Determination of Shear Modulus of an Aluminium rod using Torsion Test

AS2100 : Basic Aerospace Engineering Lab

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Introduction to the Experiment

The purpose of torsion testing usually parallels that of uniaxial tension tests. From the experiment, the shear elastic modulus (G), shear proportional stress (τ_{p}), shear yield stress (τ_{y}), and the Stress-Strain behavior in general, can be obtained. However, in contrast to uniaxial tension tests, the stresses are not distributed uniformly over the cross section.

Torsion loading results in twisting of one section of a body with respect to a contiguous section. During the test the angle of twist ϕ and the applied torque T are measured as the test proceeds. For a circular cross-section, in the absence of the other loads, pure shear stress state exists at each point. Torsional elastic shear stresses vary linearly from zero at the axis of twist to a maximum at the extreme fibers. Thus, in a solid circular bar, when the surface fibers reach the yield shear stress they are, in a sense, supported by elastic interior fibers. Consequently, the elastic resistance of the remainder of the section masks the effect of yielding of the surface fibers during their early stage of yielding. Usually, it is not until considerable yielding has taken place that any noticeable effect of nonlinearity is apparent using a simple mechanical troptometer to measure the angle of twist ϕ

To test the material in torsion the proper test procedure is needed. It involves mounting a shaft into the testing machine, applying torque incrementally and measuring both the applied torque and the corresponding angle of twist. Using the appropriate formulae, relationships and the measured dimensions, we can determine the shear stress and shear strain on the shaft. Then, one can plot the torque vs. angle of twist, and shear stress vs. shear strain from which one can find the material properties previously mentioned.

Apparatus other than testing specimen:

1. Caliper
2. Troptometer
3. Torsion testing machine
4. Measuring tape

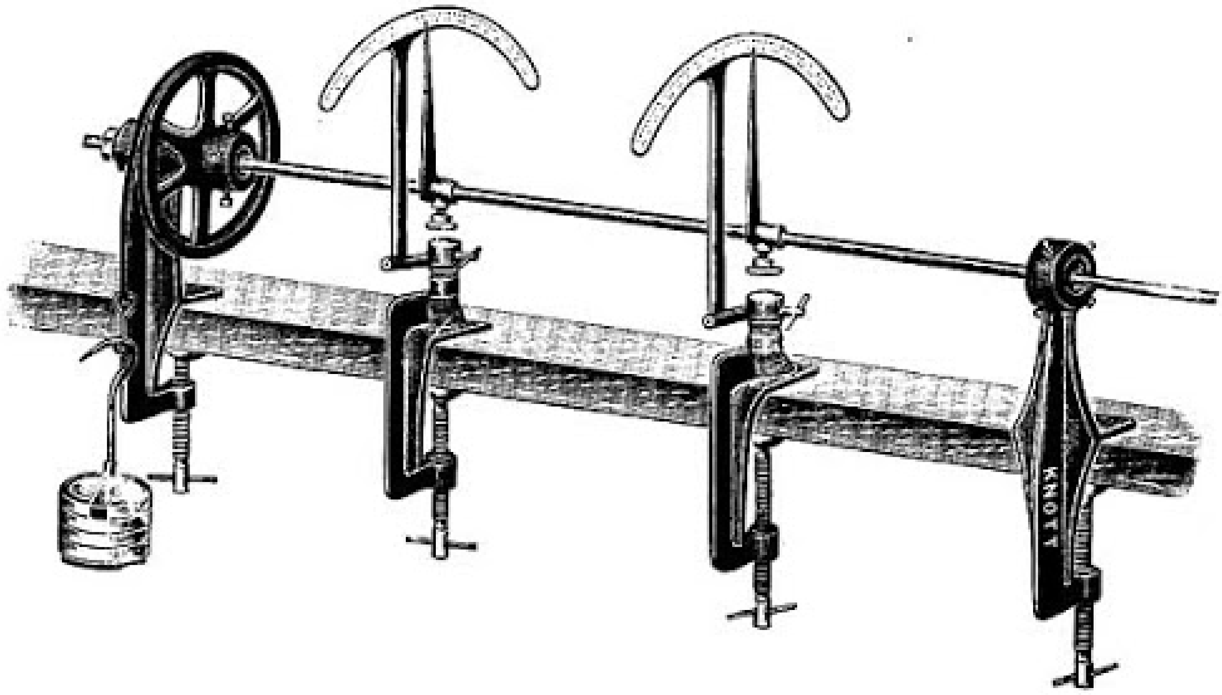
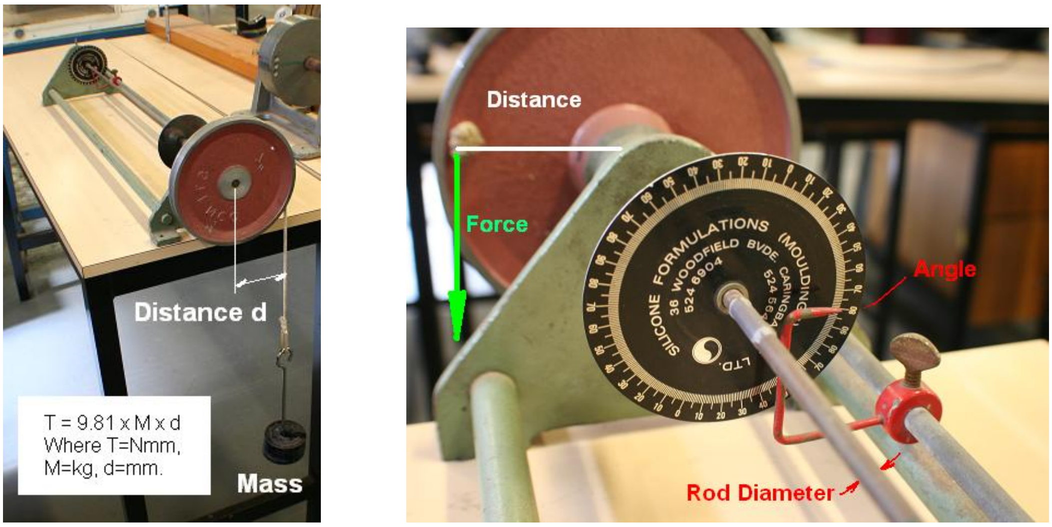


Figure 1: Torsion Apparatus

Figure 2: Torsion Apparatus Setup



(a) Lever

(b) Deflection Indicator

The assumptions made in this experiment include the following:

1. The torque is applied along the center of axis of the shaft.
2. The material is tested at steady state (absence of strain rate effects).
3. Plane sections remain plane after twisting (the circular section conforms to this condition).

Test Procedure:

1. Measure the diameter of the test specimen using the caliper (take an average of 5 measurements).
2. Slide the shaft into the Torsion Test Apparatus and tighten down the two fastening screws to the shaft.
3. Clean the clamps used to hold the shaft in place.
4. Insert the shaft into the right clamps. Tighten the clamp ensuring that the grip is very tight.
5. Adjust the Torsion Test Apparatus to the correct position pointing at 0 degree initially.
6. Measure the length between the Torsion Test Apparatus clamps Lt using the tape measure. This length is the effective length of the rod between which the Torsion takes place
7. Add weights to the lever gradually and note down the Apparatus reading of the deflection
8. After loading the lever,unload it by taking the masses off one by one and note down the Apparatus reading of the deflection.
9. Repeat this cycle 20 times to obtain sufficient data to find a good estimate of the Shear Modulus
10. Calculate the Stress and Strain with the recorded data and plot a Shear Stress vs Shear Strain Curve. **The slope of the Shear Stress vs Shear Strain curve gives the value of the Shear Modulus G**

Results :

Experimental Shear Modulus of Aluminium 6061 : 20.2659 ± 3.2629 GPa
Published Shear Modulus of Aluminium 6061 : 26 GPa

Discussion:

As we can see the Experimental value is less than the Published value. A part of this can be attributed to the fact that since we can not measure angles beyond a certain precision,we have taken a larger range of Strain to calculate the Shear Modulus.This means that the values that are obtained are not strictly confined to the proportionality limit and may exceed it.

Beyond the Proportionality Limit the Stress does not increase linearly with Strain and slope of Stress vs Strain graph decreases. Thus when we apply a Linear Fit, the slop of the Best Fit line is less than the slope of the Stress vs Strain graph in the Proportionality Limit. Due to this reason we obtain a value less than the Published value and not more than it.

Another observation we made was that the Precision of the Data given to us was higher than the Precision of the Torsion Apparatus.This suggests the use of a more precise apparatus than mentioned

Conclusion:

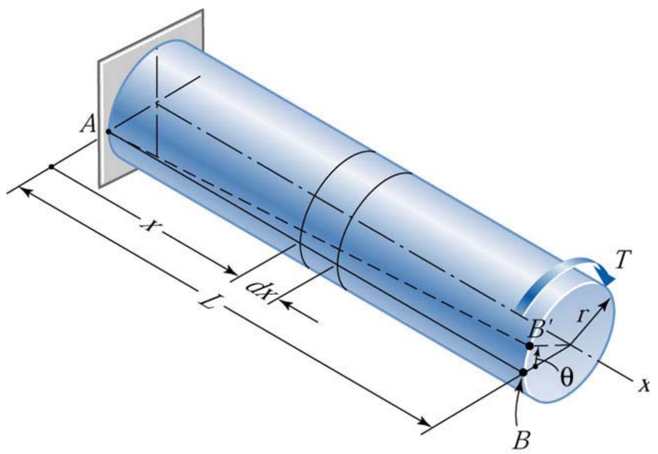
- We understood that Modulus of rigidity is the coefficient of the elasticity for a shearing force, it measure the stiffness of the particular materials in torsion test.
- Different type of material will have different elastic limit. The higher value of modulus of rigidity, G , the higher the torsion stiffness of material .
- If the material exceeds the elastic limit,permanent deformation will be occurring. On exceeding the proportionality limit the objective of this experiment will be defied.
- We learnt how Torsion Test can be used to calculate the Shear Modulus of a Material.
- Learning about the Errors that might persist during the Torsion Test and how to minimize them
- Using tools with limited technology and precision,the Torsion Test gives a decent esitmate of the Shear Modulus of a material

1.Torsion Test : Theory and Experimental Procedure

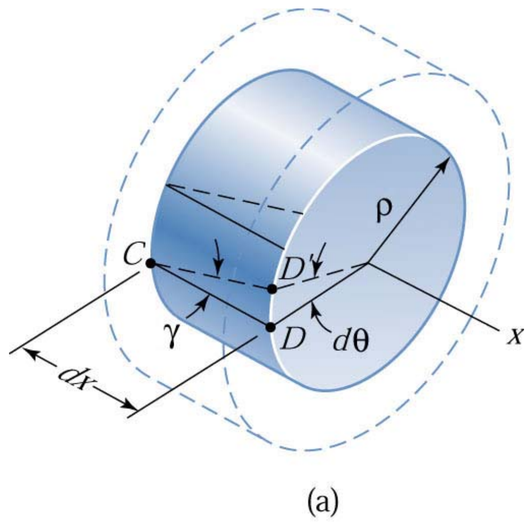
1 Theory of Pure Torsion

The Torsion Test is based on the **Theory of Pure Torsion**

If a material is subjected to twisting by the application of a couple a shear stress will be induced within the material. If a couple is applied to a cylindrical rod in such a way that the axis of the couple is coincident with the axis of the rod, then the rod is said to be subject to pure torsion. At any point within the cross-section of a rod subjected to pure torsion, there will be a pure shear stress established in a direction normal to the radius of the rod at that point.



