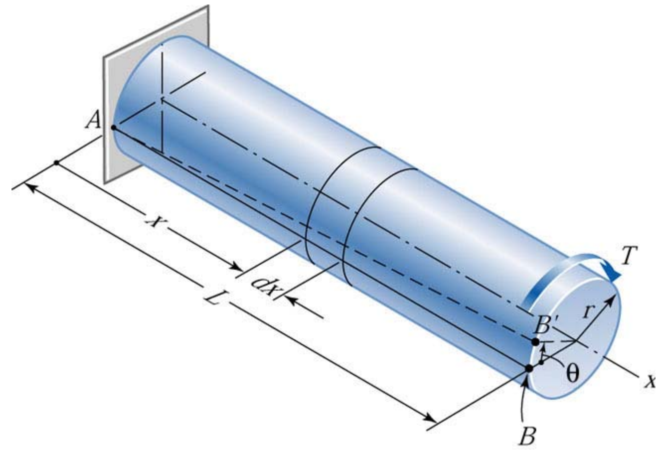
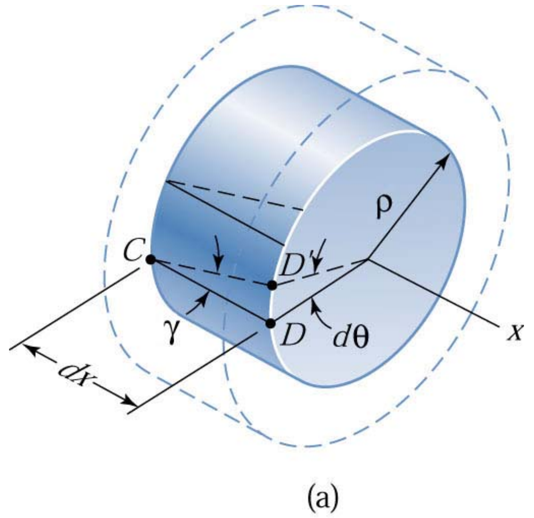
3. Relation between applied mass m and angle of deflection (θ_0) for a simply supported beam.

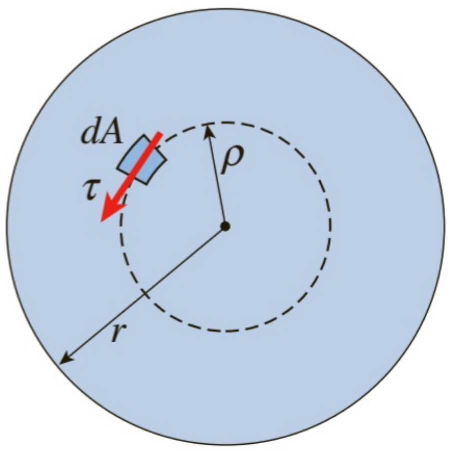
The Torsion Test is based on the **Theory of Pure Torsion**.



(a) Circular beam subjected to torque T



(b) dx section



(c) dx section

For a cylinder of diameter D and length l (see figure (a)), consider first a small annular element of mean radius rand thickness dx. The cylinder is subjected to a torque T, and this causes a circumferential shear stress T in the wall of the small element under consideration. If the torque is such that the left-hand end of this small element is twisted through an angle dθ in relation to the right-hand end, a longitudinal line AB on the surface of the element twists to position AB' (see figure (a)). For small angles of twist the shear strain γ that is developed is given by

$$\gamma = \frac{BB'}{L} \tag{1}$$

but arc BB' = rθ , therefore

$$\gamma = \frac{r\theta}{L} \tag{2}$$

Within the elastic limit, the ratio of stress to strain is constant.

$$\frac{\tau}{\gamma} = G \tag{3}$$

G=*Shear Modulus*

Substituting for γ from (2) in (3),and rearranging we have

$$\frac{\tau}{r} = \frac{G\theta}{L} \tag{4}$$

For this small element, if the thickness dx is small, it can be assumed that the shear stress is constant across the thickness of the element. Also, if dx is small, the area of the section on which the shear stress τ acts approximates to $2\pi\rho$ dρ. Hence, the total shear force acting on this element is τ*($2\pi\rho$ dρ).

The torque acting on this element is the moment of the tangential shear force about the longitudinal axis, and is $\tau\ 2\pi\rho^2 d\rho$. The torque T acting on the complete solid shaft is the sum of the moments of the tangential shear forces acting on all the small elements that go to make up the shaft, and is given by

$$T = \int_0^{D/2} \tau 2\pi\rho^2 d\rho$$

Substituting for τ from eqn(4), we have

$$T = \frac{G\theta}{L}\int_0^{D/2} 2\pi\rho^3 d\rho$$

On solving we get

$$T = \frac{G\theta}{L}\frac{\pi D^4}{32}$$

We know that for circular cross-section,

$$\frac{\pi D^4}{32} = J$$

In our experiment we produce the torque by adding mass on a lever attached to one end of the rod.

We already know the relation between the Torque applied and angle of deflection θ_0

$$T = \frac{G\theta_0}{L}\frac{\pi D^4}{32}$$

Since we know that for circular cross-section,

$$\frac{\pi D^4}{32} = J$$

In our experiment we produce the torque by adding mass on a lever attached to one end of the rod.(Length of lever = d). We apply mass m to one end of the lever which creates torque T.

$$T = mgd$$

m : *mass of load applied*
g : *acceleration due to gravity; 9.81 m/s^2*
d : *length of lever*

Substituting the value of T in the main equation,we get the relation between applied mass m and the angle of deflection θ_0

$$m = \frac{G\theta_0}{gdL}\frac{\pi D^4}{32}$$