wings are generally lighter than low aspect ratio wings, all other factors being equal. This is because a high aspect ratio wing has less structural material (such as spars and ribs) per unit of wing area.

The advantages and disadvantages of high and low aspect ratio wings must be optimized to create an optimal design for a given aircraft. A lower aspect ratio wing is more suitable for high-speed flight due to its reduced drag and better maneuverability, while a higher aspect ratio wing has better lift and stall characteristics.

Overall, the aspect ratio of a wing is an important consideration in aircraft design, as it affects many key performance characteristics. A finer analysis is needed to determine the optimal value of parameters which meet the required lift, drag and stall characteristics. The trade-off between the effect and sensitivity of parameters can be implemented to find a more optimal solution of the wing planform.

### 4.3.3 Delta Wing

A delta wing is defined by a wing with a fixed slope throughout. Using the two-point form of a line to define the wing,

$$(x_1, y_1) = (0, a); \quad (x_2, y_2) = (b, 0) \quad (4.3)$$

The Mach Cone Rule can be directly implemented here by ensuring that the slope of the wing is bounded in magnitude by the design mach cone angle.

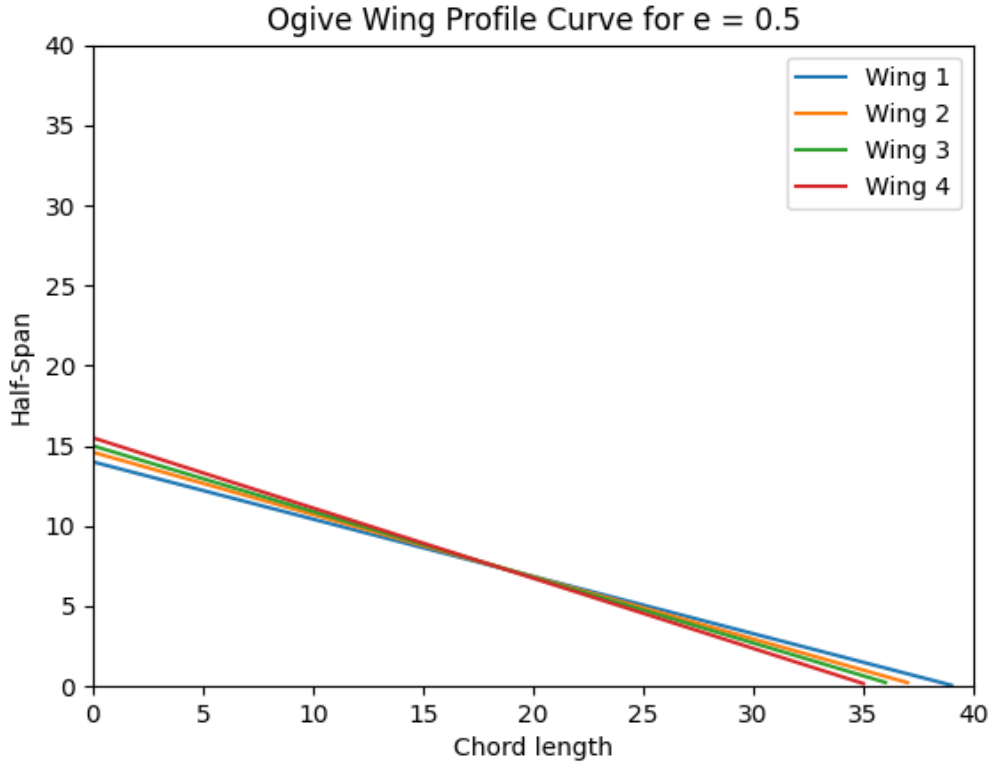


Figure 4.2: Delta Planform with desired range of Wing Area with Mach Cone Rule

	Wing Area	Aspect Ratio	a	b
<b>Wing 1</b>	274.3999 m <sup>2</sup>	0.7143	14.0	39.2
<b>Wing 2</b>	274.4799 m <sup>2</sup>	0.7766	14.6	37.6
<b>Wing 3</b>	274.4999 m <sup>2</sup>	0.8197	15.0	36.6
<b>Wing 4</b>	274.3499 m <sup>2</sup>	0.8757	15.5	35.4

Table 4.1: Delta Planform Characteristics

### 4.3.4 Ogive Configuration

A commonly used profile curve for ogive wings is the Gaussian function, given by:

$$f(x) = a e^{-(x/b)^2} \quad (4.4)$$

Implementing the Mach Cone rule along with the area requirement, narrows down the possible planforms to select from.

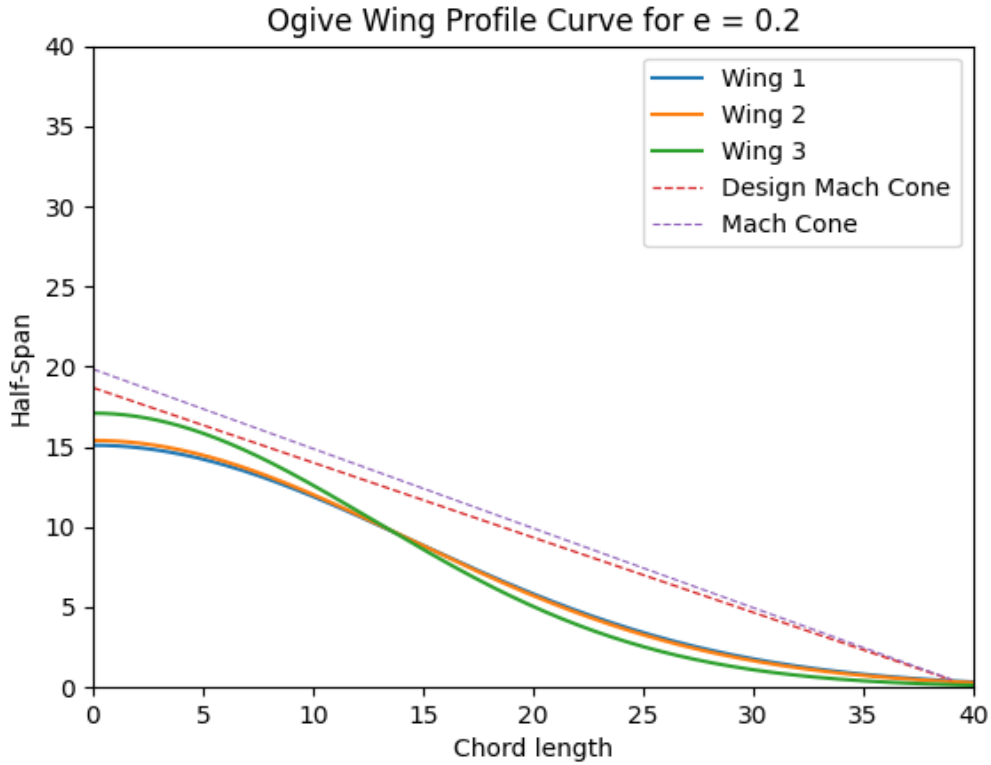


Figure 4.3: Ogive Planform with desired range of Wing Area with Mach Cone Rule

Further, reduction of the upper bound of the wing area to help minimize profile drag, we get the possible design planforms.

	Wing Area	Aspect Ratio	a	b
<b>Wing 1</b>	274.3144 m <sup>2</sup>	0.8312	15.1	20.5
<b>Wing 2</b>	274.3059 m <sup>2</sup>	0.8646	15.4	20.1
<b>Wing 3</b>	274.2811 m <sup>2</sup>	1.0661	17.1	18.1

Table 4.2: Ogive Planform Characteristics

An aspect ratio higher than this is not possible in an ogive configuration due to the fact that for a fixed wing area requirement larger span is not possible due to the mach cone rule.

## 4.4 Airfoil Selection

### 4.4.1 Airfoil Geometry

There are several key geometric parameters of an airfoil.

