

Class 02, July 10, 2021

Submission - Class 01

- .py, .m, .tex, .pdf
- No late submissions
- Due now

This is shown



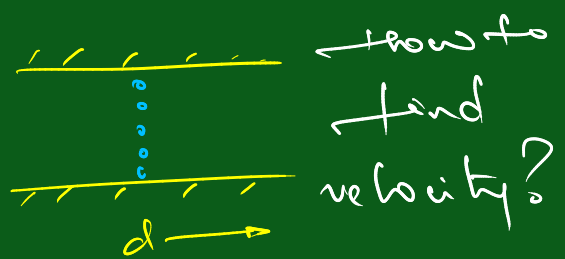
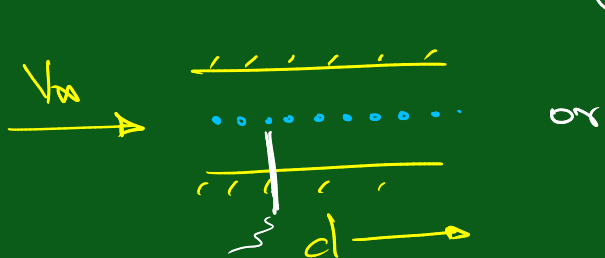
Fig. 10
Table 4

100 m/s

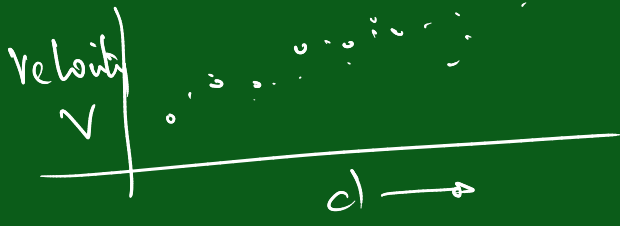
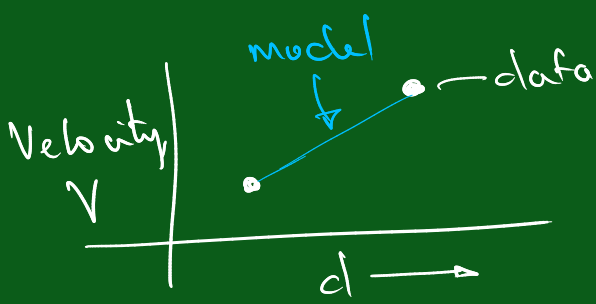
$$\begin{aligned} & \sqrt[2]{x} \\ & x^{n+2} \quad x \times x \\ & x^3 = x \times x \times x \end{aligned}$$

$$x/4 \rightarrow 0.25 \times x$$

Let us assume we are doing an experiment



- hot-wire anemometer
 - ↳ Thermal transducer
 - ↳ Single point
 - ↳ Velocity - resistance - voltage
- pitot static tube
- Venturimeter
- PIV



Eqn
↓
Tweak
↓
Least error

- linear eqn
- polynomial
- exponential

Let us assume

$$V = md + c$$

↑
to be estimated

$$V_1 = md_1 + c$$

$$V_2 = md_2 + c$$

N observations

$$V_N = md_N + c$$

$$\begin{bmatrix} d_1 & 1 \\ d_2 & 1 \\ \vdots & \vdots \\ d_N & 1 \end{bmatrix}_{N \times 2} \begin{Bmatrix} m \\ c \end{Bmatrix}_{2 \times 1} = \begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix}_{N \times 1}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}$$

$$A x = b$$

* What does $Ax=b$ mean in this context?

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{n1} & a_{n2} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}$$

$$\begin{Bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{Bmatrix} x_1 + \begin{Bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{n2} \end{Bmatrix} x_2 = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}$$

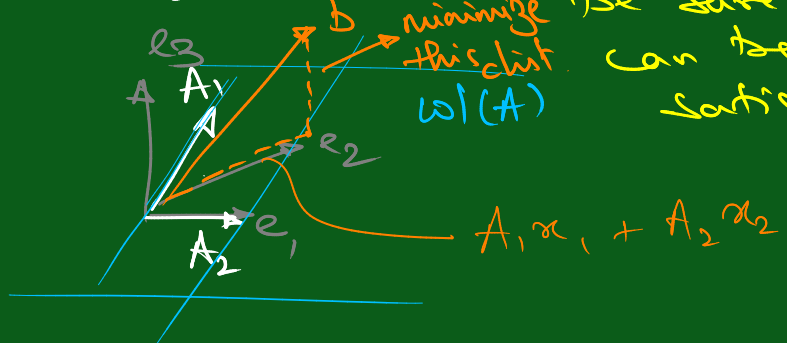
A_1 A_2

* b is a linear combination of the columns of A !

