Analysis and Simulation of a QPSK System

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Abstract

This report presents the MATLAB implementation and simulation of QPSK modulation in digital communication. Data bits are organized into transmission blocks featuring leading training sequences and guard bits. The simulation primarily investigates Bit Error Rate (BER) performance, studying variations with Signal-to-Noise Ratio (SNR). Realistic scenarios assess non-ideal synchronization and phase estimation, evaluating degradation across diverse training sequence lengths and E_b/N_0 values. Signal constellation analysis considers increasing noise effects and phase estimation errors.

The study introduces modifications, incorporating a time-varying phase shift, including a $\pi/4$ phase increase during the transmission of one data block. The conclusion involves an analysis of power density spectrum variations with different pulse shapes, emphasizing spectral characteristics and system bandwidth occupancy. Key considerations identified during the study include refining training symbol design, optimizing synchronization strategies, and addressing time-varying phase shifts.

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1 Introduction

1.1 Background

The report addresses questions about the implementation of QPSK modulation in digital communication. The answers presented are based on the complex baseband model of the system that has been simulated using MATLAB. The basic model of the channel approximates the channel to be an AWGN channel. Necessary modifications to the model have been made to answer specific questions. Oversampling has been used to model the continuous system using the discrete simulation model. This enables us to study issues related to continuous time implementation like synchronization.

The data bits are modelled to have been separated into blocks which have a leading sequence of training bits for each block. The training sequence are assumed to be known by both, the transmitter and the receiver, and hence enables synchronization and phase correction. The data bits along with the training bits are padded by guard bits on either ends. The simulation deals with the transmission of a number of such blocks. This aims to capture the nature of the system rather than obtaining results for its single realization.

1.2 Problem Formulation

The presented simulation results aim to address several key questions, shedding light on the performance and behavior of the communication system under various conditions. They are outlined in this section.

The Bit Error Rate (BER) performance, with perfect phase estimation and synchronization, is investigated. Its variation with the SNR (E_b/N_0 in dB) are simulated. The study includes the derivation of an exact expression for BER, facilitating a comparison with simulation results to identify any discrepancies.

Shifting focus to realistic scenarios with non-ideal phase estimation and synchronization, the simulation assesses the degradation in performance of synchronization and phase estimation. It is assessed for different length of training sequences across different E_b/N_0 values. In this realistic set-up the signal constellation within the receiver are examined, to consider the effects of increasing noise levels and the implications of errors in the phase estimate on it.

The simulation is modified to better mimic a real-life channel characteristic — a time-varying phase shift particularly with a phase increase up to $\pi/4$ during the transmission of one data block. — to assess its impact on system performance.

Lastly, the power density spectrum of the transmitted signal using different pulse shapes is studied, highlighting the influence of pulse shaping on spectral characteristics which determine the bandwidth occupancy of the system.

2 Methodology

The analysis undertaken to answer the above enumerated questions is partly from simulations and partly analytical. The bit-error rate (BER) performance of QPSK modulation as a function of E_b/N_0 with perfect phase estimation and synchronization has been done in the simulation code by modelling the downsampled and phase-corrected recieved symbols as a sum of the exact transmitted sequence of complex-valued QPSK symbols and complex AWGN noise. This mimics a perfect synchronization and phase-correction as there are no errors (of phase and synchronization) other than the additive white gaussian channel noise. The derived expression for the corresponding exact BER has been presented in the Appendix (Sec. [A.1]) and compared with the simulation result.

The realistic phase estimation and synchronization has been analyzed next. The extent of faithful synchronization was analyzed by comparing the $t_{samp,pilot}$ obtained by maximizing the cross-correlation on the training symbols and the $t_{samp,data}$ obtained by maximizing the cross-correlation on the data symbols. The variation of the scaled magnitude of difference in cross correlation of the data symbols at the $t_{samp,pilot}$ and $t_{samp,data}$ was used as a measure of the performance of the synchronization to the channel noise. This is a sensible measure since the data has maximum correlation at $t_{samp,data}$, while the samples being used to decode the data symbols will be taken from $t_{samp,pilot}$. Similarly, the performance of phase to noise has been analyzed by plotting the the difference in the phase estimation obtained from the pilot symbols ($\hat{\phi}_{pilot}$) and the data symbols ($\hat{\phi}_{data}$). $\hat{\phi}$ is the average difference in the phase of the transmitted and the received symbols. The performances have been analyzed for different lengths of the training sequence.