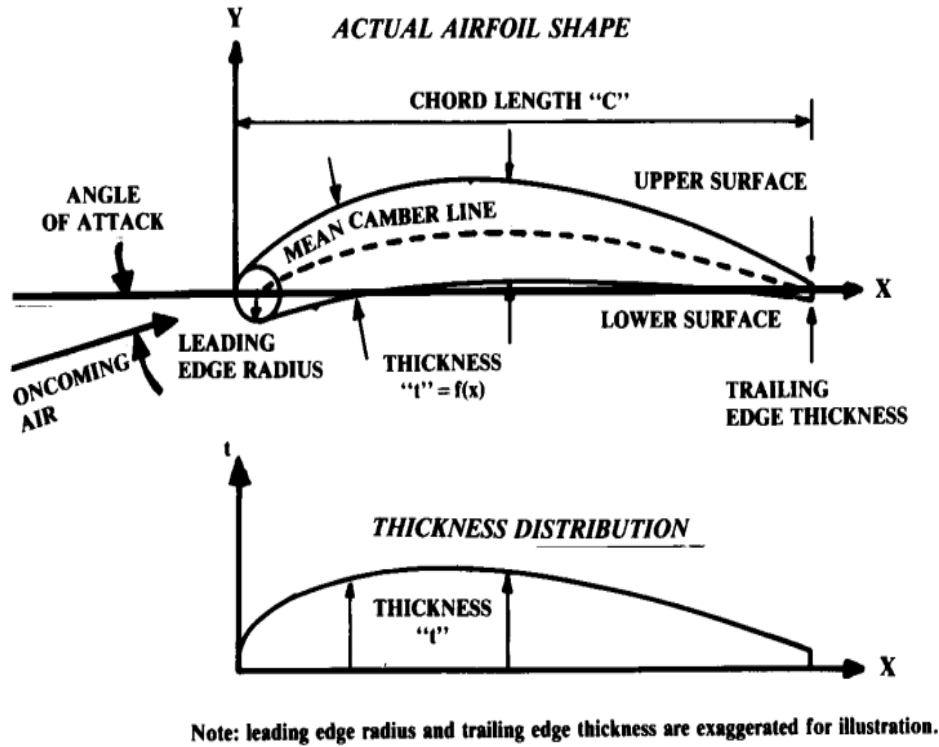


Figure 4.4: Airfoil Geometric Parameters[9]

An airfoil designed to operate in supersonic flow will have a sharp or nearly sharp leading edge to prevent a drag-producing bow shock. The use of wing sweep relaxes this condition on the leading edge, if the sweep is sufficient enough to ensure that the entire leading edge of the wing faces a subsonic flow.

It is traditional to separate the airfoil into its thickness distribution and a zero-thickness camber line. The former provides the major influence on the profile drag, while the latter provides the major influence upon the lift and drag due to lift. When an airfoil is scaled in thickness, the camber line must remain unchanged, so the scaled thickness distribution is added to the original camber line to produce the new, scaled airfoil. Similarly, an airfoil which is to have its camber changes is broken into its camber line and thickness distribution. The camber line is scaled to produce the desired maximum camber; then the original thickness distribution is added to obtain a new airfoil. This is how, an airfoil can be reshaped to change either profile drag or lift characteristics, without generally affecting the other.

### 4.4.2 Airfoil Lift, Drag and Pitching Moment

The typical airfoil lift, pitching moment, and drag characteristics are as follows :

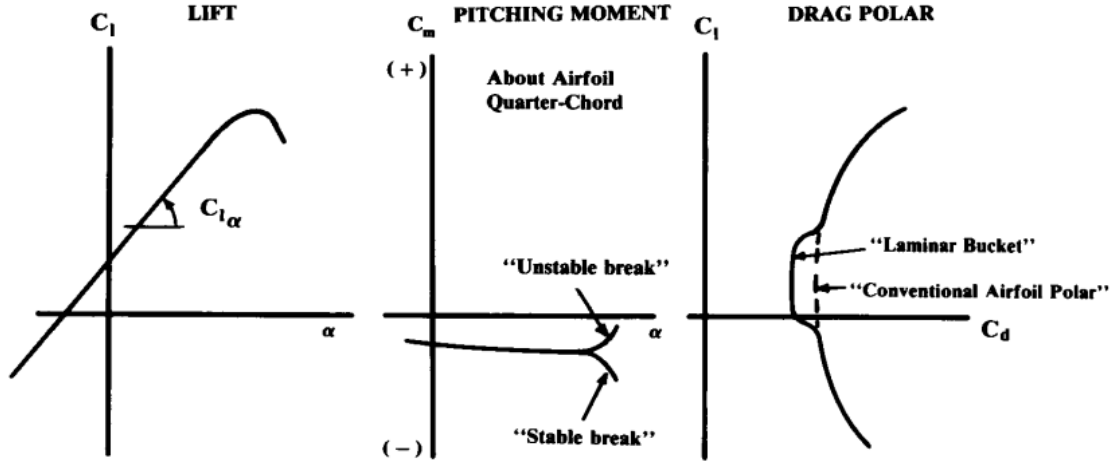


Figure 4.5: Typical Airfoil Characteristics[9]

Airfoil characteristics are strongly affected by the Reynolds number at which they are operating. The Reynolds Number is given as

$$Re = \frac{\rho V L}{\mu} \quad (4.5)$$

where  $V$  is the velocity of the fluid with respect to the object,  $\rho$  is the density of the fluid,  $\mu$  is the dynamics viscosity of the fluid, and  $L$  is the characteristic linear dimension i.e. length travelled by the fluid along the surface of the object. The Reynolds number influences whether the flow will be laminar or turbulent, and whether flow separation will occur. Thus, the Reynolds number is crucial in the airfoil selection and wing design.

Pitching Moment must also be considered in airfoil selection. Horizontal tail or canard size is directly affected by the magnitude of the wing pitching moment to be balanced. For a stable tail-less or flying wing aircraft, the pitching moment must be near zero. This requires a camber with the characteristic upward reflex at the trailing edge. Reflexed airfoils have poorer L/D than other airfoils due to their reduced wetted area.

### 4.4.3 Stall Characteristics

Stall characteristics play an important role in airfoil selection. Some airfoils exhibit a gradual reduction in lift during a stall, while others show a violent loss of lift, accompanied by a rapid change in pitching moment.

Twisting the wing such that the tip airfoils have a reduced angle of attack compared to the root ("washout") can cause the wing to stall first at the root. This provides a gradual stall even for a wing with a poorly stalling airfoil. The wing may be designed to stall at the root first. This provides good flow over ailerons for roll control at an angle of attack where the root is stalled.

Wing stall is directly related to airfoil stall only for high-aspect-ratio, unswept wings. For lower aspect ratio or highly swept wings the 3-D effects dominate stall characteristics, and airfoil stall characteristics can be essentially ignored in airfoil selection.

#### 4.4.4 Airfoil Thickness Ratio

Airfoil thickness ratio has a direct effect on drag, maximum lift, stall characteristics and structural weight. A supercritical airfoil tends to minimize shock formation and can be used to reduce drag for a given thickness ratio or to permit a thicker airfoil at the same drag level. For low-aspect-ratio, swept wings, a sharper leading edge provides greater maximum lift due to the formation of vortices just behind the leading edge. These leading edge vortices delay wing stall.

Thickness also affects the structural weight of the wing. Historic data shows that the wing structural weight varies approximately inversely with the square root of the thickness ratio. For initial selection of thickness ratio, the historical trend can be used. Supercritical airfoil would tend to be about 10% thicker (1.1 times the conventional airfoil thickness) than the historic trend.

