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ϵ^* : An Online Coverage Path Planning Algorithm

This work has been published in:

J. Song and S. Gupta, “ ϵ^* : An Online Coverage Path Planning Algorithm”, *IEEE Transactions on Robotics*, Vol. 34, Issue 2, pp 526-533, 2018.

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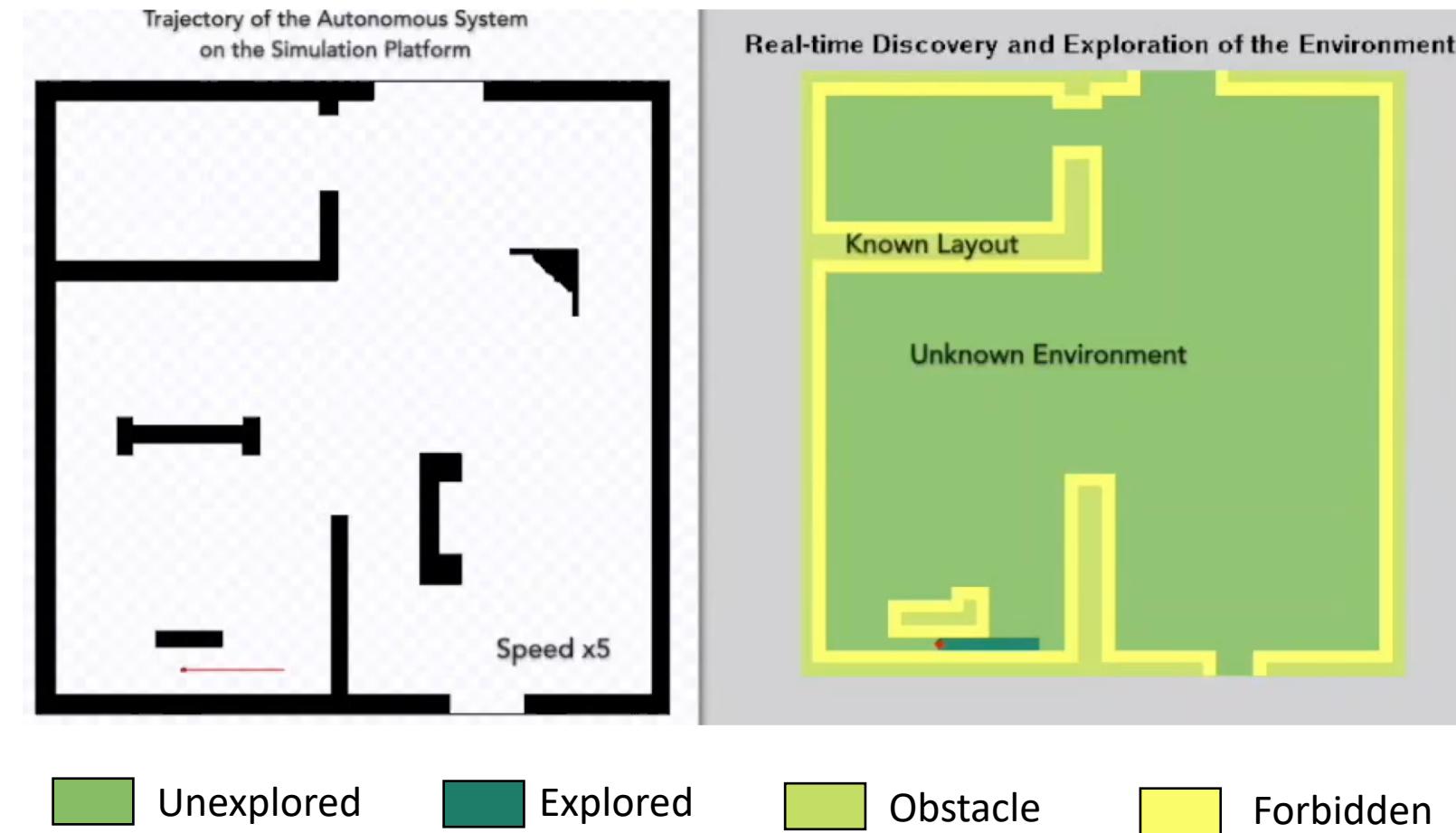
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ϵ^* : Online Coverage Path Planning in Unknown Environment

❖ **Objective:** Develop an online coverage path planning algorithm for an autonomous vehicle in unknown environment

❖ **Challenges:**

- Online detection and avoidance of unknown obstacles
- Generate back-and-forth path with minimized turns and overlappings
- Must guarantee complete coverage and prevent any local extremum
- Low computational complexity for real-world applications





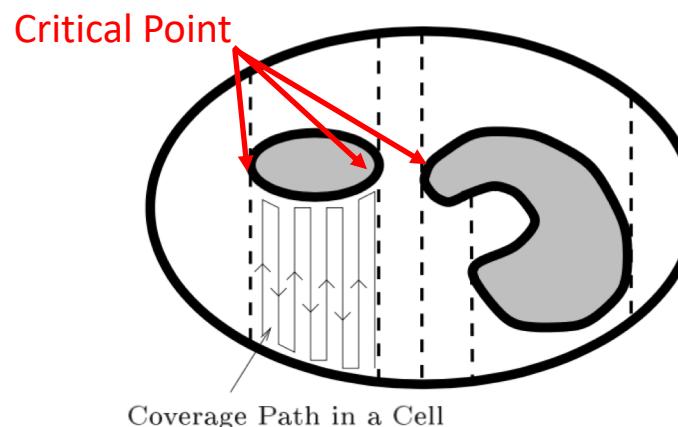
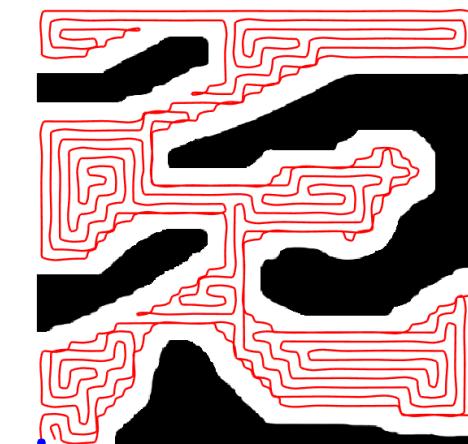
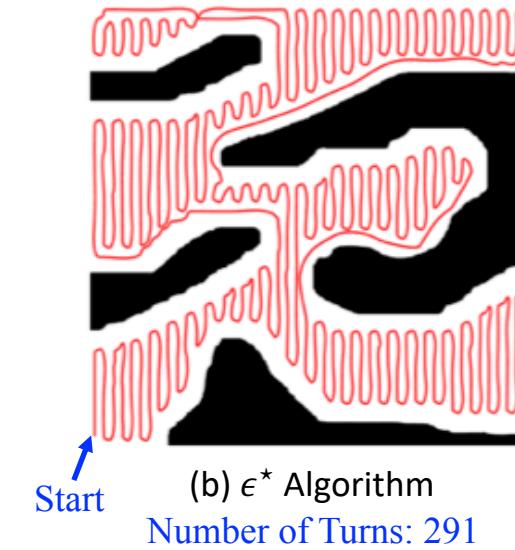
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Coverage Path Planning Algorithms

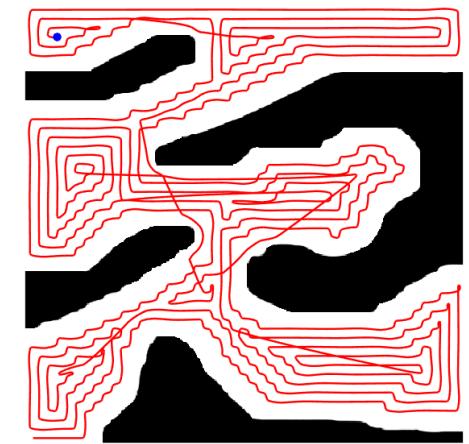
State-of-the-art and Novel Contributions

❖ Existing Approaches:

- Learning Real-time A* (LRTA*)
 - Spanning-tree Coverage
 - Backtracking Spiral Algorithm
 - Brick-and-Mortar Algorithm
 - Cellular Decomposition (*back-and-forth* path)
 - Rely on detection of critical points (detection and pairing of IN & OUT critical points are difficult in complex environment)
 - Require cycle algorithm which leads to overlappings
 - Cannot work in rectilinear environment
- Strong path overlappings*
Generate spiral path with too many turns

(a) Cellular Decomposition based Method^[1]

Number of Turns: 348



❖ Features and Novel Contributions of the ϵ^* Algorithm:

- Produces the desired *back-and-forth* path
- **Does not need critical point detection on obstacles**
- Guarantees complete coverage and prevents the local extrema problem using hierarchical potential surfaces (called MAPS)
- Capable of adapting sweep direction in known sub-regions to further reduce the number of turns
- Computationally efficient for real-time applications



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ϵ^* Algorithm

Tiling of the Search Area

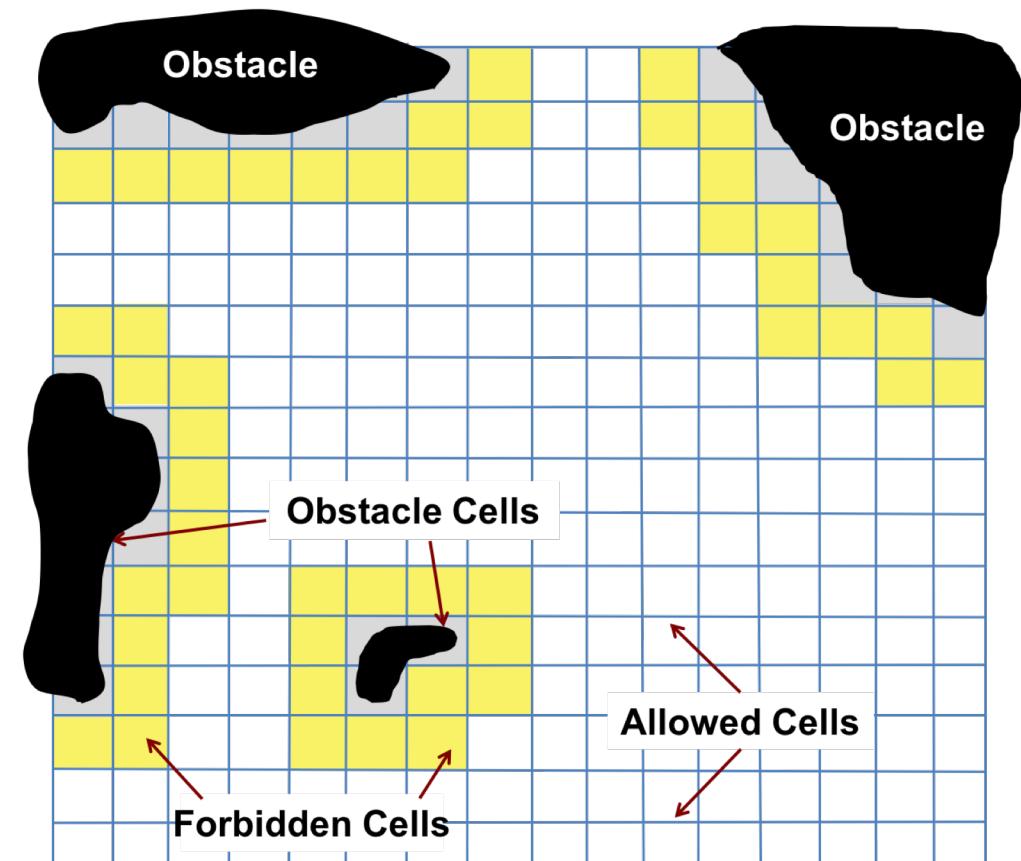
- ❖ Let $\mathcal{R} \subset \mathbb{R}^2$ be the estimated area that includes the desired area to cover.

Tiling: The set $T = \{\tau_\alpha \subset \mathbb{R}^2 : \alpha = 1 \dots |T|\}$ is called a tiling of \mathcal{R} if its elements:

- i) have mutually exclusive interiors, i.e., $\tau_\alpha^o \cap \tau_\beta^o = \emptyset, \forall \alpha \neq \beta$, where $\alpha, \beta \in \{1 \dots |T|\}$.
- ii) form a minimal cover, i.e., $\mathcal{R} \subseteq \bigcup_{\alpha=1}^{|T|} \tau_\alpha$, while removal of any tile destroys the covering property.

ϵ Cell: Each element $\tau_\alpha, \forall \alpha \in \{1, \dots, |T|\}$, is called an ϵ -cell.