The received signal constellation was studied by plotting the received signal constellation across different values of E_b/N_0. The effect of an error in the phase estimate on the received signal constellation has been analyzed by the visible phase shift in constellation.

The phase correction performance was then analyzed in the presence of a channel with a linearly time-varying phase. This is done by adding a linear phase shift which goes upto a phase shift of $\pi/4$ per block. To study the region in the block where this shift causes the most error, the number of bit errors is plot for bins of data bits across the block length. The simulation result presented was performed for the parameters

$$\text{nr\_data\_bits} = 10000;\ \text{nr\_training\_bits} = 1000;\ \text{nr\_blocks} = 500.$$

The baseband power density spectrum of the transmitted signal has been analysed by plotting it using the MATLAB function **periodogram** with different pulse shapes. The rectangular pulse shape along with the root raised cosine pulse shapes with roll-off factors (β) equal to 0.2 and 0.75 were analyzed.

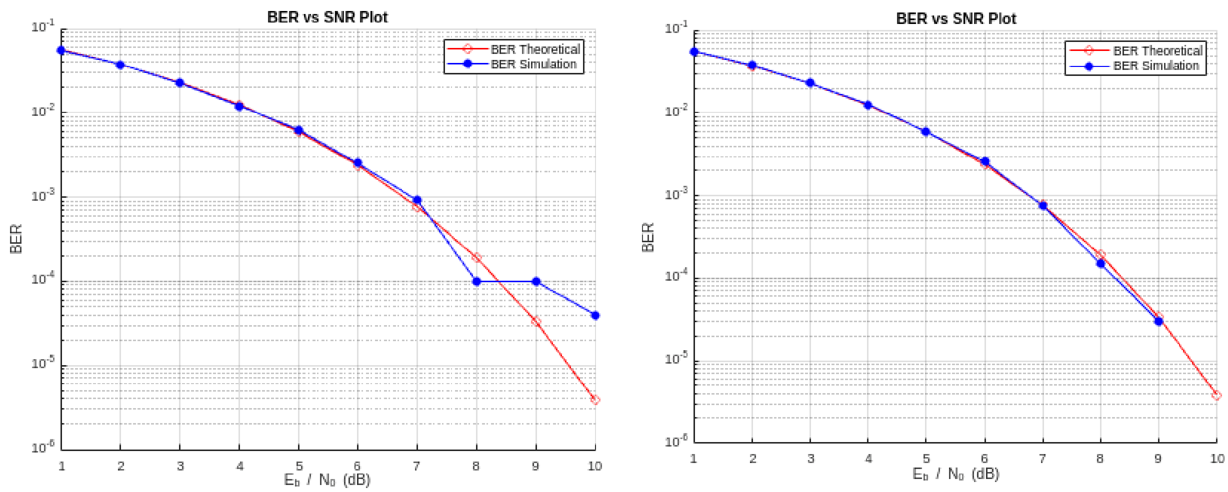
3 Results

3.1 BER performance : ideal synchronization and phase-correction

The exact expression for the BER of digital communication using QPSK modulation in an AWGN channel is derived to be

$$BER = \frac{1}{2} erfc(\frac{1}{2\sigma}) \tag{1}$$

The simulation results with perfect synchronization and phase estimation agree with the theoretical results. The approximation improves as the number of blocks simulated increases. Thus, the simulation results