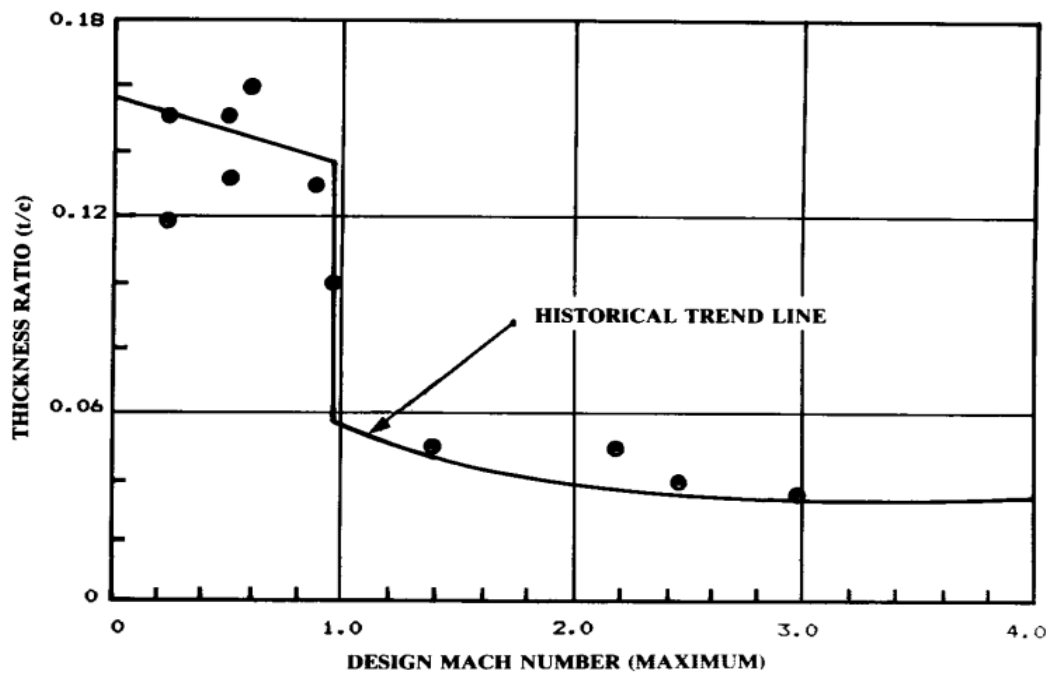


Figure 4.6: Thickness Ratio Historic Trend[9]

The thickness is often varied from root to tip. This is beneficial, resulting in a structural weight reduction as well as more volume for fuel and landing gear. This thicker root airfoil should extend to no more than 30% of the span.

For swept-wing supersonic aircraft, the NACA 64A, NACA 65A, NACA 5 digit series and NASA SC(2) series are good airfoils for initial design.[9][5] For Supersonic cruise of 2.25, we chose the airfoil with thickness ratio around 4.5% as per the above figure.



Thickness	NACA 64A	NACA 65A	NASA SC(2)
3.5%	NACA 64A035	NACA 65A035	NACA SC(2)-0035
4.0%	NACA 64A040	NACA 65A040	NACA SC(2)-0040
4.5%	NACA 64A045	NACA 65A045	NACA SC(2)-0045
5.0%	NACA 64A050	NACA 65A050	NACA SC(2)-0050

Due to lack of computational fluid dynamics capabilities, historic data was used to decide the airfoil. The wing planform was also decided using historic data of aspect ratio.

Historic data suggest having an aspect ratio of 1.6 should be needed. From the wing planform analysis done, with the selected function of Ogive wing, the maximum possible Aspect Ratio is 1.066, and hence the design with that aspect ratio was finalized. The airfoil was finalized to be **NACA24104**, due to it's thickness ratio which is around the required value.

