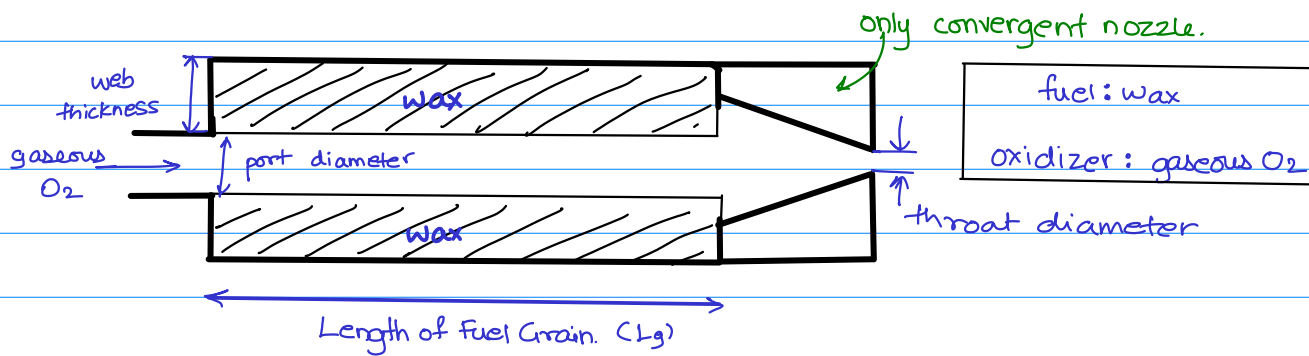


02



Data

$$\dot{r} = 0.116 G_{ox}^{0.62} \quad (\dot{r} \text{ in mm/s and } G_{ox} \text{ in kg/m}^2\text{s})$$

$$\dot{m}_o = 0.030 \text{ kg/s}$$

$$\rho_f = 900 \text{ kg/m}^3$$

$$L_g = 0.134 \text{ m}$$

$$\text{initial port diameter} = 0.009 \text{ m}$$

$$\text{web thickness} = 0.022 \text{ m}$$

$$\text{throat diameter} = 0.008 \text{ m}$$

$$\text{chamber Temp } (T_c) = 3400 \text{ K}$$

burnt gases

$$\rightarrow M = 22$$

$$\rightarrow \gamma = 1.2$$

Relations

$$\text{regression rate} \quad \dot{r} = 0.116 G_{ox}^{0.62} \quad \text{_____ (1)}$$

$$\text{oxygen mass flow rate} \quad \checkmark \dot{m}_o = A_p \cdot G_{ox} \quad \text{_____ (2)}$$

$$\text{Average port area} \quad A_p = \frac{\pi}{4} \left(\frac{\checkmark d_i + d_f}{2} \right)^2 \quad \text{_____ (3)}$$

$$\text{fuel mass flow rate} \quad \dot{m}_f = A_b \cdot \rho_f \cdot \dot{r} \quad \text{_____ (4)}$$

A_b : combustion port surface area. (taking equal to the average port area).

$$\text{conservation of mass.} \quad d_f^2 = \frac{4(\Delta m)}{\pi \rho_f L_g} + \checkmark d_i^2 \quad \text{_____ (5)}$$

To solve the given question we first need to find \dot{m}_f at different times.

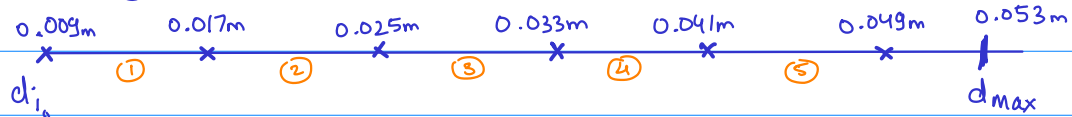
we have to take 5 such intervals where we calculate it.