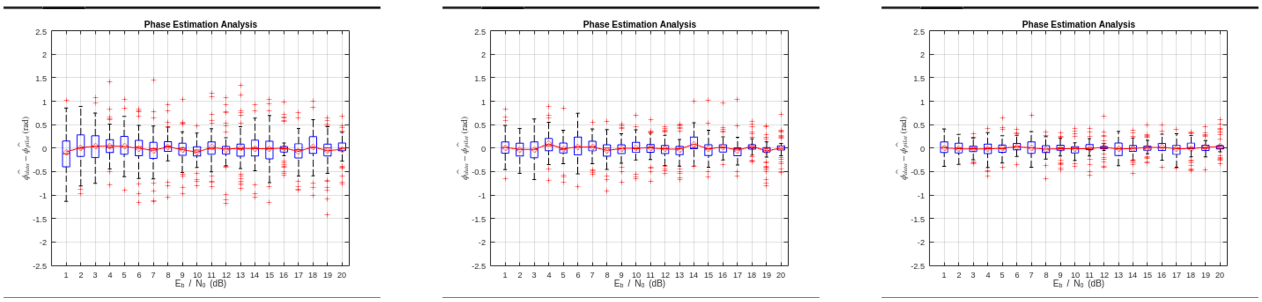
nr_blocks = 50 nr_blocks = 100

Figure 1: Comparison between Theoretical and Simulated BER

3.2 Performance of synchronization and phase-correction

The graphical representation of the parameters outlined in the methodology, spanning E_b/N_0 values from 0 to 20 dB, illustrates that, despite the absence of significant enhancement in synchronization with the increase in the number of training symbols (Sec. [A.2]), there is a notable improvement in the performance of the phase estimate as seen in Figure 2.

The enhancement is evident in the variability (spread) of the values for $\hat{\phi}_{data}$ - $\hat{\phi}_{pilot}$. Greater disparities in these values indicate the presence of blocks where the phase estimation using training symbols significantly deviates from the phase estimate of the data symbols. A phase misalignment between transmitted and received signals leads to an increase in the simulation symbol error compared to the theoretical symbol error. As the phase correction is based on the phase estimate of the training symbols, a smaller disparity in the phase estimates corresponds to a lesser increase in simulation symbol error (relative to the theoretical symbol error) attributed to the phase misalignment.



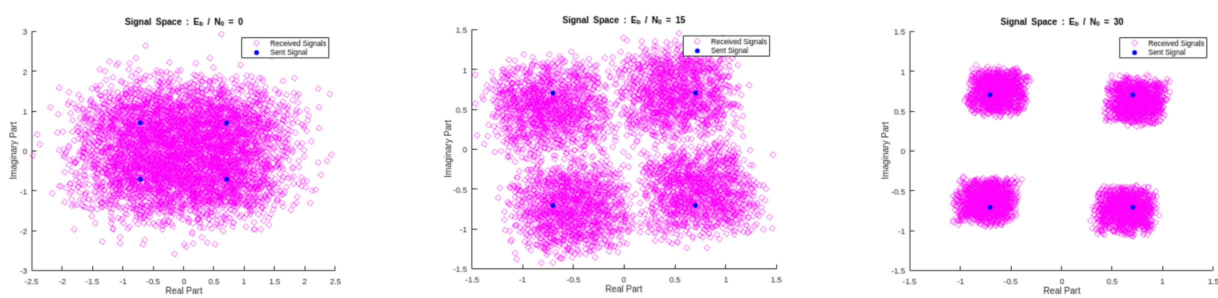
nr_training_bits = 24 nr_training_bits = 50 nr_training_bits = 100

Figure 2: Phase Estimation Performance : nr_data_bits = 1000, nr_blocks = 50

In the case of QPSK, the symbols in the signal constellation are $\pm\pi/2$ rad apart in terms of phase shift, thus the decision boundaries for equal a-priori probable symbols are at a phase shift of $\pm\pi/4$ rad from each symbol. Thus, a reasonable number of training bits would be those which keep a majority of the difference in phase estimates within [-$\pi/4$, $\pi/4$], which corresponds to the majority of the phase corrected decision region overlapping with the optimal decision regions (Sec. [A.3]). Thus, a reasonable number of training bits is around 5-10% of the number of data bits, depending on the required amount of accuracy. Furthermore, the selection of the training sequence length should be proportionate to the maximum channel impulse response (CIR) to be estimated [1].