## 4.5 Preliminary Wing Design with Placeholder Fuselage

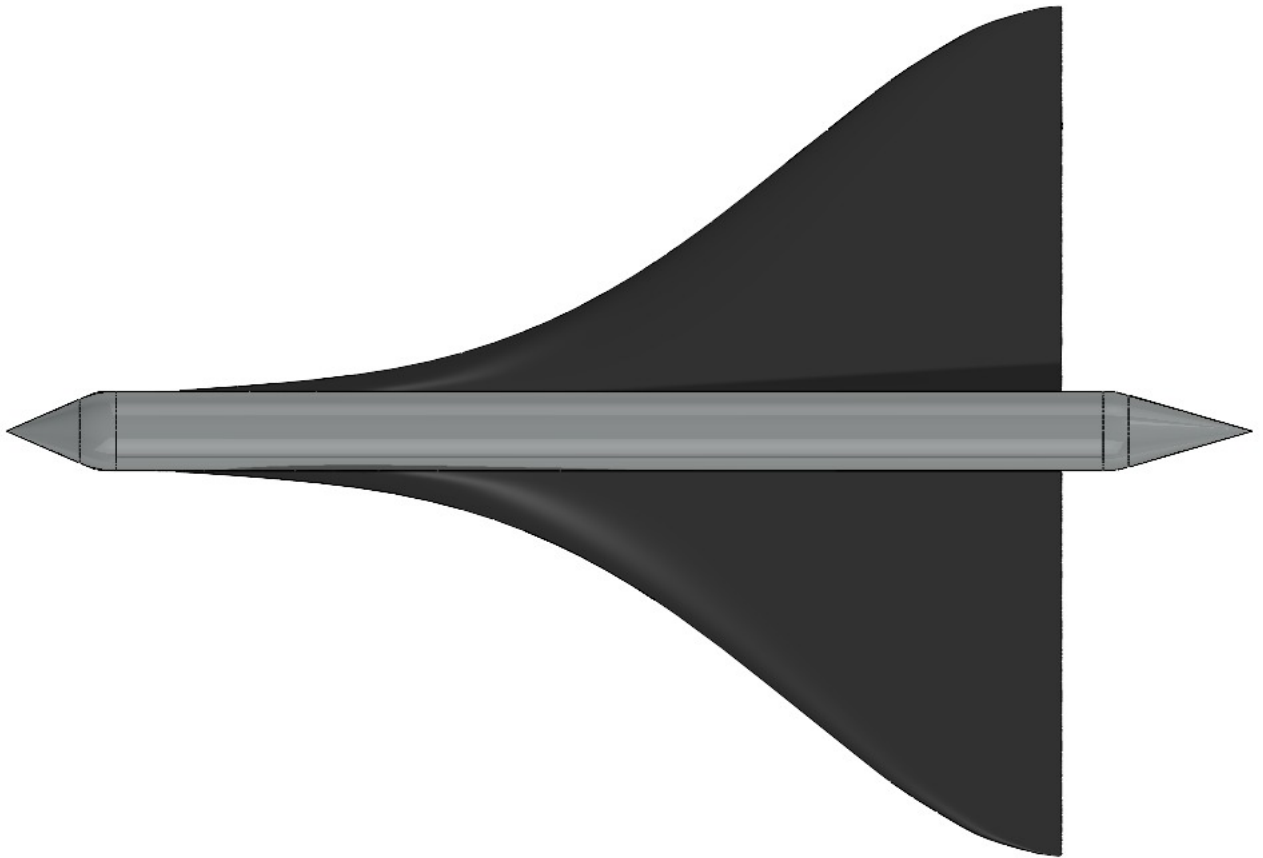


Figure 4.7: 3D Model Top View

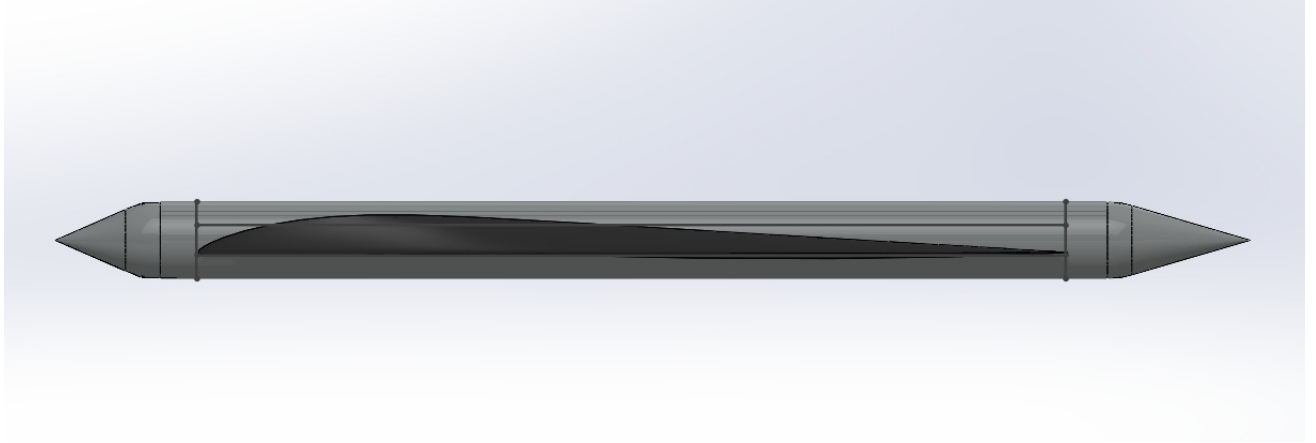


Figure 4.8: 3D Model Side View

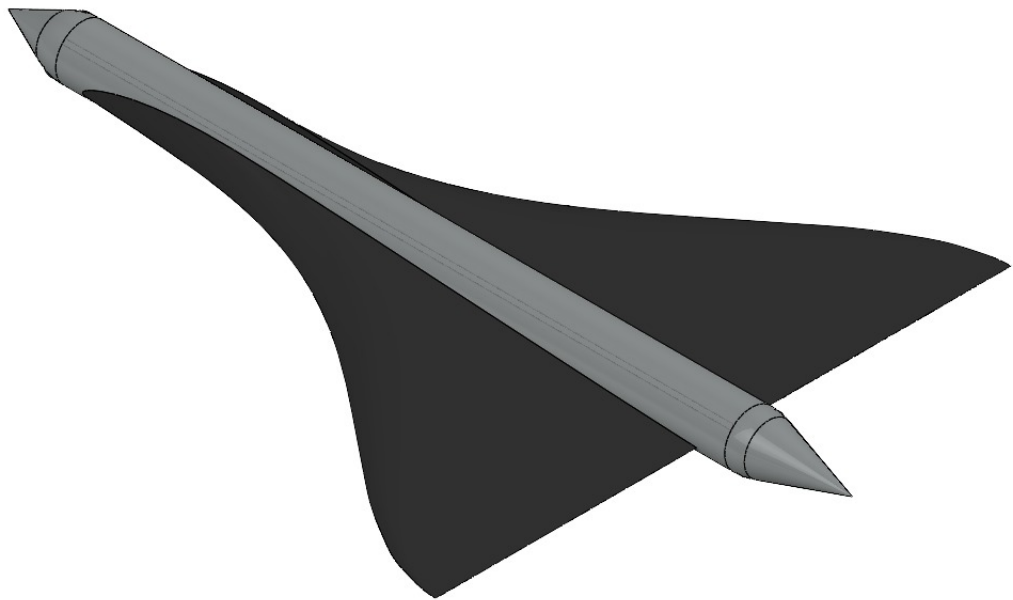


Figure 4.9: 3D Model Orthogonal View

# Chapter 5

## Fuselage Design

### 5.1 Placeholder Fuselage Calculations

The dimensions of a placeholder fuselage were calculated to find the best combination of wing planform and airfoil. All passengers were designed to be seated on the root chord of the wing, which is 40m in length. The Entry/Exit Doors were designed to be placed right after that.

The Aircraft consists of 3 different seating classes, which have different comfort levels due to different seat dimensions. The number of rows and the number of seats in each row was decided accordingly.

	Emergency Exits	Economy	Economy Plus	First Class
No. of Rows	2	x	35 - x	10
Seats/Row	4-6	6	4	4
Seat Length	1m	0.8m	0.8m	1m
Seat Width	0.45m	0.45m	0.6m	0.7m
Passage Width	0.45m	0.45m	0.45m	0.35m

Table 5.1: Aircraft Passenger Seats Description

The number of seat distribution between Economy and Economy Plus has been kept variable to be adjusted after the variation of fuselage diameter is decided to satisfy the whitcomb area rule.

The diameter of the cylindrical body of the placeholder fuselage can be calculated as

$$inside\ diameter\ (3.15m) + walls\ (0.175m \times 2) = fuselage\ diameter\ (3.5m) \quad (5.1)$$

## 5.2 Final Fuselage Design

### 5.2.1 Whitcomb Area Rule

Whitcomb's Area Rule, also known as the transonic area rule, is a scientific principle that governs the aerodynamic design of aircraft operating in the transonic speed regime, near the speed of sound. It was developed by Richard T. Whitcomb, an American aerospace engineer, while working at the National Advisory Committee for Aeronautics (NACA) in the 1950s. Whitcomb's work on the area rule had a profound impact on the design of high-speed aircraft and contributed significantly to advancements in supersonic flight.

The transonic speed regime, encompassing speeds from about 0.7 to 1.3 times the speed of sound, presents unique challenges for aircraft designers. At these speeds, airflow over certain areas of the aircraft can reach or exceed the speed of sound, leading to the formation of shockwaves. These shockwaves cause a sudden increase in air pressure and result in wave drag, a form of drag that significantly hampers the aircraft's performance and efficiency.

Whitcomb's key insight was that the distribution of cross-sectional area along the length of an aircraft directly affects the formation and intensity of shockwaves, and thus, the magnitude of wave drag. His goal was to reduce wave drag by minimizing the abrupt changes in the cross-sectional area of an aircraft.

According to the area rule, the total cross-sectional area of an aircraft should be distributed in a manner that ensures a smooth transition from one section to another along the aircraft's length. Specifically, the rate of change of cross-sectional area should be minimized at the location of maximum cross-sectional area. This means that regions of the aircraft where the cross-sectional area is expanding rapidly should be compensated by regions where it is contracting, resulting in a more streamlined overall shape.

To understand why this is important, it is necessary to examine the aerodynamics of shockwave formation. When an aircraft moves through the air at speeds close to or exceeding the speed of sound, localized areas experience supersonic flow. These supersonic regions create shockwaves, which manifest as discontinuities in air pressure and temperature.

In the case of an aircraft with a sudden increase in cross-sectional area, such as a bulge or a widening fuselage, the shockwave generated at that point is stronger and more pronounced. This intensifies the wave drag and increases the overall drag on the aircraft. Conversely, if the cross-sectional area decreases abruptly, a shockwave is formed, again leading to increased wave drag.

Whitcomb's Area Rule aims to delay or reduce the formation of shockwaves by distributing the cross-sectional area changes smoothly along the aircraft's length. By avoiding abrupt changes in area, the intensity of shockwaves is diminished, resulting in lower wave drag and improved aerodynamic efficiency. Thus, we further modify the fuselage to implement the Whitcomb Area Rule.

# Chapter 6

## Tail Sizing and Design

### 6.1 Introduction

A conventional tail stabiliser allows the main wing to be optimised for lift and therefore to be smaller and more highly loaded. A conventional fixed-wing aircraft has a horizontal stabiliser surface separate from its main wing. This extra surface causes additional drag requiring a more powerful engine, especially at high speeds. If longitudinal (pitch) stability and control can be achieved by some other method, the stabiliser can be removed and the drag reduced.

A tailless aircraft has no other horizontal surface besides its main wing. The aerodynamic control and stabilisation functions in both pitch and roll are incorporated into the main wing. A tailless type may still have a conventional vertical tail fin (vertical stabilizer) and rudder. Like other tailless aircraft, the tailless delta wing is not suited to high wing loadings and requires a large wing area for a given aircraft weight. The most efficient aerofoils are unstable in pitch and the tailless type must use a less efficient design and therefore a bigger wing. Techniques used include:

- Using a less efficient aerofoil which is inherently stable, such as a symmetrical form with zero camber, or even reflex camber near the trailing edge,
- Using the rear part of the wing as a lightly- or even negatively-loaded horizontal stabiliser:
  - Twisting the outer leading edge down to reduce the incidence of the wing tip, which is behind the main centre of lift. This also improves stall characteristics and can benefit supersonic cruise in other ways.
  - Moving the centre of mass forwards and trimming the elevator to exert a balancing downforce. In the extreme, this reduces the craft's ability to pitch its nose up for takeoff and landing.