* Example.

$$\begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix} x_1 + \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} x_2 \approx \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

how can you be sure this can be satisfied?



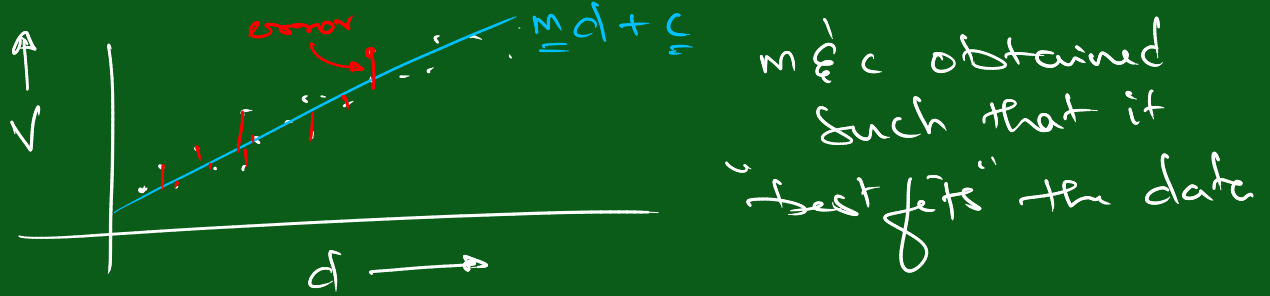
$$A_{n \times 2} x_{2 \times 1} = b_{n \times 1}$$

$$(A^T A) x = (A^T b)$$

$$x = (A^T A)^{-1} A^T b$$

↑
solution for x !

how can you be sure this matrix is invertible?



Linear Regression

$$Ax = b$$

$$\text{find } \hat{x} = \arg \min_x \|b - Ax\|$$

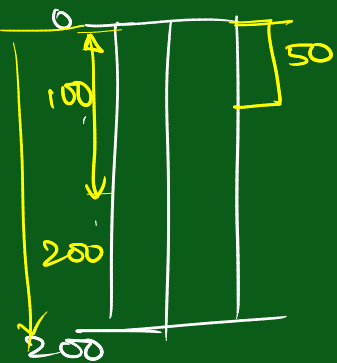
$$Ax - b = \epsilon \quad \text{minimize this term}$$

$$\hat{x} = (A^T A)^{-1} A^T b$$

$$v = m d^c \rightarrow \log v = \log m + c \log d$$

$$v = m d + c + k d^A \rightarrow \text{valid?}$$

* Task : data/scripts - class 02 - data1.txt
 200(x,y) 2.txt 3.txt



- Perform linear regression using $y = mx + c$
- Using first 50, 100, 200 points

Data	m	c
50		
100		
200		

D2		

D3		

files to be submitted

• m
• c
• pdf

- Based on the behaviour of m & c , infer the type of data.

- Do you think a better model would work instead of $y = mx + c$? What? Why?

- Plot the data a scatter plot & a line for the model. Support + previous inference

- Use octave
- Invert matrix using Gaussian-Elimination

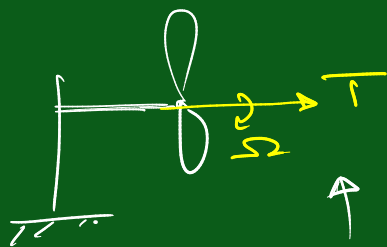
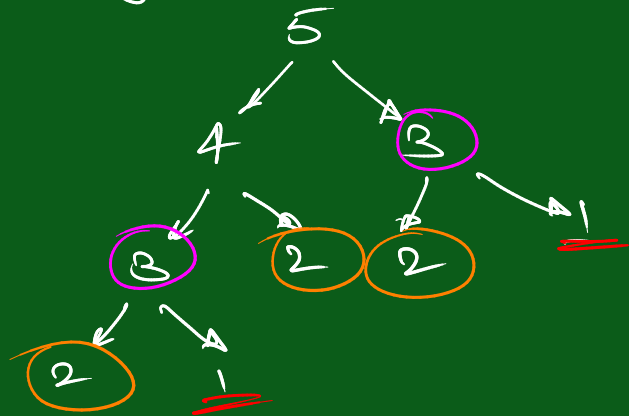
do not use $\text{inv}(A)$ or $A \setminus b$!