(a) Circular beam subjected to torque T



(b) dx section

The assumptions made in the Theory of Pure Torsion or made in deriving the equation for pure torsion are as follows:

- The material is homogeneous and isotropic
- Hooke's law is obeyed by the material.i.e deformation are withing within proportionality limit **(Important)**
- The shaft is circular in section and remains planar even after torque is applied on it
- The cross-section of the shaft remains uniform throughout.
- The shaft is subjected to pure torque only.
- The shaft is not subjected to any initial torque. i.e initial deflection is zero
- The stress of the material should not exceed the elastic limit i.e no permanent deformation is caused due to loading

The torsion equation is given as follows:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

The given equation hold for the condition of **Pure Torque**.Every diameter of the material rotates through the same angle i.e The shear strain γ varies linearly in the radial direction

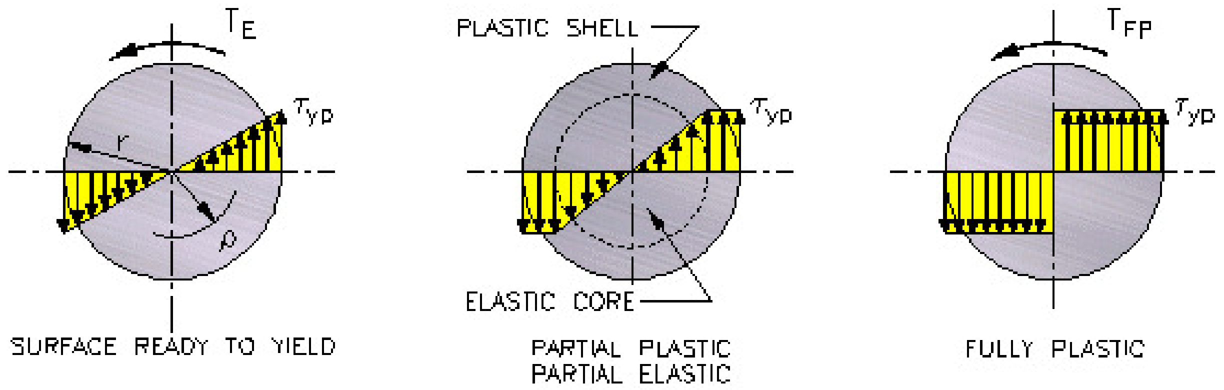


Figure 5 Torsion Stress Distributions

Figure 2: Types of Torques acting on a cross section:(a)Pure Torsion, (b) Partially Elastic Torsion, (c) Fully Plastic Torsion

Thus, continuing with the Pure Torsion Assumption,we find the Shear Modulus (G) through our experiment.

2 Procedure

1. Measure the length and diameter of the Aluminium 6061 Rod
2. Subtract the Zero Errors to obtain the true dimensions
3. Weigh the masses to be used for the experiment
4. Clamp the rod in the Torsion Test Apparatus
5. Check for any zero error present in the apparatus. If possible,set the reading to zero when no mass is attached
6. Gradually add weights and note down the reading of the deflection while loading
7. After having put all the weights,remove them one by one and note down the reading in deflection while unloading as well
8. Complete 20 such cycles of loading and unloading
9. For each reading calculate Shear Stress and Shear Strain using the formula:

$$ShearStress = \frac{Torque * RadialDistance}{J} = \frac{(mgd) * r}{J}$$

and

$$ShearStrain = \frac{Deflection(radian) * RadialDistance}{L} = \frac{\theta r}{L}$$

where,
m = mass attached to the apparatus
Radial Distance = from centroidal longitudinal axis to the outer surface
d = length of lever used to apply torque in Torsion Test Apparatus

10. Plot the Data points for each cycle
11. Sketch the best fit straight line through the points for each plot
12. Measure the Slope of the best fit line

Slope of the best fit line in a Stress vs Strain Plot gives the value of the Young's Modulus

Since we made a plot between Shear Stress and Shear Strain, the Slope of the best fit line gives us the value of the Shear Modulus (G)

2. Sources and minimization of errors in the experiment

1 Systematic Error

This type of error arises due to defect in the measuring device.It is repetitive in nature and can be removed after being calculated to get a precise value of the measurement.
1. The dimension of the specimen that has been measured may not accurate. Zero error may be occurred on the Vernier caliper or the Torsion Test Apparatus. It can be subtracted from the measured reading to obtain true value.
2. The blocks of mass used for the experiment might have some error.They should be checked and corrected before the experiment to minimize error due to them

2 Random Error

This type of error could occur due to sudden change in experimental conditions. The specimen could have exposed to undesired and unsuitable temperature and humidity. It is an accidental error and is beyond our control. Random errors, unlike systematic errors, can often be quantified by statistical analysis, therefore, the effects of random errors on the quantity or physical law under investigation can often be determined.

They are mostly positive and negative fluctuations that cause about one-half of the measurements to be too high and one-half to be too low. Sources of random errors cannot always be identified. Possible sources of random errors are as follows:

1.Observational For example, errors in judgment of an observer when reading the scale of a measuring device to the smallest division.

2. Environmental For example, unpredictable fluctuations in line voltage, temperature, or mechanical vibrations of equipment.

Repeated measurements produce a series of data sets that are all slightly different. They vary in random vary about an average value. Taking many sets of reading can minimize the random errors

3 Human Error

This type of error could occur due to the faulty procedure adopted by us. A person may record a wrong value, misread a scale(including Parallax Error), forget a digit when reading a scale or recording a measurement, or make a similar blunder.
These errors can be reduced if there are multiple different people conducting the experiment.It will reduce the Human error if not completely remove it.

1.When apply the load we must be careful otherwise on additional force will occur and it cause to the error readings.
2.This apparatus has been used for a long time for this experiment so the rod has been subjected to torque many times. It may be subject to fatigue.To avoid this, the rod must be changed at regular interval.

Many parameters may be expected to influence the accuracy of this test method. Some of these parameters pertain to the uniformity of the specimen, for example, its straightness, the uniformity of its diameter, and, in the case of tubes, the uniformity of its wall thickness.

The variation in shear modulus ΔG due to variations in diameter Δ D are given by:

$$\frac{\Delta G}{G} = -4\frac{\Delta D}{D}$$

Other parameters that may be expected to influence the accuracy of this test method pertain to the testing conditions, for example, alignment of the specimen, speed of testing, temperature, and errors in torque and twist values.

The error in shear modulus ΔG due to errors in torque ΔT are given by:

$$\frac{\Delta G}{G} = \frac{\Delta T}{T}$$

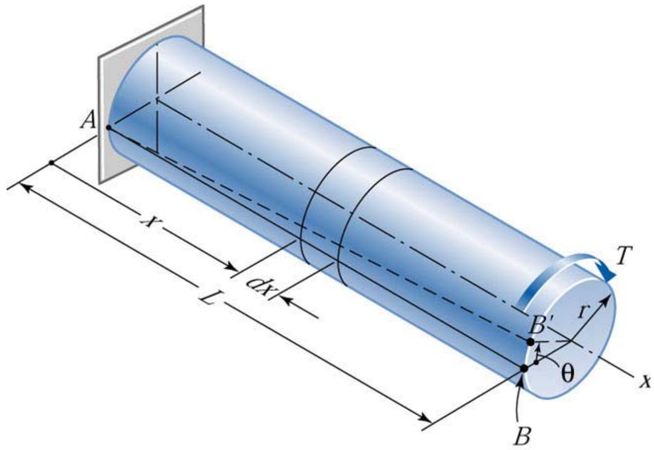
12.3.2 According to Eq 2 (see 6.1), the error in shear modulus ΔG due to errors in angle of twist Δ θ are given by:

$$\frac{\Delta G}{G} = \frac{\Delta \theta}{\theta}$$

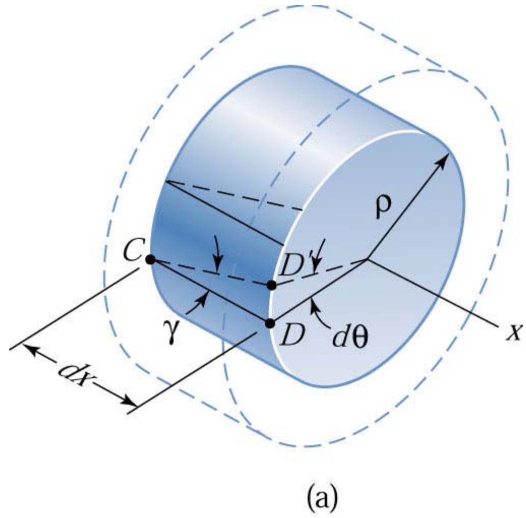
The least count of the twist gage should always be smaller than the minimum acceptable value of Δ θ. In general, the overall precision that is required in twist data for the determination of shear modulus is of a higher order than that required of strain data for determinations of most mechanical properties, such as yield strength. It is of the same order of precision as that required of strain data for the determination of Young's modulus .

3. Relation between applied mass m and angle of deflection (θ_0) for a simply supported beam.

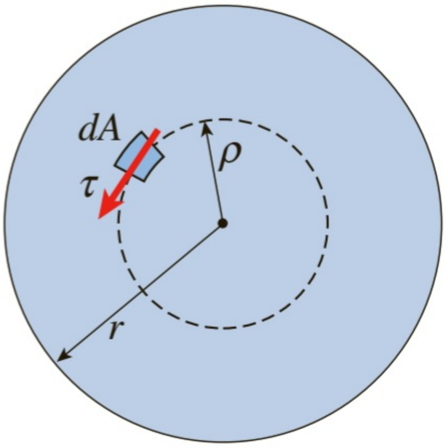
The Torsion Test is based on the **Theory of Pure Torsion**.



(a) Circular beam subjected to torque T



(b) dx section



(c) dx section

For a cylinder of diameter D and length l (see figure (a)), consider first a small annular element of mean radius rand thickness dx. The cylinder is subjected to a torque T, and this causes a circumferential shear stress T in the wall of the small element under consideration. If the torque is such that the left-hand end of this small element is twisted through an angle dθ in relation to the right-hand end, a longitudinal line AB on the surface of the element twists to position AB' (see figure (a)). For small angles of twist the shear strain γ that is developed is given by

$$\gamma = \frac{BB'}{L} \tag{1}$$

but arc BB' = rθ , therefore

$$\gamma = \frac{r\theta}{L} \tag{2}$$

Within the elastic limit, the ratio of stress to strain is constant.

$$\frac{\tau}{\gamma} = G \tag{3}$$

G=*Shear Modulus*

Substituting for γ from (2) in (3),and rearranging we have

$$\frac{\tau}{r} = \frac{G\theta}{L} \tag{4}$$

For this small element, if the thickness dx is small, it can be assumed that the shear stress is constant across the thickness of the element. Also, if dx is small, the area of the section on which the shear stress τ acts approximates to $2\pi\rho$ dρ. Hence, the total shear force acting on this element is τ*($2\pi\rho$ dρ).

The torque acting on this element is the moment of the tangential shear force about the longitudinal axis, and is τ $2\pi\rho^2 d\rho$. The torque T acting on the complete solid shaft is the sum of the moments of the tangential shear forces acting on all the small elements that go to make up the shaft, and is given by

$$T = \int_0^{D/2} \tau 2\pi\rho^2 d\rho$$

Substituting for τ from eqn(4), we have

$$T = \frac{G\theta}{L}\int_0^{D/2} 2\pi\rho^3 d\rho$$

On solving we get

$$T = \frac{G\theta}{L}\frac{\pi D^4}{32}$$

We know that for circular cross-section,

$$\frac{\pi D^4}{32} = J$$

In our experiment we produce the torque by adding mass on a lever attached to one end of the rod.

We already know the relation between the Torque applied and angle of deflection θ_0

$$T = \frac{G\theta_0}{L}\frac{\pi D^4}{32}$$

Since we know that for circular cross-section,

$$\frac{\pi D^4}{32} = J$$

In our experiment we produce the torque by adding mass on a lever attached to one end of the rod.(Length of lever = d). We apply mass m to one end of the lever which creates torque T.

$$T = mgd$$

m : *mass of load applied*
g : *acceleration due to gravity; 9.81 m/s²*
d : *length of lever*

Substituting the value of T in the main equation,we get the relation between applied mass m and the angle of deflection θ_0

$$m = \frac{G\theta_0}{gdL}\frac{\pi D^4}{32}$$

4. Effect of changing the number of cycles per experiment in the mean value of G and Effect of mass increment value Δm in accuracy or precision of the G for a given m_{max}

Accuracy is how close a measurement is to the correct value for that measurement.The precision of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions).

Precision is sometimes separated into :
• Repeatability — The variation arising when all efforts are made to keep conditions constant by using the same instrument and operator, and repeating the measurements during a short time period.
• Reproducibility — The variation arising using the same measurement process among different instruments and operators, and over longer time periods.