The main advantages of the tailless delta are structural simplicity and light weight, combined with low aerodynamic drag.

## 6.2 Stability and Control of Tailless Aircrafts

A tailless aeroplane has no separate horizontal stabilizer. Because of this the aerodynamic center of an ordinary wing would lie ahead of the aircraft's center of gravity, creating instability in pitch. Some other method must be used to move the aerodynamic center backward and make the aircraft stable. There are two main ways for the designer to achieve this.

1. Sweeping the wing leading edge back, either as a swept wing or delta wing, and reducing the angle of incidence of the outer wing section allows the outer wing to act like a conventional tailplane stabiliser. If this is done progressively along the span of the outer section, it is called tip washout. It can be achieved by giving the wing upper surface a conical curvature. In level flight the aircraft should be trimmed so that the tips do not contribute any lift: they may even need to provide a small downthrust. This reduces the overall efficiency of the wing, but for many designs - especially for high speeds - this is outweighed by the reductions in drag, weight and cost over a conventional stabiliser. The downside is that the long wing span reduces manoeuvrability.
2. An alternative is the use of low or null pitching moment airfoils, seen for example in the Horten series of sailplanes and fighters. These use an unusual wing aerofoil section with reflex or reverse camber on the rear or all of the wing. With reflex camber the flatter side of the wing is on top, and the strongly curved side is on the bottom, so the front section presents a high angle of attack while the back section is more horizontal and contributes no lift, so acting like a tailplane or the washed-out tips of a swept wing. Reflex camber can be simulated by fitting large elevators to a conventional airfoil and trimming them noticeably upwards; the center of gravity must also be moved forward of the usual position. Due to the Bernoulli effect, reflex camber tends to create a small downthrust, so the angle of attack of the wing is increased to compensate. This in turn creates additional drag. This method allows a wider choice of wing planform than sweepback and washout, and designs have included straight and even circular (Arup) wings. But the drag inherent in a high angle of attack is generally regarded as making the design inefficient.

There is a trade-off between stability and maneuverability. A high level of maneuverability requires a low level of stability. Some modern hi-tech combat aircraft are aerodynamically unstable in pitch and rely on fly-by-wire computer control to provide stability.

The solution usually adopted is to provide large elevator on the wing trailing edge. Unless the wing is highly swept, these must generate large control forces, as their distance from the aerodynamic center is small and the moments less. Thus a tailless type may experience higher drag during pitching manoeuvres than its conventional equivalent. In a highly swept delta wing the distance between trailing edge and aerodynamic centre is larger so enlarged surfaces are not required. However even in the airplanes with active pitch control, pitch control at the high angles of attack experienced during takeoff and landing could be problematic and hence additional stability is provided by additional canard surfaces.

## 6.3 Canard

Rather than use the conventional tailplane configuration found on most aircraft, an aircraft designer may adopt the canard configuration to reduce the main wing loading, to better control the main wing airflow, or to increase the aircraft's maneuverability, especially at high angles of attack or during a stall. Canard foreplanes, whether used in a canard or three-surface configuration, have important consequences for the aircraft's longitudinal equilibrium, static and dynamic stability characteristics.

Early supersonic canard delta designs, encountered stability and control problems and hence prevented widespread adoption. In 1963 the Swedish company Saab patented a delta-winged design which overcame the earlier problems, in what has become known as the close-coupled canard.

Static canard designs can have complex interactions in airflow between the canard and the main wing, leading to issues with stability and behaviour in the stall.[16] This limits their applicability. The development of fly-by-wire and artificial stability towards the end of the century opened the way for computerized controls to begin turning these complex effects from stability concerns into maneuverability advantages.[2]

Determining if a canard is necessary for an aircraft design depends on several factors, such as the aircraft's intended use, performance requirements, and overall design goals. Some considerations are :

1. Stability requirements : If the aircraft has a rearward center of gravity, a canard may be necessary to shift the center of lift forward and improve stability.
2. Control requirements : If the aircraft has poor stability or control, a canard may be necessary to improve the aircraft's handling characteristics.
3. Lift requirements : If the wing design does not provide adequate lift or control, a canard may be necessary to supplement the wing's performance.
4. Performance requirements : If the aircraft needs to perform maneuvers such as high-angle-of-attack flight, a canard may be necessary to provide additional lift and control.

In summary, a canard may be necessary if the aircraft has poor stability or control, a rearward center of gravity, inadequate lift or control from the wing, or if the aircraft needs to perform specific maneuvers. However, it's important to evaluate the impact of adding a canard on the overall design and performance of the aircraft. Adding a canard may increase weight, complexity, and cost, so it's important to consider the trade-offs of adding a canard to the aircraft's design.

It is tough to stall a well-designed canard design. If airspeed slowed down excessively, the canard would stall first. The nose would drop, airspeed would build back up, and the aircraft would recover. This only works safely if the design can assure that the canard will always be the first to stall. This is accomplished by giving the canard very high wing loading. If the wing stalled first, the aircraft's tail would sink, causing the angle of attack to increase rather than decrease. As a result, the stall would deepen and could become unrecoverable.

## 6.4 Vertical Tail Sizing and Design

Vertical stabilizers are required for yawing motion of the Aircraft.

### 6.4.1 Vertical Tail Area Estimation

The force resulting from tail lift increases in proportion to the tail area, indicating that the effectiveness of the tail is directly proportional to the product of the tail area and the tail moment arm. This product, measured in terms of volume, forms the basis for the initial estimation of the tail size using the tail volume coefficient method.

$$C_{VT} = \frac{S_{VT} L_{VT0}}{S_w b_w} \quad (6.1)$$

where,  $C_{VT}$  is tail volume coefficient,  $S_{VT}$  is area of vertical tail,  $L_{VT}$  is moment arm of vertical tail,  $S_w$  is wing area and  $b_w$  is wing span.

$$S_{VT} = \frac{C_{VT} S_w b_w}{L_{VT}} \quad (6.2)$$

The above equation has two unknowns  $C_{VT}$  and  $L_{VT}$ . To find  $C_{VT}$  we have used the graph shown below from Kroo(2013).

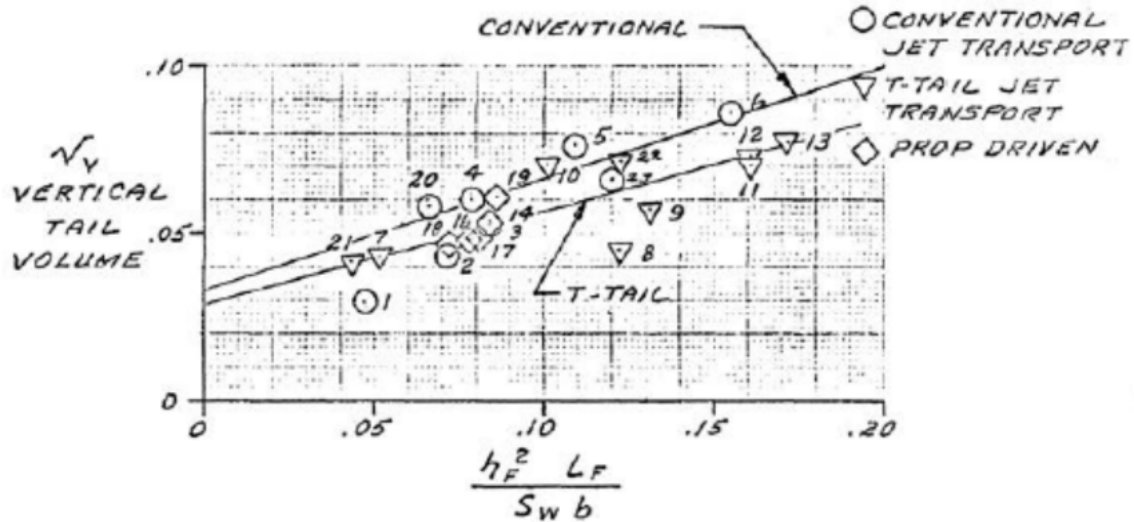


Figure 6.1: Plot of Tail volume coefficient for vertical tail( $C_{VT}$ )