3.3 Signal constellation

It is observed that as the SNR increases, the symbols in the constellation become more distinct and less susceptible to noise interference. This makes it easier for the receiver to accurately identify and decode the transmitted symbols, thus reducing symbol errors. It translates to a more densely packed signal constellation as seen in Figure 3.



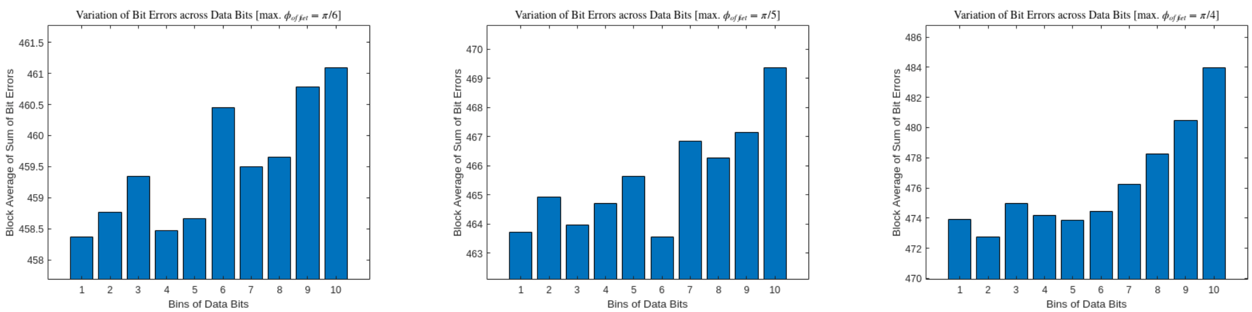
$E_b/N_0 = 0$ dB $E_b/N_0 = 15$ dB $E_b/N_0 = 30$ dB

Figure 3: Sent and Received Signal Constellation for one of the simulated blocks

When the value of the estimated phase shift is far from the true phase shift i.e. $\hat{\phi}_{data}$ - $\hat{\phi}_{pilot}$ is significant, there is a visible phase offset in the received signal constellation from the sent signal constellation. This is seen in the case of $E_b/N_0 = 15$ dB in Figure 3. As mentioned before, it increases the number of symbol errors.

3.4 Phase-correction performance: time-varying phase channel

Since the pilot symbol is at the beginning of the block, it detects an average phase upto the duration of it's transmission, and corrects for it. The number of error thus increases as we move towards the end of the block, where the phase offset is increasingly different. Furthermore, the maximum number of bit errors in the last bin increase as the maximum phase offset increases. The simulation results presented are in Figure 4



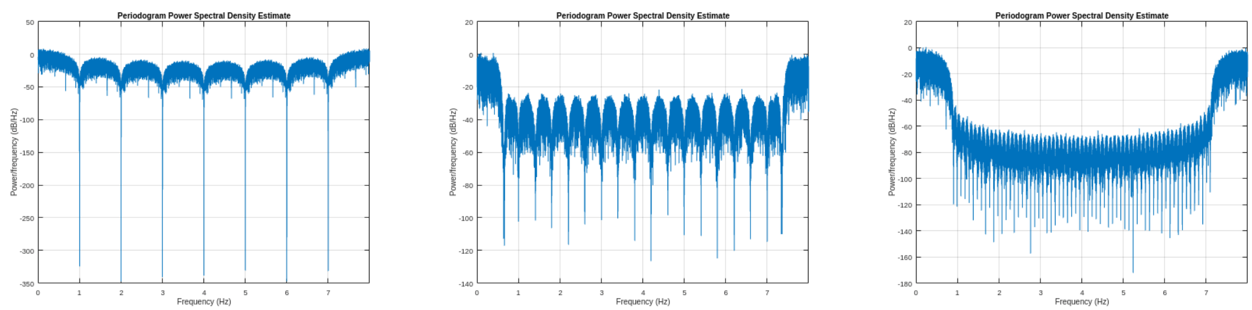
max. $\phi_{offset} = \pi/6$ max. $\phi_{offset} = \pi/5$ max. $\phi_{offset} = \pi/4$

Figure 4: Distribution of Bit Errors in Data Bits

Differential modulation can overcome this error for a slowly varying channel phase shift.