- ❖ The tiling T is partitioned into three subsets:
 - **Obstacle cells** (T^o): they are detected online.
 - **Forbidden cells** (T^f): create buffer around obstacles
 - **Allowed cells** (T^a): these are the target cells to cover



Tiling of the Search Area

ϵ^* Algorithm

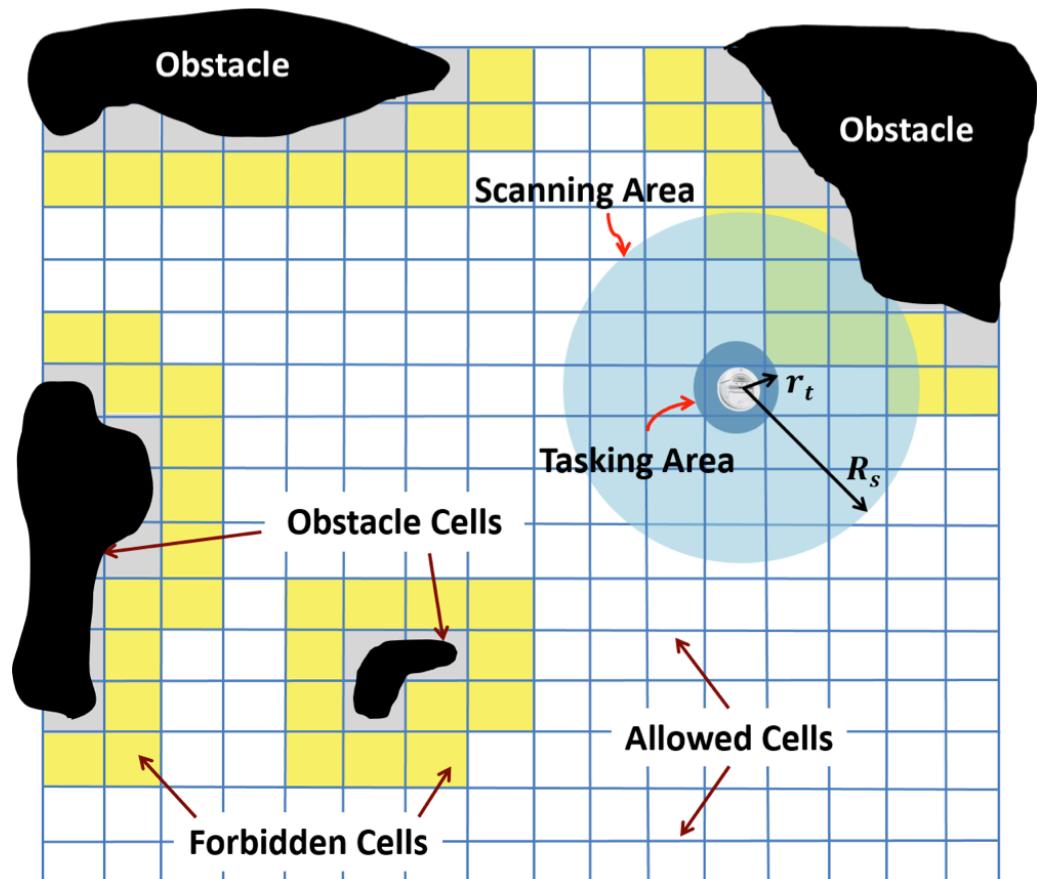
The Autonomous Vehicle and ϵ -Coverage

❖ **The autonomous vehicle is equipped with:**

1. Localization System
 - Provides vehicle location (e.g., GPS), and heading (e.g., Compass)
2. Range Detector with Sensing Radius R_s
 - Allows the vehicle to detect obstacles in the local neighborhood (e.g., laser)
3. Tasking Sensor with Radius r_t
 - Allows the vehicle to carry out certain tasks (e.g., cleaning, target detection, crops cutting) while it operates in the field

ϵ -Coverage Let $\mathcal{R}(T^a)$ denote the total area of the allowed cells. Let $\tau(k) \in T^a$ be the ϵ -cell visited by the autonomous vehicle at time k and explored by its tasking sensor. Then, \mathcal{R} is said to achieve ϵ -coverage, if $\exists K \in \mathbb{Z}^+$, such that the sequence $\{\tau(k), k = 1, \dots, K\}$ covers $\mathcal{R}(T^a)$, i.e.,

$$\mathcal{R}(T^a) \subseteq \bigcup_{k=1}^K \tau(k)$$



The autonomous vehicle and the tiling



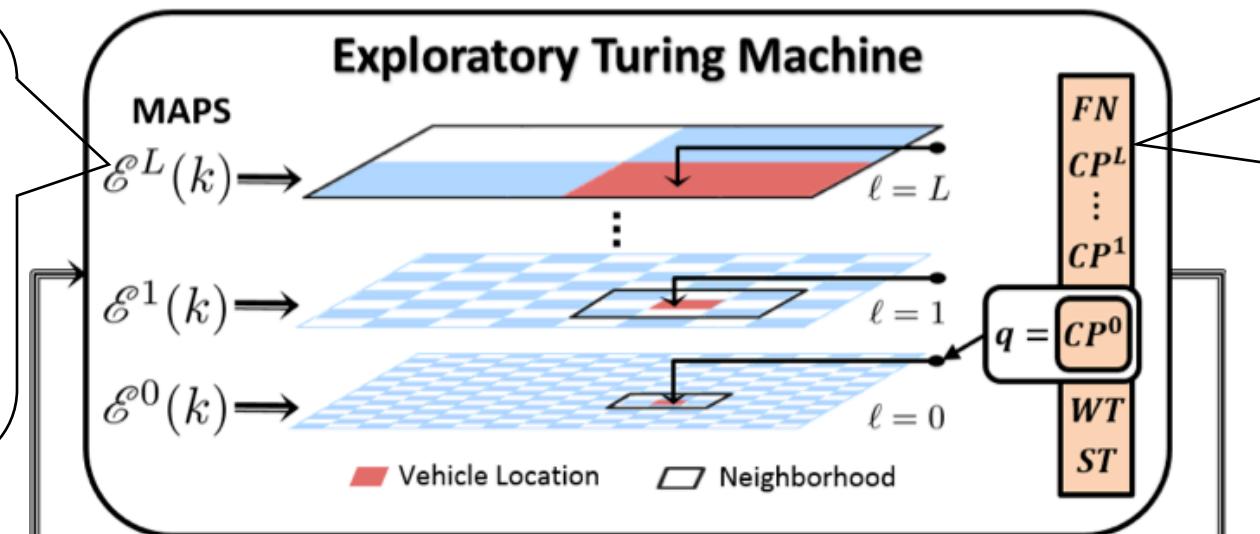
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The Supervisory Controller: Exploratory Turing Machine (ETM)

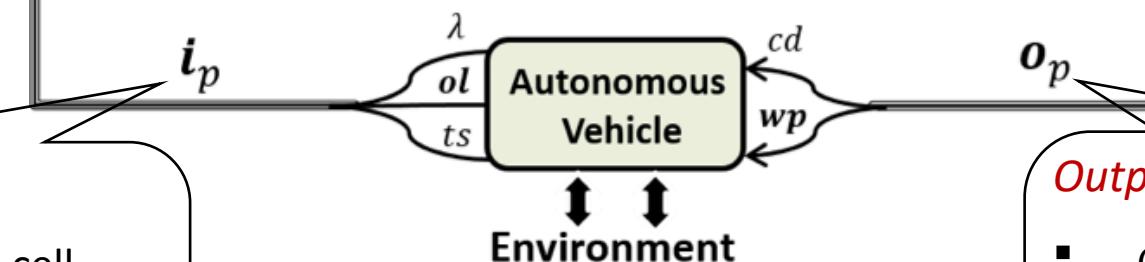
Multi-scale Adaptive Potential Surfaces(MAPS)

- \mathcal{E}^0 : time-varying potential surface at the finest level
- $\mathcal{E}^\ell, 1 \leq \ell \leq L$: time-varying potential surfaces at higher levels



Input Vector i_p

- $\lambda \in \{1, \dots, |T|\}$: index of current cell
- $ol \subset \{1, \dots, |T|\}$, indices of obstacle cells
- $ts \in \{cm, ic\}$, tasking status, where $cm \equiv Complete$, or $ic \equiv Incomplete$.



Output Vector o_p

- $cd \in \{mv, tk, id, sp\}$: command to the vehicle, where: $mv \equiv Move$, $tk \equiv Task$, $id \equiv Idle$, and $sp \equiv Stop$.
- $wp \subset \{1, \dots, |T|\}$, candidate set of new waypoints for the vehicle



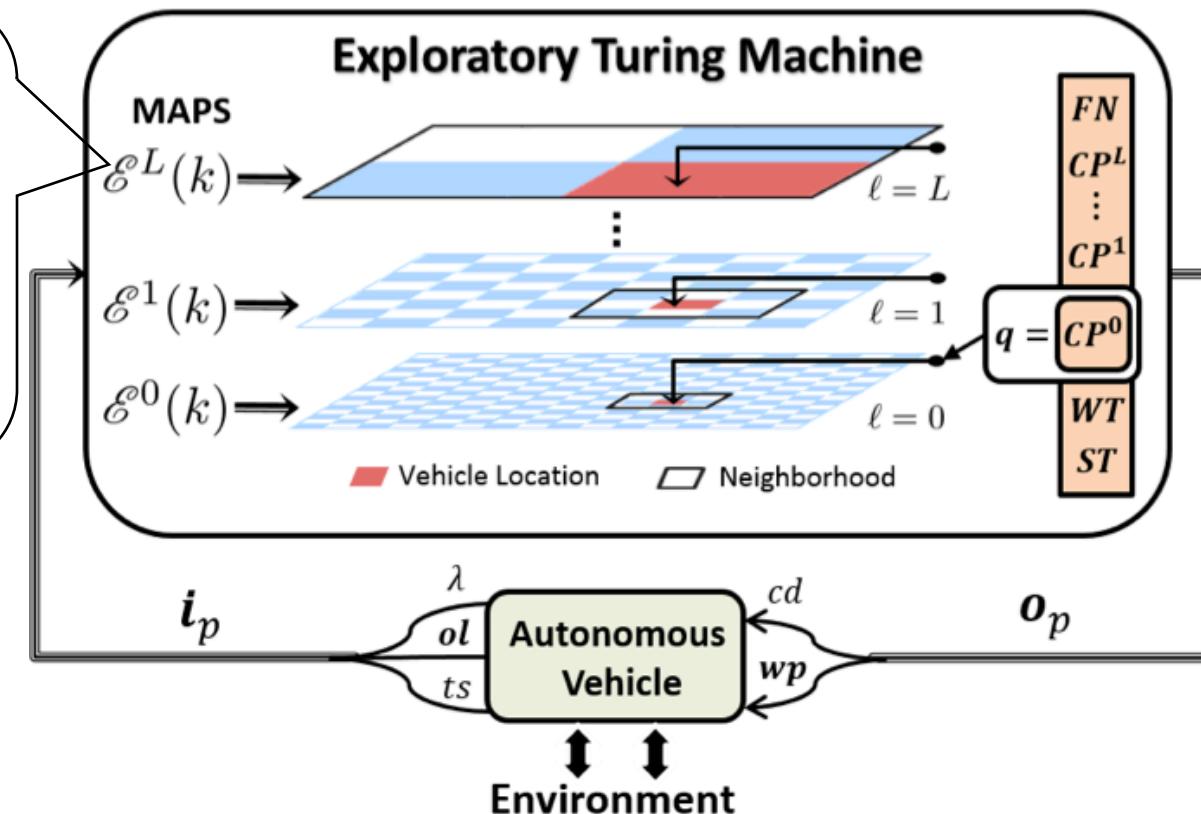
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The Supervisory Controller: Exploratory Turing Machine (ETM)

Multi-scale Adaptive Potential Surfaces(MAPS)

- \mathcal{E}^0 : time-varying potential surface at the finest level
- $\mathcal{E}^\ell, 1 \leq \ell \leq L$: time-varying potential surfaces at higher levels





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Dynamically Constructed Multi-scale Potential Surfaces (MAPS)

❖ Level 0 of MAPS

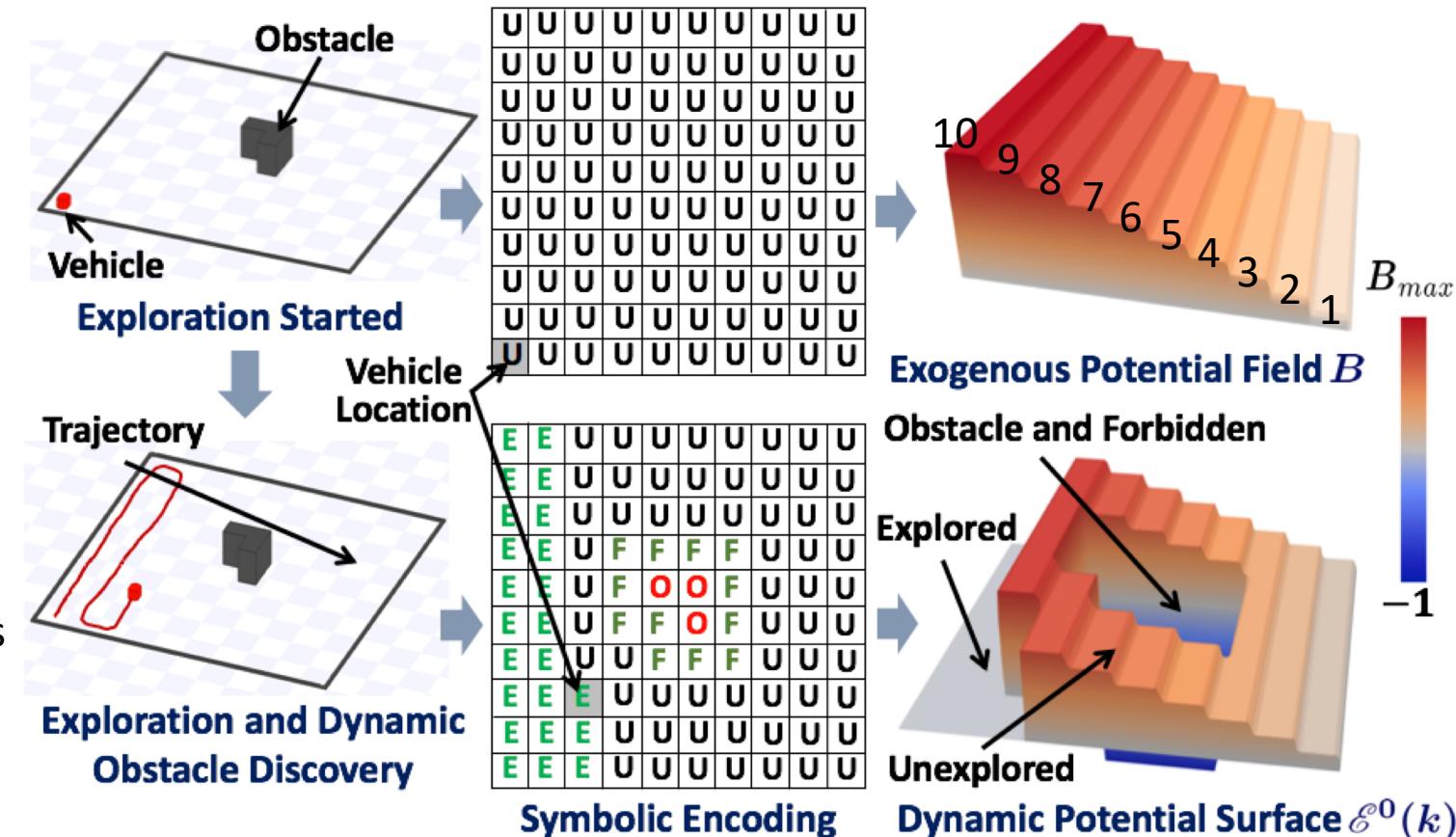
- **Symbolic Encoding:** each ϵ -cell at level 0, τ_{α^0} , is assigned with a symbolic state s_{α^0} , from below:

- O : Obstacle
 - F : Forbidden
 - E : Explored
 - U : Unexplored
- } Allowed cells

- **Potential Surface \mathcal{E}^0 :**

$$\mathcal{E}_{\alpha^0}(k) = \begin{cases} -1 & \text{if } s_{\alpha^0} = O \text{ or } F \\ 0 & \text{if } s_{\alpha^0} = E \\ B_{\alpha^0} & \text{if } s_{\alpha^0} = U \end{cases}$$

where $B = \{B_{\alpha^0} \in \{1, \dots, B_{\max}\}, \alpha^0 = 1, \dots, |T^0|\}$ is a time-invariant exogenous potential field. It is designed *offline* to have plateaus of equipotential surfaces along each column of the tiling.





