EE6417 : Advanced Topics in Control Systems

Jan-May, 2023

Coding Assignment

Instructions :

- **Submit on or before 11:59 PM, 04/02/2023**
- You have to turn in the well-documented code along with a detailed report of the results of the simulations. Any code that is required, must be in a separate file(which can be directly run in MATLAB), and not a part of the report.
- Include any plots/images you deem necessary
- Your submission must be named "RollNo.pdf". For example, if your roll number is EE21D405, your submission must have the name "EE21D405.pdf".
- Your submissions must be made on moodle. Any emailed submissions will not be accepted.
- It is required that you use LaTeX for writing your report.

1. **Averaging in Sensor Networks:** Consider the averaging algorithm in sensor networks:

$$x_i(k+1) = average(x_i(k), \{x_j(k), \forall j \in \mathcal{N}(i)\})$$

where $\mathcal{N}(i)$ is the set of nodes neighboring(connected) to the i^{th} node. Write a MATLAB code(or in any preferred language) to simulate the averaging dynamics. For the following network of sensors(given in Fig.1), plot states vs time and comment on the value the sensors converge to as $k \to \infty$. Verify whether sensors converge to the average of the initial values.

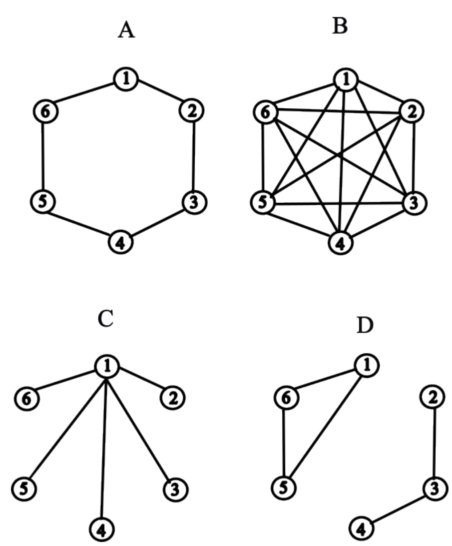

Figure 1: graph A: Cycle graph, graph B: Complete Graph, graph C: Star graph, Graph D: Disconnected graph

2. **Robots in cyclic Pursuit:** Consider an 'n-bug' system of robots restricted to move on a circle(as shown in Fig. 1.8 of the textbook). The dynamics of the system is given by:

$$\theta_i(k+1) = \mathrm{mod}(\theta_i(k) + u_i(k), 2\pi)$$

   (a) Let $u_i(k) = K\mathrm{dist_{cc}}(\theta_\mathrm{i}(\mathrm{k}), \theta_{\mathrm{i}+1}(\mathrm{k}))$, where $\mathrm{dist}_{cc}(\theta_i, \theta_{i+1}) = \mathrm{mod}(\theta_{i+1} - \theta_i, 2\pi)$ is the counter-clockwise distance from θ_i to θ_{i+1}. $K \in [0, 1]$ is the control gain. Vary the value of K and report your observations on how the 'Equilibrium' point changes. Support your answer with the help of relevant plots.

   (b) Set $u_i(k) = K\mathrm{dist_{cc}}(\theta_\mathrm{i}(\mathrm{k}), \theta_{\mathrm{i}+1}(\mathrm{k})) - K\mathrm{dist_c}(\theta_\mathrm{i}(\mathrm{k}), \theta_{\mathrm{i}-1}(\mathrm{k}))$ and repeat the process. Here $\mathrm{dist_c}(\theta_\mathrm{i}(\mathrm{k}), \theta_{\mathrm{i}-1}(\mathrm{k}))$ is the clockwise distance between the i^{th} and the $(i-1)^{th}$ bug.

   Take n=5.

3. **Kuramoto Oscillator Networks:** Consider the Kuramoto coupled-oscillator dynamics:

$$\dot{\theta}_i = \omega_i - \sum_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)$$

   where $\theta_i \in [0, 2\pi)$ is the phase angle, $\omega_i \in \mathbb{R}^+$ is the natural frequency of the i^{th} oscillator. $A = [a_{ij}]$ denote a real symmetric adjacency matrix, i.e.

$$a_{ij} \neq 0, \text{ if i is connected to j}$$
$$= 0, \text{ otherwise}$$

   (Note: if i is connected to j, then j is also connected to i, hence the symmetry.). Write a code(in MATLAB or any other programming language), to simulate the dynamics.

   (a) For A=$\mathbb{O}_{5\times5}$ (the 5×5 matrix of zeros), $\omega_i = i$ for $i = 1, 2, ...$ 5. Take arbitrary initial conditions.

   (b) Coupled as shown in Fig.2, with $\omega_i = 5$, $\forall\, i$. take initial conditions as $\{0, \frac{2\pi}{5} + \frac{1}{2}, \frac{4\pi}{5}, \frac{6\pi}{5} + \frac{1}{10}, \frac{8\pi}{5}\}$.

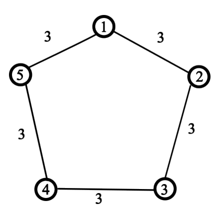

Figure 2: Cycle graph, with edge weights marked on the edges

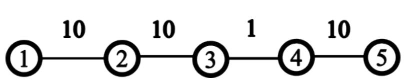

Figure 3: Line graph

(c) Coupled as shown in Fig. 3, $\omega = \{1, 1, 1, 2.5, 2.5\}$. Take arbitrary initial conditions.

Plot the states and report your observations.(Animations of how the states evolve on a circle are welcome too.)