3.5 Transmitted signal baseband power density spectrum

While the rectangular pulse is strictly contained in the time domain to ensure no ISI, it has an extremely spread out and flat frequency response. This causes it to require an extremely high bandwidth since the energy is spread across the frequency spectrum. The root raised cosine pulse with β=0 corresponds to the rectangular pulse.



Rectangular Pulse RRC with $\beta = 0.20$ RRC with $\beta = 0.75$

Figure 5: Transmitted signal baseband power density spectrum

Increasing the roll off factor causes the pulse to go beyond the rectangular pulse in the time domain, but this results in the reduction in the magnitude of the side lobes in the frequency domain, as illustrated in Figure 5, therefore, the required bandwidth for the transmitted signal reduces. For extremely high roll off factors, like $\beta = 0.75$, most of the energy is contained in the main lobe.

4 Conclusion

In conclusion, the MATLAB simulation of a Quadrature Phase Shift Keying (QPSK) communication system has provided valuable insights into critical aspects of its performance. Through systematic investigation, questions surrounding Bit Error Rate (BER) performance, considering both ideal and realistic scenarios of phase estimation and synchronization were addressed.

Transitioning to realistic scenarios with non-ideal phase estimation and synchronization, our study revealed the inherent challenges and performance degradation. While the estimation of phase was mitigated by increasing the number of training symbols, achieving precise synchronization posed to be an issue. The training symbols can be designed while considering the characteristics of the noise, the algorithm that we want to implement to achieve synchronization, and the use-case of the implementation [2]. This might lead to improved synchronization performance.

The introduction of a real-life channel characteristics allowed us to assess its impact on system performance. Since most of the real world channels are time-varying phase shift, it is important issue to address as well. To ensure better phase estimation, the phase of the pilot symbols of the next block can also be considered. An interpolation of the estimated phase between both the set of training symbols could enable a better modeling of the expected phase shift in the channel during the transmission of the data bits. Studies to analyse the optimal overhead depending on the channel at hand (i.e. on the rate and nature of channel phase shifts) and the interpolation model for lowering the BER can be done.

The examination of the signal constellation within the receiver offered deeper insights into the system's resilience under varying conditions. In cases of extremely low SNR, using learnt clustering and grouping models might help in reduction of symbol error rates. This can be done by reconstructing the sent information and using some underlying knowledge about the type of information.

Furthermore, the evaluation of the power density spectrum of the transmitted signal with different pulse shapes offered insights into the spectral characteristics influenced by pulse shaping choices, which are essential in real life band limited communications. The extent of spectrum interference is also crucial in multi-spectral communication applications in real-life scenarios like achieving high throughput through frequency sharing or frequency reuse [3].

References

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[3] R. Prasad and T. Ojanpera. “An overview of CDMA evolution toward wideband CDMA.” In: *IEEE Communications Surveys & Tutorials* 1.1 (1998), pp. 2–29. DOI: 10.1109/comst.1998.5340404.

[4] Ragnar Thobaben. *Project Assignment : EQ2310 Digital Communications*. 2023.

APPENDIX

A.1 Derivation of exact BER for QPSK Modulation

The implemented QPSK configuration and its Gray coding is illustrated in Figure 6.

