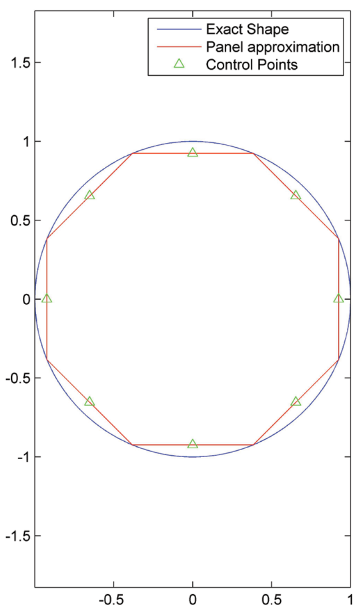
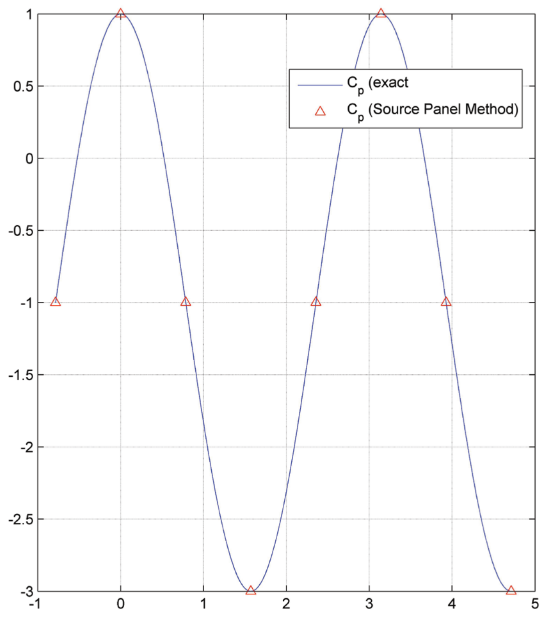
Assignment Report

Source Panel Method

Determining Pressure Coefficient in Non Lifting Flow over Cylinder





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```
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#
# PURPOSE
# - Compute the integral expression for constant strength source panels
# - Source panel strengths are constant, but can change from panel to panel
# - Geometric integral for panel-normal    : I(ij)
# - Geometric integral for panel-tangential: J(ij)
# - Compute the Pressure Coefficient at control points of the panels
#
# INPUTS
# - numP   : Number of panels in which we divide the surface
#
# OUTPUTS
# - Pressure Coefficient at control points of the panels

import numpy as np
import math as math
```

Then we define the known parameters.
In our case, the Angle of Attack(AoA) is irrelevent since a cylinder cross section as infinite symmetries

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```
R = 1                                   # Radius of the Cylinder
Vinf = 1                                # Freestream velocity
AoA  = 0                                # Angle of attack [deg]
numP = 12                               # Number of Panels (control points)
AoAR = np.radians(AoA)                  # Convert AoA to radians [rad]
```

To Compute the values of panel source strengths λ_i , We need to calculate the integrals $I_{i,j}$ and $J_{i,j}$. These are dependent only on the geometry of the object in the flow field.

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```
# %% CREATE CIRCLE BOUNDARY POINTS

# Angles used to compute boundary points (for n panels, we need n+1 boundary points)
# Create angles for computing boundary point locations [deg]
theta = np.linspace(0,360,num = numP+1)
theta = np.radians(theta)                           # Convert from degrees to radians [rad]
```

Computed boundary point coordinates are returned in *anticlockwise* direction
This orientation causes the normal to the surface to point inward (inside the body)
Since we dont want that, we reverse the co-ordinated to get panels in *clockwise* orientation, with outward normal

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```
# Reversing boundary points to get an outward normal.
XB_aclck = R*np.cos(theta)
YB_aclck = R*np.sin(theta)

XB = XB_aclck[::-1]
YB = YB_aclck[::-1]
```

We then find the other Geomtrical Characteristic Features of the Panel Geometry

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```
# %% PANEL METHOD GEOMETRY

# Initialize variables
XC  = np.zeros(numP)                # Initialize control point coordinates arrays
YC  = np.zeros(numP)
S   = np.zeros(numP)                # Initialize panel length array
phi = np.zeros(numP)                # Initialize panel orientation angle array