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ϵ^* Algorithm

Dynamically Constructed Multi-scale Potential Surfaces (MAPS)

Note: Higher levels of MAPS are used to prevent the local extrema problem.

Local Extrema: no unexplored cells are available in the local neighborhood on Level 0.

❖ Levels $\ell = 1, 2, \dots L$ of MAPS

- **Potential Surfaces** \mathcal{E}^ℓ , $\ell = 1, \dots L$, are constructed by assigning τ_{α^ℓ} the *average* potential generated by all the *unexplored* ϵ -cells within τ_{α^ℓ} , such that

$$\mathcal{E}_{\alpha^\ell}(k) = p_{\alpha^\ell}^U(k) \cdot \bar{B}_{\alpha^\ell}$$

where \bar{B}_{α^ℓ} is the mean exogenous potential of τ_{α^ℓ} , and $p_{\alpha^\ell}^U(k)$ is the probability of *unexplored* ϵ -cells in τ_{α^ℓ} .



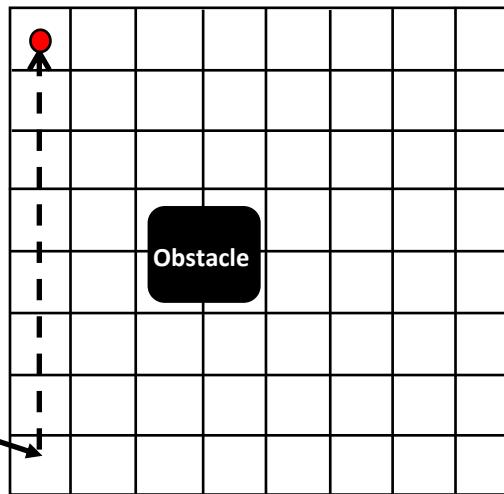
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ϵ^* Algorithm

An Illustrative Example: Updates of MAPS at Level 0

❖ An Illustrative Example

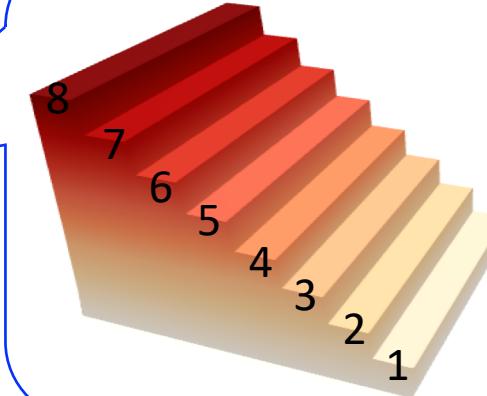
Vehicle trajectory and obstacle layout



Potentials B at Level 0

8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1

Visualization of B



MAPS update on Level 0

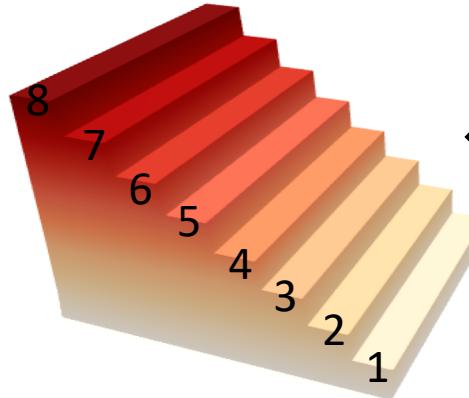
0	7	6	5	4	3	2	1
0	7	6	5	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	7	6	5	4	3	2	1
0	7	6	5	4	3	2	1



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An Illustrative Example: Updates of MAPS at Level 1

Visualization of B Potentials B at Level 0

8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1

Symbolic encodings at Level 0

E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U
E	F	F	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	F	F	U	U	U	U
E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U

Average potential \bar{B}

7.5	5.5	3.5	1.5
7.5	5.5	3.5	1.5
7.5	5.5	3.5	1.5
7.5	5.5	3.5	1.5
7.5	5.5	3.5	1.5

Update probabilities $p_{\alpha^\ell}^U$

0.5	1	1	1
0	0	1	1
0	0	1	1
0.5	1	1	1

MAPS update on Level 1

3.75	5.5	3.5	1.5
0	0	3.5	1.5
0	0	3.5	1.5
3.75	5.5	3.5	1.5



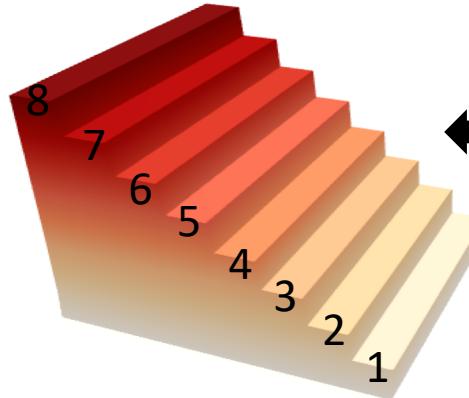


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An Illustrative Example: Updates of MAPS at Level 2

Visualization of B



Potentials B at Level 0

8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1

Symbolic encodings at Level 0

E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U
E	F	F	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	F	F	U	U	U	U
E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U

Average potential \bar{B}

6.5		2.5
6.5		2.5

Update probabilities $p_{\alpha^\ell}^U$

0.375		1
0.375		1

MAPS update on Level 2

MAPS update

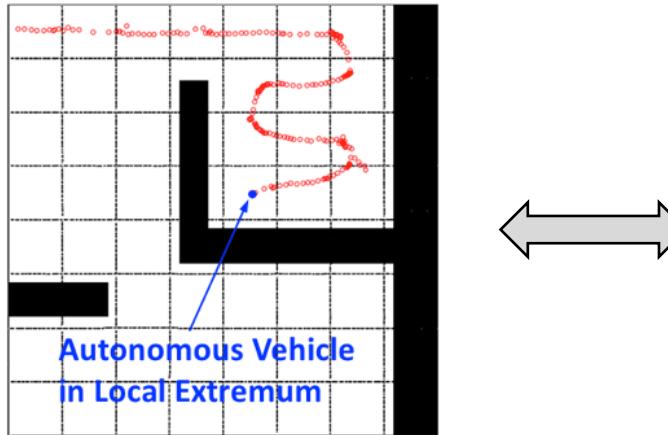
2.4375		2.5
2.4375		2.5



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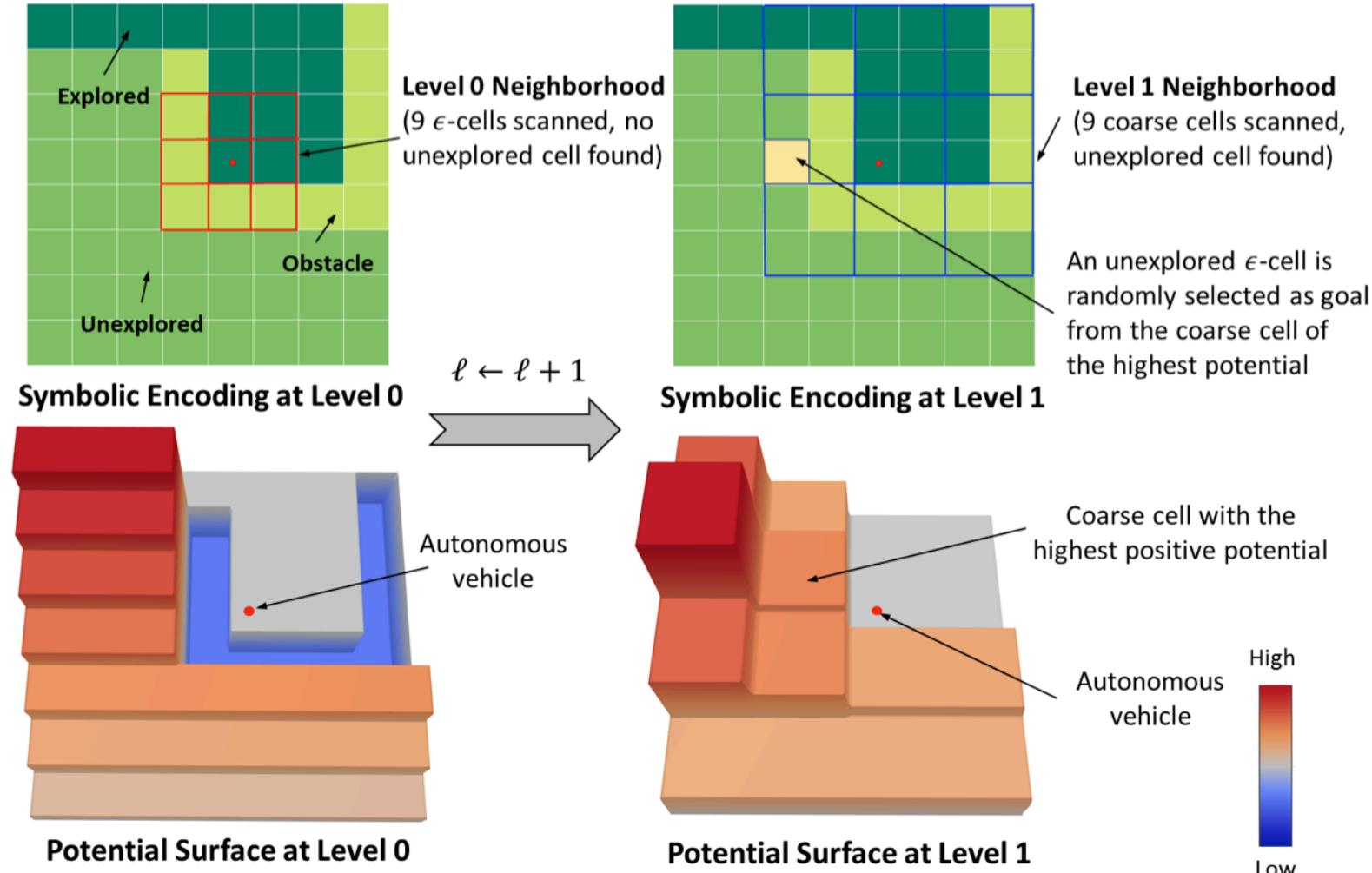
ϵ^* Algorithm

An Example of using MAPS to Prevent the Local Extrema Situation



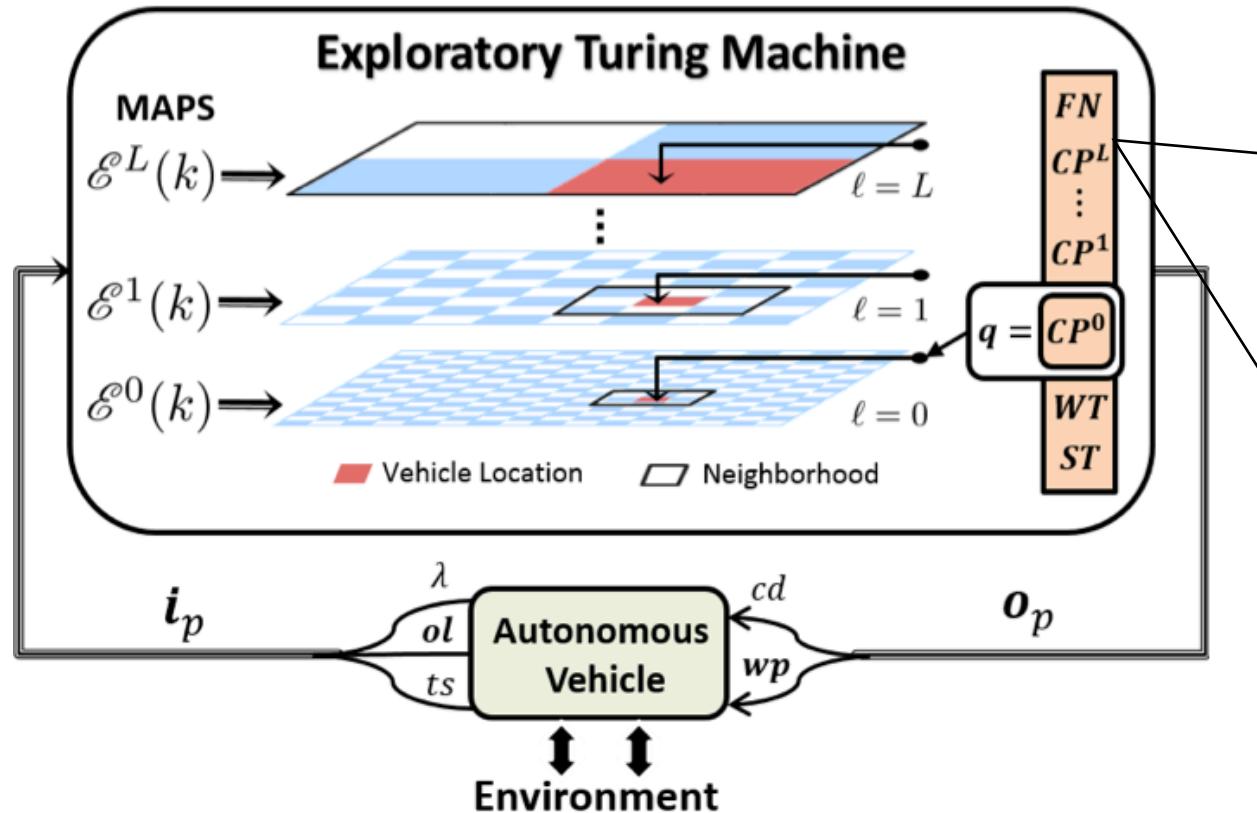
Local Extrema: no unexplored cells are available in the local neighborhood on Level 0.