- having decided the diameter intervals, we use (3) to find A_p ,
- using \dot{m}_o and A_p , we calculate G_{ox} using (2)
- from G_{ox} we find \dot{r} , using (1)
- from \dot{r} we find \dot{m}_f in that time interval, using (4)

$$d_{i_0} = 0.009 \text{ m}$$

$$\begin{aligned} d_{\max} &= d_{i_0} + (\text{web thickness}) * 2 \\ &= 0.009 \text{ m} + 0.044 \\ &= 0.053 \text{ m.} \end{aligned}$$

*To carry out calculations at 5 evenly spaced intervals, we will take them at the following intervals.



$$\dot{m}_0 = 0.030 \text{ kg/s}$$

Following the above mentioned steps, we have.... $O/F = \dot{m}_0 / \dot{m}_f$

Int.	initial diameter (in m)	final diameter (in m)	Avg. port area (in m^2)	C_{ox} (in $\text{kg/m}^2\text{s}$)	\dot{r} (in mm/s)	\dot{m}_f (in kg/s)	O/F
1	0.009	0.017	13.273 e-5	226.01885	3.342 e-3	0.39925 e-3	75.141
2	0.017	0.025	34.636 e-5	86.615	1.844 e-3	0.57483 e-3	52.1897
3	0.025	0.033	66.052 e-5	45.41877	1.236 e-3	0.73464 e-3	40.8365
4	0.033	0.041	107.521 e-5	27.90152	0.9136 e-3	0.88406 e-3	33.93424
5	0.041	0.049	159.043 e-5	18.8628	0.7167 e-3	1.02587 e-3	29.24354

from initial and final diameter, we can even find Δm , using (5)

Int.	initial diameter (in m)	final diameter (in m)	Δm (in kg)
1	0.009	0.017	0.0197
2	0.017	0.025	0.0318
3	0.025	0.033	0.04395
4	0.033	0.041	0.05607
5	0.041	0.049	0.06819

$$\dot{m} = \frac{\Delta m}{\Delta t}$$

$\therefore \Delta t$ for interval, found using $\Delta m / \dot{m}$

Int.	Δm (in kg)	\dot{m}_f (in kg/s)	Δt (in s)
1	0.0197	0.39925 e-3	49.3462
2	0.0318	0.57483 e-3	55.3659
3	0.04395	0.73464 e-3	59.82502
4	0.05607	0.88406 e-3	63.42723
5	0.06819	1.02587 e-3	66.47806

characteristic velocity (c^*)

$$c^* = \sqrt{\frac{\gamma R T}{M}}$$

$$\gamma \sqrt{\frac{2}{\gamma - 1}}$$

for our system

$$T = 3400K, \quad M = 22g/mole \quad \gamma = 1.2, \quad R = 8.314 \frac{J}{mol \cdot K}$$

for MW=22
or $\frac{P^*}{M V} = 377.9$

$$c^* = 327.222 \text{ m/s}$$

chamber pressure (P_c)

$$P_c = \frac{c^* (\dot{m}_f + \dot{m}_o)}{A_t} + P_b$$

back pressure
usually = ambient pressure

$$\dot{m}_o = 0.030 \text{ kg/s}$$

throat area

Int.	\dot{m}_f (in kg/s)	P_c (atm)
1	0.39925 e-3	2.9531
2	0.57483 e-3	2.9644
3	0.73464 e-3	2.9746
4	0.88406 e-3	2.9842
5	1.02587 e-3	2.9933

$$\text{throat diameter} = 0.008m$$

$$\text{throat area} = \frac{\pi d^2}{4}$$

$$= 50.265 e-6$$

$$P_b = 1 \text{ atm} = 101325 \text{ Pa}$$

we then calculate exit pressure using

$$\frac{A_e}{A_t} = \frac{\sqrt{\gamma} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}}{\left(\frac{P_e}{P_c} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{(\gamma - 1)} \left[1 - \left(\frac{P_e}{P_c} \right)^{\frac{\gamma - 1}{\gamma}} \right]}}$$