The random error will be smaller with a more accurate instrument (measurements are made in finer increments) and with more repeatability or reproducibility (precision).

In our experiment we determine the Shear Modulus of Aluminium 6061 by performing the Torsion Test. The experiment is carried and a value for G is obtained in that cycle. Just to be on the safe side, the experiment is being repeated (i.e multiple cycles are performed). It is highly unlikely that the second cycle will yield the same result as the first. In fact,if we run a number of replicate (that is, identical in every way) trials, we will probably obtain scattered results.

As stated above, the more measurements that are taken, the closer we can get to knowing a quantity's true value. With multiple measurements (replicates), we can judge the precision of the results, and then apply simple statistics to estimate how close the mean value would be to the true value if there was no systematic error in the system. The mean deviates from the "true value" less as the number of measurements (replicates) increases. Thus,if we increase the number of cycles per experiment ,the mean value of G obtained will be closer to the true value of G.

Yes,mass increment value Δm effects the accuracy and precision of the G for a given m_{max}.

1.If we have a smaller value of Δm ,We can plot the Stress vs Strain curve better and hence do not take those points into the calculation of G which are beyond the proportionality limit (IF The m_{max} corresponds to a strain which is beyond the proportionality limit). In this case it affects the accuracy of G.

2.Taking smaller value of delta m will reduce the random error caused in while taking measurements and hence will make our experimental value of G more precise. Thus, Δm will always affect the precision of G.

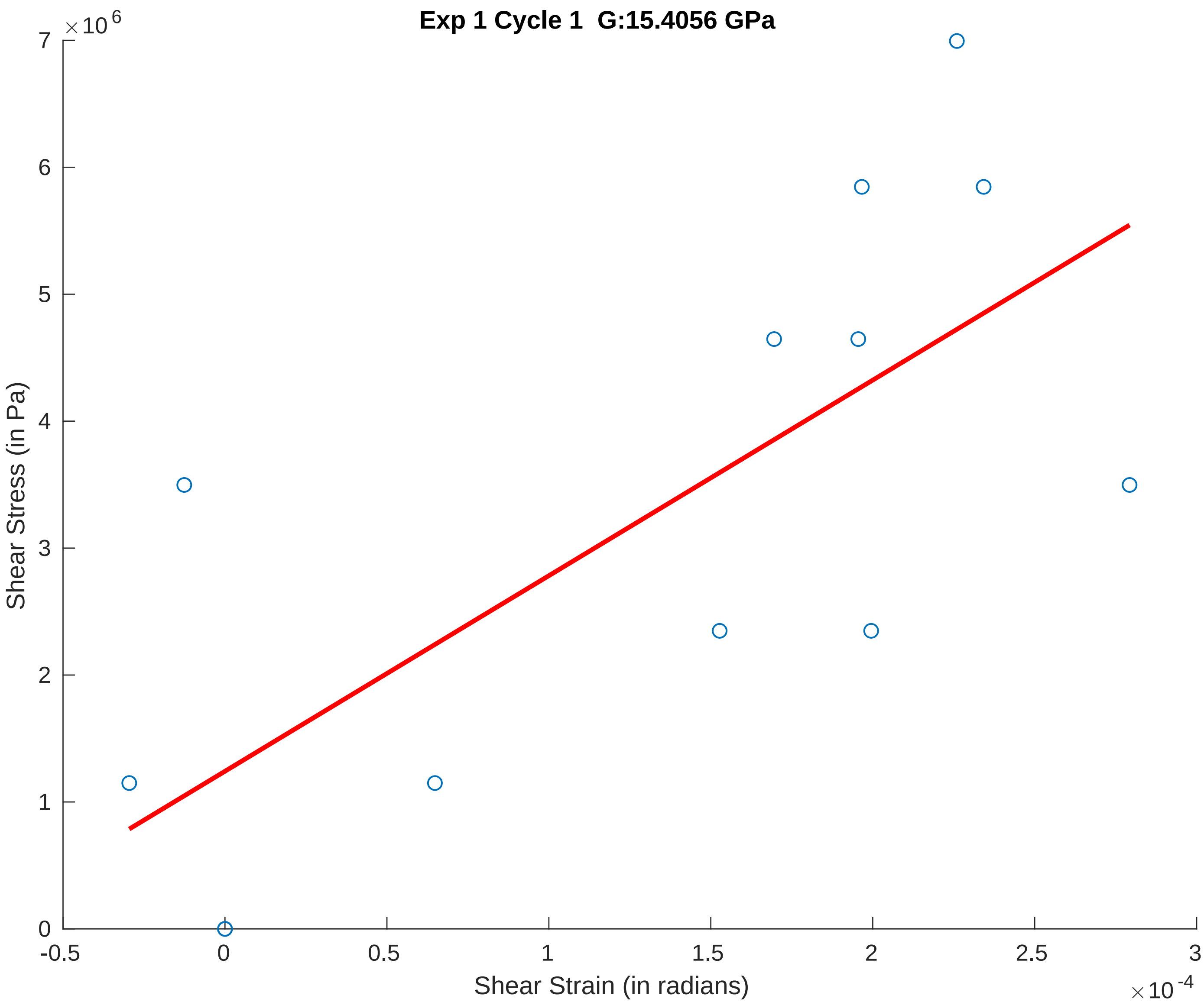
5. Shear Stress vs Shear Strain for the first cycle of each experiment using the best linear fit

MATLAB has been used to obtain the best linear fit for the given data. MATLAB finds the best linear fit by using the least square method.The slope of the best linear fit line gives the best estimate of Shear Modulus of Aluminium 6061 corresponding to the data reading of that cycle.[1]

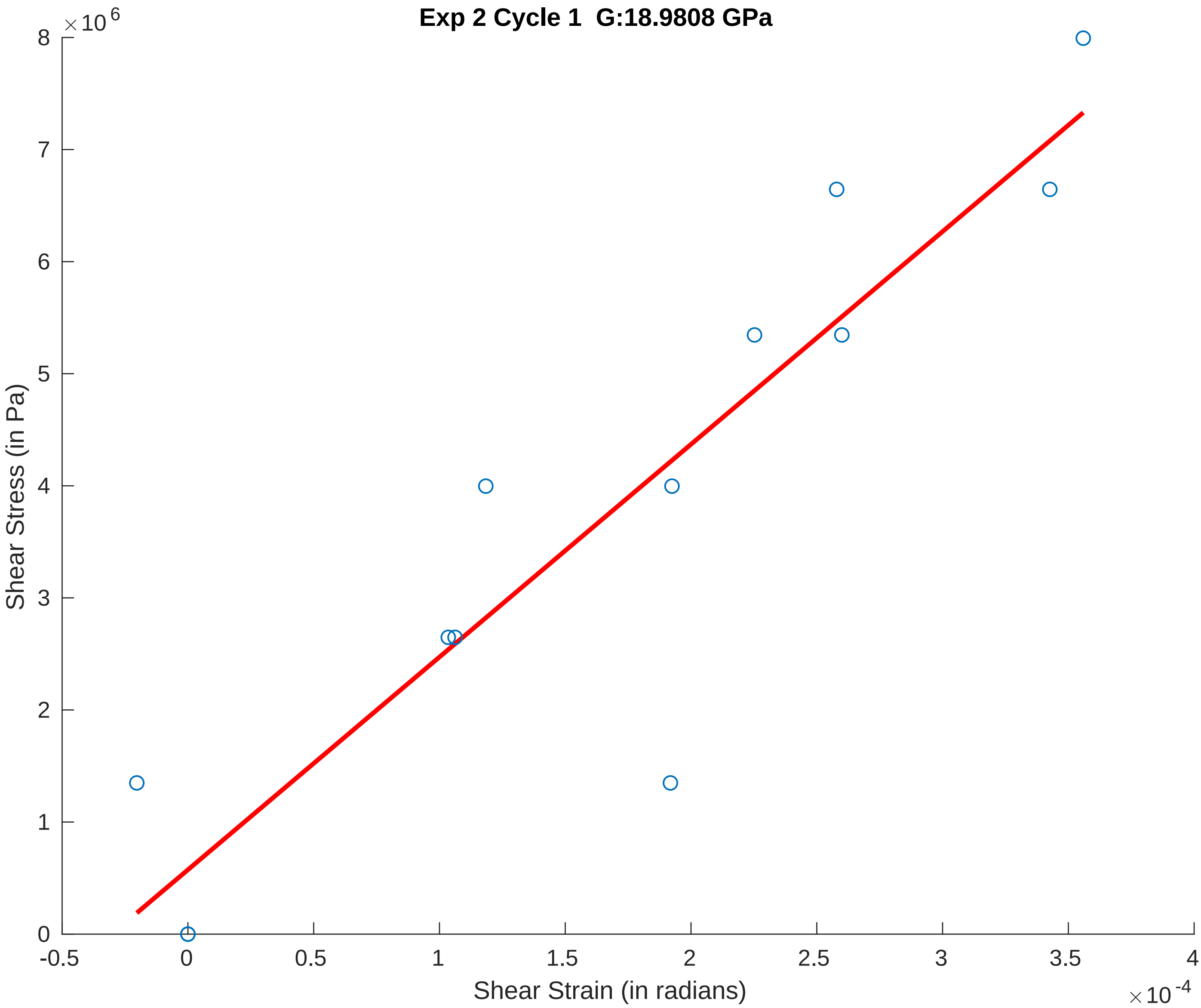
Observations Recorded :

Best Estimate of Shear Modulus of Aluminium 6061			
Experiment Number	Cycle	Slope (in Pa/rad)	G (in GPa)
1 (Group 1)	1	15405632012.205857	15.4056
2 (Group 2)	1	18980810078.480890	18.9808
3 (Group 3)	1	20547119551.370123	20.5471
4 (Group 4)	1	16281510793.680640	16.2815
5 (Group 5)	1	13602448487.800990	13.6024

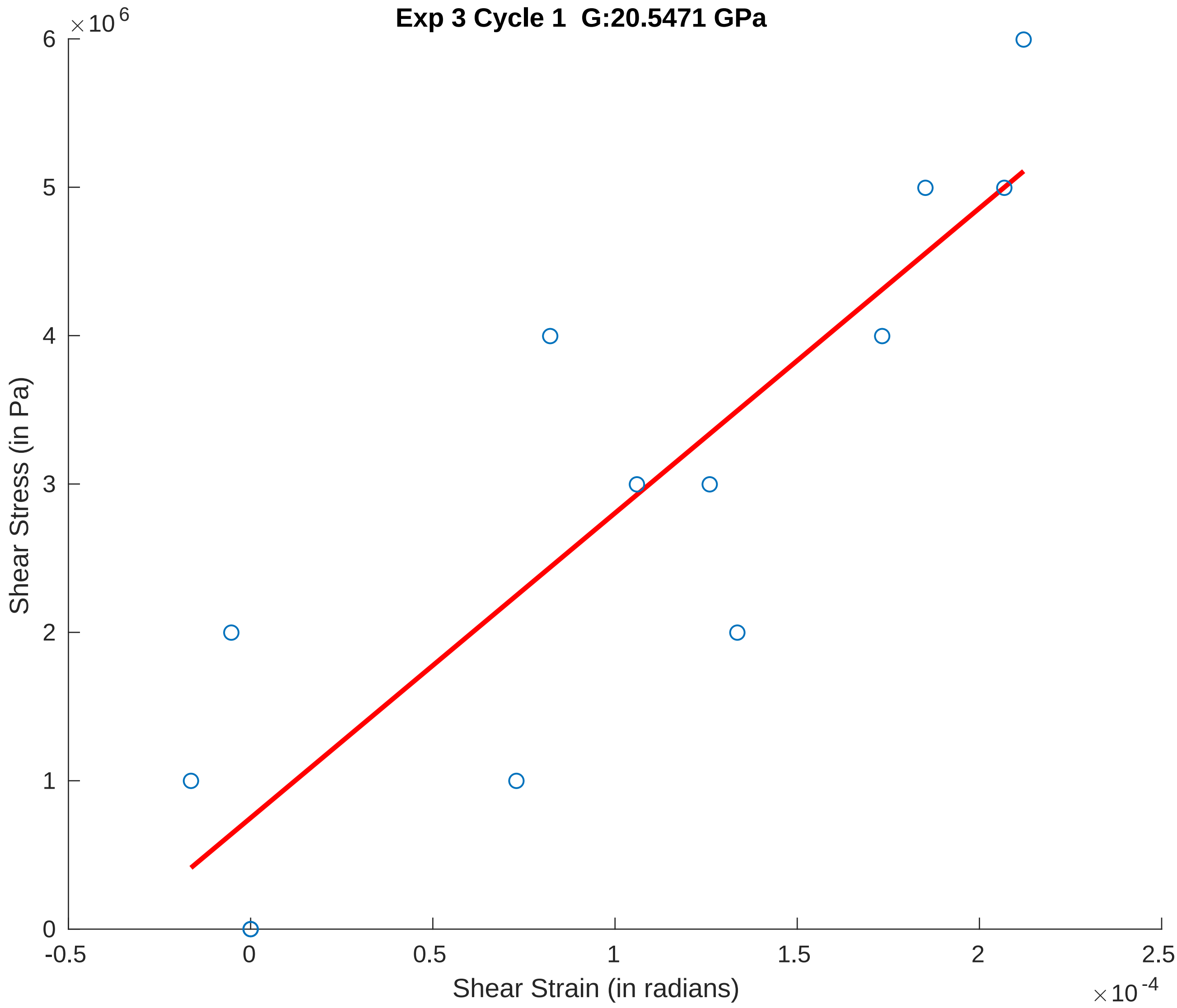
[1]The code used to arrive to these results is attached in the Archives