Low Complexity: even in the worst-case scenario, it only takes $O(|N^0| + L \cdot |N^\ell|)$ to find waypoints, where N^ℓ is the local neighborhood on Level ℓ of MAPS, $\ell = 0, 1, \dots, L$.



ϵ^* : The Supervisory Control Structure

The Exploratory Turing Machine (ETM)



Machine States

- **The Start State (ST):** start the machine and initialize the MAPS with all ϵ -cells as unexplored.
- **The Computing States (CP):**
 - CP^0 : compute waypoint wp using Level 0 of MAPS, and send navigation command cd .
 - $CP^1, CP^2 \dots, CP^L$: sequentially used to compute wp in case of a *local extremum*.
- **The Waiting State (WT):** wait for the vehicle to complete specific task (e.g., cleaning) in the current cell, until the status ts turns to *complete*
- **The Finished State (FN):** terminate the operation upon complete coverage.



ϵ^* Algorithm

The State Transition Graph

Conditions:

$$\mathcal{A}: wp = \emptyset; \quad \neg\mathcal{A}: wp \neq \emptyset$$

$$\mathcal{B}: wp \equiv \lambda; \quad \neg \mathcal{B}: wp \not\equiv \lambda$$

C: $ts = cm$:

$$i_{p_1} = (\lambda, ol, -); i_{p_2} = (-, -, ts)$$

Output Vectors:

$$\underline{\boldsymbol{o}_{p_1} = (id, -)}; \quad \boldsymbol{o}_{p_2} = (tk, -)$$

$$\mathbf{o}_{p_3} = (mv, \mathbf{w}p); \mathbf{o}_{p_4} = (sp, -)$$

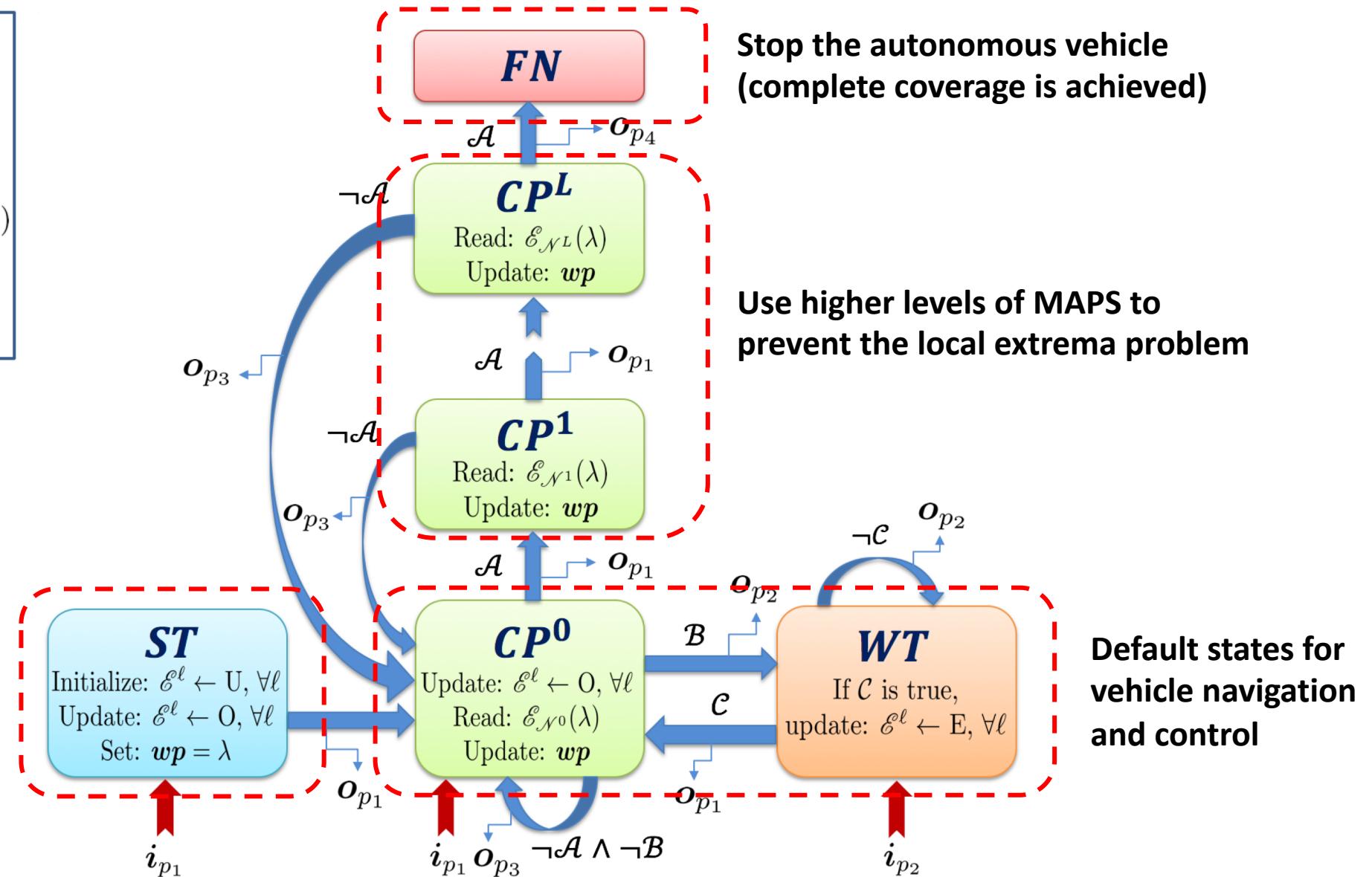
Legend:

↑ Input Vector

Output Vector

- Output vector
- State Transition

System initialization



When is it reached?

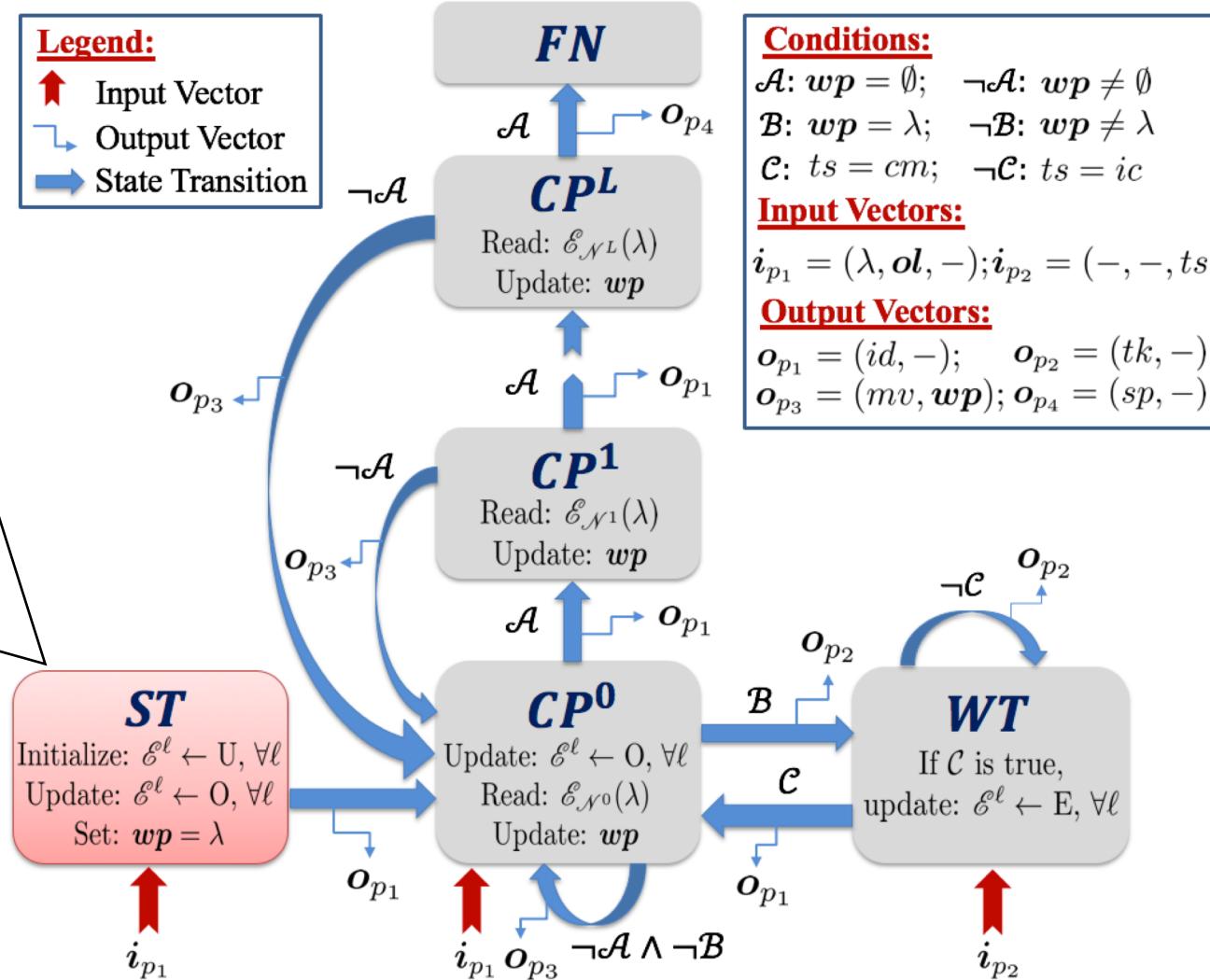
As soon as the autonomous vehicle is turned on.

What does it do?

Initialization of the system.

Operation in the *ST* State:

- **Initialization:**
 - MAPS \mathcal{E}^ℓ , $\forall \ell = 0, 1, \dots L$:
 - Level 0: initialized with U , i.e. *Unexplored*.
 - Level $1 \leq \ell \leq L$: all coarse cells $\tau_{\alpha^\ell} \in T^\ell$ are assigned potentials by substituting $p_{\alpha^\ell}^U(0) = 1$.
 - Initialize wp as the current cell λ .
 - **Input:** the input vector i_{p_1} contains the current vehicle location λ and detected obstacle locations ol
 - **Output:** Set the vehicle to idle via output vector o_{p_1} .



ϵ^* Algorithm

Operation of the ETM: The CP^0 State

When is it reached?

Either after system initialization, or when the current cell has just been tasked and needs a new wp .

What does it do?

Default state to compute for wp on Level 0 of MAPS.

Operation in the CP^0 State:

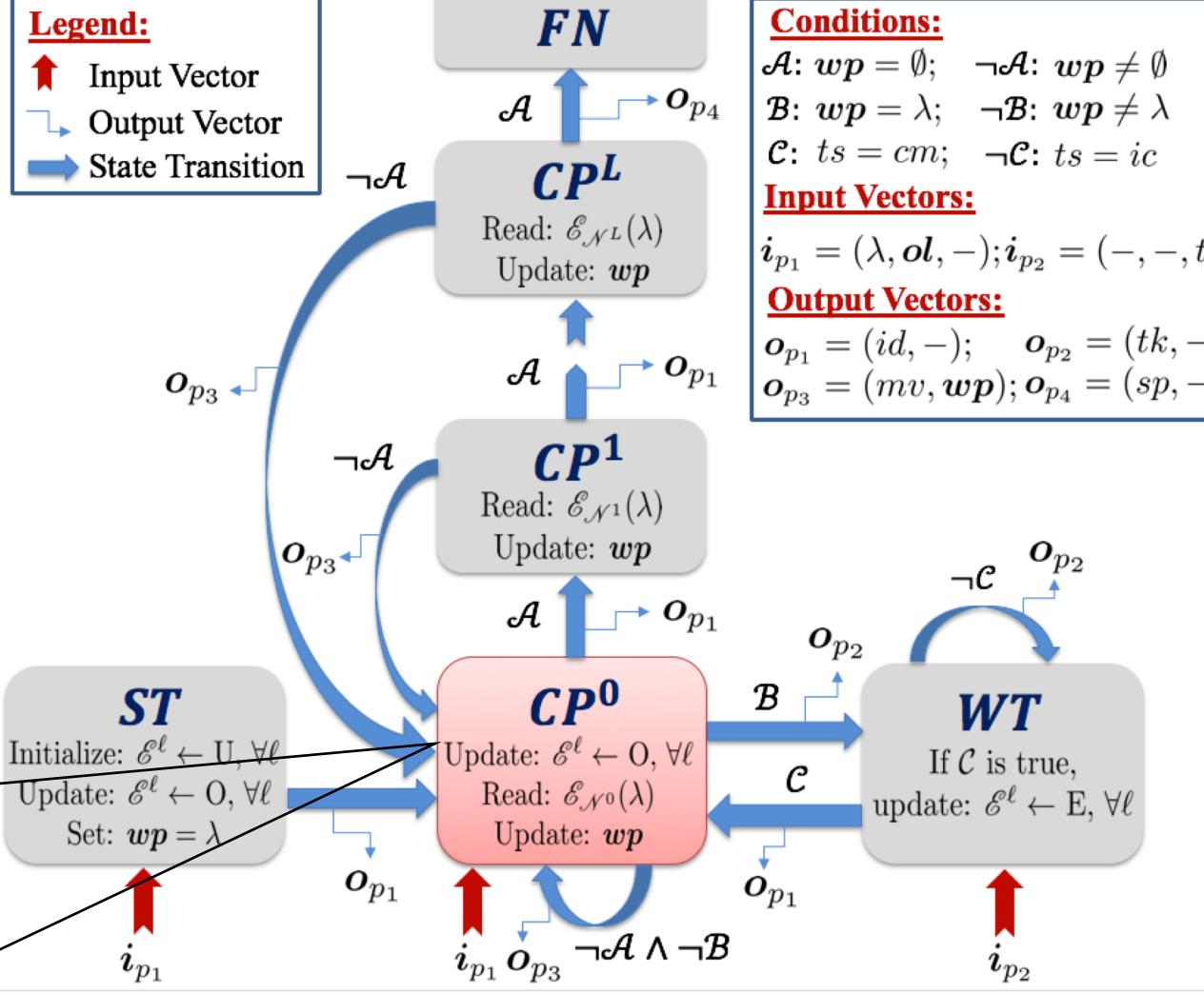
- Input:** the input vector i_{p_1} contains vehicle location λ , obstacle locations ol ; they are used to update potential surfaces $\mathcal{E}^\ell, \forall \ell$.
- Compute wp:** the directly reachable neighbor cell with the highest positive potential in the neighborhood N^0 :

$$wp(k) = \text{arcmax}_{\alpha^0 \in N^0} \mathcal{E}_{\alpha^0}$$

- Output:** If wp is found, send vector o_{p_3} to move vehicle to wp ; and upon reaching, send vector o_{p_2} to start tasking.
 - If wp not found, switch to state CP^1

Legend:

- ↑ Input Vector
- Output Vector
- ➡ State Transition



Conditions:

$$\mathcal{A}: wp = \emptyset; \quad \neg \mathcal{A}: wp \neq \emptyset$$

$$\mathcal{B}: wp = \lambda; \quad \neg \mathcal{B}: wp \neq \lambda$$

$$\mathcal{C}: ts = cm; \quad \neg \mathcal{C}: ts = ic$$

Input Vectors:

$$i_{p_1} = (\lambda, ol, -); i_{p_2} = (-, -, ts)$$

Output Vectors:

$$o_{p_1} = (id, -); \quad o_{p_2} = (tk, -)$$

$$o_{p_3} = (mv, wp); \quad o_{p_4} = (sp, -)$$



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Operation of the ETM: The *WT* State

When is it reached?