$C_{VT}$  is observed to show a good linear correlation with the ratio,

$$\frac{h_f^2 L_F}{S_w b} \quad (6.3)$$

where,  $h_f$  (3.5 m) is the height of the aircraft fuselage,  $L_F$  (60 m) is the length of fuselage,  $S_w$  (548.5 m<sup>2</sup>) is wing area and  $b$  (34.2 m) is wing span. Thus,

$$\frac{h_f^2 L_F}{S_w b} \approx 0.0392 \quad (6.4)$$



Using the geometric specifications of our aircraft and the plot above, an estimate for the  $C_{VT}$  was found to be  $C_{VT} \approx 0.04214$ .

It was noted that the  $L_{VT}/L_F$  ratio of the Concorde and the TU-144 was around 16%. This ratio was used to compute the  $L_{VT}$  for our aircraft and was found to be  $L_{VT} = 9.6$  m.

Once we have an estimate of  $L_{VT}$  and  $C_{VT}$ ,  $S_{VT}$  can be calculated

$$S_{VT} = \frac{C_{VT} S_w b_w}{L_{VT}} \approx 82.343 m^2 \quad (6.5)$$

### 6.4.2 Vertical Tail Design

For the choice of Aspect ratio, Roskam(1985) suggested values between 1.2 and 2.4, So, the average value of 1.8 is chosen.

$$A.R = \frac{h_{VT}^2}{S_{VT}} \quad (6.6)$$

thus, calculating the height of the vertical tail  $h_{VT}$

$$h_{VT} = 12.17446 m \quad (6.7)$$

For the choice of Taper ratio, we chose from the range of values suggested by Roskam(1985) i.e., between 0.37 and 1. Since the trend for the value of Taper ratio in Concorde and Tu-144 are lower than 0.37, we have decided to choose the initial choice of taper ratio to be 0.4.

Using  $S_{VT}$ ,  $h_{VT}$  and  $\lambda_{VT}$  (taper ratio) we can get an estimate for Root chord( $c_R$ ) and Tip chord( $c_T$ ).

$$c_R = \frac{S_{VT}}{0.5(1 + \lambda_{VT})h_{VT}} \approx 9.6 \quad (6.8)$$

We can use  $c_R$  and  $\lambda_{VT}$  to obtain  $c_T$

$$c_T \approx 3.84 m \quad (6.9)$$

For Leading edge and trailing edge sweep angles, we have used the quarter chord sweep angle suggested by Roskam(1985). Quarter chord sweep angle is between  $37^\circ$  and  $65^\circ$ , so the average value is  $51^\circ$ . Using the quarter chord sweep angle, we have found the Sweep angles to be

$$\Lambda_{LE} = 21.6^\circ \quad (6.10)$$

$$\Lambda_{TE} = -4.63^\circ \quad (6.11)$$

Airfoil for the vertical tail is chosen to be NACA0009. A symmetric airfoil whose thickness is small enough to reduce the drag but high enough to handle the root stresses.

# Chapter 7

## Engine Sizing

In order to size the engines for the aircraft, the thrust required for key segments of the mission profile were first calculated.

### 7.1 OEI Take-off Thrust

From equation 3.19

$$\frac{T}{W} = (5.1467e - 3) \sqrt{\frac{W}{S}} \quad (7.1)$$

For the designed wing loading,  $(W/S) = 4487.7315 \text{ N/m}^2$ . This gives us

$$(\frac{T}{W})|_{take-off} = 0.34478 \quad (7.2)$$

The thrust required is then calculated using the maximum take-off weight

$$T = 0.34478 * W_{MTOW} \quad (7.3)$$

$$= 0.34478 * 251,000 * 9.8 \quad (7.4)$$

$$\approx 848kN \quad (7.5)$$

### 7.2 OEI Climb Thrust

From equation 3.7

$$(\frac{T}{W})|_{climb} \approx 0.2858 \quad (7.6)$$

Thus, for climb weight, the thrust is given by

$$T = 0.2858 * 251000 * 9.8 \quad (7.7)$$

$$\approx 703kN \quad (7.8)$$

#### 7.2.1 OEI Divert Climb Thrust

For divert climb weight, the thrust is given by

$$T = 0.2858 * 251000 * 0.4416245 * 9.8 \quad (7.9)$$

$$\approx 310.5kN \quad (7.10)$$

### 7.3 Thrust required during cruise

From equation 3.13

$$\left(\frac{T}{W}\right)|_{cruise} = \frac{494.54}{(W/S)} + 1.691e - 6 \times (W/S) \quad (7.11)$$

For the designed wing loading,  $(W/S) = 4487.7315 \text{ N/m}^2$ . This gives us

$$\left(\frac{T}{W}\right)|_{cruise} = 0.117787 \quad (7.12)$$

The thrust required is then calculated using the maximum cruise weight as the initial cruise weight

$$T = 0.117787 * W_{MTOW} * W_{cruise}/W_{MTOW} \quad (7.13)$$

$$= 0.117787 * 251,000 * 0.9256 * 9.8 \quad (7.14)$$

$$\approx 270kN \quad (7.15)$$

Thus the total thrust needed for cruise can be approximated using analytical expressions as stated above.

The ideal calculation requires the calculation of Drag from CFD analysis. Due to limited time and computational power, the analytical expression is used.

### 7.4 Engine Specifications

From the above calculations, we can see that the thrust requirement for engines can be determined.

For One Engine Inoperational case, 3 engines are operational, and hence each engine (both supersonic and subsonic) must be able to provide 282kN of thrust.

For Supersonic Cruise, assuming only the 2 Supersonic engines are required for cruise, each engine must be able to provide 135kN of thrust.

The above requirements are met by the initial engine selection, which was used for weight calculation. Hence the engine selection is as given below:

	<b>GE90</b>	<b>CF6-80C2</b>
<b>Number of Engines</b>	2	2
<b>Intake/Fan Diameter</b>	1,803 mm	2,362 mm
<b>Engine Length</b>	8,331 mm	4,270 mm
<b>Engine Weight</b>	5,100 kg	4,144 kg
<b>Dry Thrust</b>	220 kN	210 kN
<b>Wet Thrust</b>	281 kN	276 kN
<b>TSFC (SLS)</b>	41.8 g/kN.s	9.32 g/kN.s
<b>T/W</b>	6.02	6.796*

Table 7.1: Powerplant Selection

\* the value has been calculated using the data available

# Appendix A

## Codework

### A.1 Parameter Calculation for Operating Empty Weight

```
from math import *
import numpy as np
import matplotlib.pyplot as plt

def OEW_parameters():
    W_MTOW = np.array([189830, 207000, 267619, 306247]) # W_MTOW in kgs
    W_OEW = np.array([78700, 84200, 107955, 119400]) # W_OEW in kgs

    y = np.array(np.log(W_OEW))
    x = np.array(np.log(W_MTOW))

    #find line of best fit
    c, log_a = np.polyfit(x, y, 1)
    a = exp(log_a)

    print("\n Empty Weight Analytical Expression Parameters")
    print("a : ", a)
    print("c : ", c)

def new_OEW_parameters():
    W_PP = np.array([12700, 15600, 20400, 20400])
    W_STR = np.array([66000, 68600, 87555, 99000])
    percentage_reduction = 20
    W_STR_new = W_STR*(1 - (percentage_reduction/100))

    W_OEW_new = W_PP + W_STR_new # W_OEQ in kgs
    W_MTOW = np.array([189830, 207000, 267619, 306247]) # W_MTOW in kgs

    y = np.array(np.log(W_OEW_new))
    x = np.array(np.log(W_MTOW))

    #find line of best fit
    c, log_a = np.polyfit(x, y, 1)
    a = exp(log_a)

    print("\n Empty Weight Analytical Expression Parameters")
    print("a : ", a)
    print("c : ", c)
```