Class 03, July 17 2021

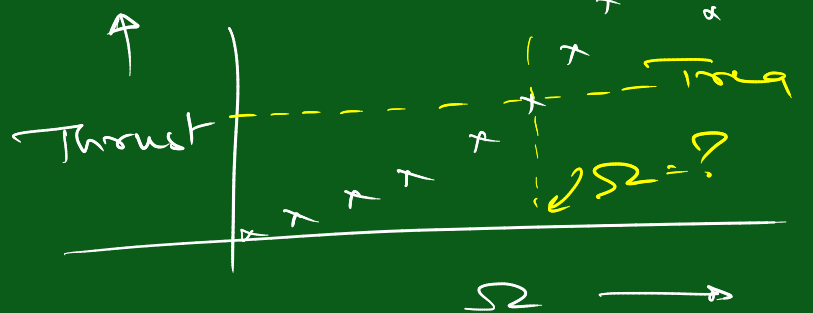
- * Demo on Python
- * Root Finding Algorithms

Submission - Class 02

- .m, .tex, .pdf
- Due now.



What is the Ω so that $T = T_{req}$?

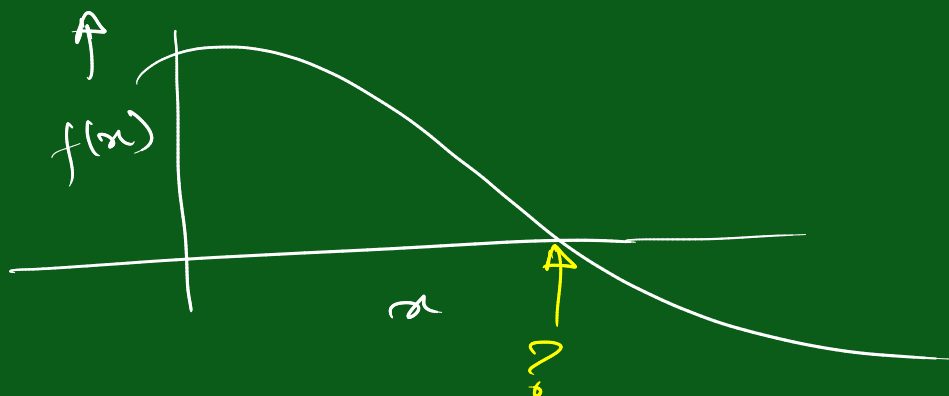


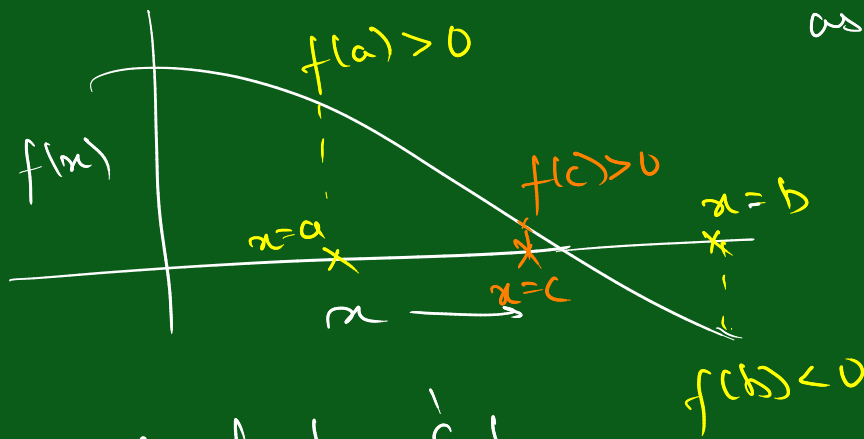
$$T(\Omega) = T_{req}$$

$$F(\Omega) = T(\Omega) - T_{req} = 0$$

- * In general we are interested in solving for $f(x) = 0$
 ↳ scalar or vector

- * Look at three methods — solving them numerically
 1. Bisection method.