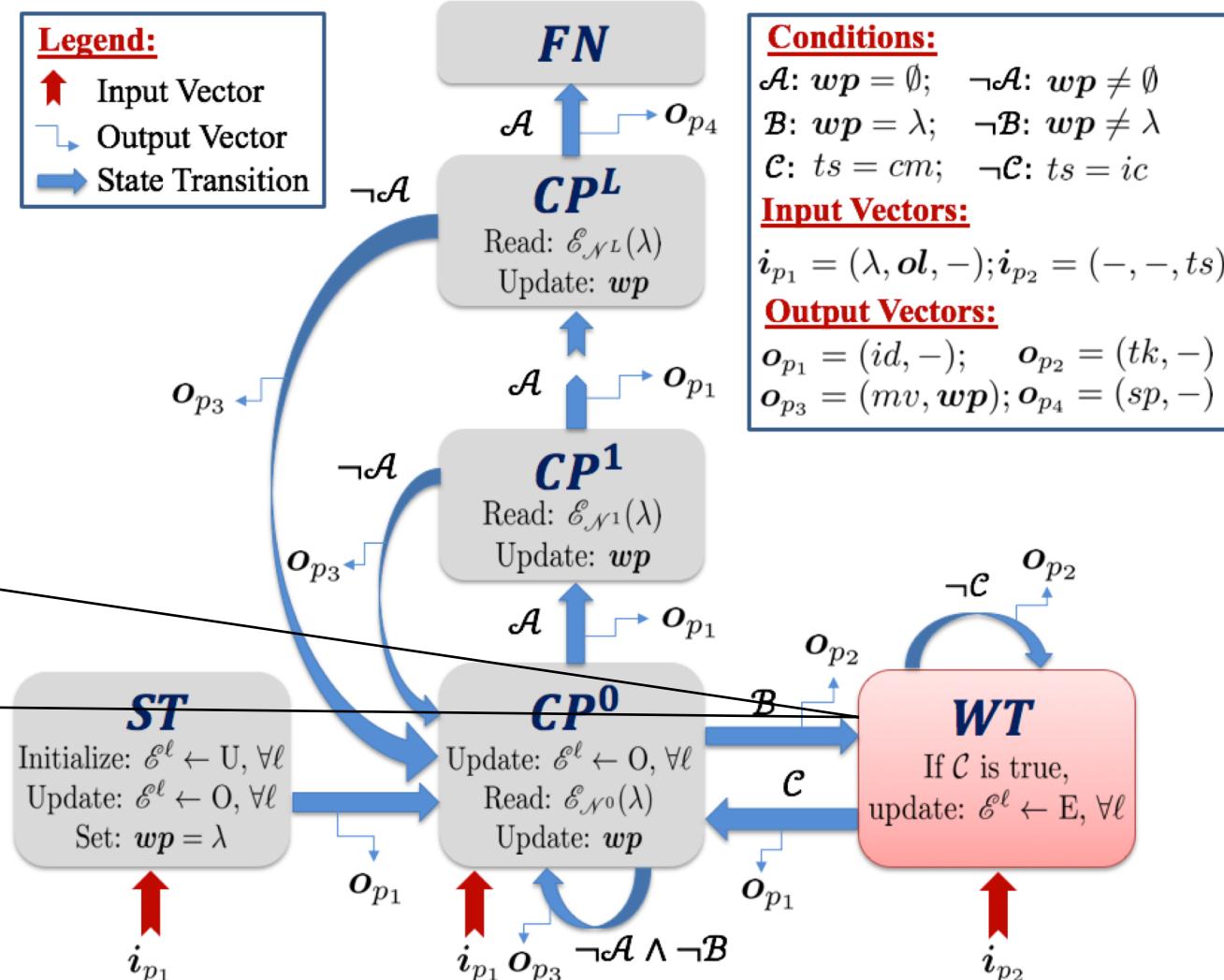
When the autonomous vehicle reaches the computed wp .

What does it do?

Command the vehicle to perform tasking (e.g., cleaning) in the current cell.

Operation in the *WT* State

- Input:** the input vector i_{p_2} contains task status ts ; if it is *complete*, update current cell λ as E , i.e., *Explored*; then update potential surfaces $\mathcal{E}^\ell, \forall \ell$.
- Output:**
 - If task status ts is *complete*, go to state CP^0 to compute for the next wp
 - Otherwise, keep on waiting.





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Operation of the ETM: The $CP^1, CP^2, \dots CP^L$ States

When are they reached?

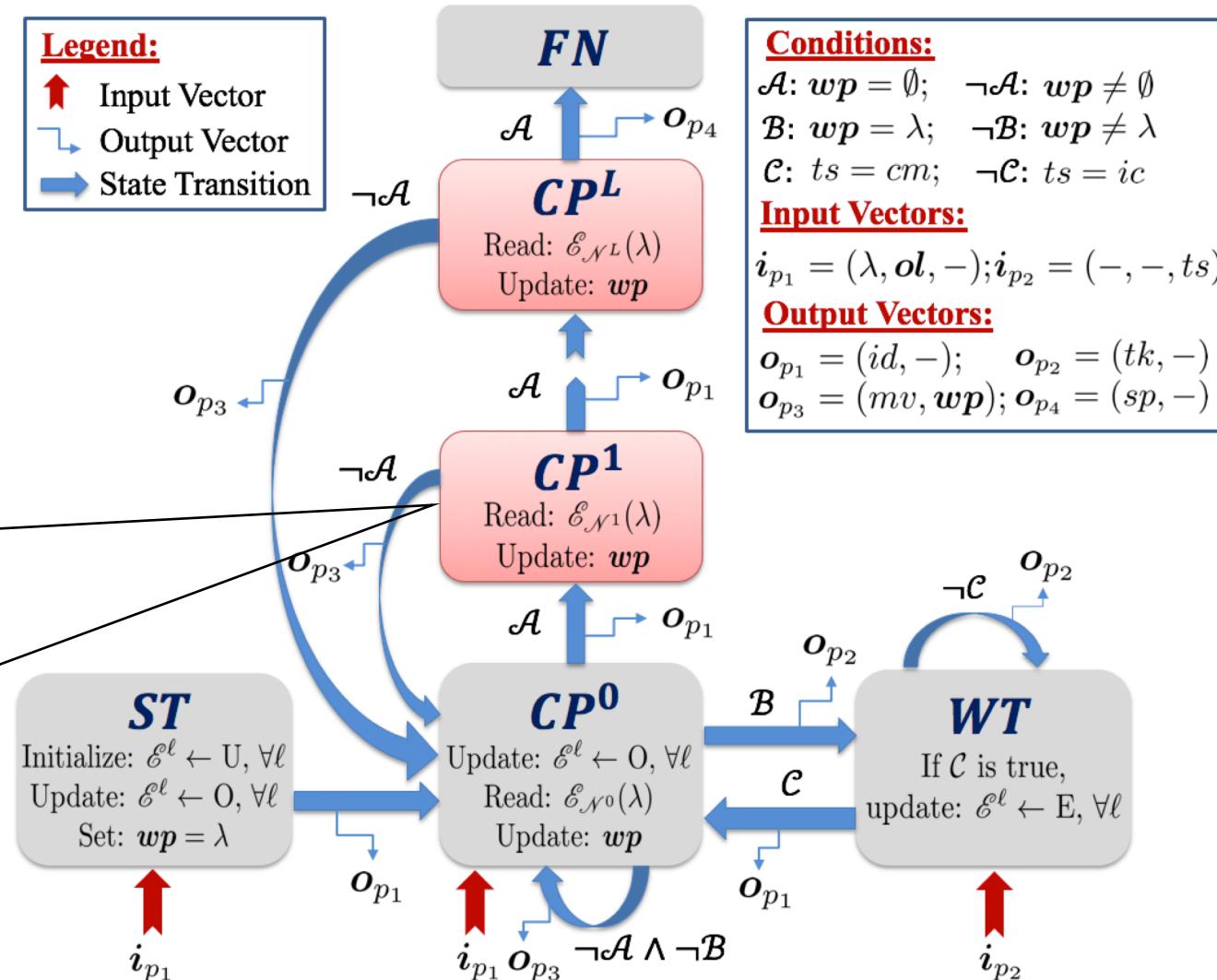
When waypoint wp cannot be found in CP^0 state.

What does it do?

Sequentially switches to higher levels of MAPS, until wp can be found at some Level $\ell \leq L$.

Operation in the $CP^\ell, \ell = 1, 2, \dots L$ States

- **Compute wp**
 - First, read the potentials in the local neighborhood on Level 1 (i.e., $\mathcal{E}_{N^1}(\lambda)$).
 - If $\exists \tau_{\alpha^1} \in N^1(\lambda)$ with positive potential, then wp is set as an unexplored ϵ -cell in τ_{α^1} .
 - Otherwise, go to CP^2 state and repeat above.
- **Output:** If wp is found, sends output o_{p_3} to move vehicle to wp ; otherwise, sends o_{p_1} to set vehicle idle.



When is it reached?

When wp cannot be found in CP^L state

What does it do?

Terminate operation since no unexplored cells are left.

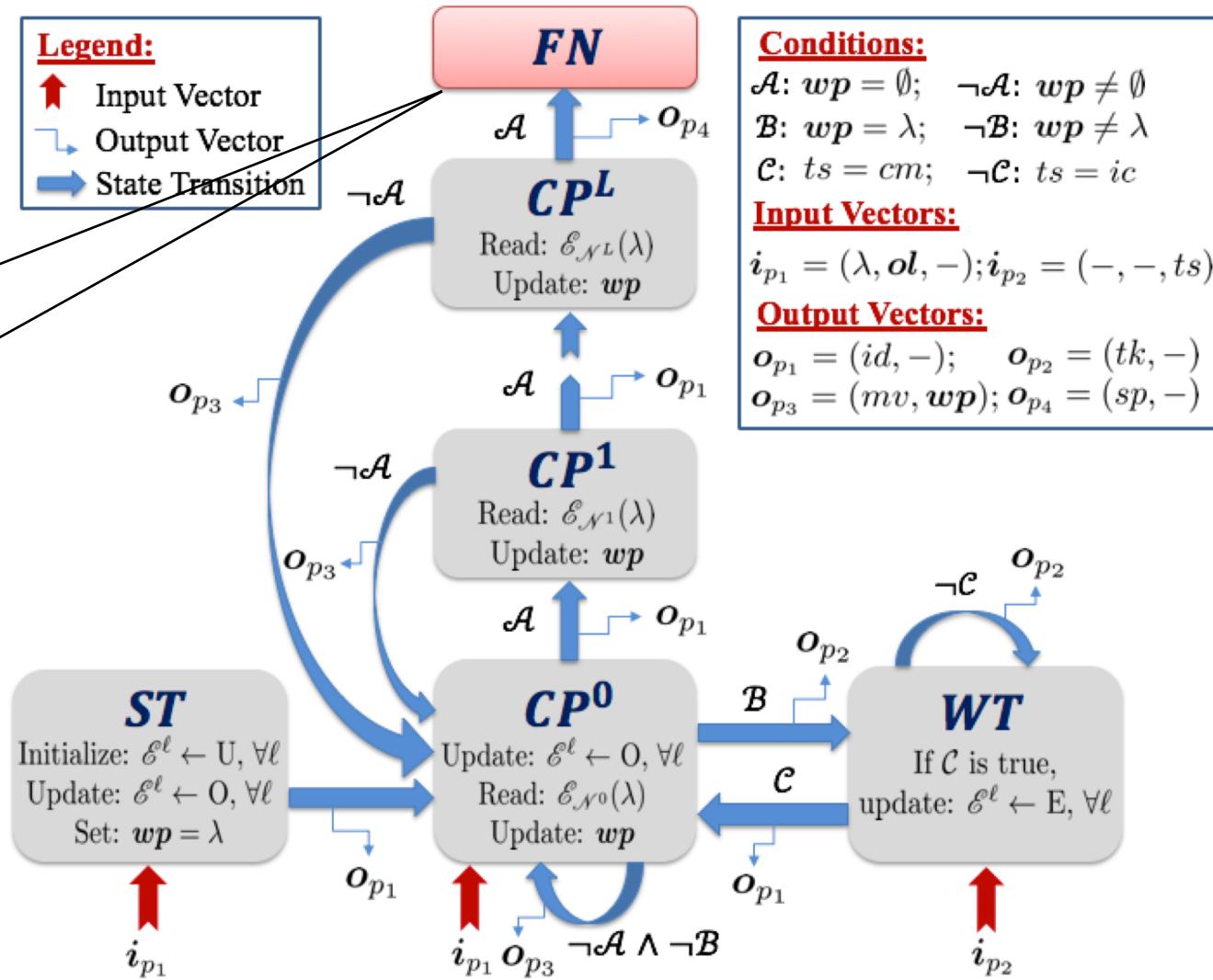
Operation in the *FN* State

- If $wp = \emptyset$ at Level L , it implies the ETM cannot find unexplored ϵ -cells at the highest Level L , then it reaches the FN state and the machinery is terminated.