# Find geometric quantities of the airfoil
for i in range(numP):                           # Loop over all panels
    XC[i]   = 0.5*(XB[i]+XB[i+1])               # values of control point
    YC[i]   = 0.5*(YB[i]+YB[i+1])
    dx      = XB[i+1]-XB[i]                     # Change between boundary points
    dy      = YB[i+1]-YB[i]
    S[i]    = (dx**2 + dy**2)**0.5              # Length of the panel
    phi[i]  = math.atan2(dy,dx)                 # Angle of panel
    if (phi[i] < 0):                            # Make all panel angles positive [rad]
        phi[i] = phi[i] + 2*np.pi
```

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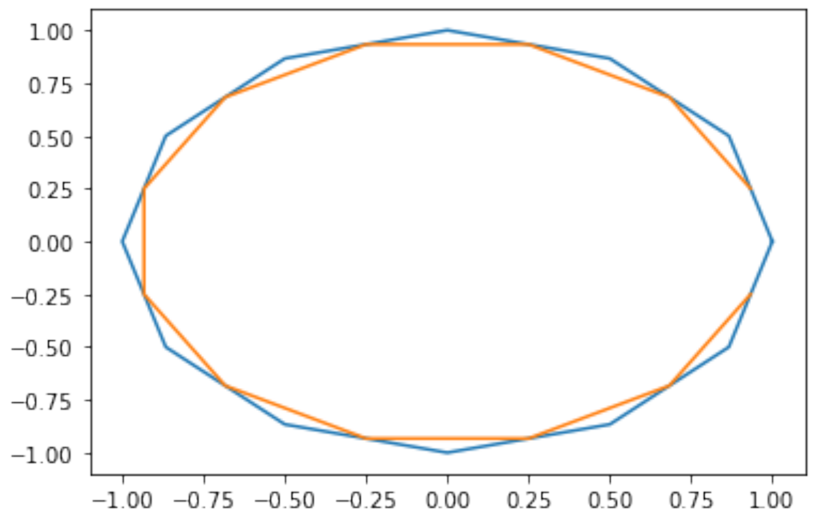
```
# To check the geometry created of the object in the flow, we plot it
import matplotlib.pyplot as plt
plt.plot(XB,YB)   #plots boundary points
plt.plot(XC,YC)   #plots control points
```

Out[130… [<matplotlib.lines.Line2D at 0x7fba8b866390>]



Now that we have the object parameters, calculating the integrals $I_{i,j}$ and $J_{i,j}$.

This section of the code is inspired from the solved example in the book :

$$Fundamentals\ of\ Aerodynamics - John\ D\ Anderson$$

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```
# Initialize arrays
I = np.zeros([numP,numP])          # Initialize I integral matrix
J = np.zeros([numP,numP])          # Initialize J integral matrix

# Computing integral by looping over panels
for i in range(numP):
    for j in range(numP):
        if (j != i):
            # Compute intermediate values
            A  = -(XC[i]-XB[j])*np.cos(phi[j])-(YC[i]-YB[j])*np.sin(phi[j])      # A term
            B  = (XC[i]-XB[j])**2 + (YC[i]-YB[j])**2                              # B term
            C = np.sin(phi[i]-phi[j])                                             # C term
            D = (YC[i]-YB[j])*np.cos(phi[i])-(XC[i]-XB[j])*np.sin(phi[i])        # D term
            E  = np.sqrt(B-A**2)                                                  # E term

            # Zero out any problem values i.e If E term is 0 or complex or a NAN or an INF
            if (E == 0 or np.iscomplex(E) or np.isnan(E) or np.isinf(E)):
                 I[i,j] = 0                                                       # Set I value equal to zero
                 J[i,j] = 0                                                       # Set J value equal to zero
            else:
               # Compute I (needed for normal velocity), Ref [1]
                term1  = 0.5*C*np.log((S[j]**2 + 2*A*S[j] + B)/B)                 # First term in I equation
                term2  = ((D-A*C)/E)*(math.atan2((S[j]+A),E)-math.atan2(A,E))     # Second term in I equation
                I[i,j] = term1 + term2                                            # Compute I integral

                # Compute J (needed for tangential velocity), Ref [2]
                term1  = ((D-A*C)/(2*E))*np.log((S[j]**2 + 2*A*S[j] + B)/B)      # First term in J equation
                term2  = ((-1)*C)*(math.atan2((S[j]+A),E)-math.atan2(A,E))        # Second term in J equation
                J[i,j] = term1 + term2                                            # Compute J integral

        # Zero out any problem values i.e if term is NON or INF, setting them to zero
        if (np.iscomplex(I[i,j]) or np.isnan(I[i,j]) or np.isinf(I[i,j])):
            I[i,j] = 0
        if (np.iscomplex(J[i,j]) or np.isnan(J[i,j]) or np.isinf(J[i,j])):
            J[i,j] = 0

  #These are values of I and J.
```

$$Normal\ Flow$$

From the Boundary Condition that no flow is across the panel, we get the equation :

$$V_{panels,n} + V_{\infty,n} = 0$$

which gives

$$\sum_{j=1,j\neq i}^{n} \frac{\lambda_j}{2\pi} \int_j \frac{\partial(ln(r_{ij}))}{\partial n_i} ds_j \ + \ \frac{\lambda_i}{2} \ + \ V_\infty cos(\beta_i) = 0$$

substituting the integral,

$$I_{ij} \ = \ \frac{\lambda_j}{2\pi} \int_j \frac{\partial(ln(r_{ij}))}{\partial n_i} ds_j$$

we get a system of n equations (corresponding to n panels) with n variables (λ_i) :

$$\sum_{j=1,j\neq i}^{n} \frac{\lambda_j}{2\pi} I_{ij} \ + \ \frac{\lambda_i}{2} \ + \ V_\infty cos(\beta_i) = 0$$

To solve these simultaneously, we use matrices.