assume $f(x)$ continuous

$$x=a, f(a) \geq 0$$

$$x=b, f(b) < 0$$

At least one root

$$x=c, c \in (a, b)$$

Step 1: find a & b

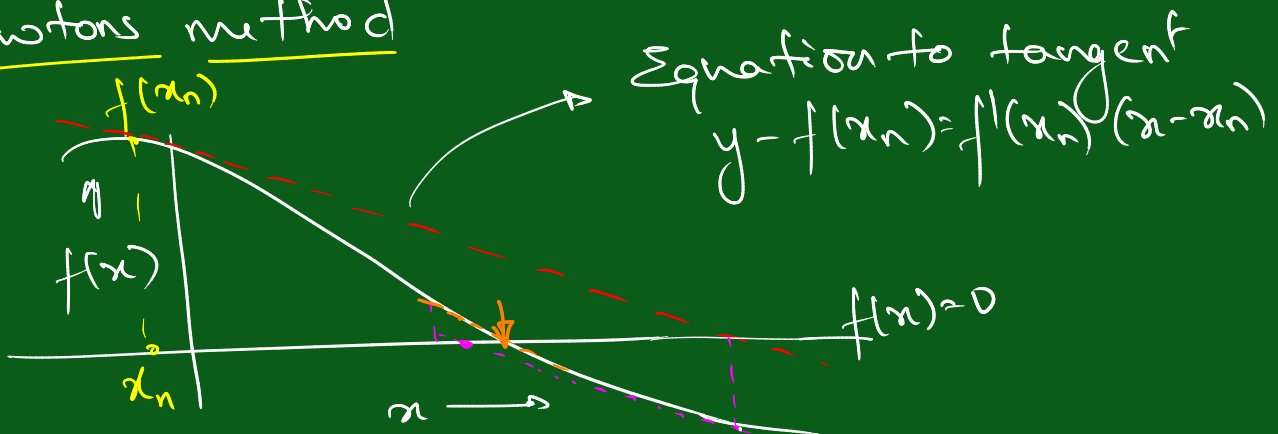
Step 2: $c = \frac{a+b}{2}$
 if $f(c) > 0$ then $a = c$
 if $f(c) < 0$ then $b = c$
 repeat till $f(c) = 0$

- limited precision

- we can play with $|f(c)| < \epsilon \sim 10^{-6}$

def bisection($f, a, b, \epsilon, N_{max}$)
 : function
 : precision
 : max step count

2. Newton's method



- we want to find $y=0$

$$0 - f(x_n) = f'(x_n)(x_{n+1} - x_n)$$

unbounded $\leftarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

can be 0 $\leftarrow f'(x_n)$

also need derivative information

Start: x_0
 x_1
 x_2
 \vdots

$$|f(x)| < \epsilon$$

3. Secant Method \rightarrow Everything in Python

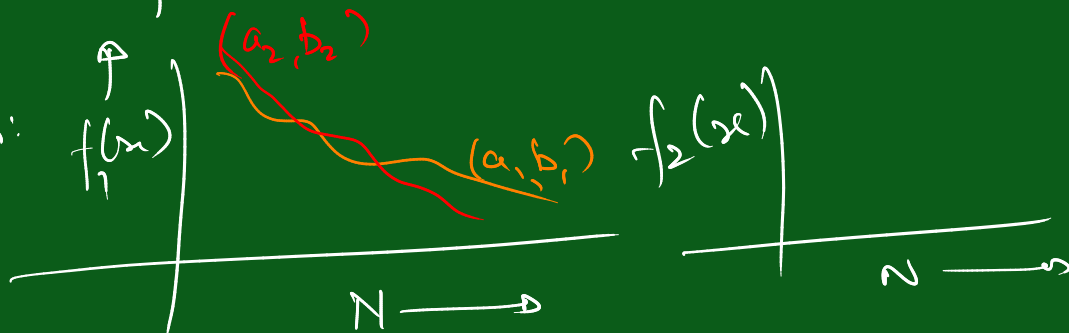
Report: Introduction (theory section with figs & equations)
Result & Analysis
 α Conclusion
 α References
must have a caption!

Choose two unique starting sets

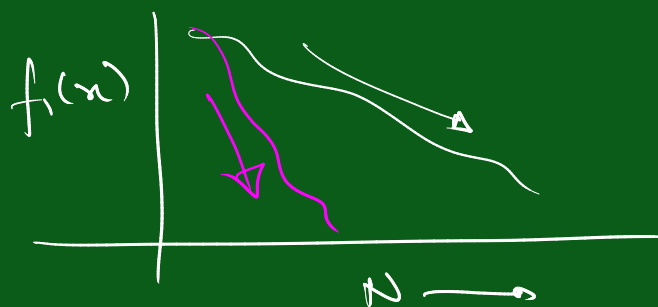
$$f_1(x) = x^3 - 3x^2 - x + 9$$

$$f_2(x) = e^x f_1(x)$$

Bisection:
Newton
Secant



$f_3(x) = x^3 - 2x + 2$
Start at $x = 0$



Class 04: July 24, 2021

Submission: Class-03

• Due now
 α .py, .tex, .pdf
multiple?