Theorem: The ETM halts in finite time^[1]

Corollary 1: Each allowed ϵ -cell is tasked only once^[1].

Corollary 2: ϵ -coverage is achieved upon halting^[1].



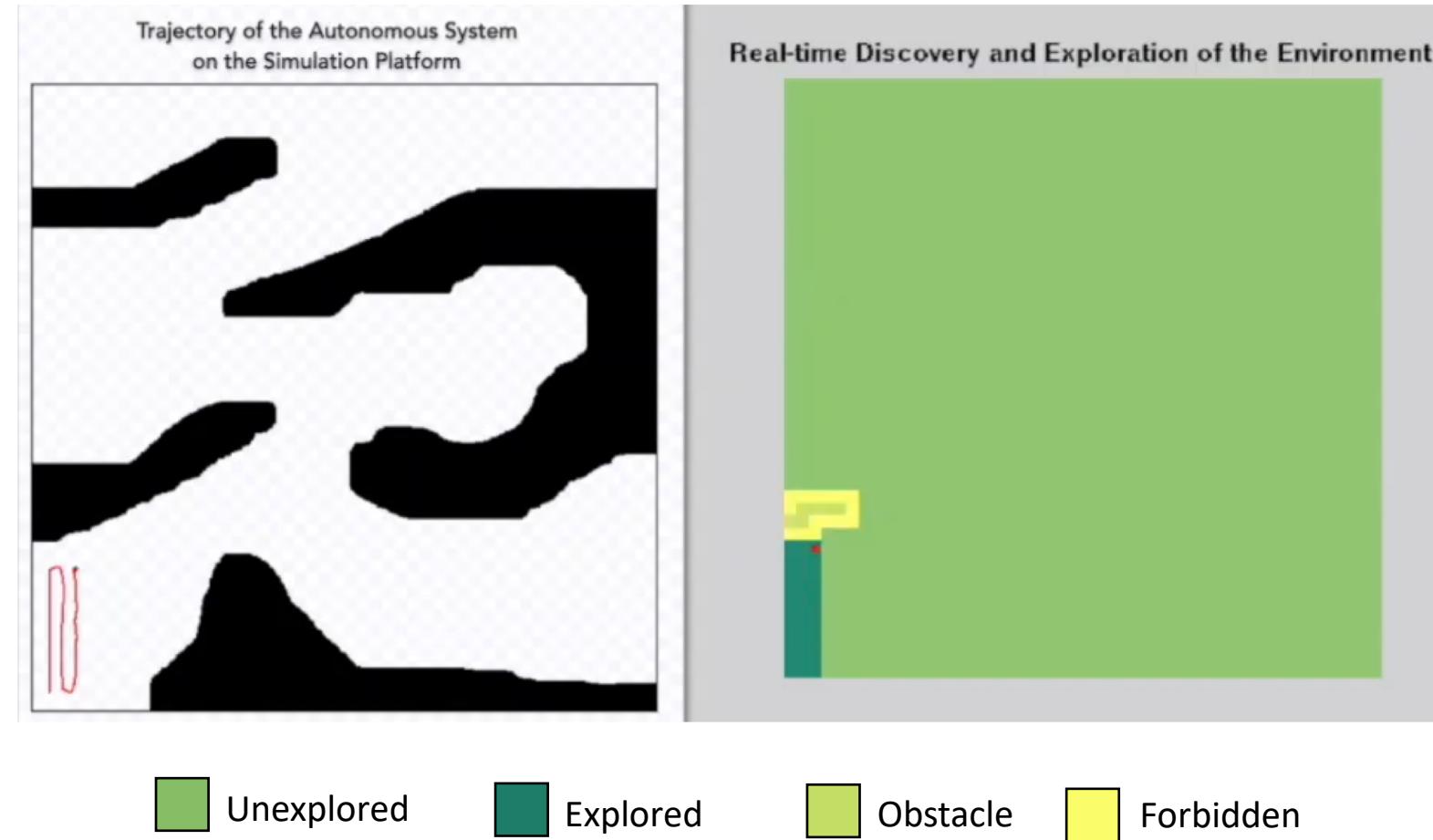
[1] J. Song and S. Gupta, “ ϵ^* : An Online Coverage Path Planning Algorithm”, IEEE Transactions on Robotics, Vol. 34, Issue 2, pp 526-533, 2018.

Simulation Validations

Validations on the Player/Stage Robotic Simulator

❖ Simulation Setup

- **Autonomous Vehicle:** a Pioneer AT2 of dimensions $0.44\text{m} \times 0.38\text{m} \times 0.22\text{m}$ was used with kinematic constraints of:
 - Top speed: 0.5m/s
 - Maximum acceleration: 0.5m/s^2
 - Minimum turning radius: 0.04m
- **Sensing systems**
 - Laser: detection range of 4m
- **Search Area:** the search area is of size $50\text{m} \times 50\text{m}$, which is partitioned into a 50×50 tiling consisting of ϵ -cells of size $1\text{m} \times 1\text{m}$. This results in MAPS with $L = 5$ levels.



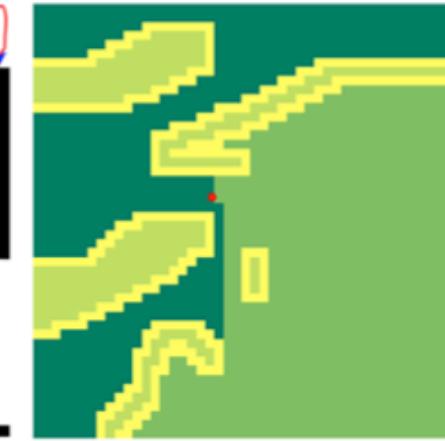
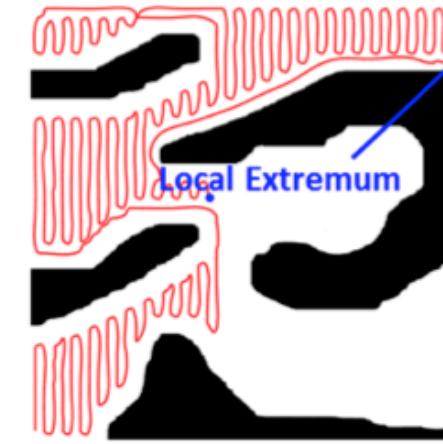
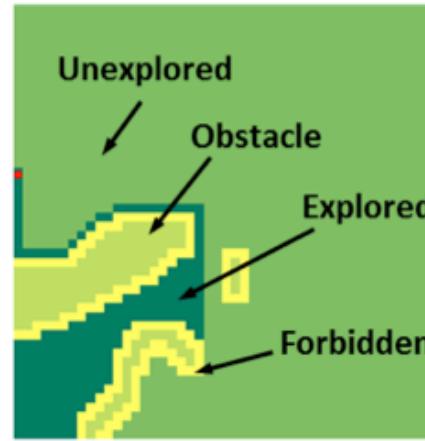
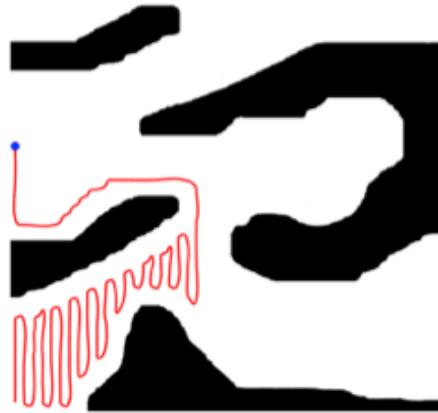


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Simulation Validations

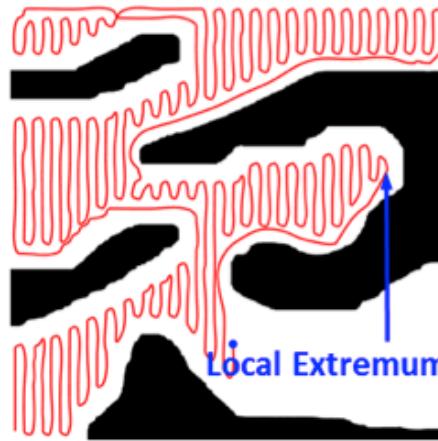
Scenario 1: Coverage Trajectories and Symbolic Encodings of the Environment

- ϵ^* incrementally builds the environment map, and complete coverage is achieved.

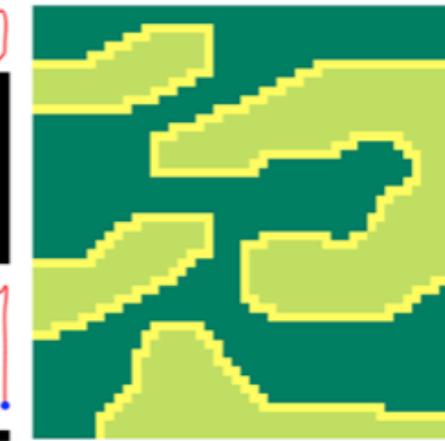
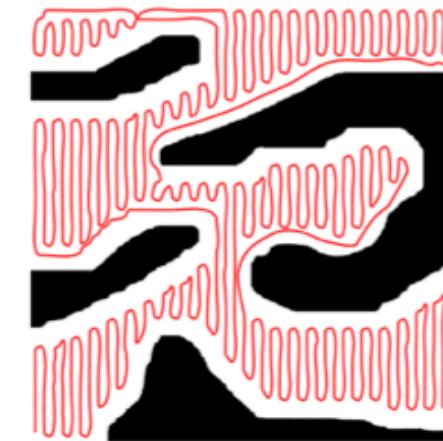


(1) Coverage started with dynamic obstacle discovery

(2) Escaping from a local extremum using MAPS



(3) Escaping from another local extremum



(4) Complete coverage achieved

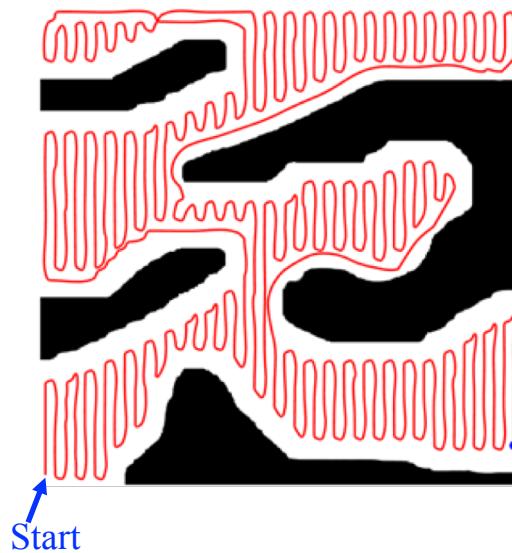


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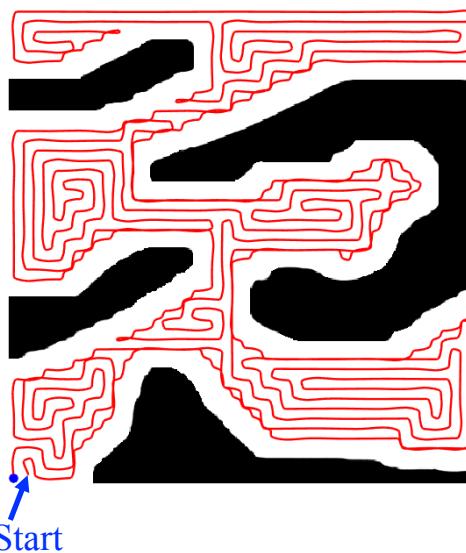
Simulation Validations

Scenario 1: Comparison with Alternative Methods

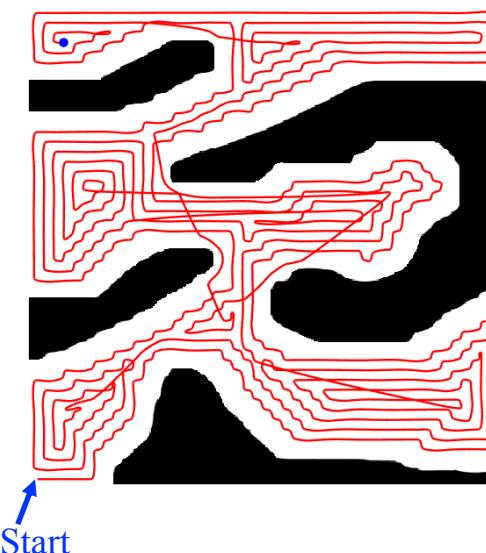
(a) ϵ^* Algorithm



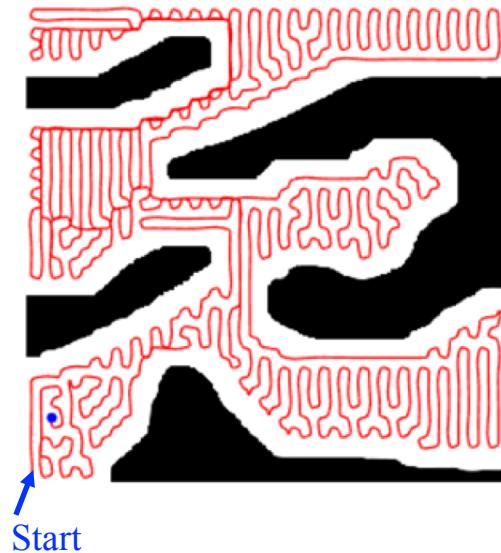
(b) Spanning Tree Coverage



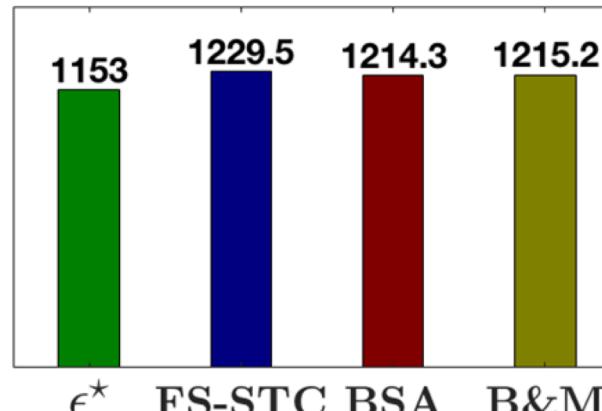
(c) Backtracking Spiral Algorithm



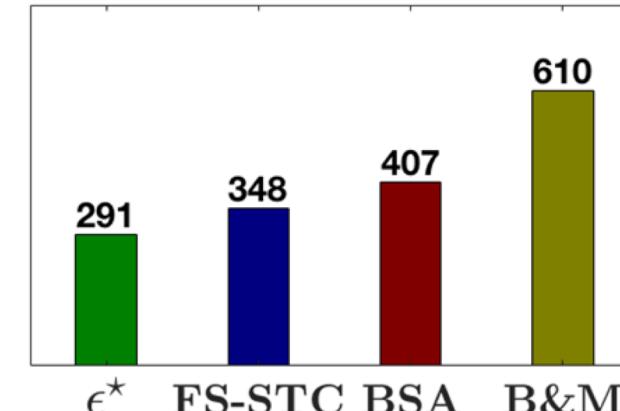
(d) Brick & Mortar



Trajectory length



Number of turns



Simulation Validations

Scenario 2: Adaptive Sweep Direction in Known Sub-regions

❖ User-controllable Sweep Direction

- If provided (partial) environment knowledge in sub-regions, ϵ^* can adapt the sweep direction to further reduce the number of turns.
- In Scenario 2 below, the layouts of all rooms are assumed **known**, but the inside obstacles are **unknown**.
- Then, the field B was designed in a manner such that the AV sweeps the top left room horizontally while the other two rooms vertically

