

Q1:

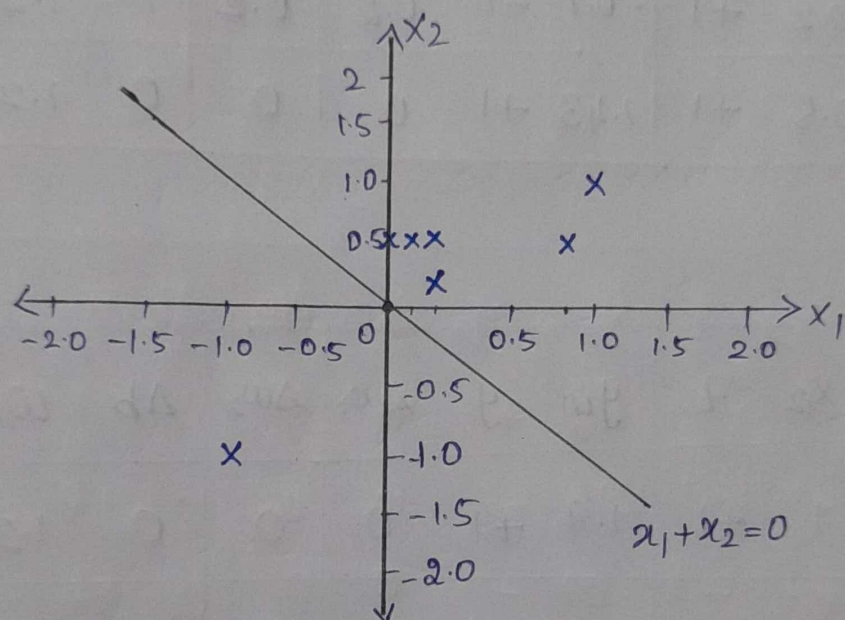
Assume weight vector of initial decision boundary $w^T x = 0$ as $W = [1, 1]$

$$\Rightarrow x_1 + x_2 = 0$$

$$b = 0$$

$$y_{in} = w_1^T x_i + b = w_1 x_1 + w_2 x_2 + b$$

Initial Decision boundary and Samples



Assume learning rate as 1

$$y = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 \end{cases}$$

$$\Delta w_1 = \alpha t x_1 \quad \Delta b = \alpha t$$

$$\Delta w_2 = \alpha t x_2$$

I)

x_1	x_2	class (t)	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	2	+1	0	0	0	1	1	0
-1	-1	-1	-2	-1	0	0	0	1	1	0
0	0.5	-1	0.5	+1	0	-0.5	-1	1	0.5	-1
0.1	0.5	-1	-0.65	-1	0	0	0	1	0.5	-1
0.2	0.2	+1	-0.7	-1	0.2	0.2	1	1.2	0.7	0
0.9	0.5	+1	1.43	+1	0	0	0	1.2	0.7	0

II)

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.9	+1	0	0	0	1.2	0.7	0
-1	-1	-1	-1.9	-1	0	0	0	1.2	0.7	0
0	0.5	-1	0.35	+1	0	-0.5	-1	1.2	0.2	-1
0.1	0.5	-1	-0.78	-1	0	0	0	1.2	0.2	-1
0.2	0.2	+1	-0.72	-1	0.2	0.2	1	1.4	0.4	0
0.9	0.5	+1	1.46	+1	0	0	0	1.4	0.4	0

iii)

x_1	x_2	t	y_{in}	y	$\Delta\omega_1$	$\Delta\omega_2$	Δb	ω_1	ω_2	b
1	1	+1	1.8	+1	0	0	0	1.4	0.4	0
-1	-1	-1	-1.8	-1	0	0	0	1.4	0.4	0
0	0.5	-1	0.2	+1	0	-0.5	-1	1.4	-0.1	-1
0.1	0.5	-1	-0.81	-1	0	0	0	1.4	-0.1	-1
0.2	0.2	+1	-0.74	-1	0.2	0.2	1	1.6	0.1	0
0.9	0.5	+1	1.44	+1	0	0	0	1.6	0.1	0