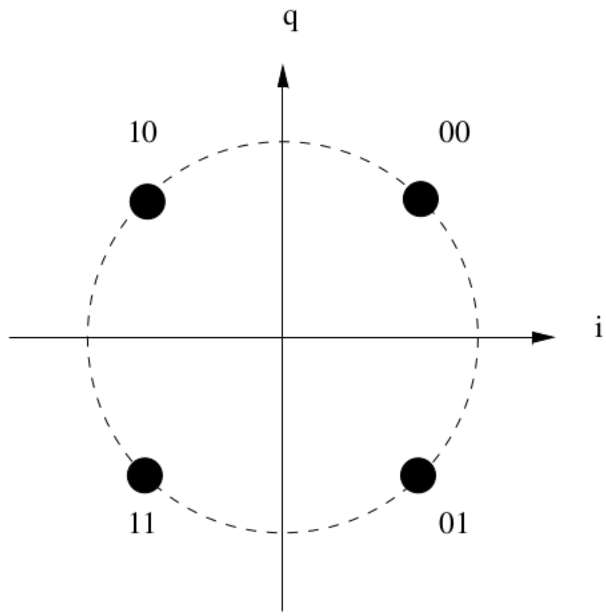


Figure 6: Gray coded QPSK configuration [4]

The QPSK constellation is defined to have unit distance from the origin, and hence the symbols on the constellation are given as

$$
\begin{aligned}
(00)\quad \mathbf{s_1} &= \frac{1}{\sqrt{2}}(1+j1)\\
(10)\quad \mathbf{s_2} &= \frac{1}{\sqrt{2}}(-1+j1)\\
(11)\quad \mathbf{s_3} &= \frac{1}{\sqrt{2}}(-1-j1)\\
(01)\quad \mathbf{s_4} &= \frac{1}{\sqrt{2}}(1-j1)
\end{aligned}
$$

Each component of the complex baseband noise is modelled as a additive white gaussian noise (AWGN), with the variance $\sigma^2 = \sigma_d^2/2$, where σ_d^2 is given in terms of the signal-to-noise ratio (SNR), the number of bits per symbol k and the samples p(n) of the pulse function g_{TX}(t) at T/Q sampling interval, where T is the time period of a each signal in the signal set and Q is the oversampling factor. For QPSK, $k = 2$.

$$\sigma_d^2 = \frac{\|g_{TX}(n)\|^2}{k\ \mathrm{SNR}} \tag{2}$$

Thus, the addition noise follows the distribution function

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{\frac{-x^2}{2\sigma^2}} \tag{3}$$

For a received signal $\mathbf{y}$ in the received signal space, the conditional probability distribution function (PDF) of $\mathbf{y}$ given $\mathbf{s_i}$ is transmitted is :

$$p(\mathbf{y}|\mathbf{s_i}) = \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{\frac{-\|\mathbf{y}-\mathbf{s_i}\|^2}{2\sigma^2}} \tag{4}$$

Consider the symbol being transmitted is symbol $\mathbf{s_1}$, and the symbol $y = \mathcal{R}_y + \mathrm{j}\ \mathcal{I}_y$ being received in the received signal space. Assuming that the all the symbols $\mathbf{s_i}$ have equal a-priori probabilities, the decision boundary while decoding are then the real and imaginary axes. The received signals in the first quadrant ($\mathcal{R}_y > 0$, $\mathcal{I}_y > 0$) are decoded as $\mathbf{s_2}$, in the second quadrant ($\mathcal{R}_y < 0$, $\mathcal{I}_y > 0$) are decoded as $\mathbf{s_3}$, in the third quadrant ($\mathcal{R}_y < 0$, $\mathcal{I}_y < 0$) are decoded as $\mathbf{s_4}$, and in the fourth quadrant ($\mathcal{R}_y > 0$, $\mathcal{I}_y < 0$) are decoded as $\mathbf{s_1}$.