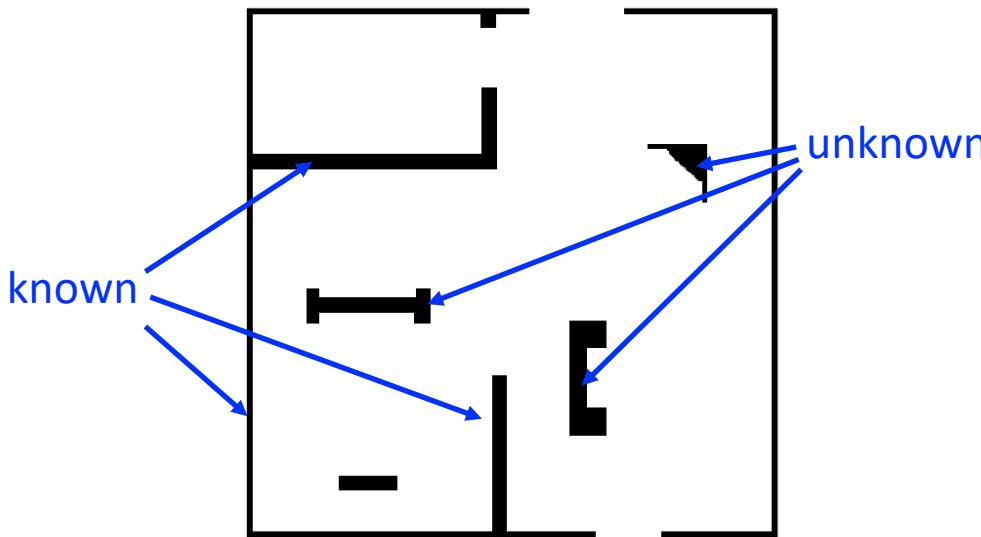


Fig. Scenario 2

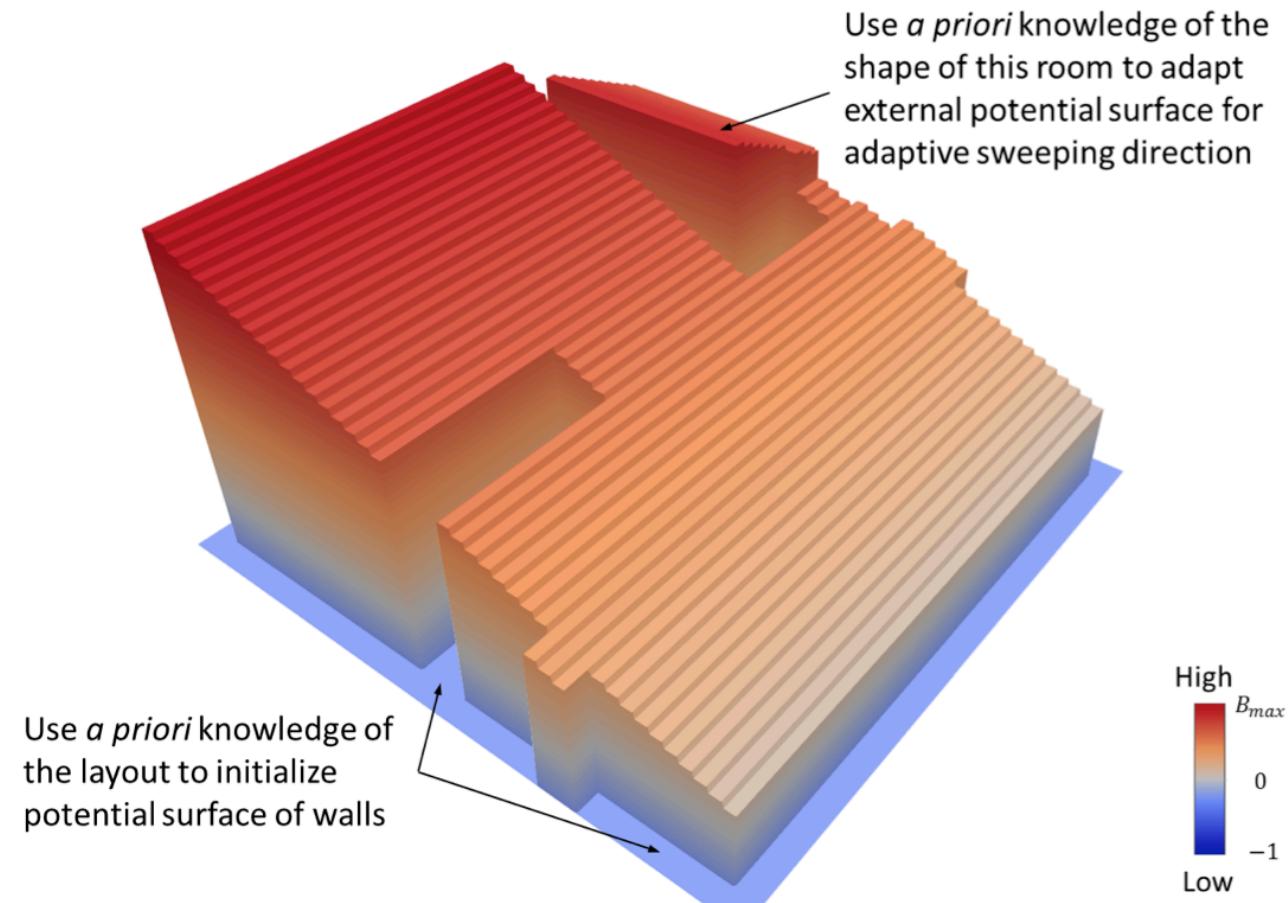


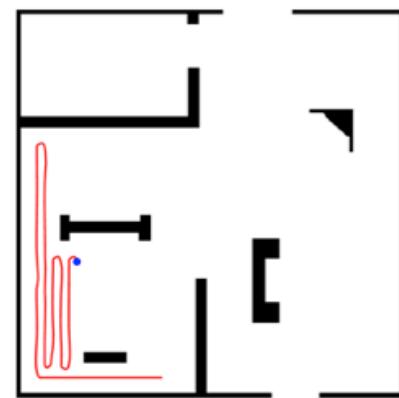
Fig. Exogeneous potential field B in Scenario 2

Simulation Validations

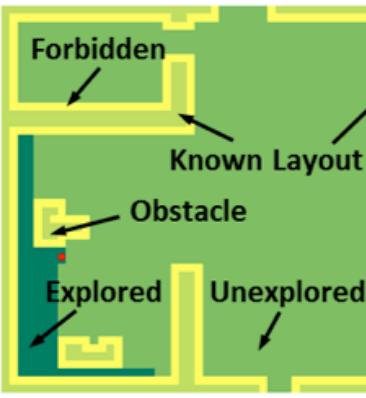
Scenario 2: Adaptive Sweep Direction in Known Sub-regions

- **User-controllable Sweep Direction:** If provided (partial) environment knowledge in sub-regions, the sweep direction can be adapted to further reduce the number of turns. This is done by altering the exogeneous potential field B .

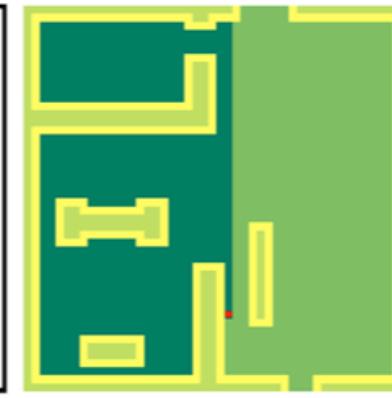
Scenario 2: Coverage trajectory of ϵ^* in an apartment scenario



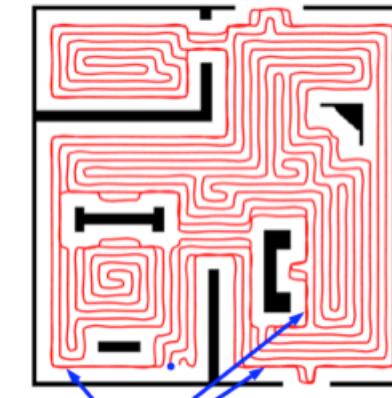
(1) Coverage started with dynamic obstacle discovery



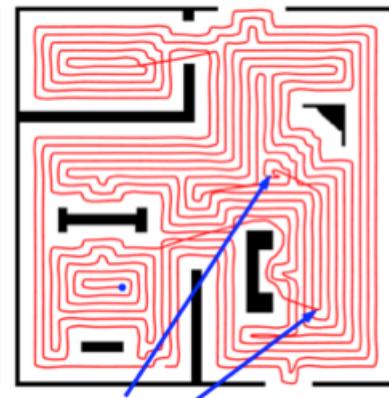
(2) Adaptive sweeping if layout is *a priori* known



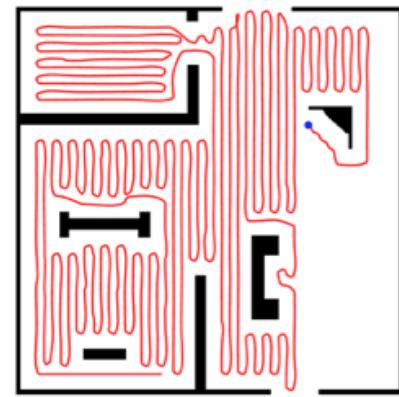
Trajectories of alternative methods



Overlaps FS-STC



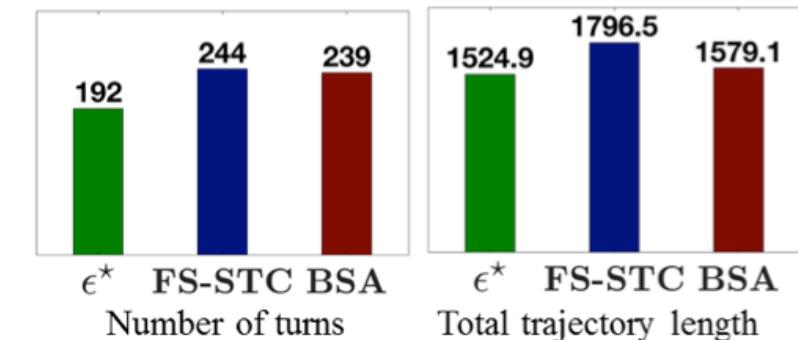
Spiral Endings



(3) Adapt to the shape of obstacle



(4) Complete coverage achieved



Performance Evaluation

Coverage Performance under Uncertainties

❖ Coverage Ratio r_c :

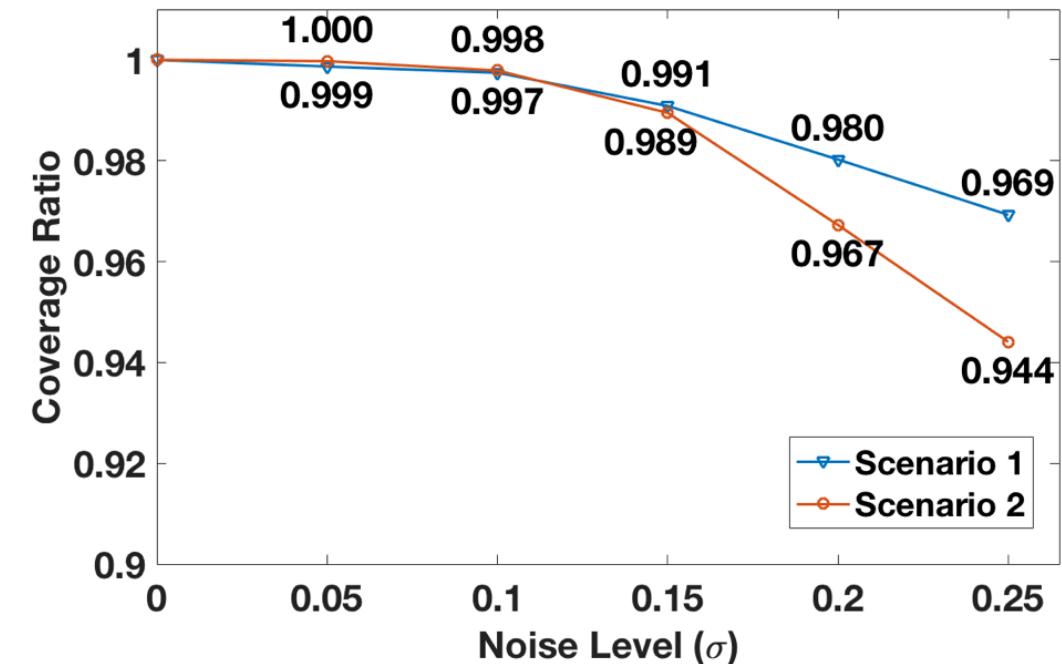
$$r_c = \frac{U_k \tau(k)}{\mathcal{R}(T^a)}$$

❖ Sources of Uncertainties:

- **Localization System:**
 - Outdoor: Real-time Kinematic (RTK) GPS can achieve an accuracy of 0.05m~0.5m^[1].
 - Indoor: Hagisonic StarGazer indoor localization system provides precision of 2cm.
- **Compass:** a modestly priced compass provides an accuracy of 1°^[1].
- **Laser Measurements:** a laser sensor typically admits an error of 1% of its operation range.