iv)

x_1	x_2	t	y_{in}	y	$\Delta\omega_1$	$\Delta\omega_2$	Δb	ω_1	ω_2	b
1	1	1	1.7	+1	0	0	0	1.6	0.1	0
-1	-1	-1	-1.7	-1	0	0	0	1.6	0.1	0
0	0.5	-1	0.05	+1	0	-0.5	-1	1.6	-0.4	-1
0.1	0.5	-1	-1.04	-1	0	0	0	1.6	-0.4	-1
0.2	0.2	+1	-0.76	-1	0.2	0.2	1	1.8	-0.2	0
0.9	0.5	+1	1.52	+1	0	0	0	1.8	-0.2	0

5)

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	1	1.6	+1	0	0	0	1.8	-0.2	0
-1	-1	-1	-1.6	-1	0	0	0	1.8	-0.2	0
0	0.5	-1	-0.1	-1	0	0	0	1.8	-0.2	0
0.1	0.5	-1	0.08	+1	-0.1	-0.5	-1	1.7	-0.7	-1
0.2	0.2	+1	-0.8	-1	0.2	0.2	1	1.9	-0.5	0
0.9	0.5	+1	1.46	+1	0	0	0	1.9	-0.5	0

vi)

x_1	x_2	t	y_{in}	y_1	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	1	1.4	+1	0	0	0	1.9	-0.5	0
-1	-1	-1	-1.4	-1	0	0	0	1.9	-0.5	0
0	0.5	-1	-0.25	-1	0	0	0	1.9	-0.5	0
0.1	0.5	-1	-0.06	-1	0	0	0	1.9	-0.5	0
0.2	0.2	+1	0.28	+1	0	0	0	1.9	-0.5	0
0.9	0.5	+1	1.46	+1	0	0	0	1.9	-0.5	0

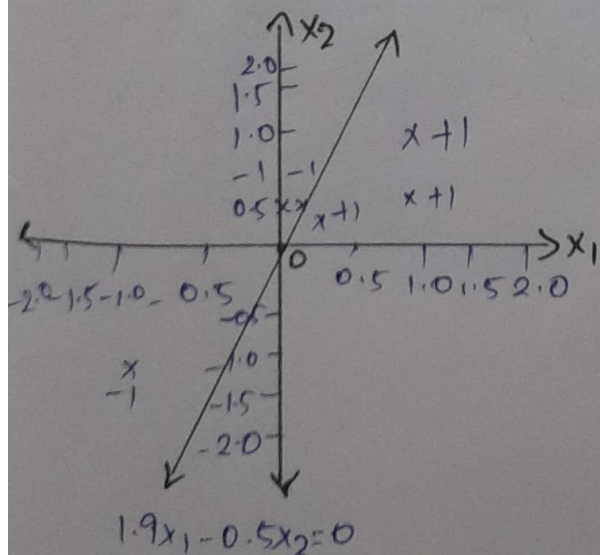
The perceptron Learning algorithm converged in 6 steps

The final weight vector of the decision boundary is $w = [1.9, -0.5]$

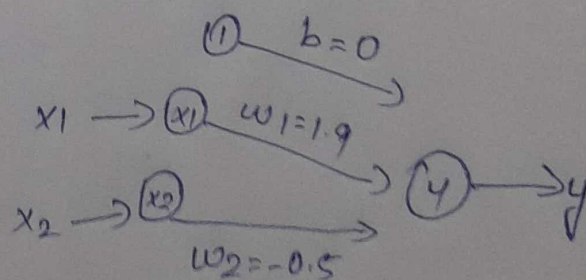
$$1.9x_1 + (-0.5)x_2 = 0 \Rightarrow 1.9x_1 - 0.5x_2 = 0$$

Let's plot the final Decision boundary

We can see that $1.9x_1 - 0.5x_2 = 0$ line separates the two classes correctly



Final Decision Boundary



Neural Network corresponding to the perceptron