\therefore only converging nozzle used $\therefore A_e = A_t$

$$\left(\frac{P_e}{P_c}\right)^{1/r} \cdot \sqrt{\frac{2r}{(r-1)} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{r-1}{r}}\right]} = \sqrt{r} \cdot \left(\frac{2}{r+1}\right)^{\frac{r+1}{2(r-1)}}$$

taking $\left(\frac{P_e}{P_c}\right)^{1/r} = x$

$$x \sqrt{(12)(1 - x^{r-1})} = \sqrt{1.2} \left[\left(\frac{10}{11}\right)^{\frac{2.124}{2}} \cdot \frac{1}{0.2} \right]$$

$$x \sqrt{12(1 - x^{0.2})} = \sqrt{1.2} \cdot \left(\frac{10}{11}\right)^{1/2}$$

$$x^2 (1 - x^{0.2}) = 0.035049389$$

$$\therefore x = 0.6209$$

$$\left(\frac{P_e}{P_c}\right)^{1/r} = 0.6209$$

$$P_e = (0.564450668) P_c$$

Int.	ṁ _f (in kg/s)	P _c (atm)	P _e (atm)
1	0.39925 e-3	2.9531	1.66686
2	0.57483 e-3	2.9644	1.67323
3	0.73464 e-3	2.9746	1.67903
4	0.88406 e-3	2.9842	1.68445
5	1.02587 e-3	2.9933	1.68959

similarly.

Thrust coefficient

$$C_F = \underbrace{\sqrt{r} \left(\frac{2}{r+1}\right)^{\frac{r+1}{2(r-1)}}}_{0.64853117} \cdot \underbrace{\sqrt{\frac{2r}{r-1} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{r-1}{r}}\right]}}_{0.64853117} + \left(\frac{P_e}{P_c} + \frac{P_b}{P_c}\right) \frac{A_c}{A_t} \leftarrow 1$$

0.6209
↓
1atm

$$= 0.9647456$$

$$C_F = 0.6256676 + (0.6209 + \frac{1}{P_c})$$

$$* C_F = 1.2465676 + \frac{1}{P_c}$$

Int.	\dot{m}_f (in kg/s)	P_c (atm)	P_e (atm)	C_F
1	0.39925 e-3	2.9531	1.66686	1.5851976
2	0.57483 e-3	2.9644	1.67323	1.5839090
3	0.73464 e-3	2.9746	1.67903	1.5827446
4	0.88406 e-3	2.9842	1.68445	1.5816632
5	1.02587 e-3	2.9933	1.68959	1.5806432

and further

$$\text{Thrust (F)} = \overset{\substack{\text{m/s} \\ \downarrow}}{C^*} (\overset{\substack{\text{kg/s} \\ \downarrow}}{\dot{m}_f} + \overset{\substack{\text{unitless} \\ \downarrow}}{\dot{m}_o}) \cdot \overset{\substack{\text{unitless} \\ \downarrow}}{C_F} \dots \text{N}$$

Int.	\dot{m}_f (in kg/s)	P_c (atm)	P_e (atm)	C_F	Thrust (N)
1	0.39925 e-3	2.9531	1.66686	1.5851976	15.768446
2	0.57483 e-3	2.9644	1.67323	1.5839090	15.8466264
3	0.73464 e-3	2.9746	1.67903	1.5827446	15.917744
4	0.88406 e-3	2.9842	1.68445	1.5816632	15.984204
5	1.02587 e-3	2.9933	1.68959	1.5806432	16.0472413

Q2 Answer

Sl. no.	Time(s)	\dot{m}_f (kg/s)	O/F	P_c (atm)	Thrust(N)
1	49.3462	0.39925 e-3	75.141	1.66686	15.768446
2	104.7121	0.57483 e-3	52.1897	1.67323	15.84663
3	164.53712	0.73464 e-3	40.8365	1.67903	15.917744
4	227.96435	0.88406 e-3	33.93424	1.68445	15.984204
5	294.44241	1.02587 e-3	29.24354	1.68959	16.0472413

as fuel is used, the port diameter inc $\therefore A_p \uparrow$

$\therefore \dot{m}_o$ is constant, $C_{ox} \downarrow \therefore \dot{r} \downarrow$

$$\dot{m}_f = \underline{A_b} \uparrow P_f \cdot \underline{\dot{r}} \downarrow$$

with $n=0.5$ i.e. n less than 0.62,

then the correlation of $C_{ox} \downarrow$ and $\dot{r} \downarrow$ will be lesser
at $n=0$,

\dot{r} will be independent of any change in C_{ox}
and will keep on increasing as $A_b \uparrow$.