$$\begin{bmatrix} \frac{1}{2} & \frac{I_{12}}{2\pi} & \frac{I_{13}}{2\pi} & .. & \frac{I_{1n}}{2\pi} \\ \frac{I_{12}}{2\pi} & \frac{1}{2} & \frac{I_{13}}{2\pi} & .. & .. \\ \frac{I_{13}}{2\pi} & \frac{I_{13}}{2\pi} & \frac{1}{2} & .. & .. \\ ... & ... & ... & .. & .. \\ .. & ... & ... & .. & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ .. \\ .. \\ .. \\ \lambda_n \end{bmatrix} = (-V_{\infty,n}) \begin{bmatrix} cos(\beta_1) \\ cos(\beta_1) \\ .. \\ .. \\ .. \\ cos(\beta_n) \end{bmatrix}$$

which we solve to get the values of strength (λ_i)

$$Tangential\ Flow$$

With the strength of the sources found, we can find the tangential velocity over the panels

$$V_i = V_{panels,s} + V_{\infty,s}$$

$$V_i = \sum_{j=1,j\neq i}^{n} \frac{\lambda_j}{2\pi} \int_j \frac{\partial(ln(r_{ij}))}{\partial s_i} ds_j \ + \ V_\infty sin(\beta_i)$$

the λ_i term drops since the self-contribution of the panel is zero in the tangential direction.substituting the integral,

$$J_{ij} \ = \ \int_j \frac{\partial(ln(r_{ij}))}{\partial s_i} ds_j$$

We solve this using matrix multiplication to get V_i

$$\begin{bmatrix} 0 & \frac{J_{12}}{2\pi} & \frac{J_{13}}{2\pi} & .. & \frac{J_{1n}}{2\pi} \\ \frac{J_{12}}{2\pi} & 0 & \frac{J_{13}}{2\pi} & .. & .. \\ \frac{J_{13}}{2\pi} & \frac{J_{13}}{2\pi} & 0 & .. & .. \\ ... & ... & ... & .. & .. \\ .. & ... & ... & .. & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ .. \\ .. \\ .. \\ \lambda_n \end{bmatrix} + V_\infty \begin{bmatrix} sin(\beta_1) \\ sin(\beta_1) \\ .. \\ .. \\ .. \\ sin(\beta_n) \end{bmatrix}$$

We can eliminate β from the equation, using the relations we obtain from the geometry of the panels

$$\beta = \phi + \frac{\pi}{2}$$

and hence

$$cos(\beta) = -sin(\phi)$$

$$sin(\beta) = cos(\phi)$$

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```
# Using the above calculated values of I[i,j] and J[i,j],
# we form a matrix equation of the system of n equations and n variables (n=numP)
mat_I = np.zeros([numP,numP])
mat_J = np.zeros([numP,numP])

for i in range(numP):
    for j in range(numP):
        mat_J[i,j] = (J[i,j])/(2*math.pi)
        mat_I[i,j] = (I[i,j])/(2*math.pi)
        if(i==j):
            mat_I[i,j]=0.5
            mat_J[i,j]=0

#and the other matrix in the equation
b  = np.zeros(numP)                # Initialize control point X-coordinate

for i in range(numP):
    b[i] = Vinf*math.sin(phi[i])
    # to remove precision error
    if(-math.pow(10,-15) < b[i] < math.pow(10,-15)):
        b[i] = 0
```

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```
# Solving for strength

# strength = np.linalg.inv(mat_I).dot(b)
strength = np.linalg.solve(mat_I, b)

sum=0
for i in range(numP):
    sum = sum+strength[i]
    # to remove precision error
    if(-math.pow(10,-15) < strength[i] < math.pow(10,-15)):
        strength[i]=0

# to remove precision error
if(-math.pow(10,-15) < sum < math.pow(10,-15)):
    sum =0

#for equal area panels, sum of strenghths = 0 . If not, solution is wrong
```

In [134…

```
# Solving for Vi

Vs   = np.zeros(numP)                # Initialize contribution of freestream velocity
V   = np.zeros(numP)                 # Initialize velocity over control points
Cp   = np.zeros(numP)                # Initialize pressure coefficients over control points


for i in range(numP):
    Vs[i] = Vinf*math.cos(phi[i])

V = np.matmul(mat_J,strength) + Vs
```

We obtain the Pressure Coefficient over the panels using the following formula:

$$C_{p,i} = 1 - (\frac{V_i}{V_\infty})^2$$

In [135…

```
# Calculating Pressure Coefficient
for i in range(numP):
    Cp[i] = 1 - math.pow((V[i]/Vinf),2)


angle_control_point = np.zeros(numP)                         # Initialize panel orientation angle array
for i in range(numP):
    angle_control_point[i]  = math.atan2(YC[i],XC[i])          # Angle of control panel
plt.scatter(angle_control_point,Cp)

# Plotting the Analytically obtained Function for reference
psi = np.linspace(-math.pi,1*math.pi,10000)
function  = np.zeros(10000)                                  # Initialize the Analytical function values

for i in range(10000):
    function[i] = 1 - 4*math.pow(math.sin(psi[i]),2)

plt.plot(psi,function)
```

Out[135… [<matplotlib.lines.Line2D at 0x7fba8b4fa050>]