The probabilities of the real/imaginary component of a received symbol y being greater than zero, when the symbol $\mathbf{s_2}$ is sent are given by

$$p(\mathcal{R}_y > 0|\mathbf{s_1}) = p(\mathcal{I}_y > 0|\mathbf{s_1}) = Q(-\frac{1}{\sqrt{2\sigma^2}}) = 1 - Q(\frac{1}{\sqrt{2\sigma^2}}) \tag{5}$$

where

$$Q(x) = \frac{1}{2} erfc(\frac{x}{\sqrt{2}}) \tag{6}$$

Hence, the probabilities of the real/imaginary component of a received symbol y being less than zero, when the symbol $\mathbf{s_2}$ is sent are given by

$$p(\mathcal{R}_y < 0|\mathbf{s_1}) = p(\mathcal{I}_y < 0|\mathbf{s_1}) = 1 - (1 - Q(\frac{1}{\sqrt{2\sigma^2}})) = Q(\frac{1}{\sqrt{2\sigma^2}}) \tag{7}$$

As per the decision boundaries, the received signal is decoded correctly only if $\mathbf{y}$ falls in the first quadrant i.e $\mathcal{R}_y > 0$ and $\mathcal{I}_y > 0$. With the gray coding, if only one of the components is incorrectly received there is a one bit error; and if both the components are incorrectly received there is error in both the bits. The expected number of bit error when symbol is then

$$\mathbb{E}(\text{number of bit errors}|\mathbf{s_1}) = \sum n_{BitError}\ p(n_{BitError}) \tag{8}$$

where $n_{BitError}$ is the number of bit errors per symbol.

$$\begin{aligned}\mathbb{E}(\text{number of bit errors}|\mathbf{s_1}) = &\ 0\ p(\mathcal{R} > 0|s_2)\ p(\mathcal{I} > 0|s_2) \\ &+1\ [p(\mathcal{R} > 0|s_2)\ p(\mathcal{I} < 0|s_2) + p(\mathcal{R} < 0|s_2)\ p(\mathcal{I} > 0|s_2)] \\ &+2\ p(\mathcal{R} < 0|s_2)\ p(\mathcal{I} < 0|s_2)\end{aligned} \tag{9}$$

The expected number of bit error per symbol in terms of $Q(\frac{1}{\sqrt{2\sigma^2}})$, is given as

$$\begin{aligned}\mathbb{E}(\text{number of bit errors}|\mathbf{s_1}) = 0\ &[(1-Q(\frac{1}{\sqrt{2\sigma^2}}))\ (1-Q(\frac{1}{\sqrt{2\sigma^2}}))] \\ +1\ &[(1-Q(\frac{1}{\sqrt{2\sigma^2}}))\ Q(\frac{1}{\sqrt{2\sigma^2}})+Q(\frac{1}{\sqrt{2\sigma^2}})\ (1-Q(\frac{1}{\sqrt{2\sigma^2}}))] \\ +2\ &[Q(\frac{1}{\sqrt{2\sigma^2}})\ Q(\frac{1}{\sqrt{2\sigma^2}})]\end{aligned} \tag{10}$$

which reduces to

$$\mathbb{E}(\text{number of bit errors}|\mathbf{s_1}) = 2\ Q(\frac{1}{\sqrt{2\sigma^2}}) = erfc(\frac{1}{2\sigma}) \tag{11}$$

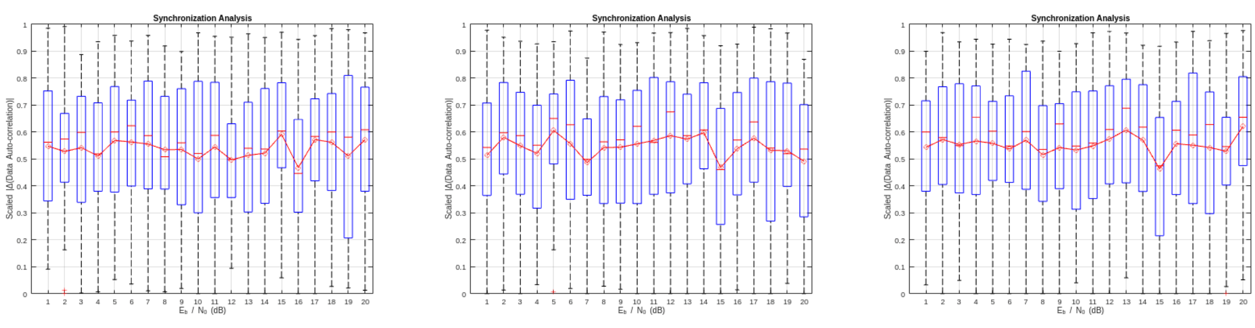
By symmetry of the constellation and the equi-probable nature of all the symbols, the expected number of bit error per symbol is