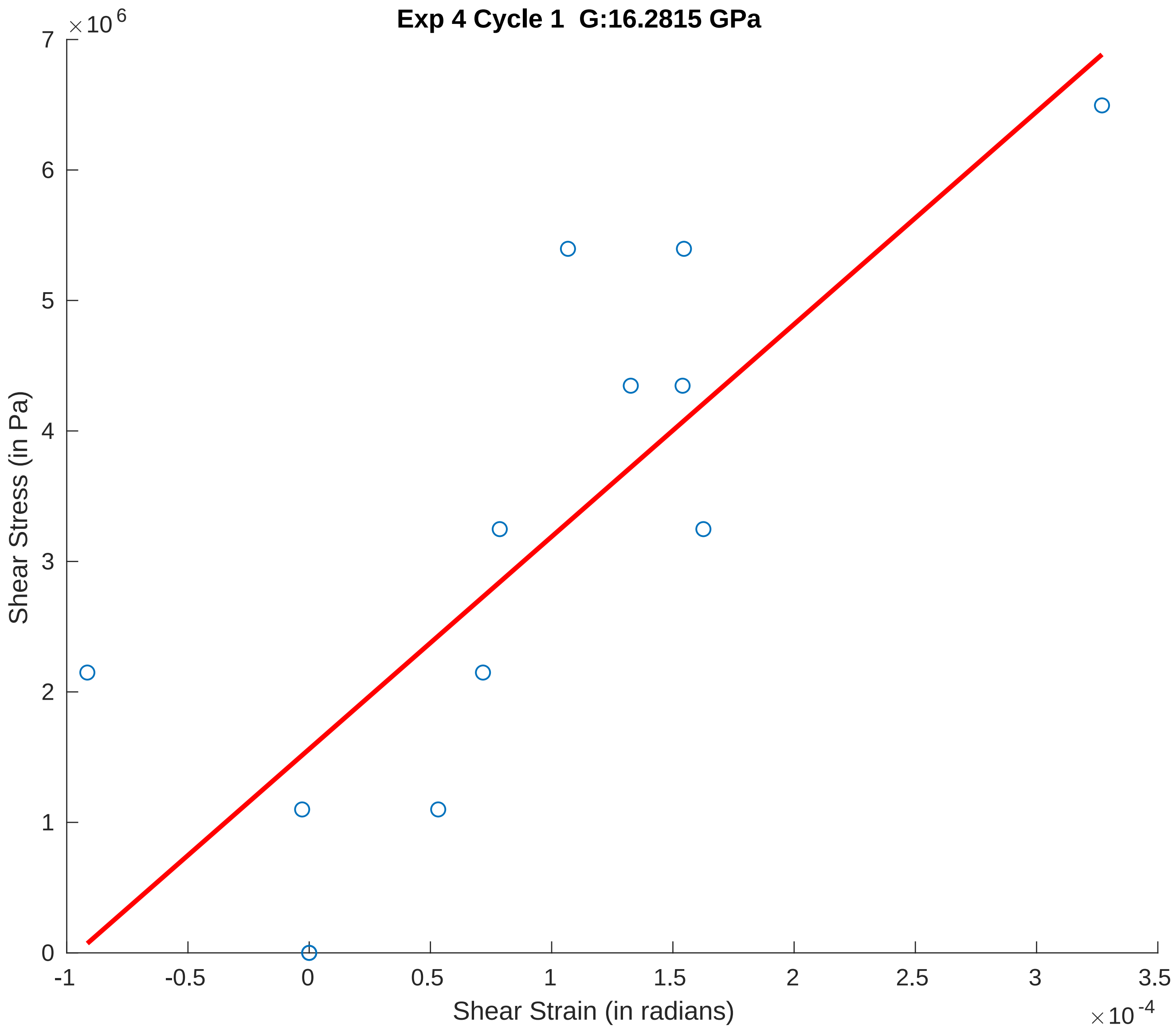


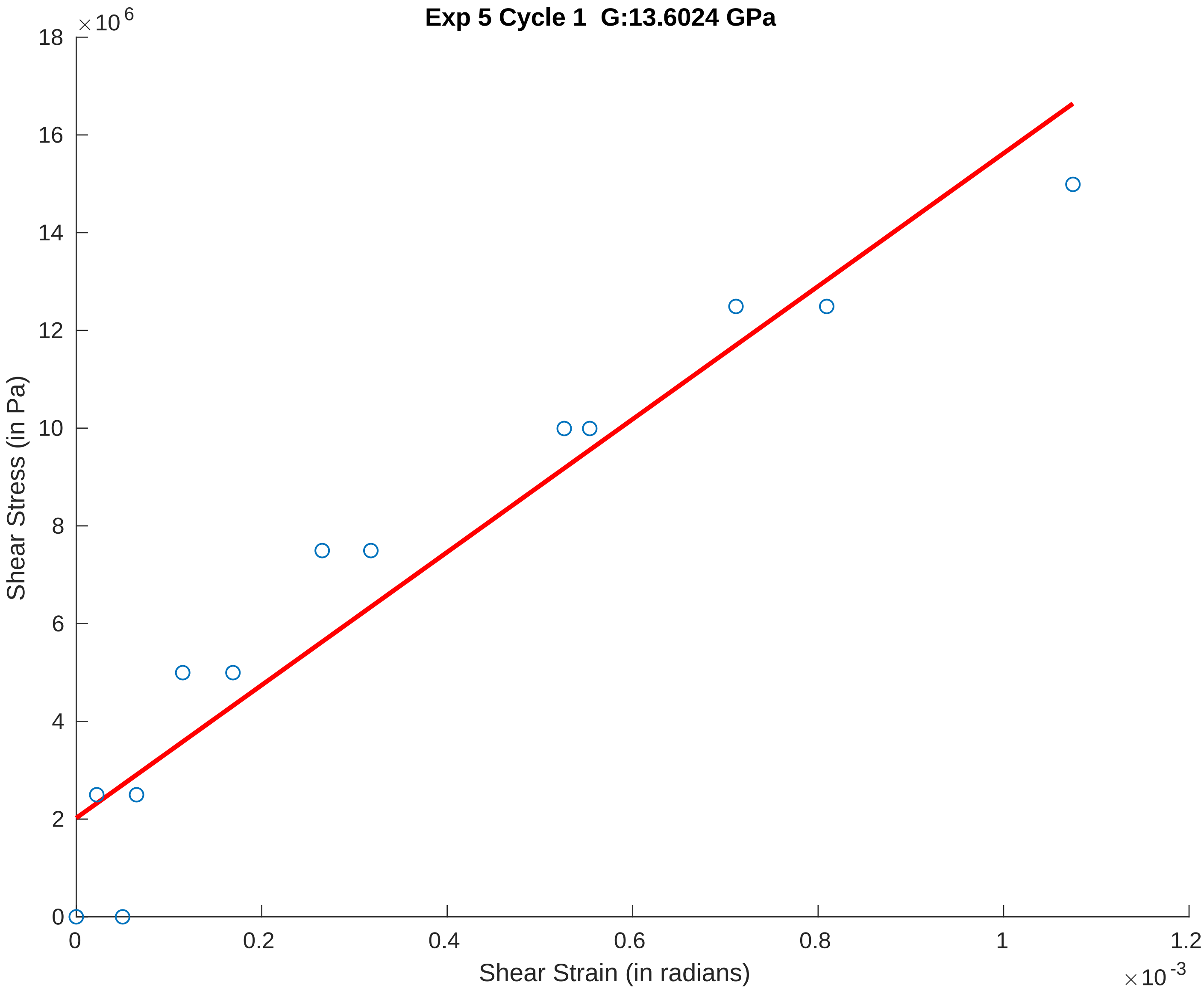
Exp 1 Cycle 1 G:15.4056 GPa
×10^6
7
6
5
4
3
2
1
0
Shear Stress (in Pa)
-0.5
0
0.5
1
1.5
2
2.5
3
Shear Strain (in radians)
×10^-4



Exp 2 Cycle 1 G:18.9808 GPa
×10 6
8
7
6
5
4
3
2
1
0
Shear Stress (in Pa)
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
Shear Strain (in radians)
×10 -4


Exp 3 Cycle 1 G:20.5471 GPa
×10^6
6
5
4
3
2
1
0
Shear Stress (in Pa)
-0.5
0
0.5
1
1.5
2
2.5
Shear Strain (in radians)
×10^-4

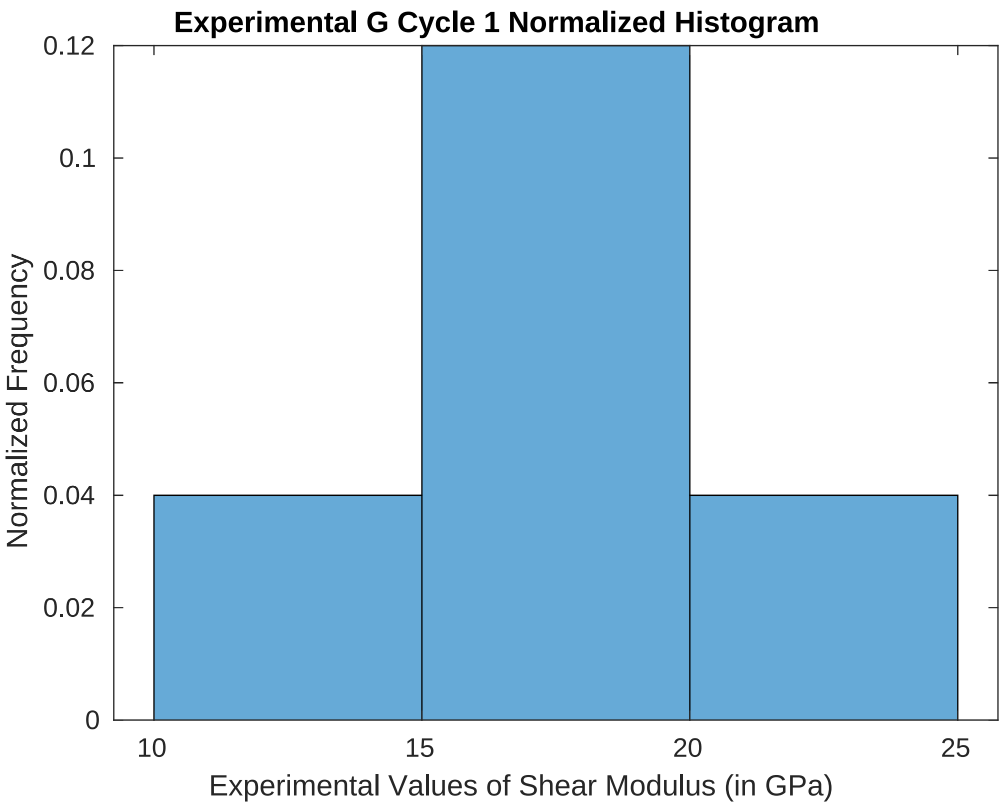

Exp 4 Cycle 1 G:16.2815 GPa
×10^6
7
6
5
4
3
2
1
0
Shear Stress (in Pa)
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
Shear Strain (in radians)
×10^-4