## A.2 Total Fuel Weight Fraction Calculation

```

from math import *
import numpy as np
import matplotlib.pyplot as plt

def TFW_calculation():
    n = 2                                # number of engines

    climb_rate = 3000                    # in feet/min
    climb_rate = climb_rate * 0.3048 * (1/60)    # in meter/second

    g = 9.8                              # in m/s^2

    sea_level_TSFC = 9.32                 # in g/(kN.s)
    cruise_TSFC = 33.8                    # in g/(kN.s)

    sea_level_TSFC = sea_level_TSFC * g * 1e-6 * n    # in 1/s
    cruise_TSFC = cruise_TSFC * g * 1e-6 * n          # in 1/s
    climb_TSFC = sea_level_TSFC/(0.8)                # taking SL_TSFC = 80% * climb_TSFC

    # Altitude Parameters
    take_off_altitude = 0                 # in meters
    subsonic_cruise_altitude = 40000     # in feet
    supersonic_cruise_altitude = 60000   # in feet
    subsonic_loiter_altitude = 3500       # in feet
    subsonic_cruise_altitude = subsonic_cruise_altitude * 0.3048    # in meters
    supersonic_cruise_altitude = supersonic_cruise_altitude * 0.3048 # in meters
    subsonic_loiter_altitude = subsonic_loiter_altitude * 0.3048     # in meters

    # L/D Characteristics
    L_by_D_subsonic = 12
    L_by_D_supersonic = 7.5
    L_by_D_loiter = L_by_D_subsonic/0.866

    supersonic_cruise_speed = 2.25        # Mach speed
    supersonic_cruise_speed = supersonic_cruise_speed * 295.5    # speed in m/s

    endurance = 25                        # endurance in minutes
    endurance = endurance * 60             # endurance in seconds
    loiter_TSFC = sea_level_TSFC           # in g/(kN.s)
    loiter_TSFC = loiter_TSFC * g * 1e-6 * n    # in 1/s near SL

    #-----Phase_P1-----#
    dW_P1 = 0.970

    #-----Phase_P2-----#
    dh1 = subsonic_cruise_altitude - take_off_altitude
    dh2 = supersonic_cruise_altitude - subsonic_cruise_altitude

    M_i = 0.85
    M_f = 2.25

    dW_P2_1 = exp(-1*(dh1 * climb_TSFC)/(climb_rate*L_by_D_subsonic))
    dW_P2_2 = (0.991 - 0.07*M_f - 0.01*M_f*M_f)/(0.991 - 0.07*M_i - 0.01*M_i*M_i)
    dW_P2_3 = exp(-1*(dh2 * climb_TSFC)/(climb_rate*L_by_D_supersonic))

    dW_P2 = dW_P2_1 * dW_P2_2 * dW_P2_3

```

```

#-----Phase_P3-----#
Range = 4700                                # cruise range in km
Range = Range * 1000                        # cruise range in meters

dW_P3 = exp(-1*(Range * cruise_TSFC)/(supersonic_cruise_speed *
↪ L_by_D_supersonic))

#-----Phase_P4-----#
dW_P4 = 1

#-----Phase_P5-----#

dW_P5 = exp(-1*(endurance * loiter_TSFC)/(L_by_D_loiter))

#-----Phase_P6-----#
dW_P6 = 1

#-----Phase_P7-----#
dh3 = subsonic_loiter_altitude - take_off_altitude

dW_P7 = exp(-1*(dh3 * sea_level_TSFC)/(climb_rate*L_by_D_subsonic))

#-----Phase_P8-----#
dW_P8 = 1

#-----Phase_P9-----#
dW_P9 = dW_P5

#-----Phase_P10-----#
dW_P10 = 1

#-----Phase_P11-----#
dW_P11 = 0.995

# calculating dW for MTOW : Maximum Take-off Weight (ignoring P1)
dW_end_by_dW_MTOW = dW_P2*dW_P3*dW_P4*dW_P5*dW_P*dW_P7*dW_P8*dW_P9*dW_P10*dW_P11
print("\n dW_end/W_MTOW : ", dW_end_by_dW_MTOW)

dW_TFW_by_dW_MTOW = 1.06*(1-dW_end_by_dW_MTOW)
print("\n W_TFW/W_MOTW : ", dW_TFW_by_dW_MTOW)

```

## A.3 Maximum Take-off Weight Calculation

```
import numpy as np
import matplotlib.pyplot as plt

def takeoffwt(ratio=0.4387, W0=100000.0, Wpay=17550.0, a=1.22355, c=0.8959):
    condition = False
    counter_limit = 5000
    counter = 0
    W=[W0]
    while(condition == False):
        We = a*(W0**c)
        Wf = 1.06*W0*(1-ratio)
        W0st = Wpay/(1-(We/W0)-(Wf/W0))
        W.append(W0st)
        if abs((W0-W0st)/W0) < 1e-7:
            condition = True
        if counter > counter_limit:
            condition = True
            print("Counter Limit Exceeded")
        else:
            W0 = W0st
            counter = counter + 1

    W = np.array(W)
    err = abs((W[1:]-W[:-1])/W[:-1])
    #Output
    print(" Payload: " , Wpay,
          "\n Empty weight:", We,
          "\n Fuel weight:", Wf,
          "\n Takeoff weight:", W0st)

    return err # For error analysis
```

## A.4 Combined Constraint Diagram

```

import matplotlib.pyplot as plt
import numpy as np

def combined_constraint_diagram():
    one = np.ones(10000)
    x = np.linspace(10,8000,10000)S

    #----Cruise_Consideration----#
    rho = 1.225                # Density at sea level
    rho_11 = 0.365             # Density at 11 km
    rho_60 = 0.1164756         # Density at 60000 ft
    M = 2.25                   # Cruise Mach number
    v = M * 295.5              # Cruise velocity
    q = 0.5*rho_60*v*v         # Dynamic pressure at 60000 ft

    gamma = 1.4                # Specific heat ratio of air
    n_inlet = 1                # Engine Inlet Efficiency

    # Density Correction Factor
    tau_rho = (rho_11/rho)**0.7 * (rho_60/rho_11)
    # Pressure Correction Factor
    tau_p = (1+(n_inlet * (gamma - 1)/2 * M*M))*(gamma/(gamma-1))

    W_20 = 0.9256 * 0.97       # W_cruise / W_MTW
    C_d0 = 0.0185              # Profile Drag Coefficient
    K = 0.052                  # Drag due to Lift Factor

    # T/W for Supersonic Cruise Consideration
    TWcruise = (C_d0*q/(x*W_20) + (K/q)*x*W_20)*W_20/(tau_p * tau_rho)
    plt.plot(x,TWcruise,label = 'Cruise')

    #----One_Engine_Inoperative_Take-off_Distance_Considerations---#
    s_takeoff = 3500           # Design Take-off Distance
    n_eng = 4                  # No. of engines
    g = 9.81                   # Acceleration due to gravity
    A = 1.687886925            # Aspect ratio of the wing

    # T/W for OEI Take-off Length Consideration
    TWtakeoff = 1.1*np.sqrt(n_eng/(n_eng-1) * (x/(A*rho*g*s_takeoff)))
    plt.plot(x, TWtakeoff,label='OEI Takeoff')
    # 5% tolerance in Take-off Distance
    plt.plot(x, TWtakeoff/np.sqrt(1.05), '--', color = 'orange')

    #-----Landing_Distance_Considerations-----#
    C_lmax = 1.2               # Maximum coefficient of lift
    W_110 = 0.42876182         # W_landing/W_MTW
    s_land = 2000              # Design Landing Distance

    # W/S for Landing Distance Considerations
    WSland = 0.8563*rho*C_lmax*s_land/W_110
    plt.axvline(WSland,0,1,color = 'green',label = 'Landing')
    # 5% tolerance in Landing Distance
    plt.axvline(WSland*1.05,0,1,color = 'green',linestyle = '--')