$$\mathbb{E}(\text{number of bit errors}) = erfc(\frac{1}{2\sigma}) \tag{12}$$

The bit-error-rate BER is defined as the number of bit errors divided by the total number of transferred bits. Each symbols transmits 2 bits, therefore the BER is given as

$$\boxed{BER = \frac{1}{2}erfc(\frac{1}{2\sigma})} \tag{13}$$

A.2 No improvement in the Synchronization Performance with the increase in the number of training symbols

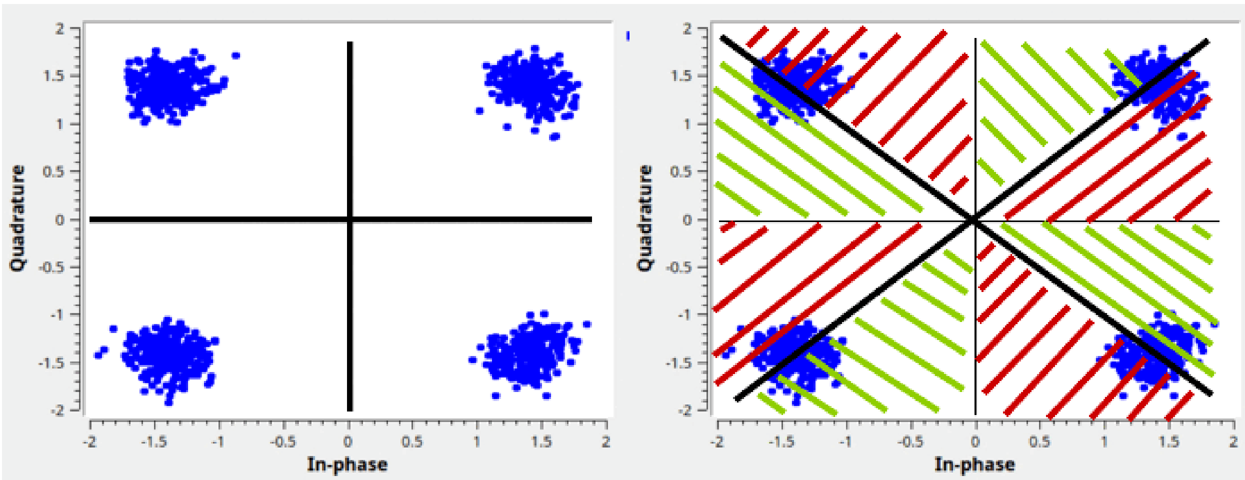


nr_training_bits = 24 nr_training_bits = 50 nr_training_bits = 100

Figure 7: Synchronization Performance : nr_data_bits = 1000, nr_blocks = 50

Despite increasing the number of training symbols, observable improvement in synchronization performance remains minimal or negligible. This limited enhancement suggests that factors beyond the sheer number of training symbols may exert a more influential impact on the synchronization performance.

A.3 Effect of Phase Offset on Decision Boundaries and Decision Regions



Optimal Decision Boundaries Phase-Corrected Boundaries and Regions

Figure 8: Decision Boundaries and Regions for a phase offset $\hat{\phi}_{data}$ - $\hat{\phi}_{pilot}$ = $\pi/4$ rad

The bold black lines correspond to the decision boundaries. The green decision regions correspond to the ones where the optimal and phase-corrected decisions overlap, while the red are cross-overlaps. For a phase offset of of magnitude $\pi/4$ rad, there is 50% overlap with the optimal decision regions as seen in Figure 8.

In the simulation, as illustrated in Figure 3, the decision boundaries remain the same, since the receiver uses the template of the transmitted signal constellation. The phase offset causes the received signal constellation to undergo a phase shift.