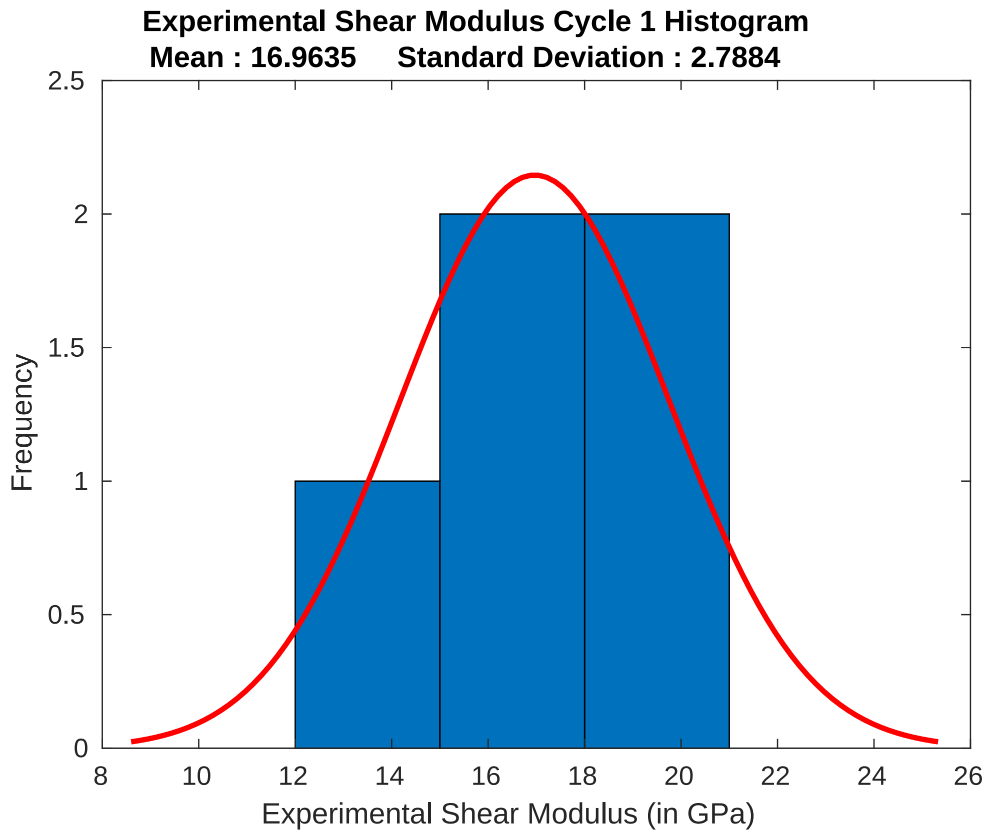


Exp 5 Cycle 1 G:13.6024 GPa
×10^6
18
16
14
12
10
8
6
4
2
0
Shear Stress (in Pa)
0
0.2
0.4
0.6
0.8
1
1.2
Shear Strain (in radians)
×10^-3

Best Estimate of Shear Modulus G of Aluminium 6061(in GPa)					
Cycle	Group 1	Group 2	Group 3	Group 4	Group 5
1	15.4056	18.9808	20.5471	16.2815	13.6024
2	23.1932	16.2989	20.9568	15.3368	14.5182
3	14.4278	18.0067	19.1010	23.6965	15.3531
4	23.6029	21.3467	28.9270	21.5365	15.4221
5	24.4523	19.9375	19.6609	15.0040	13.9876
6	19.6592	18.7894	16.4957	20.3673	14.1422
7	16.0682	21.0934	17.8017	24.9917	13.5261
8	22.3908	19.9046	24.9266	20.7837	15.7506
9	19.3626	19.9448	20.2522	26.5262	14.9680
10	19.2821	23.1883	15.2823	25.6834	13.8522
11	21.3640	19.7951	21.5843	23.2311	14.1783
12	18.5747	18.8597	21.9365	20.4039	14.0114
13	18.3262	17.0349	23.8582	19.3746	15.5855
14	22.3425	19.4617	26.1568	17.6308	13.5734
15	26.1794	15.5853	18.5172	23.6005	14.4280
16	20.4599	15.9529	17.4638	27.5469	14.5544
17	19.5354	22.2001	27.0359	16.3402	14.6347
18	19.6729	21.2489	21.3897	17.1683	12.7887
19	17.2217	22.7034	17.8797	16.3786	12.6379
20	18.6922	21.4132	17.4849	20.1685	13.6559
μ	20.0107	19.5873	20.8629	20.6025	14.2585
σ	3.0582	2.2000	3.7034	3.9131	0.8667

Table: Best Estimate of Shear Modulus of Aluminium obtained from Experiment Data set


Experimental G Cycle 1 Normalized Histogram
Normalized Frequency
0
0.02
0.04
0.06
0.08
0.1
0.12
10
15
20
25
Experimental Values of Shear Modulus (in GPa)


Experimental Shear Modulus Cycle 1 Histogram
Mean : 16.9635 Standard Deviation : 2.7884
Frequency
2.5
2
1.5
1
0.5
0
8
10
12
14
16
18
20
22
24
26
Experimental Shear Modulus (in GPa)

8. Statistical Quantities Comparison

Statistical Quantities		
	μ	σ
Experiment 1	20.0107	3.0582
Experiment 2	19.5873	2.2000
Experiment 3	20.8629	3.7034
Experiment 4	20.6025	3.9131
Experiment 5	14.2585	0.8667
Cycle 1	16.9635	2.7884

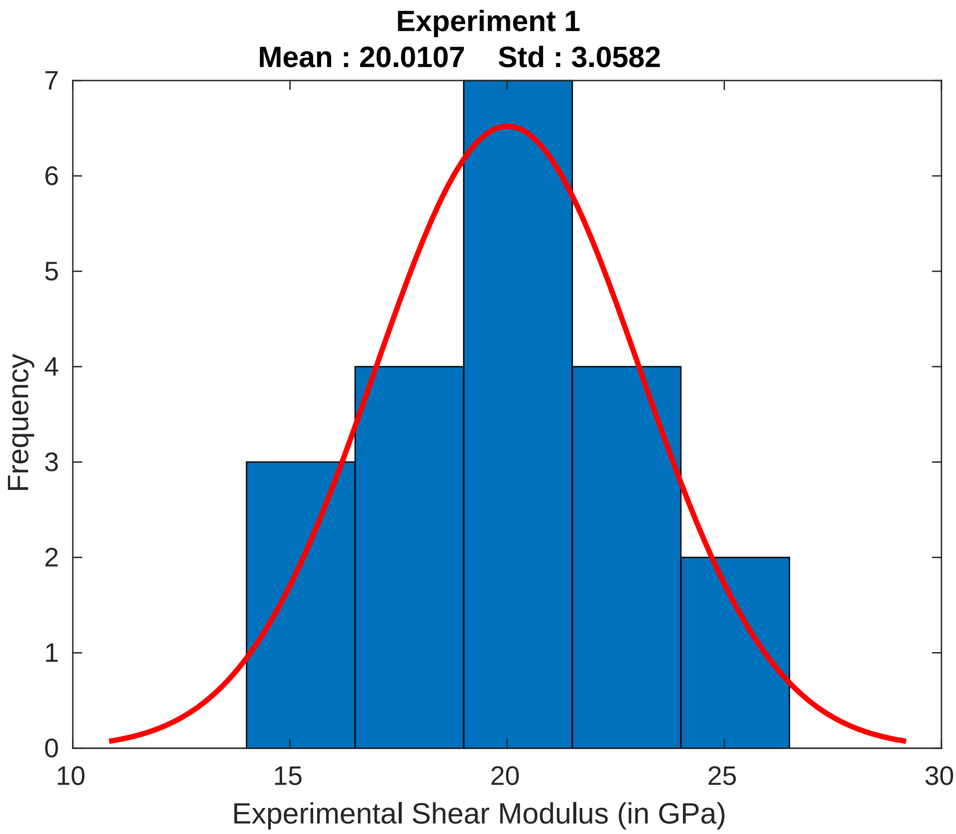
9. Anomalous Experiment

Anamoly is Experiment 5.
In Experiment 5, the load is too large which implies higher Stess and hence a higher value of strain.

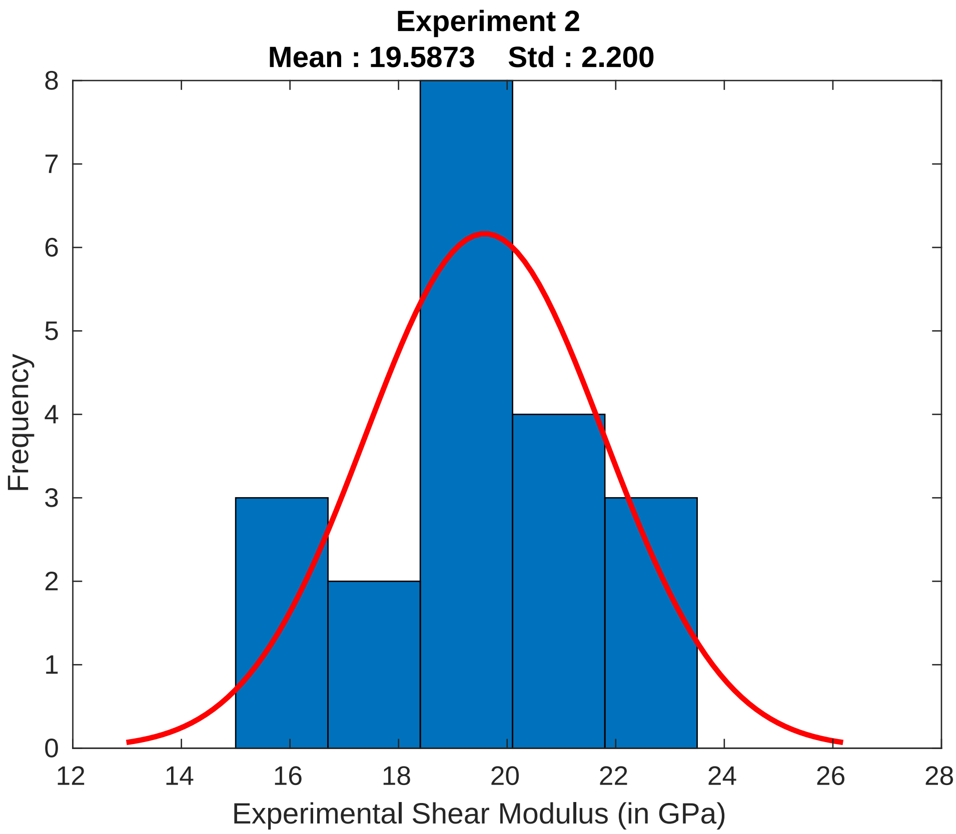
- Range of Maximum Mass for other Experiments : 1.2-1.6kg
- Maximum Mass for Experiment 5 : 3kg