- ❖ **Monte Carlo Simulations:** The sensor noise are simulated as Additive White Gaussian Noise (AWGN), with:
- **Localization System:** $\sigma = 0.05\text{m}, 0.10\text{m}, \dots 0.25\text{m}$
 - **Compass:** $\sigma_{compass} = 0.5^\circ$
 - **Laser Measurements:** $\sigma_{laser} = 1.5\text{cm}$

Coverage ratio vs. noise for ten Monte Carlo runs



[1] L. Paull, S. Saeedi, M. Seto, and H. Li, "Auv navigation and localization: A review," IEEE Journal of Oceanic Engineering, vol. 39, no. 1, pp. 131–149, 2014.

[2] J. Palacin, J. A. Salse, I. Valganon, and X. Clua, "Building a mobile robot for a floor- cleaning operation in domestic environments," IEEE Transactions on Instrumentation and Measurement, vol. 53, no. 5, pp. 1418–1424, 2004.



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Performance Evaluation

Choosing a Proper Sized ϵ

❖ Selection of the Size of ϵ :

- Should be big enough to contain the autonomous vehicle, and small enough for the tasking sensor to be able to cover it.
- Within these two bounds, the choice of ϵ depends on the following factors:
 - Smaller ϵ : provides a better approximation of the search area and its obstacles.
 - Larger ϵ : reduces the computational complexity by requiring less number of ϵ -cells to cover the area and it also provides improved robustness to uncertainties for localization within a cell.

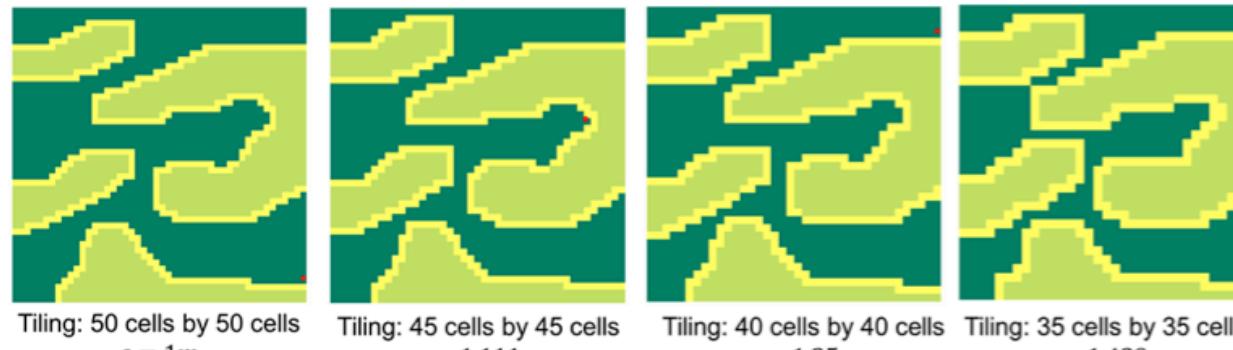
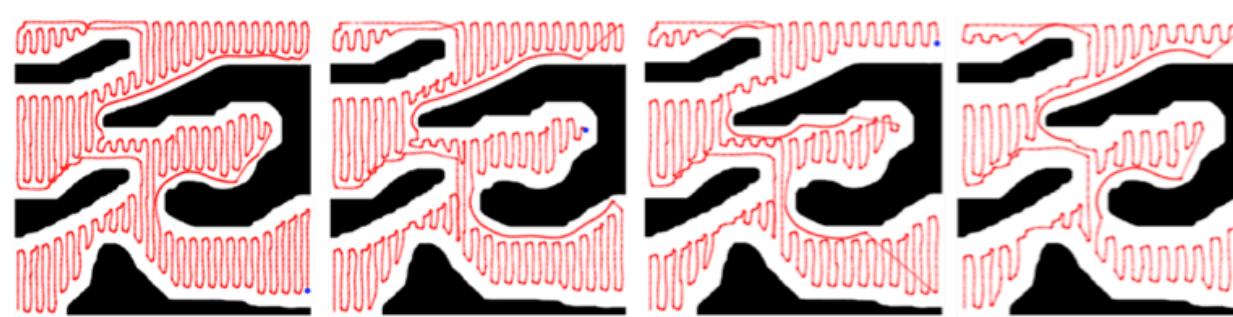


Figure 1. Scenario 1: coverage trajectories for varying size of ϵ

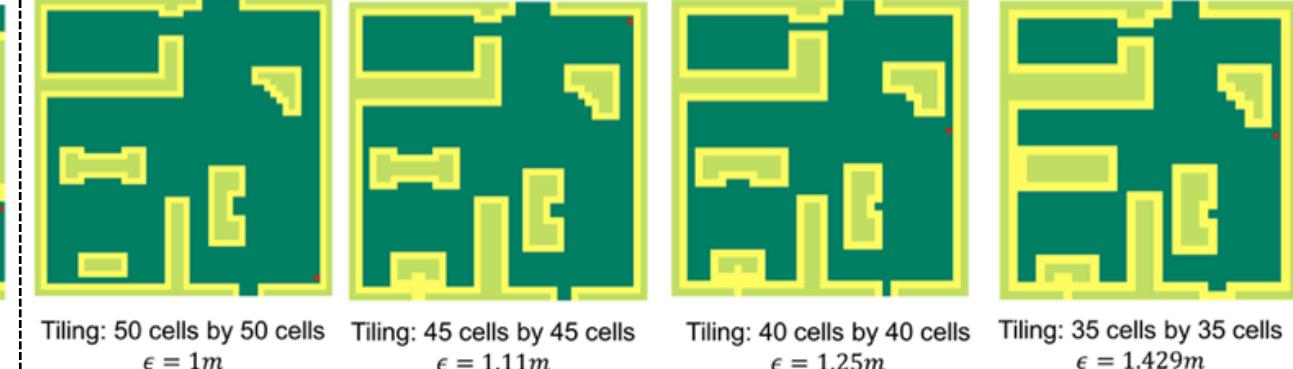
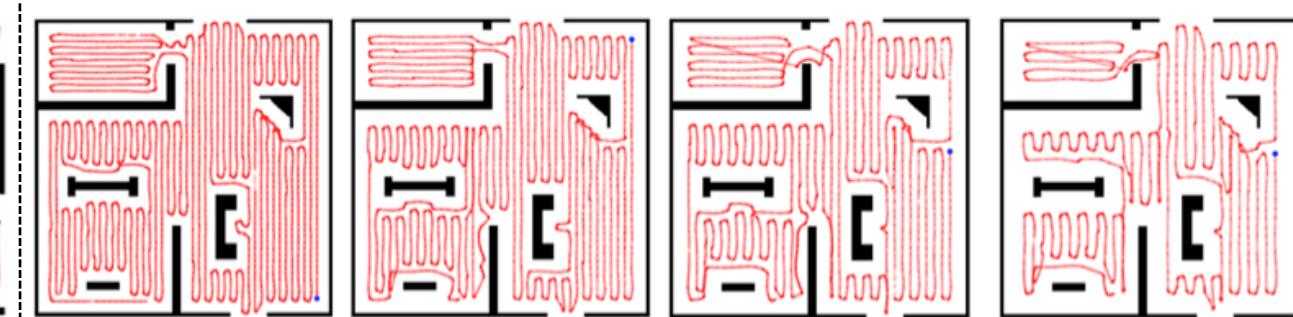


Figure 2. Scenario 2: coverage trajectories for varying size of ϵ

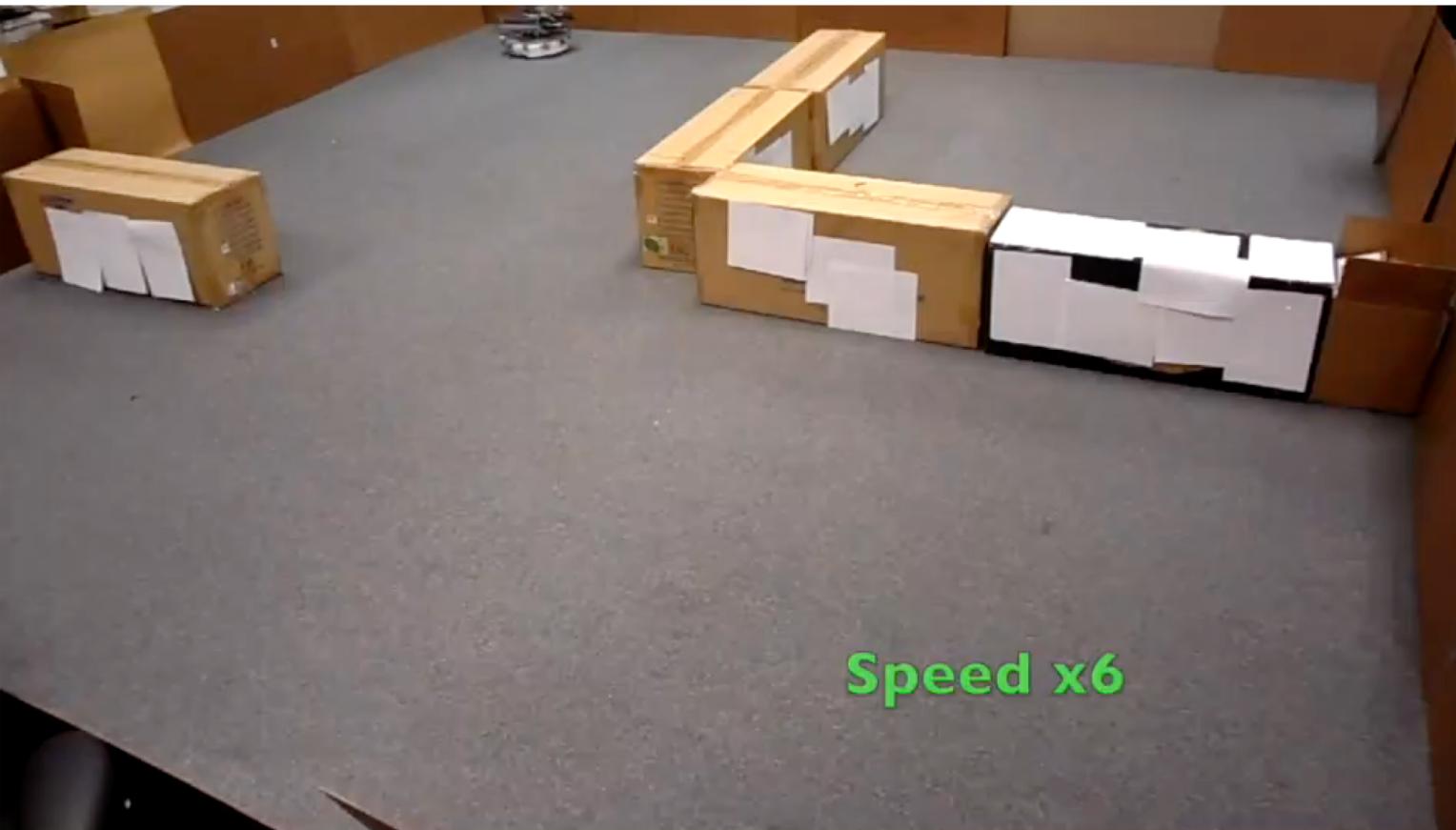


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Real Experiments

The Autonomous Ground Vehicle (AGV)

- ❖ ϵ^* algorithm was validated in real laboratory-scale experiments to address real-life uncertainties in sensing and vehicle control
- ❖ iRobot Create was used as the AGV, which is *programmable* and *controllable* using feedbacks from popular sensing devices



An AGV integrated with multiple sensing devices

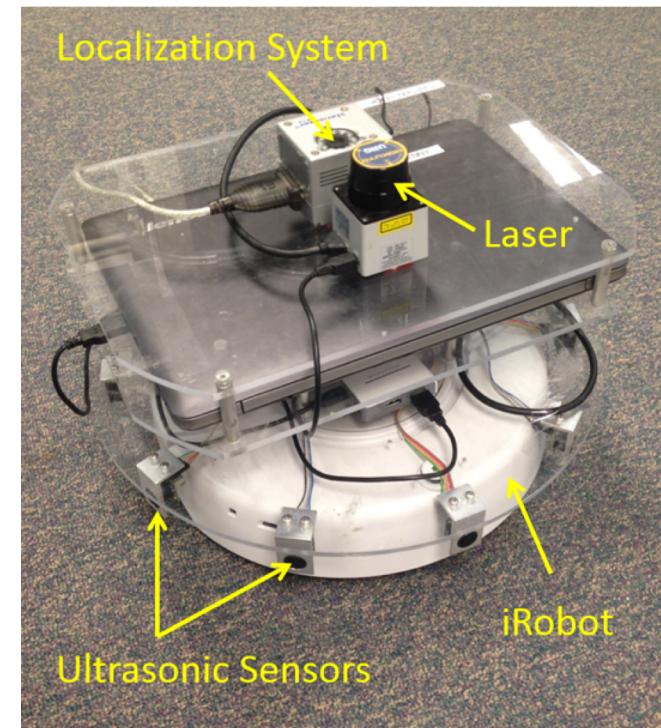


Table. Specifics of the on-board sensing systems

	Localization	Laser	Ultrasonic
Model	StarGazer	URG-04LX	XL-MaxSonar-EZ
Range	—	0.02m ~ 5.6m, 240°	0.2m ~ 7.65m
Resolution	1cm, 1°	1mm, 0.36°	1cm
Accuracy	2cm, 1°	±1% of Measurement	—