Quadrature

- \rightarrow historically - determine area
- \rightarrow Express solution in terms of integrals
- \rightarrow numerical analysis: approximation of definite integral of a function

$$I = \int_a^b f(x) dx \approx \sum_i f(x_i) w_i$$

\rightarrow weight given to the function at x_i

\rightarrow function value at x_i

Summation over finite pieces

- Used in many places - forming basic function for integration

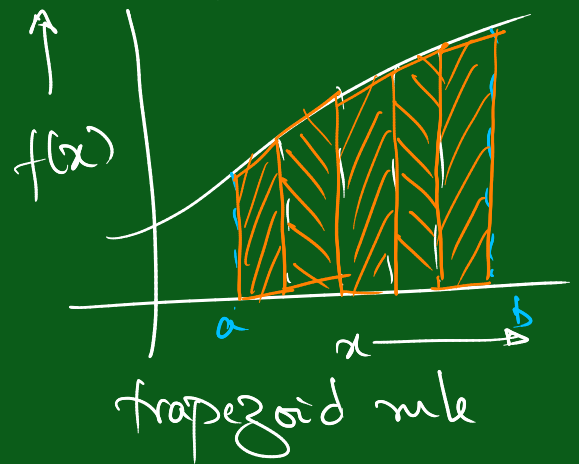
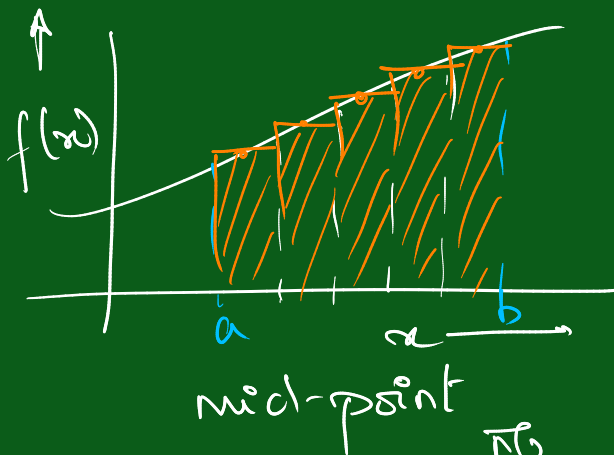
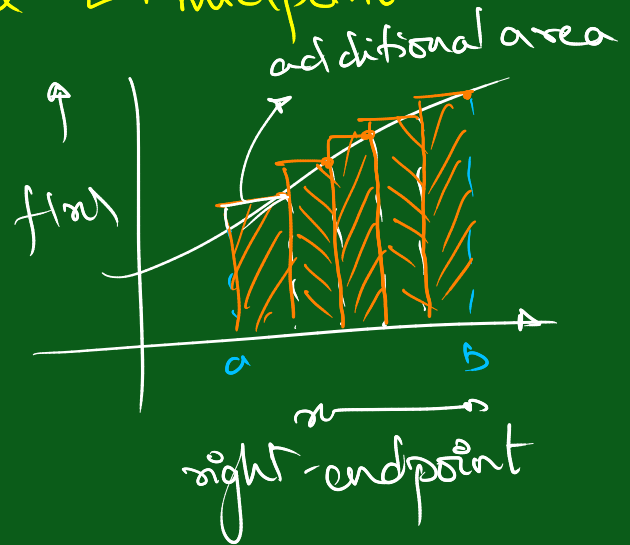
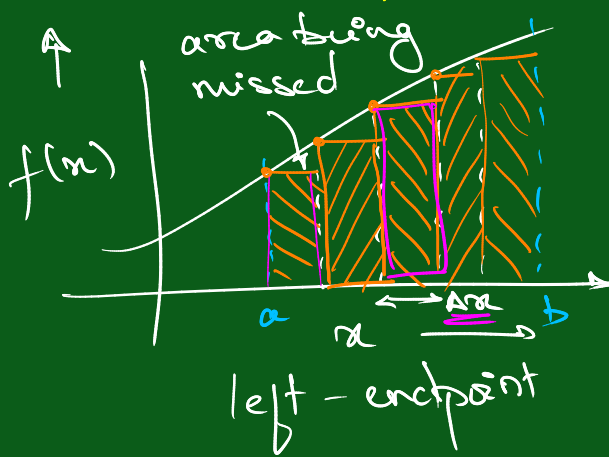
- FEM