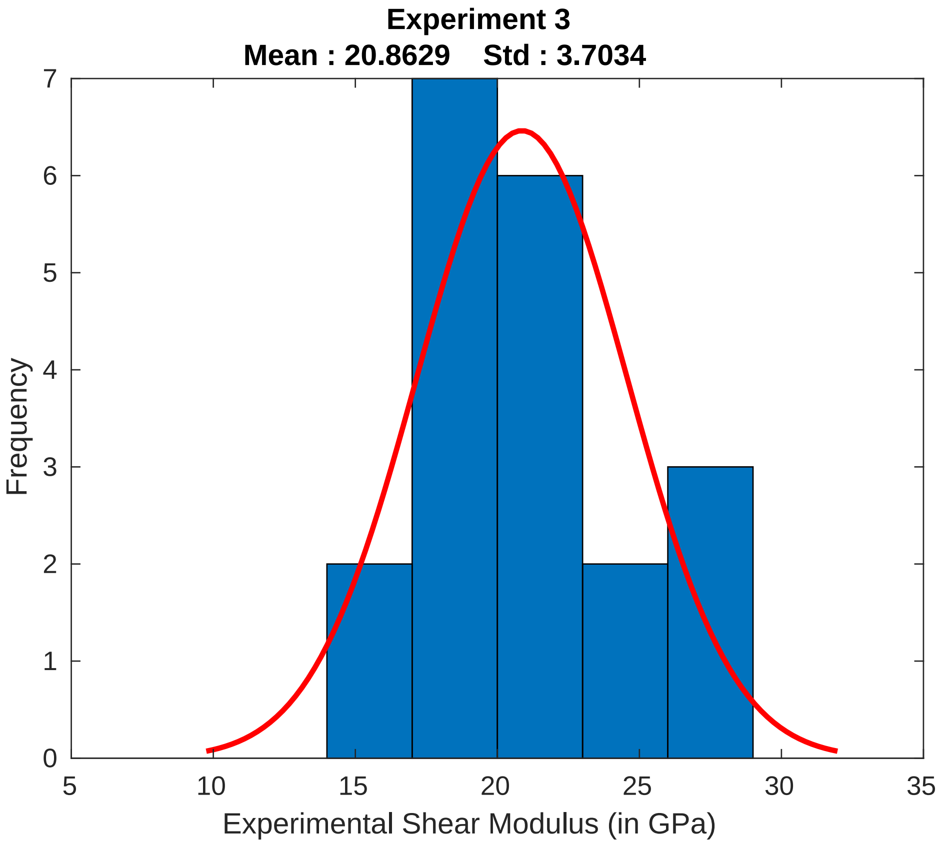
While performing this experiment we assume stress and strain to be very small and in the proportionality limit of the material. i.e Stress and Strain are linearly dependent

Although the plots obtained by the other Experiments are also not completely linear due to which we do not get the accurate value,The percentage of linearity in the plot of Experiment 5 is extremely low which makes it the anamalous experiment

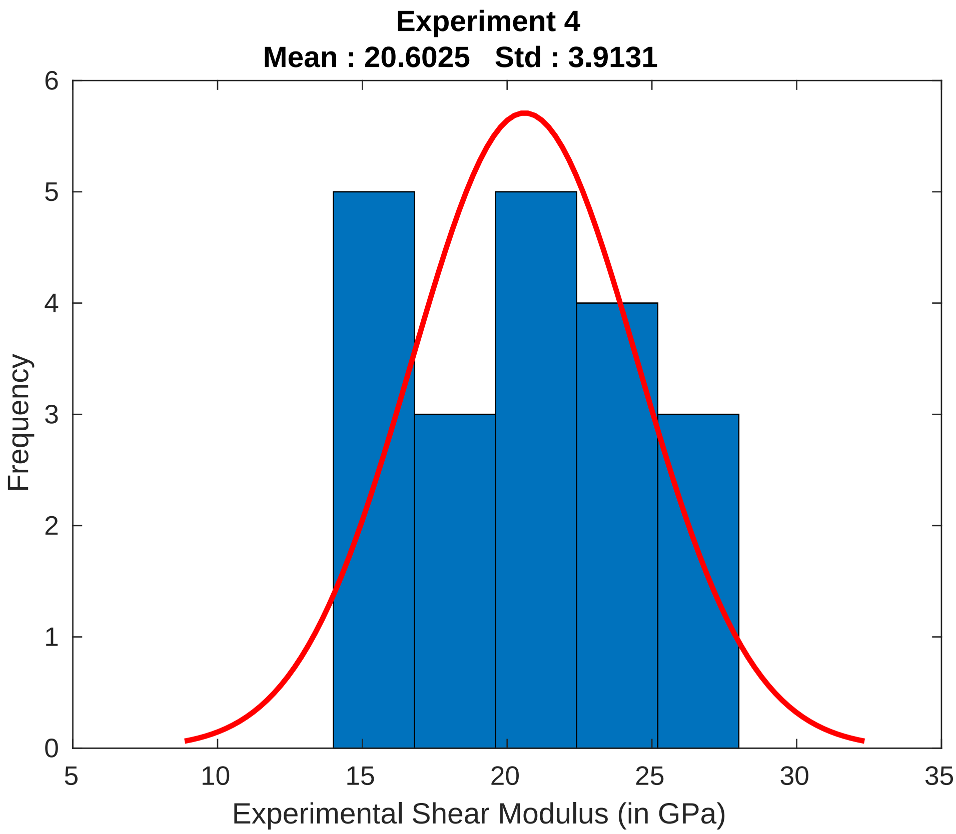


Experiment 1
Mean : 20.0107 Std : 3.0582
Frequency
Experimental Shear Modulus (in GPa)
0
1
2
3
4
5
6
7
10
15
20
25
30

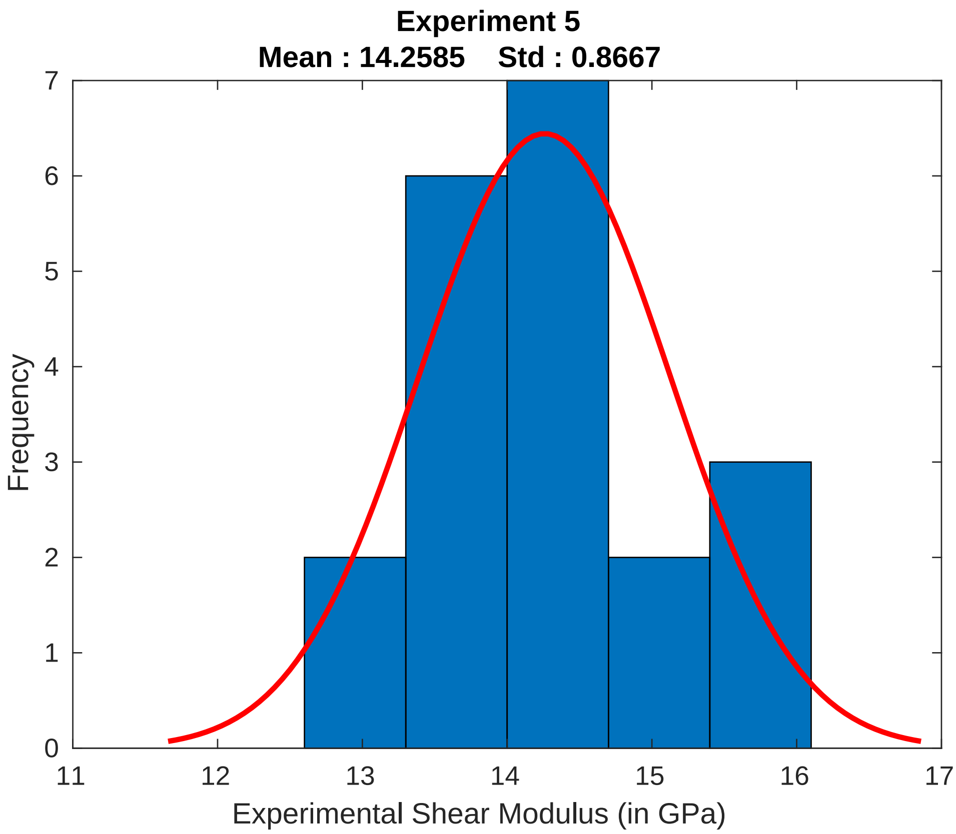

Experiment 2
Mean : 19.5873 Std : 2.200
Frequency
Experimental Shear Modulus (in GPa)
0
1
2
3
4
5
6
7
8
12
14
16
18
20
22
24
26
28



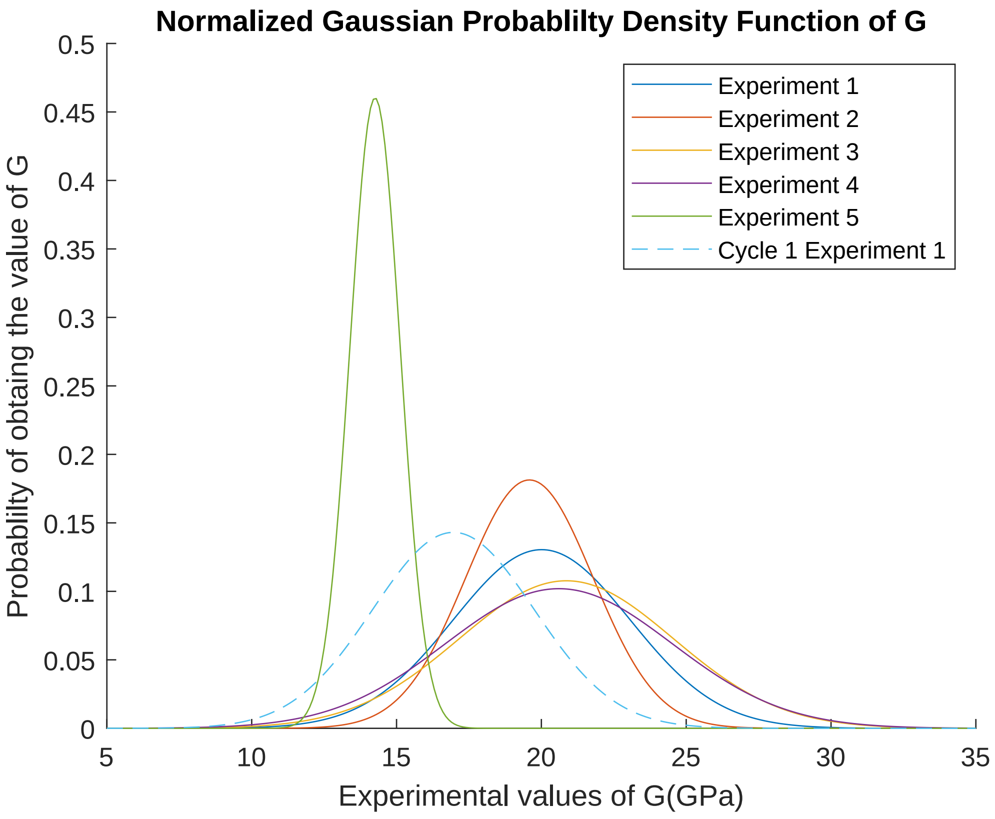
Experiment 3
Mean : 20.8629 Std : 3.7034
Frequency
0
1
2
3
4
5
6
7
5
10
15
20
25
30
35
Experimental Shear Modulus (in GPa)