```



```

#-----One_Engine_Inoperative_Climb_Rate_Considerations-----#
L_by_D_climb = 5                # L/D during Climb
gamma_SSCG = 0.024              # FAR-25 Regulations
W_10 = 0.97                     # W1/W0
TW_SSCG_OEI = n_eng/(n_eng -1)*((1/L_by_D_climb) + gamma_SSCG)*W_10

# T/W for Second-Stage Climb Gradient
plt.plot(x,one*TW_SSCG_OEI,color = 'red',label = 'OEI SSCG')
# +5% tolerance in T/W
plt.plot(x,one*TW_SSCG_OEI*1.05,'--',color = 'red')

W_50 = 0.428761682              # W5/W0
gamma_MA = 0.021                # FAR-25 Regulations
TW_MA_OEI = n_eng/(n_eng -1)*((1/L_by_D_climb) + gamma_MA)*W_50
# T/W for Missed Approach Climb Gradient
plt.plot(x,one*TW_MA_OEI,color = 'black',label = 'OEI MA')

#-----Stall_Considerations-----#
WS_stall = 4274.03
plt.axvline(WS_stall,0,1,color = '#33FFBD',label = 'SL stall')
plt.axvline(WS_stall*1.05,0,1,color = '#33FFBD',linestyle = '--')
plt.axvline(WS_stall*0.95,0,1,color = '#33FFBD',linestyle = '--')

#-----Design_Area-----#
x1 = np.linspace(10,WS_stall*1.05,10000)
y2 = 1
y1 = (C_d0*q/(x1*W_20) + (K/q)*x1*W_20)*W_20/(tau_p * tau_rho)
y3 = TW_SSCG_OEI*1.05
y4 = np.maximum(y1, y3)
plt.fill_between(x1,y4,y2,facecolor='orange',alpha = 0.3,label = 'Design Area')

#-----Design_Point-----#
plt.plot(WS_stall*1.05,TW_SSCG_OEI*1.05,'x',color = 'purple',label = 'Design
↪ Point')
plt.text(2500,0.3250,'({0:0.4f},{1:0.4f})'.format(WS_stall*1.05,TW_SSCG_OEI*1.05))
plt.legend(loc=1)
plt.title('Combined Constraint Diagram', fontsize=15)
plt.xlabel('Wing Loading (W/S) in $N/m^2$', fontsize = 15)
plt.ylabel('Thrust Loading (T/W)', fontsize=15)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
plt.ylim(0,0.75)
plt.show()

```

## A.5 Ogive Wing Planform Geometry Design

```

import numpy as np
import math
import matplotlib.pyplot as plt

def ogive_wing_planform(area):

    plt.figure()

    wing_area = area
    e_minus = 0
    e_plus = 0.2

    a_range = np.arange(15,18,0.1)
    b_range = np.arange(10,30,0.1)

    Area = []
    AR = []
    parameters = []

    wing_num = 1

    j_count = 0
    for j in a_range:
        i_count = 0
        for i in b_range:
            a = j          # affects span
            b = i          # affects root chord

            y_lim = 0.005
            x_lim = ((-1*math.log(y_lim/a))*0.5)*b

            x = np.arange(0, x_lim + 0.01, 0.01)
            y = a * np.exp(-(x/b)**2)

            area = np.trapz(y, x)
            ar = a*a/area
            if area > (wing_area - e_minus)/2 and area < (wing_area + e_plus)/2 :
                Area.append(area)
                AR.append(ar)
                parameters.append((a,b))
                label_text = 'Wing {}'.format(wing_num)
                plt.plot(x, y, label=label_text)
                wing_num = wing_num + 1
            i_count = i_count + 1

        j_count = j_count + 1

    ## Mach Cone Rule
    cruise_mach = 2.25
    safety_factor_cruise_mach = 2.25*(1.05)          # 5% safety factor

    mu = np.arcsin(1/cruise_mach)                  # mach_cone_half_angle
    mu_design = np.arcsin(1/safety_factor_cruise_mach) #
    ↪ design_mach_cone_half_angle

```

```

# Define the point and slope
x1, y1 = 40, 0
m = math.tan(- mu)
m_design = math.tan(- mu_design)

# Create x and y values for the line
x_mach_cone = range(0, 40)
y_mach_cone_design = [y1 + m_design * (xi - x1) for xi in x_mach_cone]
y_mach_cone = [y1 + m * (xi - x1) for xi in x_mach_cone]

plt.plot(x_mach_cone, y_mach_cone_design, '--', linewidth=0.9, label='Design Mach
→ Cone')
plt.plot(x_mach_cone, y_mach_cone, '--', linewidth=0.8, label='Mach Cone')

plt.xlabel('Chord length')
plt.xlim([0,40])
plt.ylabel('Half-Span')
plt.ylim([0,40])
plt.title('Ogive Wing Profile Curve for e = 0.2')
plt.legend()
plt.show()

ogive_wing_planform(548.5)

```

## A.6 Delta Wing Planform Geometry Design

```

import numpy as np
import math
import matplotlib.pyplot as plt

def delta_wing_planform(area):
    plt.figure()

    ## Mach Cone Rule
    cruise_mach = 2.25
    safety_factor_cruise_mach = 2.25*(1.05)      # 5% safety factor

    mu = np.arcsin(1/cruise_mach)                # mach_cone_half_angle
    mu_design = np.arcsin(1/safety_factor_cruise_mach) #
    ↪ design_mach_cone_half_angle

    m = math.tan(- mu)
    m_design = math.tan(- mu_design)

    wing_area = area
    e_minus = 0
    e_plus = 0.5

    # supersonic airplanes implement low Aspect Ratio and High Sweep
    a_range = np.arange(10,50,0.1)
    b_range = np.arange(30,40,0.2)

    Area = []
    AR = []
    parameters = []

    wing_num = 1

    j_count = 0
    for j in a_range:
        i_count = 0
        for i in b_range:
            a = j          # affects span
            b = i          # affects root chord

            # Define the two points
            x1, y1 = 0, a
            x2, y2 = b, 0

            # Calculate the slope and intercept of the line
            slope = (y2 - y1) / (x2 - x1)
            intercept = y1 - slope * x1

            # Create x and y values for the line
            x = np.arange(0, b)
            y = [slope * xi + intercept for xi in x]

            area = 0.5*b*a
            ar = a*a/area
            # enforcing the sweep angle to be more than design mach cone angle
            if area > (wing_area - e_minus)/2 and area < (wing_area + e_plus)/2 and
            ↪ (slope >= m_design) :
```

```

        Area.append(area)
        AR.append(ar)
        parameters.append((a,b))
        label_text = 'Wing {}'.format(wing_num)
        plt.plot(x, y, label=label_text)
        wing_num = wing_num + 1
        i_count = i_count + 1

    j_count = j_count + 1

# Define the point and slope
x1, y1 = 35, 0
x_mach_cone = range(0, 40)
y_mach_cone_design = [y1 + m_design * (xi - x1) for xi in x_mach_cone]
y_mach_cone = [y1 + m * (xi - x1) for xi in x_mach_cone]

plt.xlabel('Chord length')
plt.xlim([0,40])
plt.ylabel('Half-Span')
plt.ylim([0,40])
plt.title('Ogive Wing Profile Curve for e = 0.5')
plt.legend()
plt.show()

delta_wing_planform(548.5)

```

# Appendix B

## Simulation Results

### B.1 Parameter Calculation for Operating Empty Weight

# Bibliography

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