- CFD schemes

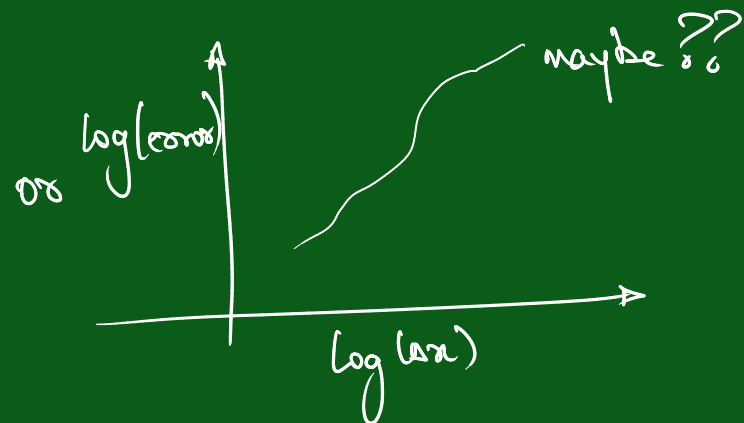
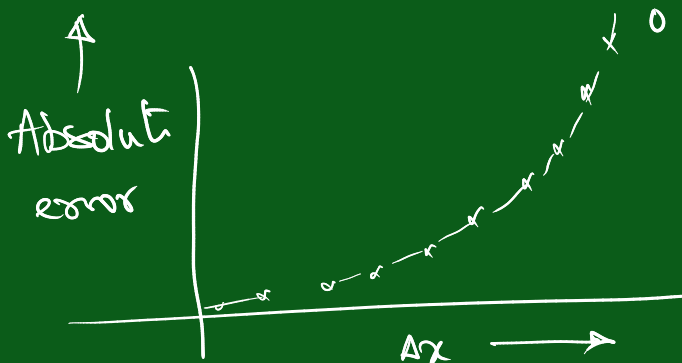
$$I = \int_a^b f(x) dx$$

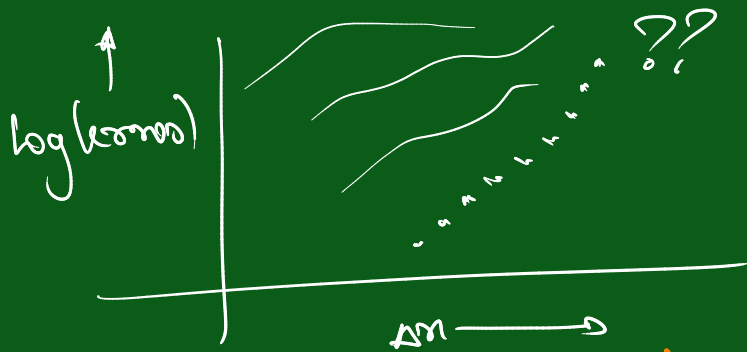
→ Simple integration

Rectangular rule → left endpoint
 → right endpoint
 Trapezoidal rule → midpoint



- Let us assume $I = \int_0^{\pi/2} \sin(x) dx$ → Exact value known

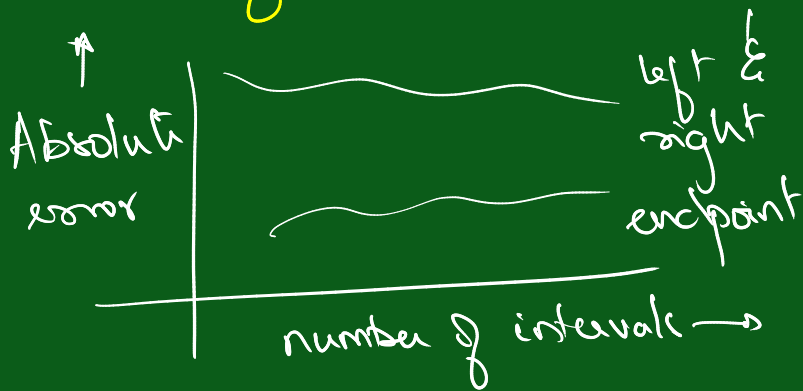




→ Fit an appropriate curve using linear regression

→ Identify the "order of accuracy" of each method.
 what does it mean?

→ Comment on the accuracy of the four methods investigated



→ Conclusion?
 lines up with any known mathematical result?

- Python, try to use functions