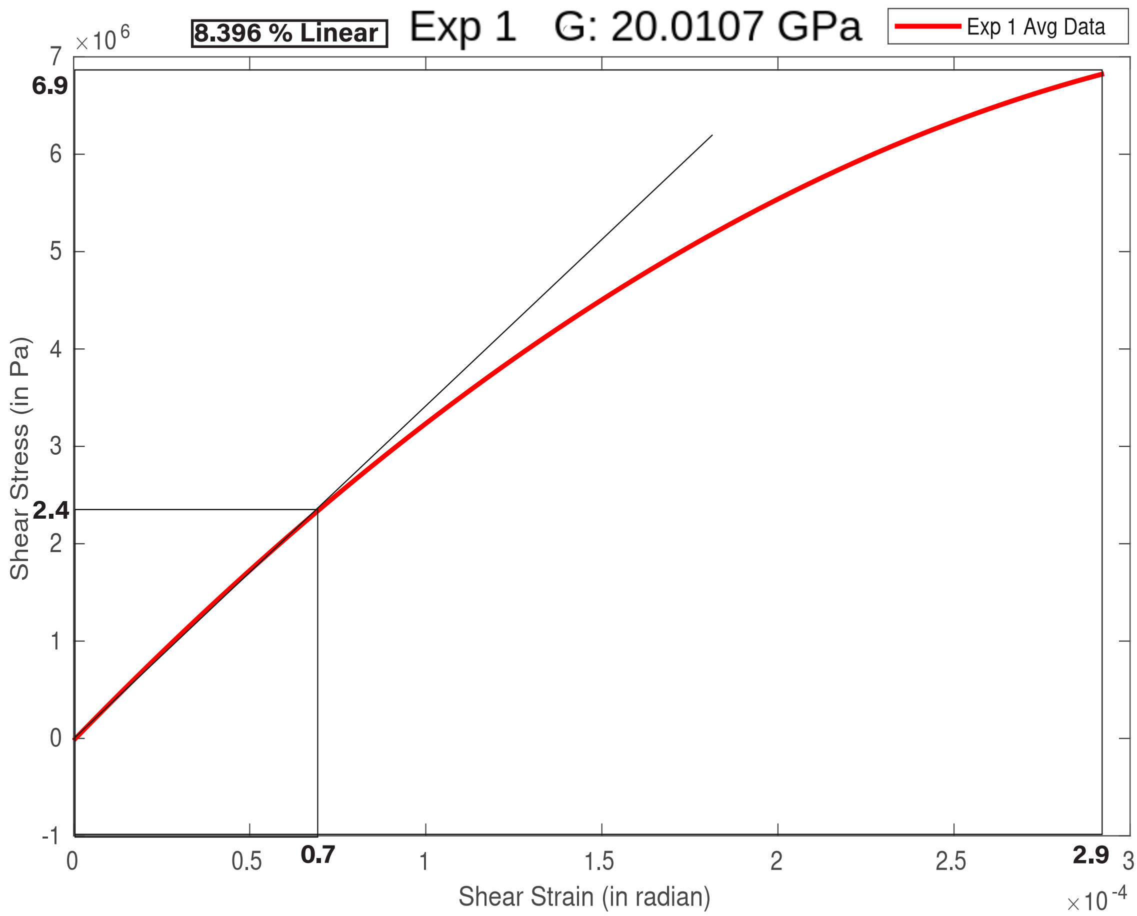
Experiment 4
Mean : 20.6025 Std : 3.9131
Frequency
0
1
2
3
4
5
6
5
10
15
20
25
30
35
Experimental Shear Modulus (in GPa)

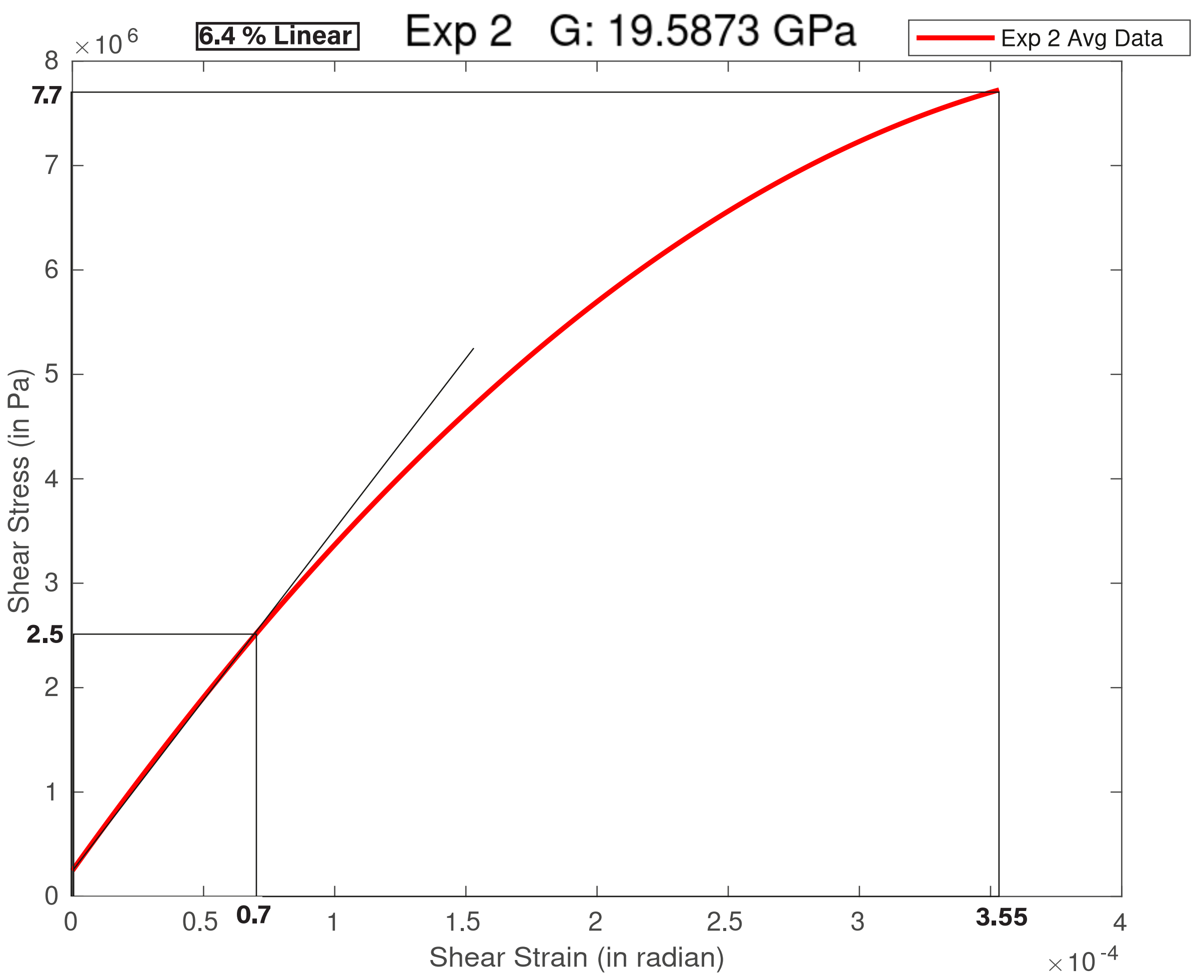


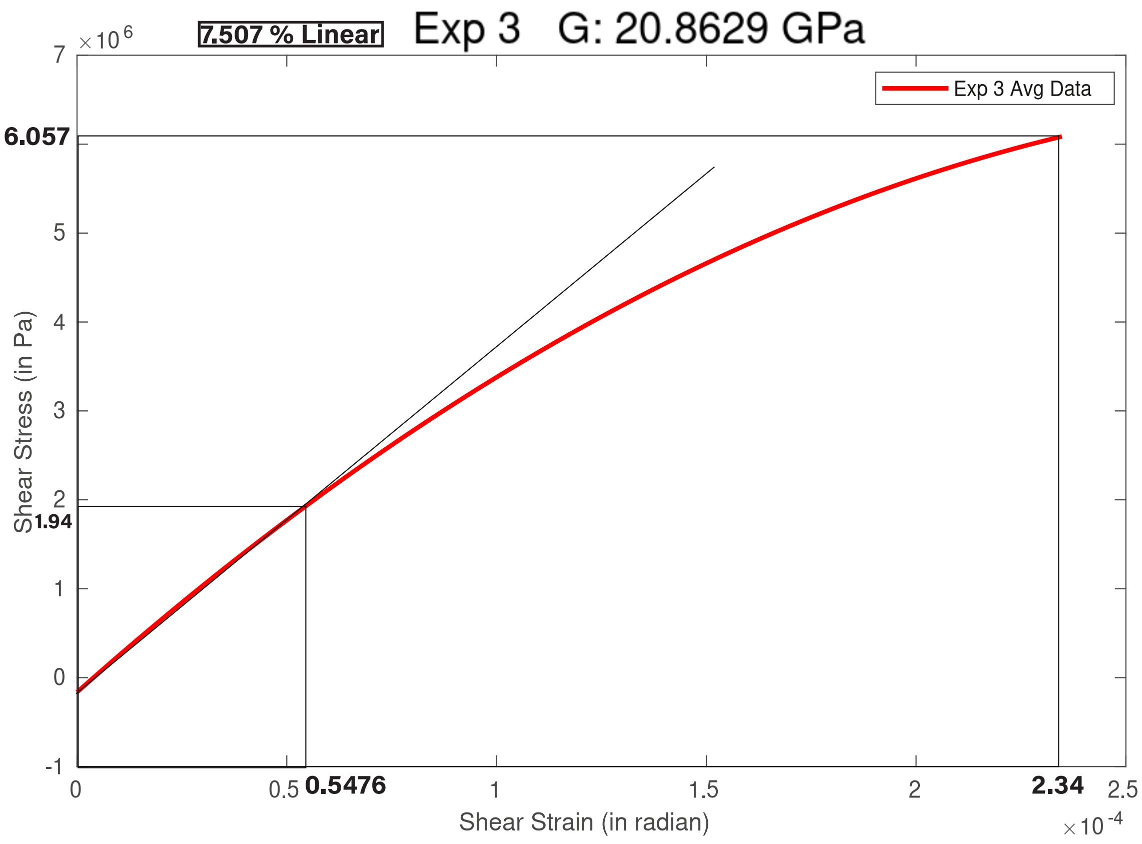
Experiment 5
Mean : 14.2585   Std : 0.8667
7
6
5
4
3
2
1
0
Frequency
11
12
13
14
15
16
17
Experimental Shear Modulus (in GPa)

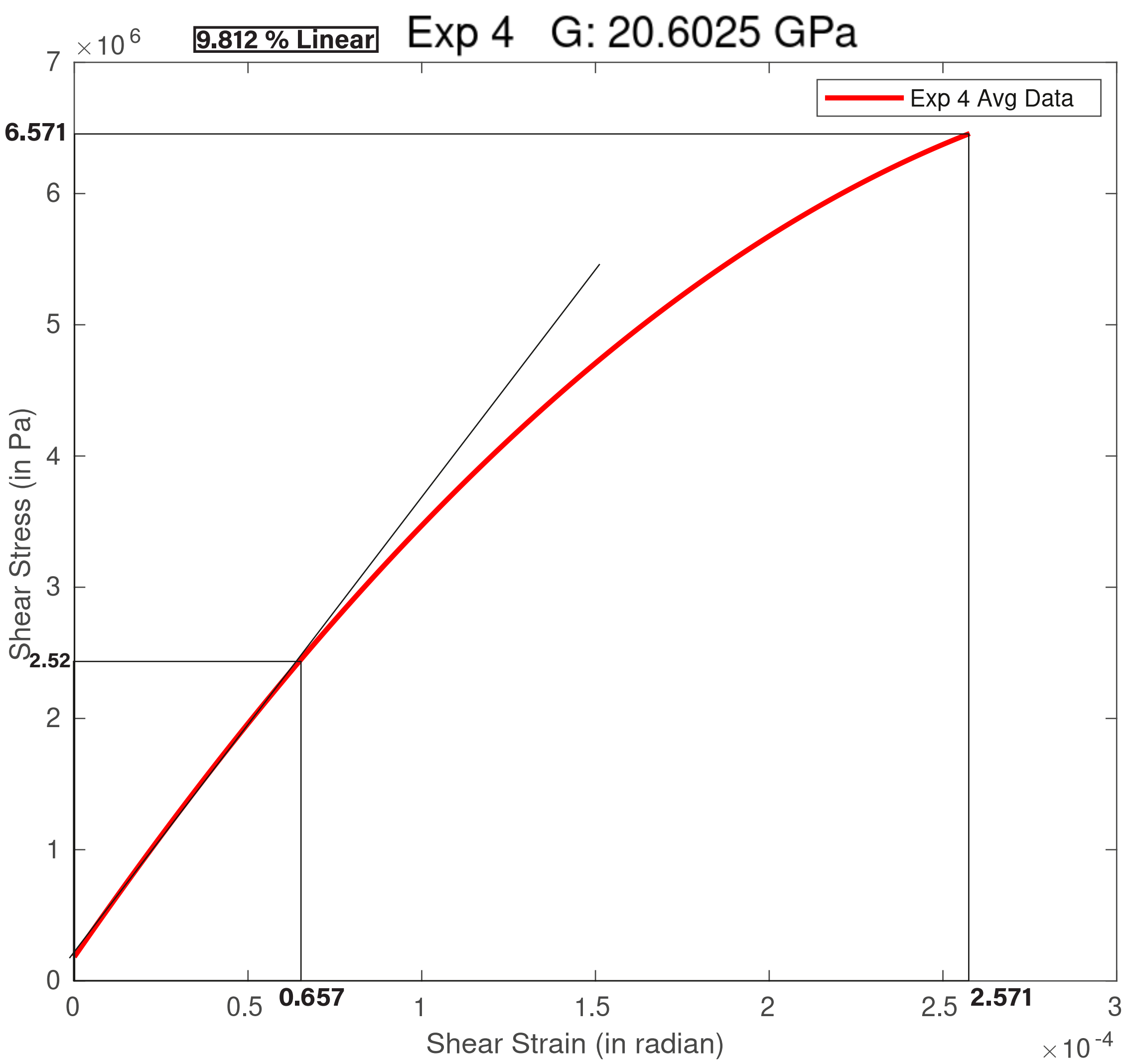


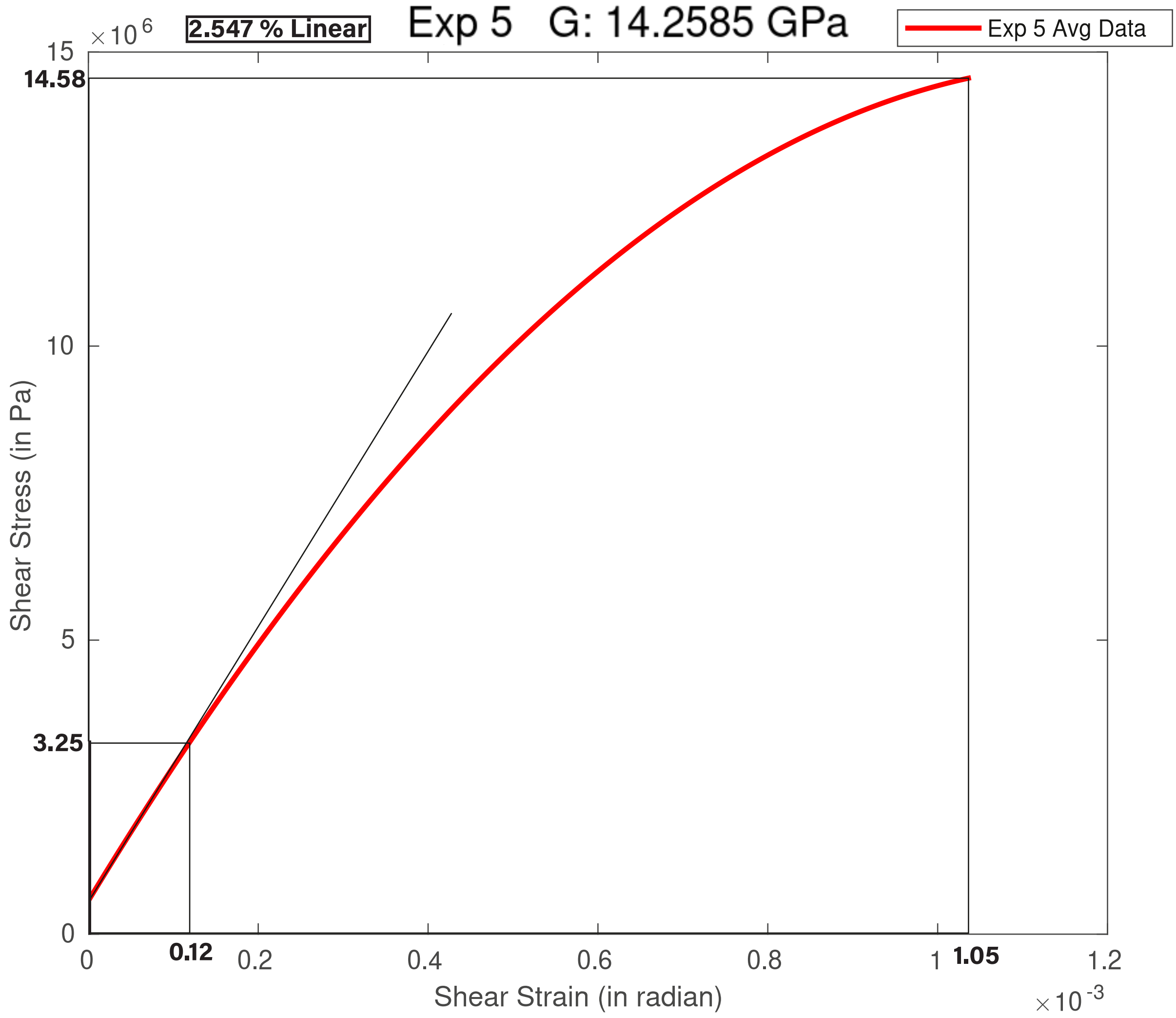
Normalized Gaussian Probablilty Density Function of G
Experiment 1
Experiment 2
Experiment 3
Experiment 4
Experiment 5
Cycle 1 Experiment 1
Probablilty of obtaing the value of G
Experimental values of G(GPa)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
5
10
15
20
25
30
35


8.396 % Linear
Exp 1 G: 20.0107 GPa
Exp 1 Avg Data
×10^6
7
6.9
6
5
4
3
2.4
2
1
0
-1
Shear Stress (in Pa)
0
0.5
0.7
1
1.5
2
2.5
2.9
3
Shear Strain (in radian)
×10^-4


6.4 % Linear
Exp 2 G: 19.5873 GPa
Exp 2 Avg Data
×10^6
8
7.7
7
6
5
4
3
2.5
2
1
0
Shear Stress (in Pa)
0
0.5
0.7
1
1.5
2
2.5
3
3.55
4
Shear Strain (in radian)
×10^-4


7.507 % Linear
Exp 3 G: 20.8629 GPa
Exp 3 Avg Data
$\times 10^6$
7
6.057
5
4
3
2
1.94
1
0
-1
Shear Stress (in Pa)
0
0.5
0.5476
1
1.5
2
2.34
2.5
$\times 10^{-4}$
Shear Strain (in radian)


9.812 % Linear
Exp 4 G: 20.6025 GPa
Exp 4 Avg Data
$\times 10^6$
7
6.571
6
5
4
3
2.52
2
1
0
Shear Stress (in Pa)
0
0.5
0.657
1
1.5
2
2.5
2.571
3
$\times 10^{-4}$
Shear Strain (in radian)



True Value of Shear Modulus of Aluminium 6061: 26 GPa

Experiment	% Linear	G_Value (in GPa)	|Error| (in GPa)
Group 1	8.396%	20.0107 GPa	5.9893 GPa
Group 2	6.4%	19.5873 GPa	6.4127 GPa
Group 3	7.507%	20.8629 GPa	5.1371 GPa
Group 4	9.812%	20.6025 GPa	5.3975 GPa
Group 5	2.547%	14.2585 GPa	11.7415 GPa

Table 1. Percentage Linearity of Average Experimental Readings and corresponding Magnitude of Error in Shear Modulus .

10. Best Estimate of Shear Modulus from all Non-Anomalous Experiments

For the Best estimate of the Shear Modulus G we find mean(μ) and standard deviation(σ) from experimental values of G obtained from all Experiments (i.e Exp1, Exp2, Exp3, Exp4) except Exp5 since it is Anomalous

Experimental Result :
Mean(μ) = 20.2659 GPa
Standard Deviation(σ) = 3.2629 GPa

Experimental Value of Shear Modulus : 20.2659 $\pm$ 3.2629 GPa

Published Value of G = 26 GPa

11. Short Comings and Alternatives

While looking at our dataset for theta, we noticed that many a times during loading or unloading the value of theta was negative. During the addition of incremental weights, the apparatus may have been allowed to unload completely as the experimenter could have been holding up the weights, causing the anomalies we see in the data.

Since in the experiment,we use the Masses and the Torsion Appratus instead of machines to do the same, The Errors in all the manual machines used adds up to the error in the value of Shear Modulus obtained due to Error Propogation.

Furthermore we are using lever and mass to produce torque but also causes a downward force to act on the rod which adds a bending moment to it. According to our assumption we assume the rod to be under pure torsion. Thus producing the Torque using a Chuck would eliminate the external bending moment to a large extent (although Bending Moment due to the weight of the rod still persists)

From the analysis of our data, we propose the following to outperform the current experiment.

- A better experiment would be using an improved apparatus such as a torsion testing machine that is equipped with a chuck to provide the torque and offer greater precision.

- Furthermore, regularly changing the testing sample after each cycle or a certain number of cycles to ensure that data remains unaffected due material fatigue can also give a better data reading.

- It is seen that during the experiments, the aluminium rod would regularly go beyond the limit of proportionality of the material

  To keep a check on when the Data goes beyond the Proportionality Limit, we can reduce the value of Δm and get more data points so that we can plot a better curve and find out where the material and data points exceed the Proportionality Limit of the given Material

Other method that can be used to find the Shear Modulus of a material is:

- **Rail Shear Test**
  This is a very popular method used to measure in-plane shear properties. This method is extensively used in aerospace industry. The shear loads are imposed on the edges of the laminate using specialized fixtures.
  There are two types of such fixtures:

  1. **Two Rail Fixture**
     The two rail shear test fixture has two rigid parallel steel rails for loading purpose. The rails are aligned to the loading direction and load (compressive or tensile) is applied to it.

     Another modification made to it is the **V-Notched Rail Shear Test**.

     2 Strain Gauges are attached to the centre of the V, such that they measure strain along the $\pm$ 45° axis. Furthermore,the positioning tools offer high reproducibility.

  2. **Three Rail Fixture**(improved version)
     Using one more rail in two rail shear test fixture it can produce a closer approximation to pure shear. The fixture consists of 3 pairs of rails clamped to the test specimen

     The outside pairs are attached to a base plate which rests on the test machine. Another pair (third middle) pair of rails is guided through a slot in the top of the base fixture. The middle pair loaded in compression

- **Four-Point Loading or Saddle Test**

  Four-point loading or saddle test is used to determine the shear modulus of a material. To accomplish this, we apply equal load force on the four corners of a square plate perpendicular to the plate. Forces along one of the diagonals face upwards while the forces along the other diagonal will be facing downward, which will result in a deflection of the plate that looks similar to a saddle.

  The vertical cross-sections along the diagonal will result in two identical parabolas, one concave upwards and the other concave downwards. The shear modulus can is calculated from the ratio of the imposed loads on the plate and the vertical deflection of the plate with respect to its geometric centre.

$$G = \frac{3Pu^2}{2wt^3}$$

  P = Load applied at each corner
  t = plate thickness
  w = deflection of a point (x,y) on the diagonal with respect to the center of the plate
  u = the Diagonal Distance from the Center to the point (x,y)
  G = Shear Modulus

The Advantage in the above Tests is that the increment in stress is almost continuous and gives a very precise,reproducible plot with minimal error.

There Error Propogation is also diminshed owing to the use of heavy machinery which gives highly precise and accurate readings.

ARCHIVES

MATLAB code used to obtain Best Estimate of G, using the best linear fit in Shear Stress vs Shear Strain plot

The following MATLAB code plots the best linear fit :

```
%Global variables
d=0.01;       %Diameter of rod in meters
d1=0.1;       %Length of lever in meters
L=1;          %Length of Rod in meters
g=9.81;       %Value of g in m/s^2
load('P8_TorsionTest.mat');

%Experiment 1

%Takes values of load from m_exp1 variable of P8_TorsionTest.mat
loads = m_exp1;
theta =theta_0exp1(:,1);     %Taking data of Cycle 1

%to make y and x of same dimensions since theta was columnar
theta=theta';

J=pi*(d^4)/32;    %Polar Second Moment of Inertia
T=loads*(g*d1);     %Torque applied on the rod
y=(T*(d/2))/J;        %Shear Stress in Pa

x=(theta*(d/2))/L;       %Shear Strain

coefficients=polyfit(x,y,1);  %finds coeff of best fit line for Data
xFit = linspace(min(x),max(x),1000);
yFit = polyval(coefficients,xFit);
G_in_Pa=coefficients(1);
%slope of the best fit line gives best estimate of Shear Modulus

G_in_GPa(1)=G_in_Pa/1e9;

%%% PLOTTING GRAPH
plot(xFit,yFit,'r-','LineWidth',2)
xlabel('Shear Strain (in radians)');
ylabel('Shear Stress (in Pa)');
title({sprintf('Exp 1 Cycle 1  G:%g GPa',G_in_GPa(1))})
figure; %%to plot different plots in different windows

%Repeat this for the first cycle for all Experiments
```